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Magnonics vs. Ferronics

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ABSTRACT

Magnons are the elementary excitations of the magnetic order that carry spin, momentum, and energy. Here we compare the magnon with the *ferron*, i.e. the elementary excitation of the electric dipolar order, that transports polarization, momentum, and energy in ferroelectrics.

1. Introduction

Much of condensed matter physics addresses weak excitations of materials and devices from their ground states. More often than not the complications caused by electron correlations can be captured by the concept of approximately non-interacting quasi-particles with well-defined dispersion relations. Quasi-particles are not eigenstates and eventually decay on characteristic time scales that depend on material and environment. In conventional electric insulators, the lowest energy excitations are lattice waves with associated bosonic quasi-particles called phonons. The elementary excitations of the magnetic order are spin waves and their bosonic quasi-particles are the "magnons" [1]. In magnetic insulators, magnons and phonons coexist in the same phase space and can form hybrid quasi-particles that may be called "magnon polarons". Yttrium iron garnet (YIG) is the material of choice to study magnons, phonons, and magnon polarons because of their longevity in YIG bulk and thin-film single crystals. For more than half a century, Professor Sergio Rezende and his team seminally contributed to our understanding of magnons, phonons, and their hybrids by experimental and theoretical research, see e.g. [2,3]. Recently we extended Prof. Rezende's Boltzmann theory of the spin Seebeck effect in magnetic insulators [4] to ferroelectric (FE) insulators by introducing "ferrons", i.e. the excitations of the electric dipolar order and explored their ability to transport heat and polarization, including the associated Seebeck and Peltier effects [5,6]. Here we expound the analogies and differences between magnets and FEs in their ground states [7] as well as in their dynamic and transport properties, thereby comparing the magnonics of ferromagnetic insulators with the "ferronics" of ferroelectric insulators.

2. Magnetism vs. ferroelectricity

2.1. Dipoles

A magnetic dipole $\mathcal{M}$ is called "Ampèrian" [8] because the intrinsic or orbital angular momentum of the electron generates a dipolar magnetic field, implying broken time reversal symmetry. The interference with a homogeneous and constant external magnetic field $\mathbf{H}$ causes the Zeeman interaction

$$E_M = -\mathcal{M} \cdot \mu_0 \mathbf{H} \tag{1}$$

which leads to the Landau–Lifshitz equation of motion.

$$\dot{\mathcal{M}} = -\gamma \mathcal{M} \times \mu_0 \mathbf{H}, \tag{2}$$

where μ_0 is the vacuum permeability, $\gamma = g_e \mu_B / \hbar$ is the modulus of the gyromagnetic ratio, μ_B is the Bohr magneton, g_e is the electron g-factor and $\hbar = h/(2\pi)$ is Planck's reduced constant.

Two local spins $\mathbf{S}_1$ and $\mathbf{S}_2$ with magnetic moments $\mathcal{M}_i = -\gamma \mathbf{S}_i$ may couple by the exchange interaction

$$E_x = -J \mathbf{S}_1 \cdot \mathbf{S}_2, \tag{3}$$

where the exchange integral J vanishes exponentially as a function of distance between the moments on an interatomic length scale. The magneto- and electro-dipolar interactions have the same angle and distance dependence, but different prefactors [9]. The interaction energy of two parallel magnetic moments with $\mathcal{M}_i = \mu_B$ at a distance r reads

$$F_M = \frac{\mu_0 \mu_B^2}{4\pi} \frac{1}{r^3} \sim \frac{\alpha^2}{4\pi} \left(\frac{a_B}{r} \right)^3, \tag{4}$$

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where $\alpha = 1/137$ is the fine structure constant and a_B the Bohr radius.

An electric dipole $\boldsymbol{\mathcal{P}}$ is called "Gilbertian" [10], i.e. a directional difference of positive and negative charges. $\boldsymbol{\mathcal{P}}$ breaks inversion symmetry, but does not change under time-reversal. A dipole interacts with a homogeneous and constant electric field $\mathbf{E}$ as

$$E_P = -\boldsymbol{\mathcal{P}} \cdot \mathbf{E}. \tag{5}$$

A homogeneous field does not exert a torque on a point electric dipole with constant modulus, hence $\dot{\boldsymbol{\mathcal{P}}} = \mathbf{0}$.

There is no such thing as an exchange interaction between electric dipoles, so they interact according to electrostatics. The coupling of two parallel electric dipoles with $\mathcal{P} \sim e a_B \sim 10^{-29}\,\mathrm{C\,m}$ scales like

$$F_P = \frac{\mathcal{P}^2}{4\pi\epsilon_r\epsilon_0}\frac{1}{r^3} \sim \frac{1}{4\pi\epsilon_r}\left(\frac{a_B}{r}\right)^3, \tag{6}$$

where ϵ_r is the relative dielectric constant of the medium that embeds the dipoles. The electrical dipolar interaction is significantly larger than the magnetic one

$$\frac{F_P}{F_M} = \frac{1}{\alpha^2\epsilon_r} \gg 1. \tag{7}$$

2.2. Ground states in the solid state

Magnets - The magnetic order in electric insulators can be described by the Heisenberg model of local spins coupled by the exchange interaction J between nearest neighbors (*n.n.*)

$$H = -\frac{J}{2}\sum_{ij}^{n.n.} \mathbf{S}_i \cdot \mathbf{S}_j. \tag{8}$$

For a positive J the ground state is ferromagnetic with a coarse-grained continuous field $M_0\mathbf{m}$, where $M_0 = |\gamma\mathbf{S}/\Omega|$ is the magnetization and Ω is the volume occupied by a single spin. The direction and position dependence of the unit vector $\mathbf{m}$ minimizes the magnetic energy that includes the exchange energy and the Zeeman interaction with applied and effective magnetic fields.

FEs - Ferroelectricity is the ordered state of a large number of microscopic electric dipoles with an associated permanent electric polarization, which appears below phase transition temperatures that can be much higher than room temperature. Roughly two types of FEs can be distinguished, viz. "displacive" and "order–disorder". In the former, the order emerges during a soft-phonon structural phase transition that breaks inversion symmetry of the unit cell, while in the latter stable molecular dipoles order during the phase transition. We can write the unit cell dipole as $\mathbf{P}_0 = Q\boldsymbol{\delta}$, where Q is the ionic charge and $\boldsymbol{\delta}$ a displacement vector. The polarization density $p_0 = P_0/\Omega$, where Ω is the unit cell volume.

Since FEs have a large dielectric constant $\epsilon_r \sim 10^3$, Eq. (7) leads to $F_P/F_M \sim 20$. The magnetostatic interactions are typically in the GHz regime. While larger than that, the electric dipolar interactions are still much smaller than ambient thermal energies. Since there is no exchange interaction that orders the dipoles, room-temperature ferroelectricity appears to be a consequence, but not the origin of the phase transition to the symmetry-broken ground state. However, the dipolar interactions are more important in governing the textures of the order parameter in FE than in magnetic structures.

2.3. Excitations

Magnonics and ferronics address the small-amplitude dynamics of the ordered states at temperatures below the phase transition.

Magnets - In magnets, the low energy excitations are spin waves, i.e. plane-wave-like modulated precessions around the equilibrium magnetization with a time- and position-dependent phase. The exchange energy contribution $\omega_k^{(x)} \sim Jk^2$ vanishes at small wave numbers k. In this limit the magnetodipolar interaction with $\omega_k^{(dip)} - \omega_0^{(dip)} \sim k$ dominates. Spin and lattice waves coexist in the same regions of reciprocal space. When their coupling by the magnetic anisotropy and magnetoelasticity is larger than the level splitting and broadening, they form hybrid states or magnon polarons that transport magnetization with the sound velocity [11]. The interaction between spins and lattice is relativistic and in general weak, so in most magnets magnon polarons form, if at all, only in relatively small volumes of phase space.

FEs - The electric dipoles in FEs are defined by the coordinates of the charged ions that have mass and are subject to electrostatic and elastic forces. In contrast to magnets, the polarization dynamics for $k \to 0$ is governed by transverse optical phonons in the THz regime. The dipolar contribution to the restoring elastic force constant can be estimated for a film geometry as

$$C_P = \frac{Q^2}{\epsilon_r\epsilon_0\Omega} = \mathcal{O}\left(10^2\,\mathrm{J/m^2}\right), \tag{9}$$

which is much smaller than typical mechanical ones. To a good approximation the elementary excitations of the FE order therefore appear to be mechanical, i.e. electrically polarized phonons rather than oscillating massless dipoles that solve a lattice dynamics equation of motion.

3. Transport

Transport of magnetic order in the form of spin currents is crucial in spintronics and spin caloritronics [12]. Surprisingly, the transport of the FE order has attracted little attention, see bibliography in [5]. Here we review our formulation of polarization transport in FEs that is inspired by the models for spin transport in magnetic insulators [4,11,13,14].

3.1. Linear response

Magnets - Magnons in magnetic insulators carry energy and spin currents when subject to unidirectional temperature and magnetic field gradients ∂T and ∂H. In the linear response regime of a homogeneous and isotropic magnet, the extended "Ohm's Law" reads

$$\begin{pmatrix} -j_m \\ j_q \end{pmatrix} = \sigma_m \begin{pmatrix} 1 & S_m \\ \Pi_m & \kappa/\sigma_m \end{pmatrix} \begin{pmatrix} \partial H \\ -\partial T \end{pmatrix}, \tag{10}$$

where $(-j_m)$ is the magnon number current density defined by the conservation law $\partial j_m = -\dot{M}$, where M is now the magnetization density. j_q is the heat current density, σ_m is a magnetization or magnon conductivity, κ the thermal conductivity, while Π_m and $S_m = \Pi_m T$ are the spin (magnon) Peltier and Seebeck coefficients, respectively. The signs are chosen such that the transport parameters are positive for simple ferromagnets.

FEs - Similar equations govern electric polarization transport, but H is replaced by an electric field E:

$$\begin{pmatrix} -j_p \\ j_q \end{pmatrix} = \sigma_p \begin{pmatrix} 1 & S_p \\ \Pi_p & \kappa/\sigma_p \end{pmatrix} \begin{pmatrix} \partial E \\ -\partial T \end{pmatrix}, \tag{11}$$

and $-j_p$ is the polarization (ferron) current density defined by the conservation law $\partial j_p = -\dot{P}$, where P is now the polarization density. σ_p is a polarization or ferron conductivity, Π_p and $S_p = \Pi_p T$ are the polarization Peltier and Seebeck coefficients, respectively. The signs are chosen such that the transport parameters are positive for simple FEs. The polarization current should be distinguished from the displacement current $\dot{P}$, which is not a transport property.

3.2. Diffusion theory

Magnets - In magnetic insulators, the diffusion picture of transport has successfully made contact with experiments with a small number of adjustable parameters. At equilibrium with field H_0 and temperature

T_0 the magnons are distributed according to the Bose–Einstein function with zero chemical potential (or Planck's function)

$$f_{BE}^{(0)}(\varepsilon) = \left[\exp\left(\frac{\varepsilon + \hbar\omega_0}{k_B T_0}\right) - 1\right]^{-1}. \tag{12}$$

where $\omega_0 = \gamma\mu_0 H_0$ is the Larmor frequency. A crucial assumption in the derivation of a diffusion theory is the local equilibration of the magnon spectral distribution to the form [15]

$$f_{BE}(x,\varepsilon) = \left[\exp\left(\frac{\varepsilon + \hbar\omega(x) - \mu_m(x)}{k_B T_m(x)}\right) - 1\right]^{-1} \tag{13}$$

in terms of a slowly varying magnetic field $H(x) = H_0 + \triangle H(x) = \omega(x)/(\gamma\mu_0)$ and temperature $T_m(x) = T_0 + \triangle T_m(x)$. In magnetic insulators, this assumption can be justified by efficient magnon-conserving magnon–phonon and magnon–magnon scattering, while magnon-non-conserving damping processes are weak [13]. The magnon number density then reads

$$-m(x) = -\frac{M(x)}{\hbar\gamma} = \int f_{BE}(x,\varepsilon)\,\rho_m(\varepsilon)\,\frac{d\varepsilon}{2\pi}, \tag{14}$$

where $\rho_m(\varepsilon)$ is the magnon energy density of states. In linear response and otherwise homogeneous systems

$$f_{BE}(x,\varepsilon) = f_{BE}^{(0)}\left[1 - f_{BE}^{(0)}\exp\left(\frac{\varepsilon + \hbar\omega_0}{k_B T_0}\right)\left(\frac{\hbar\triangle\omega - \mu_m}{k_B T_0} - \frac{(\varepsilon + \hbar\omega_0)\,\triangle T}{k_B T_0^2}\right)\right] \tag{15}$$

and

$$-m(x) + m_0 = \chi_m\left(\triangle H - \frac{\mu_m}{\hbar\gamma}\right) + \chi_T \triangle T_m, \tag{16}$$

where m_0 is the magnon density at thermal equilibrium. The susceptibilities

$$k_B T_0 \chi_m = -\hbar\gamma\mu_0 \int \left(f_{BE}^{(0)}\right)^2 \exp\left(\frac{\varepsilon + \hbar\omega_0}{k_B T_0}\right)\rho_m \frac{d\varepsilon}{2\pi} \tag{17}$$

$$k_B T_0^2 \chi_T = \int \left(f_{BE}^{(0)}\right)^2 \exp\left(\frac{\varepsilon + \hbar\omega_0}{k_B T_0}\right)(\varepsilon + \hbar\omega_0)\,\rho_m \frac{d\varepsilon}{2\pi} \tag{18}$$

parameterize the response to *constant* field and temperature changes and govern magnetocaloric effects. The finite lifetime $\tau_m\,(\tau_q)$ of the magnon (energy) density modifies the conservation relations

$$-\frac{\partial j_m}{\hbar\gamma} = \dot{m} + \frac{m - m_0}{\tau_m} \tag{19}$$

$$-\partial j_q = \dot{q} + \frac{q - q_0}{\tau_q}, \tag{20}$$

where the subscript 0 indicates the equilibrated values. Efficient magnon-conserving magnon–phonon interactions render a short τ_q and the lattice has a large heat capacity. Then $T = T_0$, i.e. the magnon temperature, equals the lattice temperature everywhere. This approximation should be reconsidered in the low temperature regime [16]. Combining Eqs. (10), (14)–(19) and assuming $\partial^2 H = 0$ we find for the steady state

$$\partial^2 \mu_m = \frac{\mu_m}{\ell_m^2}. \tag{21}$$

with magnon diffusion length $\ell_m = \sqrt{\sigma_m \tau_m/\chi_m}$. The transport parameter σ_m indicates the realm of spin caloritronics [12]. We note that the divergence of a spin accumulation is equivalent to magnetic charges and stray magnetic fields that most studies neglect.

FEs - By the same arguments, we arrive at the polarization diffusion equation for the chemical potential or polarization accumulation μ_p

$$\partial^2 \mu_p = \frac{\mu_p}{\ell_p^2}. \tag{22}$$

with $\ell_p = \sqrt{\sigma_p \tau_p/\chi_p}$. Eq. (22) relies on assumptions that are well-tested for magnetic insulators at room temperature, but not for ferroelectrics. The existence of a polarization accumulation μ_p as a driving force for transport requires a relaxation time τ_p that is much larger than the scattering life time τ_r in the conductivity σ_p. In contrast to a magnon accumulation, a polarization accumulation is not protected in the non-relativistic limit, however, so the assumption $\tau_r \gg \tau_p$ should be better justified. An example of a scattering process that limits transport but conserves polarization is intermode phonon back scattering by defects. τ_p should be measurable by the electrocaloric response to small amplitude pulsed or AC electric fields along the equilibrium polarization. Polarization accumulations would betray their presence by gradients that generate space electric charges and observable electric fields.

4. Microscopic theory

Next, we discuss theories that address the material and device-dependent parameters discussed above. In magnonics, the results compare well with the parameters fitted to experiments. No experiments are available for ferroelectrics, however.

4.1. Excitations

Magnets - The Heisenberg model is the starting point of most calculations. For large spins such as $S = 5/2$ for half-filled 3d-shells, atomistic simulations give an appropriate picture for the spatiotemporal magnetization dynamics at finite temperatures [17]. In the long wave-length limit, the Landau–Lifshitz–Gilbert (LLG) equation for the magnetization density $\mathbf{M}(\mathbf{r},t) = M_0\mathbf{m}(\mathbf{r},t)$ and $|\mathbf{m}|^2 = 1$

$$\dot{\mathbf{m}} = -\gamma\mathbf{m}\times\mu_0\mathbf{H}_{\rm eff} + \alpha_G\mathbf{m}\times\dot{\mathbf{m}} \tag{23}$$

is well established, where M_0 is the ground state magnetization density, $\mathbf{H}_{\rm eff}$ is an effective magnetic field and α_G the Gilbert damping constant. $\mathbf{H}_{\rm eff}$ contains a stochastic term with a correlation function that in equilibrium obeys the fluctuation–dissipation theorem in terms of the Gilbert damping and temperature, and can be used to compute the spin–spin correlation and response functions [17]. The linearized LLG equation, equivalent to the Heisenberg equation to lowest order in the Holstein–Primakoff expansion [1], is appropriate for low-density magnon gases, i.e. for weakly excited magnets sufficiently below the Curie temperature. Its solutions are the magnon dispersion relations, amplitudes, and group velocities, which are the starting point for transport theories such as the Boltzmann formalism. The magnetization dynamics then consists of small transverse fluctuations $\mathbf{m}_\perp$ that reduce the net magnetization relative to the ground state. When the equilibrium magnetization is along z,

$$-m_z = \frac{\mathbf{m}_\perp^2}{2}. \tag{24}$$

The (quantum) thermal average $-\overline{m_z}$ may be interpreted as the local number of magnons with density $-m = \overline{\mathbf{m}_\perp^2} M_0 / (2g\mu_B)$ in Eq. (14).

Magnetoelastic interactions mix the magnons with phonons to create hybrid magnon polarons in reciprocal space that are beyond the scope of micromagnetics. An appropriate model for small wave numbers is an ensemble of coupled harmonic oscillators with weak level repulsions at the crossing points of the magnon and phonon dispersions [11].

FEs - We focus on the symmetry-broken phase at temperatures well below the phase transition with a finite macroscopic polarization per unit cell

$$\mathbf{P}_0 = \sum_s^{\text{unit cell}} Q_s \mathbf{r}_s^{(0)}. \tag{25}$$

$\mathbf{r}_s^{(0)}$ is the location of the sth ion in a unit cell with net charge Q_s and $\sum_s Q_s = 0$. We do not address here the complications caused by surface charges [18]. Finite temperatures or external excitations induce lattice vibrations that affect the polarization. An elementary excitation of the crystal is a phonon with wave vector $\mathbf{k}$ in a band σ, polarization unit

vector $\mathbf{e}_{\mathbf{k}\sigma}$ and frequency $\omega_{\mathbf{k}\sigma}$. It modulates the position $\mathbf{r}_{ls}$ of an ion s with mass $\mathfrak{M}_s$ in the $3N$ unit cells with index $l = \{1, \dots, 3N\}$ as:

$$\boldsymbol{u}_{\mathbf{k}\sigma}(l,s) = \frac{1}{\sqrt{N\mathfrak{M}_s}}\mathbf{e}_{\mathbf{k}\sigma}(s)e^{i\mathbf{k}\cdot\mathbf{r}_l - i\omega_{\mathbf{k}\sigma}t}. \tag{26}$$

A local dipole then fluctuates according to

$$\triangle\mathbf{P}_l = \sum_{\mathbf{k}\sigma s} a_{\mathbf{k}\sigma}Q_s\boldsymbol{u}_{\mathbf{k}\sigma}(l,s), \tag{27}$$

where $a_{\mathbf{k}\sigma}$ is the phonon amplitude. The polarization $\mathbf{e}$ has components along and normal to the equilibrium polarization. The dipolar dynamics consists of a rotation by an angle θ normal to and a deformation in the direction z of the dipolar order, which modifies the projection

$$\left(\triangle P_l\right)_z = \left(P_0 + \triangle P_l^{\parallel}\right)\cos\theta. \tag{28}$$

Since harmonic oscillators do not change the average position of the ions and therefore the polarization, we have to take into account nonlinearities. In magnets, the high energy cost of changing the modulus of the magnetization leads to the transverse dynamics described by the LLG Eq. (23). In "order–disorder" FEs such as KNO_3 or $NaNO_2$ we have a similar situation, because the nitrate and nitrite molecular units are characterized by stable permanent dipoles. The low-frequency phonons are then polarized along the minimum energy path that switches the polarization without deforming the strongly bound molecular units. Small transverse fluctuations $\triangle\mathbf{P}_l^{\perp}$ reduce the polarization projection and thereby the macroscopic polarization by

$$\left(\triangle P_l\right)_z = -\frac{\left|\triangle\mathbf{P}_l^{\perp}\right|^2}{2P_0} + \triangle P_l^{\parallel} + \mathcal{O}[(\triangle\mathbf{P}_l^{\perp})^2, \triangle P_l^{\parallel}]. \tag{29}$$

Here the longitudinal fluctuations $\triangle P_l^{\parallel}$ are corrections that are disregarded in the Landau–Lifshitz–Gilbert dynamics of the magnetization (24). The transverse fluctuations or *ferrons* reduce the polarization, just as the magnons reduce the magnetization. This "ferron" approximation should be accurate for the order–disorder type as argued above, but it is as yet untested for displacive FE's.

The averaged polarization is reduced by

$$\overline{\triangle p} = \frac{1}{V}\sum_{\mathbf{k}\sigma}\triangle p_{\mathbf{k}\sigma} = -\frac{1}{2P_0V}\sum_{\mathbf{k}\sigma}\left|a_{\mathbf{k}\sigma}\right|^2\left|\mathbf{F}_{\mathbf{k}\sigma}\right|^2 \tag{30}$$

where

$$\mathbf{F}_{\mathbf{k}\sigma} = \sum_s \frac{Q_s}{\sqrt{M_s}}\mathbf{e}_{\mathbf{k}\sigma}^{\perp}(s). \tag{31}$$

After quantizing the oscillators, we arrive at the thermally averaged polarization per unit cell

$$\overline{\triangle p} = -\frac{\hbar^2}{4P_0V}\sum_{\mathbf{k}\sigma}\frac{\left|\mathbf{F}_{\mathbf{k}\sigma}\right|^2}{\varepsilon_{\mathbf{k}\sigma}}f_{BE}^{(0)}\left(\varepsilon_{\mathbf{k}\sigma}\right) \tag{32}$$

and the susceptibilities that govern the electrocaloric and pyroelectric susceptibilities $\chi_p = \partial\overline{\triangle p}/\partial E$ and $\chi_T = \partial\overline{\triangle p}/\partial T$. $-\overline{\triangle p}/p_0$ may be interpreted as an effective number of ferron excitations. In contrast to the pure magnon case, but similar to that for the magnon polaron, each quasiparticle excitation contributes with a state-dependent weight to the reduction of the polarization.

These properties can be computed by conventional lattice dynamics codes for realistic models. We may capture the essential physics by a one-dimensional harmonic oscillator model of a diatomic chain of atoms with equal masses $\mathfrak{M}$, opposite charges $\pm Q$, and force constants for longitudinal (C_L) and transverse (C_T) motions. The FE phase transition shifts the ions in each unit cell to generate a permanent electric dipole $\mathbf{P}_0 = Q\boldsymbol{\delta}$ with shift vector $\boldsymbol{\delta}$. The polarization $\mathbf{p}_0 = \mathbf{P}_0/a^3$ can point in any direction, but we focus here on a dipolar order along or normal to the chain. We have to consider one longitudinal and two transverse phonon branches in the Brillouin zone with boundaries $|k| \le \pi/a$. When the polarization is along the chain, the FE transition doubles the size of the unit cell, folding the bands at $|k| \le \pi/(2a)$ into acoustic and optical ones. We simplify this already primitive model even further by assuming that all transverse force constants are the same. The longitudinal phonons are not ferroelectrically active but contribute to the heat conductance. We may entirely disregard the high-frequency longitudinal optical phonon mode. In [6] we plot the phonon bands for the two main polarization directions. Remarkably, the polarization of the acoustic mode can be switched off completely by rotating the direction of the FE order from in-chain to perpendicular to the chain. This should affect the electrocaloric, pyroelectric, and transport "caloritronic" properties discussed in the following.

4.2. Transport

The transport coefficients can be computed straightforwardly with the above models for diffuse and ballistic transport. Rezende et al. [4] and Cornelissen et al. [13] formulated a Boltzmann equation in the relaxation time approximation, where the latter focuses on the role of the magnon chemical potential. Flebus et al.'s [11] spin Seebeck effect theory for magnon polarons can be easily adapted to handle ferrons. Meier and Loss' [14] scattering theory for magnon transport in ballistic spin chains inspired our ferron transport formulation [6].

4.2.1. Boltzmann Equation

Magnets - The starting point of the Boltzmann formalism is the non-equilibrium distribution function in real and reciprocal space $f_\sigma(\mathbf{k},\mathbf{r})$ that in equilibrium reduces to $f_{BE}^{(0)}\left(\varepsilon_{\mathbf{k}\sigma} + \hbar\gamma H_0\right)$. In the steady state, constant relaxation time τ_r, and one spatial dimension x, the distribution is modified by a field or temperature gradient as

$$\begin{aligned}\triangle f_\sigma(\mathbf{k},x) &= \tau_r v_x(\mathbf{k}\sigma)\frac{\partial f_\sigma(k,x)}{\partial x}\\ &= -\tau_r v_x\left(\frac{\hbar\partial\omega(x) - \partial\mu(x)}{k_BT} - \frac{\varepsilon_{k\sigma} + \hbar\omega_0}{k_BT^2}\partial T\right)\\ &\times\left(f_{BE}^{(0)}\right)^2\exp\left(\frac{\varepsilon_{k\sigma} + \hbar\omega_0}{k_BT}\right),\end{aligned} \tag{33}$$

where $v_x(\mathbf{k}\sigma) = \partial\varepsilon_{\mathbf{k}\sigma}/(\hbar\partial k_x)$ is the magnon group velocity and we used the expansion Eq. (15). Here the relaxation time τ_r that includes all scattering processes is shorter than τ_m in the magnon diffusion equation (21).

The magnon spin current in the magnon–polaron system for constant gradients then, for example, reads

$$-\frac{j_m}{\hbar\gamma} = \frac{1}{V}\sum_{\mathbf{k}\sigma}\triangle m_{\mathbf{k}\sigma}v_x\triangle f_\sigma(\mathbf{k}) = \sigma_m\left(\partial H - \frac{\partial\mu}{\hbar\gamma}\right) - \sigma_m S_m\partial T, \tag{34}$$

which leads to the conductivity

$$\sigma_m = \frac{\tau_r\hbar^2\gamma^2\mu_0}{k_BTV}\sum_{\mathbf{k}\sigma}\triangle m_{\mathbf{k}\sigma}\left(v_x f_{BE}^{(0)}\right)^2\exp\left(\frac{\varepsilon_{\mathbf{k}\sigma} + \hbar\gamma H_0}{k_BT}\right), \tag{35}$$

where $\triangle m_{\mathbf{k}\sigma}$ is the magnetic component in the magnon–polaron wave with index $\mathbf{k}\sigma$, which follows from diagonalizing the magnetoelastic Hamiltonian [11].

FEs - The transport coefficients for FEs can be computed under the ferron approximation analogously ($E_0 = 0$)

$$\sigma_p = \frac{\tau_r P_0}{k_BT}\sum_{\mathbf{k}\sigma}\triangle p_{\mathbf{k}\sigma}\left(v_x(\mathbf{k}\sigma)f_{BE}^{(0)}\right)^2 e^{\frac{\varepsilon_{k\sigma}}{k_BT}}. \tag{36}$$

where $\triangle p_{k\sigma}$ is the polarization (change) of a phonon in state $k\sigma$ introduced in Eqs. (30), (31). The transport coefficients in the 1D model with polarization parallel to the gradients can be obtained analytically in the high and low temperature limits [5].

4.2.2. Ballistic transport

Magnets - The Landauer–Büttiker scattering theory of transport is designed to treat transport that is limited by geometry, for example, point contacts that are connected adiabatically to large thermodynamic reservoirs. Eq. (11) then becomes

$$\begin{pmatrix} -J_m \\ J_q \end{pmatrix} = G_m \begin{pmatrix} 1 & S_m \\ \Pi_m & K/G_m \end{pmatrix} \begin{pmatrix} \triangle H \\ -\triangle T \end{pmatrix}, \tag{37}$$

where the driving forces are now the field and temperature differences, K and G_m are heat and spin (magnon) conductances, respectively, and on the l.h.s. we have currents rather than current densities. The transport coefficients are governed by the matrix of transmission probabilities $\mathbf{T}$ of magnons that propagate from the left to the right, which in the absence of scattering becomes a unit matrix in the space of propagating states. The magnon conductance of the spin chain in the low temperature limit [14]

$$G_m = \hbar\gamma^2 f_{BE}^{(0)}\left(\epsilon_m\right). \tag{38}$$

$H_{\rm eff}$ in $\epsilon_m = \hbar\gamma H_{\rm eff}$ represents applied or magnetic anisotropy fields that fix the direction of the magnetic order. $G_m \sim e^{-\epsilon_m/(k_BT)}$ vanishes exponentially when the temperature falls below the magnon gap.

FEs - In ferroelectrics

$$\begin{pmatrix} -J_p \\ J_q \end{pmatrix} = G_p \begin{pmatrix} 1 & S_p \\ \Pi_p & K/G_p \end{pmatrix} \begin{pmatrix} \triangle E \\ -\triangle T \end{pmatrix}, \tag{39}$$

where, e.g., the polarization (ferron) conductance for the dipolar chain [6]

$$G_p = \frac{1}{k_BT}\sum_\sigma \int \left(\xi_{\mathbf{p}} f_{BE}^{(0)}\right)^2 e^{\varepsilon/(k_BT)}\frac{d\varepsilon}{h}. \tag{40}$$

G_p may be compared with the conductivity σ_p of a diffuse wire (36). The latter differs by being proportional to the relaxation time and the increased importance of the group velocities in the integral.

For perpendicular FE order only the high-frequency optical branch contributes with

$$\xi_{\mathbf{p}} = -\frac{(\hbar Q)^2}{M P_0 \epsilon} \text{ for } \mathbf{p} \perp \hat{\mathbf{x}}, \tag{41}$$

while for parallel polarization with $\varepsilon \geq 0$

$$\xi_{\mathbf{p}} = -\frac{\varepsilon P_0}{4C\delta^2} \text{ for } \mathbf{p} \parallel \hat{\mathbf{x}}. \tag{42}$$

We see that the ferron conductance dramatically differs from the magnon conductance by its strong dependence on the direction of the dipolar order. When $\mathbf{p} \perp \hat{\mathbf{x}}$, G_p vanishes exponentially with temperature because only the high-frequency optical phonon branch contributes. For $\mathbf{p} \parallel \hat{\mathbf{x}}$, however, polarization transport is gapless and the ferron conductance scales linearly with low temperatures $G_p \sim T$ [6].

5. Devices

The theory can be tested by experiments with concrete devices fabricated from different materials and flexible configurations. Magnonics has a great advantage: magnon currents can be measured when injected into heavy metal contacts by means of a transverse electromotive force induced by the inverse spin Hall effect [19–21]. We are not aware of a "polarization Hall effect" that could detect a ferron current. Nevertheless, ferrons do cause observable effects. Here we introduce two simple devices that are tailored to find evidence for a polarization current in the diffuse and ballistic regimes.

5.1. Planar capacitors

A capacitor is a slab of a dielectric insulator between metal contacts. Taking advantage of the spin Hall effects, films with one or two Pt contacts on YIG have been investigated extensively. Electric charges and capacitances appear to play only a minor role in magnonics, however. Planar FE capacitors are common devices because the high dielectric constant of FE's ensures high capacitances and the switchability of the FE order allows their use in FE random access memory (FeRAM) cells. Metallic contacts screen the FE surface charges in equilibrium and residual electric fields are small [22]. A global temperature change modulates the equilibrium polarization and generates thermovoltages, while applied voltage steps heat or cool the FE spacer depending on the susceptibilities χ_p and χ_T.

A temperature *gradient* generates a directional heat and associated polarization current, but because $\partial E = 0$ in these devices, an applied voltage difference does not. We expect that a polarization current injected into a metal contact decays on a very short length scale without obvious observable signatures. However, opaque contacts accumulate a polarization on the length scale of the diffusion length ℓ_p. In an open-circuit configuration, a thermally excited polarization $p(x)$ generates a voltage over the contacts

$$\triangle V = -\int_0^L \frac{\triangle p(x)}{\epsilon} dx = -\frac{\chi_E}{\epsilon P}\int_0^L \mu(x)\, dx, \tag{43}$$

where L is the thickness of the FE barrier. In a capacitor with one contact interface opaque and another one transparent and in the limit $\ell_p \gg L$ becomes $\triangle V_{\max} = -\chi_E LS\triangle T_{\rm ext}/\left(2\epsilon_0\epsilon_r\right)$. $\triangle V$ can be used by inserting the capacitor into an electric circuit, analogous to pyroelectric devices, but operating on temperature differences rather than global temperature changes. The effects can be switched off at temperatures below the optical phonon band by rotating the polarization, as discussed above.

Returning to magnonics: a temperature gradient over a capacitor with a electrically insulating magnetic spacer generates a magnon accumulation and stray magnetic fields, which to the best of our knowledge have not yet been reported, however.

5.2. Lateral structures

More flexible than the planar capacitors are lateral structures on and of thin films. Propagating magnons can be injected and detected by narrow microwave striplines [23], while Pt contacts on YIG films allow the study of diffuse magnon transport by injecting them by the spin Hall effect or Ohmic heating and detection in another contact by the electromotive force generated by the inverse spin Hall effect from spin pumping [24].

We envisage analogous experiments in gated thin FE films. Their polarization can be perpendicular to the plane, either spontaneously or induced by gates. The ferroelectricity in van der Waals materials such as monolayer FeTe [25] and bilayer WTe_2 [26] or BN [27] is an especially interesting phenomenon [27]. In contrast to the planar capacitor, it should be easy to generate electric field gradients and electrically controlled polarization currents. Thin films may be nanostructured such that large islands are connected by point contacts or strips over which the electric field and temperature drop selectively, see Fig. 1. The scattering theory of transport is applicable and may be approximated by the single wire model introduced above. Additional local electrostatic gates should allow rotation of the FE order in the constriction. These structures are open to local probes that can measure, for instance, the significant magnetic stray fields predicted for the electric dipolar current through the constriction [6].

6. Summary and outlook

Magnonics has developed into one of the most active fields in magnetism and spintronics, while the field of "ferronics" does not yet exist. Our models are primitive, some approximations are untested, and essential parameters are not known. Experimental results are necessary to guide better theoretical models and understanding. We hope that our initial theoretical steps will motivate experimentalists with frontier spirit to test our predictions and on the way discover completely new

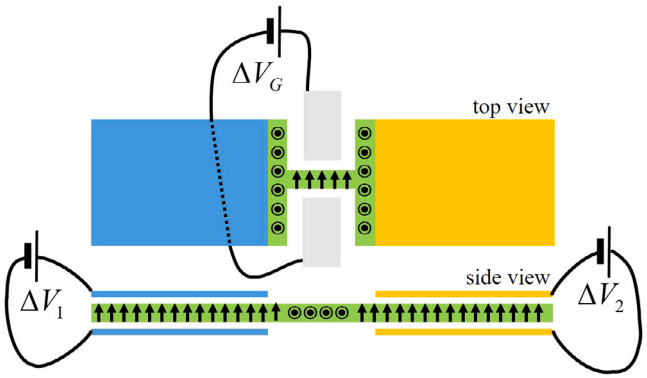

Fig. 1. Lateral polarization valve on a thin film FE. The two large FE pads are thermodynamic reservoirs assumed to be at local thermal equilibrium, but at variable electric fields and temperatures, controlled by metallic gates with voltages ΔV_1 and ΔV_2 as well as temperatures (indicated by the color). Here a voltage ΔV_G over side gates controls the FE polarization direction angle with the sample plane.

physics and applicable new functionalities. Realistic lattice dynamic calculations of the polarization amplitudes in ferroelectric materials can assess the accuracy of the ferron approximation. The polarization relaxation time and diffusion lengths are key parameters that have to be established with better accuracy. We have only scratched the surface of the field, not addressing fascinating extensions of ferronics that include antiferroelectrics, ferroelectric textures, polar metals, and multiferroics.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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