"Biton's Lost Siege Engine: Experimental Archaeology in Classical Studies"
a thesis in fulfilment of the requirements for Master of Arts by research

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Abstract:

This thesis entails an examination of several problems inherent in placing a technical treatise by the Hellenistic Greek engineer, Biton of Pergamon, at a siege of 156-154BCE, with a view to galvanising the existing case of previous scholarship through a combined approach of literary, textual, geographical, and technical analysis. Particular focus is given to the following problems: technical errors in current translations of the treatise of Biton; technical considerations in scholars' reproductions of a particular engine in the treatise; an assessment of the practical implications of the treatise *in situ* at the physical site of the ancient city of Pergamon in the second century BCE, as evidenced by archaeological findings and surveys; assessment of those implications by way of historical records of similar conflicts from the Hellenistic period; and suggesting a procedure of dimensional analysis for testing a hypothesis regarding the feasibility of the ancient engineer's recommended engines as a stand-in for the city's original defenses, in a manner that harmonises the methodologies of historicism and experimental archaeology with sound and appropriate modern engineering practice from the field of Fluid Mechanics.

CONTENTS

PREFACE

Thanks are due to the following individuals:

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ABBREVIATIONS

Abbreviations of ancient sources in this thesis largely follow the conventions set by the *Oxford Classical Dictionary*. Below is a list of those sources, alongside their Oxford abbreviations as they appear in the footnotes. Where Oxford abbreviations are not available, alternatives have been substituted, and are indicated with a check mark [×].

Author	Work	Abbreviation	Non-Oxford
Appian	*Syriaca / The Syrian Wars*	App. *Syr.*	
Arrian	*Anabasis*	Arr. *Anab.*	
Archimedes of Syracuse	*Quadrature of Parabola*	Arch. *Quad.*	×
Archimedes of Syracuse	*On Floating Bodies*	Arch. *On F.*	×
Aristotle	*Analytica Posteriora*	Arist. *An. Pos.*	
Aristotle	*De Interpretatione*	Arist. *Int.*	
Aristotle	*Physica*	Arist. *Ph.*	
Cicero	*De Republica*	Cic. *Rep.*	
Cicero	*Tusculanae Disputationes*	Cic. *Tusc.*	
Diodorus Siculus	*Bibliotheca*	Diod. Sic.	
Diogenes Laertius	*Lives of Eminent Philosophers*	Diog. Laert. *Lives*	
Euclid	*Elements*	Euc. *Elements*	×
Euclid	*Optics*	Euc. *Optics*	×
Heron of Alexandria	*Belopoeica*	Heron, *Bel.*	×
Heron of Alexandria	*Pneumatica*	Heron, *Pneum.*	
Heron of Alexandria	*Mechanica*	Heron, *Mechanica*	×
Justin	*History of the World*	Justin, *Hist.*	×
Livy	*Ab Urbe Condita / The Histories*	Livy, *Hist.*	×
Livy	*Periochae*	Livy, *Per.*	
Pausanias	*Descriptions of Greece*	Paus. *Descriptions*	×
Philon of Byzantium	*Belopoeica*	Philon, *Bel.*	
Philon of Byzantium	*Paraskeuastika*	Philon, *Par.*	×
Philon of Byzantium	*Poliorketika*	Philon, *Pol.*	×
Plutarch	*Moralia*	Plut. *Mor.*	
Polybius	*The Histories*	Polyb.	
Proclus	*Commentary on the First Book of Euclid*	Procl. *Comm.*	×
Strabo	*Geography*	Strabo, *Geo.*	×
Vegetius	*De re militarii / Military Science*	Veg. *Mil.*	
Vitruvius	*The Ten Books on Architecture*	Vitr. *De Arch.*	
Xenophon	*Cyropaedia*	Xen. *Cyr.*	

Adapted from Hornblower, S., Spawforth, A., & Eidinow, E. (2012). *Oxford Classical Dictionary.* Oxford: Oxford University Press.

1. INTRODUCTION

Before the invention of the mounted weapons that concern this thesis, hand-drawn projectile weapons like slings and bows were the only means by which warfare could be conducted from afar. The hand bow can be separated into two varieties: the relatively straight, C-shaped *euthytone*, and the roughly M-shaped, double-curved *palintone* or *recurve bow*. The former is almost universally recognisable, while the latter is somewhat more advanced, and considerably more powerful. In the *Iliad*, Teukros is armed with a "τόξον παλίντονον",[1] identifying it as a palintone or recurve bow, and he uses it to fell Hektor's chariot-driver.[2] In Homer's *Odyssey*, the intimidating size and tensioning force of Odysseus' own palintone makes the ability to string it analogous to the right to rule.[3] These weapons were powered by their "τόνοι", or their two constituent arms. These were usually composed of wood, or a combination of wood and animal horn sandwiched together, with adhesives or binding.

The scale and power of these weapons were to increase significantly from the turn of the 4th C. BCE to the 1st C. CE, beginning with the simple *gastraphetes* crossbow, and culminating in the grand *ballistae* of the late Imperial Roman period.[4] After this point, innovations in counterweight technology like the Byzantine *helepolis* – no longer a siege tower in the sense of the Hellenistic *helepolis*, but now instead referent to the trebuchet – rendered the Greek and Roman engines an expensive and overcomplicated endeavour of the past.[5]

In later literature of antiquity, siege engineers used some of the above terms slightly differently to refer to certain types of catapult. Those that used a large bow – not unlike a vastly upscaled version of the hand bows mentioned above – are referred to in modern scholarship as being 'non-torsion', in that they do not use a torsion-type spring of hair and/or sinew to provide their motive power. These are further separated into two categories as attested in the literature: *oxybeleis*, or arrow-shooters, and *petroboloi*, or stone-throwers. The latter, too, received the general term *lithoboloi*, which remained as a general term for stone-throwing engines long after torsion technology became dominant.

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[1] Hom. *Il*. 8.266, 15.443.
[2] *ibid.* 8.309-313.
[3] Hom. *Od*. 21.11 & 59 identify the bow as a palintone, and Penelope offers the challenge of stringing it to the suitors at Hom. *Od*. 21.68-79.
[4] Johnson, 1983.
[5] Dennis, 1988; deVries, 1992.

The torsion-type engines that came to be the standard for most of the Hellenistic period developed far greater power on account of the improved spring tension and elasticity of sinew over wooden bows, and were the subject of rapid, well-funded development. As such, they quickly became the predominant type, and remained essentially unchanged in their basic operation for several centuries, apart from incremental improvements in tractability, power, ease of construction, and durability. They, too, had their terms and definitions evolve through the discourse of ancient engineers. The names used for types of *tonoi*, as explained above, became referent to ammunition-types in torsion engines instead. Heron of Alexandria therefore defines the *euthytonos* in this context as an engine that fires arrows or bolts, while the *palintonos* is one that fires stones.[6]

The domination of Greek artillery by torsion-spring catapults from the turn of the 4th C. BCE onwards is almost universally accepted, as references to the non-torsion type all but completely fade away soon after the death of Alexander the Great.[7] An awkward exception to this rule is the treatise of Biton. It is addressed to a king Attalus, which firmly places it in the kingdom of Pergamon between the years of 241 BCE, when Attalus I's reign begins, and 133 BCE, at the end of the reign of Attalus III. The contents are unlike other treatises with which it is bundled in the manuscript tradition: it is tense, erratic in diction and content, affords little technical nuance, and the measurements and names of variables in geometric descriptions have been lost, jumbled together, or others substituted in, so as to make some parts of the treatise almost unintelligible when read with a view to recreate the machinery described, whether in drawing or model. Rendering the text as a cohesive, intelligible whole has been the primary focus of scholarship in this field over the last century, and so we are now at a point where a more narrow focus can be employed to make better sense of the contents of the text.

The relevance of Biton recommending outmoded technology to a kingdom as well-established as Pergamon is also not immediately apparent. The city was a major centre of cultural influence, and enjoyed greater sway with the Roman Senate than most in Hellenistic Asia Minor.[8] The Attalid kings enjoyed many military victories over the course of their illustrious careers, and the library of Pergamon rivalled that of Alexandria, attracting philosophers and, undoubtedly, some whose scope

[6] Heron, *Bel*. 74.
[7] Hassall, 1998: 23; Kern, 1999.
[8] The Attalid dynasty is well-covered in the scholarship, which will be referred to in parallel with ancient sources; see particularly chapter 2: Historical Context.

of research included that which would allow us to ascribe them the descriptor of 'engineer'.[9] For such a treatise to be addressed to an Attalus, then, suggests that it was precipitated by extraordinary events.

The differing opinions amongst scholars on the topic of Biton can be summarised as follows: Rehm & Schramm, two of the earliest scholars to tackle the text with a view to making a practical interpretation of the machines described therein, were largely nonplussed – deciding that it must surely be from the time of the earliest Attalus, and thus closest in time to the engineers Biton refers to, and that the antiquated technology it details must have simply been a curious oddity.[10] Marsden attempts to reason that the Pergamene military had some practical preference for non-torsion engines when certain circumstances demanded it, and also places Biton with Attalus I.[11] Drachmann discounts Biton altogether as a fraud, instead putting forth a theory that the treatise is merely an exercise in fantasy or technical writing, composed by a writer of the second century CE.[12]

Lewis[13] makes the most convincing case, and one that remains undisputed. By pointing to the opening lines of the text, he suggests that this treatise was written in a moment of dire urgency, when Pergamon was caught unprepared for an invasion.[14] Specifically, this must, as he asserts, have happened at a time when the hair and sinew torsion springs of the defensive artillery of the city were in disrepair or had rotted away, as they required constant maintenance and frequent replacement; to which I would add that the braiding and re-stringing of these engines' springs was incredibly laborious as well.[15] He cites several occasions where other Greek states required emergency reserves of hair and sinew.[16] He dates Biton's most recent recommendation, by one Damis of Colophon, by pointing to the use of a screw in that design.[17] Damis must therefore post-date Archimedes of Syracuse's invention of the screw-thread, placing him at least in the late 3rd C. BCE, to which we must add an allowance of a sufficient number of years for Damis to learn of and

[9] Naturally, this modern term does not account for the considerable overlap of fields which any one of these ancient intellectuals may have dabbled in. Ancient Greek engineers that wrote about their findings could be considered philosophers, but not all philosophers were engineers, and none of the above were necessarily in the direct employ of a ruler. See 2.2: The First Artillery.
[10] Rehm & Schramm, 1929.
[11] Marsden, 1969.
[12] Drachmann, 1972.
[13] Lewis, 1999.
[14] *ibid.* 162.
[15] Philon, *Bel*. 58, 61.
[16] Polyb. 4.56.3, 5.89.9.
[17] Biton, 58.

make use of Archimedes' innovation.[18] Similarly, Biton's mention of a tool called the *dioptra*[19] tentatively places him in the 2nd C. BCE, when treatises on the use of this device first appear.[20] This would correspond with the tempting supposition by the scholar Drachmann, which offers that Biton's Damis might happen to be the same individual as the Eudamos of Colophon that served as an admiral of Eumenes II, and is attested by Livy and Polybius.[21] The one threat that is levied against Pergamon during this indicated period of history is a serious one indeed, and it is here that Lewis (and Drachmann, though tentatively) places Biton.[22]

It is to this theory that some further historical context, argumentation, and a closer analysis of the engines of the treatise and Pergamon's defences can be added. Some new observations on the most widely-available English translation, and revision of standing interpretations of the treatise's technical content, should serve to better understand it as a functional manual, and more comfortably place it in this particular moment in history. Problems of methodology that may be of use to further research, too, can be addressed; and some effort be made to more tangibly link the disciplines of literary, textual, archaeological, and engineering study together in a manner that constructively directs would-be scholars of these texts towards critical, practical engagement with such dense and technical volumes as Biton's.

1.1 INTERRELATION OF ARCHAEOLOGY, CLASSICAL STUDIES, AND POLIORCETICS

The scholarly work on ancient siegeworks and siege engines has enjoyed centuries of enthusiastic attention. Some of the most committed research has been performed by German scholars of the late 1800s and early 1900s, to whom current scholarship owes a great debt for deciphering many of the disparate and corrupted medieval texts that relate the findings and designs of ancient engineers.[23] Their work is best read when accompanied by the conservation and reconstruction efforts of such state-subsidised institutions as the Pergamonmuseum of Berlin, which was established purely for the reception of artifacts uncovered by the Pergamon archaeological team.[24] The textual scholarship here relies greatly on the boom of archaeological findings made under the auspices of the German Empire. However, the German textual analyses - and indeed, relevant archaeological excavations at

[18] Lewis, 1999: 163.
[19] Biton, 53.
[20] Lewis, 1999: 166.
[21] Drachmann, 1963: 11. See examples Livy, *Hist.* 37.26 and Polyb. *Hist.* 27.7.6.
[22] *ibid.* 167.
[23] See Chapter 5: Treatise.
[24] Staatliche Muzeen zu Berlin, 2020.

Pergamon and other sites across Europe - were first hampered by the outbreak of the First World War, and then interrupted entirely by the Second.

The interrelation of this scholarship with the business of war is worth noting. The modern field of study concerning siegecraft (as part of the study of military strategy) retains the name *poliorcetics*, in deference to the Byzantine era of siege manuals or *poliorcetica,* which in turn derive their name from the Greek practice of siegework.[25] Apart from use as a general term in ancient literature, it is also the title of an exemplary work by Heron of Byzantium, who in turn makes reference to designs by a range of Hellenistic engineers.[26] Reverence for the Byzantines in modern military scholarship is likely due to the inordinate number of early scholars of classical siegeworks who were military personnel themselves. For example, Erwin Schramm, a curator of the Saalburg Museum, held the rank of Lieutenant-General and is responsible for some of the scholarship most pertinent to this thesis.[27] Wilhelm Rüstow, a Prussian officer of the engineering corps,[28] assisted philologist Hermann Köchly in the writing of another seminal work.[29]

These scholars worked closely with both textual and archaeological evidence, as well as employing principles of engineering to their work, in order to shed light on the practical ancient application of otherwise impenetrable texts. This interdisciplinary cooperation is perhaps first exemplified by the pairing of Köchly & Rüstow, whose respective careers could not have been more different, yet collided in this one respect. This dynamic was again mirrored by the pairing of the military man, engineer, and archaeologist Schramm, with the philologist, epigrapher, and archaeologist Rehm. The notable difference in the latter pair is that Schramm followed up the drafting of the machines concerned by building physical reconstructions to prove their viability. In doing so, he managed to create what one might consider the prototype model for the methodology and practice of experimental archaeology.

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[25] See Chapter 2: Historical Context.
[26] *Poliorcetica* was collated with one other work of Heron in an 11th C. manuscript housed in the Vatican Library; see Heron, *Vaticanus Graecus,* 1605.
[27] Some scholars appear to make the error of confusing Erwin Schramm's rank with that of another turn-of-the-century German historian, Percy Ernst Schramm. Ernst held the rank of Major, and served in the Wehrmacht during the Second World War. His relationship to Party ideology was unclear and he was allowed to return to his teaching post after the war - see Matikkala (2012). Erwin's service records are, on the other hand, unknown, but this does not diminish the fact that he continues to be habitually conflated with Ernst and referred to as "Major Schramm" by modern scholars - see for example Hassall, 1998: 23. On the title page of Schramm (1918) he is credited as a Generalleutnant z.D. (*zur Dienstleistung*; recalled for peacetime service).
[28] He also enjoyed a varied and dramatic career, deserving of study in its own right; see Chisholm, 1911: 937.
[29] See Köchly & Rüstow, 1852.

In the politically uneasy years following the Second World War's conclusion, scholarship on the subject of ancient siege and engines resumed amongst scholars of the western European and North American community. Despite some pauses, the field continues to incrementally grant new insights to this day. The Saalburgmuseum, which is built upon the ruins and reconstruction of a Roman fort, was under the curatorship of Schramm before the war. It eventually came under the directorship of Dietwulf Baatz,[30] who continued some of Schramm's work concerning Roman artillery, and provided a foreword to the 1980 reprint of Schramm's 1918 work[31] on the artillery and defenses of the Saalburg fort.

The Pergamonmuseum largely survived the War, and continues its conservation efforts with the assistance of the latest in technology. The grand and artistically remarkable Altar of Pergamon, constructed under the reign of Eumenes II between 197-159 BCE, then unearthed and curated under the leadership of Alexander Conze in the 1880s, has since been 3D-scanned and reproduced as an interactive model for off-site study by scholars and the public, requiring no specialised software to view in all its most intricate details.[32] German scholarship and archaeological work on Pergamon and its surrounds have once again flourished under the auspices of the Deutsches Archäologisches Institut (amongst others), which regularly publishes its findings in collaboration with the Turkish government.[33]

It can thus be said that German archaeology, textual scholarship, and Pergamon itself, have always been the springboard to the greater study of siege and its relevance to conflict, imperialism, and political developments of the ancient world, which in turn gave rise to the political landscape of Europe, and the nature of war, today.

1.2 METHODOLOGY: HISTORICISM & EXPERIMENTAL ARCHAEOLOGY

It should be made clear that this study will use a typically Historicist, strictly evidence-based epistemology, where textual and archaeological sources serve as that evidence. Source criticism in

[30] The Baatz book *Der Römische Limes* (2000) summarises some of the most important archaeological finds in this field as they were found throughout the areas of his specific expertise.
[31] See Schramm, 1918.
[32] Fraunhofer IGD, 2020.
[33] See below references to current work of the DAI in 1.3: Methodology.

the traditional sense of historicist work[34] is essential in this line of analysis. This is particularly true where ancient texts are concerned, whose contents may be either fragmentary, incorrect, or intentionally misleading on the part of the ancient author.[35]

Forming arguments from the evidence will be pursued in a manner that demonstrates self-reflexivity - interrogating the quality of argument sincerely and transparently, such that the conclusions reached in this work should be found to be without any embellishment or misrepresentation of the evidence.[36]

The evaluation of reconstructive efforts with regards to ancient processes or artefacts, as well as production of any reconstruction, will also need to follow guidelines that ensure scholarly integrity. The methodological framework of experimental archaeology aims to achieve this precise goal by making conservative efforts to interpret material and textual history through sympathetic reconstruction.[37] In order to test a hypothesis under this method, a hypothesis for the production, materials, or usage of an artifact is put forward, and a constructed model is used to determine the accuracy of this interpretation.[38]

This methodology in turn leans heavily on more traditional variations of archaeological theory; in particular, Post-Processualism, which at its heart maintains that investigation of the archaeological record should be descriptive rather than having any intent to fit the evidence to a preconceived narrative. That said, the very nature of experimental archaeology does require that it maintain strict adherence to the scientific method, and some hypothesis must be fielded in order for it to be tested. Multiple vectors of critical evaluation must be applied to the hypothesis and the experiment – requiring epistemic support from both material and textual sources.[39]

[34] Garraghan & Delanglez, 1946.
[35] Hester, 2018: 216-217.
[36] McCullagh, 1984.
[37] Paardekooper, 2019.
[38] Chippindale & Maschner, 2007: 11.
[39] Comis, 2010: 10.

1.3 METHODOLOGY:
USE OF COMPUTER-AIDED DESIGN

Some use of software - particularly computer-aided design - will be required to adequately address and demonstrate some of the technical minutiae of Biton's treatise. The use of CAD, in combination with research typical of Classical Studies, and the findings of archaeological digs, is well established. An example of this synthesis of methods can most famously be found in the latest scholarship around the Antikythera Mechanism.

Albert Rehm - the aforementioned co-author of Schramm's study of Biton, and the Köchly to his Rüstow - was the first scholar to suggest that this Mechanism may have been a calculator of astronomical movement, using observations of outwardly visible gearing and Greek inscriptions around its outward edifices. Although he did not publish his theory and much of it has since been revised as more has been discovered about the device, his notes of 1905 to 1906 survive and are eerily insightful, including some that would have served to form part of a book that was never to be published.[40] It was not until the 1970s that radiography[41] revealed the full extent of the gearing inside.

The vindication of Rehm's theory has been made complete by recent use of X-ray tomography, 3D modeling, virtualisation of the gear train, and extensive mapping of the device's known and possible functions to in turn posit working replacements for any of its missing or damaged parts. The net result of this combined study has been the creation of working replicas that accurately reproduce the ability of the original to plot and calculate accurate dates, waxing and waning moons, and the positions of our closest planetary neighbours, using one or the other set as a reference to calculate the other. In addition, this reproduction of the machine has allowed for researchers to use the assumed positions of these bodies in the device's version of the almanac relative to their positions in the last two millennia, and their relation to dates of the Greek calendar, to in turn date the device to the late 2nd century BC - a date with which some textual sources, like Cicero, appear to agree, and the century in which our engineer Biton lived and worked.[42]

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[40] Jones, 2012: 7.
[41] Wright, 2006: 28.
[42] Freeth *et. al.*, 2006: 595; Jones, 2012: 20. Cic. *Rep.* 1.22 and Cic. *Tusc.* 1.14 mention multiple similar devices being in circulation by this time period.

This pleasantly circular trail of scholarship serves to highlight not just the usefulness of modern drafting and virtualisation technology when used in concert with classical philology, but also the hitherto unrealised complexity of the multi-component assemblies and fine manufacturing precision that contemporaries of Biton were able to achieve. However, this is not an isolated case.

Previously, I mentioned the exploits of Rehm's partner in scholarship, Schramm. Not only did Schramm apply engineering theory, drafting expertise, and physical experimentation in achieving plausible reproductions of ancient engineering, but he was also involved tangentially with other similar projects. His curatorship at the Saalburgmuseum, in the region of Homburg, was most notably the backdrop for his work *Der Antiken Geschütze der Saalburg,*[43] in which he postulated the probable design for the defenses and defense weapons of the Roman fort at Saalburg whose ruins remain visible. The fort itself - thought to have been abandoned by its Legion during the Imperial Crisis of the 3rd C.[44] - was also partially reconstructed in the 1910s. Parallel to the excavations of Pergamon, the father-and-son team of Louis and Heinrich Jacobi - architects and archaeologists both - conducted extensive excavations of the fort and its surroundings.[45] Kaiser Wilhelm II, who had been witness to one of Schramm's reconstructed artillery pieces in action, provided permission for the Jacobi team to reconstruct the fort directly upon its old foundations following the documentation of their finds. Again, this reconstruction took shape with the use of multivariate evidence: ancient textual accounts, extant fort remains, the archaeological finds made on-site, and the contemporary drafting and architectural science of the day.

Current scholarship around Pergamon also leans heavily on multivariate analysis. Reconstructions of the ancient city are carried out using a combination of geospatial imaging and analysis, CAD, consultations of ancient texts, comparison with other pertinent archaeological sites, and an archaeological tradition stretching back to the 1800s.[46] The increasing digitisation and interactivity of the data collected by this scholarship is particularly useful to a study such as this, which deals with the corporeal problem of placing a literary description of a machine into a physical environment - one that must be simultaneously correlated with landscape, fortifications, historical

[43] Schramm, 1918.
[44] See Baatz, 2000.
[45] Moneta's (2018) review of the early excavations is exhaustive, and makes for easier reading than the original Jacobi accounts.
[46] See the latest report on work in the area at time of writing: Pirson (2020) *Pergamon: Die Arbeiten des Jahres 2019*.

and literary evidence, the interaction of the ancient city and its people with their surrounds, and myriad other practical considerations.

1.4 STRUCTURE OF THE THESIS

What follows is a brief summary of the chapters in this study, their content, and their relevance to the problem at hand. Addressing the *Historical Context* of a field before sallying forth is invariably necessary, and the second chapter places both the treatise, and the use of artillery in the ancient world in general, into perspective. I will appraise a variety of ancient sources in this endeavour to find some frame of reference for placing Biton in 2^{nd} C. Pergamon, and to better understand the use-cases of the technical solutions he offers. This will provide context for consideration of the problems of the treatise's content, the study of the ancient city's defences, and the deductions we can make from Biton's text.

In Chapter 3, *Reconciling Ancient Engineering*, I will make an assessment of some of the peculiarities of ancient treatises that are relevant to this discussion, and address the issue of how they are to be understood by modern readers whose grounding in theory is necessarily that of modern engineering, using the subject of anti-matériel projectile weapons as the vehicle of demonstration. This will be followed in Chapter 4 by a more specific study of the foundations of *Euclidean Geometry*, which looks at the foundations for ancient understanding of the built environment, which in turn has clear links to the methods of explanation, and the problems of the theory of scaling, in ancient engineering texts. I will then attempt to reconcile this with current theory on the properties and strengths of materials at scale. We can then apply this understanding to the problems of scale as related to us by ancient engineers and as employed in their own deployment of machines, and the problem of comparing the relative attributes of projectile weapons of the ancient world. This problem is then addressed further in Chapter 5, *Fluid Mechanics*, where a possible solution is offered for the problem of judging the suitability (and therefore placement) of Biton's recommendations, using well-established analytical tools, in such a way that conjecture can be removed and the otherwise overwhelming number of variables in the problem can be reduced. A further benefit to this methodology, as it is presented, is its potential for being used to evaluate the veracity of the accounts of ancient sources elsewhere.

Chapter 6, dedicated to the *Treatise*, brings us to the specific contents of the work written by Biton. Some observations on the currently-available translations of the Greek will be addressed, and their relationship to the prevailing reconstructions of Biton's machines will be demonstrated. The problems dealt with in the practice of ancient *poliorcetics* will need to be sympathetically interpreted with reference to modern engineering concepts, in a way that does not impress any more external influence onto the text than can be avoided. I hope to show, by some fresh interpretation of the text and re-appraisal of the evidence, that there are further clues that not only bolster the dating theory of Lewis, but also hint at the possibility of a novel mode of operation in one of these machines that would make particular sense in the suggested time and place of Biton's activity.

Chapter 7, *The Isidorus Engine*, sharpens focus on a particular machine recommended by Biton. It investigates in more detail the implications of the amended translation I have suggested in concrete terms, with relation to the production and engineering solutions available in 2^{nd} C. BCE Asia Minor, and in the particular historical context of the treatise as it has been established. The feasibility of this design will be weighed against the evidence, and we will explore its suitability for the problem at hand in this revised format. The conclusions reached here will then be transported to the city in Chapter 8, *Defenses of Pergamon*, where an overview and analysis will be provided of various extant geographical and archaeological features and finds that might indicate the possible placement and use of the treatise, and the Isidorus engine specifically, as it has been interpreted. A closer examination of fortifications, strategic considerations from the historical record, recommendations from the texts of other ancient engineers, the previous exploits of the Attalid kings, and the nature of the particular problem for which the treatise is written, will be used to sketch a fuller picture of the concrete possibilities for this text *in situ*.

The *Conclusion* will re-establish the links between these chapters as they are used to solve the particular problem of the thesis, and summarise the findings of the analysis.

There are two further addenda that may be of some use to the reader. The first appendix on Understanding Treatises by Geometry provides further reflection on the nature and problems of the history of geometry, and its relationship to understanding of the built environment, with particular reference to the ancient Greek world. The second appendix, Isidorus in Design, gives a practical-use, rather than scholarly, adaptation of this particular section of Biton, using modern design and engineering terms. It also includes Marsden's[47] and Rehm's[48] translations of this section for reference.

1.5 A NOTE ON TERMS AND MEASUREMENTS

A glossary has been appended to this thesis for any technical terms whose meaning may be unclear.

Standards of Greek measurement vary somewhat between extant examples, as the basic unit of the foot, or *pous*, can be longer or shorter depending on where it is found. Two schools of thought have arisen from this observation. One is that of the *reductionists*, who hold that there are just three distinct *podes* in the Greek world: the Attic-Cycladic, the Doric, and the Samian-Ionian. According to this view, the minor variations that have been observed within each class are held as largely inconsequential. The *permissive* school holds the view that far more variation should be recognised than is belied by the imposition of these three super-classes, and that the *pous* may have been so heterogeneous as to be substantively different from one area, cultural group, or city-state to another.[49]

We cannot be sure whether Biton, being Pergamene, refers to the Attic or Pergamene foot, or *pous*, in his treatise; the latter being a slightly greater measurement that is commensurate with other examples grouped under the Samian-Ionian banner. Marsden assumes a value that is within the general domain of Attic examples, and using either the Attic or 'common foot' standard has become something of a convention in scholarship except where comparison between people-groups is required.[50] An adaptation of Marsden's[51] conversion table has therefore been used in this thesis to avoid confusion. This table has been rewritten and abridged as necessary for the contents of this thesis, and included in the Glossary. His original conversion of the *pous* to 308.3mm has been harmonised here to 308.8mm, in order to make it divide correctly into the rest of the units in the scale. (While the precise length of the *pous* as it appears across various examples is disputed, the ratios between it and other units within the common Greek system remains constant.[52]) This number also remains relatively close to a more recent statistical analysis by Rottländer, which

[47] Marsden, 1972: 69.
[48] Rehm & Schramm, 1929.
[49] Jones, 2000: 75.
[50] Marsden, 1972: xvii.
[51] *ibid.* xvi.
[52] Jones, 2000: 73.

places the mean value of the Attic foot at 310.6mm.[53] I have included another conversion table with Rottländer's Samian-Ionian foot of 347.7mm for comparison, so that one might try the conversions with an eastern Greek standard that best represents what would otherwise have been used by Pergamene engineers. For good measure, there is also a table based on the Doric *pous*, which uses a mean value of 327mm derived from past metrology and recent archaeology.[54]

Any conversions that result in irrational numbers or ones whose decimals do not terminate within three points have been rounded to a maximum of three decimal places.

[53] Rottländer, 1996: 241.
[54] Jones, 2000: 77.

2. HISTORICAL CONTEXT

2.1 SCOPE

The foundation of the Pergamene kingdom came about in the wake of Alexander the Great's death, as his conquered territories were alternatively seized by, or distributed amongst, his successors and rivals. Beginning with its acquisition by the progenitor of the Attalid line of kings, Philetaerus, Pergamon saw its most illustrious years under this dynasty until its bequeathal to the Roman Empire by Attalus III, and subsequent decline.

The history of the Pergamene state in this time period is inseparable from the military campaigns of its kings. Inseparable, too, is the history of the mounted projectile weapon as it happens to develop and change the nature of warfare throughout this period, and particularly as it affects the ability of Greek cities and states to survive, expand, and protect themselves. As such, I will endeavour to cover these developments simultaneously, in a level of detail that is appropriate for each historical point of interest included here. We will begin at the earliest point in the timeline of the textual evidence of artillery, and work through the history of this branch of engineering until it coincides with the foundation of Pergamon. As we come to the post-Alexander period in particular, where the Attalid line of kings begins, we can assume a sharper focus on the rise of the Pergamene kingdom. From this point, the two will be dealt with in parallel, continuing along a strictly chronological line. Some brief asides will be made to explore incidences of sieges, notable uses of artillery, and incursions into Pergamene territory, that I believe are relevant to this discussion. In doing so, I hope to demonstrate the complexity of the problem of situating Biton; the nature of the Attalid kingdom, particularly its shifting allegiances and tensions with its neighbours; and to bring to light some precedents in the Attalids' behaviour which may influence the deductions that we can make from the more immediate context (and, of course, content) of Biton's treatise.

Despite their strong allyship with Rome, there is a case to be made that Pergamon's shifting rivalries with Macedon, the Seleucids, and other neighbouring states, positioned it to become the subject of aggression by the Bithynian prince Prusias II, emboldened by his own alliances forged towards the middle of the 2nd C. BCE. By this stage of technological development, the non-torsion artillery recommended by Biton are long outmoded, evidenced by the lack of reference to any such machinery in the Greek world past the exploits of Alexander the Great. Torsion-spring engines are, at this juncture, the only reasonable means of conducting ranged artillery barrages for great states like Pergamon, and the art of siegework has been distilled to what is, perhaps, its pinnacle in the Hellenistic period. This context is thus necessary for understanding the peculiarity of Biton's recommendations, and the more immediate context of the second century is necessary for understanding the reasons as to why these peculiarities may find some justification for being dated to this point in history. More importantly, the placement of Biton's treatise in this particular moment leads us to consider the defense of Pergamon with a Bithynian route of attack in mind. This is relevant to the greater thrust of this thesis, as we come to examine the possible serviceability of non-torsion engines *in situ* at the ancient site of Pergamon.[55]

2.2 THE FIRST ARTILLERY

The earliest written account of mounted,[56] large-scale artillery is by Diodorus Siculus, who claims that an array of pre-existing types of "καταπελτικόν" - as distinguished by the type of ammunition each was designed to dispense - were being further developed by skilled craftsmen working under Dionysius I of Syracuse,[57] who was building great numbers of these machines[58] in preparation for an offensive against Carthage in 399 BCE.[59] According to this account, these specialists were well-paid for this expertise and enjoyed direct patronage and favours from Dionysius.[60] In his following campaigns, Dionysius used not only the projectile engines that are the particular focus of this study, but also employed wheeled siege-towers for scaling walls, and battering-rams.[61]

Cross-references are few on this particular point, but for the purposes of this study, there is little reason to argue the veracity of this account. At some time prior to this, we can safely assume that the original designer of the *gastraphetes*, or belly-bow, flourished. Heron of Alexandria (fl. c. 10-70 CE) describes this early crossbow-like weapon as being hand-held, and having a bow at the front attached to the stock, or case, of the weapon.[62] Thus, the *gastraphetes* represents a smaller and simpler precursor to arrow- or bolt-shooting artillery. A slider runs up and down the length of this

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[55] See chapter 8: The Defenses of Pergamon.
[56] Being mounted on a framework or bipod-type arrangement, as opposed to hand-held.
[57] Diod. Sic. 14.41.3. Although he claims that certain catapult types were 'invented' here, it is more reasonable to assume that great innovation, rather than invention, was occurring.
[58] *ibid*. 14.43.3.
[59] *ibid*. 14.45.2.
[60] *ibid*. 14.42.1.
[61] *ibid.* 14.51.1. We will also briefly examine just such a siege tower as part of the study of Biton's own text; see Chapter 6.2: The Text.
[62] Heron, *Bel*. 81.

case, carrying the arrow or bolt. To load the weapon, the user ran the slider to the front of the case, attached the bowstring to a catch on the slider, and pulled the bowstring back by hand in small increments. The rear end of the case would be braced into the user's stomach – hence the name. The unwanted forward motion of the slider back towards the bow is arrested by the use of a ratchet-and-pawl system,[63] whereby each incremental motion backwards is indexed by a hooked arm slipping into the next slot in a toothed rail.

Once the slider has been pulled back as far as it can travel, a bolt or arrow can be placed in the groove of this slider. When the trigger is released, it releases the bowstring from the slider, catching the rear end of the projectile and carrying it forward. With the tension of the bow released, the pawl could be released from the ratchet by hand, and the slider moved forward again. In this way, the operator could load and dispatch projectiles using a bow far more powerful than one of the conventional type, which they would only be able to pull back with a single arm.[64] This basic functionality provided by the slider, ratchet and pawl, a motive power source at the front of the weapon, and a trigger, remains largely unchanged throughout the development of Greek and Roman *euthytone* and *palintone* engines.

Returning to the time of Dionysius, the major innovation of relevance in this period is the addition of a stand or base on which to mount a similar, albeit larger, style of crossbow-like configuration, creating the large, stationary artillery pieces mentioned above. With the addition of this mounting system, the greatest limiting factor to weapons development of this type was mitigated: the carrying, aiming, and loading capacity of the human user. Philip II of Macedon's establishment of a standing army in the mid-4th C. BCE may have been partly responsible for increased funding, attention to, and organisational efficiency around, weapon technology research.[65] Certainly, by the time of Alexander, stone-throwing artillery not only receives its earliest mention at the siege of Tyre in 332 BCE, but is already sufficiently powerful and present in great enough concentration to damage the structure of city walls.[66]

It is during this time that siege engineering is first formalised to the extent that it reaches the status of a prominent, pioneering, well-funded arm of military operations and technological research,

[63] See Glossary.
[64] Heron, *Bel*. 81.
[65] Marsden, 1977: 211. Vitr. *De Arch*. 10.13.3 mentions that wall-scaling engines (as opposed to projectile ones) were one of several that were the subject of further development under Philip II.
[66] Diod. Sic. 17.45.2.

which characterises the field as it continues to be represented by many great city-states and empires in the following centuries. Engineers specialising in projectile weapons were of significant value to military commanders.[67] These engineers, dependent perhaps on their level of skill, appeared to have multiple roles. They were primarily responsible for accompanying an army's artillery divisions while on campaign - for the purposes of supervising maintenance and making modifications to existing machines; making calculations to assist in ranging engines, scaling walls, or choosing ammunition;[68] managing logistical concerns in the same vein as modern logistics or engineering corps;[69] and building new siege engines as required for emergent battlefield conditions.[70] It is in this context that more prominent engineers like Posidonius, mentioned in Biton's treatise, accompanied Alexander as mechanicians,[71] whom we might see as the natural development of Dionysius' abovementioned assembly of craftsmen. While engineering of this particular kind was certainly bankrolled and greatly advanced in this formalised relationship, it is also fair to assume that independent, peripatetic engineers were practicing. Biton's mention of one Zopyrus of Tarentum working in two different locations might serve as evidence of this,[72] as does the fact that Dionysius' craftsmen were purportedly attracted to his service from all parts of the Mediterranean by the offer of high wages.[73]

The strategic uses of the improved range and payload of these early projectile engines surpassed that of merely picking off the troops of the enemy. At Pelium in 335 BCE, Alexander used engines to cover the movement of his troops across a river, firing indiscriminately in the direction of Glaucias' forces on the opposite side. The range of these weapons was clearly much greater than that of ordinary archers, who had to stand and fire from the middle of this river to reach a similar range. The strategy was successful; the hail of projectiles dissuaded Glaucias from making any attempt to follow.[74] The psychological effect of barrage was clearly realised.

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[67] Diod. Sic. 20.93.5 describes how Amyntas II of Macedon captured a group of renowned engineers in this field, presumably intending to add them to his retinue. Alexander amassed a significant number of them to complement his forces: Arr. *Anab.* 2.21.1.
[68] Arr. *Anab*. 2.26.1.
[69] Diod. Sic. 14.48.3; Arr. *Anab.* 2.21.1.
[70] As demonstrated, for example, by the Tyrians at Diod. Sic. 17.41.3 & 17.43.7-45.5; see also Marsden, 1977: 212.
[71] Vitr. *De Arch.* 10.13.3; Ath. Mech. 4.10.9. As per Vitruvius, Posidonius was certainly not alone, as individuals like Diades and Charias were also distinguished engineers in Alexander's service, having learned from Philip II's engineer Polyidus.
[72] Biton, 62 & 65.
[73] Diod. Sic. 14.41.3.
[74] Arr. *Anab.* 1.6.2.

As the records of these various sieges demonstrate, the development of projectile weapons progressed in parallel with that of equally important wall-breaching machines. At the siege of Halicarnassus in 334 BCE, a variety of tactics and machinery were used that are pertinent to the study of the defence of Pergamon; particularly in that Halicarnassus, though very different in many respects, also demanded that would-be besiegers make their assault up an incline. Initially, the Macedonians brought a covered ram to breach the city gates.[75] However, this move was delayed by the defenders' use of strategically-dug ditches around the city perimeter to slow and stop the wheeled 'sheds' from closing range. These armoured, moving sheds were then repurposed to bring earth and rocks forward and fill in the ditches, delaying the assault.[76] Philon of Byzantium continues to recommend the tactic of ditch-digging in the 3rd C. BCE, suggesting that the defenders also dig tunnels under the walls to draw earth away whenever the enemy tried to fill these ditches in.[77] The only recourse to this kind of assault was the use of large *lithoboloi* that could break the advancing machines, or bolts and caltrops that could be set alight and used to set the attacking force's wooden machines on fire.[78] While this multi-pronged attack strategy and its corresponding defense measures remain greatly similar across history, it is important to note that the use of long-range projectile weapons in this context remain crucially important for both attack and defense; whether it be for direct fire, or for denying the enemy the use of certain areas of the battlefield, particular machines, or other countermeasures.

One more important example that is relevant to our study of projectile weapons, and stone-throwers in particular, is that of the siege of Tyre in 332 BCE, as Alexander used scaling-ladders to transfer men from ships to the city walls. More interesting, however, was the employment of *petroboloi*, or small stone-throwers, to chip away at the city's defences.[79] The usefulness of these machines in suppressing the defenders and thus covering the Macedonians' construction of a naval mole on the approach to the city, allowing them to bring their wall-breaching measures to bear, is important to note.[80] Using engines to suppress defending infantry on the battlements of walls and remove sections of parapets – as opposed to using them expressly for creating great breaches, for the purpose of inserting an invasion force - was evidently a problem

[75] Diod. Sic. 17.24.4.
[76] Arrian, *Anab.* 1.20.2.
[77] Philon, *Pol.* 1.36.
[78] *ibid.* 3.4.1.
[79] Diod. Sic. 17.43.4 & 17.46.1, as well as Arrian *Anab.* 2.23.1.
[80] Diod. Sic. 17.42.7.

that survived well into the Hellenistic period, as Philon describes measures to counter this with iron-plated shutters and the securing of masonry.[81]

2.3 PHILETAERUS (343-263 BCE)

In the Wars of the Diadochi that followed the death of Alexander, Philetaerus was serving under the *diadochus* Lysimachus, who appointed him ruler of Pergamon at around 281 BCE (although Philetaerus may have already found himself in provisional command of the city by the authority of Lysimachus for some time beforehand).[82] Following the death of Lysimachus at the hands of the Seleucids, Pergamon was nominally part of the Seleucid empire; however, after the death of Seleucus himself shortly after, Philetaerus' fledgling state was largely left to its own devices.[83] In the years following, until his death, Philetaerus began to secure Pergamon with more comprehensive fortifications that were to be completed under the reign of his nephew, Eumenes I, and assisted neighbouring states in conflict with the Galatian tribes to the north, securing amicable relations with Pergamon's neighbours.[84]

While cities remained susceptible to all kinds of attack or subversion - particularly treachery - we can be sure that sinew-spring, torsion-type catapults had become ubiquitous in this period, thanks in no small part to the increasing danger of the deployment of engines against the walls of cities and the need for defenders to arm themselves with appropriate countermeasures. Storage records of the "frames" of catapults, indicating that the framework for holding sinew-springs was present as opposed to merely a case and a bow, survive in inscriptions dated to 307-6 BCE.[85] Similarly, inventories of catapults in storage mention their type as being that "with sinew springs".[86] Byzantium,[87] Rhodes,[88] and Perinthus[89] were well-stocked with artillery at various times; and Tyre was said to have "a wealth" of engines.[90]

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[81] Philon, *Pol.* 3.3 and 1.8 respectively.
[82] Strabo, *Geo*. 13.4.1-2.
[83] Paus. *Descriptions.* 1.10.3.
[84] Hansen, 1971: 18.
[85] *IG* II^2 1627: 328-341.
[86] *IG* II^2 1487: 89-90.
[87] Diod. Sic. 18.51.6. The Cyzicani were able to quickly and easily procure missiles from Byzantium.
[88] *ibid*. 20.84.4 & 20.97.2.
[89] *ibid*. 16.74.4 & 16.76.4 demonstrates that while trains of equipment usually moved with their armies, they could also quite easily be left behind at ill-timed moments. The Byzantines had a healthy contingent of artillery, but left it in Perinthus - adding to that city's stocks of engines and other supplies.
[90] *ibid*. 17.41.3.

Elsewhere, Demetrius I, son of Antigonus I, set about earning his epithet of *Poliorcetes*. At his attempted siege of Rhodes in 305-4 BCE, he successfully breached the walls using stones fired from a torsion *palintonos.*[91] Judging by the extant ammunition of the Rhodians in the archaeological record, and Demetrius' ability to quickly return his machines to working order despite the great volume of Rhodian fire,[92] they had no *lithoboloi* large enough in ammunition capacity to irreparably break Demetrius' engines, and had to resort to alternative means to halt his advance.[93] Using *euthytonoi* to pelt the approaching engines with fire-arrows and bolts, they set his rams and artillery alight;[94] and when he approached with a rolling siege-tower, the Rhodian engineer Diognetus pumped water and sewage into its path, causing it to be caught in the resultant quicksand.[95]

2.4 EUMENES I (c. 263-241 BCE)

Philetaerus adopted (and was succeeded by) his nephew, Eumenes I. It is during Eumenes' rule that the pharaoh Ptolemy II – the son of another *diadochus* – waged a war against the Seleucid king Antiochus I. Eumenes took the opportunity to liberate Pergamon from Seleucid influence by revolting, and led an army that ultimately defeated Antiochus at Sardis. He expanded Pergamon's territories significantly, from the foothills of Mount Ida in the north to the tail of the river Caïcus in the south, earning him the title of *basileus*.[96] However, Pergamon's troubles with neighbouring states were not over. During this entire period, it can be inferred from from the accounts of Livy and Strabo[97] that the Galatian tribes to the north were levying tributes from surrounding kingdoms – including Pergamon – on threat of invasion and sacking.

2.5 ATTALUS I (269-197 BCE)

With the end of Eumenes I's reign, we can begin to sharpen our focus on the Attalid kings. The adopted son of Eumenes I, Attalus I enjoyed a storied career, beginning with his refusal of Galatian

[91] Diod. Sic. 20.97.6.
[92] *ibid*. 20.97.2. Demetrius' engines certainly must have outclassed the Rhodians by size if we take Diod. Sic. 20.92.1 at face value.
[93] Shatzman, 1995: 57.
[94] Diod. Sic. 20.96.3-4.
[95] Vitr. *De Arch*. 10.16.7. The tactic of flaming bolts is also attested by Philon, *Pol*. 3.4.1 and Aen. Tact. 33.2.
[96] Hansen, 1971: 21-22.
[97] Livy, *Hist*. 38.16.12; Strabo, *Geo.* 12.5.1.

demands to pay tribute, and his defeat of their assembled forces at the source of the aforementioned Caïcus river in the north.[98] With the Gallic menace to the surrounding region suppressed, Attalus returned to Pergamon a hero: once again assuming the title of *basileus*,[99] attested on building inscriptions of the upper temple district of Pergamon.[100]

Not long after his return, a Gallic counter-offensive was mounted with the support of the Seleucids. The Seleucid aggressor, Antiochus Hierax, also happened to be married into the ruling family of Bithynia.[101] Attalus I successfully repelled the invasion, and seized some of the Seleucids' southern territories,[102] which were abruptly lost some years later.[103] Meanwhile, Rome and Carthage were embroiled in the Second Punic War.[104] An interesting web of diplomatic relationships is observable at this time, as Philip V of Macedon was allied with Carthage and in active hostilities with the Aetolian League, while Pergamon was allied with the League and friendly with Rome.[105] Philip V of Macedon was, in turn, allied with king Prusias I of Bithynia, who had married Philip's half-sister for reasons of diplomacy.[106] This convoluted set of relationships came to a head as the First Macedonian War led Attalus I to campaign against Philip V in mainland Greece on behalf of the Aetolian League.

In 208 BCE, Prusias I of Bithynia took the opportunity presented by Attalus' absence to make a direct attack on Pergamon, presumably to distract the Pergamenes from his brother-in-law's campaign in mainland Greece.[107] Little can be gleaned from this encounter, as the evidence is slim, and there is no particular reason to assume that Prusias could have reached any further than Philip subsequently managed to do.[108] Having attacked Samos and Chios in a renewed effort at expanding his territory, Philip V marched on Pergamon in 201 BCE, from which Attalus I had mobilised his own forces to help put down the Macedonian assault. Having had his ships run aground in a naval battle against the Macedonian fleet, Attalus was absent from Pergamon at this time.[109] Philip V attempted to attack the city, believing "he had as good as made an end of Attalus"; but instead, Polybius

[98] Strabo, G*eo*. 13.4.2; Polyb. 18.41.1-9.
[99] Hansen, 1971: 31.
[100] Boehringer & Szalay, 1937: 123.
[101] Justin, *Hist.* 28.1.
[102] Polyb. 4.48.6.
[103] *ibid*. 4.48.9.
[104] Livy, *Hist*. 23.33.
[105] *ibid.*
[106] Hansen, 1971: 49.
[107] Livy, *Hist*. 29.12.
[108] Hansen, 1971: 48. See also the Introduction to this thesis re. placing Biton.
[109] Polyb. 16.6.

recounts that he was "easily repelled" by the defending forces, and he took to destroying the sacred sites of Pergamon's surrounds instead.[110]

The only information we can glean from this encounter is that both Macedon and Bithynia certainly made some inroads into Pergamene territory, but their reach is uncertain. Polybius' words may lead one to wonder if Philip V may have had his heart set on capturing Attalid Pergamon itself, had he not been dissuaded by the city's defenses. It is unclear whether the failure of the garrisoned force to sally forth into the countryside to prevent the invaders from looting temples was either due to that force being improperly equipped to pose any meaningful threat to the Macedonian expeditionary force in open battle; or, indicative of a general Pergamene preference for adopting a turtle-like strategy in the face of harm. If we refrain from attempting to interpret what an "end of Attalus" might have entailed, we are left only with the face-value evaluation of Pergamon's defensive capabilities of this time. The purported absence of any difficulty, and Pergamon's already-established military readiness from the very outset of Attalus I's reign, make it difficult to place the theorised stopgap engines of Biton at this date; particularly when considered alongside the uncomfortably close dating of the Damis of Colophon mentioned in his treatise.[111]

Attalus I, now nearing 70 years of age, was once again drawn into a campaign on the Greek mainland as Macedonian expansionism threatened Athens in 200 BCE, sparking the Second Macedonian War.[112] Joined, this time, by forces from Rome led by Titus Quinctilius Flamininus,[113] Attalus and his allies pushed the Macedonians and their Acarnanian cohorts back with a combined naval and ground campaign.[114] The relationship fostered between Rome and Pergamon during this time was to reach far into the future.

In 197 BCE, Attalus I accompanied the military on tour and was present at a diplomatic council where Flamininus hoped to convince the Achaean League to abandon their alliance with Philip V of Macedon and side with Rome. Attalus I attempted to make his representation, and collapsed abruptly mid-speech, having to be removed from the proceedings. It appears he was the victim of a

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[110] *ibid*. 16.1.
[111] See chapter 1: Introduction; Lewis, 1999: 164.
[112] Polyb. 31.9.
[113] As an aside, the Roman ability to effectively use siege engines was evidently not yet as well-practiced as that of their Hellenic allies. On one occasion, Flamininus created an embankment upon which to drive up his siege rams and was repulsed by the defenders. He then built a siege tower, which also broke down on the journey to the walls. Livy, *Hist.* 32.18.3.
[114] Livy, *Hist.* 31.45; Hansen, 1971: 61.

stroke, as Livy describes him unable to use his "one side", and in the following days he continued to suffer "powerlessness of the limbs"[115] despite being otherwise outwardly physically healthy (his general frailty due to his age notwithstanding).

2.6 EUMENES II (221-159 BCE)

The eldest natural-born son of Attalus I, Eumenes II was crowned in 197 BCE. He came to power in unfortunate circumstances, however, as the territories gained by Attalus I abruptly shrunk.

Eumenes II began by carrying on Attalus I's work - establishing an alliance with Rome to oppose continued Macedonian efforts at expansion in Greece and Asia Minor, now under the kingship of Perseus, son of Philip V. Antiochus III of the Seleucid empire began appealing to the Greek kingdoms and city-states in Asia Minor to submit to Seleucid sovereignty, including allies of Pergamon.[116] Eumenes sent his brother Attalus (soon to become Attalus II), to Rome in 192 BCE to warn the Senate of these developments. Antiochus' diplomatic and military advances succeeded in claiming substantial territories,[117] and he moved a wing of his military west into Thrace.

Rome urged Antiochus III to return the Pergamene territories,[118] and Eumenes II refused Antiochus III's diplomatic offers of partnership.[119] Tensions continued to mount, and the subjugated cities of Smyrna and Lampsacus urged Attalus I's former Roman campaigning partner, Flamininus, to not only include them and their fellow Greeks in Asia Minor under a previous proclamation that Rome would support the "liberty of the Greeks of Hellas", but to follow through on that promise.[120] The Romans held true to their word, and a protracted conflict ensued.

It is during one of Pergamon's low points in this war that we find another worthwhile case study in Pergamene tactics, and the use (or indeed, non-use) of artillery. Seleucus, son of Antiochus III, made a foray into Pergamon's immediate surrounds, encamping his forces outside of the city and looting the surrounding area. Attalus II was present in Pergamon at this time, and made some small attempts to harass Seleucus' forces with light infantry. Unfortunately, he was alone, as Rome's

[115] Livy, *Hist.* 33.21.
[116] Hansen, 1971: 70.
[117] *ibid.* 74.
[118] Polyb. 18.47; Livy, *Hist.* 33.34.2-3.
[119] App. *Syr*. 5; Kosmetatou, 2003: 163; Gruen, 1984: 544-5; Ma, 1999: 92.
[120] Hansen, 1971: 75.

combined forces were occupied near Rhodes, and Eumenes II had not yet returned from overseeing naval operations at Samos. Antiochus III arrived with reinforcements, joining Seleucus and forcing Attalus II to withdraw behind Pergamon's walls.[121]

The breaking of the stalemate that followed is particularly interesting on account of an observation made by Livy with regards to Attalus II's tactics - or lack thereof - in the face of the invaders.[122] The Pergamenes remained behind the city walls while the enemy lazed, waited, and watched idly from the valley below, remaining out of the range of the Pergamene defense batteries. The Seleucids, at this moment, were evidently uninterested in laying direct siege to Pergamon, instead content to extract what plunder they could from the surrounding area while Antiochus III engaged in dialogue with Rome, Pergamon, and their Rhodian allies.[123] Eumenes II eventually joined his brother with a combined infantry and cavalry force, slipping into the city under the cover of night. Under similar conditions, a relatively small contingent of Achaeans under the command of one Diophanes also joined the Pergamene defense. Livy mentions that Attalus and Eumenes made little attempt to remove the encamped invaders; "none came out of the city, not even to attack the advanced posts with missiles at long range."[124] The implication, then, is that field artillery could be brought to bear *outside* of the city walls, when range demanded it. After several days of observation, Diophanes made a heroic sally despite unfavourable odds, taking the Seleucids' complacent forces by surprise. He repeated this strategy several days later, and successfully drove them from Pergamon.[125]

The difficulty of attempting to place Biton's treatise at this precise moment lies in the fact that Pergamon had, at this time, been at war for several years. The likelihood of the city's defensive torsion artillery being out of action and the city's defenses thus substantially reduced and in need of an unusual stopgap is, as Lewis points out, very small.[126] Additionally, the circumstances we shall examine below during the reign of Attalus II make for a more compelling case. Nevertheless, the possibility of fielding troops that could in turn be supported by field artillery outside of the city walls is an important point which I should like to reference later in this study.

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[121] *ibid.* 37.18.
[122] *ibid.* 37.20.
[123] *ibid. 37.19.*
[124] *ibid.* The "missiles", in this case inferred from context, are those of artillery rather than hand bows.
[125] *ibid.* 37.20-1.
[126] Lewis, 1999: 164.

Once the Roman expedition and Pergamene forces reunited, the combined army ultimately drove Antiochus back to his former territories - thus concluding the Roman-Seleucid War. Notably, Eumenes II acquitted himself exceedingly well in battle against highly-esteemed Macedonian and Seleucid troops at the pivotal Battle of Magnesia, enhancing the reputation of the Pergamene cavalry.[127] Having allowed Rome to take the bulk of the responsibility for the campaign of liberation, Eumenes II then received all the territories previously belonging to Antiochus III south of the Taurus Mountains,[128] effectively restoring a significant portion of the territory and tribute enjoyed by Pergamon under Attalus I.

Perseus of Macedon made preparations to take the field in the following years, and Eumenes attempted to warn the Senate of these developments, traveling to Rome in person in 172 BCE. On his return home via Delphi, an assassination attempt gravely wounded him, and Macedon declared him dead,[129] proving suspicions about the Macedonian preparations for war correct. Attalus II assumed the throne, believing his brother to indeed be dead.[130] Rome marched on Macedon, marking the commencement of the Third Macedonian War of 171 to 168 BCE, which ended with the defeat of Perseus and the dissolution of Macedon into four smaller states. Eumenes II, having recovered from the attempt on his life, returned to Pergamon during this time, where Attalus II returned the throne to his brother. The Roman Senate, believing that they had been tricked by the Pergamenes into entering war with Macedon, was displeased. Eumenes II attempted to reconcile with Rome, but was barred from entry into the city; not least for the reason of having remained neutral while Rome once more fought in mainland Greece on Pergamon's behalf.[131]

The final military endeavour of Eumenes II's career was a cooperative venture with Attalus II to once again put down Galatian agitators to the north of Pergamon. In 166 BCE, Pergamon was victorious, but graciously granted the defeated Galatians their sovereignty at the behest of Rome. Eumenes had secured the eastern border of Pergamon, and by the time of his death in 159 BCE, had left a safe and prosperous Pergamon to his successors.

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[127] App. *Syr.* 6.31-6.
[128] Livy, *Hist.* 38.39; Strabo, *Geo.* 13.4.2; Kosmetatou, 2003: 163; Gruen, 1984: 640-3.
[129] App. *Syr.* 1.4.
[130] Plut. *Mor.* 3.15.184a-b.
[131] Livy, *Per.* 46.1-2.

2.7 ATTALUS II (220-138 BCE)

After the death of his brother, Attalus II – who was now 61 years old - once again took the title of *basileus.*[132] However, this was a nominal title, as he performed the duty of regent on behalf of Eumenes II's young son, Attalus III, who would wait another 21 years before ascending to the throne.[133] After installing an ally for himself in Cappadocia,[134] Attalus turned his attention to the old enemies of Pergamon – Galatia and Bithynia.

From 156 until 154 BCE, the war which we are most interested in – that waged between Attalus II and the Bithynian king Prusias II – broke out. Surviving accounts of these hostilities are to be found exclusively in the *Mithridatic Wars* of Appian and fragments from Polybius. Whatever the precise circumstances might have been from Prusias' point of view,[135] it appears that the incursion of 156-154 BCE was abrupt, and there is little doubt that the Pergamenes were caught unawares. After an initial raid on Pergamene territory by the Bithynians, Attalus sent an envoy, Andronicus, to Rome to report the attack and to request their aid,[136] citing "circumstances of dire emergency".[137]

The Roman Senate, after some resistance, sent legates to Pergamon with the intent of issuing an official prohibition to Prusias on escalating hostilities against Pergamon. This was to be delivered at a parley on the border between the two kingdoms. Attalus and his Roman allies appeared to be either unsuspecting of foul play, or helpless to put measures in place to combat it, as they arrived at the frontier to discuss the matter with Prusias and his army having brought a complement of only a thousand soldiers.[138] It may be that this is all Pergamon had at the time, as no standing army was active, and none had been recruited from the citizenry.[139] Prusias disregarded the Roman prohibition, and advanced with the entirety of the force he had assembled there, cutting down the majority of Attalus' retinue, and forcing him to flee to the capital. The Bithynians then set about looting the Pergamene sanctuary of Athena, the Nikephorion, and plundered surrounding temples much as Philip V had done nearly 50 years earlier.[140]

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[132] Plut. *Mor.* 184b & 489f.
[133] Strabo, *Geo*. 13.4.2.
[134] Polyb. 32.10 & 32.12; Diod. Sic. 31.32-32b; App. *Syr*. 47; Justin, *Hist.* 35.1.
[135] Sadly, no account of Prusias' motives is given in the extant sources.
[136] Polyb. 32.16, 2.
[137] *OGIS* 323.
[138] App. *Mith.* 1.2-3.
[139] Lewis, 1999: 167.
[140] App. *Mith.* 1.3; Hansen, 1972: 134.

Attalus thus returned to Pergamon, most likely suspecting a siege to be Prusias' next step. At this moment in time, the last siege Pergamon had been threatened with was under Philip V, and the last military engagement they had to seriously consider was with the threat of Antiochus III in the late 190s BCE. It is fair to suggest, then, that the artillery of Pergamon was not prepared for an invasion.[141]

Two legates returned to Rome with news of Prusias II's disregard for Roman orders, his hostilities towards Pergamon, and those towards his neighbouring city-states. Unwilling to allow the Bithynians to run rampant any further, Rome sent commissioners to put an end to the war and to compel Prusias II to compensate Attalus for the damages wrought on Pergamon and its surrounding territories. Nevertheless, Attalus spent 155 BCE amassing a large army, hiring Cretan mercenaries from Aptera, and calling upon the aid of his various allies.[142] During Attalus' preparations, the Roman commissioners met with Prusias, who refused to submit to their demands. The commissioners then returned to Attalus and encouraged him to continue solidifying defences of Pergamene territory, but to do so without inciting any further hostilities with the Bithynians.

Meanwhile, the Roman commissioners returned to the Senate with reports on the situation in Asia Minor. When the Senate had heard the report, three new envoys were dispatched to Prusias to end the war. Rome ordered Prusias to cede twenty ships to Attalus at once and to pay him five hundred talents over twenty years as compensation, along with a reimbursement of a hundred talents for damages done to the territories of Pergamon.[143] Once a peace treaty had been drawn up between the two states, Attalus withdrew his forces, bringing the war with Bithynia to an end in 154 BCE. In 149 BCE, however, aware of Prusias' concern over the growing popularity of his son, Nicomedes, as a potential candidate for the Bithynian throne, Attalus amassed an army, and invaded in support of Nicomedes. Prusias fled to the capital city of Nicomedia. He was betrayed by his own citizens, who opened the gates for the Pergamene forces. As a last resort, Prusias fled to the Temple of Zeus, where he was stabbed to death by emissaries sent by his son.[144] Attalus II thus not only exacted revenge on Prusias, but also successfully installed Nicomedes II on the Bithynian throne, ensuring that the state remained friendly to Pergamon from that point onwards.

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[141] Lewis, 1999: 164.
[142] Hansen, 1971: 134.
[143] Polyb. 33.13.6-9.
[144] App. *Mith*. 1.7.

Despite his advanced age, the remainder of Attalus II's reign in Pergamon saw some further military action, including another successful campaign against the Galatians. "For the rest of the reign of Attalus II, and for that of his successor, we are left, apart from the comparatively meagre epigraphical evidence, with literary references that are either fragmentary or incidental, or both".[145] This meagre epigraphy - an inscription serving as a dedication of Attalus II[146] - provides a date of 145 BCE for Attalus II's last foray into Thracian territory in pursuit of Prusias' son-in-law, Diegylis.[147] This is confirmed by Strabo to be his last major military action, as Attalus thereafter leaves the throne to his nephew.[148] As such, the last few years of Attalus' reign are largely undocumented, save that which we might infer from other evidence. In 138 BCE, Attalus II died and Attalus III Philometor, the son of Eumenes II, took the throne of Pergamon.

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[145] Allen, 1983: 83.
[146] *OGIS* 330.
[147] Allen, 1983: 82.
[148] Strabo, *Geo*. 13.624.6.

3. RECONCILING ANCIENT ENGINEERING WITH MODERN CONVENTIONS

Where Biton's or other *poliorcetic* treatises preserved by the medieval manuscript tradition are not missing their accompanying illustrations,[149] they have often had them rendered impossible to understand by abstraction of their components or proportions in ways that suggest either (i) simple confusion or misunderstanding on the part of scribes, (ii) a set of norms or conventions for interpretation of technical drawings that has been lost to time, or (iii) a loss or omittance of elements of the diagram in one common antecedent copy of the document - whether accidental or intentional.

Barriers to understanding Hellenistic engineering manuals can therefore be separated into two categories: problems of the text, and problems of the diagrammatic accompaniment. Both of these are in turn made up of several constituent issues, which interact with each other in various ways. In order to decrypt the contents of Biton's treatise, some understanding of the underlying geometry and how it is to be reconciled with modern engineering is required.

3.1 HOW TREATISES ARE WRITTEN & PROBLEMS THAT ARISE

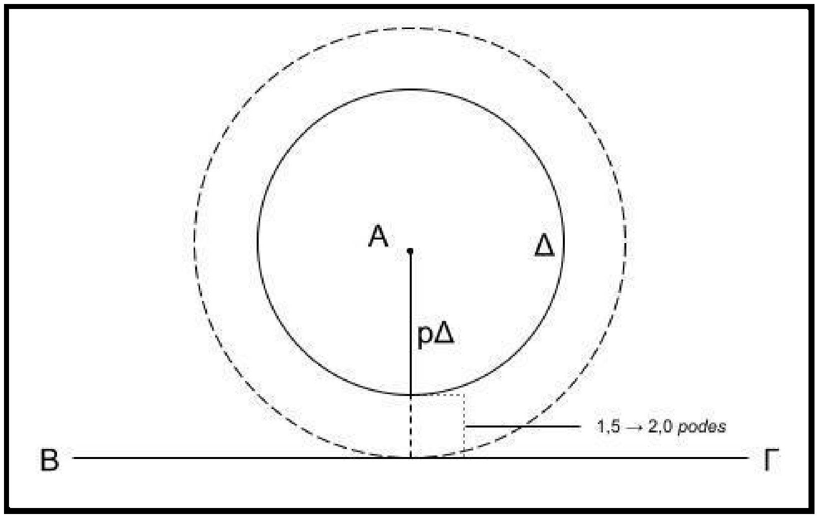
Much of the confusion arising from studying the original manuscripts comes from deciphering faulty or incomplete formulae for describing the construction plans. This is only exacerbated by the way in which the plans are written. Mathematical variables, referred to using Greek letters, are given geometric constraints with relation to one another and to simple geometric constructs; e.g. 'parallel to', or 'perpendicular to'.[150] For example, they may solidly bind two lines, shapes, or entire assemblies together along one face or vertex; or, they may describe the arc that a free-moving part takes around its parent assembly. In order to demonstrate this using an example that is immediately accessible, we might consider a swing hanging from a tree. The swing seat (the free-moving part) is able to move in a pendulum-like arc around the tree branch (the parent assembly), and the length of the rope that hangs the swing determines the radius of the arc that the swing takes as it travels through the air around the branch. Thus, one could say that the length of the rope is the constraint that determines the radius of the arc drawn by the swing. Other constraints in action here are that

[149] Specific examples are explored below.
[150] Constraints, in the fields of drafting and engineering, might here be defined as delimiters or restrictions that define the relationship between two bodies or elements. See entries for "constraint" and "degrees of freedom" in Atkins & Escudier, 2013: 490 and *ibid.* 531 respectively.

one end of the rope is affixed to the tree branch, and the other to the swing seat, affixing the two to one another permanently and preventing them from separating and creating a new arc of movement entirely (or at least, we should hope so, so that the hypothetical swing's rider is not made involuntarily airborne).

In determining the length of the rope for a garden swing and describing the construction, an imaginary ancient engineer might write the following instruction to his builder:

> *Consider the area of construction as though viewing it down the grain of the branch upon which you desire to affix the swing (such that the rider of the swing will, in following their path of swinging, will have their flank facing you, and will draw a half-arc before you). Where the desired point of affixing the swing to the branch of the tree is referred to as point Α, and the line created by the earth below is referred to as line Β-Γ, draw a circle concentric to Α such that it is tangential with Β-Γ. Ensure that the circumference of this circle does not impede on any other boughs or structures in the construction area. Once satisfied, determine the radius and subtract two* podes [two Ancient Greek feet or 0.616 modern metric metres] *to allow for the user's hanging limbs in the case of a swing for adults or teenagers, or one and a half* podes *in case of a swing for children, naming this new radius line ρΔ. Use ρΔ to draw a new circle Δ, erasing the previous circle described. As the previous step will have ensured that a circle greater than Δ will not cause harm to the user or their environment, you can be confident that the shorter radius of Δ, ρΔ, can now be used to determine the safe length of your swing-rope except in the case of very tall users.*

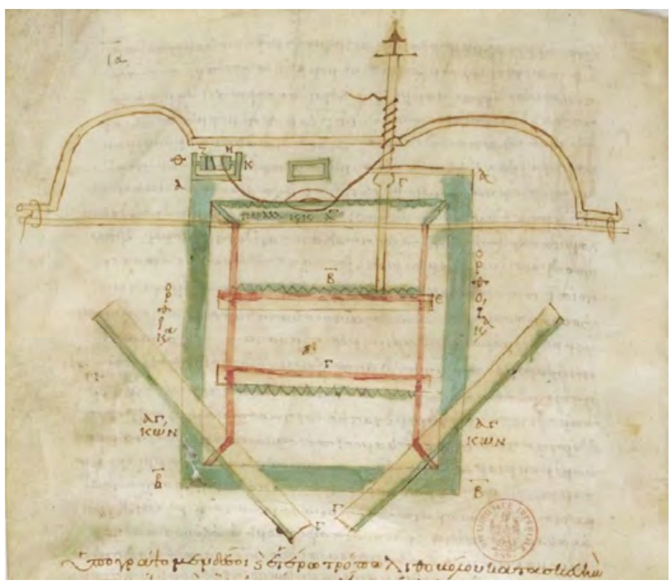


In this way, a textual description functions as a step-by-step guide to drawing the individual components of a construction's - or indeed, siege engine's - most important parts. It can also do this without necessarily providing strict measurements, but rather a set of relationships between parts that can be scaled proportionally and infinitely.[151] This stands in stark contrast to modern engineering practice where scaling of a design is, in most cases, explicitly and vociferously disallowed. This difference is partly due, of course, to current standards of safety and accountability that prohibit any modification of a design without review by the design engineer. More interesting, however, is the role played by the conventions and limitations of ancient Greek geometry as it is employed in siege technology, and the faithful adherence of Hellenistic siege engineers in particular to popular principles of proportionality that were specifically aimed at allowing the reader of a siege manual to scale machines up and down to suit the peculiarities of any given combat situation.[152]

The limitations of the Greek conventions for creating an engineering drawing from text are immediately apparent when one considers how one might apply this method of description to a three-dimensional part of any complexity. The plane of construction is merely a perfectly flat surface, and is *pre-Cartesian*.[153] Without the luxury afforded by being able to map a two-dimensional

[151] Note that the length of the swing rider's limbs are dictated in terms of *podes*; this was not a formalised measurement as we think of imperial feet today, but would have varied regionally with the proportions of the inhabitants (although this is largely due to the fact that strict formalisation of measurement standards only became necessary in the 20th C.).
[152] See discussion of Philon below.
[153] Prior to René Descartes' work *La Géométrie*, one book within the greater work of *Discours de la méthode*. Atkins & Escudier 2013: 297.

space with points, as per Descartes, we can have no coordinates or point of origin[154] from which to describe or create constructions. Considering our swing example, we are unable to make either the tree branch, or the intersection of the tree trunk with the ground, an origin point; and instead of referring to points on an ($_0$; $_0$) basis, we are only able to refer to objects we have instantiated, like lines or circles, by their assigned letters. Without a coordinate system, it is also impossible to refer to a axis, and hence we are unable to describe the length of the swing seat without beginning a new drawing that views it from the perspective of the front, rear, above, or below. There is no way, therefore, to describe depth or accurately communicate the minutiae of a complex three-dimensional shape without describing each face thereof in turn. Any of those faces that are not a regular shape must be laboriously described in terms of lines and angles before one can even begin to consider the features that are raised from, or cut into, that face. Even then, the reader who recreates this textual description is merely presented with multiple profile views[155] of the object described. Add to this the practical problem of a tired or impatient scribe or apprentice who erroneously copies or substitutes an unclear or missing variable name - say, confusing the lowercase letter *o* for σ - and the true meaning of the instructions can be lost completely.



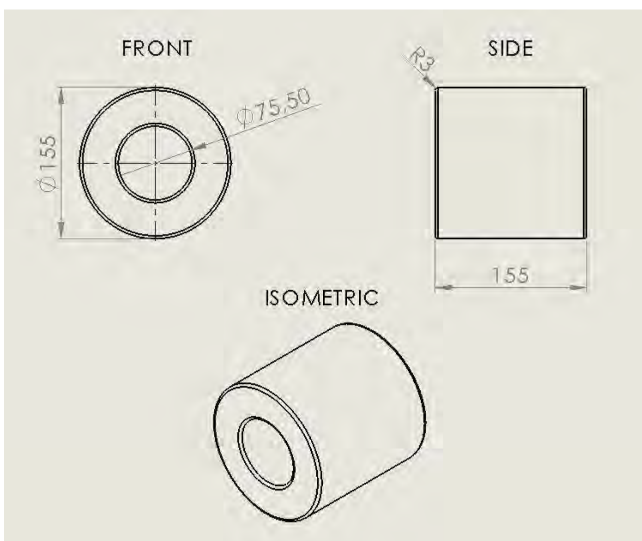
The engineering drawing of Charon's stone-thrower per Biton 45f, as rendered in 10th C. CE manuscript Supplément Grec 607: f.26v. The entire assembly is shown, but is out of proportion and only

[154] Although the two perpendicular numbered axes that are popularly ascribed to Descartes do not appear in his seminal work, the fundamental notion of using a numbered line to refer to points in a construction, and thus describing a shape in terms of an algebraic formula, is first described in *ibid.* 313.

[155] "Profile views", as defined in engineering design, are representations of an object 'dead-on' from the point of view of one side at a time. An equivalent in portraiture, for example, would be the face-forward and profile mugshots used by law enforcement.

shows references to variable letters mentioned in the text. Components are not shown in their relationships that they would display in reality; the two sawtooth constructions, for example, are horizontally instead of vertically constrained.

In today's industry, when describing a complex assembly, multiple views or *projections*[156] are used to demonstrate a range of measurements from different viewpoints. For a moving assembly, multiple drawings of the same assembly in the same projection - each drawing displaying it in a possible position within its total range of movement - can further demonstrate the textual instructions in such a way that any unclear variables can be seen to interact in the manner the designer intended. The confusion arising from faulty ancient textual descriptions might be mitigated if the scribe and their readers were able to refer to a drawing like the one for our garden swing above, which shows the constraints between variables diagrammatically.[157] Most of the surviving copies of Biton are not only missing accurate assembly drawings of complete machines, but also their component-level drawings that would otherwise show these simple relationships within, and between, discrete components.



A modern engineering sketch of an individual component showing three views. One is a flat view of the front of the single component, one is of the side, and the final one is a projection in the isometric format; intended to show the component in three dimensions, 'al vif'*.*

Thus, a hypothetical rogue lowercase σ can be corrected by the scribe (or an attentive reader) to an *o*, when it is seen that following a written instruction like "*connect point* σ *to point* κ *by means of a*

[156] As defined in engineering design; "a representation of the features of a three-dimensional object on a two-dimensional engineering drawing" Atkins & Escudier, 2013: 935. Specifically, this is a representation that views the object from a realistic perspective; for example, from an elevated three-quarter view.
[157] For more on the significance of diagrams esp. as re. Euclid, see chapter 4: *Euclidean Geometry* below.

straight line" would result in creating an element that clearly did not exist in the drawing, such as placing a wall straight through the floor plan for an open courtyard.

The immediate question that comes to mind, then, is how and why ancient manuals came to be written and illustrated using these conventions, and how we are to understand them as the ancient reader might have done.

3.2 THE SCRIBAL PROBLEM

From a Classicist's perspective, the ability of Greco-Roman art to accurately represent three-dimensional objects and spaces in two dimensions is unquestionable. Roman Second Style frescoes demonstrate considerable ability in consistent and accurate projection. The cubiculum of Synistor at Pompeii, beautifully preserved, shows elevation views with well-rendered perspective and disappearing point:

Wall fresco as reproduced in Kleiner (2007: 31)

Clearly, this was not limited to the Roman tradition: Vitruvius describes the Greek ability to accurately relay the dimensions and the arrangement (or assembly order) of a scene or subject,[158] and asserts this to be the origin of Roman methods.

[158] Vitr. *De Arch.* 1.2.2.

A manuscript of Heron of Alexandria's *Mechanica*,[159] translated into Arabic in the 9th C. CE by the scholar Qusta ibn Luqa, is another, earlier example of a manuscript crucially impaired in its illustrations. Importantly, however, it is one of the earliest to show component drawings in multiple views. The original Greek version from which ben Luka made his translation has since been lost. Projections shown in this manuscript are often bizarre, and it is again unclear whether this is due to a lack of clear standards for the process of projection, or the work of less-skilled copyists. The fact that it is such an early example, and has many more component views included than other ancient engineering texts, might indicate that purer forms of the diagrammatic component of these other manuscripts did originally exist.

However, this slow distortion of the ancient texts is not indicative of a linear degradation of the skill of representation on a universal level. Architectural sketches in the handbook of Villard de Honnecourt from the 13th C. CE,[160] for example, demonstrate that certain groups or sects were certainly able to match this level of technical accuracy. Evidently, some methods of detailing or description had been lost in the evolution of ancient siegeworks manuscripts from their original forms to their descendent copies in the Middle Ages. The means by which some learned individuals managed to preserve those representations, while others did not, deserves some explanation.

[159] Heron, *Mechanica.* cf. MS Harley 5589 & MS Harley 5605 (The British Library).
[160] MS Fr 19093. See discussion below.

Perspective architectural drawing of Villard de Honnecourt in an approximation of an isometric projection, c. 13th C. CE (MS Fr 19093: f.6v)

3.3 SCRIBES OF THE MIDDLE AGES

Many surviving ancient (and most medieval) pictorial representations of siege weapons and other precisely-engineered machines are largely unrepresentative of true-to-life proportions, measurements, or even strict functionality. They are symbolic, demonstrative of a limited few elements or functions that the artist - or the writer of the accompanying text - wished to emphasise above all others. This is not necessarily all due to a loss of skills or artifice in the field of realism in Europe’s post-Classical and pre-Enlightenment era, as is commonly and mistakenly thought.

Academics in Medieval Studies and Art History are well equipped at this point in the scholarship to account for a conscious shift in preference towards an expressionist style: exemplified by the Christian monastic orders, which were the most prolific text preservationists during this time, made up of exhaustively-trained career professionals. Perhaps the most conspicuous example of the expressionism that characterises the era is the representation of animals, as the medieval bestiary formed an entire genre on its own. However, rather than merely illustrating and describing animals for study and entertainment in a purely secular, scientific manner,[161] the medieval bestiaries often have these illustrations formatted around biblical references, scenes, or iconography; they also include fully-fledged biblical scenes of creation, and the text accompanying illustrations is often concerned with biblical or religious themes.[162] The religiosity of these texts and their tendency to expressionism are almost covariant. The deviation from realism is due in part to ecclesiastical concerns over the possibility of idolatry arising from depictions placed in religious texts,[163] the impiety implicit in an artist's perceived attempt to imitate or even better the creations of the Christian God himself,[164] or adherence amongst the clergy to a view that the vernacular definition of beauty should transcend (and perhaps, eschew) materialism almost to the point of exclusion, emphasising intangible spiritual qualities instead.[165] It is not until the 15th century that illustrations in a strictly realist style, not only of animals, but also of man-made constructions, begin to take prominence - most likely inspired by surviving Byzantine and Classical texts that came to be rediscovered in the path towards the Renaissance.[166]

Prohibitions on imitating the divine in illustration were not shared in the fields of engineering, construction, and architecture, however. Despite the Church's characteristic disdain for sciences that undermined the primacy of the ecclesiarchy and its canon, the fields of mathematics and geometry were not considered scandalous - rather, in the sense of "sacred geometry",[167] they enjoyed a certain proximity to divinity. They were given special dispensation for practice through skilled trades, particularly inasmuch as they served a religious aim in the glorification of the divine. However, engineering knowledge and ability was more thinly spread amongst the clergy than that pertaining to liturgy, as these practices remained primarily the reserve of private artisans. This is not to say that monastic brotherhoods were at all incapable of conducting their own architectural design and masonry, as they were perfectly capable of flat floor plan projections on a precise scale as early as the 9th century,[168] and Benedectine monks were successfully executing these plans as

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[161] Examples of bestiaries found in Europe often contain animals not endemic to the continent, and attempt to catalogue those that would not be commonly available to see first-hand but were known to be kept at the courts of rulers in the Middle East, Asia, and Africa. Multiple examples exist of European monarchs who attempted to do the same by importing and keeping various animals of interest (Cuttler, 1991: 163).
[162] The most famous example perhaps being the Ashmole Bestiary (MS Ashmole 1151).
[163] Hassig, 1990: 142.
[164] *ibid.* 144.
[165] *ibid.* 145.
[166] Cuttler, 1991: 167.
[167] Not unlike the proximity of geometry to the inherent godliness of the universe in the imagination of Hellenistic science; see Euclidean Geometry, chapter 3.
[168] Reginbert, Cod. Sang. 1092.

early as the 12th - though not without difficulty. Surviving records show disputes between the brothers instructed to do the labour and the bishopry; the chief concerns being the physical scale of the grueling projects to be undertaken by a group of (mostly) intellectuals, and the interference of this work with their regular duties, which normally included a full itinerary that encompassed not only technically superfluous community services and religious observations, but also tasks essential to the survival of the monastery.[169]

It is no coincidence that secretive fraternities aimed at execution and preservation of engineering skills and knowledge rose to prominence in parallel with organised Christendom, and experienced heightened tension with the latter as their proximity to divine knowledge - mixed with that same self-preservationist secretiveness - was liable to attract great suspicion along the lines of possible heretical practice, or withholding of privileged knowledge that might allow them to undermine the authority of the Church. On a more pedestrian level, the protection of skills through the system of apprenticeship, coupled with the regulatory and bargaining power of organised unions or guilds amongst skilled trades, allowed for these organisations to bargain equitably with the Church while maintaining the secrecy necessary to preserve their market position. A prominent example of these norms in action is the murder of an 11th century bishop by a master stonemason, following the bishop's successful attempt to elicit protected trade secrets regarding the foundation layout for cathedrals through bribery of a tradesman's apprentice.[170] Despite funding for grand projects being put forward both by the Church and by private or noble donors, the latter produced fewer surviving manuscripts when it comes to private-use buildings. Extant manuscripts largely feature religious buildings, and this is most likely due to the fact that they are preserved by the same religious *scriptoria* mentioned above. Meanwhile, work carried out by artisanal societies was necessarily secretive and their scripts more jealously guarded; or, more readily discarded, to prevent leaking of trade secrets.

One of the earliest manuscript examples from one of these technically skilled tradesmen is the 13th-century sketchbook of Villard de Honnecourt. This collection of parchments covers a jumbled variety of unrelated topics and is clearly intended for personal reference rather than publication. Despite the frenetic organisation and informality of the document, many of the drawings demonstrate accuracy in, and a familiarity with, scale and projection that is missing from the

[169] Horn & Born (1986: 18) make a very good appraisal of records of labour disputes between the bishopry and the brothers of the monastery called upon to carry out the work at St. Gall.
[170] Schwartz & Bok, 1990: 142.

Benedictine examples. To make a fair comparison, a cathedral floor plan penned by Honnecourt - in the margins of an unrelated piece which dominates the page - shows a far more precise scale, along with the addition of wall depth, columns, and reinforcements. It also appears to be accompanied by a 'setting-out' site plan for foundation digging,[171] information adjacent to that which resulted in the murder mentioned above.

The simplest explanation for the poverty of technical accuracy in manuals copied from the ancients, therefore, is that any accurately projected drawings that were extant at the time of copying were disregarded and omitted. Christian monastic orders, the primary inheritors of manuscripts of all kinds, were not sufficiently well-versed in the precepts of mechanical engineering or drafting to recreate missing three-dimensional projections or component-level drawings very well, or at all; and the copying of partial or damaged ones must have been challenging as well. The few examples of those who might have been able to do this, like the Benedictine brothers described above, were limited in their ability. Where correction and supplementing of drawings, variables, or measurements were required, their level of proficiency with the required ancient languages prevented them from making these changes in successive copies of manuscripts, as it appears that a limited number of scriptorial monks held linguistic proficiency beyond copying letters. It is from this that the near-mimetic phrase "Graecum est, non legitur" found its way into manuscripts copied under the auspices of the monks from the Western (i.e Latin) wing of Christendom. Ironically, this is mirrored in texts preserved by scribes of the Greek tradition in Eastern Europe, when working with Latin texts.[172]

Manuscripts that contain Biton are thought to stem from several common ancestor texts, mostly from the Byzantine tradition of *poliorcetica*. Byzantine scholars of this period are known particularly for their texts on siege and warfare in general, and it is by them that many concomitant ancient Greek texts survive in thematically consistent collections of works by disparate authors. Despite this level of sharpened focus, and particular familiarity with scholarly Greek by their place in the Eastern wing of the Christian Church, a number of illustrative and textual corruptions occur through this line of the manuscript tradition; not least by the conversion of an earlier Ionic Greek version of the treatise to a Byzantine version, evidenced by the discovery of manuscripts from parallel traditions that do still retain these inflections.[173] Further corruption by way of these

[171] de Honnecourt, MS Fr 19093: 14v.
[172] Troje, 1971: 292.
[173] See Chapter 6: Treatise.

manuscripts being lost, damaged, or falling into the hands of Latinist scholars whose copies may have suffered from the aforementioned lack of familiarity with Greek, particularly in the scattering of manuscripts during and after the events of the Crusades and Ottoman expansion, is a distinct possibility for some of the errors and omissions in Biton. Scribal attempts to reverse this corruption of the manuscripts may, too, cause further issues of accuracy and legibility.[174]

In the Latin tradition, the few scribes that did have such linguistic proficiency were called upon to perform 'higher tasks' in copying and translating scripture.[175] It is fair to say, then, that technical manuals such as Biton's may have been relegated to less experienced, less skilled, and less attentive members of the order - particularly one such as this which contains designs that, to the medieval siege engineer who was already well-versed in counterweight weapon technology,[176] were not only defunct but also less effective and more costly to execute than any other 'make-do' methods of artillery that would have been available to them. The same reason may account for the fact that errors were not found - or perhaps were ignored or dismissed as not worth wasting more parchment on - by the *correctores* whose task it was to proofread the work of the scribes. Thus, the textual components of copied treatises suffered accelerated incremental degradation between copies. Finally, the specialist scholars and tradespeople best equipped to interpret, correct, translate, and reproduce these technical drawings were of a discrete (though related) array of fraternities who were characterised by their isolation, and by the protected nature of their rites and writings. A manuscript as obscure as Biton's would necessarily be difficult for them to procure, and would need to make its way into the hands of a specialist well-versed in Ancient Greek, in a further moment of serendipity.

[174] Burney MS 69: f.361r has, in a procurement note by a 19th. C. CE. auction house, mention of its provenance via a Greek copyist called '*Porion'*; however, the condition of its text of Biton is not very greatly improved over other versions of the same manuscript lineage.
[175] Horn & Born, 1986: 33.
[176] Most notably, the trebuchet.

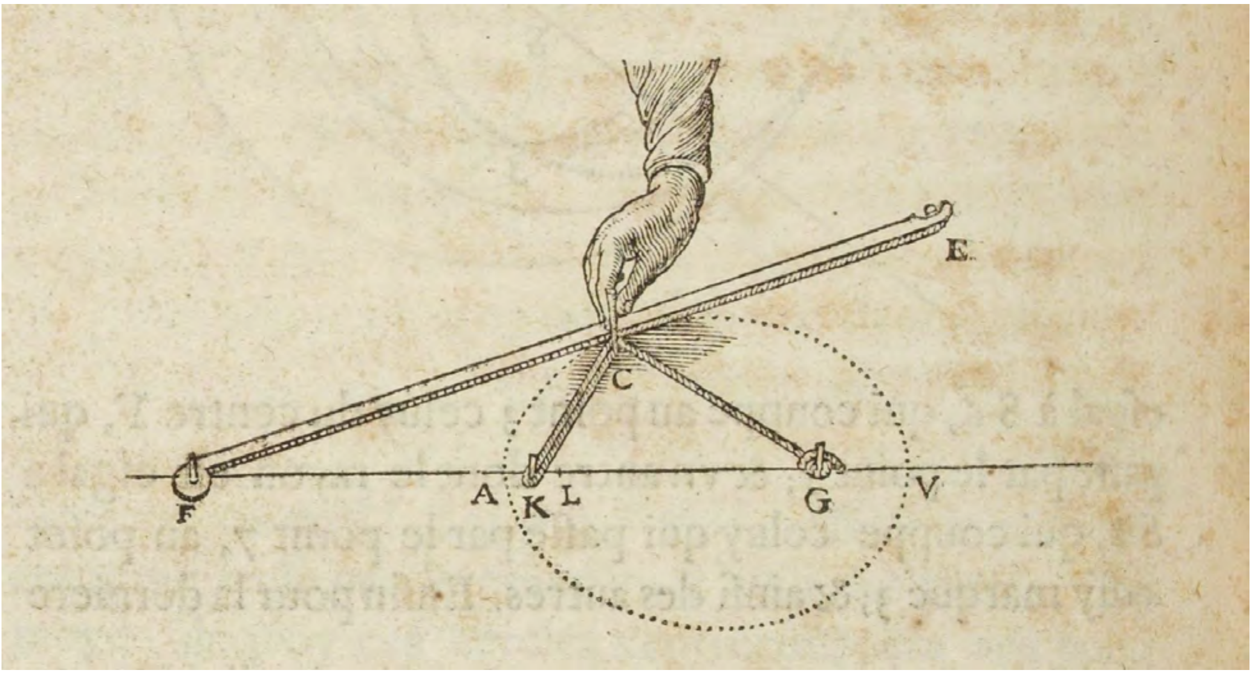
4. EUCLIDEAN GEOMETRY

For Biton and his contemporaries, the primary means of understanding and communicating elements in the world of two- and three-dimensional space was through use of Euclidean geometry. A significant hurdle to any fresh attempt to study these texts is the fact that they are best understood with prior grounding in this, the engineers' own frame of understanding. I find it necessary to establish the most pertinent cornerstones of this framework on account of the fact that the significance of the precedents set by Euclid in terms of his methods, and communication of concepts, is severely understated or even ignored by scholars of siege engineering. They are usually, at best, mentioned in passing. This makes deciphering the meaning of these obtuse texts a needlessly punishing task, and as such, the topic deserves some exploration.

Euclid of Alexandria (fl. c. 300 BCE) is responsible for a number of treatises that are extant, and may well also be responsible for a number of others that have not survived but are positively attributed to him. These treatises cover a wide variety of topics, but only two need be visited for the purposes of studying siege manuals. The first is the *Elements*, which contains the Euclidean theory of geometry which informs Biton's principles of design and construction. The second is the *Optics*, which may shed light on the possible content of Biton's own now-missing work of a similar name, which he mentions in his extant treatise.[177] Specifically, it may enlighten us as to some of the principles he may have been familiar with as regards proportion in mechanical design, and the principles required for backwards-engineering siege engines from purpose (that is, from the interaction between observable geometry of a landscape, and the performance of the desired payload).

To best understand the *Elements* as it pertains to the problems of reading engineering treatises, it is most useful to approach it by situating it within the greater framework of the history of the understanding of geometry and the built environment. The rules and methods described by Euclid are practiced almost exclusively on a two-dimensional plane, and are intended to be carried out with only a straight-edge (simply an unmarked ruler) and a compass.[178] As we have explored above, the realm in which his constructions exist is one where the relationships between geometric elements are dictated by mechanical constraints between one another.

[177] See discussion of Chapter 5: Treatise.
[178] "La reigle & le compas" in Descartes (*Discours de la méthode* f. 381).



Example of a straight-edge and compass (the latter substituted here with dowels and string) in a uniform two-dimensional area from Descartes (Discours de la méthode, La Géométrie Livre 2, f. 356). The method used is largely indistinguishable from that of Euclid.

The innovations of Descartes allowed for these constraints to be expressed as algebraic equations, in a manner that would lay the foundation for Isaac Newton's calculus. As each geometric element in the drawing could be assigned an and value, their position and entire movement throughout the drawing can be plotted using one brief equation. Furthermore, more complex curves - like parabolas - could be precisely described, without needing to clumsily express them as sections of another object.[179] For example, in a treatise dedicated to calculating the area of a parabola, Archimedes of Syracuse (c. 287-212 BCE) dissects a parabola with progressively smaller triangles to find an approximate number. This treatise is loosely based on a work by Euclid, now lost, from which the first three propositions are directly derived.[180] However, even though Archimedes' work is most likely a considerable furtherance of Euclid's, he does not provide a method for drawing a parabola, other than describing it as a section or sliver of a conoid (three-dimensional) shape. The definitions for these geometric elements and the relationships between them - lines, triangles, circles, parallelism, perpendicularity, and so forth - remained largely unchanged from Euclid to Descartes, in that each object or relation was defined in terms of *axioms*. The desire to explain geometric

[179] *ibid.* 331.
[180] Archim. *Quad.* Prop. 1-3.

constraints from first principles in this way is characteristic of Greek engineers, and it is essential to all following theories of geometry that attempt to create a unifying thesis for the underlying structure of the observable universe. The idiosyncrasies of Greek philosophy and language are stubbornly imprinted in these theses, and therefore transfer these idiosyncrasies to writing of the kind in siege manuals.

Euclid's theory of geometry in the *Elements* is, therefore, *axiomatic* - that is, it is constructed entirely within the constraints set by a number of statements, or rules, that are held to be true in all circumstances, apart from those which incorporate special modifiers which take effect where specific conditions are met. These statements of logical truths, expressed in the natural language[181] of contemporary Greek, stipulate the various possible relationships between geometric formations, beginning with definitions of a point and straight line and culminating in more complex descriptions such as that of the relationship between parallel lines, angles within a triangle, and so on.[182] These possible relationships are in turn underwritten by asserted truths about the tangible physical world that are seen to be self-evident, in the spirit of Aristotle's *Posterior Analytics*.[183] Thus, in order to avoid having to stipulate a regressive spiral of rules and supporting rules, each of which might require demonstrative proofs,[184] Euclid's axioms are simplistic by design - and assume rigid limitations on observable geometry that a modern mathematician might consider reductive or even totally unrepresentative of observable phenomena.

4.1 A NOTE ON DIAGRAMS

Euclid's axioms do not appear to make much sense - or any at all – at first reading, without reference to diagrams. Similarly to Aristotle's position that some things are obvious and can be seen, seeing the output of a logical equation in action can allow one to disqualify erroneous output - for example, outliers on a graph. This is much like if Plato were to show his students, in addition to his definition of man, a monkey and say, 'a man is also much like this, but not'.[185] This is in turn almost indistinguishable from the basic logic of a machine learning algorithm, using related, observable

[181] Natural language as defined in the field of Linguistics; that is, any spoken language whose vocabulary and rules have come to exist organically. (Lyons, 1991: 68).
[182] Euc. *Elements* I, Post. 5, pertains to parallel lines and is perhaps most infamous - it will be explored below.
[183] Arist. *An. Pos.* I.1
[184] A mathematical proof, defined as "one in which all of the rules that can be used are set out in advance so carefully as to leave no room for interpretation or subjectivity, and in which each step of the proof uses one of these rules" (Miller, 2007: 2).
[185] Dio. Laert. *Lives* IV.40. See Appendix 1: Understanding Treatises by Geometry.

data to act as heuristics for disqualifying false syllogisms.[186] This would otherwise need to be programmed-in manually, as Plato did when he added the qualifier of 'broad, flat nails' to his definition for a man. However, this still does not fully satisfy requirements of a reliable algorithm, as Diogenes might then bring a monkey to the academy and propose that that, too, were a man. Thus, 'like a monkey, but not a monkey' combined with 'like Plato, but not only Plato' could provide a complex set of criteria in otherwise small packages of logic. Naturally, this does not quite satisfy the level of "first-orderedness" that we would ideally like to have; and given the many steps and rules that are glossed-over in this comparative and deductive logic, there are many errors that might come out of it.

Nevertheless, it remains that Euclidean diagrams, and by extension, the diagrams of engineers who studied Euclid, function as a kind of logic or logic assistant for the purpose of heuristics. This might have been the nature of Biton's original diagrams for us, which would have been in place of those medieval artists' representations of the machine as a whole, and may have been far less artistic and far more technical in style.

4.2 CRITIQUES OF EUCLIDEAN GEOMETRY

A highly visible result of the problematic relationship of Euclid's legacy with mathematical theory is that, when embarking on a study of Euclid with principles of real-world design application in mind, the Humanities student is unavoidably greeted by a gargantuan corpus of scholarship from the direction of our colleagues in mathematics and philosophy. This collection of books and publications often cite such ominous problems with Euclid's axioms as "distance errors", plagued by the "curse of dimensionality",[187] and much ado is made about how Euclid is to be 'fixed'.[188] The size and scope of this literature can be worrisome to a Classicist in particular, who may not have the grounding in mathematics that would allow them to follow the proofs and argumentation in this body of work. Little, if anything, has been published in an attempt to explain the source of the controversy in a manner that is understandable to Classical Studies students, which might alleviate

[186] See Appendix 1: Understanding Treatises by Geometry.
[187] Aggarwal *et al*. (2001: 2) uses both these terms judiciously.
[188] A long-standing nuisance that endures today; though he may not be mentioned or referred to as explicitly as in earlier work on the subject (e.g. Daus' (1960) paper *Why and how we should correct the mistakes of Euclid*), the theoretical bones of Euclid continue to be tossed around under other names as they pertain to problems derived iteratively from calculus logic that incorporates Euclidean axioms (e.g. *On the Surprising Behavior of Distance Metrics in High Dimensional Space* by Aggarwal *et al*. (2001)). This is not to say that first-hand work on Euclid has abated; for example, Miller's (2007) work on better accounting for Euclidean logic by addressing the ways in which it intersects with the inferential qualities of his diagrams.

these worries and provide a springboard for those who wish to investigate the relationship between Euclid's writings and their interpretation and implementation, both in the ancient context and today. A simple explanation of the controversy - however brief - is useful not only to our study of the application of Euclid in Biton, but also to demonstrate the merit of the work of scholars of ancient languages, whose ongoing philological work in communicating an accurate translation of the implicit logic of these texts has provided the basis for the advanced logics of calculus, and the algorithm logic of computer programming and machine learning. This discussion has been placed in an addendum,[189] which may be read in connection with some of the concepts to which I shall refer.

4.3 RELEVANCE TO THE *POLIORCETICS*

How do these observations affect our study of Biton and his contemporaries? On the macro-level, we can certainly see that mathematics, and the philosophy thereof, influences how the artifacts of these engineers are noted down, represented in textual description and drawings, and how their descriptions are thus communicated for reproduction elsewhere. The efficiency with which we can interpret their writing depends highly on whether we are familiar with the principles that the ancient writer took for granted as common knowledge in his reader. This condition is no different from the level of familiarity one is expected to have, say, when reading a manufacturer's factory service manual that is intended for technicians trained by that manufacturer.

On the other hand, it is also true that there are deficiencies in this theoretical background that frustrate the reader for reasons other than mere unfamiliarity. There are limitations to the ability of ancient Greek engineers to communicate their ideas or plans effectively, which are in turn imposed by the limitations of the peculiar models of understanding that they apply to physics, and the relatively rudimentary tools and expressions available in their theory of geometry. The result is that the textual descriptions we are left with are understandably very odd in their turn of phrase and in the imprecision of their descriptions. They also lack specialised terms for particular components, and are forced to use approximate terms which can become muddled or seem vague in context.[190]

On the level of specificities, it is clear that the precedents set by Euclid and other geometers of his school are what informed this style of writing, and the style used to communicate three-dimensional ideas through natural language. The tradition of Aristotelian terms provides the

[189] See Appendix 1: Understanding Treatises by Geometry.
[190] Hacker (1968: 41) explores some of these terms, which at the time of Nossov (2005: 133) are still unclear; we will also investigate some of Biton's similarly unclear terms.

template for the axiomatic definitions of theory, which in turn imparts certain limitations to understanding. Axiomatic theory and Aristotelian logic constructions have their problems, therefore, and criticism of them is valid. Iterating them led to many centuries of building on false assumptions mistakenly taken as universally true, and translating them from natural language, though it removes confusion, does not shake off the faulty logic or assumptions that they carry. Argument from first principles using algebraic operators in current Set Theory now strives, in a sense, to include considerations for the underlying structure of the universe itself in a manner that Greek philosophers might have approved of. Recursive definitions are, nevertheless, sadly necessary.

As we can see by the evolution of geometric descriptions in Ancient Greece to algebra in the work of Descartes, Euclid's postulatory style on its own does not allow for curves to be calculated and mathematically notated, let alone to any fine level of resolution. This is largely due to their existence on a non-notated plane, and their expression through natural language. As such, complex curves and parabolas are not described exhaustively or at all. The exception to this rule is any curve that happens to be describable as a section of the radius of a circle, which still forecloses on the possibility of describing a progressive curve, like a hyperbolic function or a parabola. It is conceivable that a relatively accurate parabola could be described in the manner of Archimedes – as a section of a cone – but this cone would have to be described in three dimensions, and the particular section from which the parabola is to be drawn must be described in terms of a straight line drawn across two points on the larger circular termination of the cone. From there, the reader might slice a cone and transfer it to paper, so as to trace their parabola to their construction drawing or workpiece.

This is entirely too laborious to consider for most constructions, and the absence of this method from engineering descriptions might be indicative that a similar sentiment was felt by Greek engineers. The lack of a coordinate system or sufficiently complex algebra therefore prevents engineers like Biton from making their descriptions more word-efficient, and requires that they rely on diagrammatic accompaniment for the reader to make sense of their step-by-step, point-by-point construction of a complex design. Conversely, those diagrams are rendered less effective in their demonstrative value by the lack of a coordinate system or other supporting theory to make their message clear. Without this accompaniment – or with it sufficiently corrupted – a vital heuristic is removed from the text.[191] The precise curves of a palintone bow, for example, are to be made according to the reader's interpretation of diagrams, first-hand experience of similar bows, and trial-and-error testing, rather than precise geometric constraints.[192]

The Greek emphasis on proportionality, as exemplified by Philon of Byzantium, informs a spirit of relativity that seems to be evident throughout their engineering texts. Standardisation in this discipline of engineering is unlike ours, in that the only standardised concepts were those theories of geometry and physics as they were expressed by earlier writers, who are in turn referenced often enough by their successors that they appear to have been 'required reading' for the budding engineer. Measurements in the text that are not deemed absolutely vital are variable and estimable. The only proviso was that that measurement be kept consistent throughout one construction, and that all measurements are kept within proportion.[193] Compare this practice with those standards we are familiar with,[194] where every measurement – whether a millimetre or an inch – is governed by international standards. The all-important rule of proportionality also circles us back to the problems of Euclidean distance, and the specific problem of scaling Biton's machine for the purposes of studying its usefulness in the context of a siege at Pergamon.

The outward relevance of planar distortion and problems of high-dimension data science may seem thin. After all, it goes without saying that Euclidean vectors could not take factors like distortion of distance and straight lines through space and gravity into account, in that they predate the requisite advances in the theory of space-time by more than two millennia. That said, the visible effects of distortion by distance and distortion were plainly seen and accounted for in civil engineering, and in dioptric treatises like Euclid's *Optics*. The Parthenon at Athens, for example, has its floor convexly structured in a manner that is equitable with the visual distortion of a straight line at range along the curvature of the earth, concordant with our observations of how a line drawn along a Euclidean plane behaves in reality. The effect that is created for the distant observer is that the foundation of the Parthenon appears to be uncannily straight indeed.[195] Euclid himself appears to be aware of this

[191] See Appendix 1: Understanding Treatises by Geometry, as well as Crippa's (2009) *Inferential Role of Euclidean Diagrams*.
[192] See for example the comically rounded curves of the palintone bow accompanying Biton's text in Burney MS 69: f.10v.
[193] Philon, *Bel*. 56 describes how, for each construction, a yardstick must be made, simply to ensure inward consistency.
[194] The very subject of proportionality in measurement and in the Greek theory of machinery as a whole is worth a formal comparison with modern engineering standards.
[195] Vincent, 1974: 76.

distortion in his theory of proportions and perspective, which predates and informs the theory used in Vitruvius' instructions on ensuring the correct appearances are maintained for the upper façade of a temple. Similar to the convex floor, the trick Vitruvius details is one that involves angling the façade downwards towards the viewer, such that it is no longer plumb and square with the building, but does appear to be perfectly square as it is positioned at a point of direct perpendicularity to the eyes of the viewer.[196] Why, then, the discord between observation and practice, and theory?

The Aristotelian conception of physics dismisses, with the theoretical equivalent of a hand-wave, the tendency of matter to form around a sphere.[197] Knowledge of the spherical shape of our planet had already had proofs formulated for it, and was commonly accepted as commensurate with the precedents set by Aristotle's ideas of the structure of the universe.[198] However, this knowledge was not to find its logical conclusion in the conception of a non-Euclidean plane in antiquity. This is perhaps partly due to the semi-sacred status Euclid's geometry occupied, such that his many commentators and reviewers largely accepted his axioms, apart from minor emendations and commentaries, even as late as Pappus of Alexandria, active around the 4th C. CE.[199] His axioms satisfy the quintessential requirement of reduction to a Platonic Form level of simplicity, to the extent that Epicurean detractors were said to have labeled at least one axiom as obvious even to a donkey.[200] The apparent need for ancient engineers and architects to flout the rules of this geometry in order to satisfactorily accomplish their work was, it seems, not sufficient evidence for them to prompt an exhaustive revision of those rules in antiquity. The possible reasons for this shall be explored below.

The closeness of Euclid to current issues of data ordering and manipulation[201] has implications outside of the body of literature and proofs that I have attempted to summarise here. I do maintain that being cognizant of the relevance of ancient logic to current affairs in other subjects like data science, mathematics, and programming is useful on its own, and it is just as useful to be similarly well-versed in the underlying mathematics that make it so troublesome for scholars in those fields.

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[196] Vitr. *De Arch*. 3.5.
[197] Arist. *Ph.* 194a.
[198] Kahn (1960: 115-8) investigates the origin of the understanding as the earth as a sphere. Diog. Laert. *Lives* 8.48 & 9.21-2 attributes it to Parmenides via Theophrastus. Dicks (1970: 72-73) traces it ultimately to Pythagoras.
[199] Pappus reviewed the parallel postulate in an effort to simplify it, but did not seek to fundamentally change it at all. See Clapham & Nicholson (2009).
[200] Procl. *Comm.* 251.
[201] See also Appendix 1: Understanding Treatises by Geometry.

More importantly, however, I believe this is the perfect point of entry for addressing what appears to be a blind spot in the methodology and practice of experimental archaeology as it has been performed since the overturning of Euclidean fundamentals, and specifically with regard to scaling and proportion in ancient devices.

4.5 SCALING OF SIEGE WEAPONRY

Philon of Byzantium, amongst others,[202] provided a theorem for scaling torsion engines according to the desired projectile weight (in the case of a stone-throwing palintone), or the length (for an arrow-shooting euthytone).[203] The question posed was, essentially, as follows: if a given engine throws a projectile of weight or length to distance, how should the construction of the engine be scaled up to throw a projectile of (2·) to the same (^{1}) distance?[204] It was posited that increasing the volumetric size of the engine's various components - and doubling the volume of the sinew-spring bundle which lent its force to the projectile, in particular - would in turn double the maximum projectile size for a given target range. By judging a variety of spring volumes in relation to shot weights, a highly approximate relationship was found, and a formula derived from it, which remains remarkable for being both an early organically-derived algebraic formula as well as a basic logarithm (before the algebraic theory for log tables had been devised). The variable that was used as a shorthand reference to a spring's overall volume, and therefore power, was its diameter. Every other component of the engine was measured and plotted from the calculated "spring-hole" diameter through which that spring would be loaded into the frame, and so the diameter would correspond to the overall volume[205] in a predictable way. The latent assumption was that the performance of an engine scaled more-or-less *linearly* with the *volume* of its motive power, and that its frame components should necessarily do so as well.

The first step was to use the theorem of two mean proportionals[206] to calculate a cube root for the weight or length variable that one wished to use. Without the ability to compute complex numerals, finding a root number is a tricky problem, and finding a root of that, too, is even more so. A highly approximate method was deduced from geometric construction; with this being ever the more troublesome on a Euclidean (that is, pre-Cartesian) plane, where neither complex algebra nor

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[202] Philon, *Bel.* 51 is a revised version of Heron, *Bel*. 113.
[203] See Glossary.
[204] Philon, *Bel.* 51.
[205] Of this, essentially, cylindrical bundle.
[206] Philon, *Bel*. 52.

points can be used to streamline the process. Rather, the known measurements of the construction must be manipulated using a combination of drawn shapes, axiomatic rules, and simple arithmetic. Philon mentions one particular part of this process which happens to involve bouncing a ruler with a loose hand around the table on which the construction is plotted.[207] The solution to his first step, therefore, was already somewhat approximate and therefore prone to some margin of error.[208] Nevertheless, this root number was then multiplied or divided by a constant to find the spring diameter;[209] and from this point, multiple diameters could be calculated for different ammunition, and engineers assembled their own quick-reference tables for these figures. This method of scaling may have worked as a very rough rule-of-thumb for ensuring that one's catapult was at least nominally capable of what one wished it to do, but was by no means perfect. Coupled with some level of observation and trial and error, this formula's shortcomings seem to have been recognised and accounted for between Heron, Philon, and eventually Vitruvius, each of whom tweaked the numbers in their quick-reference tables to stray from the precise value that would otherwise be produced by the formula. [210] Marsden theorises that these reference tables simply gave the point at which the economy of scale drops off, and diminishing returns set in for each weight-class of projectile; and that the bracketed scaling system is an attempt to economise on production and performance.[211] I disagree here, as there is no evidence for the use of any kind of stringent multivariable, control-protected scientific method that would render these results in the way Marsden claims. Rather, an observed impact point that was within an approximate area of the forecast performance according to Philon's principles may well have been deemed 'good enough' to the engineer. The tables and their erratic adjustments are an indication of the imprecision with respect to placing a weight and spring size into a 'good enough' range; not a reflection of the precision of an imagined set of field tests, and regimented assessments. Certainly, the scientific method was in its infancy at this time, and evidence of the employment of the hypothesis and experimentation model that we might traditionally associate with the Greeks is thin on the ground when reading ancient sources like Heron.[212] It is also important to note that, for most situations,

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[207] *ibid*. 52.

[208] *ibid*. 51. The procedure is well demonstrated in an eminently readable format by Coxeter & van de Craats (1993). As it does not pertain directly to the point I would like to make, I will not describe the cube formula in as much detail as others I have mentioned. However, it is of relevance to note that 'doubling' the cube is a bit of a misnomer for the mathematics at work.

[209] See graphs below for this formula in action.

[210] Vitr. *De Arch* 10.11.3 and Heron, *Bel.* 113.

[211] Marsden, 1969: 38.

[212] For example, the water tank and siphon arrangement from the *Pneumatica* which Heron proves incorrect; Landels (1978: 192-193) agrees to the rarity of this level of scrutiny. *Methodos* is mentioned many times by Philon, but really only in terms of the construction, not the testing of weapons: *Bel.* 50.15ff: "it was necessary

these flawed calculations may have been just close enough to appear reliably true. The use of principles in Euclid's *Optics* appears to have been used to gauge the distance of a besieged city's walls, or – if the distance were known – the height of those walls, such that an engineer could then select the appropriate ammunition size and corresponding engine scale to be within an acceptable margin of range and efficacy.[213] That acceptable margin was evidently good enough to ultimately land a shot close enough to that target such that minor adjustments of position and equipment configuration would be enough to correct for any imprecision.[214] However, this margin of error was most likely much greater than we might otherwise assume.

Referring to incremental errors that impact performance in siege engines, Philon relates the story of Polykleitos the sculptor.[215] The gist of this anecdote is that small discrepancies from the design and production processes add up to larger shortcomings in ultimate performance. Of course, a margin of error did exist in the sense of the 'Polykleitos' phenomenon. But there was another in the mathematical method for carrying out the formula, and yet another was present in the assumption of the possibility of linear scaling of material properties by volume. The Polykleitos story, therefore, seems to me to be a stubborn echo of the civil engineer's insistence that the human eye was the source of problems of distortion. In a curious mirroring of the clash of theory with observable results in another field, the military engineer was placing the responsibility for variance in outcome on cumulative errors in the human inability to attain a godly standard of mathematical purity, when instead the mathematical theory was, largely, the source of any cumulative errors that might have led to the issue at hand. If the testing procedure found that a relatively tight grouping of shots was recorded at a distance point that is within that approximate zone, then that tight grouping would only better serve to rule out the possibility in the engineer's mind of any idiosyncrasies in the machine or the operator, which might have otherwise indicated faulty calculations. In other words, the reliability of the machine's power delivery is not in question here so much as its power delivery relative to its size. Note, here, that I do not mean to imply that engineers of the calibre of Heron or Philon might have intentionally fudged their numbers. Nor do I imply that they were content with

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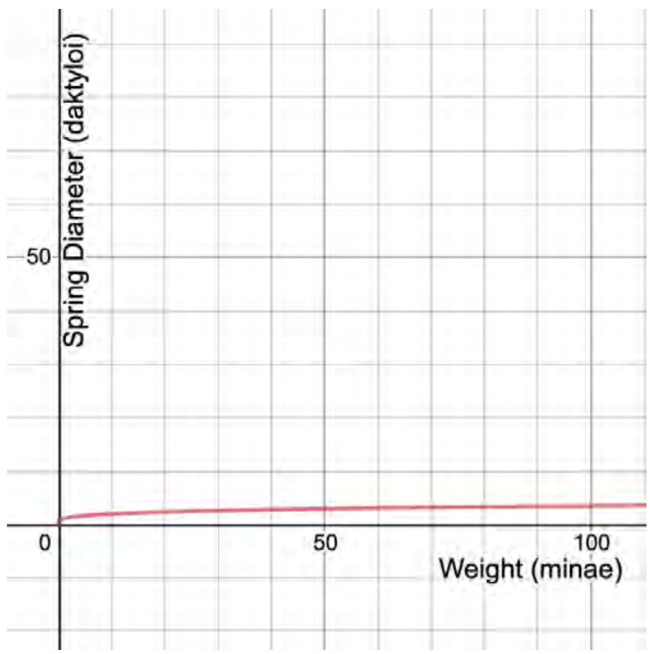
for this to be grasped not by chance or at random, but by a fixed method"; 52.21ff: "This too must not be drawn at random, but by a method". Marsden (1969: 38) speculates about how that experimentation might have been performed, but his ideas are entirely conjectural.

213 Vincent, 1858: 348.

214 See Chapter 2: Historical Context re. siegeworks.

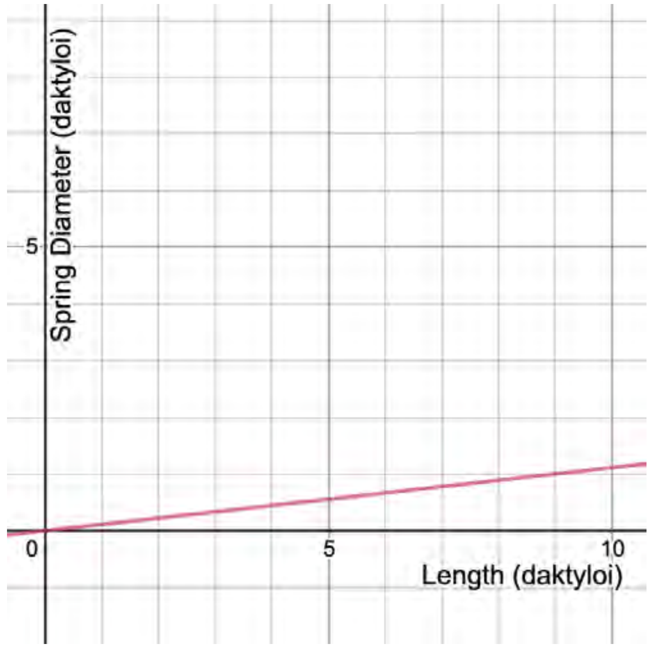
215 Philon, *Bel*. 51.

approximate results, as opposed to repeatable pinpoint accuracy.[216] However, I do suggest that attaining a range within one standard deviation of Philon's predictions would have certainly been good enough to serve as proof to the Greek engineer that the golden, Platonic ideal embodied in the mathematics was true, and that any shortcomings of the machine were likely human rather than built into the design, let alone present in the notion of linear scaling by volume. In other words, the ancient engineer may have been completely unaware of the margin of error built into the principles underlying the mathematics of the calculations, and simply chalked up discrepancies in the field to errors in their own implementation.



Philon's formula for calculating spring diameter for a stone-throwing palintone engine, graphed. Spring diameter in daktyloi *( ) is expressed as a function of the projectile weight ( ) in* minae*. The curve may be described as (* $= 1.1\sqrt[3]{\ }$ *). Graphing calculator for educational use provided by Desmos:* https://www.desmos.com/calculator/yfe4i548gv

[216] Within reason, as machines required constant readjustment of springs, angle, pullback, etc. as parts wore and other variables like temperature and humidity fluctuations affected performance. However, this wear was, following Heron (via Philon), 'resettable': Philon, *Bel.* 61.



Philon's formula for calculating spring diameter for an arrow-shooting euthytone engine, graphed. Spring diameter in daktyloi *( ) is expressed as a function of the projectile length ( ), also in* daktyloi*. The line may be described as ($= \frac{}{9}$) . Graphing calculator for educational use provided by Desmos:*
https://www.desmos.com/calculator/wyiiff8jwl

The formula for an arrow or bolt engine's spring results in a straight line when graphed. An example of this concept in action at a very elementary level is in the conversion of Celsius temperature to Fahrenheit. Unlike a straight multiplication or division conversion, like that of inches to centimetres, the formula for Fahrenheit requires some additional arithmetic to account for its equivalent zero-point in Celsius being 32°F.[217] If their degree values were plotted on a Cartesian plane, therefore, the lines for Celsius and Fahrenheit would diverge as they both increased, but each remains a straight line. The ascension of Fahrenheit in comparison with Celsius is, therefore, an essentially linear one.

The initial sharp upswing of the weight curve for a stone-throwing engine demonstrates that the ancient engineer was aware that overcoming the inertia of a heavy stone requires a great step up in power. Naturally, the initial torque required to launch a stone will be much greater than that of a light, aerodynamic bolt. Similarly, the flattening of the curve after that may indicate that he was aware that diminishing returns occur after that initial struggle of physics, such that only a much smaller increase in spring diameter relative to weight is required after this point. However, the

[217] C=59×(F-32).

curve function from the point of about 10 *minae* onwards is so flat - or, rather, linear - that it is strikingly close to an approximate straight-line function of the order of around ($= (\frac{}{50}) + 2$) until it reaches the point of about 100 *minae*, or (very) roughly, one and a half *talents*. This is where I have cut off the graph, as it represents a tapering-off of the more common of the large anti-matériel engines of the period. There are multiple reasons why this relatively flat predictive line does not seem to mirror the reality of material scalability (to be addressed in the next section).

In modern recreated engines, experimental archaeologists have been unable to reliably reach the predicted ranges, reaching inconclusive and unsatisfying findings that are not useful in illuminating the historical record apart from proving the tractability of a basic engine layout.[218] Scholars have attempted to explain this by way of a variety of theories. One explanation might be that we are simply not as accomplished in our carpentry, or selection or seasoning of timber, to build a machine as efficient and as well-balanced as our ancient counterparts; in other words, we fail to meet Polykleitos' standards.[219] Another might be that the cumulative errors in measurement conversions are as regionally approximate as one might expect for an ancient stadion or a talent.

There are many factors that affected the ancients' projectile weapon performance that they were aware of; the rest, they appear to have attributed to human error. We make the same mistake when analysing the Biton problem, by presuming to measure and compare engines – including those that are non-torsion - by shot weight, which is an unreliable and, frankly, misleading metric, as we do not have any reliable measurement of exact range per bracket of spring diameters. Marsden and Schramm make this error in their assessment and recreation of Biton's recommendations,[220] and it presents a problem for us where we wish to match Pergamon's likely original engines with a suitable replacement from Biton. Biton, too, mentions that scale can be used to make an engine fit for the task at hand, rendering shot weight a dependent, rather than independent, variable. Thus, recommended shot weight alone does not tell us whether machines are comparable – instead, a dimension that cuts across all others is required in order to tell us whether engines are comparable. From there, we can make a reliable estimate of the scale of engine that would need to be deployed by Biton for it to be competitive – and therefore, allow it to be placed – in the time period we have

[218] Marsden (1969: 86) made some such reconstructions; Schramm (1918: 27ff) published an exhaustive volume with them and noted the range results for each machine.
[219] Hassall, 1998: 23.
[220] See Chapter 6: The Treatise.

put forward. Dimensional analysis can cut through these extraneous variables and faulty assumptions, and has other benefits to this field of study as well.

It should go without saying that scaling the proportions of any machine or structure should not reliably result in an identical scaling of the work it performs. Unlike a relatively simple, univariate problem, like measuring the force of leverage with different lengths of lever,[221] a complex machine like a siege engine's performance is influenced by many variables of complex interplay, and the volume of its limbs are but one of them. While the performance of a siege weapon might not be a high-dimension consideration in terms of ^{50}or greater,[222] there are certainly enough dimensions to make sound analysis an intensely frustrating endeavour.[223]

If we restrict ourselves to the factors taken into account by Philon so as to attempt to better his theory, we can start by looking exclusively at the sinew-springs of a catapult as a function of projectile weight and maximum projectile travel distance.

Ordinarily, a multivariate analysis of this data would require that we take into account that no spring is 'ideal' - that is, one that always obeys the principle of Hooke's Law.[224] Within the boundaries of Hooke's Law, a material may have stress applied to it - measured in the field of physics by Young's modulus, notated as E and measured in GPa[225] - and return to its original form. Once we pass the boundary of force within that material's limit of stress, we begin to enter the region of strain, measured by the shear modulus, notated as G. In this region, the material does not return to its original shape. In other words, it has exceeded its elasticity, and will assume a new plastic state if the force acting upon it is removed. The density of a material, notated as ρ and measured in mg/m^3, has some influence on that material's modulus of elasticity. Depending on the material and whether it is composite (i.e. mixed with other materials to influence one or more of its combined attributes), this elasticity and its boundaries will change. This is the case with both the sinew-spring[226] and bow-type catapults of Greek invention. The higher the modulus and density of a material, the more inflexible it tends to be. High-density and high-modulus materials, like low-carbon 'mild' steels (of a rough analogue to the iron of antiquity),[227] have a small stress region

[221] As in the lever study of pseudo-Aristotle.
[222] See 4.2: Critiques of Euclidean Geometry.
[223] Examples of considerations are wood, seasoning, binding of the springs, the type of sinew or hair used, the temperature or humidity, etc. See 5.1: Recommended Application of Dimensional Analysis.
[224] Atkins & Escudier, 2013: 703.
[225] Gigapascals; that is, one billion Pascals, or Pa.
[226] Marsden's (1969: 68) study seems to demonstrate that multiple composite options may have existed for torsion springs; hair, sinew, and a half-half composite.
[227] With a [ρ] value of around $8000kg/m^3$ and [E] of about 200 GPa, per the chart below.

of high resistance before ‘yielding’ to plastic deformation by strain. This region of plastic deformation is large, and it takes significant shear force to fracture that material. Purely for the purposes of this discussion, and for meaningful comparison on the graph below, we may choose to substitute an elastomer-type material - like rubber or silicone - for the springy, elastic sinew of an ancient catapult. (At least, until the modulus of appropriately-prepared sinew is accurately ascertained.) Elastomers are a remarkable sort of material, in that their density and elasticity tend to have a slightly asymmetric relationship: their modulus of elasticity can remain very low, despite their density being high.[228] More importantly, the boundary between their stress and strain regions is almost imperceptible, making their testing and measurement (in a scientifically-repeatable fashion) extremely difficult. Under normal operating conditions, an elastomer’s stress curve rises steadily. Under extreme temperatures, this curve becomes almost a straight line – the elastomer is permanently strained, or abruptly fails and snaps, with very little stress applied to it.

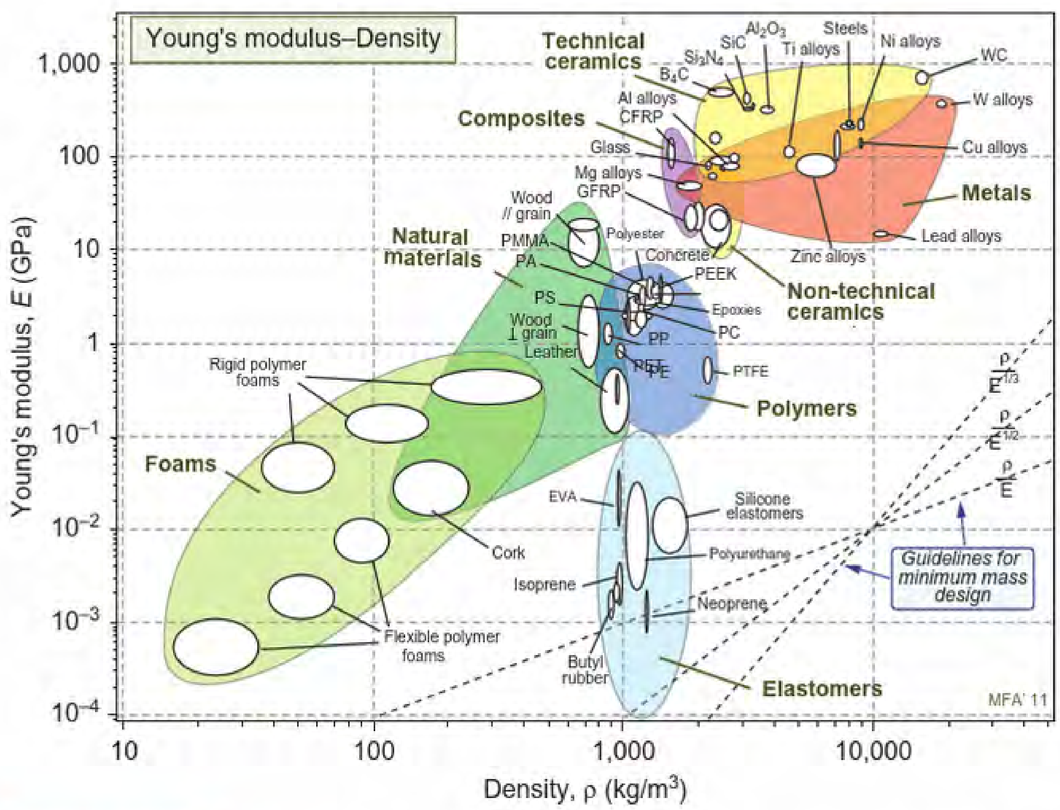


FIGURE 9.12 *The Modulus–Density chart: the one for stiffness at minimum weight*

Ashby & Johnson’s (2014) reference for elasticity relative to density.

[228] A typical elastomer being around 10^{-2} GPa and $1700 kg/m^3$.

We can be almost certain that the extreme conditions of a siege engine hurling boulders at an enemy will exhaustively test a material beyond the limits of its elastic stress region - not least by the token of ancient sources that report constant readjustment of the springs being necessary as they cumulatively wear and assume new plastic states.[229] Philon describes this phenomenon but, importantly for our consideration, claims that the original elasticity is restored by vertical (rather than torsional) re-adjustment, after which the form and properties of the sinew spring returns. This is perhaps the most interesting theory that deserves testing by this route of enquiry. Moreover, the amount of wasted energy that is lost during the storing of energy (drawing-back of the projectile slide), and releasing it (setting loose the projectile) is directly influenced by the material properties. A by-product of stretching and releasing an elastic material may be - as anyone who has idly played with a rubber band may be aware - heat. The amount of heat that is generated and then dissipated into the air by convection will be a factor of the material's innate efficiency, and the ratio of its outward-facing area relative to its volume, which allows for more or less effective dissipation into the surrounding air to occur and for more energy to be lost. Any formula that attempts to scale the spring force effectively should, when plotted on a graph, move through acute and outwardly obvious up- and down-trends.

To some extent, then, Philon alludes to a valid point: volume is a useful metric for ascertaining the effectiveness of a spring, but only when that material's density is considered alongside it, and when a number of other considerations like energy wastage and temperature are considered.[230] As we observed earlier, it may be that his formula is applicable only to an average-case scenario, and errors can only multiply the further one strays from a 'Goldilocks' region that is not anywhere near as accurate as we might hope it to be.

Plotting these data points, however, would require analysis of each dimension's possible effect, at various data points and values, on every possible value of the data points in every other dimension. The result is a brain-twister, a whirlpool of possibilities. We have run into the high-dimensionality problem.[231] Even if we were to conduct the experiments necessary to gather the data to make such a thorough plotting of the dimensions at work, the chances are that we would run into one or both of the following complications. One would be the issue of Euclidean distance in high dimensionality.

[229] Philon, *Bel.* 61.
[230] Archim. *On F.* 1.3 posits that density can be separated into categories of denser than, less dense, and similar in density to water; however, the phenomenon is not explored further than measurement.
[231] See Appendix 1: Understanding Treatises by Geometry.

The other would be in formulating the logic of the algorithms used to sort that data and draw inferences from it. An accurate multivariate analysis of this kind is well out of our computing grasp without extensive effort in experimentation and data-gathering. Nevertheless, it bears mentioning on account of the ideal tool it could represent to solve the problem, and for the pleasant circularity with which it brings us back to problems of Euclid.

Instead, it would be far preferable to employ a method that cuts through the incidentals and allows us to accurately measure and predict the effects of scale on these engines in such a way that cost, labour, and unnecessary complexity in experimentation and data interpretation can be avoided. Ideally, we should be able to replace Philon's simple formula with one of our own, such that we can easily plot his scaling predictions against a more accurate version; and do so using a method that can also be applied to the analysis of machines like Biton's, such that we can compare scale and performance across propulsion types without needing an infeasible number of experimental models, produced at great cost.

Dimensional analysis is just such a technique, and is an experimental and mathematical process employed under multiple branches of the study of mechanics, but most commonly in the field of Fluid Mechanics.[232] Using this method, one can observe the outward effects of the dimensional scale of a model on the other variables that interact with the model. In this way, a desired property or dimension of the material or model which would normally scale at an unpredictable ratio to its physical size can be expressed as a ratio of the other variables at play.[233] In order to find the dimensional properties of a model that fit a scale that we desire – say, to double the energy stored and exerted by a machine and find out the requisite size to do so – we then need only graph the ratio equations of each observed relationship to determine the overlapping region where the desired criterion is true. This 'solution' for a dimension is called the *unit variable*, or a *dimensionless number*.

The first step to finding the unit variable is to describe the known variables upon which a physical phenomenon's measurable change appears to depend, but whose functional relationships to the scaled size of the observed object are unknown. In other words, we need to determine what the outwardly-obvious *primary dimensions* are, and their own constituent *secondary dimensions or fundamental dimensions* that contribute to their values. The dimensions that one might use vary according to the case at hand. If we discard, for now, the various other components of the siege

[232] Atkins & Escudier, 2013: 643.
[233] *ibid.* 545.

weapon, and focus on its power delivery system which provides the force to propel a projectile, we can narrow our example down to a manageable equation. Much like Philon, we will look for the dimensions associated with the spring or bow that stores the energy of the weapon, and the weight of the projectile.

5.1 RECOMMENDED APPLICATION OF DIMENSIONAL ANALYSIS

5.1.1 Introduction & Identifying Units

For this analysis, we will use the following SI (International System) units. Note that we will not be using Newton-Metres as a measurement of torque, despite using some torsion springs in this analysis, as I do not wish to measure any forces here as a function of their distance travelled. It is important not to confuse the division of metres by Newtons in some parts of this analysis as a measurement of Newton-Metres. Also note that Δ here refers to a distance measurement rather than time or temperature, as is traditional in Fluid Mechanics equations.

> **Newtons**: measure of force equal to mass x acceleration. Mass is measured here in kilograms, and acceleration in [G]. Therefore = ·
> **Metres**: standard metric metre, distance measurement, in [m].
> **Kilograms**: standard metric kilogram, weight measurement in [kg].
> **Young's Modulus**: In order for it to be used alongside the above measurements in equations, we will let refer to the unit measurement in [Pa] or Pascals ($\times\ 10^9$) rather than [GPa] or gigapascals.

Using these units, we can begin to construct our dimensions. We can now look to the forces acting on and within the catapult during the process of its operation, to make an educated guess as to what will best encapsulate and account for the entirety of the variables that translate to the ultimate performance criteria. At the same time, we need to take care not to get carried away and over-dimension our problem. For the purposes of this analysis, I have ignored the variable of air-drag, and assumed that energy is conserved. Although this may be a problem with a large-enough catapult, we will assume, at this early stage, to work within a reasonable range.[234] Let:

[234] As per the 17th C. study of cannon trajectory and gunpowder charge by Edmond Halley; see Nahin, 2007: 167.

= spring stiffness, as measured by force divided by distance [—]. See next section for an explanation of spring stiffness and the calculation thereof.

Δ = draw length, or the distance by which the slide of the catapult draws back the bowstring [m]

= mass of the projectile [kg]

Δ = flight distance [m]

= modulus of elasticity of the spring material [Pa]

Here, E of the spring material has been added after-the-fact to make solving the equations for the dimensionless number(s) easier - providing us with an extra primary dimension to cancel exponents against in the process of derivation.

5.1.2 Calculating the Modulus of Elasticity of a Material

Where a material’s modulus is not known, it can be calculated by measurement.

Pascals are calculated as force exerted over an area, or [$= \frac{\quad}{\quad^2}$]. Therefore, we can also say that that they are equal to [$\frac{\cdot}{\quad^2}$]. Using substitution and derivation:

$$= \frac{\times}{\frac{\quad^2}{\quad^2}}$$

$$= \frac{\cdot}{\quad^2} \times \frac{1}{\quad^2}$$

$$= \frac{\cdot \frac{1}{\quad^2}}{\frac{}{1}}$$

$$\therefore \quad = \frac{}{\quad^2}$$

Thus, it is possible to calculate the modulus of a material using experimentation with a constructed model. In this case, as we are dealing strictly with the spring material, [kg] would be referent to the kilogram force applied to the spring, while [m] would be the length of deflection or draw [Δ], and seconds would be that number of seconds that it takes for the spring to snap back to its original plastic form.

It is possible to then use this modulus to calculate a select few extra figures without experimentation at all.

5.1.3 Types of Springs and Calculating Stiffness

Now, we can turn our attention to calculating the stiffness of non-torsion (bow-type) engines using the theory of deflection of a beam as our guiding model.

To calculate the stiffness [k], we can construct it as though within the theoretical realm for calculating the deflection of a beam. For simplicity's sake, all of our trial calculations here will depend on a theoretical model based on one side of the catapult – in other words, half of the bow in the case of a non-torsion engine, and one of the torsion springs for a torsion one – with perfect symmetry assumed in the actual, complete construction. Each side of the catapult will contribute half of the total pullback force [p] measured at the bowstring; so, effectively, we are still creating an accurate model if we consider that we are still measuring total force of the catapult, and remember that we are merely using the geometry of one side for the input into our stiffness equation.

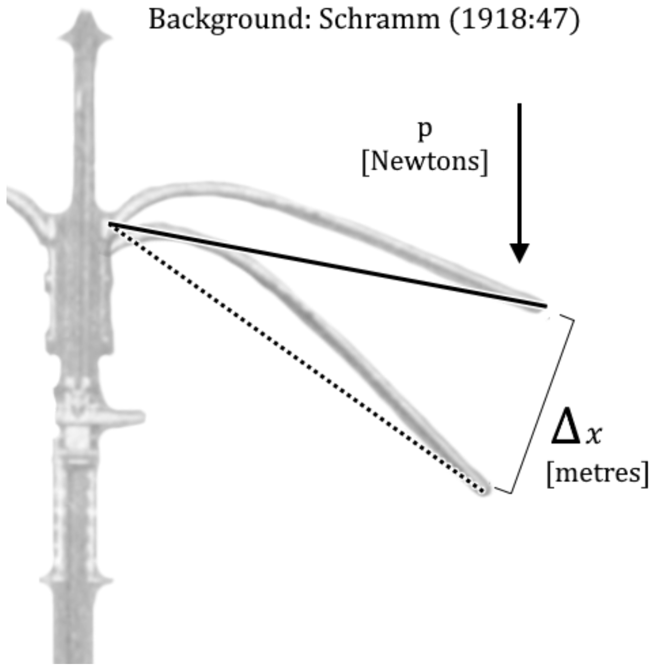
The formula for stiffness is:

= —

Where [k] is our nominally arbitrary unit of stiffness, calculated by the load [p] divided by the distance travelled [ ]. Here, we will use Newtons for [p] and metres for [ ]. Therefore:

= —

Calculating the distance [m] of the deflection of the bow is not necessary if it can be measured on an experimental model – however, this would not be as simple as with the torsion engine detailed below, where the bow-arm moves in a single plane. We can also be sure that it will not rotate in a perfectly circular motion like the arms of a torsion engine, as a bow flexes inwards on itself when draw force is applied to it. The bows of Schramm[235] and Marsden[236] both curve upwards and to the rear of the machine, such that their movement must be measured by a straight-edge clamped to the engine casing, and following along the new plane created by the bow's deflection as it is observed in a constructed model.



The ideal measurement conditions on a hand-operated gastraphetes, *where the deflection is relatively constrained to one plane; unlike the* lithobolos *where the bow is also curved up towards the gunner.*

Next is to find the Newtons [N] measurement for [p]. Let us consider the standard formula for Newtons, which is [×] or indeed, [×]. With a force-meter measuring the number of kilograms of force applied to the bowstring when the spring is winched back, we then need only to measure the acceleration of the bow arm as it travels back to its original plastic position. This can be done in experimentation with a device (an accelerometer), or it can be done with the standard formula for acceleration [÷]. In this case, this would require measuring the metres-per-second [/] speed of the bow-arm as it snaps back into place, and dividing that by the number of seconds [s] taken for it to do so. Naturally, this will be a number of many decimal places; but measuring it in anything other than seconds will create difficulties for the derivation of our equations in SI units.

[235] Rehm & Schramm, 1929: 31.
[236] Marsden, 1972: 81.

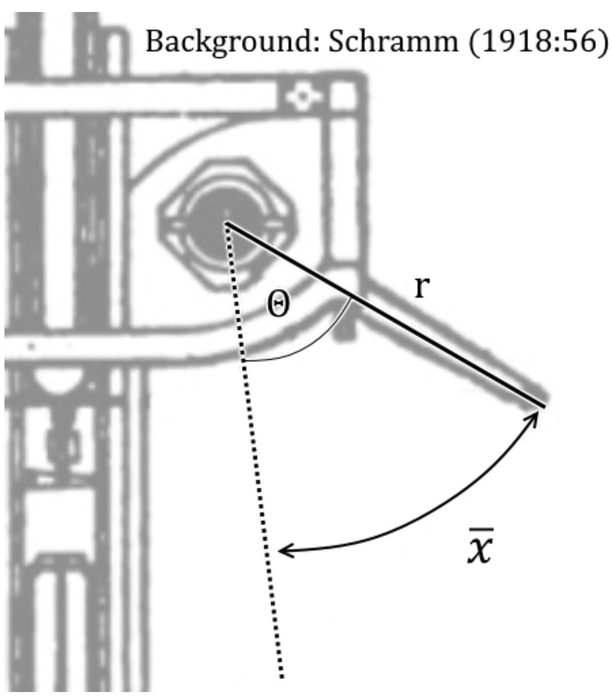
With these measurements, we can determine our [k] value for the spring-stiffness of the torsion catapult in a manner that is equitable with that measured for the torsion catapult.

5.1.4 Calculating Stiffness of Torsion Springs

Again, we will consider the catapult's springs as though they are one, and assume perfect symmetry. Let us find, once again, the measurements for our stiffness formula; with our units substituted in, it remains:

$$= \quad —$$

To find the [m] distance travelled by the spring in the course of normal operation and therefore the metres measurement for [   ], we can use the section of a circle that the catapult's arm traces during the winching-back of the catapult's slide. Once again viewing the spring from directly above, this time looking down the centre of the column created by the bound package of sinew, we will make this centre the origin. From there, we can measure the number of degrees [Θ] through which the spring makes its travel – in other words, how many degrees of drawback are observed between the arm's resting position and its position at the full draw length of the slide.



Measuring the movement of a torsion-powered arm.

This number of degrees (annotated with degree symbol [°]) must then be converted to radians (annotated with symbol [ᶜ]):

$$1° = \frac{\pi}{180 \cdot}$$

$$\therefore 1° = 0.0174...$$

$$\therefore \Theta = \Theta° \cdot 0.0174...$$

We also need the length of the arm of the catapult's bow-arm, measured from the centre point of the torsion spring to the point of attachment for the bowstring. This, effectively, is the radius of the circle being traced by the bow-arm. We will measure this in metres. With this information, we can substitute into the formula for the section of a circle and find our [m] distance, which we can label [¯] in the format of the circle-section formula.

$$\bar{} = \cdot \Theta [\ ]$$

Finally, we need the Newtons [N] measurement for substitution of our [p] force in the stiffness equation. The method of measurement for this will be identical to that of the non-torsion catapult, as both engines' slides operate in a straight line. With that, we will have a [k] stiffness measurement for the spring assemblies in both catapult designs.

5.1.5 Choosing Repeater Variables & Calculating Dimensionless Groups

With our dimensions established, we can begin the Buckingham Pi method[237] of dimensional analysis proper. The first task is to determine how to group our variables that we have identified in such a way that we can create meaningful ratios between them.[238] First, we need to select our *repeater variables* – those which, if grouped together, are *not* constituted by dimensions that can cancel one another out in an equation. If they cannot be dissolved within an equation, they cannot become dimensionless within that group – thus, grouped dimensions that are of the inverse property are referred to as *dimensionless groups*.

[237] As per Langhaar, 1951: 18f.

[238] We know that we have identified five variables, and we have three primary dimensions (M, L, and T as below) by which to categorise them. We therefore have two possible dimensionless groups: Langhaar, 1951: 29.

With some trial and error, it is evident that , Δ , and are our repeater variables. In order to dissolve them into our two dimensionless groups, we can now substitute in the *primary* or *fundamental dimensions* that constitute each of our variables. Next, we can arrange each dimensionless group as a ratio to the repeater variables. Finally, we follow the steps of the Buckingham method to cancel out exponents for each fundamental dimension and reduce each ratio to its most elementary components.

We will substitute into our formulae with the three primary dimensions of MLT (Mass, Length, and Time) as according to the method.

Symbol	Description	Unit	Dimensions
k	Spring Stiffness	—	$^{1} \; ^{-2}$
Δ	Draw Length	m	$^{1} \; ^{-2}$
	Mass of Projectile	kg	^{1}
Δ	Flight Distance	m	^{1}
E	Modulus of Elasticity	$(\times\ 10^{9})$ Pa	$^{-1} \; ^{-2}$

Let us begin with our repeater variables multiplied into [Δ], and solving for each dimension's exponent value where the primary dimension's base value is equal to one. Using substitution and standard derivation:

$$(\quad \cdot \Delta \quad \cdot \quad) \times \Delta \; [\quad ^{1}] \, [\quad ^{1}] \, [\quad ^{1} \quad ^{-1} \quad ^{-2}] \, [\quad ^{-1}]$$

$$^{0} = \quad + \quad = 0$$
$$\therefore \quad = 0$$

$$^{0} = \quad - \quad - 1$$
$$\therefore \quad = \quad - 1$$

$$^{0} = \quad - 2$$
$$\therefore \quad = 0$$

We can then carry these exponent values back to our original equation, and using substitution and standard derivation we find that:

$$(\quad \cdot\Delta \quad \bullet \quad)\times \Delta$$
$$^{0}\cdot\Delta \quad ^{-1} \bullet \quad ^{0}\times \Delta$$
$$\therefore \frac{\Delta}{\Delta} \qquad \pi 1$$

Next, we can substitute into our second group equation, using our repeater variables and [k]. We will need to order our solutions slightly differently this time in order to find values for exponents [a], [b], and [c] that can be substituted back into those same solutions. Using substitution and derivation:

$$(\quad \cdot\Delta \quad \cdot \quad)\times \quad [\quad ^{1}] \, [\quad ^{1}] \, [\quad ^{1} \quad ^{-1} \quad ^{-2}] \, [\quad ^{1} \quad ^{-2}]$$

$$^{0} = \quad - \; 2 \quad - \; 2$$
$$- \; 2 \quad = \; 2$$
$$\therefore \quad = \quad - \; 1$$

$$^{0} = \quad -$$
$$- \; (- \; 1) = \; 0$$
$$\therefore \quad = \quad - \; 1$$

$$^{0} = \quad + \quad + \; 1$$
$$- \; (1) + (1) = \; 0$$
$$\therefore \quad = \; 0$$

Once again, we substitute these exponents back into our original equation to find our second ratio:

$$\left(\quad \cdot\Delta \quad \bullet \quad \right)\times$$
$$^{0}\cdot\Delta \quad ^{-1} \bullet \quad ^{-1} \times$$
$$\therefore \frac{}{\cdot\Delta} \qquad \pi 2$$

5.1.6 Conclusion

Using the Buckingham Pi method of dimensional analysis, we can identify the most outwardly-obvious variables at work, and derive two dimensionless numbers which can be used to compare and scale ancient siege engines. As we have identified base variables that are independent of the type of propulsion used, and are instead derived from universally-measurable phenomena using standardised units (like spring stiffness, in [—]), torsion and non-torsion catapults can be compared on an ‘apples-to-apples’ basis. These dimensionless numbers are calculated as a ratio of these visible, measurable data points, and that data can be manipulated in various ways.

$$\pi 1 = \frac{\Delta}{\Delta} = \frac{h\,[\ \ ]}{h \qquad [\ \ ]}$$

Suggested description: engine metrics to performance ratio.

$$\pi 2 = \frac{}{\cdot\Delta} = \frac{[—]}{[\ \ ]\cdot \quad h \qquad [\ \ ]}$$

Suggested description: spring metrics to performance ratio.

Once experimental data has been collected in the manner suggested in the sections above, and has been inputted into these equations, we can plot the dimensionless results for [π1] and [π2] as functions of each other in a graph format to observe the nature of the relationship between these ratios.

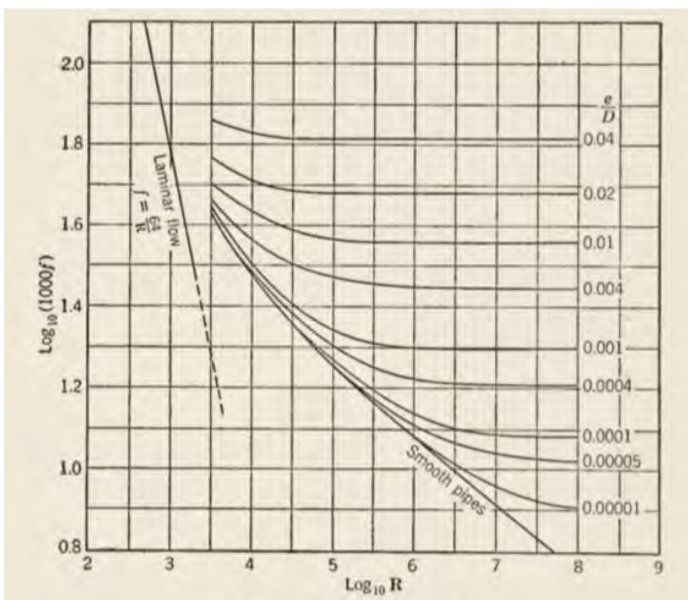


Fig. 2. Pipe Friction Factors

The ‘Moody graph’ as printed in Langhaar (1951: 22). This observes the functional relationship of the diameter of a pipe and the roughness rating of its interior surface, as mediated by the dimensionless Reynolds number [Re]. Depending on the variable value selection, the value of the [Re] number will change, and this value can be used to predict the turbulence of flow through the pipe. Conversely, pipe diameter or pipe roughness can be selected as the driving variable in finding a desired outcome.

With this understanding, we can make accurate predictions for the necessary attributes of a machine to fit whichever variable we choose. For example, if we were to try to find a non-torsion engine whose performance matches that of chosen torsion engine, and thus find the requisite scale of construction of the former, we can find the overlapping region between the graphs of [π1] and [π2] for both torsion and non-torsion [π], where the variable (here, Flight Distance) is true in its desired value.

Similarly, if we wished to scale a machine accurately according to another variable – in this case, we have identified [Δ], or Flight Distance in [m], as a usable one – we can substitute this desired value into the equation for [π2] as a function of the base value which it is to replace. For example, if we wish to double the range of an engine to match another whose performance is known, we can find the requisite [k] value (and therefore the dimensions of the spring, or bow, required) by substituting [2 · Δ] in a replacement variable.[239]

5.2 RELEVANCE

The procedure, then, according to the dimensional analysis outlined above, would be to select a machine and reproduce its power source – in this case, a sinew spring or wooden bow - in several scales. Measuring the energy output from each of these scaled models, along with covariate data points, such as force exerted in tensioning the source to store that energy, will allow us to ascertain what that model's ability to store energy happens to be as a function of its scale. Dimensional analysis is essentially a shortcut to accurate predictions, cutting through all other considerations. By using observed data from experimentation, the properties of materials in the experiment are effectively skipped over.

From this analysis, we can determine what size of bow or spring would be required to meet the performance criteria of Philon or, indeed, any other ancient source, and determine whether their predictions in doubling weight capacity by way of doubling material volume are at all accurate. We can also ascertain a much more precise value for the discrepancy caused by the use of the Greek spring theorem at each corresponding level of scale, by applying this same analysis to models scaled using their method and measuring their performance. In addition, we can interpolate using a variety of construction volumes as a function of range to find the drop-off point in the economy of

[239] See calculation of dimensions from a complete dimensionless set: Langhaar, 1950: 40.

scale applied to maximum ranges as provided by larger and larger power sources.[240] It remains, however, that building physical models according to ancient instructions is still necessary to achieve this; after that, however, the analysis is complete and this mathematical model can be applied as a universal reference. The practice of dimensional analysis in a similar methodological process has long been applied to the manufacture of hand-bows for recreation and sport,[241] and I believe that the model of experimentation outlined above demonstrates that the same principles may be applied to the study of siege weapons as it pertains to the Greek method of selecting and scaling artillery.

The construction of models in the case of those machines whose core operating principles are still uncertain – as they are in fragmentary accounts, like much of Biton – still has value in assessing the feasibility and tractability of a concrete interpretation of the ancient text. For designs whose core operating principles are certain,[242] the application of the scientific method in a more rigorous manner, and collecting more measurements, will lend some extra credibility to the endeavour and provide useful input to further analysis. The goal on a broader scale may be to assist in moving experimental archaeology away from strict adherence to a methodology of using ancient procedures to the extent of also employing an approximation of ancient observation, scaling, and analysis of data - where more stringent collection and processing of that data may grant us some new insights. Cost and labour in this field of study can also be greatly reduced. Experimentation and data collection can be performed with only a few models to acquire a range of infinitely-scaling (or, shall we say, vectorised) data points, instead of making costly full-scale reproductions for every iteration that we might wish to test. In this way, a classicist or historian can make a more precise and insightful assessment on the veracity or implications of data given by an ancient text, in the same way that engineers do to determine ideal solutions for multiple variables – including problems of non-linear scalability - in industry. Thus, we can better address problems like the Biton question by determining whether or not a stop-gap solution in the form of non-torsion propulsion is viable in the proposed era we put forward for Biton's activity. We would, in other words, be able to compare apples with apples, and with some reduced risk of conjecture.[243]

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[240] Marsden (1969: 38) supplied a graph of range as a function of spring diameters for a constant weight, but this is, by his own admission, not based on experiment and is entirely an estimation.
[241] Langhaar, 1953: 81.
[242] Like Heron's hand *gastraphetes*, which is well studied. de Camp, 1961: 241.
[243] See discussion of some concerns regarding substitutions offered by Marsden and Schramm in Chapter 6: Treatise.

In conclusion, it seems that the assumption of linearity of scale that Philon relies on has clear origins in Euclid's axioms of the straight line and – by extension – parallelism. As we have observed, engineers like Vitruvius, who quotes a number of Greek sources including Philon, demonstrate in their practice that dimensional scale does not, in fact, work very well when applied linearly. To combat planar distortion, civil engineers were modifying Euclidean rules for their work. The discrepancy between what was measured and what was seen by the eye was, if anything, dismissed as an error of human vision,[244] and thus it would seem that engineers never saw fit to deviate from Euclidean traditions of constancy. Despite the Aristotelian position that matter conformed, in its most primordial state, to a sphere,[245] Greek engineers were seemingly unwilling to reconcile this observation with the appealing linearity of Euclid. This erroneous assumption in the scaling of measurement was therefore transferred to the scaling of force and energy, in turn by way of the scaling of materials without dimensional analysis. As with their inability to reconcile Euclid's straight lines with visible distortion in civil works – instead preferring to lay blame on the human eyeball - confirmation bias may likely have made siege engineers more willing to assume that human errors in production were the problem, rather than questioning the tenets of the sacred geometry and its own dogged adherence to linear scale. The resultant margin of error in ranges reached by siege weapons (as opposed to what one might hope to achieve from the use of Philon's theorem, or indeed his table), is not a misrepresentation of egregious magnitude. However, it does remain a very real and measurable source of discrepancy, and more importantly, there is a very real consideration for our methodology here where siege engines, or indeed the study of any ancient engineering that relies on these concepts, are concerned.

[244] Siebert, 2014: 110.
[245] See Chapter 4: Euclidean Geometry.

6. TREATISE

6.1 PROVENANCE & SCHOLARSHIP

The single surviving work of Biton comes to us through a manuscript tradition. The original document almost certainly was written and later circulated as its own work – it does not make any internal reference to attachments or appendices, apart from diagrams that are meant to be in-line with the text. As it survives, however, it is bundled or bound together with writings by other ancient Greek engineers from across the centuries, in an apparent attempt to make topical collections of works. These collections of siegecraft were sufficiently prominent to garner their own genre description, by which they remain most commonly referred - the Byzantine Greek term *poliorcetica*. Inversely, this has somewhat unfairly resulted in some of the writers included therein being reductively categorised as 'siege engineers', when their expanded works – preserved elsewhere – reveal them to be well-rounded experts in various fields, from city planning to waterworks and agricultural equipment.[246] While Biton does refer to a work on the topic of *Optics* that he claims to have written,[247] we know him only through his *Construction of War Machines* (κατασκευαὶ πολεμικῶν ὀργάνων καὶ καταπαλτικῶν).

The 'standard edition' of Biton for almost a century - alongside authors such as Athenaeus Mechanicus, Philon of Byzantium, and Heron of Alexandria - is Thévenot's 1693 work,[248] which contains much cleaner and clearer reproductions of earlier manuscript illustrations, and significantly more legible Greek transcriptions, alongside a Latin translation and commentary. However, this work is based solely on one poliorcetic manuscript, the *Parisinus Graecus* 2435, dating to the 16th century CE. As more manuscripts with similar contents came to light from their hiding-places in private collections[249] and previously shuttered institutional libraries,[250] these contents could be compared and more comprehensive, more stringently corrected versions could be synthesized. Wescher's 1867 compilation *Poliorcétique des Grecs* makes a great advance in the paleography of this manuscript group, and has deservedly become a standard reference in its own right. Considerable labours were undertaken by Wescher to compare the extant manuscripts, identify the chronological order of their creation, and then determine what the relationships

[246] Whitehead, 2016:15.
[247] Biton. 53.
[248] Thévenot (1693) *Veterum mathematicorum opera.*
[249] Such as the Harley and Burney collections, donated to the British Library in the late 19th century.
[250] Like that of the Vatican, from which at least one version of Biton can now be found publicly available in the *Vaticanus Graecae* 1904.

between them might be in terms of the tradition of scribal copying.[251] Both Wescher and Thévenot's editions remain relevant, and due to the immense variation in pagination between Middle Ages manuscripts, it is still necessary to be aware of their respective line-referencing standards which make these manuscripts more easily inter-referenced. By their system, line references are not parameterised by separate works, but by the total manuscript, and the units refer to sections – for this reason, references to Biton start at 43 and end at 68. The German scholarship on *poliorcetica* goes hand-in-hand with their early 20th C. archaeological work in Asia Minor and parts of Europe, and Hermann Diels' (1914) emendations of Wescher's attempt to harmonise the manuscripts and correct the corruption of the original Greek was closely followed by the philological and reconstructive work of Rehm & Schramm (1929). Both of these parties use a combination of Wescher and Thévenot's line references to perform their work.

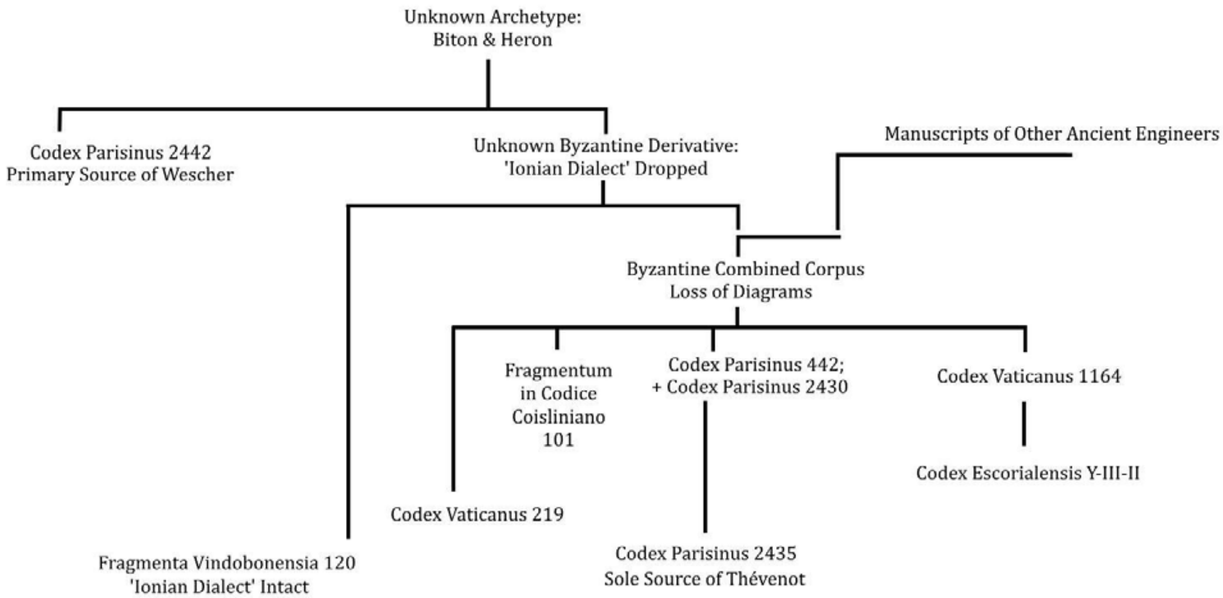
This translation and interpretation is of particular interest to us as it was further developed with the partnership of Rehm with Schramm's military engineering expertise, and it is with their 1929 work[252] that we have the extant diagrams of the manuscript tradition re-interpreted as line sketches in multiple elevations with a scale, and a very plausible set of component-level analyses aimed at producing a working model of the projectile engines in particular. The next most notable landmark in this field, to which I will most closely be referring, is the two-volume work *Greek Artillery* by Marsden. The first volume, *Historical Development,*[253] is, as the title suggests, concerned with placing the poliorcetic authors in their historical contexts, examining other references to their work in both texts and the archaeological record, and making an account of and commentary on the apparent course of the development of technology and innovations in the design and execution of artillery. *Technical Treatises*[254] focuses on the source texts themselves, and contains refreshed translations of each author with commentary, including suggestions by Diels, Rehm & Schramm, and Wescher. Marsden's translation of Biton into English uses Wescher's line-referencing system, and this is the convention to which I adhere throughout this thesis. The depth and scope of Marsden's work has made him an authority on the subject, but some notes and clarifications must be made on his translation and interpretation of Biton, particularly as it is this engineer whose work has perhaps suffered the most from procedural degradation through manuscript copying. Since it is currently the most widely available translation in English, it deserves all the more scrutiny. By

[251] See Chapter 3.2: The Scribal Problem.
[252] Rehm & Schramm (1929) *Biton's Bau von Belagerungsmaschinen und Geschützen.*
[253] Marsden, 1969.
[254] Marsden, 1972.

Marsden's own admission, there are sections of his work where missing or corrupted parts of the text have necessitated the application of some 'mechanical probability' to fill in the blanks. There are of course some limitations to this method, as we must exercise great caution to limit ourselves to what the text does, in fact, say, and to what we can reliably infer from context. For this reason, I have highlighted some areas of his translation which I believe deserve some commentary, given the particular focus of this thesis. I have also read the text alongside the more faithful (that is merely to say, less inferential) translation of Rehm (1929). Important points in the harmonised Greek, as it stands amicably emended between Diels, Rehm, and Marsden, have been extracted further down in this analysis and given special attention. At this point in the wider scholarship of *poliorcetics*, I have found the remaining usefulness of consulting the original manuscripts has become limited to that of consulting better images of the text and, more importantly, its accompanying diagrams, than those seen in the somewhat grainy and distorted plates rendered by the natural limitations of printed books and articles.

It seems then that, over the course of the 20th C., the work carried out on Biton and his manuscript-fellows falls into three distinct categories. The first was in better reconciling the translation of rarely-seen technical terms with the denotation that could be inferred by the additional contextual clues provided in the increasing number of other technical texts that continued to surface in the scholarship of this time period. The second was in restoring the texts from their corrupted state by comparing manuscripts and diagrams to better ascertain their place in the scribal tradition. The third was to further clarify the nature of the machinery described by reconstructing the recommendations in line with archaeological evidence, and with the application of engineering practice, creating what would become the antecedent of experimental archaeology methodology.[255] Today, the basic chronology of the manuscript tradition is more-or-less in agreement. The relationships between the manuscripts are best summarised in the following 'family tree' which I have re-drawn from a version by Marsden,[256] and to which I have added the full names of the manuscripts rather than their abbreviations.

[255] See Chapter 1: Introduction.
[256] Marsden (1972: 13).



Once again, for our study, we are using Rehm and Marsden's versions of Wescher's text, with some diagrams taken from other sources for illustrative purposes.

6.2: THE TEXT

The treatise of Biton is remarkably short in comparison with some of its manuscript companions; the Harley manuscript[257], for example, has only a four-folio section dedicated to Biton, including diagrams. The language is short and terse, and scant in details. Nevertheless, it manages to cover a number of machines in short order with these - presumably intentionally abridged - descriptions. None of these machines, six in all, is Biton's own original design, and he instead presents each one to his reader using the name and place of residence of their original designer as a reference. The descriptions are book-ended by an introduction that hails one King Attalus and briefly describes the contents, and a conclusion with a short nicety on the hopeful suitability of the weapons and the possibility of their adaptation. Where conversions to metric dimensions are made, I have rounded the conversions (whether they be in metres or millimetres) to their first three decimal points.

6.2.1 INTRODUCTION 43-44

By way of preface to the treatise, Biton makes a business-like statement on his intention to describe stone-throwing engines (or *lithoboloi*) in particular, with a passing reference to his benefactor,

[257] Harley MS 6317.

Attalus. Though short, this passage is densely-packed with information. He urges the reader not to "scoff", per Marsden,[258] should he make some recommendations that are not to this description – which he does. This is followed by an assurance that the machines he describes will specifically be of use in repelling the "engines employed in the offensives of [Attalus'] enemies", in a "counter-attack".[259] The stone-throwing engines, here, will be the subject of our closest analysis.

6.2.1.1 COMMENTS

The inferences we can draw from this are that Biton was writing for a wartime audience, and that his use-case was one of reactionary anti-matériel measures. For whatever reason, he seems also to have some recommendations for Attalus that are not of immediately obvious utility for the problem at hand.

6.2.2 CHARON OF MAGNESIA, AT RHODES: 45-48

He then slips into the plural "we" (ἀρξόμεθα "we will begin")[260] to make his first recommendation: a *lithobolos* designed by Charon of Magnesia while at Rhodes. Again, Biton makes a qualifying statement; he urges the reader to "double-check" the dimensions he provides.[261] The machine, as per its name, is designed to hurl spherical stone projectiles. The total length of the case is some 6 *podes*, which is to say about 96 *daktyloi* or 1.853m. The style of description for each component is quintessentially Euclidean, in that each component is defined by a letter and then given its dimensions. The two beams that are fastened together to make the case are introduced as "straight beams AB" (ὀρθους κανόνας τοὺς *AB*), and their dimensions defined as one would expect for a geometric construction: "Let these have a length of 6 *podes*, breadth and height of half a *pous*".[262]

Biton stipulates a pair of reinforcing beams with iron cladding at their ends be built into this case, and the "saw-teeth"[263] be affixed to these. Evidently, he is referring to the slider, and the ratchet teeth that will secure the slider in place while winching it back. There are two of these ratchet rails, each with a corresponding pawl on either side of the slider, which will – necessarily – be released by the trigger system. He takes great care to explain that the ratchets must be precisely and uniformly machined to shape and positioned in a perfectly parallel relationship such that the slider is winched

[258] Biton, 43.
[259] *ibid.* 44.
[260] Or, alternatively, a more beseeching "let us begin".
[261] Biton, 45.
[262] *ibid.*
[263] *ibid.* 46.

back and secured – presumably to avoid the slider either 'walking' left or right and jamming in the slide, or to simply ensure that both pawls engage at the same moment with each tooth of travel. Above this, he describes a "sling"[264] to be made of hair, strong enough to "withstand the stone-shot", which we can only take to be the bowstring of the weapon. He places a bow of undetermined size at the front of the case for propelling the shot, merely specifying that it be "fitted in",[265] and describing its arms as bending inwards towards the case when tensioned.

Suddenly, Biton refers to new components which have not been previously defined, referring to their letters. He describes the action of pulling the slider back as being done "by means of the beams ZH incorporated into the stanchions K[TH]", which Marsden and Rehm both take to be referent to the hand-operated windlass at the rear of the case. This is somewhat confirmed by the accompanying diagram as it survives in *Supplément Grec 607*, although this shows the windlass attached to the front of the machine rather than the rear (see below). Once again referring back to the main case, Biton stipulates that it should be braced with iron plating at the rear, such that it be "strong enough to withstand the discharges".[266] He closes by pointing to the illustration.

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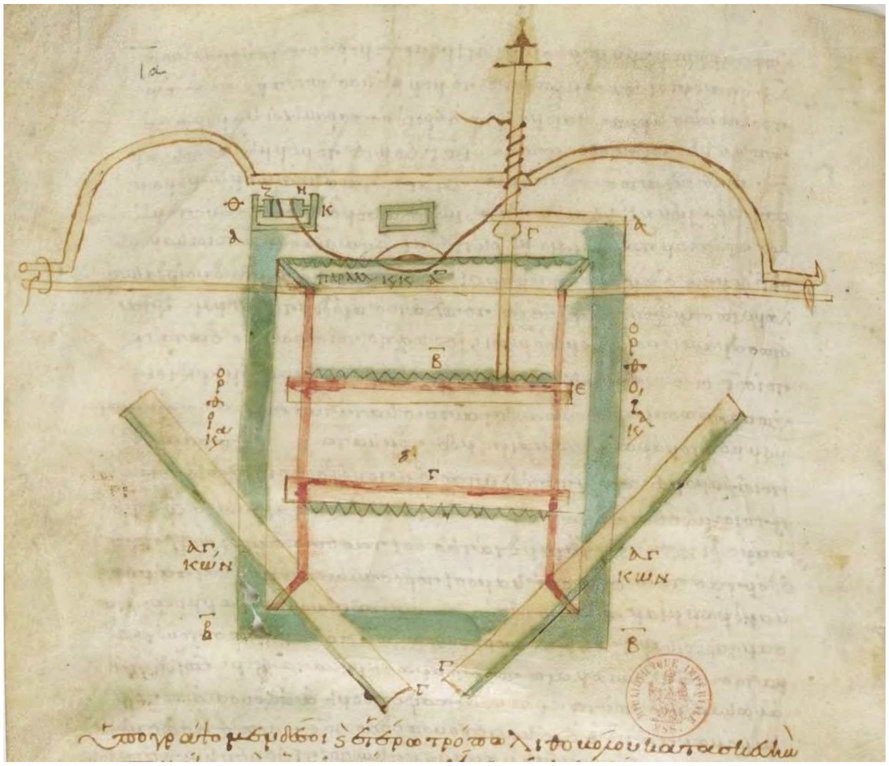
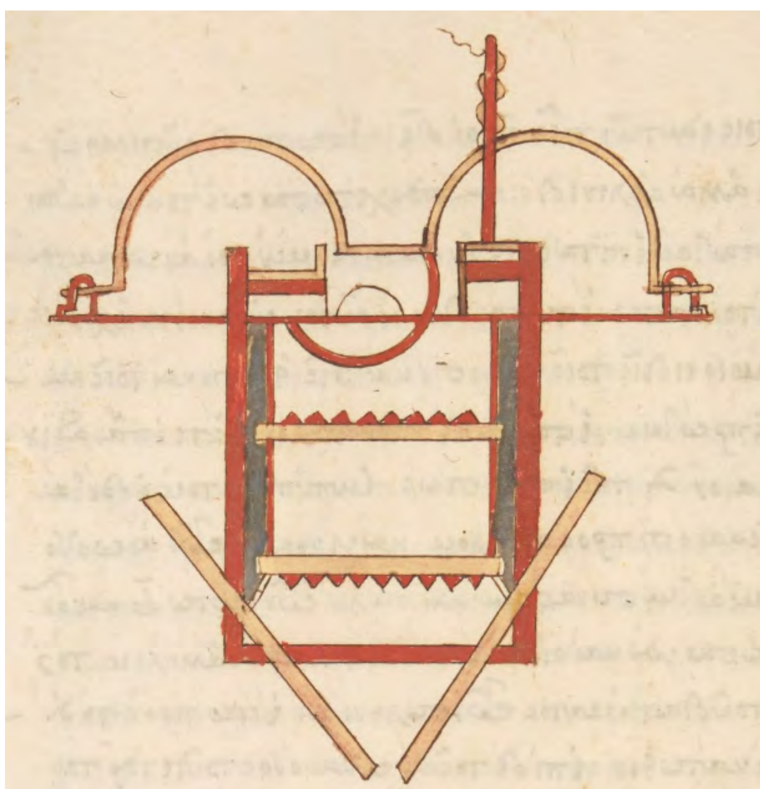
[264] *ibid.* 47.
[265] *ibid.* 47.
[266] *ibid.* 48.



Supplément Grec 607: f.26v



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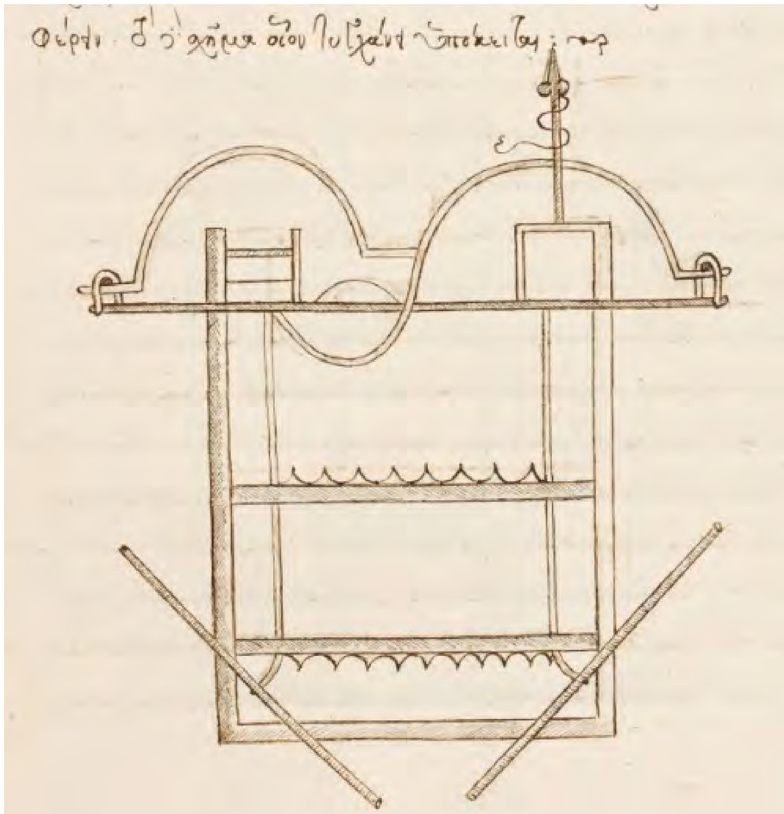
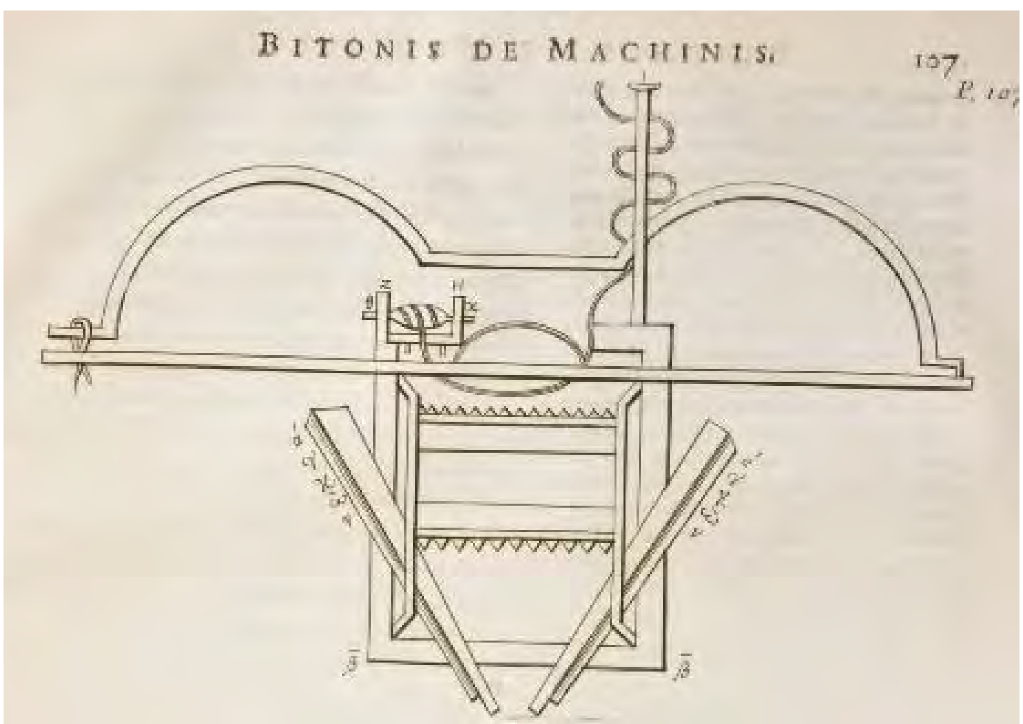


Harley MS 6317: f.9r



Veterum Mathematicorum: 107

6.2.2.1: COMMENTS

The qualifying statement regarding the checking of measurements, which I mention above,[267] is an interesting aside by Biton. The implication seems to be that if a machine is found suitable as judged by its outward dimensions and core operation, it must be further queried through the engineer, who presumably would need to refer again to an external text or record for the rest of its description and specific details of measurements and construction. The sense one might get from this treatise is that it is a design brief, offering the engineering equivalent of abstracts for each machine. I think it is reasonable to suggest that Biton's sudden reference to a previously uninstantiated variable near the end of this section is indicative that he is writing a shorthand version of another, more comprehensive drawing or description, in an antecedent text; perhaps by Charon himself. Once again it seems that he is attempting to give the reader a sense of the overall dimensions of the machine and its most important functional components, and has skipped over a number of others in an apparent hurry. Should the intended audience have selected that machine for the use-case at hand, based on its size and basic function, those variables may have been explained in depth.

The diagrammatic accompaniment is confusing on many points, and in every manuscript iteration. The only means by which a front-windlass construction could be achieved is if the carriage-winching cable is routed around a peg at the rear of the machine and brought forwards again to the windlass in front of the case. This would only complicate the design, and render it less efficient. The additional routing-peg or "idler" roller[268] would also present an additional point of possible equipment failure. Reconstructing these designs with the windlass at the rear, therefore, is a perfectly sensible interpretation of the text, and would provide a clear antecedent to the Zopyrus engines nearer the end of the treatise with their rear double-windlass construction (which are also far better preserved in their diagrams - see below). Moreover, the two "parallel beams"[269] of the slider are represented in the diagram as being perpendicular to the case – evidenced by their saw-toothed ratchet sections. Naturally, this arrangement would render them useless, and the layout of the Zopyrus engines provides us with a better model of interpretation.

Comparing the manuscript illustrations of the first two engines with those of the last two gives the impression that the former are most likely not in any way connected to Biton's original diagrams. It

[267] Biton, 45.
[268] In the sense of an idler pulley in a belt-routing system.
[269] Biton, 46.

would appear that they were reconstructed from the textual description only, by someone unfamiliar with projectile technology or engineering in general, in a parent manuscript predating those extant. The aforementioned problem of the perpendicularly-arranged ratchets is not the only indication that this might be the case. Although it is not pointed out by any other scholar that I can find, I believe that the copying scribe for *Supplément Grec 607* made an error – as can be plainly seen by a third row of ratchet-teeth inexplicably added parallel to the other two, in between the bow and the bowstring. It appears, too, that there was an attempt to cover up this error, as it has been daubed-over with the same green paint used to denote the solid beams of the weapon's case. Why this scribe - and others - did not refer to the Zopyrus illustrations for a better sense of the technical realities of this machinery and correct the drawings is, I think, most likely the result of the same level of uncertainty evidenced by amateurish mistakes like the ratchet error. Had this manuscript fallen into the hands of another, more learned scribe,[270] the surviving diagrams may have been quite different.

The lack of a measurement for the bow arms is repeated in the other non-torsion designs, and given that the diagram is *scenographically* 'folded-out' and proportionally different in every iteration, it is impossible to infer the proportional scale of the bow from the case or slider. The precise shape of the bow is clearly meant to be of the palintone type,[271] which can be confirmed by similar shapes in the Zopyrus diagrams. This would indicate that the Hellenistic machine likely used a composite bow construction. Schramm's reconstruction uses a spring-steel bow,[272] which does not fit well with the text. Not only is this material not mentioned in the text, but Biton stipulates ash wood in his introduction[273] and only mentions iron components as an exception to this rule.[274] From a testing perspective, there is some utility to a spring-steel bow for modelling purposes. The repeatability of test measurement is assured by the relative resilience of steel, where a composite bow may delaminate and procedurally wear, modifying its modulus of elasticity. The caveat, obviously, is in ensuring that the steel analogue is constructed in a manner that imitates precisely the spring force [k] of a composite bow reconstruction at the desired scale.[275] Furthermore, the scaling of the steel analogue alongside the scaling of the machine must be done in a way that imitates the force exerted by the bow which would otherwise have accompanied that scale of machine. In other words, a

[270] See Chapter 3.2: The Scribal Problem.
[271] As the definition pertains to the shape of hand-bows, not to torsion-type stone-throwing engines.
[272] Schramm, 1918: 47.
[273] Biton, 43.
[274] That is, where cladding is concerned, e.g. Biton, 48.
[275] See discussion of dimensional analysis for experimental archaeology, Chapter 5.1.

dimensional analysis is required for both material-types of bow, which would in turn require constructed models of both for testing. Thus, one might beg the question: why bother? For full-scale testing of a model over hundreds of test iterations, this might be a useful trade-off. Where dimensional analysis is used to predict performance at various scales, the initial cost and labour effort of building several composite bows and performing only one analysis seems to be the more efficient route.

I suspect the reason for Schramm's use of spring steel is that a leaf-spring construction would, naturally, substantially reduce the scale size of bow required to impart the same amount of spring tension. This would circumvent the problem of how to fit a bow of any diameter great enough - that is, more than five *daktyloi* or 96.5mm, to hurl stones equal to or greater than five to six *minae*[276] - into a case that is only half a *pous* (or 154.4mm) tall, and already has half of that taken up by the internal dovetail slide and slider assembly proper. Marsden asserts that the centre of the bow should be able to be made much slighter in diameter than the *tonoi*, or arms, thereof, thus making a larger bow fit into a smaller cut-out in the case. However, this does pose a problem. In the making of hand-bows, this area is traditionally reinforced on account of it being the mechanical moment at which the stress of the tensioning of the bow is most concentrated. He points to the fact that this area may be cut from the case, the bow placed in, and reinforced with iron plate as per Biton's recommendation – and this, as he mentions, may lend some support to the bow. However, this does not account for the fact that the centre of the bow still encounters flexion as it imparts rigidity to the rest of the limb. The combination of a reduced mass at the centre of the bow, and placing iron plate flush around the diameter in this same area, will likely only create a stress riser in the same manner as when one breaks a twig across their knee. In addition, there is a mechanical problem in the plan to fit the bow into the case by means of a circular hole, where it is assumed that the bow's cross-section, too, is circular. Gluing the bow in this position before plating is not guaranteed to prevent the bow from rolling forwards in this slot after a shot is discharged, particularly if the engine is exposed to direct sunlight and heat. For this reason, Rehm & Schramm's option of a rectangularly-profiled bow is preferable if using the through-case method. However, Rehm's translation of Biton's instruction – "befestig" – is preferable to Marsden's "fitted into",[277] as it does not imply that the bow necessarily be passed through the frame, and is thus closer to the Greek. It is entirely possible, therefore, that this bow could be intended to be bracketed to the frame instead, which would afford one the opportunity of upsizing it significantly if necessary.

[276] Marsden, 1972: 79.
[277] Biton, 47.

In their reconstructions, Rehm & Schramm's bow is gently curved into the palintone shape, and they deemed a measurement of approximately 2.75m across to be appropriate.[278] Marsden, on the other hand, uses 150 daktyloi or approximately 2.895m, obtained by comparison with the later bolt-firing *gastraphetae* of the treatise.[279]

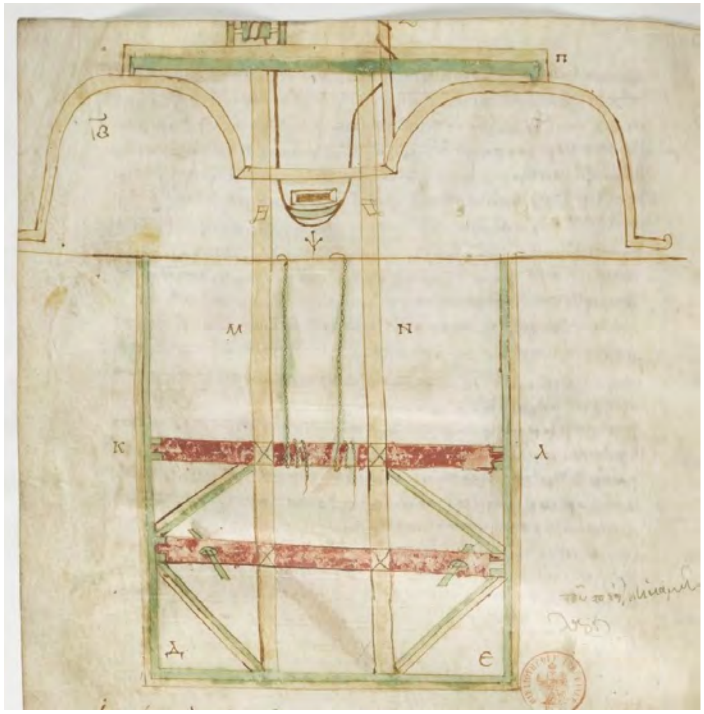
6.2.3 ISIDORUS OF ABYDOS, AT THESSALONICA: 49-51

Biton proceeds immediately to his second *lithobolos*, for the reason that "often local conditions do not favour the same types of engine".[280] Designed by one Isidorus of Abydos, the case dimensions are much larger than Charon's – 15 *podes* or 4.632 metres in length, and 2 *podes* or 617.6mm in square cross-section. It, too, is plated in iron. The core operating principles are the same as Charon's, but the design differs in the next few lines, as the wooden windlass we are familiar with is replaced with an "iron roller" (*kochlias*)[281] above and below the slider. We might understand these to be two separate windlass reels – the lower being for the rather ordinary purpose of drawing the slider *forward* for the loading of ammunition and re-attaching of the bowstring, facilitated by operating the windlass in the opposite direction of rotation to that used to winch back and tension the bow. Biton orders that the rollers be "fitted in bearings".[282]

Next, Biton delves into a dizzying list of beams with their corresponding letter designations and their constraints relative to the surrounding framework. Several of them are braced with iron supports, which are in turn secured with iron plates. Neither their form nor function is very clear until he describes replicating one of them opposite to itself, and adorning them with iron teeth, revealing this particular pair to be either the case's internal slider or the slider proper.[283] A sliding beam is placed on this assembly such that it can move freely up and down the case, and curiously, is "plated flush with iron".[284] Again without specific dimensions, the bow is added – but this time, Biton specifically instructs that it be fixed "through the section of [the case] that is left exposed"; that is, to cut a hole through the case and insert the bow through it. Biton then finishes the design by

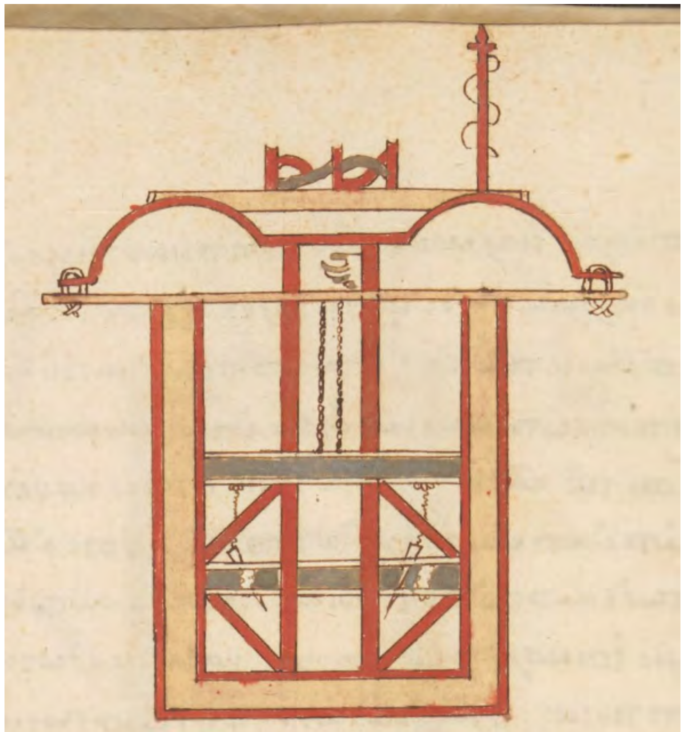
[278] Approximate as per the drawing scale, which lacks component dimensions: Rehm & Schramm, 1929: 29.
[279] Marsden 1972: 80.
[280] Biton, 49.
[281] *ibid.* 50.
[282] *ibid.* This translation will be discussed in depth in Chapter 7: The Isidorus Engine.
[283] *ibid.*
[284] *ibid.*

describing the “sling”[285] or bowstring, which is pulled back by “other little hooks” on the winching rope.

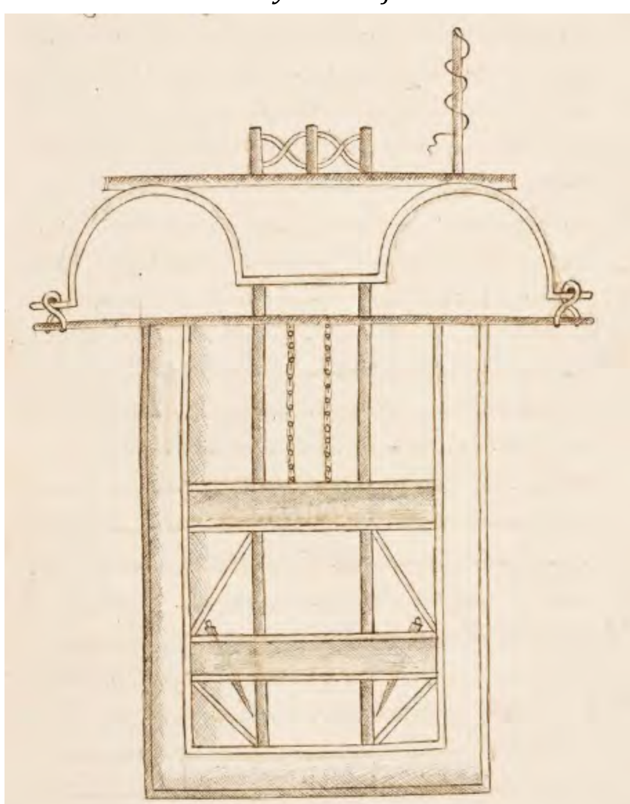


Supplément Grec 607: f.27v

[285] *ibid.* 51.



Burney MS 69: f.11v



Harley MS 6317: f.10r

6.2.3.1 COMMENTS

Again, Rehm & Schramm[286] use a rectangular cross-section for their bow, which eliminates the potential for roll in Marsden's[287] design.

Interestingly, this machine seems intended to survive great stresses commensurate with its size. The slider is plated "flush with iron"[288] at its mating-face with the case's slide, presumably so that there is no timber-to-timber contact which might ordinarily cause binding to occur. This is especially true where swelling or other movement of wooden limbs might cause these moving parts to snag against each other and cause a catastrophic - and possibly dangerous - equipment failure. Judging by the size of the machine, the possibility that it was used in open-air artillery positions is very good, which might expose it to water ingress by rain. Beams like that of the main case, being over half a metre in cross-section as mentioned above, could easily swell or warp to the point of ruining the fine fitment between case and slider. Elsewhere, the iron plating is considerably more generously-applied than on Charon's machine.

One can surmise that the iron rollers in place of a wooden windlass was a decision made for a similar reason. The great tension of the bow presumably caused incredible stress where the winching-rope was secured, and this required reinforcement. Using solid iron rollers of a diameter of "one third of a foot"[289] or roughly 101.904mm (if using 0.33 *podes* for simplicity) does seem excessive, however. Since the text does not suggest otherwise, it may be that the original intention was for these to be iron-plated parts – in other words, a cylindrical iron sleeve fitted over a wooden core.

Biton's use of "other little hooks"[290] is as bizarre as the translation sounds. The full sentence, here is as follows: "Let the hooks on the withdrawal-ropes have other little hooks, which draw back the string of the bow in the course of tightening the rollers." All surviving illustrations, moreover, show the winching-rope as being connected to the bowstring directly. If the winching-rope that pulls back the slider is also used to pull back the bowstring with a hook, then the only means by which the shot can be propelled forward is if the entire slider moves with it. In other words, this layout would

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[286] Rehm & Schramm, 1929: 31.
[287] Marsden, 1972: 83.
[288] Biton, 51.
[289] Biton, 49.
[290] *ibid.* 51.

indicate that the trigger releases the ratchet-pawl rather than the bowstring. Power would therefore be transferred by the bowstring pulling the slider forward, and launching the projectile by effectively allowing momentum transferred by the slider to carry it past the muzzle of the weapon. The great muzzle velocity that might otherwise be achieved stands to be significantly dampened this way; firstly by the friction losses between the mating surfaces of the slider and case, and secondly by the cumulative resistances in the unwinding of the rollers and winch-ropes. Finally, the shock created by the sudden halting of the slider at the end of the case could cause significant damage over repeated shots - not only by the impact of the slider against whatever is to be its stop, but by material fatigue to area around the impact site, joints of the case framework, and the winch-ropes. Neither Schramm nor Marsden mention or reflect on this functionality in their reconstructions, and so it seems prudent to explore it further.

Perhaps this is why Biton specifies that the rollers run in bearings, and why the slider is "plated flush with iron" to ease in movement. If executed precisely such that there were no binding-points in the travel area or the bearings, and both were lubricated with a suitable oil or fat of low viscosity, this combined effort would greatly reduce the magnitude of the friction losses in the firing of the weapon. The extensive reinforcements to the case by way of iron supports might also be intended to withstand the repeated hammering of the slider's kinetic energy being transferred to the frame. This would also explain the "inclined faces"[291] at the end of the ratchet rails – presumably meant as a stop for the slider, rather than facilitating this immensely stressful moment by means of the winch-ropes, which surely could not endure much of this kind of abuse before failing. The necessary caveats that must be kept in mind for this design to have any chance of success, however, are as follows. First, the clearance of the slider and the case must be immaculately finished and perfectly uniform along the entire length of the travel area; this could be a distance as great as 12 *podes* or 3.706m, which Biton cites as the length for the slider.[292] Any variation could cause a jam, with potentially campaign-ending effects for the artillery piece, and danger to its crew. Secondly, the slider would have to be of an extremely low mass relative to the rest of the machine to prevent it parasitically sapping the total work exerted by the bow's release of energy, which would be inevitably wasted in its braking moment. I would suggest that the 12-*podes* framework Biton indicates as having saw-teeth is a static slider; and the unspecified size of beam that he mentions being placed on top of this arrangement and plated flush with iron is a much smaller version of a slider - a shot-carriage, perhaps. Finally, the large case-slider must remain permanently static, and

[291] *ibid.*
[292] *ibid.* 49.

only the lightened carrier be able to move, both during pullback and release. Once again, there is certainly no evidence in Biton's text to indicate the actual size of this carriage, so we are free to make an educated guess.

If this is indeed the design Isidorus had in mind, it may have been an attempt to solve a problem of bowstring breakage on *lithobolos*-type engines by spreading the load across a wider area of bowstring. It is more likely, however, that it is intended to correct a tendency of stones to go wayward during their projection forward without some kind of means to keep their position constant relative to the centreline of the case – a problem that could easily occur if the bow's arms were not perfectly equal due to some variation in the wood grain or craftsmanship, and one that would be solved by simply holding the stone securely along a linear rail for the entirety of its travel while on the machine.

6.2.4 POSIDONIUS THE MACEDONIAN: 52-56

Biton now begins to stray from his original directive, describing a "giant siege-tower"[293] (*helepolis*) built for Alexander, son of Philip, which can only be Alexander the Great. He attributes this specific siege-tower design[294] to one Posidonius of Macedon. A list of wood-species to be used as components to this moving tower is provided, with each being suited for particular purposes. Biton takes a moment to make an aside here, pointing out that the siege tower must, of course, be taller than the wall it is assaulting, and mentioning that there is a "technical method" for ensuring this.[295] He indicates that that method is given in another treatise he has written, the *Optics*, and that he is (per Marsden) "knowledgeable in the art of surveying".[296]

The treatise then turns back to the subject of Posidonius' siege-tower. Biton describes the axles for the wheels that will roll the tower toward a city, and the underlying structure around which the armoured sides must be built. These sides, too, are described as being reinforced with joists and posts, before being "covered in" with wooden boards and rags.[297] Above the wheels and axles, which until now have been driven by "men pushing",[298] Biton describes a platform upon which a capstan

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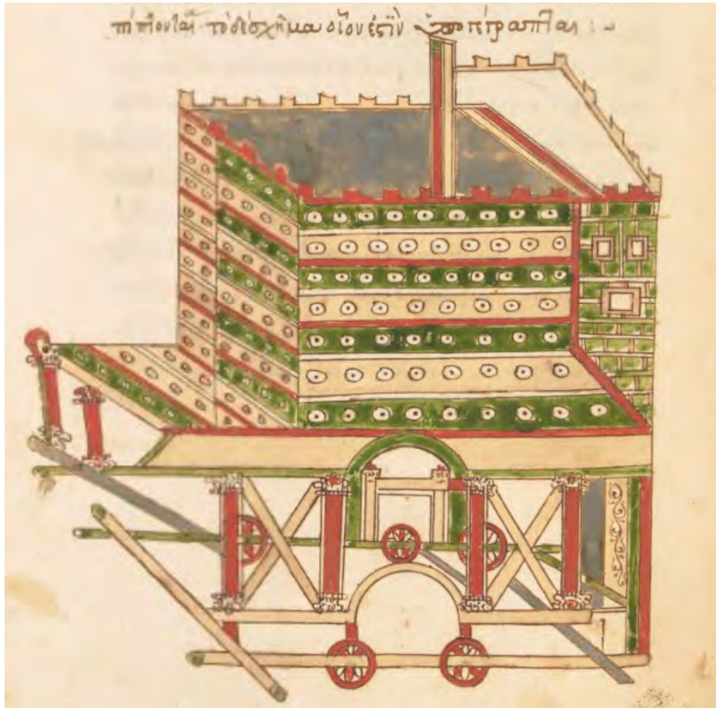
[293] *ibid.* 52.
[294] Naturally, siege towers of various unique designs were present by this time, and as such the originator of the siege tower as a *general* concept is another matter entirely. See Chapter 2: Historical Context.
[295] Biton, 53.
[296] *ibid.*
[297] *ibid.* 55.
[298] *ibid.* 53.

must be mounted to make the axles “easier to turn”.[299] The drivetrain, then, is capable of transferring vertically-arranged rotational motion of the capstan into horizontal rotational motion for the purpose of applying drive to both – for Biton uses the plural – axles.

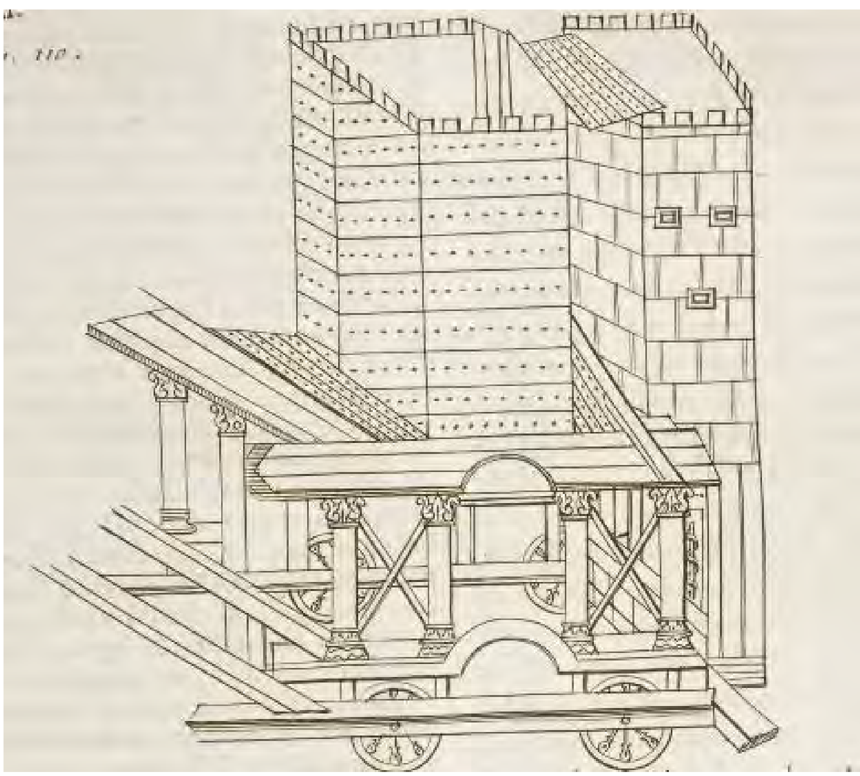
Above this ‘engine room’, so to speak, with its supports and protective walls, Biton calls for a tower to be built, with multiple vertical floors, and a drawbridge at the topmost floor where the tower is to meet the city wall – evidently so that troops can pour out onto the battlements of the besieged. This upper tower is to be additionally covered in fleeces, to catch any projectiles launched at it.[300] Biton then points, again, to an illustration.

Supplément Grec 607: f.28v

[299] *ibid.* 55.
[300] Biton, 56.



Burney MS 69: f.12v



Veterum Mathematicorum: 110

6.2.4.1 COMMENTS

The resulting machine is monstrous. Although its height is dictated by the walls of the city it is intended to take, the axles are given a length of 50 *podes*, or 15.44m. The breadth of this mobile fortress must naturally be this, plus the wheels and armoured sides. Marsden estimates, based on an extrapolation of proportions from a similar invention by Epimachus of Athens for Demetrius Poliorcetes at the siege of Rhodes,[301] that the finished breadth and width of Posidonius' tower may have been intended to be some 48 by 48 cubits, or 22.234m for each.[302] Height, naturally, was dictated by the walls of the city which the tower was intended to assault, and Biton does not mention at what particular city or cities Posidonius may have deployed the original design. Purely for illustration, we can observe some roughly contemporaneous examples like that given by Diodorus Siculus, who mentions that a tower deployed at the siege of Rhodes - perhaps, also, that of the aforementioned Epimachus - had a height of 100 cubits, or 46.32m.[303]

While the manuscript illustrations are roughly correct in their proportions, to judge from what we can glean from ancient sources, several of them have been adorned with fantastic accoutrements that make them appear more like decorated cakes than weapons of war. The reconstructions of Marsden and Schramm seem far more harmonious with the text.

The tower, however, is secondary in interest to the brief moment of insight Biton grants us into his background. It is one of few glaring exceptions to his general rule of brevity when Biton makes his aside regarding his *Optics* (Ὀπτικά)[304]. He specifically refers to his familiarity with the use of a *dioptra* or "τοῦ διοπτρικοῦ", although Marsden translates this as "knowledgeable in the art of surveying". It is interesting that Biton would see fit to take a moment to pin this particular medal on his chest in the midst of an otherwise terse treatise. We might make the conjecture that, unlike his more well-rounded colleagues in antiquity, Biton was a man of civil works in particular, and the art of war was simply not his specialty. This brief word on optics might be a kind of veiled apology for his unfamiliarity with the subject at hand, indicating that his particular field of study was elsewhere, and that any errors in his recommendations should be ascribed to this fact. But why, then, would he be consulted or otherwise offer advice where he was not strictly qualified to do so, particularly when the Attalids evidently had one or more competent artillery engineers on staff? Were they on

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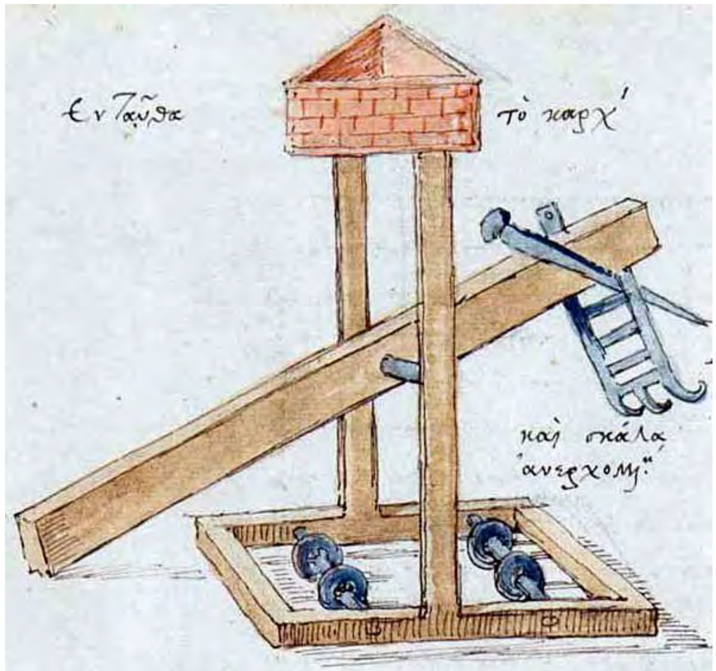
[301] Vitr. *De Arch*. 10.16.4.
[302] Marsden, 1972: 84. The cubit in this case being derived from, and equal to, the Greek *pechys.*
[303] Diod. Sic. 20.91.1-4.
[304] Biton, 53.

campaign, and the treatise addressed to Attalus as a formality of the court in his absence? This seems both unlikely, and a stretch of the imagination.

6.2.5. DAMIS OF COLOPHON: 57-61

Biton's next description is also a device for surmounting walls, which he refers to as a *sambuca* or "σαμβύκης". This, too, is mobile, its wheels running on two "parallel axles".[305] However, there are no walls nor armour plating here. Instead, there is a framework of iron-plated joists and trestles, and mounted upon them is a horizontal "roller"[306] some 15 *podes* or 4.632m long and 19 *daktyloi* or 336.7mm in diameter. It is equipped with "a capstan instead of the bearing", so that it can be manually rotated by hand.[307] Mounted to this roller is a bracket, through which the titular *sambuca* – a great, wall-scaling ladder – is passed, and fastened such that one end is shorter than the other. The gigantic rolling see-saw that is thus created is then equipped with a box of weights at the short end, and a gangway at the other, so that it stands – at rest – with the ladder in the air and the weighted end trailing on the ground.[308] The desired order of operation, Biton says, is that one can order an assault squad to climb onto the gangway, and then roll the machine forward and drop the *sambuca* down onto the battlements of a city wall by turning the capstans. He points to the illustration, once again.[309]

[305] *ibid.* 58.
[306] This translation is also disputed (see Drachmann 1972: 490 and Lewis 1999: 163), but this is not relevant for the purposes of this explanation and the focus of this thesis.
[307] Biton, 59.
[308] *ibid.* 60.
[309] *ibid.* 61.



MS Hunter 220: f.7v

6.2.5.1 COMMENTS

The few accessible manuscript illustrations that accompany the *sambuca* are much more interpretive than they are technical. Once again, this particular machine is not our primary interest, but the implicit problem-solving capabilities of the ancient engineer are worth noting. Normally, infantry might place a ladder against the walls of a fortress and rely on covering fire from engines and archers to keep the defenders from firing down or finding some means of knocking them off of their ladders during their vulnerable ascent. Once at the top, there is every possibility of being speared or pushed off during the process of clambering off of the ladder and onto the ramparts, righting oneself, and then readying a weapon. The *sambuca* negates this disadvantageous position by depositing standing troops directly onto the wall via the gangway, with weapons at the ready, such that a beachhead can be established and reinforcements brought forth without hindrance.

6.2.6 ZOPYRUS OF TARENTUM, AT MILETUS: 62-64

Biton then returns to projectile weapons, though not of the kind we are primarily concerned with. He switches back to the singular "I", here, and introduces this one as a *gastraphetes* (γαστραφὲτες), or "belly-bow" in Marsden's translation, describing it as "the one which Zopyrus of Tarentum designed at Miletus".[310] However, it is immediately apparent that this is not a *gastraphetes* of the type that Heron introduces us to in his *Belepoieca.*[311] Instead of a small crossbow that is pulled back by hand and braced against the stomach, this appears to be a frame-mounted weapon. Biton describes this mounting base before giving the weapon a "hollow beam, the usual type for catapults"[312] which we can safely assume is the case of the weapon. The length of this case is 9 *podes* or 2.779m, placing it roughly halfway in length between the Isidorus and Charon engines. Although we are not given a breadth and height for the cross-section of the case, we can assume it is more slender than the *lithobolos*-type engines, as Biton then reveals that this mounted *gastraphetes* is meant to fire bolts with a "perimeter of cross-section of 15 *daktyloi*", or 0.29m.[313] Marsden and Rehm both make emendations here to correct the text which would otherwise specify a *diameter* of 15 *daktyloi*, which would not fit the other dimensions at all and would create a bolt sized and shaped in the manner of a modern artillery shell - one of many such errors in the text. Instead, the estimate is a bolt of 2 *daktyloi* or 38.6mm in diameter.[314] The length given by Biton is 6 *podes* or 1.853m.

Having dealt with the case, Biton adds a "strong bracket" at the front of the machine to hold "the appropriate bow". This time he provides dimensions: "a periphery of 9 *podes*, and a perimeter of cross-section of 4 *daktyloi*"[315] – some 2.779m wide and 77.2mm in diameter. Interestingly, Biton also stipulates that "the bow have perfectly balanced springs (*tonoi*)".[316] Immediately after this comes a bizarre development. We are instructed to make "two apertures", "parallel to the slider" (a constraint that we should take to mean 'collinear'), "through which the bolts will be discharged". Zopyrus' machine is double-barrelled, firing two bolts at a time.

Biton then describes a system of strings and cords that interact with both the bowstring and slider, eventually terminating at a pair of "wheels" at the base of the engine. It seems that here he makes up for his earlier lack of discussion on the routing of cables by making explicit reference to the slider and its connection to the windlass via the winching-rope. With this, he points to the diagram.

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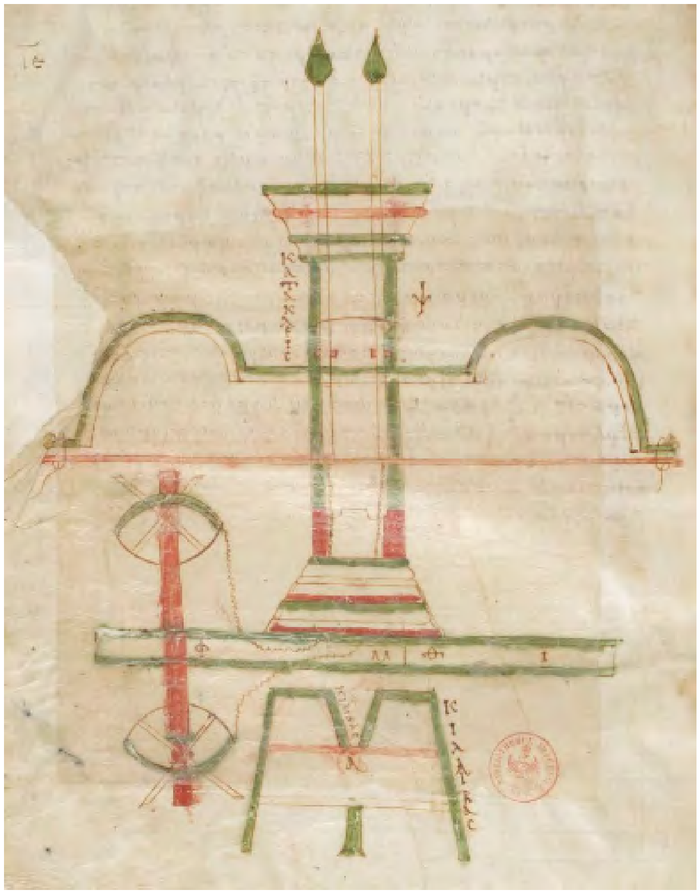
[310] Biton, 62.
[311] Heron, *Bel.* 76f.
[312] Biton. 62.
[313] *ibid.* 63.
[314] Marsden, 1972: 89.
[315] Biton 63.
[316] *ibid.* 63.



Supplément Grec 607: f.30v

6.2.6.1 COMMENTS

Here, elements of truth in the diagrams are plainer to see. A recognisable case and rear-mounted double-windlass, with centrally-aligned bolts loaded into the slider, inspire some confidence in the otherwise fanciful double-barrelled system.

Judging by Biton's comment regarding the balancing of the bow, one can only assume that an imbalance between the two sides or *tonoi* of the bow would cause an uneven distribution of power and the shot would go wide. This may lend credence to the theory presented here regarding the Isidorus machine's possible operation, but would directly contradict the corrections made by Marsden[317] and Schramm,[318] who either pass over the point or discount it as an error, choosing to have their Isidorus reconstructions' winching-cable connect to the slider alone.

[317] Marsden, 1972: 81.
[318] Rehm & Schramm, 1929: 31.

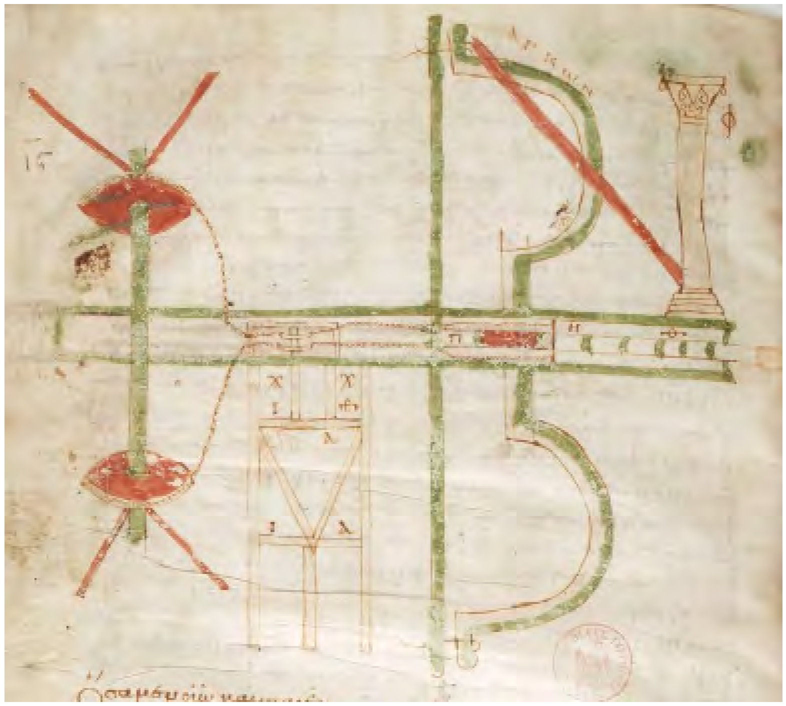
6.2.7 ZOPYRUS OF TARENTUM, AT CUMAE: 65-67

Biton's final machine is another designed by Zopyrus of Tarentum, on this occasion while residing at Cumae. He shifts once again to the plural "we" while making this recommendation. It is described as a 'mountain *gastraphetes*' and has a case whose dimensions are a "length of 5 *podes* [1.544m], a height of 1 *pous* [0.31m], a breadth of 3 *podes* [0.926m], hollow inside".[319] The bow of this model is also bracketed, and is 7 *podes* or 2.162m in width, and 9 *daktyloi* or 173.7mm in perimeter of the cross-section; about 55.29mm in diameter.

Additional equipment is then specified for the engine; at a distance of 3 *podes* or 926.4mm from the front of the case, a "cup-bracket extending up to the curvature of the bow", and "a column… 3.5 *podes* [1.080m] away from the bow, performing the function of an axle, which, when positioned, will stand everything carried above it."[320]Although the precise nature of the assembly is unclear, it seems that Biton is recommending equipping the gun with the equivalent of a bipod on a curved rail, presumably such that it can be depressed or elevated sufficiently for effective aiming in mountainous terrain.

Finally, Biton describes the rear of the machine in what seems an oddly trite explanation of an obvious, well-established component. He labels the winch and roller as a "wheel fitted on an axle" and connected via cord to a hitherto uninstantiated beam labelled T, such that "the missile… be projected more powerfully and further".[321] The illustration is once again pointed to.

[319] Biton, 65.
[320] *ibid.* 66.
[321] *ibid.* 67.



Supplement Grec 607 f.31v

6.2.7.1 COMMENTS

This engine is smaller by case size than Charon's, and the smallest of the treatise. The sense one might have, then, is that the 'mountain' designation is not an indicator of size, but of use-case. The ease of movement, set-up, and breakdown of a small, lightweight anti-personnel field gun is of obvious utility to infantry trekking through a mountainous environment.

The usefulness of the explanation and the diagrams, in this case, are undoubtable: here, we are treated to a recognisable explanation of a double-windlass and winching-ropes, and a proportionately sensible case. A single bolt is placed in the correct area of the case. Less useful, however, is the scribe's interpretation of a front "column" – an architectural feature has been added above the case, rather than a wooden brace that we might otherwise recognise as an underslung attachment.

6.2.8 CONCLUSION: 68

Biton makes his closing statement with characteristic compactness. He asserts that all those engines which he – or rather, "they" – have considered of use to the reader have been thus related. Oddly, they are clearly not meant for use in precise scale. He asserts that the reader may "work out similar designs", and that one should not be dissuaded from doing so because of his provision of "definite measurements". Instead, Biton seems aware that the use-case may be fluid, or different from that intended by the designers of each weapon that has been relayed: "If you wish to construct larger or smaller engines, do so; simply try to preserve the symmetry".[322] He ends by pointing, once again, to his diagrams.

6.2.8.1 COMMENTS

This closing statement is once again useful for determining the context for this treatise. In the absence of more information, we are forced to extract what little we can through the use of deduction. The intended audience, it seems, is one that is at least passably familiar with the operation of siege weapons; or at least the theory of their construction. Their precise needs are uncertain, and the engines described by Biton are intended to be modified for purpose through the use of measurement scaling and, possibly, other modifications. As we are now aware by our study of scaling in Philon, Heron, and Vitruvius, the intention here is almost entirely centred around designing for the ammunition one wishes to use in a given machine.

The use-case, as we learn from the introduction, is repelling other engines of unknown type, and it is for this reason that Biton makes his recommendations of two kinds of *lithobolos* before any other. The reason for his inclusion of wall-scaling equipment is unclear; it may have been useful in counter-manoeuvres, or it may be that this was a later addition to round out the treatise in the style of others. The latter is doubtful, however, as the sections on these machines are in the same hurried, clipped style of writing as the others, indicating the continued presence of time-pressure.

Nevertheless, it is the projectile engines, which are most suited to repelling an invader, that are our focus. They are unified by their use of non-torsion bows for moving a projectile; as we are now aware, however, this is highly irregular for the period of activity of any known King Attalus.

6.3 COMMENTS ON TRANSLATION

With regards to the English translation by Marsden, there are some technical issues – and some literary – that could stand to be commented on. While there are few quibbles I can make regarding Marsden's harmonisation of the Greek in terms of variables and measurements, some of the terms

[322] Biton, 68.

used to communicate the intention of the text, as they have been relayed above, might be potentially misleading for the would-be designer of siege weapons, and for the scholar attempting to place Biton.

Marsden's translation is, as a general rule, aimed at expanding the text in order to illuminate unclear details. While this is a welcome goal for such a dense text, expansion may also be unnecessary in some places where tweaking of the diction can convey the meaning appropriately.

In multiple places, the German translation of Rehm is a useful comparison, and for this reason, I will provide his version alongside the Greek passages I wish to examine as they appear in Marsden. With some commentary and small re-interpretations of the text, I hope to communicate some of the otherwise unappreciated details that remain here.

6.3.1 BITON'S INTRODUCTION, 43-44

The introduction provides us with the most important contextual clues of the treatise, and deserves closer inspection.

Biton's words, per Marsden, read as follows:

> "Λιθοβόλου ὀργάνον κατασκευὴν ἐπιβέβλημαι γράψαι, ὦ Ἄτταλε βασιλεῦ· καὶ μὴ σκώψῃς, εἴ τινα ἑτέραν αὐτοῦ εἰς ὑπόθεσιν πίπτοντα τυγχάνει ὄργανα... δι'ὧν πέπεισμαι, ὅτι ταῦτα τὰ κατὰ τὰς προσβολὰς τῶν πολεμίων ὄργανα ῥᾳδίως ἀναστρέψεις, ἀντιστρατευόμενος ταῖς ὑπογεγραμμέναις μεθόδοις. πειρῶ δὲ ταῖς ἐπιστήμαις χρῆσθαι· χρή γὰρ χρῆσθαι καὶ τοῖς μέτροις καὶ ἒτι τοῖς ῥυθμοῖς τῶν προβεβλημένων." (Biton, 43-44)

Rehm & Schramm[323] provide the following interpretation in German:

> "Die Konstruktion eines steinwerfenden Geschützes habe ich unternommen zu beschreiben, o König Attalos; und spotte nicht, wenn einige Maschinen zu einer davon verschiedenen Aufgabe gehören… Durch diese bin ich überzeugt, daß Du leicht diese zum Angriff bestimmten Kriegmaschinen zurückschlagen wirst, indem

[323] Rehm & Schramm, 1929.

> Du nach den unten beschriebenen Methoden Dein Gegenmaßnahmen einrichtest. Versuche es aber die Wissenschaften heranzuziehen. Denn man muß sowohl die Maße wie die Formen (entsprechend) dem Vorgeschlagenen zur Anwendung bringen."

Marsden's translation in English:

> "I have set out, king Attalus, to describe the construction of a stone-throwing engine; and do not scoff at me if some engines perhaps belong to a type different from this. I am convinced that, with their assistance, you will easily repulse those engines employed in the offensives of your enemies, if you counter-attack by the methods described below. Endeavour to apply general technical knowledge, for it is necessary to use both the measurements and, furthermore, the proportions of the engines here considered."

My own reading, in English, is as below:

> "I have undertaken to describe the construction of a stone-throwing machine, O King Attalus; and jeer not if another machine differs from this one… through this I am convinced that you will be able to easily field a counter with this war-machine, while you, in following the method described below, manage your countermeasures. But try to make use of common sense (or, experience). Because one must take both the dimensions and profiles (to correspond to) the given application."

A minor criticism is in Marsden's "do not scoff at me"; though fairly accurate "jeer not", in the style of Rehm's "spotte nicht", is better. More to the point, however, the all-important situation that requires the treatise be written in the first place seems somewhat misrepresented. "Repulse" and "zurückschlagen" seem both to imply that an active offensive be mounted, but I read "ἀντιστρατευόμενος" in this context to be more akin to "fielding a response", in that it does not necessarily mean sally forth so much as it does to merely prepare to meet an enemy in the current position.

Marsden’s “offensives of your enemies” seems to imply complex manoeuvres on multiple fronts by an organised enemy, while “zum Angriff” is a better means of conveying that these are non-specific aggressions by an enemy whose organisation or lack thereof is an unknown quantity. Next, Marsden’s absorption of Rehm’s “Gegenmaßnahmen” into his “counter-attacks” seems hasty, and I add the English equivalent “counter-measures” here in support of Rehm’s reading. Both Rehm’s translation - and, I hope, this one – convey that more than one strategy is enabled by the use of the weapons in this treatise, and mounting a march on the offensive is not necessarily the only one available.

Finally, the use of “Wissenschaften” in the decision-making process regarding proportions and measurements is only applicable insofar as the proportionality of Euclidean geometry is, to the ancient Greek, science. For this reason, Marsden’s “general technical knowledge” conveys a better sense for the layman’s interpretation of what geometry is in our current environment. I have settled instead for “common sense”, as the application of proportionality might be, considering that the audience here is at least technically-minded.

6.3.2 BITON’S LOVE OF THE DIOPTRA, 53

As mentioned in our close reading above, Biton’s aside regarding surveying is an unusual one. The Greek is copied here below:

> “ἒστι δὲ καὶ τοῦτο μεθοδικὴ θεωρία, ἣν διείλεγμαι ἐν τοῖς Ὀπτικοῖς· ἒγκειται γάρ μοι τὸ γένος τοῦ διοπτρικοῦ.” (Biton, 53)

Rehm’s reading is appropriately expressive of Biton’s apparent feelings on the matter:

> “Auch dies beruht auf einer methodischen Theorie, die ich in der Optik entwickelt habe. Denn mir liegt die Dioptrik am Herzen”.

Marsden’s, however, is comparatively stark:

> “There is a technical method for achieving this which I discussed in my Optics, for I am knowledgeable on the art of surveying.”

My own reading is as follows:

> "This is based on a methodology which I developed in the Optics. This is because I am devoted to (or, wrapped up in) the family (or, the art) of using the *dioptra*."

The Marsden translation does not adequately convey how enthusiastic Biton apparently is for the use of the *dioptra* – the ancient precursor to the theodolite used for surveying in civil works, and occasionally for siegeworks, as in this context where an enemy fortress-wall must be dimensioned. This, I think, lends credence to the notion that Biton is not primarily a man of war. Instead, his 'kin', in the Homeric sense, is that of users of the *dioptra*, a tool that occupies a symbolic place, perhaps in a similar sense to the compass of a Masonic order.[324]

6.3.3 CLOSING STATEMENT, 67-68

Biton's final words are also a valuable source of peripheral information. The Greek, once again, is as follows:

> "Ὅσα μὲν οὖν μάλιστα ἐνομίζομέν σοι ἁρμόζειν, ἀνεγράψαμεν. πεπείσμεθα γάρ, ὅτι σὺ διὰ τούτων τὰ ὁμοειδῆ ἐξευρήσεις. μὴ παραταραχθῇς δέ, ὅτι ἱσταμένοις μέτροις κεχρήμεθα, μήποτε καὶ σὲ δεήσῃ τοῖς αὐτοῖς μέτροις κεχρῆσθαι. ἐάν τε γὰρ βούλῃ μείζονα κατασκευάζειν, ἐπιτέλει, ἐάν τε ἐλάσσονα· μόνον πειρῶ τὴν ἀναλογίαν φυλάττειν. τὰ δὲ σχήματα καὶ τὰ μέτρα προγέγραπται." (Biton, 67-68)

Rehm & Schramm make a somewhat obtuse reading:

> "Wir haben Dir alles aufgeschrieben von dem wir annahmen, es möchte Dir am meisten passen. Wir sind ja überzeugt, daß Du selbst vermittelst dieser Beschreibungen das Gleichartige ausfindig machen kannst. Laß Dich aber nicht dadurch in Verwirrung bringen, daß wir feste Maße verwenden, als ob Du auch dieselben Maße verwenden müßtest. Denn wenn Du größer bauen willst, so tue es,

[324] See Chapter 3.2: The Scribal Problem for a comparison with the educated class of engineers in the Middle Ages.

> und auch, wenn Kleiner**:** Du brauchst lediglich auf die Analogie Acht zu geben. Die Zeichnungen und die Maße sind ja vorn angegeben."

And Marsden's translation follows Rehm's relatively closely:

> "Whatever engines we have considered particularly suitable for you, we have now described. We are convinced that you will be able to work out similar designs by means of the ones provided. Do not be worried by the thought that, because we have used definite measurements, it will be necessary for you to use the same measurements, too. If you wish to construct smaller or larger engines, do so. Simply try to preserve the symmetry."

My own, clipped, is as follows:

> "We have described to you everything which we (assume/judge) will suit you best. We are convinced that you yourself can (mediate/navigate) these descriptions, and make a similar discovery… Try only to consider the (proportions/overall model design)."

I do not dispute the use of 'larger and smaller' here in both Marsden and Rehm – it is an appropriate and important echo of the introduction's clues regarding intended usage of the designs assembled here. The ability of the reader to make their own inferences is again reinforced, which leads one to assume that this treatise intended for someone with at least some familiarity with the concepts and style of description that Biton uses. Both Marsden and Rehm attempt to do some justice to the Greek "τὴν ἀναλογίαν", which we might translate as the "dimensionless features of the model". The German, I believe, is more accurate; "die Analogie" conveys the sense of a Platonic model of the machine as a dimensionless, infinitely-scalable item. Marsden's "symmetry" is, on the other hand, less evocative of that concept. For the sake of space, I substitute "proportions", or "design", but I do not believe that either of these quite reflect the sense of the original text.

7. THE ISIDORUS ENGINE

We have established that non-torsion engines were thoroughly outclassed in range by torsion ones, and that this creates a major difficulty for us in placing one of Biton's machines at Pergamon in the second century BCE.[325] However, we can tell from Biton's descriptions that one of these engines was, perhaps, large enough in scale to make for an adequate stopgap.

I have attempted to write a line-by-line comparative version of the passage from Biton's instructions for building this machine in lines 48-51, using some current design terminology. This is entirely for the sake of interest as it inserts, removes, and moves elements around from the original text to suit my own interpretation. That said, the basic order of operations is the same as it appears in Biton. Due to its inadmissibility as a scholarly piece, I have made an addendum of it. It may be of use for reading this analysis, or for attempting a reconstruction yourself.[326]

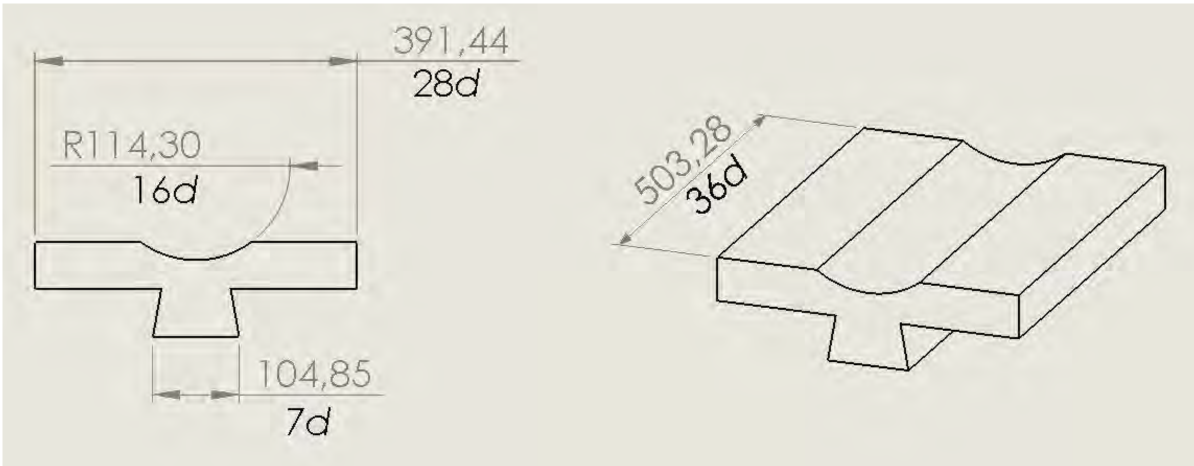
7.1 BITON'S RECOMMENDATIONS IN CONTEXT

Biton's note regarding equal tension of bow *tonoi* in his description of the Charon engine seems to indicate that inaccuracy as a result of bow tension issues may have been a legitimate issue. On an engine the size of Isidorus', the possibility of variation in the craftsmanship and natural defects in the timber over such a great breadth of bow can only have exacerbated any such issues. An engine whose shot goes wide - particularly when it is a great stone-shot of considerable destructive potential - may do serious harm to one's own fortifications, equipment, or crew.

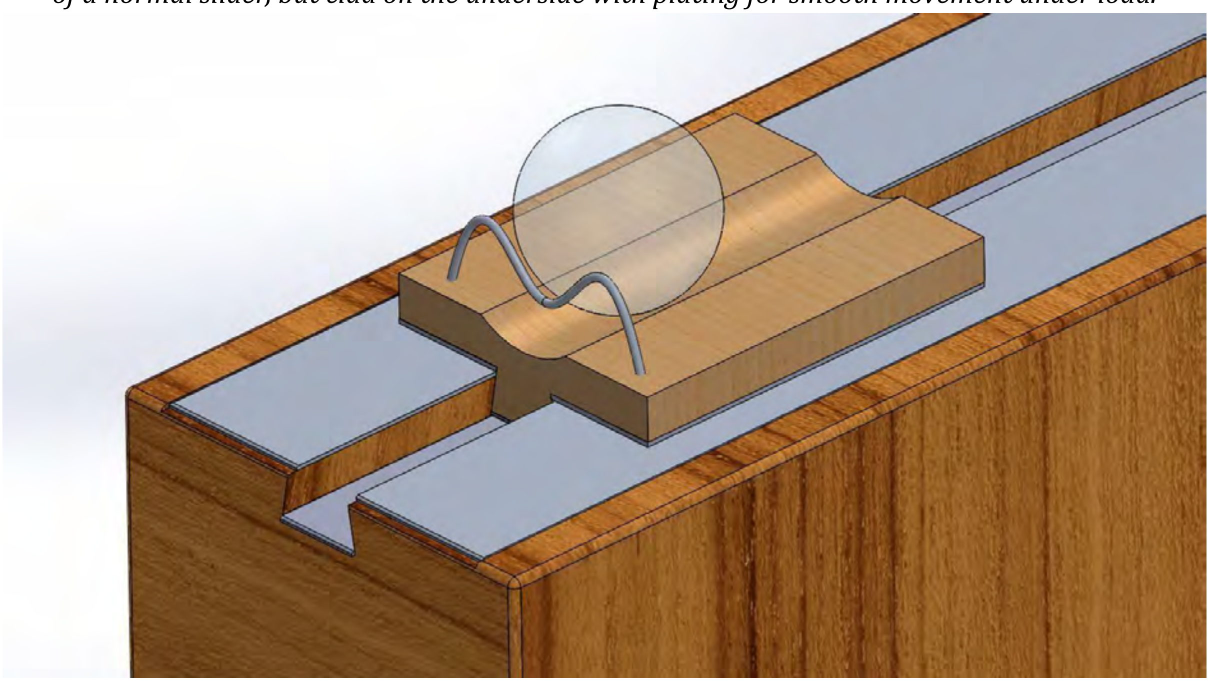
If Biton's description is correct, and the Isidorus engine did indeed have its bowstring hooked to the winch-rope, marrying it to the slider, the use of this 'shot-carriage-slider' configuration would carry the stone shot in a perfectly linear motion, so long as the clearance between slider and case is not so tight or so loose that it be allowed to jam. The 'inclination' at the end of the ratchet that Biton specifies would function as a stop, and presumably, the pawl would function as the trigger. Of course, there would be no safety mechanism now that the ratchet-and-pawl is able to be released. Then again, the great tension placed on the pawl by the stored energy of the bow would make for an incredibly heavy trigger-pull; something of a safety measure in itself. Moreover, we should not assume that these engineers of war were as concerned with trigger discipline and operational

[325] See Chapter 1: Introduction.
[326] See Appendix 2: Isidorus In Design.

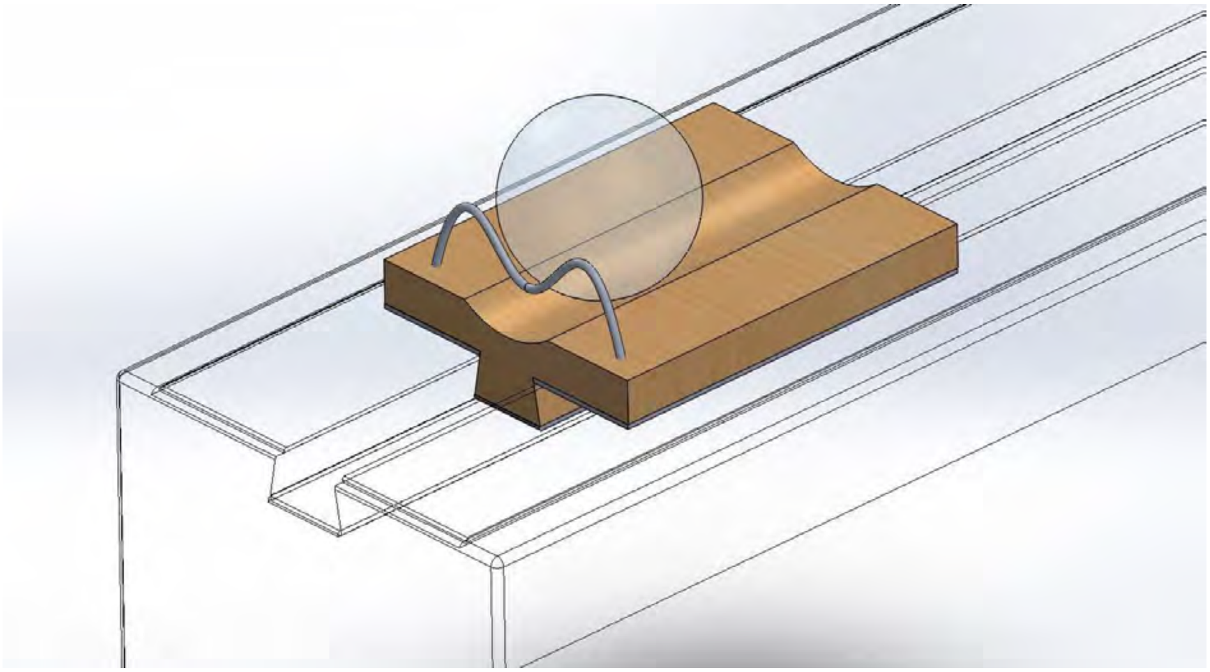
health-and-safety as we. Finally, the use of 'bearings' would be entirely justified, as would iron-plated winch rollers, considering that the winching system and slider are now made to be part of the train of power delivery.



Possible dimensions for the 'shot-carriage-slider' in millimetre and daktyloi *[*d*]. A fraction of the length of a normal slider, but clad on the underside with plating for smooth movement under load.*



A visualisation of the principle at work in CAD. Here I have taken the liberty of cladding the case's internal slider as well, to negate the possibility of the carriage fouling on wood burrs.



The same three-quarter view, but with the case in wireframe. The visualised shot is the 9" Marsden estimation, but in the closest Greek equivalent of 16 daktyloi*.*

Finally, a front elevation with exaggerated clearances; note the cladding is recessed, here, but this is not necessary in practice if the dovetail is routed to a greater depth.

The placement of the Isidorus machine second to Charon's in the treatise might indicate that Biton was aware of the difficulty of the extra considerations required to make that machine work – that is, the quantity of plating and fine tolerances in finishing and fitment that are required. We would be thinking too little of Greek manufacturing capabilities to discount the machine along these lines, however. Biton explicitly mentions the toothed ratchet rails be made “without joins”,[327] which would imply the availability of flat bar-stock in raw form from which parts could be made; or, at least, pattern-welded together by hammer forging.[328] Hardening the ‘ways’ of this machine would be entirely possible in a large-enough quenching trough. The potential gain in accuracy by using this design, along with the starting bulk of the Isidorus engine in its standard format,[329] would bring it much closer to being able to effectively fire weights of stone shot closer to the greater proportion of ammunition type found at Pergamon.

Estimating the precise size of shot fired by Isidorus’ engine, and its range, is very difficult without knowing the specific dimensions of the bow and its construction. Given the sheer size, it is all the more likely that it may have been a composite of wood, horn, and other reinforcements. We can make some estimation of the girth given how much of the case is left after the various components have been fitted, but this is useless without a measurement of length. We could also linearly scale the bow from Zopyrus’[330] machines, whose measurements we do have, to give ourselves a vague guess. This has been Marsden and Schramm’s inclination.[331] However, we know from our brief study of the relationships between material density, modulus of elasticity, and spring tension, that this is not a satisfactory means of scaling a spring, which may have many other variables impacting on its ability to store and release energy. Instead, our best hope at ascertaining the precise size of the Isidorus bow is by making a dimensional analysis of several composite bows analogous to that of Zopyrus’. Potentially, this analysis may allow us to make a far better estimation of the scale of bow required to meet the spring tension necessary to fire one or more sizes of stone, as opposed to a bolt, at various ranges. Due to Biton’s specific instructions on fitting the bow,[332] the limiting factor on scale will be the cross-section dimensions left for us to fit it into the engine’s case without considerably undermining its structural integrity.

Without further research involving a reproduction and analysis, then, we cannot have a sure answer on the measured variables necessary to make a good comparison of the engine. However, we can provisionally use the estimate by Marsden as an indication. His estimate of the ammunition used by

[327] Biton, 50.
[328] In the style first attributed to Glaucus of Chios, the welder, attested by Paus. *Descriptions* 10.16.1. Diod. Sic. *Hist.* 5.13.1-2 mentions the chain of refinement from iron ore to iron sold at markets.
[329] Assuming it was a tested and proven design.
[330] See Chapter 6: Treatise.
[331] By Marsden’s admission (1972: 80).
[332] Biton, 50.

the Isidorus machine is 40lbs or 18kg,[333] which in turn is based on an analysis of the estimated volume, and therefore mass, of the projectile.[334] His source for this data takes a protracted route. First, he estimates the capacity of the Charon engine from the diameter of the bowstring specified by Biton as suitable for the stone's size.[335] By comparing the case widths between the two machines and finding the Isidorus engine to be double Charon's in that dimension, he makes the conclusion that the diameter of the stone shot placed on it must, too, be double. However, there are some caveats to this estimation on account of the multiple variables in play; scaling of mass, spring tension, and the estimate of Charon's bow by first using one scaled from Zopyrus. Nevertheless, we shall endeavour to make a convincing conclusion in spite of this missing data.

7.2 A NOTE ON "BEARINGS"

In this analysis, I refer to the use of "bearings" in the Isidorus machine, and the usefulness of this design feature should we construct it in the manner Biton appears to have intended – that is, with the slider (or "shot-carriage") of the weapon delivering the stone shot by direct interface with the bow. In this case, we depend heavily on the ability of the rollers to move freely under high tension.

Drachmann is right to criticise Marsden's use of "bearing" for the text,[336] where the Greek *epitonion* is better translated as a 'pin' or 'axle'. 'Bearing' would imply that there is a sub-assembly of rollers, of a ball- or needle-type, between the face of the roller, or *kochlias,* and a circular cut made into beam *K* to accommodate it. We might cite the evidence for the possibility of their integration by way of the cast bronze bearings of the Nemi archaeological finds; a pair of ships, upon which a rotating assembly was found, which uses bronze bearings in a hardwood race.[337] However, these are tentatively dated to the 1st C. CE, and are thus very far from the likely time of Isidorus, if we are to place him around the foundation of Thessalonica in the late 4th C. BCE. The other is that refining a bronze casting for use in this relatively complex assembly would require time and labour that might not be available in a crisis.

The shorthand style of Biton's text does not provide us with any precise indications of the fine relationships between components. Neither do the illustrations have any component views or side

[333] Marsden, 1969: 15.
[334] Marsden, 1972: 83.
[335] Biton 47.
[336] Drachmann, 1972: 490.
[337] Ceccarelli, 2019.

elevations that can assist us. Both Marsden and Schramm devised a plan for the Isidorus machine, but never reconstructed it to test its tractability. Schramm places the rollers into curved cutouts in his open-case design,[338] but this does not follow the unit-construction of the main case as it is described by Biton, and will allow the roller to continue to experience problems of binding against the bearing face, rather than allowing for unrestricted movement, when tension is applied to the windlass assembly by the bow. Marsden retains Biton's solid beam case construction,[339] but does not incorporate "bearings" of any kind, preferring to have the rollers ride in an unclear location directly cut into the frame. They are partially-exposed, as a result of his interpretation whereby the top beams of the frame are partially sunk into the case, which unfortunately leaves no room for the rollers to fit. Thus, it seems, they are likely to bind in place when experiencing applied stress. The result is that neither of the above reconstructions are able to adhere fully to the text, and I suspect that both would encounter operational problems in their respective roller assemblies.

However, let us accept the implication of Marsden's translation for a moment. These components, as described by Biton, are gigantic in scale; the roller has a diameter of one third of a *pous* or 101.94mm, and is fitted into a beam whose breadth is 1 *pous* or 308.8mm, but no other dimensions are specified. (We might assume that it is square in cross-section, as Schramm and Marsden have done, but there is no evidence to support this either way.) Let us assume the roller is inserted breadth-wise into the beam. We will be conservative and presume to insert it only 200mm into the 308.8mm body. The contact area between roller and beam is, effectively, that of an open cylinder of 101.94mm diameter and 200mm length; roughly 64cm^2. Considering the size of the components, and the fact that this roller is also inserted into a beam at its other end, the potential kinetic friction[340] on this part is considerable. If the wood of the beam were to swell and close-up the fitment between roller and hole,[341] no amount of lubrication would prevent it from becoming totally immovable - preventing the slider from operating, and rendering the machine and its crew dead in the water.[342] Specifying iron for the roller, as Biton does, will greatly reduce the incidence of this problem by ensuring that only the frame of the catapult has the ability to shift. However, a small change in temperature or humidity could still potentially render the machine inoperable. It is quite unbelievable that this would be tractable at all, and there are simple solutions to combat it. We have established that the omission of details from Biton's text, here, may well be made along the theme of

[338] Rehm & Schramm, 1929: 31.
[339] Marsden, 1972: 83.
[340] Atkins & Escudier, 2013: 759.
[341] Thus increasing the force applied over the area of friction.
[342] See Childs (2019:877) for tolerances and fits between parts.

what the audience might be expected to know should they have some technical expertise, or, fuller plans to refer to once a choice of machine is made. It would be foolish, then, not to investigate the solution we might use for reconstruction, and to do so in a way that is sympathetic to the ancient engineering tradition.

If the roller's ends are reduced in diameter, but the nominal diameter of its main body retained, we can reduce the friction between the components dramatically. Let us, for illustration's sake, make the reduced 'bearing face' 50mm in diameter, roughly half that of the main body, for the entire insertion length of 200mm. The shape of the roller thus becomes more akin to a rolling-pin, and the effective contact area of the roller is greatly reduced. Although this will alleviate the problem of sticking rollers and also satisfy the text, it remains that the frame of the catapult can still pinch the roller with relative ease.

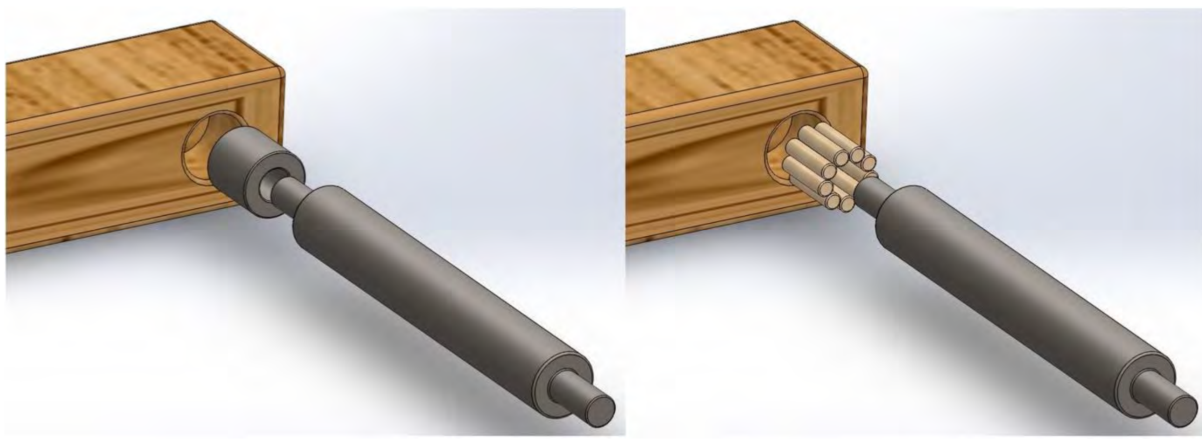
The solution I would offer, which is a midpoint between Drachmann's uncompromising approach to the text, and Marsden's more interpretive one, is a simple bushing. By reaming out the roller's bearing-face in the top beam to the full 101.94mm diameter as per the text, and reducing the roller's ends to 50mm, we are left with a void between the two that can be occupied by a bushing.[343] If the engineer executing the plans were knowledgeable, he would use one made of cast bronze for its properties of lubricity and malleability.[344] The alternative, if using a plain roller, is that the entire upper frame of the catapult will need to be discarded when worn beyond operability. However, if we take this to be too much of a stretch of the evidence, we can presume to insert an iron one instead. This way, only the rollers and bearing would require replacing; and most probably only after the wooden structure of the catapult has already begun to lose its structural integrity. The operating principle is simple enough to be available to an engineer of Isidorus' time – a simple iron peg in an iron tube, inserted flush into the wooden frame – and would greatly reduce the problem of frictional losses in the engine's firing cycle.

An alternative solution that we might be tempted to offer is that of a primitive needle-roller bearing, but here we begin to deviate from the evidence. Substituting hardwood pegs instead of iron for our needle-rollers would still leave us in a difficult spot for the date of this innovation in technology, and

[343] See Glossary.

[344] The ability to cast complex bronze parts is unquestionable; the costly process is complained about by Philon, *Bel*. 62, and confirmed by archaeological finds. Baatz (1978: 6) shows a bronze roller no different from the bushing I describe here.

the *epitonion* Biton repeatedly specifies would have to be textually corrupt to the point of being interchangeable with *ta epitonia.* In addition, the use of these rollers would preferably be accompanied by a race – in the form of a thin, tube-like bushing – placed in the wooden frame. The complexity of this assembly would beg the question as to why an ancient engineer would bother with this solution when the bushing suggested above might well suffice. Nevertheless, it is interesting to visualise on account of the fact that it would make the *kochlias* smoothly operable in a manner that entirely belies the mass, fitment, and strain placed on the part. If we presume the roller to be fitted in the manner illustrated, the two halves of the enclosing frame would ensure that the roller keeps the needle-rollers snugly ensconced in their bearing-race.



An illustration of the proposed solution in CAD. The roller is stubbed at the ends (like a lathe-centre) but remains dimensioned in the rest of its diameter as per the text. On the left is the iron bushing solution; on the right, the (perhaps wishful) roller bearing solution.

Serendipitously, then, the use of 'bearings' as a translation in Marsden happens to correspond to a realistic solution to a problem revealed by a more literal reading of the text - specifically, where the slider, or shot-carriage, appears to move with the stone-shot as it is propelled forward. The bulk of the frame can be significantly reduced if we are to take the lack of dimension for this part of the frame as license to modify it. The engine as a whole would be far more attractive as an option for the Pergamene defence effort: able to sustain much greater projectile weights, with greater accuracy, and with less attention required to ensure perfect balance in the work of the bowyer, in an already-hurried time of crisis.

7.3 THE BOWYER PROBLEM

If we are to assume that the need for a non-torsion engine was the result of a lack of twisted sinew-springs, our next question ought to be whether replacing them with a bow is of any time-saving value at all. If Pergamon's craftsmen could not process the timber in preparation for action, they would be in almost as difficult a predicament as the original one - that is, a lack of sinew and/or hair. The worst-case scenario would be the requirement to build a number of the enormous bows required for Isidorus' engine. The availability and processing of horn for reinforcement of the bow's composite construction (as per Philon's notes on the matter) is, sadly, an unknown quantity as of yet.

One might be tempted, therefore, to speculate on the time-frame required for carving such a weapon, which is ultimately unknown without experimentation. Our estimate, per bow, might be days of non-stop carving and sanding. However, due to the scale of the bow and its long, thin profile when placed on a set of saw-horses, it would be possible to position a very large team of carvers around the bulk of it to bring the roughing-out time down considerably in this armchair calculation. Given Marsden's width estimation of 4.572m,[345] and estimating the average width of a Pergamene carpenter, with sufficient elbow-space not to knock his fellow craftsmen at 1m, there is plenty of space to fit a team of ten around the span of the bow in order to get the most labour-intensive and least-skilled task out of the way.

Refining and balancing a bow of this size after the roughing-out process is the real question. This step would surely require an experienced bowyer, and could not be rushed. Measurement of the *tonoi* in every aspect and at regular intervals, would have to guide the refinement process of the bow's balance[346] – a long and arduous process, and restricted to one or two personnel to ensure precise control over the final product. Based on our placement of Biton at some hundred and fifty years (at least) after the last serious deployment of non-torsion catapults, it is entirely possible that the requisite skills for this scale of bow-making were either vanished, or in critically short supply.[347] However, this is where the particular design of the Isidorus machine comes in useful. If we are to assume that the above conclusion based on the text is correct, and that the shot was guided to the very end of the machine by a carriage, this would eliminate a large factor of error where unbalanced *tonoi* would be concerned. The design would be greatly preferable, then, to the finicky and lengthy process of refinement required for the Charon and Zopyrus machines – the latter of which, in particular, Biton makes a special note to balance properly. The tradeoff, here, is that significant

[345] Marsden, 1972: 83.
[346] Heron, *Bel.* 81.
[347] Xen. *Cyr.* 8.2.5 makes the observation that if specialised skills were to be found anywhere, it would be in a large city (like Pergamon).

ironwork in making the sliding 'ways' would be taken on by the blacksmithing workshops of Pergamon. However, divesting some of the man-hours involved in the operation amongst the skilled workers of the city – requiring less refinement, removing some lead-time from bow production – would only make the roll-out of a fresh artillery battery that much faster.[348]

7.4 CONCLUSION

The impression we are left with is that of a hulking brute of a machine. If a rudimentary method of scaling by 170% is used to make a rough estimate of the ultimate size of Isidorus engine required to replace the 60-*minae palintonoi* of Pergamon's original defense battery,[349] the dimensions we are left with are truly gigantic. Schramm's reconstruction of the engine (see below) places it on a base, and puts the assembled machine at a height of 3m if fixed, as he suggests, at a single elevation.[350] The bow, too, is roughly 3m wide in this scale. The upscaled estimate would naturally be in the range of 5m tall, with a bow 5m wide. For illustration, this machine would be approximately two stories in height, and the palintone bow in the diagrams below would be close to the length of the average family-oriented four-door sedan. The 60-*minae* ball, I estimate, was likely intended to reach a range of 1 *stadion* [185.28m] at its absolute maximum in this upscaled version of the engine, based on analysis of the defenses; although whether this design were able to achieve that precise range is unknowable without testing.[351]

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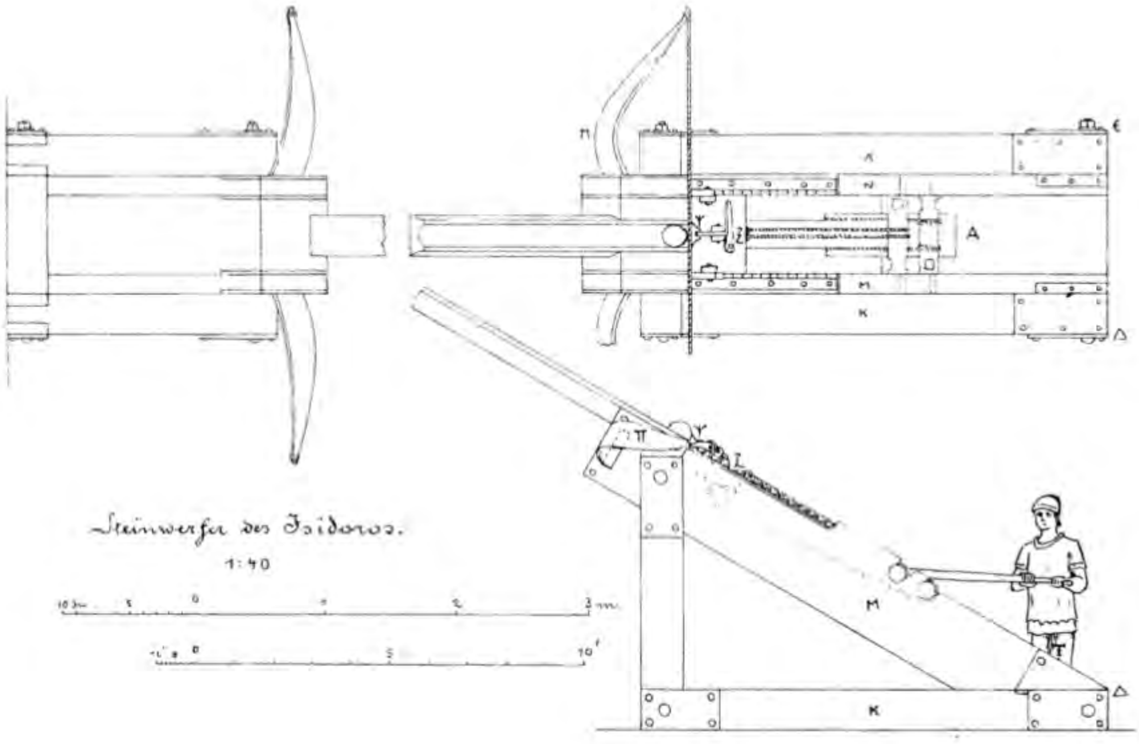
[348] It was not unusual for individual workshops to be contracted and reorganized into a production line for the purpose of building weapons of war in the Greco-Roman world. See Veg. *Mil.* 2.11.
[349] See Chapter 8.1: The Arsenal.
[350] In a departure from Schramm, I would presume instead to place the machine on a base akin to that of the *palintonos*, whereby the elevation and traversal of the artillery piece can be more easily changed.
[351] As per the testing method described in Chapter 4.1: Recommended Application of Dimensional Analysis; and as per analysis of defences in Chapter 8.4: Other Emplacements: The *Geschützstellung.*



Schramm's (1929:31) reconstruction of Isidorus' engine.

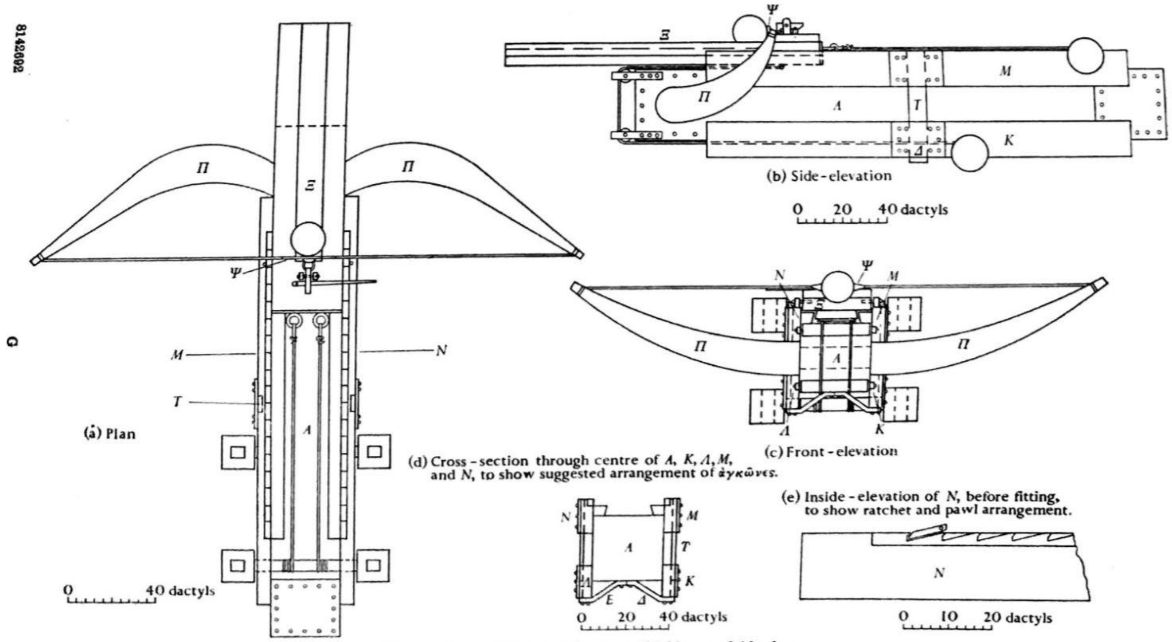


DIAGRAM 2. The stone-thrower of Isidorus of Abydus.

Marsden's (1972:81) reconstruction.

8. THE DEFENSES OF PERGAMON

If we are to establish whether an artillery piece is to be used for the defense of a city, we must first find out whether it can be brought to bear at all. The first criterion we can consider is whether we have ammunition to load into that artillery piece, and for that, our closest estimates must come from that most shaky of sources which has been analysed earlier in this thesis: the recommended shot-weight to engine-scale tables.[352] In order to place Biton's machines at Pergamon, we can examine the places where one might, in the most literal sense, place them. A valid argument against placing the use of Biton's machines at Pergamon might be their scale, as the sheer size of the Isidorus *lithobolos* demands some explanation. Due to the level of degradation of the walls and guard-towers around the archaeological site of ancient Pergamon, we cannot be sure of the height of the chambers in which artillery pieces were stationed. We only have available to us the length and breadth of the floor plans, as per the archaeological survey maps.

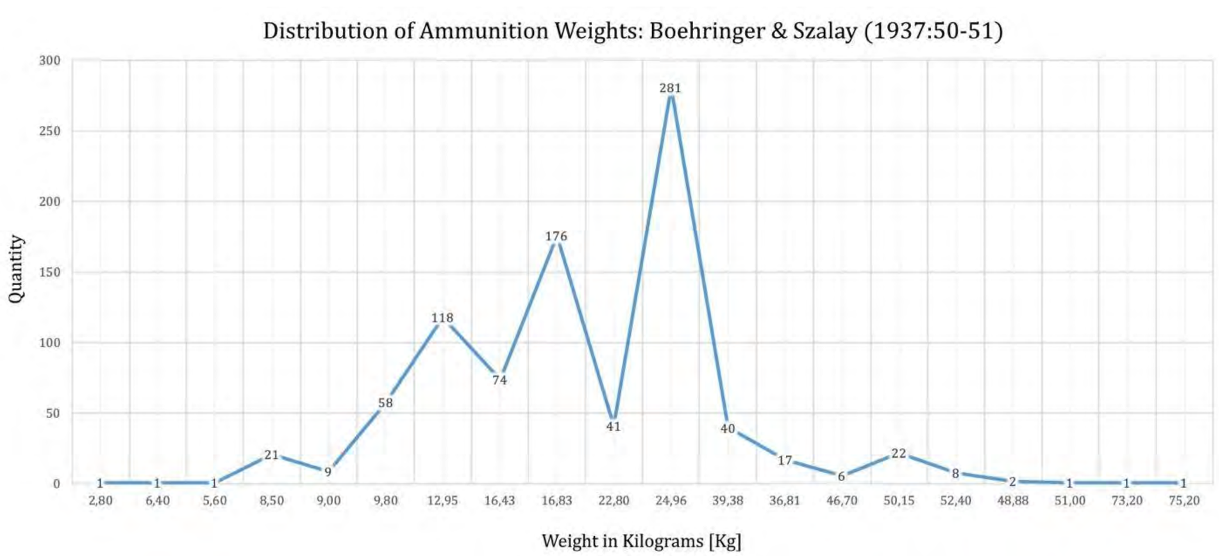
8.1 THE ARSENAL

During the early 20th century excavations of the city, Boehringer & Szalay surveyed the northernmost tip of the city, the area of the Arsenal, within which a number of storage warehouses were found. The dating of the area was confirmed to be Hellenistic through the identification of pottery fragments, with no evidence of Roman or Byzantine disturbance.[353] Conversely, areas with evidence of Byzantine activity were bare of ammunition.

Their grouping of ammunition weights (or "kaliber", a misnomer given that they are measured by mass, and not diameter) can be misleading, and the format of their table is difficult to interpret.[354] According to Boehringer & Szalay's records, nine hundred and sixty-one stones were found, which they elected to separate into thirteen distinct size-groups according to weight in kilograms. There are some truly gigantic examples here that must have been fired by appropriately fearsome weapons - but they are few. Several stones in the region of seventy-five kilograms and up to roughly three talents [77.4kg] were present. The latter would require a machine that - if scaled by Philon - would have required springs of 27 *daktyloi* [0.521m], which would be built into a case of roughly

[352] See Chapter 4.4: Scaling of Siege Weaponry.
[353] Boehringer & Szalay, 1937: 48.
[354] As per graph note, see Boehringer & Szalay (1937: 50-51) for table of weights, calibre, and sizes; Tafel 31(b) for photographs.

9.9m long, and supported by a crew of artillerymen. The archaeological team asserted that the largest stones were more novel than anything else, and that the Pergamene military must not have made use of small *lithoboloi* either.[355] The bulk of the armory's ammunition is, instead, clustered into three distinct groups towards the middle of the distribution, one of which was – I believe erroneously - previously assimilated into the 60-*minae* group.[356] Note that some errors of precision in Boehringer & Szalay's measurements were acknowledged at the time of their study, due to the unavailability of an adequate weighing scale from the nearby Turkish village, and as such the readings are, up to several hundred grams (a few decimal points), inaccurate.[357] I have graphed the findings below as a function of the two most pertinent criteria for us: weight, and number found.



Using this grouping of data instead of the archaeological survey's one, we can see three distinct spikes in ammunition types: the 12.95kg; the 16.83kg; and the 24.96kg. The troughs below and above each of them are what may be adjusted sizes for the machines that adhere to that strict size rating. Similar stone shot from the excavation of a Salaminian tumulus also show a discrepancy of up to 3kg within one weight class, as is helpfully indicated by inscriptions of the shot weight on their surfaces, placed there by the artillerymen of Salamis who used them.[358] In the Pergamene case, the 12.95kg standard might be swapped-out for the lighter 9.8kg shot, which could be used to maximise range and velocity; concentrating the kinetic energy of the projectile into a smaller area, and making for the rough equivalent of an armour-piercing round in a modern battle tank. The

[355] *ibid.* 53.
[356] Shatzmann, 1995: 56.
[357] Boehringer & Szalay, 1937: 49.
[358] Shatzmann, 1995: 58.

heavier 16.83kg could be used for short-range, high-damage applications, where dispersing the kinetic energy of the shot around the initial impact site is preferred; the equivalent of a high-explosive round in modernity. I would therefore place three main ammunition types, and therefore engines, at Pergamon. In ascending order, with their closest round *minae* number:

12.95kg (30 *minae*); 16.83kg (40 *minae*); and 24.96kg (60 *minae*).[359]

We can now consider the sizes of torsion engines used in the Pergamene defense as per the tables of Philon and Vitruvius;[360] the latter used only where dimensions are missing from Philon, given his later (Roman) period of activity. Marsden[361] uses a total combined case length of (30 × .) to account for the extra accoutrements that accompany a fully-assembled torsion catapult. This is far above the dimension for the bare case given by Philon or Vitruvius at (19 × .), but gives us an acceptable maximum dimension for accommodating windlass movement, the crew's loading and aiming, and thoroughfare through the emplacement pit.

Shot (Minae)	**Spring Dia. (Daktyloi)**	**Shot (Kilograms)**	**Spring Dia. (Millimetres)**	**Case Length (19×Spring Dia.)**	**Operating Length (30×Spring Dia.)**
10	11	4.3	212.3	4033.7	6369
15	12.75	6.45	246.075	4675.425	7382.25
20	14	8.6	270.2	5133.8	8106
30	15.75	12.9	303.975	5775.525	9119.25
50	18.75	21.5	361.875	6875.625	10856.25
60	20	25.8	386	7334	11580
150	25	64.5	482.5	9167.5	14475
180	27	77.4	521.1	9900.9	15633

The 40-*minae* grouping is interesting on account of being an intermediate size. Philon recommends the most care be taken of one's 30-*minae* batteries, as they are the most versatile,[362] but the data

[359] Note that this 24.96kg measurement is roughly 3kg below what we would expect for 60 *minae* in the Attic conversion. This cannot be accounted for by the discrepancy mentioned by Boehringer & Szalay above. Instead, the presence of stone shot labeled by Hellenistic masons as 60 *minae* in Tel Dor (also underweight by 3kg), seems to indicate that either an alternate Greek *mina* was in place for that group, or a convention amongst artillerymen that preferred the 60 *minae* to run slightly underweight. See Shatzmann, 1995: 57.
[360] Philon, *Bel.* 55ff and Vitr. *De Arch*. 10.10 respectively. The numbers here have been slightly amended through analysis of the manuscript tradition; this version is the most well-attested by Drachmann (1953: 279), and edited for the Hellenistic period from the corrupted manuscript text as per Marsden (1972: 158), and therefore may differ slightly from other scholar's calculations.
[361] Marsden, 1969: 34.
[362] Philon, *Pol.* 95.49.

appears to demonstrate that this was Pergamon's least popular engine. Then again, it is possible that the popularity of the 30-*minae* is what led to depletion of those ammunition reserves, and the spiriting-away of any popular surplus ammunition during the decline of the city in the late Hellenistic period. However, this exact situation would also have to have occurred at Carthage around the same period, as the Punic arsenal and surrounding area was also found to be distinctly lacking in 30-*minae* examples from the survey conducted in the early 1900s.[363] The more likely conclusion to be made, then, is that an intermediate size was preferred for small artillery, and the larger sizes were the most popular overall by far.

If we look to Marsden's 18kg estimate for the non-torsion engine, we can see an immediate problem: there are none of this exact size, only examples above and below. This can mean one of several possibilities: Marsden's prediction is incorrect; or he was neither correct nor incorrect and the Isidorus machine was never deployed; or he could be correct with regards to the design, but not to the deployment scale of the weapon, as the stopgap nature of the Isidorus solution would not have necessitated the creation of an entire new ammunition class for the weapon at all. Considering Biton's encouragement to scale the engine if need be in the final lines of his treatise,[364] I am tempted to fall in with this view. If such an engine were employed at Pergamon, it makes best sense for a beleaguered Pergamene military to pick and scale a weapon for the ammunition they already have, rather than both build an engine as well as carve out and refine its ammunition.

It is entirely possible, then, that the Isidorus engine – if deployed – was using a smaller or larger shot than we might otherwise assume from Biton's description. Here, we can consider Philon's recommendation with regards to defending walls: to dig trenches in front of one's defensive walls, out to a range of 163m, just under 1 *stadion*.[365] By disturbing the approach to a city in this way, the defender can ensure that the one-talent *lithoboloi* of the besieging force - deployed specifically for breaching walls - would not be able to set up near enough to those walls to make a meaningful impact if the approach terrain is compromised.[366] We can surmise that firing from further away would require a steeper angle of elevation, with the shot thus needing to fight against gravity whilst also expending its kinetic energy in flight; and evidently, at this range, these effects were enough to make the endeavour more laborious or frustrating than might otherwise be the case. Thus, the very

[363] Rathgen, 1909: 236.
[364] Biton, 68.
[365] Philon, *Par.* 84.
[366] This choice of engine for breaching walls is corroborated by Diod. Sic. 20.83.

rough estimate of range we need to meet is in excess of 163m, with a stone shot of sufficient size and kinetic energy to do significant damage to the wooden structure of a 1-talent (60-*minae*) *palintonos*. From the elevated position of Pergamon and its walls, this is made somewhat easier for a defender whose rounds are aimed in a shallow angle of depression. However, the ammunition required for successfully dismantling an enemy artillery piece - high impact force, with a great dispersion of that force - will necessarily be on the larger side. For this to be the case, the Isidorus machine will need to be scaled up rather than down.

The best probability for choice of a large engine - that is still common enough to presume that it was used around the entire defensive perimeter of the walls of Pergamon - would be the 60-*minae,* or one talent, *palintonos* - having the largest share of the distribution of ammunition. This seems reasonable given attestations to the anti-matériel capability of this engine, and is a pleasingly symmetrical solution for suppressing or destroying the probable one-talent opposing engines sent to breach the defences. It is also not that far, in the greater scheme of the data, from the approximate region of Marsden's estimate – being a shot of some 24.95-25.8kg, or roughly 170% of his initial guess. If one were tempted to try, they could make an extremely flawed estimate of the size of the upscaled Isidorus case from this figure. Philon himself discourages this course of action whereby an engine is scaled by multiplying out its frame components instead of its motive power;[367] nevertheless, it makes for a fascinating aside. If an Isidorus case, in stock form, comes to an absolute minimum unassembled length of 4.632 metres, then an upscaled version of 170% would be 7.874 metres in length. Coincidentally, this happens to be almost precisely one and a half *podes* shorter[368] than the total assembled case length of the original 60-*minae palintonos* that the Isidorus machine would, following our theory here, be contracted to replace. It is extremely tempting, then, to place the Isidorus here – and perhaps even to surmise that the Isidorus bow could be retrofitted (either cut into the case, or bracketed in the style of a Zopyrus machine), directly to the original *palintonos* case, with it having had its spring-retaining framework removed.

As interesting as it may be, making an estimate of the scale of the Isidorus engine based on this data is ultimately speculative – a guess extrapolated from another guess. Instead, we can look to the size of the *palintonos* it replaces to determine where it would be installed. The kind of positions that the Isidorus machine would be occupying, then, would be any in which it would replace a 60-*minae*

[367] Philon, *Bel*. 55.

[368] It is precisely 0,5404m in difference; one and a half *podes* are 0,4623m. This would make the bare case only 78mm or 4 *daktyloi* shorter than the assembled case of the original 60-*minae* case.

engine. Given the conversion tables of the artillery engineers above, we are looking for places that would accommodate any machine of a nominal case length of 7.334m and above; which in turn requires us to find a longitudinal space of anything up to or over the worst-case scenario of 14.475m.

8.2 THE WALLS

The most obvious location for an engine is on the battlements and towers of the city walls, where it has the advantage of height, cover, and a great arc of fire to pick off would-be besiegers. The most well-preserved of the ancient defenses at the archaeological site happen, fortunately, to be those from the era which we wish to investigate. Referred to on German survey maps as the *Eumenische Mauern*, for the king who oversaw the completion of their construction,[369] the remains of these walls remain visible across the countryside and are easily distinguishable from later Roman and Byzantine fortifications.

The Attalid construction is distinctive by way of its typically Hellenistic, *isodomic ashlar* stonework.[370] The scale of these individual blocks is much greater than any other present in the area, and they are honed to regular rectangular shapes that fit together with evident precision. Later additions are easily told apart by their composition of sharp, irregularly-shaped rock pieces that are fitted together *ad-hoc*. The same methods are cross-referenceable with other Hellenistic towns in Asia Minor, like Sillyum;[371] the difference in Pergamon being, however, that we have no extant artillery embrasures or other slits through which an artillery piece would be pointed, preventing us from making an inference from that data. The Hellenistic outer circuit wall is expansive, and loops around from the hilltop where the arsenal, barracks, and palaces are ensconced, down and around the lower city and the southernmost flat plain where the basin of this valleyed region begins, and back up to the arsenal. Remains of a smaller, internal circuit are also visible. Using the most recent digitized survey, we can measure the usable internal space of the guard towers, and walkways of the walls, that belong to these early circuits.

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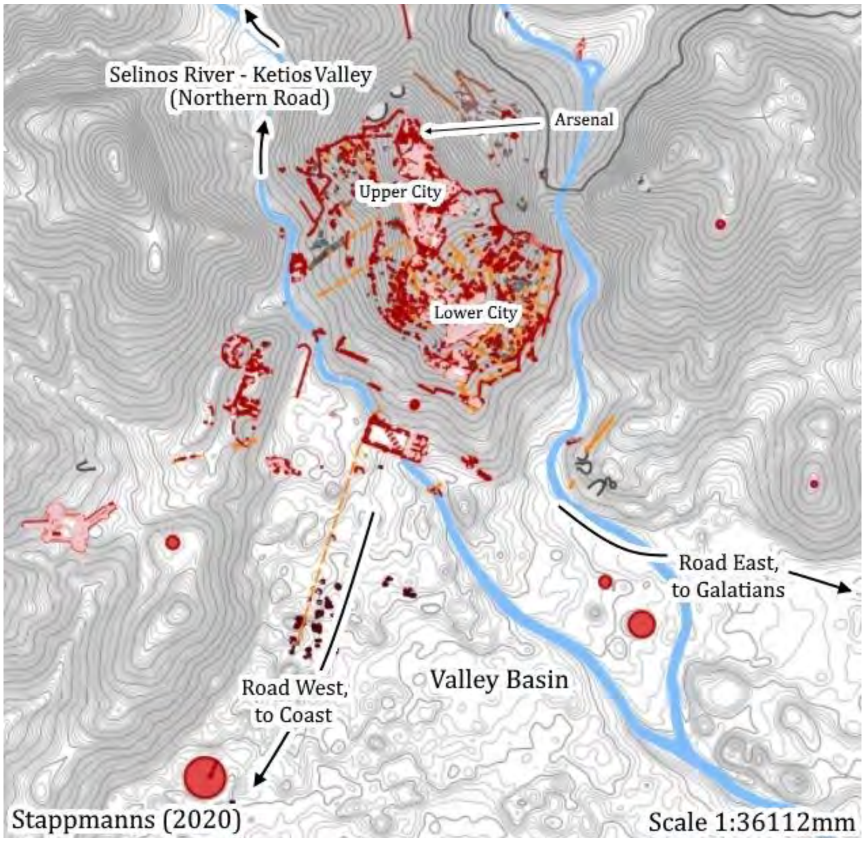
[369] That being Eumenes I; the project was initiated by Philetairos. Hansen (1971: 17), Radt (1999). See also Chapter 2: Historical Context.
[370] A method of construction described by Vitr. *De Arch.* 2.8.7, and Plin. *HN.* 36.171.
[371] McNicoll & Milner, 1997: 150.



Adapted from Stappmanns' (2020) map. Note that the eastern fork of the river is not accessible by road; only the Ketios Valley provides a road to the north, and necessarily passes by the western circuit of walls due to the impassibility of the steep mountain face protecting the Upper City and Arsenal.

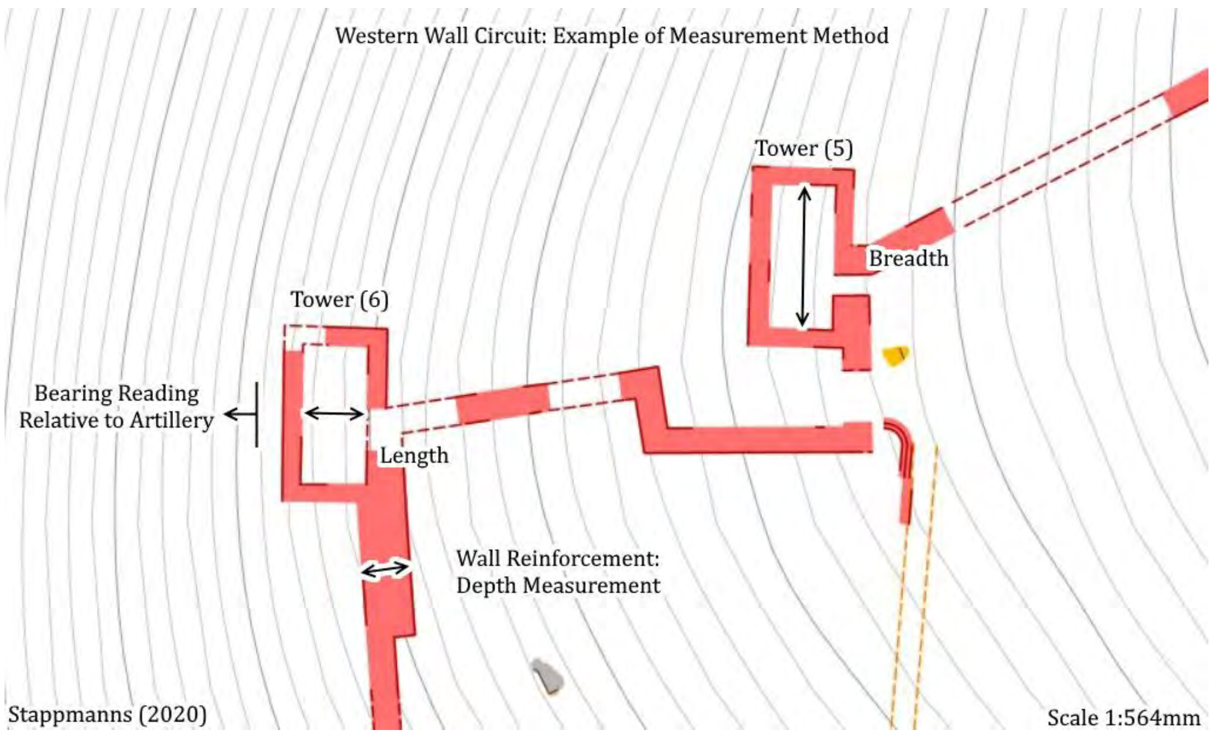
The measurements that follow below have been taken using the Geographical Information System (GIS) survey of the Pergamon Micro-Region, as published by the Deutsches Archäologisches Institut.[372] The German names for each of the sites identified in this survey have been retained in order to avoid confusion in inter-referencing scholarly work.

The outer circuit of the *Eumenische* city walls has been separated into East and West sections for this study, and the *Eumenisches Tor* complex - the grand gatehouse at the main entrance of the city - has been placed in its own table. The latter exists as one large central hub, off of which are two guard towers and one larger guard room. These have been arranged in order from west to east in an anti-clockwise direction. There are therefore a total of three tables included here.

The remains of the towers are restricted to their lowest, ground-level sections, identifiable by their isodomic ashlar construction. Thus, only the floor plan can be deduced from the remains. The

[372] Stappmanns, V. (2020) 'Digitale Karte von Pergamon 1.1', Hochschule Karlsruhe Technik un Wirtschaft & Karlsruher Institut für Technologie.

‘length’ here is referent to the measurement by which the towers jut out from the wall; in other words, the direction in which the occupants would look out onto the surrounding terrain, and the axis along which the case of an artillery piece might be aligned. This is reflected in the ‘bearing’ reading given for each tower. The ‘breadth’ is the lateral space in each tower. Some of the towers' ground levels have an opening inward towards the interior city, indicating that one entered on the ground floor and ascended a set of stairs or ladders to reach the upper floors. Others are closed all around on their ground floor, indicating that they were most likely accessed from the adjacent wall walkways above, or from an external scaffolding. For each of these tower sections, an average wall depth reading has been taken, but not tabulated. This is referent to the depth of the walls that link the guard towers together, and upon which the Pergamene defence would have arranged its forces, and transported personnel and materials from tower to tower.



Naturally, we cannot assume to fill the space to its utmost with an engine that is too large; that is, one that would either require the operator to dangle off of a wall to sight it (in the case of an open-air position), or one that affords no manoeuvrability around it for loading, aiming, and maintenance (in the case of those positioned within a guard tower). It would also be somewhat pointless to assume an artillery piece that is too small, as its usefulness in repelling an assault by an opposing force’s engines in the surrounding countryside would greatly depreciate with diminishing range and payload. Having a machine able to match the range of enemy field batteries, fire a stone that is large enough to maximise the likelihood of hitting the (relatively) small target of an enemy engine, and that will be more likely to cause irreparable damage should it hit the target, seems more preferable. Something in the 'middle' then, as it were, would be the most accurate inference from the data at hand.

Towers of the Western Outer Circuit; from Arsenal to *Bau Y*

No.	Lat/Long	Bearing	Desc.	Length [m]	Breadth [m]	Area [m^2]
1	39.134; 27.181	N 74° 54' 48" W	Square, open	4.99	6.79	33.882
2	39.134; 27.18	N 3° 26' 50" E	Square, closed	5.4	4.93	26.622
3	39.134; 27.18	N 3° 54' 1" W	Square, closed	5.16	4.84	24.974
4	39.134; 27.179	N 88° 43' 11" W	Rectangular, open	5.12	11.28	57.754
5	39.134; 27.178	N 89° 45' 17" W	Rectangular, closed	4.09	10.81	44.213
6	39.133; 27.178	S 74° 59' 42" W	Rectangular, closed	5.26	9.03	47.498
7	9.132; 27.178	S 65° 28' 59" W	Rectangular, closed	5.69	10.68	60.769

Notes to above table:

- Average floor plan length: 5.10m
- Average surrounding wall depth: 2.11m
- Towers 1-2 bridged by a deeper, 2.79m wall.
- Tower 6 followed on south side by 3.81m wall depth, extending 14.28m towards bearing S 4° 43' 28" E.

***Eumenisches Tor* of the Southern Gate, South of Lower Agora**

No.	Lat/Long	Bearing	Desc.	Length [m]	Breadth [m]	Area [m^2]
1	39.124; 27.186	S 47° 35' 49" W	Square, open, west side	5.48	4.41	24.1668
2	39.124; 27.186	S 30° 42' 46" E	Square, open, south side	7.3	7.25	52.925
3	39.124; 27.187	N 59° 25' 50" E	Rectangular, open, east side	9.25	14.85	137.3625

Notes to above table:

- Average floor plan length: 7.34m
- Average surrounding wall depth: 3.44m

Towers of the Eastern Outer Circuit; from *Eumenisches Tor* to *Felsheiligtum Ost*

No.	Lat/Long	Bearing	Desc.	Length [m]	Breadth [m]	Area [m^2]
1	39.124; 27.187	S 24° 17' 27" E	Square, unclear	7.26	8.07	58.588
2	39.125; 27.188	S 40° 25' 20" E	Square, closed	6.56	7.14	46.838
3	39.126; 27.189	S 35° 40' 11" E	Square, open	6.87	8.19	56.265
4	39.127; 27.19	S 75° 42' 23" E	Square, closed	6.52	6.71	43.749
5	39.127; 27.20	S 79° 18' 54" E	Square, closed	6.11	5.71	34.888
6	39.128; 27.19	S 84° 26' 49" E	Square, open	8.05	6.85	55.143
7	39.13; 27.191	N 44° 2' 27" W	Square, closed	6.26	6.6	41,316

Notes to above table:

- Average floor plan length: 6.80m

- Average surrounding wall depth: 2.81m

As we can immediately tell from the data, the floor plans of these chambers seem too small for the uninhibited storage and operation of a 60-*minae palintonos*. Tower 6 of the eastern circuit is only just large enough to fit the 7.334m case of a 60-*minae* - let alone crew, windlass, and space for manoeuvring around the machine, up to our high estimate of 14.475m. The third room of the *Eumenisches Tor* gatehouse appears large enough to fit a 60-*minae* so that it faces south towards the valley basin[373] – however, closer inspection of the survey shows three columns placed at 3m intervals through the centre of the room, blocking any possibility of traversal of the engine between targets to its left or right.[374] The breadth of Towers 4-7 from the western circuit could fit the bare case of the 60-*minae*, but this would be a pointless endeavour: they would be facing either another section of wall or another guard tower, and would be impossible to spin around in this small chamber to face in any other direction without being dismantled.

The western circuit, despite having smaller floor plans and shallower walls than the eastern side, shows evidence of reinforcement between Towers 1 and 4. So does the south side of Tower 6, which is closest to - and has the first available line of sight on - any approach by the bend of the Selinos River[375] flowing from the north. There are the skeletal remains of a tower closer to the river, also visible on the survey,[376] but there is not enough remaining of it to make any convincing estimate of its size or purpose. The entire eastern circuit appears to have been built more sturdily from the point of its initial design, having both deeper walls and larger towers on average. This I believe we can safely assume is due to the relatively naked and topographically flat approach to this side of the city, from the Seleucid interior of Asia Minor.[377]

The layout of these defences is puzzling on initial inspection, given our conclusion on the basis of the archaeological evidence of available ammunition, when cross-referenced with Philon. The lack of outwardly obvious 60-*minae* emplacements seems to render the greatest proportion of Pergamon's ammunition unusable, as the torsion *palintonos* built to fire it – per Heron and Philon's scaling recommendations – would also be unable to fit in these rooms. Clearly, then, there is more to be considered.

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373 Winter, 1994: 39.
374 Possible explanation for these columns provided further below.
375 Strabo, *Geo*. 8.7.5
376 Stappmanns, 2020.
377 Lüdwig, 2020: 20.

Judging by the other remains of ammunition, there were certainly smaller artillery in use – in particular, the 30- and 40-*minae* machines. There is a small quantity of 20-*minae* ammunition, the corresponding engine for which would certainly fit in these towers. Composed of a 5.114m case and requiring a worst-case length of 8.106m for easy operation, this engine could conceivably fit in Towers 1 and 6 of the eastern circuit, or in Tower 2 of the southern gatehouse. However, we would be stretching the evidence: there are only 21 rounds of 20-*minae* shot in the survey, and it would be less likely for the Pergamene defence to employ one or two engines of a different type rather than standardise their array. The interior of these guard towers were more likely occupied by archers, lookouts, and *euthytonoi* – that is, arrow-shooting weapons. We can infer from the available space in the western circuit (5.10m) and eastern circuit (6.8m) that a lowest-common-denominator *euthytonos* engine at Pergamon could have had the following form factor:

We know that Philon's formula for a *euthytonos'* spring is ($= \frac{}{9}$), where [ ] is the spring bundle diameter, and [ ] is the length of the bolt ammunition in *daktyloi*.[378] We also know that the length of an engine case is equal to (19). However, if we take Marsden's (30) clearance suggestion to be correct, we can use the tower layout to find the probable length of an engine case, and therefore the size and type of engine, employed in the space. Using the above information, let us find out what engine could be used in a 5.1m (or, 5100mm) space:

$$\frac{5100}{30} = \quad = 170$$
$$\therefore \frac{}{9} = 170$$
$$\therefore \quad = 1530$$

The result is a 1.53m or 5 *podes* bolt, with a 170mm - or approximately 9 *daktyloi* - spring bundle. The unadorned case length of (19) would therefore be 3.23m. This does not seem an unreasonable estimate, given the iron bolt tips found in the Arsenal excavation that measure almost precisely one tenth of this bolt length, in the region of 160mm.[379] Ammunition of this size could easily suffice as a fearsome anti-personnel weapon, and seems an appropriate choice for the Pergamene watchtowers. Biton specifies a bolt for his first *gastraphetes* - Zopyrus' dual-bolt machine – that is 6 *podes* or 1.852m.[380] The second *gastraphetes* does not have a bolt length provided, but does have a case of 5

[378] See Chapter 4.4: Scaling of Siege Weaponry.
[379] Boehringer & Szalay, 1939: 117.
[380] Biton, 62.

podes, the same length as the bolt computed above.[381] The proximity of the calculation seems to be more than mere coincidence.

However, if we assume that the towers were bristling with *euthytonoi*, then what of the *palintonoi*? Even if a sprinkling of 20-*minae* weapons were applied around these areas, they were clearly an exception. There are virtually no smaller ammunition sizes available, and our most common *lithoboloi* are only of larger sizes. Where, then, were these artillery pieces stationed?

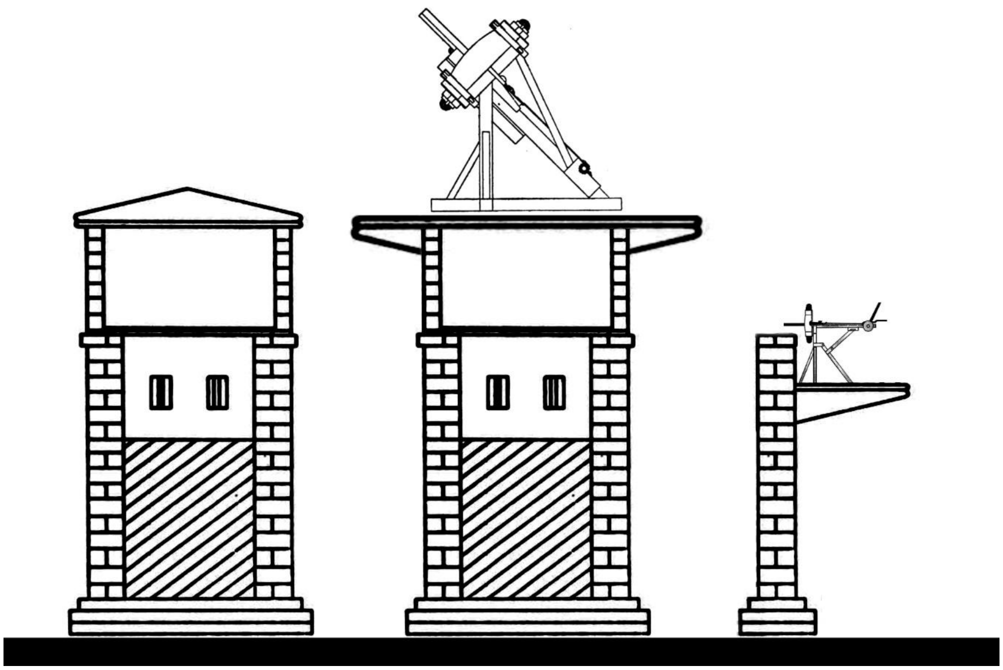
8.3 OTHER EMPLACEMENTS: PLATFORMS AND CANTILEVERS

A similar problem presents itself in analysis of the artillery emplacements at Tel Dor in Israel, where the 30 to 60-*minae* weapons attested by the ancient ammunition reserves have no clear placement. A possible solution is wooden staging,[382] which is to say that the ramparts of the open walls between towers could be extended by means of cantilevered decks towards the interior of the city, providing the depth required to place artillery. This seems quite feasible, given that the deepest walls of Pergamon – those extending 14.28m southwards of Tower 6 on the western circuit – are up to 3.81m thick. It is conceivable that a supporting structure could extend this to as much as double the depth, or 7.62m, which would be sufficient for placing more of the *euthytonoi* most likely used in the guard towers as above. However, the only *lithoboloi* that might fit this space are the 10 to 20-*minae*, for which we have no evidence of use at Pergamon. The 60-*minae* and the Isidorus replacement, therefore, cannot possibly have fit here. The textual evidence supports this conclusion, as weapons on the walls were preferably chosen for their ease of disassembly and removal in case of inclement weather.[383]

The towers themselves might be further built upon to provide more than just a lower guardhouse and an upper parapet. Philon describes the addition of multiple floors above a tower's guardhouse, supported by beams and arches, upon which additional artillery platforms and other countermeasures can be placed.[384] What is lacking from this account, however, is any evidence that the floor space of the tower can be increased by jettying the upper floors outward, so as to place larger artillery there than on the floors below.[385] This might otherwise be a tempting theory, as the largest of the towers at Pergamon – for example, Tower 1 of the eastern circuit – would only need its

[381] Biton, 65.
[382] Shatzman, 1995: 65.
[383] Diod. Sic. 20.54.2.
[384] Philon, *Par.* 83.15.
[385] In the style typical of European Renaissance and late medieval buildings.

total length of 7.26m to be increased by 4.32m, or 2.16m to the front and rear, to accommodate the recommended clearance of a 60-*minae* at 11.58m. However, the remainder of the towers noted above would need far more drastic building-out to achieve this floor area; the more representative Tower 7 of the western circuit – the largest on this side, but still relatively small compared to Tower 1 in the east – would need 5.89m to be added with cantilevered scaffolding strong enough to support an artillery piece, crew, and ammunition dump. With any wooden structure long since decomposed, the evidence for this kind of provision at Pergamon is slim.



Example illustration of a standard tower; one with jettying to increase the floor plan length by 160%; and a rampart walkway on a city wall supported by joists. Adapted from tower illustration by Marsden (1969:163), palintonos by Schramm (1918:56), euthytonos by Marsden (1972:197).

Shatzman[386] also suggests in his analysis that the largest artillery pieces were employed outside the walls, most likely supplemented with ditches, wooden emplacements, and supporting troops. This would corroborate an allusion to this kind of tactic provided by Livy’s account of the actions of Diophanes the Akhaian (and the *inaction* of Attalus) in front of the southern gatehouse of Pergamon.[387] The open terrain around Pergamon is eminently usable for positioning field artillery on the southern and eastern approaches; and but not so on the western side, which combines a sheer topographical slope down from the *Eumenische* wall circuit into the valley below - through which the northern access road to Pergamon passes - and directly up again, into the next range of

[386] Shatzman, 1995: 64.
[387] Livy, *Hist.* 37.20-21; see also Chapter 2: Historical Context.

foothills of the Madra Mountains. The steep, rocky terrain downwards would make traversal by infantry or horses difficult, let alone the flat and secure deployment of a machine as large as the Isidorus or a 60-*minae palintonos*. While terrain like this may be the reason for Biton's recommendation of the lighter, more easily-deployed 'mountain *gastraphetes*' of Zopyrus,[388] it remains that there must be some means of deploying Pergamon's most popular *lithoboloi* here as well; after all, a major approach vector of the city surely could not have been left bare to attack. Evidently, some other solution for placing and orienting these gigantic engines must be found.

8.4 OTHER EMPLACEMENTS: THE *GESCHÜTZSTELLUNG*

It is at an apparent junction of the *Eumenische* walls that the German archaeological surveyors have noted a rectangular section of foundations that extend nearly 30m south, and are approximately 15m deep as they recede into the hill of Pergamon. This area has been tentatively labelled as a *Geschützstellung*, or artillery emplacement. It appears that, at the time of writing, no further excavation nor investigation has occurred – most likely on account of the proximity of this site to a public road and private dwellings. The following tabulated information has been collected and represented in the same manner as those earlier:

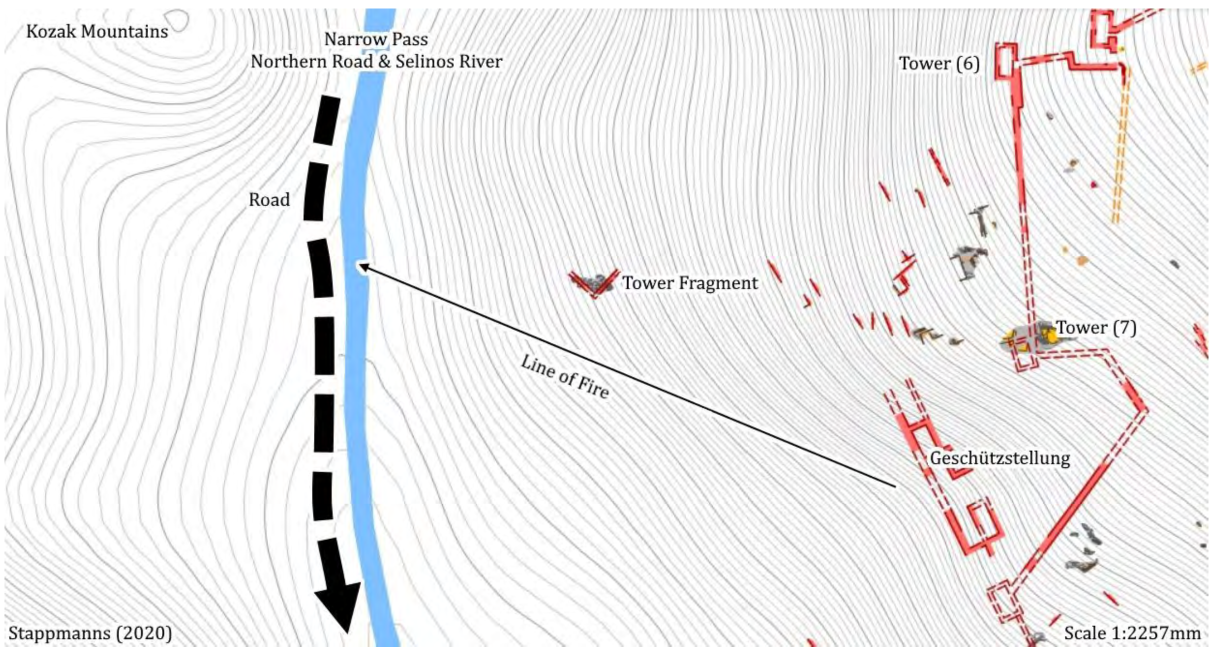
The *Geschützstellung* of the Western Circuit

Lat/Long	Bearing	Desc.	Length [m]	Breadth [m]
39.132; 27.178	S 63° 57' 40" E	Northern half of continuous foundation	10.32	29.99
39.132; 27.179	S 63° 57' 40" E	Southern half	17.26	39.61

Nevertheless, the *Geschützstellung* label seems appropriate. The segmented foundations of this area follow a ladder-like, grid-type layout, spanning the gap between two peninsulas of the western outer circuit walls, and blocking off a topographical feature that would otherwise constitute a more easily-scaled gap in the rocky approach to the city. The foundations are strikingly similar to those found at the warehouses of the Arsenal,[389] whose interlocking ladder structure provides a more comprehensive support for the wooden plank floor upon which the tonnes of stone ammunition for Pergamon's artillery were stored and retrieved. The excavators make a convincing reconstruction of this warehouse from the foundations, placing a "holzerner oberbau"[390] over the isodomic construction. If we ascribe the same method of construction to the *Geschützstellung*, we can deduce that a single large platform of more than 69.6m in breadth and a length of at least 10.32m, built to

[388] Biton, 65.
[389] Boehringer & Szalay, 1937: 141.
[390] *ibid.* 152-153.

withstand a great weight, was in this position. If the fragmentary remains of further foundations behind the rearmost lateral wall are indicative of increased depth, then the platform built on top of this must have been in excess of 17.26m deep – more than enough to accommodate a 60-*minae* or Isidorus engine, with supporting troops or fortifications. Due to the lack of visible through-holes in these foundation structures, perpendicular to ground level[391] as seen on the Arsenal foundations,[392] we cannot be sure that wooden joists or columns were placed on this foundation to support walls and a roof. It is entirely possible that this was added later by an external framework by trusses, but it is equally probable that this was an open-air deck.

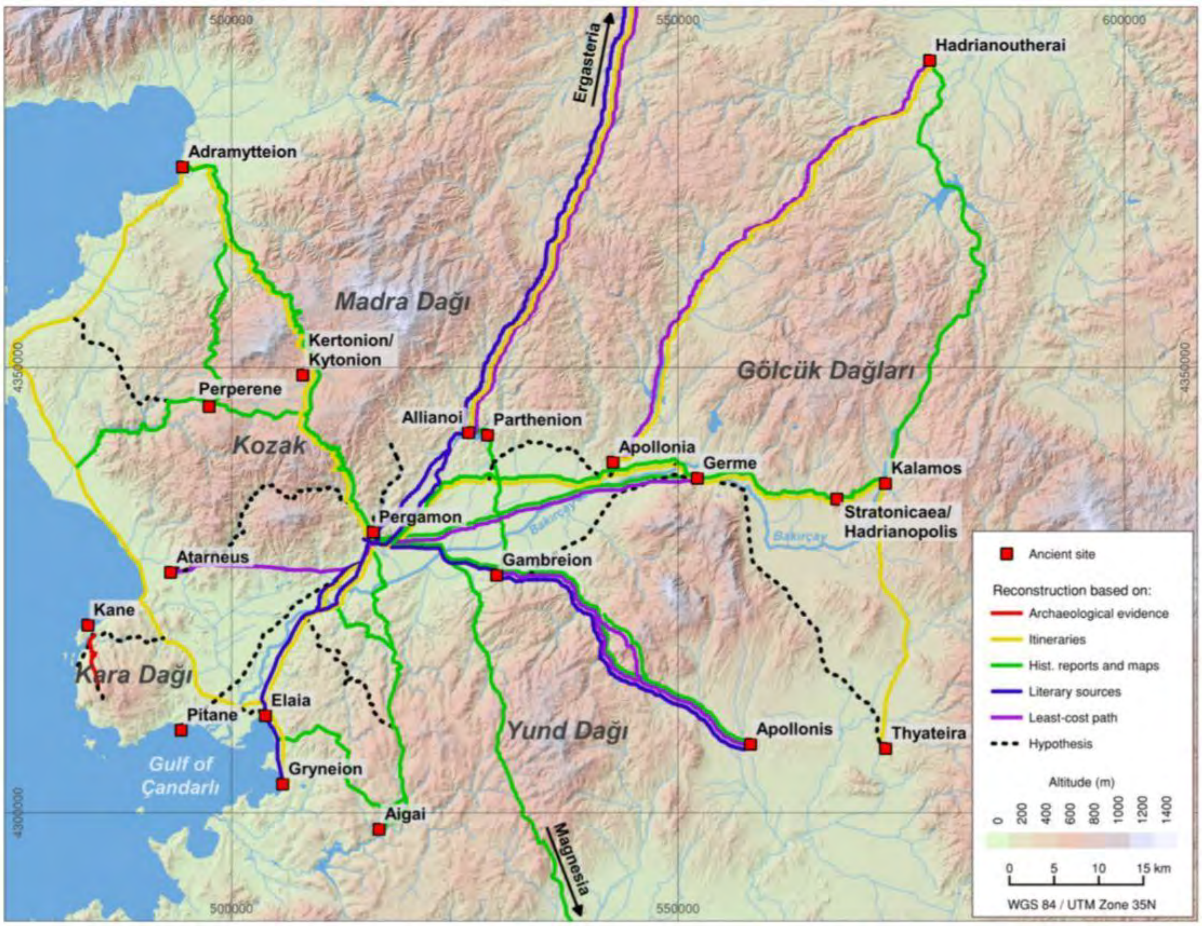


Adapted from Stappmanns' (2020) map: placement of the Geschützstellung *in relation to the northern road and the western circuit.*

The *Geschützstellung* is, quite possibly, a strategic position worth further study. We can be sure that Prusias' primary force was in the southern and western region, as this would be necessary for his brief attempt to take Elaea, and for his sacrifice in, and subsequent looting of, the sanctuary of Asklepios just outside of Pergamon.[393] However, the clear and open geography of the basin would play to the strengths of the Pergamene cavalry, and this is likely the point at which Prusias' incursion was stalled. Moving any closer would require an assault on the well-fortified guardhouse and barracks of the southern gate, from which Diophanes the Akhaian had led his noteworthy

[391] Stappmanns, 2020.
[392] Boehringer & Szalay, 1937: 149.
[393] Hansen, 1972: 133.

counter-attack years before.[394] Consulting another recent GIS survey[395] of the ancient routes around and into Pergamon, it appears that the valley in which the *Geschützstellung* is constructed makes for a useful access point to Pergamon from the northern interior. Below the *Geschützstellung*, at the base of this valley, is a river which feeds the verdant basin south of Pergamon. Across the river is a modern road, which appears to follow the same course as the ancient road to Pergamon. It stands to reason that a contingent of Bithynians – or indeed, any army – would march along the established roads for the sake of its wagons, horses, supply line, and the feet of its infantry.



Lüdwig's (2020:30) GIS survey, indicating the most likely routes in and out of Pergamon from a multivariate analysis.

This road makes for a straight line between Nikomedia of Bithynia, and Pergamon, via the Ketios Valley - which cuts through the Kozak Mountains north of Pergamon.[396] Given that it snakes around the hilltop acropolis to the western side of the city, it may have made another possible avenue by

[394] Livy, *Hist.* 31.20.
[395] Lüdwig, 2020: 9.
[396] Lüdwig, 2020: 29.

which the Pergamenes could have suspected that Prusias would make an approach; whether it be to make some kind of diversionary attack, infiltrate the acropolis by stealth, or bring to bear some supplies or reinforcements not provided by his' navy from the western, coastal route. (In such a situation where approaching from the interior of Asia Minor was required, then approaching from the east and following the foothills of the Madra Mountains might also seem to be a logical choice, as the road to Ergasteria leads directly to the eastern reaches of Bithynian territory. However, bringing men or supplies through this territory - a much more circuitous route - seems an unlikely choice.)

It also appears that the ancient Pergamenes may have taken due diligence to adequately shore up this vector of attack. As we can tell from the deeper walls that link Towers 1 and 2 of the western circuit, and the doubled depth of wall immediately south of Tower 6, it appears that some level of buttressing was applied to the defensive towers and walls along this same route. In addition to the armaments within the guard towers, it is possible that this buttressing formed the base for cantilevered decks upon which a larger complement of ranged troops – or, indeed, more machines – could be placed.[397] Alternatively, this could be an attempt to shore up the walls in case of attack by the 1-talent (i.e. 60-*minae*) anti-matériel weapons used to breach walls as Philon mentions above.

The positioning of the *Geschützstellung* in this area would enable a significant artillery battery – several large *palintonoi* standing side-by-side – to fire directly on a force that might wish to traverse the valley in order to reach the flat southern approach, where previous invaders made their encampments[398] and Prusias had set about the business of looting. Furthermore, its placement on the nearer bank of the river, far below the amphitheatre, effectively demands that any would-be passers-by would need to cross it, slowing down and wearying their troops or horses, before ascending the bank to disable this emplacement. Should that position be taken, we know that the Attalids were not averse to lighting their own equipment on fire to prevent matériel from landing in the hands of the enemy;[399] after which, the inner circuit of walls and guard towers on the hill above would remain able to deliver a sustained barrage on the invaders.

In a position like this, a 60-*minae* engine could be operated with relative ease. It is therefore certain that the Isidorus engine in its standard form, with a nominal case length of 4.632m, would also be able to fit in the place of the 7.334m case with its 11.58m operating zone. If in its standard form, it

[397] Shatzman, 1995: 65.
[398] See Chapter 2: Historical Context.
[399] Livy, *Hist.* 32.23.11.

would almost certainly be able to fire the second-most popular ammunition in Pergamon – the 40-*minae* 16.83kg ball – to some effect. However, given the obvious ancient preference for 60-*minae* ammunition in the case of anti-matériel purposes, Biton's scaling recommendation, and this size occupying the greatest proportion of Pergamon's ammunition, I believe the evidence shows that a 60-*minae* model of the Isidorus may well have been deployed in the same or a similar position. It stands to reason, after all, that a non-torsion case length similar to that used on a 60-*minae* *palintonos* could also be placed and satisfactorily operated in this location. It is also interesting to note that the average distances from the nearest corner of the *Geschützstellung* to the near and far riverbanks are in the approximate regions of 185m and 220m respectively; just over 1 *stadion*, conceivably within effective range of both *lithoboloi* and *euthytonoi*. The usefulness of this position as an artillery emplacement, therefore, cannot be overlooked.

8.5 A NOTE ON THE MAP OF BOEHRINGER & SZALAY

Boehringer & Szalay, in their survey of the greater Arsenal area, identified a zone to the East whose purpose was uncertain. They tentatively labelled it as a support structure and battery of unclear purpose, but possibly for artillery, as it adjoined the barracks to the South as well as the Arsenal and its courtyard to the West.[400] There is certainly more than enough space here for any size of engine one could dream of, and it is therefore tempting to place one here. However, this location's status as an artillery battery seems hopeful at best, and misguided at worst.

We are aware, from Heron's *Belopoietics*, that sighting an engine happens directly "down the case" towards the enemy.[401] This would indicate that direct fire was the preferred method, and certainly, there is no textual evidence for this kind of high-parabola method of artillery barrage. All other extant fortifications' embrasures show that artillery was pointed at a relatively flat elevation towards the enemy.[402] Another problem for this theory is that Euclidean geometry does not allow for the calculation of firing arcs in parabolas; this, in turn, requiring trigonometric functions unavailable at this stage of the development of mathematics.[403] The height of a *palintonos*' base relative to the length of its case, too, would prevent it from being aimed at any great ascension, let alone the near-vertical degree required to make an effective parabolic arc over a fortress wall to any respectable range. The problem that seems to be closer to the forefront of the ancient mind in this

[400] Boehringer & Szalay, 1937: vi.
[401] Heron *Bel.* 86.
[402] Winter, 1993: 37.
[403] Nahin, 2007: 165; see also Appendix 1: Understanding Treatises by Geometry.

respect is how to achieve greater depression of the weapon through an embrasure, as the ground forces approaching a city wall inevitably draw too close for the artillery to be of any further use. A lack of evidence for any method of working around the calculation of parabolas or increasing ascension of a gun in the *poliorcetic* texts would indicate, I think, that gunners continued to use line-of-sight until their modern successors in the age of the breech-and-cartridge artillery piece.[404]

[404] This includes considerations of the Byzantine *helepolis*, or trebuchet, as these too relied on line-of-sight rather than indirect barrage to hit their targets. Although the projectile was released at a point further from the ground than a Greek *lithobolos*, the trajectory of the projectile remains similarly flat. See Hacker, 1968.

9. CONCLUSION

The history of the Attalids at Pergamon reveals a tradition of military campaigning in Asia Minor and mainland Greece, a close relationship with Rome, subtle manipulation of the political landscape, and keen familiarity with matters of war. This kingdom's political prominence and wealth would have attracted various kinds of skilled individuals to its capital, and the character of the Greek engineer would certainly have been one of them. There is no doubt that the Pergamene court, like every other Greek polity of this period, appointed engineers for various tasks related to the city and its security, and engineers specialising in projectile weapons for Pergamon's defence and for remote campaigns would be included under this category. Vastly more numerous than these official appointees, we might reasonably assume, were those independent engineers and other learned practitioners of various technical disciplines that would have operated in the greater city; whether from their own workshops, or as itinerant consultants. This is not to mention the myriad skilled craftsmen and technical specialists that carried out the bulk of the labour, and who largely receive no fanfare.

It is against this backdrop that Biton takes the stage in our analysis. The treatise attributed to him is terse, densely-packed, and highly technical. He makes his recommendations to a king Attalus, who must have been familiar enough with matters of war and siegecraft to have made sense of this text. Attalus II, according to our historical study, was just such a character - so devoted to his campaigning that he continued to participate in military affairs well into old age, much like his father before him. The particular period Attalus II inhabited saw a number of conflicts, but one in particular seems to match what we can infer from the text of Biton's treatise. Attalus attempts to bargain with the help of Rome, before his retinue is slain and he is forced to retreat to Pergamon. The Pergamon-Bithynian war breaks out abruptly, and appears to take Attalus by surprise, if we are to take his slow re-arming and failure to mount a defense as evidence. This retreat is similar to those conducted by the Pergamenes in the face of Philip V, the Seleucids, or the older Prusias of Bithynia before. However, this movement is crucially different in that it happens at a unique time in Pergamon's history - one in which their defensive artillery has stood silent for just over a decade, and raising an army to respond to the threat was an effort that required several years of preparation after the fact. Corroborating evidence points to the Attalids treating this as a moment of particular distress, unlike previous threats of siege. Considering that Pergamon had not been on a war footing for quite some time, it may be that Biton - perhaps an engineer of primarily civil disciplines, or simply not a specialist in projectiles, but nevertheless of some utility to the situation - was called upon to make a submission of his recommendations of *lithoboloi* from whatever antiquated designs could be found in the city library. It is at this point in history that Lewis asserts that Biton wrote his treatise, and I believe this analysis of the text and other evidence supports this reading of the events. If the treatise were indeed from this time, and for this purpose, the remaining question as to whether it was actually put to use and its designs constructed in the period of 156-4 BCE is currently unanswerable. Certainly, once negotiations had been opened with the help of Rome, some calm may have been restored within Pergamon; but in 155 BCE, Prusias had yet to come to terms, and Pergamon's defenses were still being shored up.[405] Once more time had passed for Attalus to continue raising his army, which would also have given the city the opportunity to acquire the appropriate materials to re-fit its torsion artillery, the usefulness of the treatise becomes less apparent. However, there remains that brief moment in 156 - in which Attalus made his retreat and Prusias moved into Pergamon's surrounds - that an emergency stopgap may, even if only briefly, been considered.

The designs related in the treatise are antiquated by Attalus II's reign, but making a changeover to this kind of technology would solve the unique problem of obtaining horsehair and sinew for the torsion catapults of contemporary high technology. One could speculate that this treatise simply contains short summaries of designs that may have been - in their original form - more comprehensive. In other words, each machine's description was summarised from readings of full-length works by each of the engineers mentioned; in which case, Biton's treatise as a whole gives the impression of a work that is meant to pass on a cross-section of recommended designs, from which the addressed individual - that is, the mentioned Attalus - could choose appropriate options for the engineer to consult on further. All but two of the designs in the treatise have clear possible uses in repelling a besieging force. Judging by the use of arrow-shooting *euthytonoi* in conflicts over the previous and following centuries, Biton's arrow-shooting *gastraphetes* could have been intended to replace the arrow-shooting torsion engines that would otherwise have been used to set alight the engines of the enemy. This would explain their inclusion in a treatise initially aimed at *lithoboloi*; they, too, could repel the wooden engines of the enemy, but by using fire - much like those used by the Rhodians against Demetrius.[406] This recommendation makes all the more sense when we consider the estimates we can make using the tower floor layouts at Pergamon, where it seems that large *euthytonoi* of 9-daktyloi springs with 80-daktyloi bolts were likely used. Without

[405] See Chapter 2: Historical Context.
[406] Diod. Sic. 20.83.

sinew-springs, this array of weapons would be useless, and the Zopyrus models that Biton describes could ostensibly be used to replace them at short notice.

I would suggest that the primary subject of the treatise - the stone-throwing *lithoboloi* - are for the purposes of replacing the torsion *palintonoi* which would have been used to disable the opposing *palintonoi* brought forward to breach the walls of the city, as per the tactics used by besiegers of previous conflicts, and recommended by prominent engineers like Philon. Through an assessment of the ammunition of the city arsenal, comparative studies with other Hellenistic fortifications, and a survey of the city's walls, towers, open internal areas and surroundings, we can hypothesise several likely engines and their emplacements that were in use at Pergamon before the crisis. Using this data, we can be sure that the greatest proportion of Pergamon's ammunition - the 60-*minae* stone shot - was used by machines stationed either outside the walls, at the southern gatehouse, or at similar installments to the *Geschützstellung,* near the western wall circuit. The only weapon which Biton recommends that is comparable to Pergamon's most popular 60-*minae* artillery piece is the engine of one Isidorus of Abydos, and our analysis here of the description of this engine appears to show that the current translations and reconstructions of this piece carry some flaws. The text points to the possibility of the weapon using a unique carriage-and-slider design, which may solve several problems that present themselves when attempting to build a reliable non-torsion engine in a short period of time. This is particularly true of the time period in which Biton is to be placed, where skills of bow-making were in short supply, compared with the period which has been investigated around the time of Alexander the Great's early campaigns. Considering the suitability of this design for the job at hand, then, this thesis should help to demonstrate that the text does, in fact, have some bearing on this time period, and on the problem I theorise it to be aimed at solving.

The walls of Pergamon indicate that a complex, varied strategy was available to the defenders. Literary evidence indicates that the Attalids were certainly comfortable with waiting behind their walls for an enemy to lose interest, but at the same time, it is evident that sallying forth into pitched battle was an option that was considered. More relevant is the tactic whereby larger engines could be brought to bear outside the walls or on artillery platforms, such that the advancing engines of the enemy, designed to undermine the defenders' fortifications, could be disabled.[407] Furthermore, the evidence of the extant fortifications indicates that the likely approach vectors taken by the Bithynians were pro-actively reinforced. The Ketios Valley route, where GIS analysis of the terrain

[407] Winter, 1994: 33; Livy, *Hist*. 37.20-21.

indicates that the would-be besiegers of the war of 156-4 BCE could have made some kind of approach, is also the one at which the aforementioned *Geschützstellung* happens to be placed. The walls behind this position have portions that are almost doubled in thickness, and may well have had platforms similar to those theorised to be present elsewhere in the Hellenistic world. It is not unlikely that some of the larger towers that we have seen in this analysis - specifically those of the eastern circuit, the southern gatehouse, and the reinforced section of the western circuit – had wooden platforms built upon them, providing broader and deeper open-air platforms on which larger artillery pieces could be placed than the interior space of the towers might otherwise indicate.[408]

The overall picture of Pergamon's defenses that can be derived from this analysis is therefore this: the gatehouse, in particular, if built upwards such that the extant lower columns supported upper floors of artillery in the manner described by Philon,[409] could quite easily support at least two 60-*minae palintonoi* and their non-torsion equivalents. Elsewhere, the eastern and southern approaches to Pergamon were more likely covered by field batteries using direct fire. The evidence for the use of large artillery in this format is not only provided by the lack of alternative, workable positions, but also by the open-air *Geschützstellung* that appears to have formed part of the western wall circuit. The rocky and treacherous western side of the city, which any attacker or infiltrator from the narrow northern valley pass would be forced to engage with, would require an artificial flat surface to be made on which to place large engines. This platform, then, would provide the place for the field battery of that approach to operate from.

The way forward regarding Biton's engines is, I think, quite clear. On the subject of texts, further research into the veracity of Biton's work by way of his reference to the *dioptra* is greatly needed through investigation of, and comparison with, treatises on optics elsewhere.[410] This, too, would benefit most from an even-handed use of both engineering and philological expertise, as has always been the tradition in the literature of this field. The use of dimensional analysis on samples of both torsion- and non-torsion springs, sympathetically recreated in the methodology of experimental archaeology, will allow us to judge the dimensions of a motive power source needed to adequately perform the task of a non-torsion *lithobolos*, and thus to ascertain at what scale the engine of Isidorus might be comparable to the performance of the torsion *palintonos* indicated to have been

[408] Winter, 1994: 34; Ober, 1992: 148.
[409] Philon, *Par.* 83.15.
[410] Lewis, 1999: 165.

originally employed at Pergamon. In particular, we can ascertain precisely what scale of Isidorus' engine provides the necessary spring-tension to launch the aforementioned 60-*minae* projectile to a similar range as that of a 60-*minae palintonos*, as determined by its own dimensional analysis. To this end, I have suggested a methodological framework for this very operation, which uses principles of modern production engineering to solve an ancient question. The step-by-step guide to dimensional analysis of these weapons I have offered here hopefully demonstrates how this research can be conducted in a practical, realistic sense, and in a manner that is cost-effective and incisive. In this way, we might even be able to ultimately exonerate the engineer Biton from allegations of fraud.[411]

On the level of general interest, I hope to have reconciled some aspects of modern and ancient engineering, and to have done so in a fashion that is agreeable to scholars of both fields. With the integration of more stringent testing procedures in the field of experimental archaeology, in such a way that makes use of proven engineering principles from industry, more useful research can be carried out. The use of advanced GIS analysis provides an insight into the physical bricks and pathways of the ancient world as the ancients themselves understood them, and in turn allows us to understand their own motivations and actions. A real sense of the interconnectedness of ancient cities, and the interaction of ancient people with their landscape, is enabled by the use of this kind of study, and it deserves closer integration with both our research and our methods of teaching. The modelling of physical solutions to textual problems with CAD has also been indispensable for this thesis. The increasing availability of this kind of software, the increasing ability of commonly-available consumer hardware to execute it, and the rapidly increasing democratisation of training and information for its use, signals a bright array of opportunities for study ahead. The possibility of more researchers being able to sketch and model objects from the ancient world, using a combination of textual and archaeological evidence, provides another potential source of interesting and engaging work for current and future graduates. Finally, the ability to bring relevant models of material culture into the lecture hall can serve to draw forth discussion, and provide a tangible link to - and a window into - the texts we teach and the contexts they come from.

In conclusion, this thesis shows that a combined study of fortifications, the ancient text, geographical survey, archaeological evidence, literary research, and principles of engineering, provides further argumentation and evidence for the time of Biton, in favour of the dating theory

[411] Drachmann, 1972.

put forth by Lewis. It also demonstrates that the study of Biton is not yet over, and we have yet more to learn from this controversial text.

APPENDIX 1: UNDERSTANDING TREATISES BY GEOMETRY

1. AXIOMS AS A PROGRAMMING LANGUAGE

The most common complaints with Euclidean axioms[412] are rooted firstly in the fact that they are written with the same conventional background as the Aristotelian term logic, also formalised in *Posterior Analytics*.[413] The fundamentals of term logic as they apply to this study are brief: each axiom consists of one or more *propositions*, which in turn consist of two *terms* that are linked together by a logical relationship. One may then deduce a *syllogism* from the product of two or more propositions, which may itself be either true or a logical fallacy; depending on whether each proposition makes a universal or specific claim about its terms (some, all, or none); whether it does so in either the affirmative or negative (are, or are not); and whether more complex quantifiers are being used to distinguish the terms (each, alternating, sometimes, etc). This line of logic may be expressed in a variety of syntactical patterns, but is perhaps most recognisable to a Humanities student, outside of philosophy, in their experience of high school mathematics. The Euclidean method of describing rules, geometric features, and problems, are all antecedent to the “hypothetico-deductive method of modern mathematics”[414] which continues to inform the phraseology of mathematics instruction everywhere. A claim is made to be assumed true - for example, “*let equal 5”* - and the rest of the construction is iteratively deduced from that claim using the prescribed axioms.

The possibility of the created syllogism from some outwardly straightforward propositions being clearly and reasonably false is a well-established problem.[415] For our purposes, this might be an occasion when an intersection of logical rules pertaining to a physical construction, expressed through natural language, creates the possibility of one or more geometric elements that do not, should not, or cannot exist in a design, in the observable universe, or in the three-dimensional space as described by the geometry as a holistic system. Herein lies the first major issue with Euclidean axioms that can lead to confusion, particularly when dealing with texts that are incomplete or corrupt: simplistic axioms that express their logic through natural language naturally attract contradictions and confusion, which is not to mention any errors of logic or lack of foresight on the part of the theorist that formulates the rules. Philosophers are well-equipped for catching errors in

[412] Daus, 1960: 578.
[413] Arist. *An. Pos.* I.3.
[414] Daus, 1960: 576.
[415] Arist. *An. Pos.* I.7f; see also the footnote below on Bobzien (2020) and Uzquiano (2020).

the logic itself before they happen - using categorisation of the propositions' inputs and operators, a statement that will lead to a false syllogism may be caught.[416] However, it remains that problems arising from faulty assumptions in the *input* to this logic are trickier to avoid.

A simplistic example of both errors would be if Diogenes' infamous chicken-man moment[417] were explained in terms of axiomatic shortcomings and syllogistic fallacy. The definition that Plato offered of 'a man' might be expressed so:

Proposition 1: No man has feathers.
Proposition 2: All men are bipedal.

Thus Diogenes' conclusion :-

Syllogism: A plucked chicken is a man.

Lacking context, definitions, and rules, there are a multitude of logical errors and fallacious deductions that could occur in this instance, and many that can be inferred from Diogenes' tongue-in-cheek reply. The task in 'debugging' for this output relies on finding out what is missing from the axiomatic logic, and what problems arise from assumptions that are made by the human beings responsible for the logic. The assumption that the "*set*"[418] of featherlessness must necessarily include that that condition be naturally-occurring, and not merely an observed characteristic, might be considered an oversight in the axiomatic ruleset. So is the Platonic notion of what feathered and featherless bipeds are, and how they might look. The possibility of Diogenes formulating a subcategory of previously-feathered but currently-featherless creatures, that may contain entries such as the plucked chicken, is made possible by an incomplete roster of rules and definitions.

[416] The antecedent to the formalised AEIO method of disqualifying false syllogisms follows from the AEIO or *square of opposition* in Arist. *Int.* 6 (the need for which is first hinted at in *An. Pos*. I.32). For an introduction to fundamentals of Aristotelian term logic, see Bobzien (2020), who also provides a concise primer to more complex constructions using predicables (2.2), and compounded syntax (2.3). For complex quantifiers and their effect on the AEIO method of qualifying syllogisms, see Uzquiano (2020). 'Existential import' as a philosophical term is perhaps first properly formalised in Russell's (1905) paper *The Existential Import of Proposition*. This is also one of the more eminently readable introductions to the subject.
[417] Dio. Laert. *Lives* IV.40. The thrust of the joke lies in the wordplay of the original Greek and Plato's final revision of his definition, but the usefulness of the set-up - the ridiculous definition - remains.
[418] "Set" here refers to this concept of categories as it occurs in philosophy and mathematics (see below). In computer programming, this is often called an array.

Further abstracting this logical construction by translating it into algebraic functions, and thus eliminating the problem of natural language, is not enough, so long as the faulty axioms remain.

2. AXIOMS AS ALGEBRA

Demonstrating this fact - that natural language is but one of the problems facing axiomatic logic - is simple. In mathematics, the algorithmic process of propositions and syllogisms is best represented in algebraic methods like those used in subject-predicate calculus. Where Descartes used algebra to define geometric elements and constraints in his proto-calculus, subject-predicate calculus extends this to logic constructions. For our purposes, therefore, it suffices to define these as differential equations which use the logic of Aristotelian terms expressed in the syntax of mathematics instead of being expressed with natural language, including translating the logical operators encoded in words.[419] The usefulness of this conversion becomes especially apparent where it can be applied to our testing of textual information in ancient sources, like engineering treatises, using the luxuries afforded to us by the analysis of modern algebra and calculus.[420] Consider the following example[421] of Plato's propositions being translated into a mathematical algorithm. First, let us rephrase the propositions so:

"In the category of all men, it is also true that there is bipedalism (men as determined by a function of featherlessness and humanity)."

Consider that quantifiers are notated like so:
[∀] stands for "for all"; as in, "For all men, featherlessness is a characteristic."
[∃] stands for "there is", or it is true that; as in, "For all bipedal things, there is one that is man."

And that the variables are notated like so:
[m] stands for "man" or "men".
[isBipedal] stands for the formal constant of 'being bipedal' as being true.
[f] stands for featherless things.

Thus the equation: ∀ (∃ ((,)))

[419] Like those given as examples of quantifiers above.
[420] See Chapter 5.1: Recommended Application of Dimensional Analysis.
[421] Adapted from Dwyer's (2016: 30) introduction to predicate calculus, to fit our Diogenes problem.

Or, if using universal instantiation to solve for a specific object [ ] that may belong to any of these categories, we might use the following notation. Where the object [ ] belongs to a class of Men [ ], and objects like it are also members of the class Featherless [ ] and Bipedal [ ]:

$$(\forall\ )(\quad \Rightarrow (\quad \& \quad))$$

This logical relationship can then be algebraically manipulated in various ways. The principles of universal generalisation, for example, would allow us to notate the notion that the class of featherless things is a greater class that *contains* the sub-class of men. So, for any one man ( ), we know that they can be attributed to class featherless ( ): . Therefore, it follows that for the entire class of men, all can be attributed to that class: (∀) .

Thus, the subjects and predicates of a proposition are encoded as algebraic variables, and the logic applied to them is performed by algebraic operators. In this way, it is possible to build more complex constructions that transcend the true/false dichotomy and instead explore variables that scale against one another (for example, a rule to govern two diverging lines). Taking this notion further, one can plot the exploration of a complex logic problem in graph form as one normally might in calculus. The beauty of translating this logic into algebraic functions lies in its ability to be applied to more complex problems with much greater sets of more varied data. Furthermore, the reduction of logical functions to pure mathematics allows for a quasi-human process of reasoning to be applied in programming, providing the basis for artificial intelligence.[422] However, the fact remains that we are dealing with axioms that are translated from natural language, and/or may be faulty in their logic in other ways. In the equation above, featherlessness is *still* not expressed as a function of whether that featherlessness is naturally-occuring or not, and suffers from the same existentialist definition that results in Diogenes' conclusion.[423]

It can be said, then, that the conclusion that Plato expected to come of his definition relies on a number of assumptions that are external to the logic system or the information contained in the proposition, rather than inferences drawn directly from a combination of more

[422] Pearl (1988: 21) provides a succinct example of reducing logic statements in natural language to mathematical notation for calculating probabilistic reasoning, as opposed to simple classification or true/false statements like our chicken-man problem.
[423] Dwyer, 2016: 26.

robustly-constructed propositions and the simple logic of the philosophical equation.[424] Further definitions - the spiraling recursiveness that Aristotle sought to dispel by claiming that some things are obvious, and that Euclid attempted to fend off with his exhaustive proofs - are required. Paradoxically, as with our Plato example, this still does not prevent Euclid or his students from making false Aristotelian assumptions further up in their proofs.

3. EUCLIDEAN DISTANCE

This brings us to the most infamous error of Euclid that has prompted the great body of work referred to earlier: the axiom (or rather, postulate) of parallel lines. The first reason for its infamy is in the awkwardness of its phrasing:

> "… if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles."[425]

The postulate attempts to show that two lines, drawn infinitely, will eventually cross unless they are both at right angles to another line drawn through them; thus demonstrating parallelism by the inverse. It is unwieldy at best, and cannot be easily represented in logic; hence attempts to improve or replace the postulate with more elegant versions, such as Playfair's axiom.[426]

The second problem with Euclid's enduring notion of parallelism is the implicit assumption that the observable universe could be mapped to a rigid hypothetical space where *distances are constant*. For the purposes of this discussion, we need to visit one of Euclid's ancillary rules:

> "In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle."[427]

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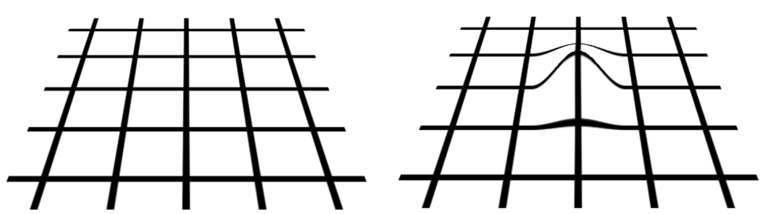
[424] Arist. *An. Pos*. II.5 makes clear how missing steps in the logical equation rely on assumptions a human being might make due to their experience of the world, and seeks to separate that in a manner that might serve to create a pure-logic system.
[425] Euc. *Elements* I Post. 5.
[426] Clapham & Nicholson, 2009: 608.
[427] Euc. *Elements* 1 Prop. 47.

This proposition provides a proof for the Pythagorean theorem, which could be used to draw a line between two points in a construction and calculate the distance between the two as though it were a hypotenuse to an imaginary right-angled triangle.[428] While this may appear accurate on an entirely mythical surface like Euclid's drawing table, populated using a straight-edge and compass, there is no formal proof that can be provided from outside of this theoretical construction.

The mathematicians Lobachevsky and Bolyai,[429] writing in the first half of the 19th C., introduced the notion that two parallel lines could each be described as not merely a straight, perfect function of two points in the traditional sense of $(_1 ; \ _1) \rightarrow (_2 ; \ _2)$. Instead, each could be defined as two mirrored curve functions that could curve away from or towards one another. When translated into three dimensions, the inference drawn from this is that the plane itself upon which these lines lie is not flat, but curved. This forms the basis of *non-Euclidean* geometry; the notion of a geometry in which the realm of construction's , , and planes are not constants in relation to one another.

In the geometry of Bolyai-Lobachevsky, the cuboid space in which Euclidean geometry is mapped is replaced with a spherical one. Much like if one were to attempt to build an infinitely-wide temple by placing a foundation slab flat along ground level, it would be found that that slab would, in fact, be curved co-radially with the curvature of the earth. If one were to lay a very long tape measure along the slab's top, or outer curvature, the distance measurement would be longer than if it were measured as a true straight line directly through space from the one outer vertice of the slab to the other - which is how Euclid would have us measure it.



A Euclidean and non-Euclidean plane as visualised with construction lines.

[428] Hence the familiar formula used in high school geometry for the distance between two points, where distance is equal to the hypotenuse of a triangle where the two other sides thereof are made up of the distances between the two points' respective horizontal and vertical positions.
[429] See Lobachevsky (1914), a later translation of his original 1840 work.

This curvature or distortion is almost immeasurably tiny in small constructions, such that Euclidean notions of perfectly flat, consistent space appear to hold true when viewing, say, a small altar. However, viewing a similar supposedly flat plane several kilometres long (particularly from a great vertical distance) will quickly demonstrate the opposite in the same way that viewing the horizon from the crow's nest of a ship might do for the horizon. Similarly, a column that extends infinitely, directly into space, will increase in diameter the further away it extends. The gravitational pull of objects in space - the same force which conforms the earth to a sphere - is what dictates the shape of non-Euclidean planes of construction.[430]

Thus, if we attempted to notate that very long slab as a purely Euclidean function - a straight line - and re-create it elsewhere, we would describe a slab that becomes more and more inaccurate the larger (or the higher in dimensionality) it is. We could attempt to circumvent this problem by notating it as a section of a radius, where that radius is co-radial to that of the earth or other body around which the construction is taking place. We might, then, have an acceptably approximate set of coordinates to map out the top and bottom faces of the slab's length. However, describing the construction becomes more complicated once we consider that we would have to do the same in order to map the widths. This is then followed by the problem of describing the height of the slab, which becomes greater the thicker it is, and the further away from the centre of our spherical plane it happens to travel. Each vertice of the slab will also lean outwards the further away from the origin they are, resulting in a slab that cannot be explained in terms of right angles. It is quickly apparent that attempting to describe each of these features using Euclidean rules in a non-Euclidean space is a fool's errand.

The third issue with the axiom of parallelism lies in the problems it creates for accurate calculation of area and volume of a curve or curved structure respectively. For the purposes of geometers in particular, it may be important to measure the area under or above a curve - whether it be to estimate the required volume of materials for a construction, or, say, the approximate throughput of a waterway given its cross-section.

[430] An extreme example would be the visible inversion of Euclidean rules in the space around the event horizon of a black hole.

Assuming that we are operating in an environment like Biton's, with only two dimensions and a manageable set of data, the problem we are faced with is simply that of how to measure the area carved out by this curve of points. Isaac Newton solved this problem by creating a formalised calculus whereby a quadratic equation can use the and values of each known data point on the curve to draw an imaginary rectangle, each one terminating at the border of the rectangle created by its closest neighbouring data point. The rectangles denoted by each ; pair could have their areas measured, and all of them added together to create an approximation of the total area.[431] However, the obvious problem with this method is that each rectangle necessarily excludes a portion of the area we wish to measure as the curve passes over its flat top to meet the next data point, creating a cumulative error that only becomes greater with the larger the construction becomes or the fewer data points there are available. Those unmeasured areas can, in turn, each have the same process applied to them - at a great cost of effort, with diminishing returns. While this innovation is attributed to Newton, the factor of error in it can ultimately be filed under the category of errors created by the Euclidean construction of space, as the plane of construction and the axioms used to construct and calculate the area of Newton's rectangles remain essentially Euclidean in origin.

Nevertheless, it remains that a curve can thus be mathematically notated using the quadratic equation, which provides the rise-over-run (or over values) of a curve's points on a plane. This is a significant step forward from Euclid's inability to describe a complex curve outside of expressing it as made up of sections of spheres and straight lines. However, the more complex the curve and the more points used to plot it, the more stored data and calculations are required in order to plot it. The result is that a curve of high resolution - whether it be for representing statistics, or a purely geometric, constructed shape - becomes very unwieldy in short order. Furthermore, when another dimension is added such that the and coordinates are joined by a to denote their place in three-dimensional space, the sets of data points grow by one exponent; literally, from a squared to a cubed number. In other words, where the number of sets - like , , and - might be notated as , a simple straight line with only -values would number only ^{1} points. In three-dimensional space, the Cartesian plane becomes a cube, and the number of data points becomes ^{3}. Assuming that we remain within the realm of engineering where ^{3}is true, it remains that calculating the volume of

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[431] See Bos' (2020) essay on Newton and Leibniz, particularly the chapter beginning p.54.

this construction with the Newton-Euclid method suffers from compound inaccuracies thanks to the imprecision of the method.

Therefore, when trying to map a construction over a great multi-dimensional area, errors of Euclidean distance and assumptions about plane geometry prevent us from storing or retrieving accurate information. This applies strictly to representations of physical objects, but may apply to other data as well. When visualising a complex interplay between variables in a graphical form, other dimension(s) may be added as sets instead of, or alongside, , , and . For example, temperature, velocity, and volume of an observed element might be plotted in a three-dimensional non-Euclidean space in order to visualise its transformation in graph form. However, if and are presumed to be rigidly constrained and without *tensors*, where the nature of the data's environment may require the opposite, the plane on which this transformation is represented may be drawn in such a way that is distorted from reality and will cause the representation of all other variables to be inaccurate.

This refers specifically to constructions which involve representation of a physical thing in the first three dimensions. When it comes to constructions that are purely made up of other data, however, we are liable to run into problems of *high dimensionality*, caused by an *over-dimensioned* space. This is an issue that does not necessarily affect the work of Biton or his contemporaries in any measurable way, but it is one that is mentioned often in current literature with reference to Euclidean distance. It is also, as mentioned previously, an important parallel to keep in mind when investigating physical constructions whose accuracy, performance, or some other measurable characteristic is influenced by a variety of interacting variables. For that reason, I believe it is necessary to explain the root of this complaint so that it can be contrasted with problems of Hellenistic thought or design.

To illustrate this problem, let us imagine that an ascending curve is plotted on a Cartesian plane, where each point on the curve can be plotted by means of a linearly-increasing -value and its corresponding -value. The more infinitesimally small the gradations between each point (and therefore, the more points there are), the more precise the notation of the curve and the more accurately it can be recreated. Conversely, the greater the overall scale of the data set, the more data points we may expect there to be in it, if it is to be accurate. Herein lies the source of the aforementioned 'curse of dimensionality'. If a third dimension is added to this large construction to create a cube filled with data points, the differential between the minimum and maximum distance measurements between any two data points across sets becomes smaller. Simply put, dissecting the space into more dimensions and adding more data points means that the sheer spatial density of plotted data points makes comparison and measurement between those points more complicated, and less meaningful, due to the difficulty of handling infinitesimally small distances. Particularly in the worlds of statistics, logistics, or engineering design computing, a construction of dimensions may have the exponent run into hundreds of sets, and our cuboid space will be dissected into a multidimensional space, or hypercube. As approaches 50 or more, the differential of maximum to minimum distances between points tends, asymptotically, towards 0, as the space becomes overpopulated and bristles with data points.

However, the bulk density of these points tends to concentrate at the spherical centre of our hypercube. Outlier values are, by their nature, few and far between, and the outer corners of the hypercube therefore remain sparsely populated while the centre increases in density at a much greater order of magnitude. Even in randomised data sets, the phenomenon of *covariance* demonstrates that varied data tends to vary in similar ways, congregating in 'clouds' of data points when mapped graphically.[432] The distribution of points, therefore, becomes greatly skewed towards the centre, and accurate calculations can no longer be made under the assumption that distribution of data (or *cosine similarity*)[433] has remained the same. The logic of an algorithm that is constructed from axioms to perform a kind of human-like task of intuition tends to fail at making meaningful inferences from these high dimensions.[434] Hence the 'curse of dimensionality', and the fact that "higher norm parameters provide poorer contrast between the farthest and nearest neighbour".[435] This contrast becomes even poorer where the axiomatic logic that guides our calculations for distance between data points is itself flawed, as explored above.

Naturally, high dimensionality is a problem conceivable only far beyond Biton's time and place. However, there is a real consideration for the methodology of experimental archaeology as it

[432] Clapham & Nicholson, 2009: 193.
[433] In other words, the measure of similarity between two vectors (or in this case, lines of data sets which dissect our hypercube). The more dissections that are made, the smaller the difference between each dimension and their respective data sets.
[434] Domingos, 2012: 82.
[435] In the words of Aggarwal *et al*. 2001: 6.

pertains to investigating the scaling of machinery used in siege, and the multiplicity of variables that are to be kept in harmony for experimentation to be as accurate as it could be.[436]

To combat the inaccuracy of Euclidean space relative to the many possible shapes planes might take in reality, late 19th C. scholars like Riemann[437] attempted to create a unifying geometry that would work not only in the specific curved, spherical space created by our earth's gravitational pull and our position on it, but one that could be applied to any imagined non-Euclidean space. Every point in Riemannian geometry is given not only a Cartesian position, but is assigned sets of *tensor* variables which describe the shape of the plane where that same point occurs.[438] Thus, mapping points in a graphical form that is accurate becomes possible, as each point is accompanied by its tensors, and more elaborate area and volume calculations than Newton's can be used to find accurate approximations of important values.[439] This theoretical framework of course provides the basis for Albert Einstein's theory of general relativity, which posits that the position of any one point of an object is determined not only by space, but by the forces of time and gravity. Furthermore, it would follow that 'distance' is not a strict term, but interpretive; and is indeed variable, dependent on the movement of the plane.

Thus, it is true to say that Euclid's fifth postulate of parallel lines - or any other revision of it, including Playfair's - cannot be true, inasmuch that a perfect vacuum that is also devoid of gravity and time cannot exist either. Similarly, Newtonian physics, which relies on the expression of its laws through a three-dimensional space mapped in the constant - - of Descartes, is also rendered inaccurate. A description of an object using Euclidean rules will always be an imperfect facsimile of a real phenomenon, and this in turn will create imperfect results when mapping high-dimensionality logic problems to graphs or models, creating the great Euclidean hullabaloo that greets us when researching the application of his work.

It also useful to note that these problems demonstrate to us that not all observable phenomena, or indeed the proofs we may seek to make for them, can be accurately and succinctly axiomatised (that

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436 See Chapter 5.2: Relevance.

437 Riemann's 1854 lecture formalising this theory was transcribed and published after his death in 1868. The translation thereof by Clifford (1998) is eminently readable.

438 Clifford, 1998: 13.

439 See for example Leisenring (1951), who calculates area by inference, superpositioning similar shapes about the perimeter of the actual hyperbolic geometrical shape we wish to measure, and dissecting it into manageable pieces that can be added together.

is, recursively dismantled to the point of a single, or handful of, axiom(s)). At their most basic logic level, they require more complex proofs expressed in formulae which in turn must have their operators, variables, and rules carefully formalised under a *set theory*. Ideally, this should be performed in such a way that *all* phenomena and constructions can be explained, or proofs provided for them, using that single unifying theory. The academic field of Set Theory arises from the study of this issue, and attempts to create, verify, and improve sets for use in the various applications of mathematics. The most relevant example of this problem in action and an attempt to solve it - as it pertains to the interests of Humanities scholars - is expressed by Russell's Paradox.[440] In appraising the shortfalls of Naive Set Theory, which also uses natural language like Euclid's to describe sets and rules, Bertrand Russell lays the foundation for a theory that eschews natural language entirely. Russell's efforts in describing a symbolic logic, using pure mathematics, would in turn create the necessary conditions for the evolution of term logic into machine logic. The significance of this step is not only in the advancement of theory towards a more precise set of tools for solving real problems, but also in the first step of an evolution of the (still quintessentially Aristotelian) logic system into our closest approximations of a universal logic.

Diogenes' false (or rather, unpredicted but nonetheless logically-derived) syllogism exposes a faulty logic, made so by an incomplete set of axioms, theory, and definitions. Similarly, the axioms of Euclidean geometry can create mathematical and geometric inconsistencies that are inaccurate in a realist sense. Euclid's attempt to create a proof - a system of geometry from first-order principles, such that no further definitions nor assumptions would be required - is a feat in itself and deserves due recognition for attempting to provide a unifying body of theory to account for the entirety of worldly constructions. Ironically, the problem of sets and attempts at unification of geometric theory seem themselves prone to infinite recurse. Philosophers of mathematics have attempted to solve the problem for good by searching for a universal set theory for all applications, but the translation of natural-language axioms to a pure logic remains a sticking point.[441] A perfect framework still does not exist, and set theories are generally deployed according to the requirements of the field or the job at hand.[442]

[440] See Russell's *The Principles of Mathematics* (1903) .
[441] Beginning with the initiative *Hilbert's Program* shortly after Russell's work.
[442] Versions of Zermelo-Fraenkel (ZF or ZFC) set theory provide some of the most persuasive candidates for a universal set theory. See Weisstein (2003) for a neat appraisal of current literature on ZF theory and the problems currently facing the unification of this logic system amongst mathematicians.

APPENDIX 2: ISIDORUS IN DESIGN
THE TEXT OF BITON 48-51

Here I have used Marsden's translation loosely, including Greek measurements. For each line as I have divided it, I have also added my own re-interpretation using modern vernacular in brackets [ ], together with modern metric measurements. This is intended to serve as an instruction for design in the same manner as Biton originally intended. Biton 48f. (transl. Marsden):

> Ὑπαγράψομεν δέ σοι καὶ ἑτέρῳ τρόπῳ <λιθοβόλου> κατασκευήν. πολλάκις γὰρ αἱ τῶν τόπων θέσεις οὐκ ἐπιδέχονται τὰ αὐτὰ τῶν ὀργάνων. ἔστι δὲ τοῦτο κατεσκευασμένον ἐν Θεσσαλουίκῃ ὑπο Ἰσιδώρου τοῦ Ἀβυδηνοῦ. εἶχε δὲ καταβολὴν τῆς ἀρχιτεκτονίας τοιαύτην.
>
> *We shall describe for you also the construction of a stone-thrower by another method; for often local conditions do not favour the same types of engine. This one was designed in Thessalonica by Isidorus of Abydos. It had the following basic design.*
>
> κανὼν ἦν τετράγωνος, ἔχων τὸ μῆκος ποδῶν ΙΕ´, τὸ δὲ πλάτος ποδῶν Β´, καὶ τὸ πάχος ὁμοίως· ἦν δὲ εἰργασμένος ὁμαλὸς πανταχόθεν· εἶχε δὲ καὶ περισεσιδηρωμένα τὰ ἄκρα ἄνωθεν καὶ κάτωθεν ἐπὶ πόδας Β´. καὶ ἔστω οὗτος ὁ κανὼν ὁ Α.
>
> *There was a beam of square cross-section, having a length of 15* podes*, a breadth of 2* podes*, and thickness the same; it was planed level all round; it had its ends plated with iron at the front and rear for a distance of 2* podes*; let this beam be A. (1)*

[New sketch. Make a square cross-section of 617.6mm. Extrude this to 4632mm. In the detailing phase, add 1mm sheet-metal cladding around both ends, also up to 617.6mm. Label this part A.

Note: It is possible that this beam be "hollowed" in the same manner as Charon's stone-thrower; that is to say, a central groove routed from the body, into which moving components can be installed. Biton may, as he often does, have forgotten to mention it. For this reason, do not regard part A as necessarily solid; extruded cuts of all kinds can be made in it to fit various components so long as it is not structurally undermined. Even then, Biton later adds iron strapping (most likely flat-bar) in great quantities, to brace the design.] (1)

εἶτα ἀπὸ τῆς σιδηρώσεως ἦν ἕτερος κανὼν τὸ μῆκος ἔχων ποδῶν ΙΒ´ ὁ Κ, ἔχων τὸ πλάτος ποδὸς Α´. εἶτα ὁμοίως καὶ ἐν τῷ ἄλλῳ μέρει ἀπὸ τῆς σιδηρώσεως ἒτερος κανὼν ἰσομεγέθης τῷ Κ ὁ Λ.

Next, starting from the iron plating, was another beam, K, having a length of 12 podes, *and a breadth of 1* pous*. Next, similarly, on the other side, starting from the iron plating, was another beam, L, of a length equal to K. (2)*

[New sketch: rectangular section, labelled K. Give it a square cross-section of 308.8mm and extrude to 3705.6mm. Place it into an assembly with A, collinear with A's length. Mate to upper or lower z-face. (You may presume it to be flush with the edge of this face, or otherwise, as this build takes shape.) Make sure it is positioned immediately behind the cladding.] (2)

καὶ διὰ τῶν ΚΛ κανόνων ἦν διωσμένος κοχλίας σιδηροῦς ἐνηρμοσμένος ἐν ἐπιτονίοις. εἶχε δὲ τὴν διάμετρον ὁ κοχλὶας ποδὸς τρίτου μέρος. ἐχέτωσαν δὲ οἱ ΚΛ κανόνες ἀγκῶνας σιδηροῦς τοὺς ΔΕ.

… and through the beams KL was pushed an iron roller fitted in bearings. The roller had a diameter of one-third of a pous*. Let the beams KL have iron braces, DE. (3)*

[Mirror part K across the centreline of A. Label it L. Make an extruded round cut of 101.94mm diameter in both, to a depth of 200mm, to accommodate the upcoming bearings. (Position of cut is to be inferred from later instructions; make it undefined for now.)

New sketch labelled Roller. Cross-section a cylinder of diameter 101.94mm. Its extruded length is driven by the distance between the inside faces of K and L. On both ends, add a central stub (a lathe centre) of 50mm diameter and 200mm length. (This will be the bearing face of the roller.)

New sketch. Plain bushing of ±50mm I.D., 101.94mm O.D., and extrude to 200mm.

(Clearance it, and the roller body, appropriately so that the bearing assembly does not bind under load.)

‘Roller’ will need to roll – like a conveyor roller – positioned between the beams of K and L. Insert it, with bushings, into the assembly.

The beams of K and L are to have iron braces, but we are unsure what kind. In detailing phase, add flat bar supports as deemed appropriate.] (3)

> εἶτα ἀπὸ ποδὸς Α΄ τοῦ Κ κανόνας ἐπὶ τὸν Α κανόνα ὀρθὸν ἕτερον κανόνα ὑποθήσομεν ἴσον τοῖς ΚΛ τὸν Μ· εἶτα τούτῳ ἕτερον κανόνα ἴσον, τὸ αὐτὸ διάστημα ἀπέχοντα ἀπὸ τοῦ Λ ὅσον ὁ Μ ἀπὸ τοῦ Κ. ἔστω δὲ οὗτος ὁ κανὼν ὅπου τὸ Ν. καὶ διώσθω δι’ αὐτῶν κοχλίας ἕτερος ἰσομεγέθης τῷ ἐπάνω· καθηρμόσω δὲ καὶ οὗτος ἐν ἐπιτονίοις.
>
> *Next, 1* pous *distant from K, we shall place against the beam A another straight beam, M, equal to K and L; next, another beam equal to this, at the same distance from L as M is from K. Let this be where N is. [In other words, let this be N.] And let there be pushed through them (i.e. M and N) another roller equal in size to the one above; let this, too, be fitted in bearings. (4)*

[Add two more parts labelled M and N, also mirrored across the centreline of A, dimensionally identical instances of the K/L pair.

Place a second instance of Roller, with bushings, into M and N’s extruded cuts. Roller length dimension is again driven by the variable distance between M and N.

Constrain the M and N assembly’s centre-point to be 308.8mm away from K and L in any axis; we know only that it is ‘below’ it in the Y-axis. Roller (2) is therefore the ‘lower roller’. Place parts as you deem appropriate; no other information available at this time.] (o4)

> εἶτα ἀγκὼν ἄλλος φερέτω σιδηροῦς ἀπὸ μέσης τοῦ Μ κανόνος ἐπὶ τὸν Δ σιδηροῦν ὁ Τ, ἀπέχων τοῦ κάτω κοχλία πόδα Α΄, ἕτερος ἀγκὼν διωσμὲνος διὰ μέσης τοῦ Ν κανόνας ἔστω. καὶ ἐνδεδεμένος δὲ ἔστω ἑκάτερος σιδηραῖς λεπίσιν.
>
> *Next, let another iron brace, T, be run across the middle of beam M to the iron brace D, at a distance of 1* pous *from the lower roller. Again, at a distance of 1* pous *from the lower roller, let*

another brace be pushed through the middle of beam K, let each be fastened on with iron plates. (5)

[These braces may provide the bracketing for the total assembly of A + KL + MN, so feel free to continue to position these parts in any format that will allow for a catapult-shape to form in the viewer, and leave bracing for the detailing step.

We may presume the following, though. The flat-bar braces run across M and N. They remain 308.8mm distant from Roller instance (2), in all axes. Another brace is fastened with plates, 308.8mm distant from Roller (2).] (5)

πρὸς τὸ ἀκινήτους δὲ εἶναι τὰς λινέας, ὅ τε Ν κανὼν καὶ ὁ Μ ἐχέτωσαν κὸρακας σιδηροῦς ἀδιαιρὲτως παντὶ ἔργῳ στερεμνὶους ὡς ὅτι μάλιστα, ἐπ' ἄκρων ἔχοντας δύο ἀνακαμπάς.

To ensure that the withdrawal-ropes are immovable, let the beams N and M have iron teeth, as far as possible solid enough, without joins, to stand all strains, and having at their ends two inclined faces. (6)

[The N-M assembly is the slider along which the carriage for the shot travels. Give it ratchet teeth that pawl-in from front to back. Pattern them as appropriate for scale. Both ends of these ratchet rails must incorporate a stop to prevent the carriage from coming off of the slider.] (6)

εἶτα καὶ ἕτερος κανὼν ἐπικείσθω ἐπὶ τῶν ΜΝ κανόνων, βεβηκὼς δὲ κατὰ τοῦ Α, ὁ Ξ, σιδήρῳ ἐνδεδεμένος ἀραρότως.

Next, let another beam, X, rest against the beams MN, sliding along A, and plated flush with iron. (7)

[New sketch, X. This carriage which carries the shot, X, is not dimensioned, but presume its width to be driven by the distance between M-N's parallel faces. The length is also undetermined. Here we diverge from other scholars. I estimate 694.8mm (36 *daktyloi*). The height is also undetermined. It need only be tall enough to perform the following functions:

X is intended to slide freely up and down NM, and has an attached pawl to interface with the ratchet rail of NM, which prevents it from traveling **forwards** unless released.

Conventionally, one can cut a dovetail into NM, and add one to X, such that they interface like the 'ways' of a lathe or mill.

X is cladded with sheetmetal on the face that slides along NM, presumably to aid in high-speed operation.] (7)

> διὰ δὲ τοῦ μέρους τοῦ κανόνος τοῦ ἀπολαμβανομένου ὑπὸ τῶν ΜΝ διακείσθω τόξον τὸ Π.
>
> *Through the section of beam A left exposed by M and N, let the bow, P, be fixed. (8)*

[Near the front of A, make an extruded cut to hold a bow. Make the extrusion rectangular in cross-section, but feel free to radius all edges.

New sketch, P. Sketch a 'palintone' bow shape (viewed from above). Presume a total width of 4800mm. Extrude it to a depth whose dimension is driven by the height of the extruded cut made in A. Radius edges as well, to imitate hand-carving. Add it to the assembly.] (8)

> ἐχέτωσαν δὲ οἱ κόρακες οἱ ἀπὸ τῶν λινεῶν ἄλλα ἀγκίστρια, <ἃ> ἐκτείνει τὴν νευρὰν τοῦ τόξου ἐν ταῖς ἐπιτάσεσι τῶν κοχλιῶν.
>
> *Let the hooks on the withdrawal-ropes have other, little hooks, which draw back the string of the bow in the course of tightening the rollers. (9)*

[Important Note: We diverge again from previous scholars' reconstructions. They assume the bowstring is released by the trigger, and only interfaces with the stone shot, carrying it away.

Instead, we will assume that the bowstring attaches to the carriage X. The trigger releases the ratchet pawl. The bowstring then carries X, and the stone shot, to the end of A. X is stopped by the stop on the ratchet rail. The stone shot is thus flung off of the front of the catapult.

It is only necessary here to add “hooks”, or a hooked bracket, to the rear face of X, so that a bowstring can be hooked into it.] (9)

> εἶτα ἔστω ἐν τοῖς MN κανόσι σφενδόνη κατηρτισμένη ἐκ τριχῶν, ὥστε δύνασθαι τὸν πέτρον βαστάζειν, ἡ Ψ, τὸ δὲ σχῆμα οἷον τυγκάνει ὑποτέτακται.
>
> *Next, let there be a sling, F, on the beams MN prepared from hair, strong enough to cope with the stone-shot. The design, as it might be, has been drawn below.” (10)*

[Sketch new part F. Use dummy assembly as a model before adding to final assembly. Add cable running from the ends of the bow to hooks on carriage X.] (10)

GLOSSARY OF TERMS

ABBREVIATIONS OF SOURCES:

AE – Atkins, T. & Escudier, M. (2013). *Oxford Quick Reference: A Dictionary of Mechanical Engineering*. Oxford: Oxford University Press.

CN - Clapham, C. & Nicholson, J. (2009). *Oxford Concise Dictionary of Mathematics.* Oxford: Oxford University Press.

UNIT CONVERSIONS:

Units of Length/Distance (Attic)

	Greek	Metric [m]
daktylos		0.0193
pous	16 *daktyloi*	0.3088
pechys	24 *daktyloi*	0.4632
plethron	100 *podes*	30.88
stadion	600 *podes*	185.28

Units of Length/Distance (Pergamene/Samian-Ionian)

	Greek	Metric [m]
daktylos		0.0217
pous	16 *daktyloi*	0.347
pechys	24 *daktyloi*	0.5205
plethron	100 *podes*	52.05
stadion	600 *podes*	312.3

Units of Length/Distance (Doric)

	Greek	Metric [m]
daktylos		0.0204
pous	16 *daktyloi*	0.327
pechys	24 *daktyloi*	0.4905
plethron	100 *podes*	32.7
stadion	600 *podes*	196.2

Units of Weight

	Greek	Metric [kg]
mina		0.43
talent	60 *minae*	25.8

DEFINITIONS OF TERMS:

Bearing - *A device that supports a component which rotates (a shaft), slides, or oscillates in or on it. The principal types are sliding bearings and rolling bearings. Sliding bearings can be designed to support either radial loading or thrust loading, while rolling bearings can support a combination of the two*. (AE, 406) **See Chapter 6.2: A Note on Bearings**

Bearing face -

1. In the absence of a complex (half-moon section, in this case of ball bearings) race, a simple bearing face is a flat surface against which another surface or moving component slides or rotates.
2. The inside surfaces of a component into which a bearing – loose, or caged - is inserted; usually cut or cast into the face of a component.

Bowyer – a bow-maker.

Bushing – "*A bush is a cylindrical sleeve forming a bearing surface for a shaft, a ball bushing being a ball bearing that permits axial movement of the shaft".* (AE, 406)

Capstan - A spindle around which rope or chain is wound, with the assistance of (usually hand) power.

Case - The 'case' is referent to the largest structural members that provide the shell within and around which the ancient artillery piece's loading, tensioning, and firing mechanisms are built. The equivalent in modern gunsmithing might be the term 'platform', which is used to denote the central casting around which various versions of the same weapon are assembled to suit different use-cases or conditions.

Dimensional Analysis – *"A systematic procedure for determining the* k-*independent nondimensional groups that are equivalent to the* [n] *dimensional variables with* [j] *independent dimensions that describe a particular physical problem. According to Buckingham's Π (pi) theorem, where Π indicates product, (* $=$ $-$)*. When the variables on which a phenomenon depends are known but not the functional relationship between them, dimensional analysis reveals the non-dimensional groups that are important*". (AE, 545)

Engine - Almost any siege weapon or complex device used in siege or defense. Most commonly used to refer to artillery pieces, but can also be referent to siege towers and battering rams. Interchangeable with the German scholarship's use of *maschinen*.

Euclid's Axioms – *"The axioms Euclid set out in his famous text, the Elements, are:*

1*. A straight line may be drawn from any point to any other point,*

2*. A straight line segment can be extended indefinitely at either end,*

3. *A circle may be described with any centre and any radius,*

4*. All right angles are equal.*

5. *If a straight line (the transversal) meets two other straight lines so that the sum of the two interior angles on one side of the transversal is less than two right angles, then the straight lines, extended indefinitely if necessary, will meet on that side of the transversal.*

He also stated definitions of geometrical entities like points and lines, and five 'common notions', which are:

1*. Things which are equal to the same thing are also equal to one another.*

2. *If equals are added to equals, the sums are also equal.*

3*. If equals are subtracted from equals, the remainders are also equal.*

4. *Things that coincide with one another are equal to one another.*

5. *The whole is greater than the part.”* (CN, 285-6)

Euthytone / ***Euthytonos*** -

1. An arrow- or bolt-firing torsion-spring artillery piece.
2. The C-shaped profile of an ordinary, unadorned hand-bow.

Flexion – The bending or extending forces, or the effects thereof, placed on a component.

Fluid Mechanics – *“The study of fluids in motion (fluid dynamics) or fluid statics where there is no relative motion between fluid particles. Fluid statics concerns primarily the variation of pressure with altitude or depth; it includes aerostatics and hydrostatics. It involves the application of the laws of mass, momentum, and energy conservation”.* (AE, 643)

Gastraphetes -

1. An ancient hand crossbow or ‘belly-bow’, so named because the slider is winched back by bracing the weapon against the stomach.
2. Specific to the text of Biton: any non-torsion engine.

Hooke’s Law – *“Where the deflexion of an elastic body is proportional to the applied load, so Hookean deformation is linear elastic behaviour that is reversible and path-independent. In terms of uniaxial stress and strain, Hooke’s law is $\varepsilon x = \sigma x/E$ where εx is the normal strain along the x-axis, σx is the normal stress and E is Young’s modulus”* (AE, 703)

Helepolis -

1. A siege tower.
2. (Byzantine) A trebuchet, most commonly.

Leaf Spring – *“A beam-like spring made up of thin independently acting plates placed over one another and held together in a buckle”.* (AE, 771)

Load – *“The force applied to a component or structure”.* (AE, 780)

Lithobolos - A stone-throwing engine of indiscriminate type.

Material Fatigue – The cumulative strain on a component that leads to accelerated onset of its yield.

Modulus of Elasticity – *"A term that usually refers to Young's modulus of an isotropic solid, although there are also the shear modulus and the bulk modulus".* (AE, 819)

Non-torsion - Any siege engine that does not use a torsion spring as the source of its motive power; most commonly, a bow.

Palintone / *Palintonos* -

1. A stone-throwing torsion spring artillery piece.
2. The shape of a recurve hand-bow; that is, one whose limbs bend away from the archer before bending rearwards again like a conventional bow.

Pawl –

1. "*A pivoted hook-like component which engages with a ratchet wheel* [or rail]*. It is used to prevent reverse rotary motion".* (AE, 876)
2. In the context of siege weapons specifically, this arrests unwanted forward movement of the slider during the winching-back of the slider.

Plastic – *"A term used where materials have been loaded beyond the yield point into the plastic range of the stress–strain curve so as to be permanently deformed".* (AE, 899)

Race – "*Either of the inner or outer hard-steel rings in a ball or roller bearing".* (AE, 949)

Ratchet –

1. "*A wheel or ring* [or rail] *with inclined teeth that engage with a pawl, resulting in one-way motion with reverse motion being prevented until the pawl is released".*
2. In the context of siege engines specifically, this is a toothed rail that travels the length of the case. It indexes with the pawl of the slider, preventing it from being pulled forward by the bowstring.

Roller – Any component which resembles a rolling pin in its operation; consisting of a central shaft, and providing a rolling surface of any shape.

Sambuca - a wall-scaling device designed by Damis of Colophon.

Scaling -

1. *To change the magnitude of a physical variable by multiplying it by a constant factor (scaling factor, scaling ratio), such as the ratio of a dimension of a model to the same dimension for a full-size version.*
2. *To generalize a physical problem by converting the variables to non-dimensional form using quantities which characterize the problem, such as a length scale, time scale, velocity scale, and temperature difference, together with material properties such as density. (AE, 1002)*

Slider –

1. The sliding central rack of a projectile engine. The slider slots into the case and travels along a dovetailed rail. The bowstring of the weapon is attached to this slider, and the slider is then winched back to the rear of the machine, tensioning the source of power.
2. *"A simple bearing consisting of two almost parallel flat surfaces, with a narrow gap filled with a lubricant".* (AE, 1024)

Torque / Torsion – *"The twisting moment of a force… about an axis, which results in torsion".* (AE, 1157)

Torsion Spring –

1. A braided and wound spring made of hair, sinew, or a combination of both, used as the standard source of motive power for both palintone and euthytone torsion engines of the ancient world.
2. *"A helical spring to which torque can be applied at the ends."* (AE, 1159)

Windlass - a winch.

BIBLIOGRAPHY

Abbreviations:
MS: Manuscript

OGIS**:** *Orientis Graeci Inscriptiones Selectae*

Primary (Manuscripts):

Aelianus. *Tactica*, or *Grec 2442*. Bibliothèque nationale de France. Stable Link: https://gallica.bnf.fr/ark:/12148/btv1b84582634/f3.image

Apollodorus, Biton. *Supplément Grec 607*, or *Codex Parisinus 607*. Bibliothèque nationale de France. Stable Link: https://gallica.bnf.fr/ark:/12148/btv1b8593585j/f11.image.r=.langFR

Athenaeus, Biton, Leo VI. *Military Treatises,* or *Harley MS 6317.* British Library. Stable Link: http://www.bl.uk/manuscripts/FullDisplay.aspx?ref=Harley_MS_6317

Biton. *Burney Manuscript 69*, or *De constructione bellicarum machinarum*. British Library. Stable Link: http://www.bl.uk/manuscripts/Viewer.aspx?ref=burney_ms_69_f009v

de Honnecourt, Villard. *MS Fr 19093* or *Album de dessins et croquis*. Bibliothèque nationale de France. Stable Link: https://gallica.bnf.fr/ark:/12148/btv1b10509412z/

Descartes, René. *Discours de la méthode*. Bibliothèque nationale de France. Stable Link: https://gallica.bnf.fr/ark:/12148/btv1b86069594/

Hero of Byzantium. *Vaticanus Graecus 1605*. Biblioteca Apostolica Vaticana. Stable Link: https://digi.vatlib.it/view/MSS_Vat.gr.1605

Heron, Apollodorus, Athenaeus, Biton, Euclides, & Leo VI. *Collectio Tactica Et Mechanica*, or *Codices Philologici et Philosophici graeci 120* (ed. O. Ghislain de Busbecq, 1325). Stable Link: http://data.onb.ac.at/rec/AC14452104

Poliorcetici Graeci, Treatises on Engines and Weapons, or *MS Hunter 220.* University of Glasgow Library. Stable Link: https://www.gla.ac.uk/myglasgow/library/files/special/exhibns/month/aug2008.html

Reginbert. *Cod. Sang. 1092* or *Plan of Saint Gall*. St. Gallen Stiftsbibliothek. Stable Link: https://www.e-codices.unifr.ch/en/list/one/csg/1092.

Boivin, J., de La Hire, P & Thévenot, M. (1639). *Veterum mathematicorum Athenaei, Apollodori, Philonis, Bitonis, Heronis et allorum opera graece et latine.* Universidad Complutense de Madrid. Stable Link: https://babel.hathitrust.org/cgi/pt?id=ucm.5327726584&view=1up&seq=11

Primary (Ancient):

Appian. *Syriaca* (transl. H. White.). 1899. *Appian, The Foreign Wars*. New York: MacMillan.

Archimedes. *Complete Works* (transl. T.L. Heath). 1897. *The Works of Archimedes.* Cambridge: Cambridge University Press.

Aristotle. *Analytica Posteriora* (transl. G.R.G. Mure). in Ross, W.D. & Smith, J.A. (eds.) 1928. *The Works of Aristotle*. Vol 1. Oxford: Oxford University Press.

Aristotle. *De Interpretatione* (transl. E.M. Edghill). 2004. *On Interpretation*. Whitefish: Kessinger.

Aristotle. *Physica* (transl. H.G. Apostle). 1969. *Aristotle's Physics.* Bloomington: Indiana University Press.

Cicero. *On The Republic* (transl. C.W. Keyes). 1928. *On The Republic. On The Laws.* Loeb Classical Library. Cambridge: Harvard University Press.

Cicero. *Tusculan Disputations* (transl. J.E. King). 1927. Loeb Classical Library. Cambridge: Harvard University Press.

Diodorus Siculus. *Bibliotheca* (transl. C.H. Oldfather). 1933. *Library Books 8-32.* Loeb Classical Library. Cambridge: Harvard University Press.

Diogenes Laertius. *Lives of Eminent Philosophers* (transl. R.D. Hicks). 1972. Cambridge: Harvard University Press.

Euclid. *Elements* (transl. J.L. Heiberg). in Heath, T.L. (ed.) 1968. *The Thirteen Books of Euclid's Elements.* New York: Dover.
Stable Link: http://www.perseus.tufts.edu/hopper/text.jsp?doc=Perseus:text:1999.01.0086

Euclid. *Optics* (transl. H.E. Burton). in 'The Optics of Euclid'*.* 1943. *Journal of the Optical Society of America.* Vol. 35. No. 5. pp. 357-372.

Heron of Alexandria. *Belopoeica*. (transl. E.W. Marsden). 1971. Oxford: Clarendon Press.

Heron of Alexandria. *Pneumatica*. (transl. J.G. Greenwood, & B. Woodcroft). 1851. London: Taylor, Walton, and Maberly.

Heron of Alexandria. *Mechanica*. (transl. W. Schmidt). 1900. *Herons v. Alexandria: Mechanik und Katoptrik.* Leipzig: Teubner.

Justin. *History of the World*. (transl. by J.S. Watson). 1876. *Justin's History of the World, Extracted from Trogus Pompeius*. London: Henry G. Bohn.

Livy. *Ab Urbe Condita* or *History of Rome.* Books 1-36. (transl. W.M. Roberts). 1912. *History of Rome in Six Volumes*. London: J. M. Dent & Sons, Ltd.

Livy. *Periochae*. Books 46-142. (transl. W.A. McDevitte, *et al.*). 1850. *History of Rome by Titus Livius: the epitomes of the lost book*s. London. Henry G. Bohn.

Pausanias. *Descriptions of Greece.* (transl. W.H.S. Jones, H.A. Ormerod). 1918. *Descriptions of Greece in 4 Volumes*. Cambridge: Harvard University Press.

Philon of Byzantium. *Belopoeica.* (transl. E.W. Marsden). 1971. *Greek and Roman Artillery: Technical Treatises*. Oxford: Clarendon Press.

Philon of Byzantium. *Paraskeuastika* and *Poliorketika.* (transl. D. Whitehead). 2016. *Philo Mechanicus: On Sieges.* Historia. Einzelschriften, 243. Stuttgart: Franz Steiner Verlag.

Plutarch. *Moralia.* (transl. F.C. Babbit). 1961. *Moralia in 15 Volumes*. Loeb Classical Library. Cambridge: Harvard University Press.

Polybius. *The Histories*. (transl. W.R. Paton). 1922. *Polybius: The Histories*. Loeb Classical Library. Cambridge: Harvard University Press.

Proclus. *A Commentary on the First Book of Euclid's Elements.* (transl. G. Morrow). 1992. New York: Princeton Uniersity Press.

Strabo. *Geography.* (transl. H.L. Jones). 1924. *Geography of Strabo, Books 6-14*. Cambridge: Harvard University Press.

Vegetius. *Epitome of Military Science.* (transl. N.P. Milner). 1993. Liverpool: Liverpool University Press.

Vitruvius. *The Ten Books on Architecture*. (transl. M.H Morgan). 1914. Cambridge: Harvard University Press.

Xenophon. *Cyropaedia.* (transl. W. Miller). 1914. *Xenophon in Seven Volumes.* Cambridge: Harvard University Press.

Secondary (Books, Articles):

Aggarwal, C.C., Hinneburg, A., & Keim, D.A. (2001). 'On the Surprising Behavior of Distance Metrics in High Dimensional Space'. *International Conference on Database Theory*. pp. 420-434.

Allen, R.E. (1983). *The Attalid Kingdom: A Constitutional History*. Oxford: Clarendon Press.

Ashby, M.F. & Johnson, K. (2014). *Materials and Design: The Art and Science of Material Selection in Product Design*. Third Edition. Oxford: Butterworth-Heineman.

Atkins, T. & Escudier, M. (2013). *Oxford Quick Reference: A Dictionary of Mechanical Engineering*. Oxford: Oxford University Press.

Baatz, D. (2000) *Der Römische Limes: archäologische Ausflüge zwischen Rhein und Donau.* Berlin: Gebr Mann Verlag.

Bachman, M. *et al.* (2014). 'Die Stadtmauern des hellenistischen Pergamon: Bericht über die 47. Tagung für Ausgrabungswissenschaft und Bauforschung vom 16. bis 20. Mai 2012 in Trier.' Stuttgart: Koldewey-Gesellschaft.

Bobzien, S. (2020). 'Ancient Logic'. In Zalta, E.N. *The Stanford Encyclopedia of Philosophy*. Stanford: Stanford University Press.
Stable Link: https://plato.stanford.edu/archives/sum2020/entries/logic-ancient/

Boehringer, E. & von Szalay, A *et al*. (1937). *Altertümer von Pergamon, Band X. Die hellenistischen Arsenale. Garten der Königin*. Berlin: De Gruyter.

Bos, H.J.M. (2020). 'Newton, Leibniz and the Leibnizian Tradition'. Pp. 49-93. In Grattan-Guinness, I. (ed.) *From the Calculus to Set Theory 1630-1910: An Introductory History*. Oxfordshire: Princeton University Press.

Campbell, D.B. (2011). 'Ancient Catapults: Some Hypotheses Re-examined'. *Hesperia: The Journal of the American School of Classical Studies at Athens*, Vol. 80, No. 4, pp. 677-700.

Carman, J. (1999). *Ancient Warfare: Archaeological Perspectives*. Gloucestershire: Sutton.

Ceccarelli, M., Ceccarelli, S., Conti, C. E., & Martines, G. (2019). 'Ball Bearings from Roman Imperial Ships of Nemilake.' *Advances in Historical Studies*. Vol. 8. pp. 115-130.

Childs, P.R.N. (2019). *Mechanical Design Engineering Handbook.* Second Edition. Oxford: Butterworth-Heineman.

Chippindale, C. & Maschner, H.D.G. (eds). (2007). *Handbook of Archaeological Theories*. Maryland: Altamira Press.

Chisholm, H. (ed.) (1911). Encyclopædia Britannica entry for Rüstow, Wilhelm. 13th ed. Cambridge: Cambridge University Press.

Clapham, C. & Nicholson, J. (2009). *Oxford Concise Dictionary of Mathematics.* Oxford: Oxford University Press.

Clifford, W. C. (1998). 'On the Hypotheses Which Lie at the Bases of Geometry'. *Nature.* Vol. 8, No. 183-184. pp. 14-17; pp. 36-37.

Conze, A. *et al* (1912). *Altertümer von Pergamon: Stadt und Landschaft.* Berlin: Georg Reimer Verlag.

Coxeter, H.S.M. & van de Craats, J. (1993). 'Philon Lines in Non-Euclidean Planes'. *Journal of Geometry*. Vol. 48.1, No. 2, pp. 26–55.

Crielaard, J.P. (2009). 'Cities', pp. 349-372. In Raaflaub, K.A. & van Wees, H. *A Companion to Archaic Greece*. Malden: Blackwell Publishing.

Crippa, D. (2009). 'To Prove the Evident: On the Inferential Role of Euclidean Diagrams'. *Teorie Vědy/Theory of Science.* Vol. 31, No.2, pp. 101-112.

Cuttler, C.D. (1991). 'Exotics in Post-Medieval European Art: Giraffes and Centaurs'. *Artibus et Historiae.* Vol. 12, No. 23, pp. 161-179.

Daus, P.H. (1960). 'Why and how we should correct the mistakes of Euclid'. *The Mathematics Teacher*. Vol. 53, No. 7, pp. 576-581.

Dennis, G.T. (1998). 'Byzantine Heavy Artillery: The Helepolis'. *Greek, Roman, and Byzantine Studies.* Vol. 39, pp. 99-115.

DeVries, K.R. (1992). *Medieval Military Technology.* Toronto: University of Toronto Press.

Diels, H. (1914). *Antike Technik.* Leipzig und Berlin: Verlag B.G Teubner.

Dicks, D.R. (1970). *Early Greek Astronomy to Aristotle.* Ithaca: Cornell University Press.

Domingos, P. (2012). 'A Few Useful Things to Know About Machine Learning'. *Communications of the ACM*. Vol. 55, No. 10, pp. 78-87.

Dou, Z. & Staples, S.G. (1999). 'Maximizing the Arclength in the Cannonball Problem'. *The College Mathematics Journal*. Vol. 1, No. 99. pp. 44-45.

Drachmann, A.G. (1963). *The Mechanical Technology of Greek and Roman Antiquity: A Study of the Literary Sources*. London: Hafner Publishing Co.

Drachmann, A.G. (1970). 'Greek and Roman Artillery: Historical Development by E. W. Marsden'. *Technology and Culture*, Vol. 11, No. 4, pp. 620-623.

Drachmann, A.G. (1972). 'Reviewed Work(s): Greek and Roman Artillery: Technical Treatises by E. W. Marsden'. *Technology and Culture* Vol. 13, No. 3, pp. 487-494.

Dwyer, B. (2016). *Systems Analysis and Synthesis: Bridging Computer Science and Information Technology*. Amsterdam: Elsevier Inc.

English, S. (2010). *Sieges of Alexander the Great*. Barnsley: Pen and Sword.

Floris-Clohen, H. (2010). 'Achievements and Limitations of Fact-finding Experimentalism'. Pp. 445-508. In Floris-Clohen's, *How Modern Science came into the World: Four Civilizations, One 17th century Breakthrough.* Amsterdam: Amsterdam University Press.

Fraunhofer I.G.D. (2020). *Der Pergamonaltar 3D: Der Altarsaal im Pergamonmuseum.* Retrieved September 15, 2020 from http://3d.smb.museum/pergamonaltar/

Freeth, T., Bitsakis, Y., Moussas, X., Seiradakis, J.H., Tselikas, A., Magkou, E., Zafeiropoulou, M., Hadland, R., Bate, D., Ramsey, A., Allen, M., Crawley, A., Hockley, P., Malzbender, T., Gelb, D., Ambrisco, W., Edmunds, M.G. (2006). 'Decoding the Antikythera Mechanism'. *Nature*. Vol. 444, No. 7119, pp. 587-591.

Freeth, T. (2012). 'Building the Cosmos in the Antikythera Mechanism'. Paper presented at *From Antikythera to the Square Kilometre Array: Lessons from the Ancients* in Kestari, Greece. 12-15 June 2012.

Garraghan, G.J. & Delanglez, J. (1946). *A Guide to Historical Method*. New York: Fordham University Press.

Gorman, S. (2016). *The Technological Development of the Bow and the Crossbow in the Later Middle Ages*. Dublin: Trinity College.

Grainger, J.D. (1995). 'The Campaign of Cn. Manlius Vulso in Asia Minor'. *Anatolian Studies*. Vol. 45, pp. 23-42.

Green, P. (1990). *Alexander to Actium: The Historical Evolution of the Hellenistic Age*. California: University of California Press.

Gruen, E. S. (1984). *The Hellenistic World and the Coming of Rome*. Berkeley: University of California Press.

Hacker, B.C. (1968). 'Greek Catapults and Catapult Technology: Science, Technology, and War in the Ancient World'. *Technology and Culture,* Vol. 9, No. 1. pp. 34-50.

Hall, B. S. (1973). 'Review: Crossbows and Crosswords. Reviewed Work(s): Greek and Roman Artillery: Historical Development by Eric William Marsden; Greek and Roman Artillery: Technical Treatises by Eric William Marsden'. *Isis*, Vol. 64, No. 4, pp. 527-533.

Hansen, E. V. (1971). *The Attalids of Pergamon*. Ithaca: Cornell University Press.

Hassall, M. (1998). ‘Perspectives on Greek and Roman Catapults’. *Archaeology International.* Vol. 2, pp. 23-26.

Hassig, D. (1990). ‘Beauty in the Beasts: A Study of Medieval Aesthetics’. *RES: Anthropology and Aesthetics*. Vol. 19-20, pp. 137-161.

Hester, R. (2018). *Historical Research: Theory and Methods*. Essex: Ed-Tech Press.

Hornblower, S., Spawforth, A., & Eidinow, E. (2012). *Oxford Classical Dictionary*. Oxford: Oxford University Press.

Horn, W. & Born, E. (1986). ‘The Medieval Monastery as a Setting for the Production of Manuscripts’. *The Journal of the Walters Art Gallery*. Vol. 44, pp. 16-47.

Johnson, S. (1983). *Late Roman Fortifications.* London: Batsford.

Jones, A. (2012). ‘The Antikythera Mechanism and the Public Face of Greek Science’. Paper presented at *From Antikythera to the Square Kilometre Array: Lessons from the Ancients* in Kestari, Greece. 12-15 June 2012.

Jones, M.W. (2000). ‘Doric Measure and Architectural Design 1: The Evidence of the Relief From Salamis’. *American Journal of Archaeology*. Vol. 104, No. 1. pp. 73-93.

Kahn, C.H. (1960). *Anaximander and the Origins of Greek Cosmology*. New York: Columbia University Press.

Kawerau, G. & Wiegand, T. (1930). *Die Paläste der Hochburg.* Staatliche Museen zu Berlin. Berlin und Leipzig: Walter de Gruyter & Co.

Kern, P.B. (1999). *Ancient Siege Warfare*. Bloomington: Indiana University Press.

Kleiner, F.S. (2007). *A History of Roman Art*. Belmont: Thomson and Wadsworth.

Köchly, H. & Rüstow, W. (1852). *Geschichte des griechischen Kriegswesens*. Uarau: Verlags Comptoir.

Kosmetatou, E. (2003). 'The Attalids of Pergamon'. pp. 159-175. In Erskine, A. *A Companion to the Hellenistic World*. Malden: Blackwell Publishing.

Landels, J.G. (1978). *Engineering in the Ancient World*. California: University of California Press.

Langhaar, H.L. (1951). *Dimensional Analysis and Theory of Models.* New York: John Wiley & Sons, Inc.

Lawrence, A.W. (1983). 'A Skeletal History of Byzantine Fortification'. *The Annual of the British School at Athens*. Vol. 78, pp. 171-227.

Leisenring, K. (1951). 'Area in Non-Euclidean Geometry'. *The American Mathematical Monthly,* Vol. 58, No. 5, pp. 315-322.

Lewis, M.J.T. (1999). 'When Was Biton?'. *Mnemosyne*. Vol. 52, No. 2, pp. 159-168.

Lewis, R. (2020). 'Ascalon, a Landscape of Conflicts: Some Landscape Archaeology Perspectives from the Days of the Latin Kingdom of Jerusalem'. In Shotten-Hallel, V.R. & Weetch, R. *Crusading and Archaeology: Some Archaeological Approaches to the Crusades.* New York: Routledge.

Lobachevsky, N. (1914). *Geometrical Researches on The Theory of Parallels* (transl. G.B. Halstead). La Salle: Open Court Publishing Company.

Ludwig, B. (2020). 'Reconstructing the Ancient Route Network in Pergamon's Surroundings'. *Land*, Vol. 9, No. 241. pp. 1-39.

Lyons, J. (1991). *Natural Language and Universal Grammar*. New York: Cambridge University Press.

Ma, J. (1999). *Antiochos III and the Cities of Western Asia Minor.* Oxford: Oxford University Press.

Marsden, E.W. (1969). Greek and Roman Artillery: Historical Development. Oxford: Clarendon Press.

Marsden, E.W. (1971). Greek and Roman Artillery: Technical Treatises. Oxford: Clarendon Press.

Marsden, E. W. (1977). 'Macedonian military machinery and its designers under Philip and Alexander'. *Institute for Balkan Studies*. Vol.6, pp. 211-223.

Matikkala, A. (2012). 'Percy Ernst Schramm and Herrschaftszeichen'. *Mirator,* Vol.13, pp. 37-69.

McCullagh, C.B. (1984). *Justifying Historical Descriptions*. Cambridge: Cambridge University Press.

McNicoll, A. & Milner, N.P. (1997). *Hellenistic Fortifications from the Aegean to the Euphrates*. Oxford: Clarendon Press.

Miller, N. (2007). *Euclid and His Twentieth Century Rivals: Diagrams in the Logic of Euclidean Geometry*. Stanford: Center for the Study of Language and Information.

Moneta, C. (2018). *Die Saalburgkastelle: Auswertung der Altgrabungen*. Darmstadt: Verlag Philipp von Zabern GmbH.

Nahin, P.J. (2007). *When Least is Best: How Mathematicians Discovered Many Clever Ways to Make Things as Small (or as Large) as Possible.* Princeton: Princeton University Press.

Nossov, K. (2012). *Greek Fortifications of Asia Minor 500–130 BC: From the Persian Wars to the Roman Conquest*. Oxford: Osprey.

Ober, J. (1987). 'Early Artillery Towers: Messenia, Boiotia, Attica, Megarid'. *American Journal of Archaeology*, Vol. 91, No. 4, pp. 569-604.

Oleson, J.P. (2008). *The Oxford Handbook of Engineering and Technology in the Classical World.* Oxford: Oxford University Press.

Olsen, B., Shanks, M., Webmoor, T., & Witmore, C. (2012). *Archaeology: The Discipline of Things.* California: University of California Press.

Pearl, J. (1988). *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. Burlington: Morgan Kaufmann.

Perez-Gomez, A. (1982). 'Architecture as Drawing'. *JAE*, Vol. 36, No. 2, pp. 2-7.

Pirson, F. (2020) 'Pergamon: Die Arbeiten des Jahres 2019.' *Forschungsberichte des DAI*. Vol. 3, pp. 111-121.

Radt, W. (1999). *Pergamon: Geschichte und Bauten: Einer Antiken Metropole*. Darmstadt: Primus Verlag.

Rance, P. (2013). 'Philo of Byzantium'. pp. 5266–5268. In Bagnall, R. S. *et al. The Encyclopedia of Ancient History.* Malden: Blackwell Publishing.

Rathgen, B. (1909). 'Die Punischen Geschosse des Arsenals von Karthago und die Geschosse von Lambaesis'. *Zeitschrift für historische Waffenkunde.* (5) pp. 236-244.

Rehm, A. & Schramm, E. (1929). *Biton's Bau von Belagerungsmaschinen und Geschützen*. München: Bayerische Akademie der Wissenschaften.

Rottländer, R.C.A. (1996). 'New ideas about old units of length'. *Interdisciplinary Science Reviews*. Vol. 21, No. 3, pp. 235-242.

Russell, B. & MacColl, H. (1905). 'The Existential Import of Propositions'. *Mind* (14) (55) pp. 398-402.

Russell, B. (1903). *The Principles of Mathematics.* Cambridge: Cambridge University Press.

Schatzmann, I. (1995). ‘Stone-Balls from Tel Dor and the Artillery of the Hellenistic World’. *Scripta Classica Israelica*. Vol. 14. pp. 52-72.

Schramm, E. (1918). *Die antiken Geschütze der Saalburg*. Berlin: Weidmannsche Buchhandlung.

Schwartz, G. & Bok, M.J. (1990). *Pieter Saenredam: The Painter and His Time*. New York: Abbeville Press.

Shoe, L.T. (1939). ‘Review: Altertümer von Pergamon, Band X, Die Hellenistischen Arsenale by Akos von Szalay and Erich Boehringer’. *American Journal of Archaeology*. Vol. 43, No. 1, pp. 166-167.

Siebert, H. (2014). ‘Transformation of Euclid’s Optics in Late Antiquity’. *Nuncius*. Vol. 29, No. 1, pp. 88-129.

Singor, H. (2009). ‘War and International Relations’, pp. 585-603. In Raaflaub, K.A & van Wees, H. *A Companion to Archaic Greece*. Malden: Blackwell Publishing.

Sivan, H. (1992). ‘The historian Eusebius (of Nantes)’. *The Journal of Hellenic Studies*. Vol. 112, pp. 158-163.

Small, J.P. (2019). ‘Circling Round Vitruvius, Linear Perspective, and the Design of Roman Wall Painting’. *Arts*, Vol. 8, No. 118, pp. 1-28.

Snodgrass, A.M. (1971). ‘Review: Ancient Artillery, Reviewed Work(s): Greek and Roman Artillery: Historical Development by E. W. Marsden’. *The Classical Review*, Vol. 21, No. 1, pp. 106-108.

de Camp. L.S. (1961) ‘Master Gunner Apollonios’. *Technology and Culture*, Vol. 2, No. 3, pp. 240-244.

Staatliche Museen zu Berlin. (2020). *History of the Staatliche Museen zu Berlin.* Retrieved September 15, 2020 from https://www.smb.museum/en/about-us/history/.

Stappmanns, V. (2020) 'Digitale Karte von Pergamon 1.1' *Hochschule Karlsruhe Technik un Wirtschaft & Karlsruher Institut für Technologie*. Stable Link: https://geoserver.dainst.org/maps/5548/

Steele, B.D. (1994). 'Muskets and Pendulums: Benjamin Robins, Leonhard Euler, and the Ballistics Revolution'. *Technology and Culture.* Vol. 35, No. 2, pp. 348-382.

Sullivan, D.F. (2000). *Siegecraft: Two Tenth-Century Instructional Manuals by "Heron of Byzantium"*. Washington: Dumbarton Oaks.

Taylor, M. (2016). 'The Attalid victory at Magnesia on a lost plaque from Pergamon'. *Anatolian Studies*. Vol. 66, pp. 81-90.

Theotokis, G. & Yildiz, A. (2009). *A Military History of the Mediterranean Sea: Aspects of War, Diplomacy and Military Elites*. Leiden: Brill.

Thonemann, P. (2013). *Attalid Asia Minor: Money, International Relations, and the State*. Oxford: Oxford University Press.

Thibodeau, P. (2016). 'Ancient Optics: Theories and Problems of Vision'. Pp. 130-144. In Irby, G.L. *A Companion to Science, Technology and Medicine in Ancient Greece and Rome.* Hoboken, NJ: John Wiley & Sons, Inc.

Tobin, R. (1990). 'Ancient Perspective and Euclid's Optics'. *Journal of the Warburg and Courtauld Institutes.* Vol. 53, pp. 14-41.

Troje, H.E. (1971). '"Graeca leguntur" : die Aneignung des byzantinischen Rechts und die Entstehung eines humanistischen "Corpus juris civilis" in der Jurisprudenz des 16. Jahrhunderts, Cologne-Vienne'. *Forschungen zur neueren Privatrechtsgeschichte*. Vol. 18, p. 292.

Trundle, M. (2008). 'OGIS 1 266: Kings and Contracts in the Hellenistic World'. pp. 103-116. In McKechnie, P & Guillaume, P. *Ptolemy II Philadelphus and His World.* Leiden: Brill.

Uzquiano, G. (2020). ‘Quantifiers and Quantification’. in Zalta, E.N. (ed.) *The Stanford Encyclopedia of Philosophy*. Stable Link: https://plato.stanford.edu/archives/sum2020/entries/quantification/

Vincent, A.J.H. (1858). ‘Extraits des MSS relatifs a la geométrie pratique des grecs’. *Notices et extraits des MSS de la Bibliotheque impériale* (19) (2) pp. 348-415.

Vincent, J. B. (1974). *The Parthenon*. New York: W. W. Norton & Company.

Watson, G.R. (1974). ‘Review: Treatises on Ancient Artillery, Reviewed Work(s): Greek and Roman Artillery: Technical Treatises by E. W. Marsden’. *The Classical Review*, Vol. 24, No. 2. pp. 243-245.

Weisstein, E.W. (2003). *Zermelo-Fraenkel Axioms*. Wolfram Research Inc. Stable Link: https://mathworld.wolfram.com/Zermelo-FraenkelAxioms.html

Wescher, C. (1867). *Poliorcétique des Grecs*. Paris: Imprimerie Impériale.

Whitehead, D. (2016). *Philo Mechanicus: On Sieges*. Historia. Einzelschriften, 243. Stuttgart: Franz Steiner Verlag.

Williams, E. (1956). ‘Hooke's Law and the Concept of the Elastic Limit’, *Annals of Science*, Vol. 12, No. 1, pp. 74-83.

Winter, F.E. (1994). ‘Problems of tradition and innovation in Greek fortifications in Asia Minor, late fifth to third century B.C.’ *Revue des Études Anciennes*. Vol. 96, no. 1-2. pp. 29-52.

Wright, M.T. (2007). ‘The Antikythera Mechanism reconsidered’. *Interdisciplinary Science Reviews,* Vol. 32, No. 1, pp. 27-43.

Zimmer, T. (2014). ‘The Basileia: The Palace District of Pergamon’. In Pirson, F. & Scholl, A. *Pergamon: A Hellenistic Capital in Anatolia*. Istanbul: Yapı Kredi Yayınları.