A Comparison of Sensitivity Analysis Techniques for Complex Models for Environmental Management

[1]Ravalico, J. K., [1]H. R. Maier, [1]G. C. Dandy, [2]J. P. Norton and [2]B. F. W. Croke

[1]Centre for Applied Modelling in Water Engineering (CAMWE), School of Civil & Environmental Engineering, University of Adelaide, SA 5005, Australia

[2]Mathematical Sciences Inst./Integrated Catchment Assessment & Management Centre, Australian National University, ACT 0200, Australia

Email: jravalic@civeng.adelaide.edu.au

Keywords: *Sensitivity analysis; Integrated assessment modelling; Decision-making.*

EXTENDED ABSTRACT

Computer based modelling methods are being used increasingly to replicate natural systems in order to review both large and small scale policy measures prior to their implementation. Integrated Assessment Modelling (IAM) incorporates knowledge from several different disciplines into one model in order to provide an overarching assessment of the impact of different management decisions. The importance of IAM is that the environmental, social and economic impacts of management choices can be assessed within a single model, further allowing assessment in relation to sustainability criteria.

The considerable detail facilitated by these models often requires the inclusion of a large number of parameters and model inputs, many of whose values may not be known with certainty. For this reason and because models do not always behave intuitively (in particular when there are non-linearities involved), sensitivity analysis (SA) of the model to changes in its parameters and inputs is an important stage of model development.

Current SA methods have not kept pace with rapid increases in computing power and availability and more importantly the resultant increases in model size and complexity. Also related to the complexity is increased difficulty in finding and fitting distributions to all parameters. Further, the complex nature of integrated models requires SA that is flexible and can be implemented regardless of model structure.

This research aims to establish new criteria for SA used in the context of integrated models for environmental management and decision-making. These criteria are believed to reflect the current requirements specific to this type of modelling. Desirable criteria are identified as: high computational efficiency; ability to take into account higher order parameter interactions; ability to account for model non-linearities; not requiring knowledge of parameter probability distributions; and use in decision making.

SA of an integrated model of the Namoi River catchment is performed using the Fourier Amplitude Sensitivity Testing (FAST) method, Morris method, method of Sobol’, and regression and correlation coefficients. The results from these analyses are used as a basis for comparing the SA methods by the new criteria outlined above. The Namoi model is a combination of a flow model with a non-linear component, a policy model, an economic model and an extraction model. It can be used for assessing management options for the river. SA of two different potential management options for the catchment is undertaken to facilitate comparison of sensitivity between two slightly different models.

Comparison of the different SA methods shows that none of the methods meet all of the criteria and, in particular, there are no methods that are effective for use when comparing management options. This lack of an adequate SA method for integrated models indicates that development of a new method of SA specifically for integrated models for environmental management is desirable.

The FAST method is shown to meet the criteria most effectively, being able to account for model non-linearity and non-monotonicity, requiring only parameter ranges (not distributions), and being relatively computationally efficient (although this does come at a loss of some resolution). Results from the FAST SA of the Namoi model show the model to be sensitive to several parameters within the non-linear loss module. Further, one management option shows sensitivity to the decision variables within the model while the other does not. This means that the first management option clearly corresponds to the more controllable form of the model.

1. INTRODUCTION

Computer-based modelling methods provide an important means to review policy choices prior to their implementation. Integrated Assessment Modelling (IAM) incorporates knowledge from several different disciplines into one model in order to provide an overarching assessment of the impact of different management decisions. The importance of IAM is that the environmental, social and economic impacts of management choices can be assessed within a single model, further allowing assessment in relation to sustainability criteria.

Such modelling methods allow for the inclusion of considerable detail and require the input of numerous parameters from varying sources, many of which may not be known with certainty. Further, environmental and integrated modelling has to contend with poorly understood sources of uncertainty, as well as the inability to take large samples or repeat experiments (Norton, 1996). For these reasons, and because model outputs do not always behave in an intuitive manner, an important stage of model development is sensitivity analysis of the model to changes in parameter values.

Sensitivity analysis (SA) methods have not kept up with the rapid increase in available computational power and, more importantly, the resultant increase in model size and complexity. An important objective of IAM is to increase the understanding of the directions and magnitudes of change under different management options, in order to allow differentiation between outcomes with confidence. SA plays a large role in developing this confidence.

Another consequence of the complexity of IAM is difficulty in finding and fitting probability distributions of all uncertain model parameters and inputs, a feature commonly required in SA (e.g Sobol' (Sobol', 1993)). Further, the complex nature of integrated models requires a SA approach that is flexible and can be implemented regardless of model structure.

When used as a decision-making tool to identify the strengths and weaknesses of management options, a model gives outcomes which can be ranked effectively according to given criteria. The necessity for rankings of model outputs to be robust and accurate makes it vital that any sensitivity of these rankings to changes in parameter values is known. This requires the use of SA that is easily understood, as well as simple to perform.

Given the rapid changes in the use, size and complexity of integrated models for environmental management, it is necessary to establish new requirements for SA and to assess the current methods of SA against these requirements.

The following research establishes criteria that are believed to reflect the current requirements for SA in the context of integrated models as decision-making tools. Based on these criteria, an assessment of commonly used SA methods is conducted, thereby highlighting both their advantages and deficiencies. This assessment is important in identifying areas where SA methods should be improved to keep pace with current advances in modelling techniques.

In order to complete this assessment, an integrated model of the Namoi River catchment (NSW) is used. The model combines a flow model with a non-linear component, policy model, economic model and extraction model in order to represent the operation of the entire system, including human activity. The non-linear component of the model and the interactions between different parts of the model make it an ideal case study for assessing SA methods against their requirements.

2. SENSITIVITY ANALYSIS CRITERIA

When comparing different methods of SA it is important to define criteria by which the different methodologies can be assessed. Five key criteria have been identified for use in comparing commonly used SA techniques. The following criteria form the basis for the comparison between SA methods undertaken in this research.

Criterion 1. Computational Efficiency

The importance of computational efficiency is apparent when considering the necessarily large and complex nature of integrated models, which can have in excess of 100 parameters with varying ranges, (Saltelli et al., 2000).

Criterion 2. Parameter Interactions

The nature of integrated modeling is such that parameters may be used in several different processes, with their effects often being larger than anticipated due to interactions with other parameters. This gives rise to the need for SA methods to be able to account for interactions between parameters, preferably with the ability to evaluate all parameter interactions, not just pairwise or third-order effects.

Criterion 3: Data Requirements

It is desirable that SA methods used for integrated models do not require knowledge of parameter probability distributions in order to accurately assess the sensitivity of the model to variations in the parameters, as often knowledge of the system being modeled is incomplete and determining accurate parameter distributions can require more data than are available.

Criterion 4: Model Non-Linearities

Environmental systems rarely behave in a linear fashion, nor are they generally monotonic. Often the behaviour of a system can change from one mode to another, significantly different, mode given different combinations of parameter values. This property of models used for environmental management requires that SA methods are able to handle both the non-linearities and non-monotonicity present in these models.

Criterion 5: Use in Decision-Making

For SA outputs to be useful in decision-making, they must be in a format that is easily understood and applied by decision-makers. SA is ineffectual if it cannot be interpreted. SA model output is in this case considered to be easily understood if there is some physical meaning to the outputs given. While ranking parameters by those that outputs are most sensitive to can be useful, this type of SA output does not give the modeller any information on the values that parameters may take, and can be deceptive when differences between the sensitivity to various model parameters are small.

3. SENSITIVITY ANALYSIS METHODS

Several global SA methods have been compared in this study to investigate their ability to meet the desired SA criteria as outlined above. These are described in the following sections.

3.1. Morris Method

The Morris method (Morris, 1991) is a one-factor-at-a-time (OAT) method using randomized sampling matrices, which allow direct observation of elementary effects. This guarantees that meaningful information can be extracted from each parameter, without mistakenly attributing effects to that parameter. Sensitivity estimates of the total effects due to a single parameter are produced, with a final output of the mean and standard deviation of the SA estimates produced in each model run.

3.2. Regression Analysis and Correlation Measures

There are several regression analysis and correlation measures that can be used for SA. The Pearson Product Moment Correlation Coefficient (PEAR), the most commonly used linear correlation coefficient, gives a measure of how strongly correlated each individual parameter is with the output and, as such, an assessment of how sensitive the output is to each model parameter. Standard Regression Coefficients (SRC) quantify the effects caused by changing a model parameter from its mean by a fraction of its variance, while all others are kept at their mean values. This measure relates directly to the sensitivity of the model outputs to the model parameters. Partial Correlation Coefficients (PCC) provide a measure of the linear relationship between any given model parameter and the model output. If a model is non-linear, the regression models used by these three methods are not particularly effective at approximating the model. To redress this, a rank transformation of the model outputs and parameters is used, replacing the model parameters and outputs with their respective rankings and then performing the same analysis. While this can then be effective in replicating the model, it should be noted that SA is now actually occurring on a model different from the one under scrutiny. Further, the rank transformations become less effective when models are non-monotonic, as they are less able to approximate the output through linear regression methods.

3.3. FAST and Extended FAST

Fourier Amplitude Sensitivity Testing (FAST) (Cukier et al., 1978) and its successor, Extended FAST (Saltelli et al., 1999), are variance-based global SA methods, which compute the Total Sensitivity Indices (TSIs) of the model inputs. The TSIs measure the main (first order) effect of each individual or group of inputs on the model output, as well as all higher order effects that can be attributed to that parameter. Both the FAST and Extended FAST methods use a transformation function to sample the parameter space, and hence approximate the variance of the model output, with the main difference between the two methods being the choice of transformation function. FAST provides independence from model structure, while having the ability to capture the influence of the full range of variation and interaction effects, as well as the ability to group factors in order to assess their sensitivity collectively. However, estimating the total sensitivity of each factor by using the partial variance of the complementary set of factors tends towards lumping of results. FAST assumes a uniform parameter probability distributions which is advantageous when distributions are unknown, however can be a disadvantage when it is known that distributions are not uniform.

3.4. Sobol' method

The method of Sobol' (Sobol', 1993) computes TSIs similarly to the FAST and Extended FAST methods. However, rather than using Fourier methods, the Sobol' method uses a unique decomposition of the model into summands of increasing dimensionality. All terms within the decomposition can then be calculated using multiple integrals.

While the TSIs obtained using the Sobol' method measure the same property of the model parameters as in the FAST method, the Sobol' method does not use the transformation function employed to generate parameter combinations as part of the FAST method, and as a result is less computationally efficient. Due to the sampling method used, the Sobol' method requires the distributions of the various parameters in order to compute the TSIs, while FAST only requires knowledge of the parameter ranges.

4. CASE STUDY: NAMOI RIVER MODEL

To investigate the efficiency and accuracy of the SA methods outlined in Section 3, they have been applied to a case study of the Namoi River catchment.

The model used is a simplified version of the integrated water-use policy model presented by (Letcher, 2002). The integrated model incorporates numerous interactions, including streamflow, rainfall, land use, crop profits and water extraction policy. The original model incorporates considerable complexity, but to perform an initial and thorough SA, the model has been simplified, while maintaining its integrated nature. This enables the evaluation of SA tools for complex models.

4.1. Model Outline

The model used consists of IHACRES, a flow model with a non-linear component (Croke et al., 2004), a policy model that determines allowed extractions based on flow, an economic model which incorporates land use, and an extraction model which calculates the actual extraction based on a combination of land use, allowed extractions and river flow. The model is run to simulate one year, with flow calculated daily.

In the context of its use for decision-making and determining appropriate management options, two versions of the policy model, representing different management options, have been employed. The first option bases the allowed irrigation extractions on the level of flow in the river, giving three different allowed extractions for each of three minimum flow levels. The extractions occur on a daily basis. The areas planted with irrigated and dry crops are then determined based on the amount of water available for extraction in that year. The second policy option does not limit the extraction, but requires that a minimum percentage of the area be planted with the dry crop so as to limit the level of irrigation. The crop areas are based on the amount of water available in the river, assuming there is no limit on extraction, beyond being able to remove what is currently there. Consideration of two different policy models allows assessment of changing sensitivities as the management options are altered. The importance of this rests in the necessity to use models to assess management options whilst ensuring that each assessment has the same level of accuracy.

The Namoi flow network consists of several sub-catchments, each identified as a particular node. A single node of the model will be used to assess the SA methods against the proposed criteria in this research.

5. ANALYSES CONDUCTED

The SA software SimLab 2.2, an updated version of that outlined by (Giglioli et al., 2000), was used to facilitate the analysis. SimLab 2.2 consists of a preprocessor module that allows the user to select between various methods of parameter combination generation, a model execution module allowing the user to run either an internal or external model with the parameter combinations generated, and a postprocessor module which performs both uncertainty analysis and SA.

Two versions of the model using differing management options to maximize the environmental flows in the river, while also maintaining profit levels among farmers, were investigated. The two models utilize the same parameter values for the flow and economic models; however, the policy and extraction models have different parameter values. This analysis allows testing as to whether the alteration in parameter values increases the model's sensitivity to the common parameters. It also allows investigation of how effective each SA method is at providing a comparison between different versions of the model with different parameter values. Management option 1 uses flow levels to determine maximum allowable irrigation extraction from the river, with three specific levels set (L1, L1 + L2 and L1 + L2 + L3) and corresponding allowed extractions (M1, M1 + M2, M1 + M2 + M3). The total allowed annual extraction is then used to determine the area planted with irrigated or dry crops, by planting as much of the irrigated crop as possible with the water available for irrigation. There is also an upper limit of annual extraction. Management option 2 sets a minimum requirement for the percentage of the area which must be planted with the dry crop. In this case, given the flow in the river, as much water as possible may be removed. The area of each crop is determined in a similar way to management option 1, but with a minimum area requirement of dry crop to be planted.

SA of the two versions of the model are conducted using the Morris method, two different sampling strategies (Monte-Carlo and Latin Hypercube Sampling) to generate regression and correlation coefficients, the FAST method, and the method of Sobol'.

Five model outputs are considered in this analysis, these being: total annual flow before extractions, total annual flow after extractions, number of days with zero flow (< 1 ML), and total profit generated over the whole area under study. These were chosen to represent key outcomes that would be potentially desirable to alter through manipulation of the system, as well as providing an assessment of the sensitivity of each component of the model.

With the exception of the profit per unit area of each crop planted and the maximum annual extraction (management option 1 only), the sensitivity to the entire set of static model parameters was analyzed. These two parameters were not selected for SA due to the direct relationship between them and the model outputs. Distributions for all parameters were chosen to be uniform, with the exception of the time constant of the linear flow module, which has been shown to have a normal distribution.

Ranges for the non-linear loss module parameters (f, e, d, τ_q) were selected based on model calibration studies, while the ranges of the decision variables of the model (L1, L2, L3, M1, M2, M3, DCR, WR) were based on values used by (Hicks, 2003). The parameter ranges are given in Table 1 and 2 below.

Table 1. Uniformly distributed parameters.

Parameter	Use	Lower bound	Upper bound
F	Non-linear loss module	0.5	1
E	Non-linear loss module	0.15	0.2
D	Non-linear loss module	100	400
L1 (ML)	1st flow level	15	45
L2 (ML)	2nd flow level	375	450
L3 (ML)	3rd flow level	800	1000
M1 (ML)	1st extraction limit	10	25
M2 (ML)	2nd extraction limit	5	15
M3 (ML)	3rd extraction limit	10	25
DCR (%)	Percentage of dry crop required (management option 2)	0	50
WR (ML)	Water requirement per unit area for irrigated crop	15	45

Table 2. Normally distributed parameters.

Parameter	Use	Mean	Standard deviation
τ_q	Time constant of the linear flow module	2	0.4

6. RESULTS AND DISCUSSION

6.1. Comparison of Methods Against Criteria

This research investigated at most 11 model parameters for the Namoi model and the model took less than 1 second of computing time to run. Hence, computational efficiency, while desirable, is not as significant an issue as in models with many more parameters and longer run times. For these models, a small reduction in computational efficiency can lead to a significant increase in SA run times. None of the methods trialled perform well with significant numbers of parameters and any increased computational efficiency in the methods comes at the expense of their ability to sample the parameter space. FAST seems to perform the most efficiently due to the cyclical nature of the curve used to select samples, while the Sobol' method is less computationally efficient. The computational cost of the Morris method is proportional to n = 2kr, where r is the selected size of each sample and k is the number of input factors (parameters). Consequently, as the number of parameters increases, the required number of samples increases significantly, thereby reducing the computational efficiency of the method. The choice of sampling method (e.g. Monte Carlo, Latin Hypercube) dictates the computational efficiency of the regression and correlation methods, placing increased importance on the sampling method chosen.

The difference between the first order sensitivity indices and the TSIs computed using both the FAST and Sobol' methods shows that there are higher order interactions present in the Namoi model. While FAST and Sobol' are able to take these into account, the regression methods do not take higher order interactions into account at all and the Morris method only considers the main effect of each parameter, but does provide an overall measure of interactions and curvatures. In the Namoi model, the parameter with the highest sensitivities is identified by all methods tested. However, methods that identify interaction effects alter the order of lower-ranked parameters. For example, considering the annual flow after extractions using management option 2, when using the regression methods, the time constant τ_q is considered to be among the three parameters with highest sensitivities, whereas in fact when higher order interactions are taken into account, it is not, and the f parameter from the non-linear loss module shows greater sensitivity. In this case, had the higher order interactions not been taken into account, the model's high sensitivity to parameter f would not have been captured.

FAST is the only method tested that does not require knowledge of the parameter distributions, as it assumes a uniform distribution for all parameters. Despite this advantage, the relation of the variance of the parameter to its range dictates that the method is sensitive to the parameter range selected. In fact, sensitivity indices for both the first and total order varied considerably when parameter ranges were altered. The sampling methods used for the Sobol', Morris and Regression and Correlation methods all require knowledge of the parameter distributions. This requirement was problematic for the Namoi case study, as there is little information on the distributions of parameters for the non-linear loss module, and also as the model parameters that were actually decision variables do not have distributions. In this case, a uniform distribution was assumed for all parameters with unknown distribution, however, this may have meant that some sensitivity data were lost due to poor sampling of areas with a high density of likely samples. These data requirements mean that none of the tested methods, with the exception of the FAST method, meet criterion 3.

The use of linear regression by the regression and correlation coefficients means that non-linearities in the model are poorly taken into account. This is apparent from the low R^2 values of the regression returned during the SA of the Namoi model. While use of the rank regression and correlation coefficients shows improvement in the R^2 values, it must be noted that this method alters the model to perform SA and hence results are not as accurate. For example the lack of account of non-linearity leads to incorrect sensitivity results from the regression and correlation measures when considering the number of days with zero flow in management option 1. The regression measures underestimate the sensitivity of parameter d from the loss module, which could potentially lead to less emphasis being placed on the accuracy of that parameter, and hence a reduction in overall model accuracy.

While the Morris method can account for non-linearity, there is no differentiation between the effects caused by non-linearities in the model and parameter interaction, and it has an assumption of monotonicity, which is not always valid. The two variance based methods, FAST and Sobol', are the only methods able to account for non-linearities and non-monotonicity in the model.

Criterion 5 is not met in any of the tested cases. All of the methods tested output sensitivity in a way that needs to be interpreted considerably by the modeller. The output of all SA methods does not allow effective comparison between the sensitivities of parameters in the two different management options, and no methods give sensitivity values that have clear physical meaning.

Comparison of the SA methods investigated against the five criteria outlined in Section 2 shows that while each method has different strengths and weaknesses, the FAST and Sobol' variance-based methods meet more of the criteria more effectively than the other methods tested.

6.2. Sensitivity Results for the Namoi Model

The sensitivity results obtained using the extended FAST method for management option 1 and 2 are shown in Figure 1 and 2 respectively. The results obtained using the method of Sobol' are very similar to those obtained using FAST in all instances. There is little variation in the sensitivity indices of the various model outputs. The analyses show that for management option 2, the number of days with zero flow (els0), annual flow after extraction (sumY) and total profit are all extremely sensitive to the crop water requirement (WR). Hence, in considering this management option, care needs to be taken when assessing crop water requirements, in particular as this value varies for different crops. Thus, the accuracy of the model is dependent on the choice of crop used in the model and may require modification of the model to include more than one type of crop.

The model behaved as expected in relation to the sensitivity of the annual flow before extraction (sumQ), with virtually all of the variance in the model being accounted for by the parameters of the flow model (d, e and f). Further, there is some similarity in the ratio between the sensitivity indices of d, e and f, the parameters from the non-linear loss module, in all of the model outputs for management option 2. It is apparent in Figure 2 that this similarity carries over to the annual flow before and after extraction for management option 1 and also to some extent the number of days with zero flow.

The sensitivity to the loss module parameters indicates a necessity for careful calibration of the parameters in order to ensure that the model is behaving as accurately as possible.

Results indicate that the annual flow after extraction, as well as the total profit of the system in management option 1, are both sensitive to the decision variables within the model, in particular the minimum level at which extraction is allowed and the maximum extraction allowed at that flow. This indicates that decision-makers do have considerable control over the system in setting these variables and as such must be cautious to ensure that these are set appropriately. Analyses also showed that the sensitivity of the annual extraction was identical to the sensitivity of the total profit from crop sales.

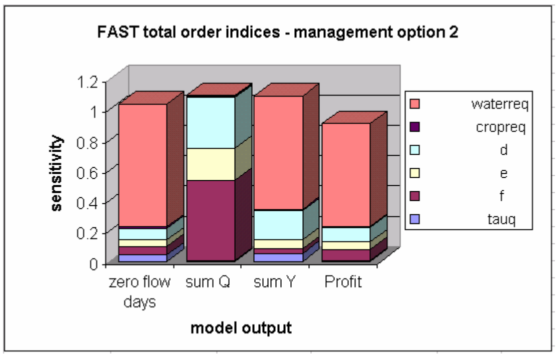

Figure 1. Extended FAST total order indices for management option 2

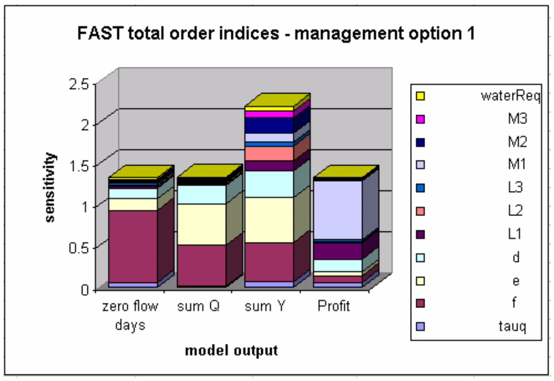

Figure 2. Extended FAST total order indices for management option 1

Comparison between the SA outputs for each management options shows that both options are consistently sensitive to parameters from the loss module, but generally (for outputs other than the flow before extraction), management option 2 is most sensitive to the crop water requirement while the strong sensitivities in management option 1 are to the decision variables. Ultimately this means that there is more potential for controlling the flow of the river and the profit of the system using management option 1.

7. CONCLUSIONS

While some of the criteria for SA were met by the methods tested, the lack of a SA method that meets all the criteria indicates that there is a need for new methods of SA for integrated models used for decision-making. In particular, the lack of any SA method that provides results meeting criterion 5 indicates that current methods of SA are not adequate. Overall the FAST method performed the best of the four methods.

The Namoi model shows the expected sensitivities to the flow model parameters, with higher sensitivity to the loss module parameters. When management option 1 is run through the model, the sensitivity to the decision variables shows that this option is more easily able to control the flow levels within the river and as such is a more effective management choice than option 2, provided that the decision variables are set appropriately.

8. REFERENCES

Croke, B. F. W. and Jakeman, A. J., A catchment moisture deficit module for the IHACRES rainfall-runoff model, *Environemental Modelling and Software*, 19, 1-5, 2004.

Cukier, R. I., Levine, H. B. and Shuler, K. E., Nonlinear Sensitivity Analysis of Multiparameter Model Systems, *Journal of Computational Physics*, 26, 1-42, 1978.

Giglioli, N. and Saltelli, A., ASimLab 1.1, Software for Sensitivity and Uncertainty Analysis, tool for sound modelling, Joint Research Centre, European Commission, Institute for Systems Informatics and Safety, Ispra, Italy, 2000.

Hicks, B., A Sensitivity Analysis Exploration of Integrated Models, Honours, Australian National University, Canberra, 2003.

Letcher, R., Issues in Integrated Assessment and Modelling for Catchment Management, Thesis for the Degree of Doctor of Philosophy, Australian National University, Canberra, 2002.

Morris, M. D., Factorial Sampling Plans for Preliminary Computational Experiments, *Technometrics*, 33(2), 161-174, 1991.

Norton, J. P., Roles for deterministic bounding in environmental modelling, *Ecological Modelling*, 86, 157-161, 1996.

Saltelli, A., Tarantola, S. and Chan, K. P. S., A quantitative model-independent method for global sensitivity analysis of model output, *Technometrics*, 41(1), 39-56, 1999.

Saltelli, A., Chan, K. and Scott, E. M., *Sensitivity Analysis*, John Wiley & Sons Ltd., West Sussex, 2000.

Sobol', I. M., Sensitivity Estivmates for Nonlinear Mathematical Models, *Mathematical Modelling & Computational Experiment*, 1, 407-414, 1993.