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Charge multiplicity in relativistic heavy ion collisions: A statistical model approach

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We use fireball geometry and a statistical model to calculate charged particle associated multiplicity in relativistic heavy ion reactions. General expressions for multiplicity distributions based upon this model are derived. The constraints of charge and baryon number conservation are shown to lead to modified Poisson distributions. The expressions developed are applicable to all nonstrange particles and composites. Comparison is made with existing data.

[NUCLEAR REACTIONS Relativistic heavy ion collisions; charge multiplicity; Fireball; statistical model.]

I. INTRODUCTION

In this work we compare the available data on charge multiplicity in relativistic heavy ion collisions with calculations based on a statistical model. Such a model has two parts to it: the geometry of clean cuts[1] and an assumption of thermal and chemical equilibrium[1,2] in the overlapping parts of the target and the projectile. Actually, the two assumptions are independent. The first one can be justified intuitively from energy arguments and also from fragmentation data. If the time scale in which the ordered motion (kinetic energy) is converted into various exit channels is large compared to reaction rates, the second assumption will follow. Another way of stating this is to say that the fireball loses the memory of how it was formed; then all parts of the available phase space are equally probable. Temperature and chemical potentials are introduced for ease of calculation, i.e., such that one can work with the grand ensemble. One may hope to test separately the two ingredients of the model from multiplicity data.

One advantage of the statistical model is that it can make predictions for any experimental result. Thus it predicts simultaneously proton, pion, triton, deuteron, and any other spectra; the same calculation will yield multiplicity distributions. We do not know of any other model that is this versatile. For example, the collective tube model makes excellent predictions for pion multiplicity,[3] but has not been used to describe proton spectra. This is not to say that the statistical model is the only approach one should pursue; however, we should confront it with many different facts in the hope of learning how to improve upon it.

The paper is organized as follows: In Sec. II, we compare our calculation with known associated multiplicity data. We also point out what experiments could test separately the "clean cut" assumption as opposed to the thermodynamic assumption. In Sec. III, formulas for multiplicity distributions are derived. The basic formula for Sec. III was presented previously in Ref. 4; however, it had to be extended and modified for practical computation. Section IV contains our calculation for pion multiplicity. In Sec. V we give our results for deuteron multiplicity, although no experimental data are available to us. Conclusions are presented in Sec. VI.

II. ASSOCIATED MULTIPLICITY

The average number of charged particles emitted in a heavy ion collision is given by

$$\langle M\rangle = \frac{\int N_{\rm ch}(b)2\pi b\, db}{\int 2\pi b\, db}, \tag{1}$$

where $N_{\rm ch}(b)$ is the number emitted at a given impact parameter b.

If one assumes a clean cut and neglects the possibility of pion production, composite particle production, etc., then $\langle M\rangle$ will be independent of energy. The fireball[1] model or the firestreak[5] model would then given identical results. However, because particles (pions, deltas) and composites are also produced, $\langle M\rangle$ is a function of energy. Despite this, actual numerical computation showed that both fireball and firestreak models give nearly the same result for $\langle M\rangle$. In our calculation we use fireball geometry throughout.

Experiments at the Bevalac[6,7] measure associated multiplicity. Specifically, charges are measured provided that in the event there is a proton at 90° to the beam axis. We therefore calculate

$$\langle M_a \rangle = \frac{\int [N_{ch}(b) - 1](dN/d\Omega)(\pi/2, b) 2\pi b\, db}{\int (dN/d\Omega)(\pi/2, b) 2\pi b\, db}, \qquad (2)$$

where $(dN/d\Omega)(\pi/2, b)$ is the number of protons per unit solid angle at 90° from a collision at the impact parameter b. We have

$$\frac{dN}{d\Omega} = \int dE \frac{d^2N}{dE d\Omega}, \qquad (3)$$

where

$$\frac{d^2N}{dE d\Omega} = (E^2 - m^2)^{1/2} \frac{E' N e^{-\beta E'}}{4\pi m^3} \times \left[\frac{2K_1(\beta m)}{(\beta m)^2} + \frac{K_0(\beta m)}{\beta m}\right]^{-1},$$

with E the total lab energy, and E' the total energy in the fireball c.m. of a free nucleon of mass $m = 939$ MeV, and $1/\beta$ the fireball temperature. (This expression is derived in Sec. IV of Ref. 1.) We include the production of pions and deltas, as well as all the composites and resonances tabulated in Refs. 2 and 8. As in Ref. 8, a critical density of $\rho = 0.12$ fm^{-3} is used throughout this paper. (Units are $\hbar = c = 1$.)

Figure 1 compares our calculation with data. The calculation should overestimate the data at the lower energy end as we have imposed no energy cutoff in the detected fragments; thus the absorption in the wall of the scattering chamber is not taken into account. (We do consider cutoff effects due to the limited pass band of the 90° detector telescope; thus we ignore those undetectable events in which the proton trigger is emitted with lab kinetic energy less than 5 MeV or greater than 200 MeV.[7]) Although we have shown only the case of Ne on U, other projectile/target combinations were also computed, with similar agreement.

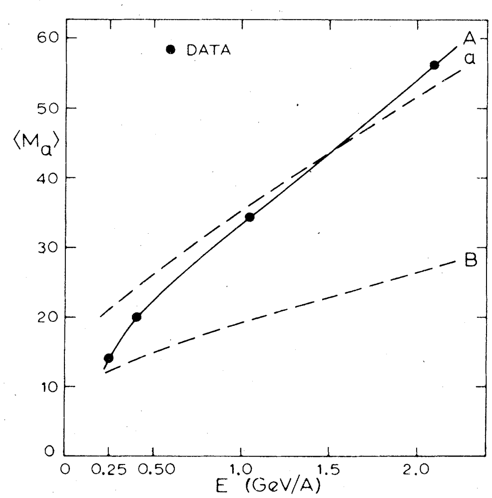

FIG. 1. Associated multiplicity for Ne on U; projectile is Ne with energies 250 MeV/A, 400 MeV/A, 1.05 GeV/A, and 2.1 GeV/A. The curve A is drawn through the data points; curve a is the theoretical calculation [Eq. (2) in the text]; curve B is a calculation for charge multiplicity without weighting [Eq. (1) in the text].

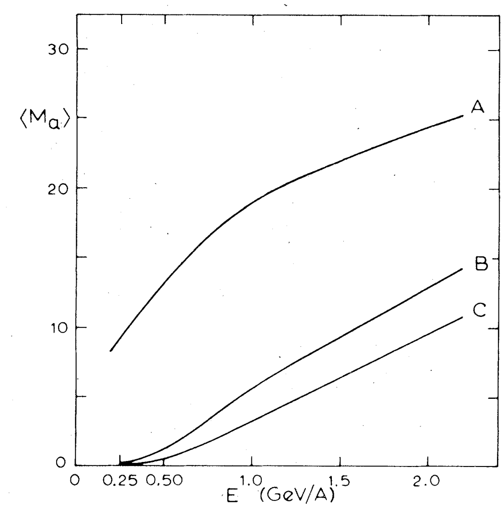

FIG. 2. Calculated associated multiplicity for protons A, negative pions B, and positive pions C for Ne on U at various incident energies. Note the change of scale from the previous figure.

Although this agreement with experiment is pleasing, for reasons to be mentioned in Sec. III it may be fortuitous. The major contributors to $\langle M_a \rangle$ in Eq. (2) are the protons; at high energies, pions become important. In Fig. 2, we show each of these contributions. Ne + U has a neutron excess over protons, so there are more negative than positive pions produced.[9] The composite production decreases as the energy increases. The net result is a slight increase in the number of protons with energy. It would be interesting to have separate experimental data for both protons and pions; if the slope of the increase of protons as a function of energy differs from what we calculate, it may mean that the "cut" varies with energy. The pion production, however, is strongly influenced by thermodynamics as well as by the cut.

III. MULTIPLICITY DISTRIBUTIONS IN THE STATISTICAL MODEL

At each impact parameter, the participating fractions of the target and the projectile define a certain number of baryons, a total charge, a fixed total energy E, and a fixed momentum P which is zero in the center of mass of the fireball. The assumption of the statistical model is as follows: Let there be n_1 particles (or composites) of one kind, n_2 of another, and so on; if such a partition conserves the total baryon and charge numbers, then the probability of obtaining it is proportional to the phase space available to it:

$$P_{n_1 n_2 \cdots} \propto \frac{1}{\Pi n_i!} \int \delta(E - \Sigma\epsilon_i)\delta(\Sigma\vec{p}_i)\Pi d\vec{p}_i \,, \tag{4}$$

where the indices run over all participating species.

For relativistic particles, the computation of this microcanonical phase space is very hard; we work instead with the canonical ensemble. Thus one defines a temperature which is adjusted to give the correct average energy. In the c.m. of the fireball, the total momentum is also, on average, zero. At a given temperature, the probability that we obtain a state which has n_1 particles in eigenstate A, n_2 particles in eigenstate B, etc. is

$$P_{n_1 n_2 \cdots}^{AB\cdots} = \frac{e^{-\beta E_A(n_1)} e^{-\beta E_B(n_2)} \cdots}{\sum\limits_{\substack{AB\cdots \\ m_1 m_2 \cdots}} e^{-\beta E_A(m_1)} e^{-\beta E_B(m_2)} \cdots} \,. \tag{5}$$

The probability of obtaining n_1 particles of one kind, n_2 of another, etc., is then just

$$P_{n_1 n_2 \cdots} = \frac{\sum\limits_{AB\cdots} e^{-\beta E_A(n_1)} e^{-\beta E_B(n_2)} \cdots}{\sum\limits_{\substack{AB\cdots \\ m_1 m_2 \cdots}} e^{-\beta E_A(m_1)} e^{-\beta E_B(m_2)} \cdots} \,. \tag{6}$$

The quantity $\sum_\alpha e^{-\beta E_A(n i)}$ is, however, the canonical partition function for n_i particles. It is given by

$$\sum_A e^{-\beta E_A(n i)} = \frac{1}{n_i!} (Z_i)^{n_i} \,, \tag{7}$$

where Z_i is the partition function of one particle of species i,

$$Z_i = V \int \exp[-\beta(m_i^2 + \vec{p}^2)^{1/2}] p^2 dp\, d\Omega \,. \tag{8}$$

Thus we have

$$P_{n_1 n_2 \cdots} = \frac{\frac{1}{n_1!}(Z_1)^{n_1} \frac{1}{n_2!}(Z_2)^{n_2} \cdots}{\sum\limits_{m_1 m_2 \cdots} \frac{1}{m_1!}(Z_1)^{m_1} \frac{1}{m_2!}(Z_2)^{m_2} \cdots} \,. \tag{9}$$

Equation (9) was derived in Ref. 4. The approximations leading to the above are detailed there.

The summations in Eq. (9) are restricted by baryon and charge conservation. In view of the fact that we have neutrons, protons, pions, deltas, and various composites and resonances, this constrained summation is hard to carry out in practice. With suitable approximations, the sums can be computed. We illustrate this with an example where only neutrons, protons, and the three pions are considered important: Let n_1, n_2, n_3, n_4, and n_5 stand for the number of protons, neutrons, π_+, π_-, and π_0, respectively, that are present. Then

$$\begin{aligned} n_1 &= Q - n_3 + n_4 \,, \\ n_2 &= B - Q + n_3 - n_4 \,. \end{aligned} \tag{10}$$

Here B is the baryon number and Q the charge number carried by the participating parts of the target and projectile for a given impact parameter. The quantity $n_3 - n_4$ is the difference between the number of π_+'s and π_-'s. Usually this is small so we can write

$$\begin{aligned} n_1! &= (Q - n_3 + n_4)! \simeq \frac{Q!}{Q^{n_3 - n_4}} \,, \\ n_2! &\simeq \frac{N!}{N^{n_4 - n_3}} \,, \quad N = B - Q \,. \end{aligned} \tag{11}$$

Equation (9) then reduces to

$$P_{n_1 n_2 \cdots n_5} = \frac{\frac{1}{n_3!}\left(Z_3 \frac{Q}{Z_1}\frac{Z_2}{N}\right)^{n_3} \frac{1}{n_4!}\left(Z_4 \frac{Z_1}{Q}\frac{N}{Z_2}\right)^{n_4} \frac{1}{n_5!}(Z_5)^{n_5}}{\sum\limits_{m_3 m_4 m_5} \frac{1}{m_3!}\left(Z_3 \frac{Q}{Z_1}\frac{Z_2}{N}\right)^{m_3} \frac{1}{m_4!}\left(Z_4 \frac{Z_1}{Q}\frac{N}{Z_2}\right)^{m_4} \frac{1}{m_5!}(Z_5)^{m_5}} \tag{12}$$

The right-hand side of Eq. (12) does not contain n_1 or n_2. If n_3 and n_4 are given, then n_1 and n_2 are fixed by Eq. (10). At this stage, we introduce $\bar{n}_3$, $\bar{n}_4$, and $\bar{n}_5$ from the grand canonical ensemble. In this ensemble, the average number of particles is given by

$$\bar{n}_i = e^{\beta\alpha_i} Z_i \,,$$

where

$$\alpha_1 = \mu_1 \,, \tag{13}$$

$$\alpha_2 = \mu_2 \,, \tag{14}$$

$$\alpha_3 = \mu_1 - \mu_2 \,, \tag{15}$$

$$\alpha_4 = \mu_2 - \mu_1 \,, \tag{16}$$

$$\alpha_5 = 0 \,. \tag{17}$$

In the above, n_3 and n_4 are determined through the law of chemical kinetics, e.g., $p+n \rightleftharpoons n+n+\pi_+$ means that at chemical equilibrium $\mu_{\pi_+}=\mu_p-\mu_n$.

We can rewrite Eq. (12) as

$$P_{n_1 n_2 \cdots n_5}=\frac{\prod_{i=3,5}\frac{(\zeta_i \bar{n}_i)^{n_i}}{n_i!}}{\sum_{m_3 m_4 m_5}\prod_{i=3,5}\frac{(\zeta_i \bar{m}_i)^{m_i}}{m_i!}}, \tag{18}$$

where $\zeta_3=\bar{n}_2 Q/(\bar{n}_1 N)$, $\zeta_4=\bar{n}_1 N/(\bar{n}_2 Q)$, and $\zeta_5=1$. Although Eq. (18) is strictly valid only for $|n_3-n_4| \ll Q$ or N, we use it throughout since at least $|\bar{n}_3-\bar{n}_4| \ll Q$ or N. We then have in general for pions, deltas, and deuterons (both to be introduced below)

$$P_{n_i}=\frac{1}{n_i!}(\zeta_i \bar{n}_i)^{n_i} e^{-\zeta_i \bar{n}_i}, \quad i=3,4,\ldots,9 . \tag{19}$$

We will also make occasional use of a better approximation than Eq. (11). For small n and large Q, one can use

$$(Q-n)! \simeq \frac{Q!}{Q^n}\left(1-\frac{n}{Q}\right)^{1/2-n} \tag{20}$$

or better yet

$$(Q-n)! \simeq \frac{Q!}{Q^n}\left(1-\frac{n}{Q}\right)^{1/2-n} f(n,Q),$$

where

$$f(n,Q)=1-\frac{n^2}{2!\,Q}-\frac{2}{3!}\frac{n^3}{Q^3}+\frac{1}{4!}\left(\frac{3}{Q^2}-\frac{6}{Q^3}\right)n^4+\cdots$$

is generated from the Taylor series expansion of $e^n(1-n/Q)^Q$.

The production of $\Delta(1232)$'s[9] can also be included in the formalism. In obtaining single proton or single pion inclusive spectra, it was deemed necessary[8,9] to include them. We have to modify Eqs. (10) to (19). Let n_6, n_7, n_8, and n_9 stand for the number of Δ_{++}, Δ_+, Δ_0, and Δ_-, respectively. Therefore,

$$\begin{aligned} n_1 &= Q-n_3+n_4-2n_6-n_7+n_9 ,\\ n_2 &= N+n_3-n_4+n_6-n_8-2n_9 . \end{aligned} \tag{21}$$

An approximation similar to that of Eq. (11) is now made. In addition to Eqs. (13) to (17) we now have

$$\bar{n}_i=e^{\beta\alpha_i}Z_i ,$$

with

$$\alpha_6=2\mu_1-\mu_2 , \tag{22}$$

$$\alpha_7=\mu_1 , \tag{23}$$

$$\alpha_8=\mu_2 , \tag{24}$$

$$\alpha_9=2\mu_2-\mu_1 . \tag{25}$$

Carrying through the same argument as before, we find the expressions for P_{n_3}, P_{n_4}, and P_{n_5} unchanged. For the Δ's, we find that the probabilities are given by Eq. (19) with

$$\zeta_6=\frac{\bar{n}_2}{N}\left(\frac{Q}{\bar{n}_1}\right)^2 , \tag{26}$$

$$\zeta_7=\frac{Q}{\bar{n}_1} , \tag{27}$$

$$\zeta_8=\frac{N}{\bar{n}_2} , \tag{28}$$

$$\zeta_9=\frac{\bar{n}_1}{Q}\left(\frac{N}{\bar{n}_2}\right)^2 , \tag{29}$$

To calculate π_- multiplicity distributions, the appropriate decays of the Δ's have to be taken into account: Δ_- decays into $n+\pi_-$, but Δ_0 has a probability a $(=\frac{1}{3})$ of decaying into π_-+p, and probability b $(=\frac{2}{3})$ into π_0+n. Thus, n negative pions may come from n, or any higher number, of Δ_0's. It is not difficult to see that with Δ's included, the π_- multiplicity distribution becomes

$$P_n=\sum_{n_4+n_8+n_9=n} P_{n_4}P_{n_9}\tilde{P}_{n_8} , \tag{30}$$

where $\tilde{P}_{n_8}=a^{n_8}P_{n_8}e^{b\zeta_8\bar{n}_8}$ and P_{n_8} and ζ_8 are given by Eqs. (19) and (28).

It is clear that similar techniques can be used to calculate the multiplicity distribution of any composite. As an example, we calculate that of the deuteron in an effort to understand what could be learned from such a study. We have used Eq. (19) both with

$$\zeta_d=\frac{Q}{\bar{n}_1}\frac{N}{\bar{n}_2}, \quad \bar{n}_d=e^{\beta(\mu_1+\mu_2)}Z_d , \tag{31}$$

and

$$P_{n_d}=\frac{\frac{1}{n_d!}(\bar{n}_d\zeta_d)^{n_d}\left(1-\frac{n_d}{Q}\right)^{n_d-1/2}\left(1-\frac{n_d}{N}\right)^{n_d-1/2}}{\sum_{m_d}\frac{1}{m_d!}(\bar{n}_d\zeta_d)^{m_d}\left(1-\frac{m_d}{Q}\right)^{m_d-1/2}\left(1-\frac{m_d}{N}\right)^{m_d-1/2}}$$

Equation (31) uses Eq. (11); Eq. (32) uses Eq. (20).

IV. PION MULTIPLICITY

We use the development of the previous section to calculate negative pion multiplicity for the reaction Ar on Pb_3O_4; the argon projectile has kinetic energy 1.8 GeV/A in the lab.[10] This particular set of data has been the subject of considerable theoretical effort.[3,11-13]

In Fig. 3 we show the results of our calculation for $P_n(b)$ as a function of b and n for Ar on Pb. For large n, $P_n(b)$ maximizes near $b=0$ but is always numerically small. The reason is obvious. If for a given b many pions are produced [$\bar{n}_4$ of Eq. (19) is large], P_n is distributed among many n

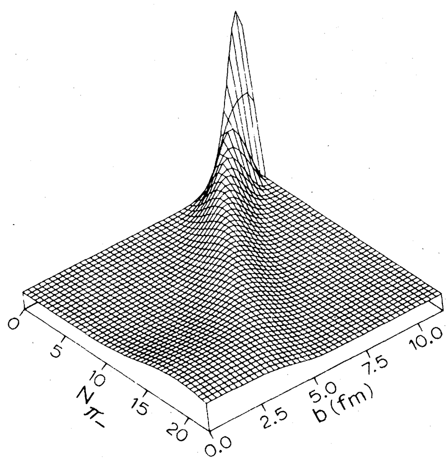

FIG. 3. Calculated values of $P_n(b)$ for negative pions as a function of b for various values of n are plotted; the vertical dimension refers to the values of $P_n(b)$. The projectile is Ar on Pb at 1.8 GeV/A.

values and individually each of them is small. On the other hand, if $\bar{n}_4$ is small, P_n for large n is necessarily small. For small n, P_n is determined primarily by the edge, i.e., large impact parameter. Thus in order to obtain information on what happens for near head-on collisions, one has to determine P_n for large n. These numbers will be small and are harder to measure.

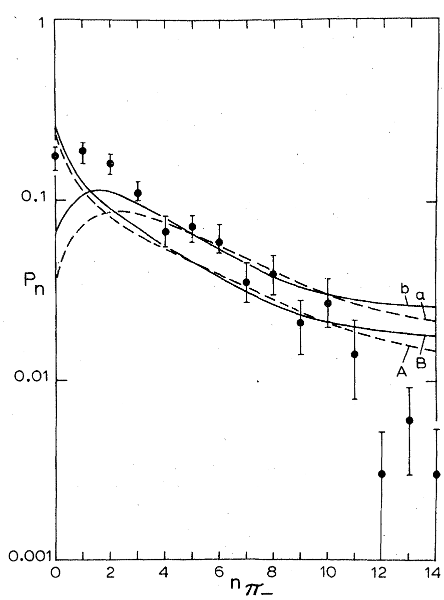

FIG. 4. Calculation of P_n for negative pions compared with experimental data. Ar projectile on Pb_3O_4, incident at 1.8 GeV/A. Curve A: Δ's excluded, no trigger bias. Curve B: Δ's included, no trigger bias. Curve a: Δ's excluded, trigger bias. Curve b: Δ's included, trigger bias. For the trigger bias used in the calculation, see Sec. IV.

Figure 4 compares our results with the data. Two calculations are shown: The first assumes only pions, nucleons, and composites are present; the second includes the production of pions by deltas. In the experiment,[10] the trigger of the streamer chamber was biased such that events in which very few charged particles were produced were not recorded. This bias is included in the calculation[12,13] by demanding that if less than five charges are involved in the collision, it is ignored in the computation of the multiplicity distribution. This amounts to the imposition of a cutoff in the impact parameter at less than the sum of the two radii of the ions. The cutoff has a significant effect on P_n for n small (Fig. 4). Compared to our fit of associated multiplicity data, the agreement here is less than ideal and we now try to understand the reason for this. One could argue that the fireball model may still be quite valid for low values of b, the impact parameter, and fails when b is large (only a few nucleons participate in the collision). But there is another explanation which we feel is more natural. Generally if one uses a fireball-type model to calculate one pion inclusive spectra, one finds that the theoretical cross section is too high by a factor of about 2 (see Refs. 8 and 14). Although the inclusive spectrum is not known at all angles or energies, this suggests that the overall normalization may be overestimated. We have therefore

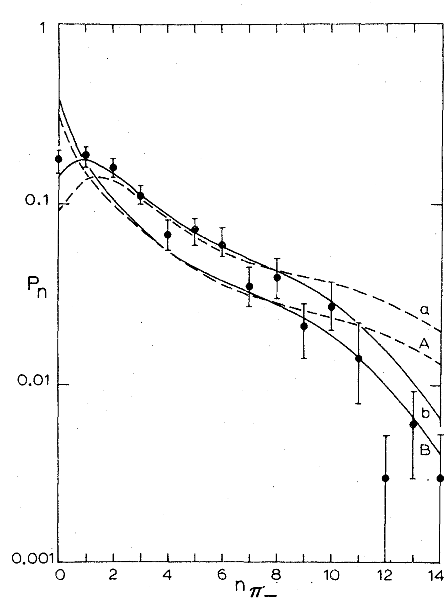

FIG. 5. Same as before except that a normalization correction has been made. See Sec. IV.

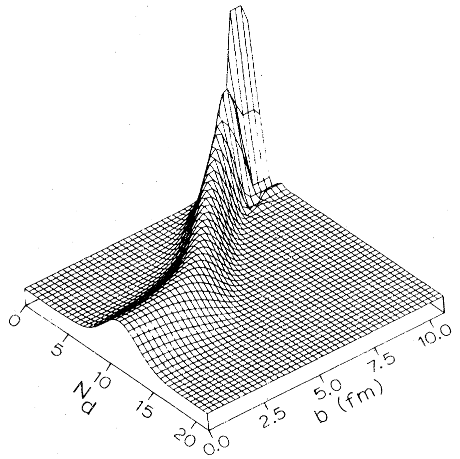

FIG. 6. The calculated values of $P_n(b)$ for deuterons as a function of b for various values of n are plotted. The vertical dimension refers to the values of $P_n(b)$. The projectile is 400 MeV/A Ne, incident on U.

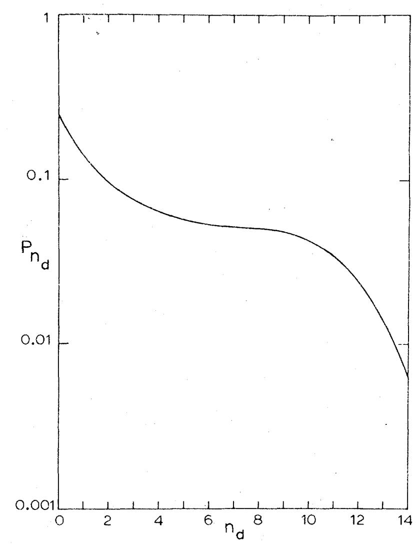

FIG. 7. Calculated P_n for deuterons for Ne on U at 400 MeV/A.

done a calculation in which $\bar{n}_4$, $\bar{n}_8$, and $\bar{n}_9$ [Eqs. (16), (24), and (25)] are computed correctly, but are then arbitrarily halved in the calculation of the multiplicity distribution [Eqs. (19), (28), and (29)]. The good agreement (Fig. 5) suggests that it is the wrong normalization which is responsible for the error. A further confirmation of this would come from a measurement of negative pion associated multiplicity (Sec. II).

Although the approach of Ref. 12 is similar in spirit to ours, there are significant differences. That calculation is valid only for $N=Z$ systems; also the temperature was parametrized, rather than being determined more rigorously from energy considerations as in our case. Of course when we make these simplifications, we reproduce the results of Ref. 12.

In Ref. 11 a multiple-collision model is developed and applied to the same data set. In that analysis, no account is taken of the experimental triggering condition mentioned above, so any comparison with our calculations should be made with our "untriggered" results. (Recall that the shape of the distribution changes significantly when this experimental constraint is imposed.)

V. DEUTERON MULTIPLICITY

As an example of multiplicity distributions for composites, we have calculated the case for deuterons. This is shown in Figs. 6 and 7. One interesting feature is the plateau between $P_{n=6}$ and $P_{n=9}$, and we have verified that this remains even if $\bar{n}_d$ of Eq. (31) is changed significantly. Any trigger bias present in the experimental set up will change the shape of Fig. 7; however, it should be possible to incorporate such biases in the calculation. Unlike the pion case, here P_n for $n=6$ or 7 already senses the central region of the collision.

VI. SUMMARY AND CONCLUSIONS

Based on the statistical model, we have obtained expressions for charged particle multiplicity distributions in relativistic heavy ion reactions. The formalism is quite general and applies equally well to, for example, pions or deuterons.

Compared to experimental data on negative pions, the difference between experiment and theory can probably be traced back to an overestimation of normalization in the theory. The agreement with data for associated multiplicity is good.

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