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Electromagnetic realization of topological states in one-dimensional arrays of bianisotropic particles

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Abstract. We propose a strategy to realize one-dimensional electromagnetic topologically protected states by modifying on-site properties of particles keeping linear equidistant geometry of the array. Based on the discrete dipole approximation, we demonstrate the existence of non-trivial topology of photonic bands and mapping to the Su-Schrieffer-Heeger model. We investigate the properties of an isolated ceramic disk to optimize its electromagnetic response, namely, the splitting of electric and magnetic dipole resonances due to bianisotropy. Using full-wave simulations we demonstrate the presence of the topological interface state in the microwave spectral range.

1. Introduction

The rapid development of nanophotonics requires effective ways of decreasing the scattering and dissipation of electromagnetic radiation. Topological photonics offers a solution to this problem by using topologically protected edge and interface states of light, which are robust to disorder in a system [1]. Initially, topological concepts were exploited to describe the properties of electronic systems, e.g. quantization of Hall conductance was connected to a topological invariant, which characterizes the global properties of electronic wavefunctions in a solid [2]. Further works showed that topological band structures are a ubiquitous property of waves propagating in a periodic medium, including electromagnetic waves [3].

The origin of electromagnetic edge and interface states is related to the non-trivial topology of photonic bands, which can usually be designed by proper tuning of the array geometry [1]. There were recent studies (e.g., [4, 5]), which exploited magneto-electric coupling (bianisotropy) to open the band gap and which considered the domain wall between the two arrays with different signs of bianisotropy. Such geometries were shown to give rise to well-celebrated Jackiw-Rebbi-type states [5]. Taking a step forward, we demonstrate that staggered bianisotropy can lead to the non-trivial topology of photonic bands. Furthermore, we develop a theoretical model which describes the origin of topological states. Our full-wave numerical simulations show that the magnitude of frequency splitting caused by bianisotropy is sufficient to observe edge and interface states in an experiment.

2. Theoretical model

To get some insight into the properties of the band structure, we first consider an infinite array depicted in figure 1.

As a building block of the array, we consider a disk with broken mirror symmetry with respect to the xy-plane [5]. Such configuration features a bianisotropic response to an external electromagnetic field, i.e.

$$\begin{aligned} \mathbf{d} &= \hat{\alpha}^{ee}\mathbf{E} + \hat{\alpha}^{em}\mathbf{H} \\ \mathbf{m} &= \hat{\alpha}^{me}\mathbf{E} + \hat{\alpha}^{mm}\mathbf{H}\text{'} \end{aligned} \tag{1}$$

where $\mathbf{E}$, $\mathbf{H}$ are the external electromagnetic fields, $\hat{\alpha}^{ij}$, $i,j = \overline{e,m}$ are the 3x3 polarizability tensors.

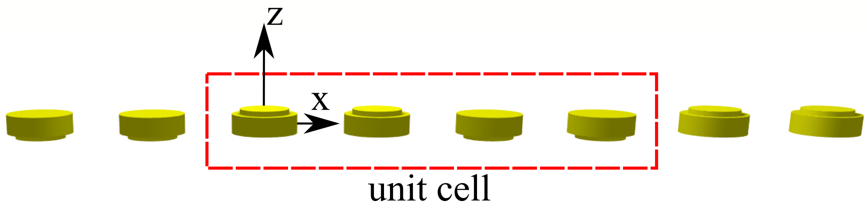

Figure 1. Array of bianisotropic particles (the distance between the neighboring disks $a = 5\ cm$)

Rotational symmetry of the disk with respect to the Oz axis and mirror symmetry in the Oxy plane impose some restrictions on the structure of polarizability tensors [4]. Besides that, we assume a pole approximation of the main values of $\hat{\alpha}^{ee}$ and $\hat{\alpha}^{mm}$ in the vicinity of electric and magnetic dipole resonances; resonant frequencies are supposed to be equal: $\omega_e = \omega_m = \omega_0$. Also, we neglect the z-oriented dipole contributions to the disk response in the considered spectral range.

Induced dipole moments can be expressed in terms of the dyadic Green's functions [6]:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \hat{G}^{ee}(\mathbf{r})\mathbf{d} + \hat{G}^{em}(\mathbf{r})\mathbf{m} \\ \mathbf{H}(\mathbf{r}) &= \hat{G}^{me}(\mathbf{r})\mathbf{d} + \hat{G}^{mm}(\mathbf{r})\mathbf{m} \end{aligned} \tag{2}$$

For simplicity, we neglect radiation losses exploiting the quasistatic form of Green's functions.

Based on the symmetry of the disk, we introduce the following form of the inverse polarizability tensor [7]:

$$\begin{bmatrix} \hat{\alpha}^{ee} & \hat{\alpha}^{em} \\ \hat{\alpha}^{me} & \hat{\alpha}^{mm} \end{bmatrix}^{-1} = \begin{bmatrix} u^{ee} & 0 & 0 & -iv_n \\ 0 & u^{ee} & iv_n & 0 \\ 0 & -iv_n & u^{mm} & 0 \\ iv_n & 0 & 0 & u^{mm} \end{bmatrix}, \tag{3}$$

where $u = (\omega - \omega_0)/A$, $v_n = \beta/(\alpha^2 - \beta^2)$ and n is the number of the disk.

As a result, we recover the following equations:

$$\begin{bmatrix} \hat{\alpha}^{ee} & \hat{\alpha}^{em} \\ \hat{\alpha}^{me} & \hat{\alpha}^{mm} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{d}_n \\ \mathbf{m}_n \end{bmatrix} = \sum_{l=n\pm1} \begin{bmatrix} \hat{G}^{ee}(\mathbf{r}_{nl}) & \hat{G}^{em}(\mathbf{r}_{nl}) \\ \hat{G}^{me}(\mathbf{r}_{nl}) & \hat{G}^{mm}(\mathbf{r}_{nl}) \end{bmatrix} \begin{bmatrix} \mathbf{d}_l \\ \mathbf{m}_l \end{bmatrix} \tag{4}$$

The system (4) can be recast in the form of the eigenvalue equation $\hat{H}\,|\psi\rangle = (\omega - \omega_0)\,|\psi\rangle$, where $|\psi\rangle = (\ldots\ \ d_{ix}\ \ m_{iy}\ \ \ldots\ \ d_{iy}\ \ m_{ix}\ \ \ldots)$, $i = 0, 1, 2, 3$ is a "state" vector with electric and magnetic dipole moment components and $\hat{H}$ is a 16x16 "Hamiltonian", which splits into two 8x8 blocks corresponding to the independent polarizations (d_x, m_y and d_y, m_x) with identical spectra. Denoting $2va^3 = \mu$, which describes the strength of the bianisotropic response of a disk, we obtained the dispersion of the considered array (the ranges of allowed and forbidden frequencies for a finite array are depicted in figure 2). The analysis of the spectrum allowed us to make some conclusions about the symmetries of our system. For example, the spectrum is symmetric with respect to the frequency of electric and magnetic dipole resonances of an isolated disk (we refer to this frequency as "zero energy"), which suggests chiral symmetry of the system.

Switching to the circularly polarized basis ($|\psi\rangle = (p_x \pm im_y, p_y \pm im_x)^T$), we can rewrite the initial equations (4) for one of the polarizations (e.g., p_x, m_y) in the following form:

$$\begin{bmatrix} \varepsilon - \mu_n & 0 \\ 0 & \varepsilon + \mu_n \end{bmatrix} \begin{bmatrix} p_x + im_y \\ p_x - im_y \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} p_x + im_y \\ p_x - im_y \end{bmatrix} \tag{5}$$

In the limiting case $\mu >> 1$, these equations are reduced to the two copies of the well-celebrated Su-Schrieffer-Heeger model - a conventional example of a topologically non-trivial one-dimensional system with the known value of the topological invariant [9]. In our case, the effective coupling constants of the corresponding Su-Schrieffer-Heeger array are $J_1 = 1$ and $J_2 = 3$.

3. Calculations for the finite chain

As an example, we consider an array of N = 241 disks and construct a 2N x 2N effective Hamiltonian. The results of calculations are shown in figure 2 for the fixed value of the bianisotropy parameter $\mu = 5$. Next, we identify the eigenfrequencies within the frequency gaps. The associated state is topologically protected. The corresponding electric dipole moment distribution shows the edge localization of this state.

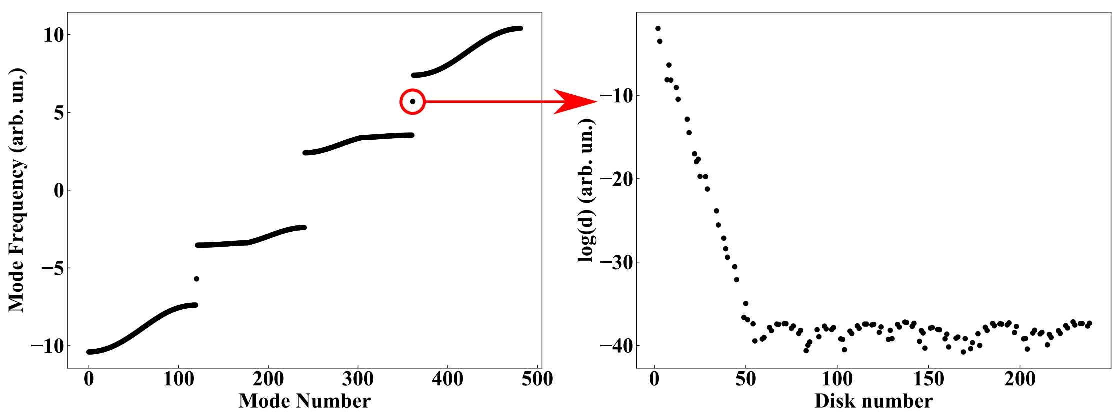

Figure 2. Results of eigenfrequency calculations and localization of the edge state ($\mu = 5$)

4. Numerical simulations

To estimate the real values of μ and the associated bandwidth, we considered a ceramic disk. Numerical simulation shows that the splitting of the dipole modes due to bianisotropy $\Delta f \approx 260$ MHz. The corresponding geometry and field distributions are shown in figure 3.

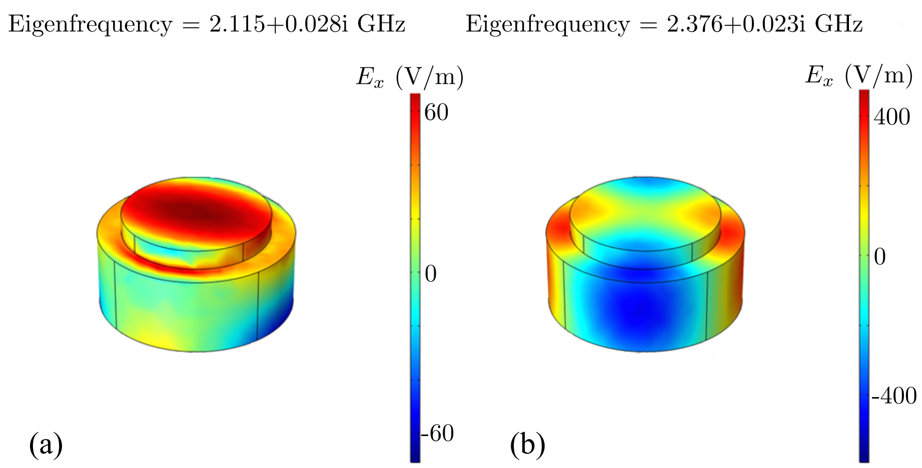

Figure 3. Hybridized dipole eigenmodes of a bianisotropic disk ($R_1 = 14.55\ mm$, $R_2 = 11\ mm$, $H = 11.6\ mm$, $h = 3\ mm$, $\varepsilon = 39$)

Based on this result, we estimated the value of μ for this particle: $\mu \propto \Delta f a^3 \approx 13$, which, as we saw, is enough for the observation of edge and interface states. Simulation of the domain wall shows the existence of an interface mode (figure 4).

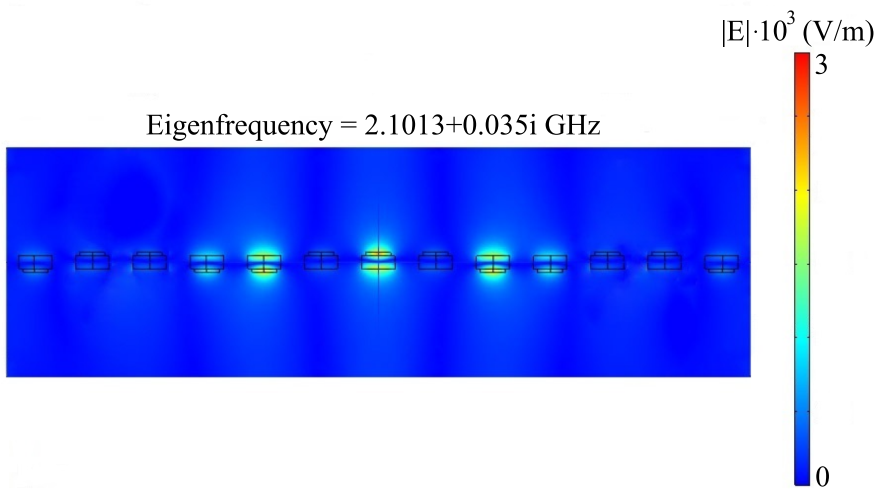

Figure 4. Interface state in the domain wall geometry ($N = 13$ disks)

5. Conclusion

We have proved that the realization of non-trivial photonic band topology and associated topological states is possible by proper tuning of the bianisotropic response of the particles with fixed linear equidistant geometry of the array. The existence of topological edge states was confirmed by studying the eigenmodes of the finite chain. Simulation of real disks showed that the splitting of dipole resonances due to bianisotropy is sufficient to observe edge and interface states, which were explicitly demonstrated by simulating domain wall geometry.

Acknowledgments
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References
[1] Ozawa T *et al.* 2019 *Rev. Mod. Phys.* **91** 015006
[2] Thouless D J, Kohmoto M, Nightingale M P and Nijs M 1982 *Phys. Rev. Lett.* **49** 405-8
[3] Haldane F D M and Raghu S 2008 *Phys. Rev. Lett.* **100** 013904
[4] Khanikaev A, Mousavi H, Wang-Kong T, Mehdi K, McDonald A and Shvets G 2013 *Nat. Mater.* **12** 233-9
[5] Gorlach A, Zhirihin D, Slobozhanyuk A, Khanikaev A and Gorlach M 2019 *Phys. Rev.* B **99** 205122
[6] Slobozhanyuk A, Poddubny A, Miroshnichenko A, Belov P and Kivshar Yu 2015 *Phys. Rev. Lett.* **114** 123901
[7] Novotny L and Hecht B 2012 *Principles of Nano-Optics* (Cambridge: Cambridge University Press)
[8] Slobozhanyuk A *et al.* 2017 *Nature Phot.* **11** 130-7
[9] Su W P, Schrieffer J R and Heeger A J 1979 *Phys. Rev. Lett.* **42** 1698-1701