NOTES AND CORRESPONDENCE

Testing for Trend in North Atlantic Hurricane Activity, 1900–98

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ABSTRACT

The detection of a trend in hurricane activity in the North Atlantic basin has been restricted by the incompleteness of the record prior to 1946. In an earlier paper, the complete record of U.S. landfalling hurricanes was used to extend the period of analysis back to 1930. In this paper, a further extension is made back to 1900. In doing so, the assumption in the earlier paper of an exponential linear trend is relaxed and the trend is estimated nonparametrically. The results show no significant trend in basinwide hurricane activity over the period 1900–98.

1. Introduction

The record of annual counts of North Atlantic hurricanes is incomplete prior to the advent of regular aircraft reconnaissance in 1946 (Neumann et al. 1993). This incompleteness has restricted efforts to identify a secular trend in North Atlantic hurricane activity to the postwar period (Elsner and Kara 1999; Landsea et al. 1999). In a recent paper (Solow and Moore 2000, hereafter SM), we proposed a method for testing for trend in a partially incomplete hurricane record. This method makes two assumptions about the quality of the observational record over the period of analysis: (i) there is no misclassification of hurricanes as tropical storms and vice versa and (ii) the record of hurricanes making landfall in the United States is complete. The method also assumes that the probability that a hurricane makes landfall in the United States is constant over the period of analysis. These assumptions outlined above also underlie the earlier informal analysis of Fernandez-Partagas and Diaz (1996).

In addition to these assumptions, SM further assumed that the sighting probability for hurricanes that do not make U.S. landfall is constant over the incomplete part of the record and that any trend in mean hurricane number follows an exponential linear model. To ensure the reasonableness of these assumptions, SM extended the analysis back only to 1930. It would be useful on both scientific and statistical grounds to extend the period of analysis further back. However, in doing so, it is important to check these additional assumptions and, if they are found to be unreasonable, to modify the method. The purpose of this paper is to take a step in that direction by extending the period of analysis back to 1900.

The remainder of this paper is organized in the following way. In the next section, the basic statistical model is outlined. In section 3, some results are presented suggesting that the assumption of constant sighting probability remains reasonable back to 1900 and the model is fit to the data using a modification of the method proposed in SM to accommodate nonparametric estimation of the secular trend in mean hurricane number. Section 4 contains some concluding remarks.

2. Statistical model

The general model outlined in this section is the same as in SM. Consider the observation period $t = 1, 2, \ldots, n$ and let Y_t be the true number of hurricanes in year t. We will assume that Y_t has a Poisson distribution with mean μ_t. In this paper, interest centers on testing the null hypothesis H_0 that μ_t is constant versus the general alternative hypothesis H_1 that μ_t is not constant. Let the random variable X_t be the number of landfalling hurricanes in year t. We will assume that the conditional distribution of X_t given $Y_t = y_t$ is binomial with parameters y_t and p, where the unknown

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landfalling probability p is assumed to be constant over the observation period. Although the incompleteness of the record precludes a direct test of this assumption, SM did perform such a test for the period 1946–98, when the record is complete, and found no evidence of a nonconstant landfalling probability. Finally, let the random variable Z_t be the observed number of hurricanes in year t that did not make landfall. We will assume that the conditional distribution of Z_t given $Y_t = y_t$ and $X_t = x_t$ is also binomial with parameters $y_t - x_t$ and q_t, with $q_t = 1$ for $t = m + 1, m + 2, \ldots, n$. That is, the first m years of the record are incomplete, but the last n–m years are complete.

Let $\{\mu_t\} = (\mu_1, \mu_2, \ldots, \mu_n)$. Inference about $\{\mu_t\}$ can be based on the likelihood function, which is defined as the joint probability mass of the observed values (x_t, z_t) of (X_t, Z_t), $t = 1, 2, \ldots, n$, regarded as a function of the unknown parameters. The contribution to the likelihood of the observation in year t is

$$L_t(p, q_t, \mu_t) = \sum_{y_t=x_t+z_t}^{\infty} \binom{y_t - x_t}{z_t} q_t^{z_t}(1 - q_t)^{y_t-x_t-z_t}\binom{y_t}{x_t} \times p^{x_t}(1 - p)^{y_t-x_t}\mu_t^{y_t} \exp\frac{(-\mu_t)}{y_t!}. \quad (1)$$

The product of the first three terms in this summation represents the conditional binomial probability that $Z_t = z_t$ given $X_t = x_t$ and $Y_t = y_t$. The product of the next three terms represents the conditional binomial probability that $X_t = x_t$ given $Y_t = y_t$. The product of the final three terms is the unconditional Poisson probability that $Y_t = y_t$. The lower bound of the summation reflects the fact that the total number of hurricanes cannot be less than the observed number. Note that in the complete part of the record (i.e., $t > m$) for which $q_t = 1$, (1) reduces to

$$L_t(p, 1, \mu_t) = \binom{y_t}{x_t} p^{x_t}(1 - p)^{z_t}\mu_t^{y_t} \exp\frac{(-\mu_t)}{y_t!}. \quad (2)$$

The likelihood function for the full set of observations is given by

$$L(p, \{q_t\}, \{\mu_t\}) = \prod_{t=1}^{n} L_t(p, q_t, \mu_t) \quad (3)$$

where $\{q_t\} = (q_1, q_2, \ldots, q_n)$.

3. Model fitting and results

To fit the model described in the previous section, it is necessary to specify models for the sighting probabilities $\{q_t\}$ and the trend $\{\mu_t\}$. As noted, in extending the period of analysis back to 1930, SM assumed that q_t is constant over the incomplete part of the record. In extending the analysis further back to 1900, it is important to consider the possibility of nonconstant q_t. However, preliminary analysis suggests that the assumption of constant q_t remains reasonable. For example, of 107 hurricanes recorded during 1900–29, 64 (or 60%) made U.S. landfall. The corresponding numbers for 1930–45 are 76 and 51 (or 67%). By a standard chi-squared test (Cox and Snell 1989), this difference in observed landfall rate is not significant ($\chi^2 = 0.98$, 1 degree of freedom, p value $= 0.32$). Moreover, under the most likely alternative, q_t increases with t. This would result in a declining observed landfall rate. In fact, the rate is slightly higher in 1930–45 than in 1900–29. On the basis of these results, we will assume that $q_t = q$ for $t = 1, 2, \ldots, m$ and that $q_t = 1$ for $t = m + 1, m + 2, \ldots, n$.

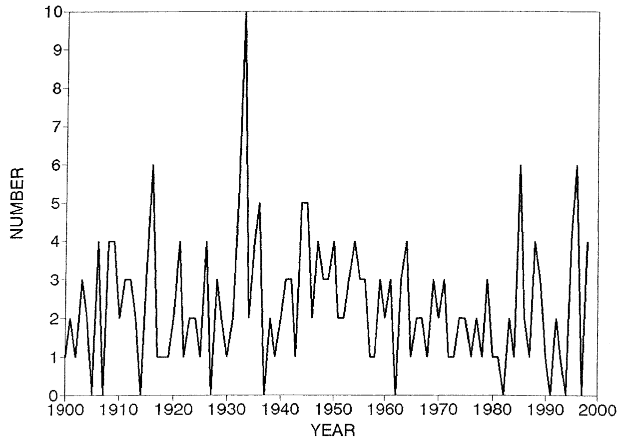

FIG. 1. Annual number of U.S. landfalling hurricanes, 1900–98.

It was also assumed in SM that $\mu_t = \exp(\mu_0 + \mu_1 t)$. In Fig. 1, the time series of landfalling hurricanes is plotted over the period 1900–98. Under the model outlined in the previous section, X_t has an unconditional Poisson distribution with mean $p\ \mu_t$. While the decline between 1930 and 1998 found in SM is apparent in Fig. 1, there is some evidence of lower numbers in the early part of the record, casting doubt on the adequacy of the exponential linear model. One possibility is to consider a higher-order exponential polynomial model. We have chosen instead to adopt a nonparametric approach and to assume only that $\{\mu_t\}$ is smooth.

One way to estimate $\{\mu_t\}$ nonparametrically is by local likelihood estimation (Tibshirani and Hastie 1987). Briefly, this would involve (i) adopting a locally parametric model for μ_t; (ii) with p and q fixed, fitting the model for each value of t by maximum likelihood using data in a neighborhood of t; and (iii) repeating the procedure for different values of p and q until the overall likelihood is maximized. The second step in this procedure is computationally demanding, particularly in conjunction with the parametric bootstrap outlined below. As an alternative, we adopted the following approximate approach. With q fixed, construct the series as follows:

$$\hat{y}_t(q) = x_t + \frac{z_t}{q_t} \qquad t = 1, 2, \ldots, n \quad (4)$$

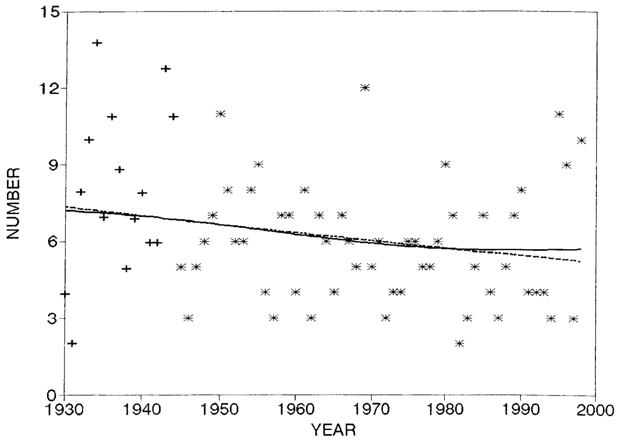

FIG. 2. Nonparametric (solid) and parametric (dashed) estimates of mean hurricane number, 1930–98. Also shown are reconstructed values (+) for 1930–45 and observed numbers (×) for 1946–98.

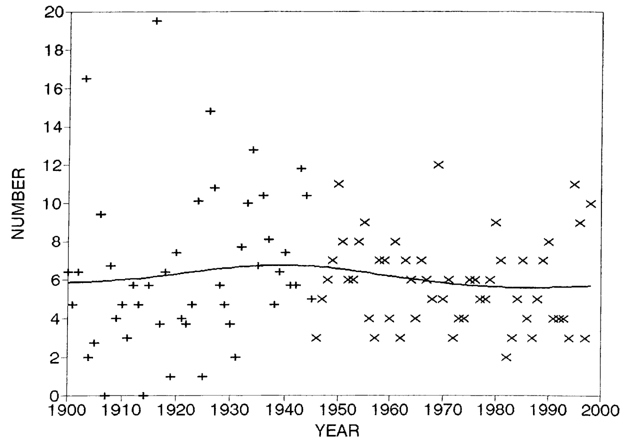

FIG. 3. Nonparametric estimate of mean hurricane number, 1900–98. Also shown are reconstructed values (+) for 1900–45 and observed numbers (×) for 1946–98.

and form an estimate $\{\hat{\mu}_t(q)\}$ of $\{\mu_t\}$ by smoothing this constructed series. With q and $\{\hat{\mu}_t(q)\}$ both fixed, find the value $\hat{p}(q)$ of p that maximizes $L(p, q, \{\hat{\mu}_t(q)\})$. Finally, find the value of q that maximizes $L(\hat{p}(q), q, \{\hat{\mu}_t(q)\})$. Computational economy is gained over local likelihood estimation because smoothing is much quicker than fitting n local models each by maximum likelihood.

To assess the performance of this approach, we applied it to the data for the period 1930–98 and compared the results to those of SM. In the smoothing step, we used the simplest Gaussian kernel estimate:

$$\hat{\mu}_t(q) = \frac{\sum_{s=1}^{n} w_s(t)\hat{y}_s(q)}{\sum_{s=1}^{n} w_s(t)} \tag{5}$$

with $w_s(t) = \phi[(s - t)/h]$, where ϕ is the standard normal probability density function and h is a bandwidth that controls smoothness. More complicated smoothers could be used. The degrees of freedom of the kernel estimate is approximately

$$\mathrm{df} = \sum_{t=1}^{n} \frac{w_t(t)}{\sum_{s=1}^{n} w_s(t)} \tag{6}$$

(Hastie and Tibshirani 1990). To ensure comparability to the results of SM in which a two-parameter log-linear model was used for μ_t, we chose $h = 17$ so that df = 2.1. In Fig. 2, the smooth estimate of $\{\mu_t\}$ is shown along with the estimate found in SM. In both cases, the point estimates $\hat{p}$ and $\hat{q}$ of p and q are 0.39 and 0.35, respectively. Also shown in Fig. 2 are the constructed values $\hat{y}_t(\hat{q})$ for t = 1930–45 and the complete counts y_t for t = 1946–98.

To assess the significance of the nonparametric estimate, we used the following pseudolikelihood ratio statistic:

$$\Lambda = 2[\log L(\hat{p}, \hat{q}, \{\hat{\mu}_t(\hat{q})\}) - \log L(\hat{p}_0, \hat{q}_0, \{\hat{\mu}_0\})], \tag{7}$$

where $\hat{p}_0$, $\hat{q}_0$, and $\hat{\mu}_0$ are the estimates of p, q, and μ under H_0. These estimates were given in SM as 0.40, 0.43, and 1.80, respectively. The expression in (7) is called the pseudolikelihood ratio statistic because, while it has the same form as the ordinary likelihood ratio statistic, the estimates under H_1 are not strictly maximum likelihood estimates. We used the following parametric bootstrap procedure to assess the significance of the observed value λ of Λ. Pairs of values (x_t^*, z_t^*), $t = 1, 2, \ldots, n$ were simulated from the model fit under H_0. Briefly, for each t, this involved simulating a value y_t^* from the Poisson distribution with mean $\hat{\mu}_0$; simulating a value x_t^* from the binomial distribution with parameters y_t^* and $\hat{p}_0$: and simulating a value z_t^* from the binomial distribution with parameters $y_t^* - x_t^*$ and $\hat{q}_0$ (with $\hat{q}_0 = 1$ for $t = m + 1, m + 2, \ldots, n$). Using the simulated pairs, the model was fit under both H_0 and H_1 and the value of Λ was formed. The procedure was repeated 200 times and the significance level of λ was estimated by the proportion of simulated values of Λ that exceeded λ. For the fit shown in Fig. 2, the value of Λ is 3.8 with an estimated significance level of 0.12 with a standard error around 0.02. This is close to the significance level of 0.09 found by SM. The nonparametric results are broadly similar to the parametric results of SM. While this comparison certainly does not constitute a formal assessment of the nonparametric approach, it does suggest that the method works well.

Finally, we applied the nonparametric method to the data for the period 1900–98. In this case, to accommodate the potential nonmonotonic in μ_t behavior in a parsimonious way, we chose a smoothing bandwidth of $h = 15$ so that df = 3.1. The estimates of landfall probability and sighting probability were $\hat{p} = 0.38$ and $\hat{q} = 0.37$, respectively; and the estimate of $\{\mu_t\}$ is shown in Fig. 3, along with the constructed values $\hat{y}_t(\hat{q})$ for the period 1900–45 and the observed counts y_t for the period 1946–98. The trend estimate increases from around 5.9 hurricanes per year in 1900 to around 6.8 hurricanes per year in 1939 before falling to around 5.7 hurricanes per year in 1998. In this case, the value of Λ is 3.86 with a significance level estimated by the parametric bootstrap of 0.26 with a standard error of around 0.03. The estimates of p, q, and μ_0 for the full record under H_0 that were used in the parametric bootstrap procedure were 0.39, 0.40, and 1.80, respectively.

This analysis provides no evidence of a trend in mean hurricane number over the period 1900–98. This result is not sensitive to the choice of h. The slightly stronger evidence for a trend over the period 1930–98 can be explained in the following way. The reconstructed hurricane counts in Figs. 2 and 3 suggest a period of slightly elevated hurricane activity in the mid-1930s. This is reflected in the nonparametric estimates of μ_t in these figures, with both estimates reaching a maximum in this period. This period of elevated activity occurs near the lower boundary of the observation period in Fig. 2, but in the middle of the observation period in Fig. 3. As a result, the lower activity prior to the mid-1930s causes the estimate of μ_t to be flatter in Fig. 3 than in Fig. 2, resulting in a higher significance level.

4. Discussion

Incompleteness of the North Atlantic hurricane record poses a serious challenge to understanding long-term historical variability in hurricane activity. While there is no substitute for good data, statistical methods can be used to make the most of the data that are available. Here, we have used the record of U.S. landfalling hurricanes to extend the period of analysis beyond the period of completeness of the overall record. In doing so, we have verified the assumption of constant sighting probability of nonlandfalling hurricanes and we have relaxed the assumption of earlier work of an exponential linear trend. The results suggest little evidence of a trend in overall hurricane activity.

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