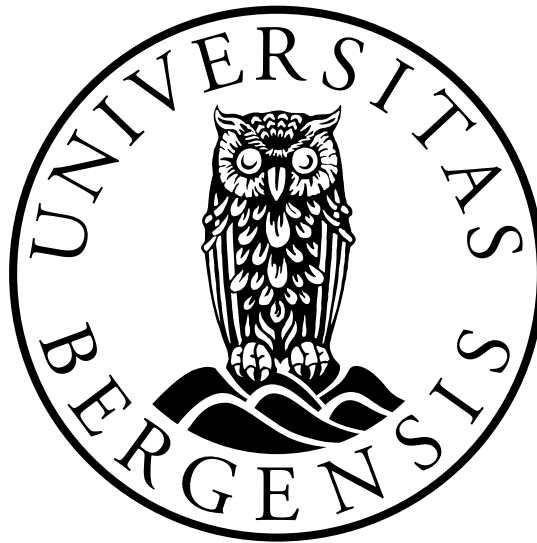


UNIVERSITY OF BERGEN



Geophysical Institute

MASTER'S THESIS

Optimizing maintenance plans of offshore wind farms by calculating the likelihood of future turbine failures

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Abstract

Although offshore wind power shows promising energy potentials, high cost of operating and maintaining offshore wind farms concerns investors. Different maintenance strategies are applied by wind farm operators to overcome this drawback. A mixed integer optimization model is developed to find the optimal maintenance plan for an offshore wind farm. The proposed model include probabilistic failure times, multiple components per wind turbine, route decisions and imperfect maintenance. That is, aspects usually studied individually in the literature. Maintenance actions are scheduled based on the calculated likelihood of future turbine failures. Results from numerical experiments show that applying an imperfect preventive maintenance strategy, as opposed to a preventive replacement strategy, is preferable in most scenarios. An additional heuristic algorithm is presented. Close to optimal solutions with optimality gaps between 1% and 3% prove that the heuristic algorithm yields good solutions.

Acknowledgment

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List of abbreviations

CM Corrective Maintenance

GW Giga Watt

LCOE Levelized Cost of Energy

MIQLP Mixed Integer Quadratic and Linear Program

MW Mega Watt

OM Operation and Maintenance

OWF Offshore Wind Farm

PDF Probability Distribution Function

PM Preventive Maintenance

SDG Sustainable Development Goals

Chapter 1

Introduction

Wind occurs as a result of the about 120000 terawatts (Jaffe and Taylor, 2018) from solar radiation, that is absorbed by the earth. Most of this energy is located at hard to reach areas, as in the upper atmosphere, or in deep water locations (Jaffe and Taylor, 2018). Consequently, there are great potentials (Bosch et al., 2018) in offshore wind. Nevertheless, high cost of operation and maintenance of Offshore Wind Farms (OWFs) challenge the development of the energy source (Stålhane et al., 2020; Tusar and Sarker, 2021). We stress the importance of making offshore wind an even more competitive energy source. We expect wind to take a significant part in reaching the 7th UN sustainability goal. That is, sustainable energy for all by 2030 (UN Summary of the Secretariat, 2021). This Master's Thesis belongs to the field of mathematical optimization. A model is developed to find the optimal maintenance plan at an OWF, in order to maximize expected net profit from turbine electricity production.

1.1 Problem statement

The failure of a turbine at an OWF results in running production losses from the time of failure to the time of repair. While loss of income favours a visit to the offshore wind farm to perform maintenance, this trip is demanding. For example, the maintenance requires costly vessel transport, a specialised crew, spare parts and expensive equipment (Ren et al., 2021; Van Bussel and Zaaiker, 2001). Travel distances are often long, and the weather conditions often challenging, making it difficult to access the OWF (Tusar and Sarker, 2021; Ren et al., 2021; Van Bussel and Zaaiker, 2001). It goes without saying that redundant trips must be avoided. Important decisions to be made include when to visit the offshore wind farm, which turbines and components need attending, and whether or not the components need to be fully replaced. There exist several mathematical models in the literature for optimizing maintenance schedules (Sarker and Faiz, 2016; Lu et al., 2018; Ding and Tian, 2012). How-

ever, to the best of our knowledge, few models present a solution where

1. the time of turbine breakdowns are not predefined,
2. both full replacement and minor maintenance can be performed,
3. multiple components are considered for each turbine, and
4. the decisions regarding maintenance *routes* are incorporated simultaneously.

It will become clear what is meant by a maintenance route in the following sections.

1.2 Research questions with hypothesis

In this Master's Thesis, we address the following research questions.

1. How can we maximize the net profit from turbine electricity production at an OWF?

We propose a mixed integer optimization model, with both quadratic and linear constraints. The model solution provides a schedule for maintenance and operations at an offshore wind farm. The trade-off between a possible gain in income from power production, and the expenses of repairing any broken turbines is captured. Some optimization models assume that information about future turbine failures are known a priori, but this assumption gives an unrealistic advantage (Gutierrez-Alcoba et al., 2019). As Gutierrez-Alcoba et al. (2019) address, such information is not known in advance (Gutierrez-Alcoba et al., 2019). We propose a model where no such information is given in advance. Maintenance decisions are instead based on the calculated likelihood of future turbine failures. We allow each turbine to consist of multiple components, for which we calculate individual probabilities of operation, based on their age. We use a Weibull distribution function to calculate the probability that each turbine is operating, hence producing power. Throughout the planning horizon, components are maintained to increase their probability of operation. Wind farm operators either fully replace a component, leaving it in perfect condition after the maintenance event, or perform a less demanding maintenance action, to increase the components probability of operation, without fully renewing it. Additionally, we assume that the wind farm operators have some predefined *routes* to chose from. In our model, each route has limitations regarding which turbines they visit, which components that can be maintained, and to which extent those components can be maintained with the route.

2. Can we solve the proposed optimization model for realistic wind farm sizes?

A good optimization model for scheduling maintenance at an OWF should be able to model existing and future wind farm sizes. We expect the solution space to grow significantly when the number of turbines, components and maintenance actions increase. It is reasonable that large instances of such a model will be demanding to solve to optimality. Existing models in literature show examples on cases where 10 turbines with four components (Ding and Tian, 2012), five turbines (Besnard et al., 2009), five turbines and four components (Tian et al., 2011), 1 turbine (Hao et al., 2020), 50 turbines with 4 components Sarker and Faiz (2016), 125 turbines (Gutierrez-Alcoba et al., 2019). We note that the number of turbines and components that are considered for each wind farm depend heavily on the model formulation.

3. Should OWF operators perform imperfect- or perfect preventive maintenance tasks?

By *imperfect preventive maintenance* we refer to maintenance actions that improve the operating state of a turbine to some extent, without fully renewing it (Ding and Tian, 2012). Similarly, we refer to *perfect maintenance* as maintenance where the operating state of a turbine is returned to a perfect operating condition after the maintenance event (Ding and Tian, 2012). We remark that the third research question is sensitive to the model instance. We expect its answer to vary with the relation between income from turbine production, failure rates, the expense of performing each maintenance action and the cost incurred by a maintenance route. As addressed by Sørensen (2009), cost models should be based on actual data from practise, and unfortunately such information can be very difficult to obtain. However, we generate different cost scenarios and perform numerical experiments to explore typical behaviour for each data instance. More specifically, we vary the cost of transportation, the cost of carrying extra spare parts/equipment, and the cost of performing imperfect maintenance in each scenario.

1.3 Thesis outline

We give motivation and background information on wind energy, its potentials, and its challenges in the first few chapters of this Master's Thesis. We start (Chapter 2) by emphasising the pressing need for competitive renewable energy sources, to reach the UN sustainability goals of both clean energy, and decent work for all by 2030. Thereafter (Chapter 3), we discuss wind power, and more specifically offshore wind power, as a significant part of the solution. We look at recent additions and ongoing projects in offshore wind, discuss challenges and advantages, and further argue for the high energy potentials in offshore wind. Moreover (Chapter 4), we consider different aspects of maintaining an offshore wind farm, as opera-

tion and maintenance account for a large proportion of the cost of offshore wind power. We mention typical failures at the different turbine components, and the maintenance actions and equipment needed to perform the required actions. Moreover, maintenance strategies and terminologies are introduced. We give (Chapter 5) an introduction to theory that is relevant for the optimization model. Further (Chapter 6), we present an overview of existing maintenance optimization models, and the current work is set side by side to existing literature. A substantial amount of the workload of this Master's Thesis is allocated to developing and implementing the proposed maintenance model (Chapter 7). We also explore the benefit of introducing a heuristic approach to minimize time of solving the model. The heuristic algorithm (Chapter 8) is meant as a starting point for further development. Several experiments are introduced (Chapter 9) to answer the research questions at hand. We present (Chapter 10) findings and results, and thereafter (Chapter 11) give the discussion, before (Chapter 12) we draw a conclusion.

Chapter 2

Tripling wind capacities by 2030

"We, the Heads of State and Government and High Representatives, meeting at United Nations Headquarters in New York from 25 to 27 September 2015 as the Organization celebrates its seventieth anniversary, have decided today on new global Sustainable Development Goals." (UN General Assembly, 2015).

So goes the declaration, introducing 17 Sustainable Development Goals (SDGs), adopted by the UN in 2015. The goals are made to favour humanity and the planet by balancing the economic, the social and the environmental prospects of sustainable development (UN General Assembly, 2015).

We find that the following of the SDGs (UN General Assembly, 2015) are especially relevant for the topic in this thesis,

SDG 7. *Ensure access to affordable, reliable, sustainable and modern energy for all,*

SDG 8. *Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all (UN General Assembly, 2015).*

The initial deadline of 15 years has decreased to 8 remaining years at the time of writing.

"We commit ourselves to working tirelessly for the full implementation of this Agenda by 2030." (UN General Assembly, 2015).

2.1 High Level Dialogue on Energy, 2021

On the 24th of September 2021, more than 130 Global leaders gathered with the intention of promoting the energy related goals of the 2030 Agenda for Sustainable Development. It is empathised that 760 million people worldwide still do not have access to electricity, and 2.6 billion are without access to clean cooking methods ([UN Summary of the Secretariat, 2021](#)). The expected outcome of the dialogue was a global plan accelerate the pursuing of the 7th SDG by 2030, henceforth the net zero emissions by 2050¹ and the 1.5 degrees goal of the Paris Agreement². Several commitments were made by participating parties. The summary report ([UN Summary of the Secretariat, 2021](#)), from the dialogue on Energy, stresses that SDG7 is within reach. To get there, UN Deputy Secretary-General Amina Mohammed points out **tripling solar and wind capacity by 2030**, as a priority for decarbonising the energy sector ([UN Summary of the Secretariat, 2021](#)). Another milestone stated to be achieved by 2030 is to reach 100 million jobs in the energy sector ([UN Summary of the Secretariat, 2021](#)).

2.2 Scaling up wind power production

There is clearly global motivation, and a pressing need to further develop and scale up renewable energy production. As pointed out by [UN Summary of the Secretariat \(2021\)](#), the energy sector still accounts for about 75 % of total greenhouse gas emissions. It is also made clear that the wind power industry has a significant role in order to pursue SDG7 ([UN Summary of the Secretariat, 2021](#)). Additionally, investments in renewable energy contribute to achieving SDG8 by creating new job opportunities.

We introduce wind power, especially offshore wind power, in the following chapter. We focus on its potential, along with its challenges. Advantages and disadvantages with offshore and onshore wind power are discussed. We argue that lowering expenses related to offshore wind, is necessary to accelerate wind power production, and further contribute to the 2030 agenda, net-zero 2050 and the 1.5 degree goal in The Paris Agreement.

¹Bring carbon dioxide emissions related to energy to net zero by 2050 ([IEA, 2021a](#))

²The legally binding agreement between 196 parties that aims on limiting effects of global warming to 1.5 degrees, compared with pre-industrial temperature levels. ([UNFCCC, 2022](#))

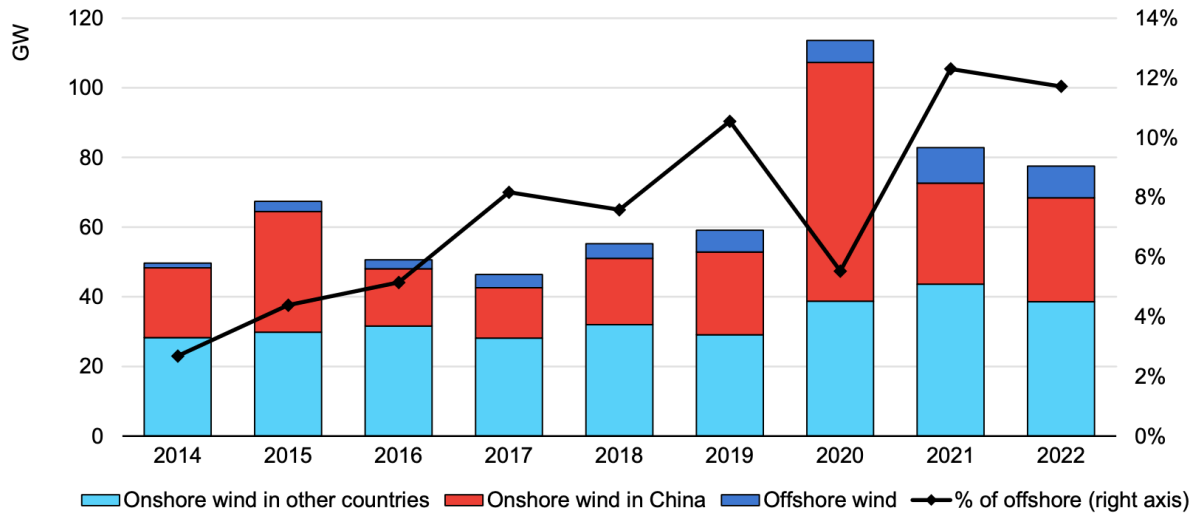
Chapter 3

Wind energy background

We begin (Section 3.1) the current chapter by presenting wind capacity additions in recent times, and some ongoing projects are mentioned. We observe that onshore wind additions still surpass offshore additions, and discuss (Section 3.2) challenges and advantages with offshore wind energy. Lastly (Section 3.3), we motivate the development in offshore wind, and a paper looking into the global power potential of offshore wind is commented on.

3.1 Wind capacity additions in recent times

Wind energy additions are rapidly increasing. The report by IEA (2021b) finds wind power to be the fastest growing source of renewable electricity generation in 2020. Almost 114 Giga Watts (GW) of new wind capacity was added globally, with China alone accounting for two-thirds of the additions in 2020 (IEA, 2021c). Further, the report (IEA, 2021c) from EIA forecasts that onshore wind capacity accelerates even further in Europe 2021 and 2022 due to large additions in France, Sweden and the Netherlands. Figure 3.1 shows the annual wind capacity addition given in Giga Watts, from 2014 to 2022. We observe that onshore additions are constantly greater than offshore additions. Nevertheless, the proportion of offshore additions is also increasing with time.



IEA. All rights reserved.

Figure 3.1: The graph (IEA, 2021c) shows wind capacity additions from 2014 to 2022.

Despite onshore additions still dominating globally, there are several finished and planned offshore projects globally. The offshore wind farm "Hornesea One" in UK, has a total capacity of 1.2 GW. The wind farm has a total of 174 turbines distributed over an area of 407km². Each turbine has a capacity of 7 Mega Watts (MW), and the final turbine was installed in 2019 (Ørsted, 2022). Moreover, "Horns Rev 3" located in Denmark, and "Kriegers Flak" located in the Baltic Sea between Denmark and Germany, are offshore wind farms with electricity capacities of 407 MW electric and 604 MW electric, respectively (Vattenfall, 2022a,b). "Kriegers Flak" was finished in 2021, while "Horns Rev 3" was officially opened in 2019 according to Vattenfalls company websites (Vattenfall, 2022a,b). Further, the areas (see Figure 3.2) named "Utsira Nord" and "Sørlige Nordsjø II" are planned for planned offshore wind farms at the Norwegian coastal line (Ministry of Petroleum and Energy, 2020). As shown in Section 3.3, the Norwegian coastline has great potential for offshore wind. The project Utsira Nord is planned to be of floating wind technology. The area spans over 1010km² (Ministry of Petroleum and Energy, 2020). Sørlige Nordsjø II spans 2591km², and the shallower water depths enable bottom-fixed turbines (Ministry of Petroleum and Energy, 2020).

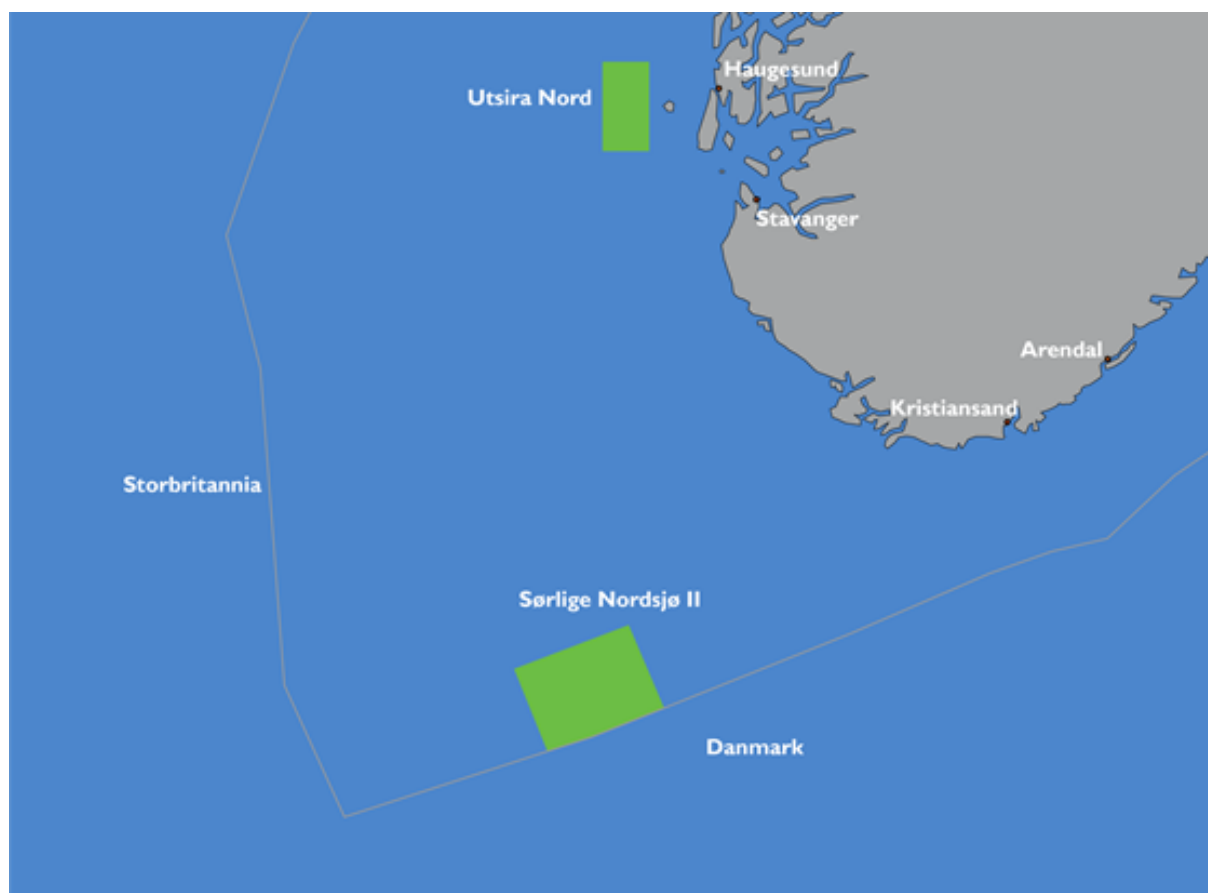


Figure 3.2: The figure (Ministry of Petroleum and Energy, 2020) shows areas along the Norwegian coastal line that are planned for offshore wind technologies.

3.2 Challenges and advantages with offshore wind

Although the difference has decreased over the years, high levelized cost of energy (LCOE) compared with onshore installations keeps investors from investing in offshore wind (Tusar and Sarker, 2021; Ren et al., 2021). Moreover, Ren et al. (2021) state the cost of OM constitutes a significant share of the LCOE of an offshore wind farm, hence lowering this cost is an effective approach to control LCOE. The maintenance actions required offshore are generally more expensive than onshore. For example,

1. specialised and costly maintenance equipment is needed,
2. harsher weather conditions result in higher failure rates for the installed turbines, and
3. high production losses occur when maintenance is postponed due to weather conditions (Ren et al., 2021).

Looking past the economical challenges, there are several reasons (Tusar and Sarker, 2021; Ren et al., 2021; Bosch et al., 2018) why offshore wind is growing, and is expected to continue

to grow in the near future. The areas used for offshore wind are often far from civilisation. As a result, they impose less visual impact and noise pollution (Tusar and Sarker, 2021; Ren et al., 2021). That is, bigger turbines can be installed, generating more power than smaller turbines (Tusar and Sarker, 2021). Moreover, the areas are often unused coastal lines that do not compete with any other usages (Tusar and Sarker, 2021; Ren et al., 2021; Bosch et al., 2018). Additionally, as complex operations are needed to install, operate and maintain the OWFs, offshore wind creates new jobs (Tusar and Sarker, 2021). Finally, deeper sea wind resources are often greater than onshore, again leading to higher power generation (Tusar and Sarker, 2021; Ren et al., 2021).

3.3 Global offshore power potential

We look at the global potential in offshore wind to motivate further development in the field. In their paper, Bosch et al. (2018) present a method for estimating the global energy potential from offshore wind power. They assume offshore wind power is built where possible. The distance to a grid connection, water depth and developed *capacity factors* are constraints used to determine areas where offshore wind is an option. Capacity factor is commonly defined as the ratio between actual power output and the maximum power output of the perfectly operating system (Jaffe and Taylor, 2018). In their results shown in Figure 3.3, Bosch et al. (2018) present the electricity generation potential from offshore wind in different countries.

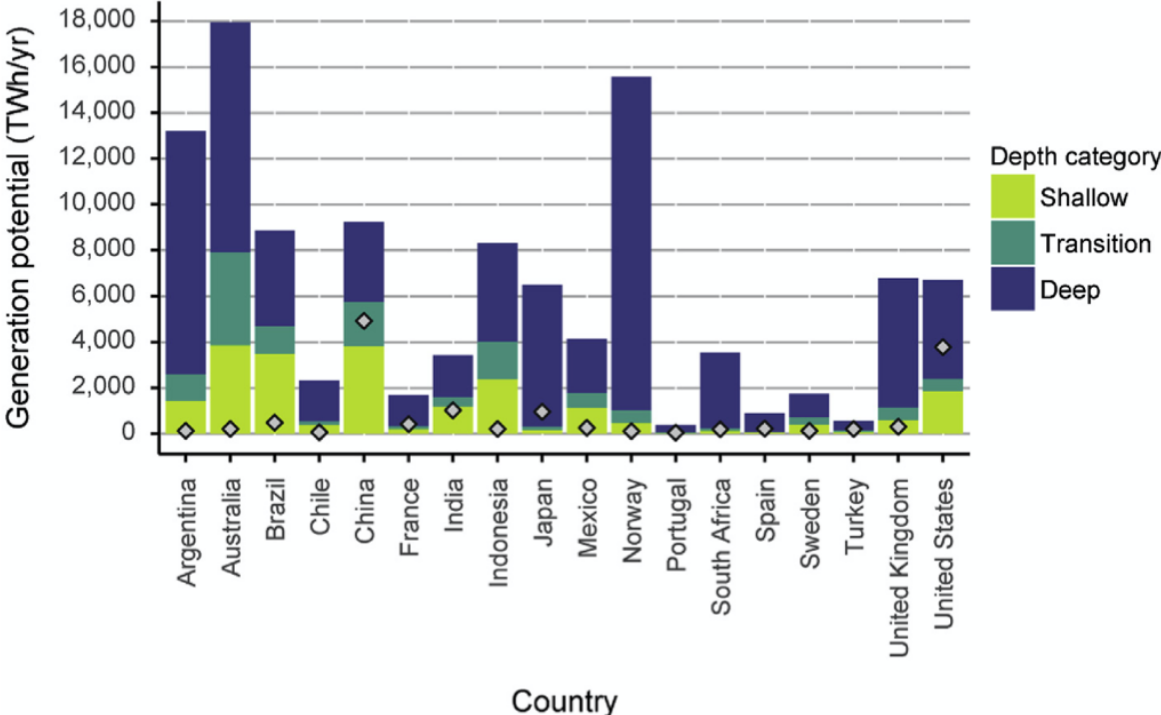


Figure 3.3: Bar graph (Bosch et al., 2018) of annual average energy production potential of offshore wind. *Shallow* water dept is defined as 0–40m, *transition* is defined as 40–60m and *deep* water depth is defined as 60–1000m. The authors include the total electricity demand of each country from 2015 as a point on each bar, for comparison (Bosch et al., 2018).

The study finds a great potential in offshore wind. It is particularly high in deep water locations of the Norwegian coast. The potential of offshore wind in Norway is estimated to be roughly 30 times the primary energy consumption in Norway in 2019, which was about 500TWh (Bosch et al., 2018; Our World in Data, 2022).

Chapter 4

Maintenance of offshore wind farms

We mention (Section 4.1) typical wind turbine components, with corresponding failures in the current chapter. Different maintenance policies are applied by OWF operators to reduce the expense of OM. We therefore introduce (Section 4.2) necessary terminology and common maintenance policies.

4.1 Components and required maintenance

There exist different types of turbines, where horizontal-axis are the most common ([Jaffe and Taylor, 2018](#)). We refer to horizontal-axis wind turbines in the following sections. Figure 4.1 illustrates a typical wind turbine. Typical components include the rotor blades, gearbox, anemometer, generator, yaw motor, control system, and the foundation ([Yu, 2021](#)).

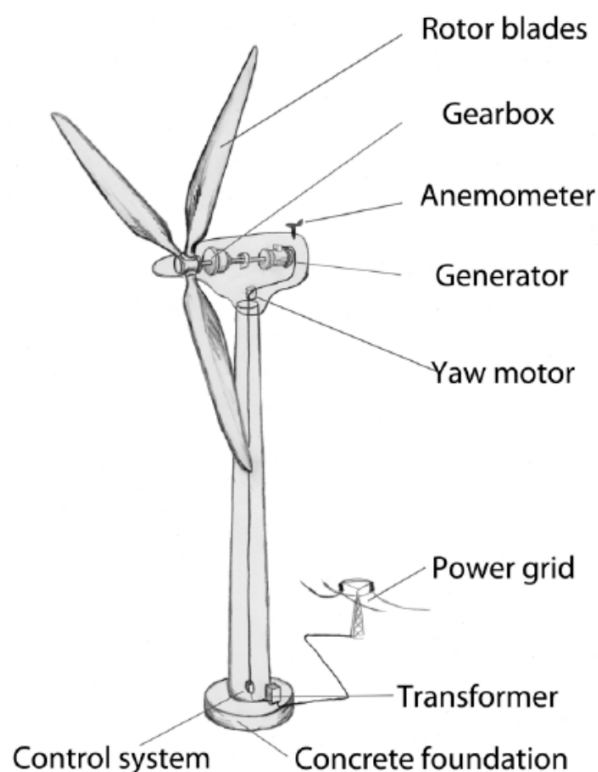


Figure 4.1: Illustration (Yu, 2021) of a typical horizontal-axis wind turbine.

We briefly discuss the role of the component and typical failures that occur at the rotor, the gearbox and the generator.

Rotor (and blades)

There are usually three blades attached to the rotor of horizontal-axis turbines. The kinetic energy is transformed to mechanical power when wind causes the blades, and therefore the rotor, to rotate. Deterioration, adjustment error, rotor imbalance, corrosion and cracks are typical failures observed at the rotor and the blades (Ren et al., 2021).

Gearbox

The function of the gearbox is to increase the rotational speed before the generator (Ren et al., 2021; Yu, 2021). Failures can occur due to wearing, gear tooth damage, oil leakage, too high oil temperature and poor lubrication (Ren et al., 2021).

Generator

The generator converts mechanical power to electrical power. Common failures are due to overheating, wearing, excessive vibration and winding damage (Ren et al., 2021).

In addition, Lu et al. (2018) emphasise the *pitch system*. The system is attached to the rotor blades, and can adjust the angle of the blades, to control the output power from turbines (Lu et al., 2018). The gearbox, the rotor, the generator and the pitch system are components that especially contribute to high maintenance cost (Lu et al., 2018). These four components are

critical as they either fail more frequently (pitch system, generator), or require a lot of time or costly equipment to repair (rotor, generator) (Lu et al., 2018). In their paper, Van Bussel and Zaaier (2001) explain that the frequent need for an expensive external crane vessel is the main cause for high OM costs for offshore wind farms. They refer to the day-rate cost of an general purpose offshore lifting equipment being at least 10 times the equivalent on-shore equipment. A *lifting operation*, performed with a crane, is required whenever a major component needs replacing or maintenance. Examples of such major components are the gearbox, the blades and the generators (Ren et al., 2021). Figure 4.2 shows an offshore lifting operation.



Figure 4.2: The figure (Shaun Campbell, 2014) shows a lifting operation

4.2 Maintenance policies and terminology

We find some inconsistency in the classification of the maintenance policies (Ren et al., 2021; Rausand and Høyland, 2004; Lu et al., 2018). One classification is introduced by Ren et al. (2021). They define, *proactive maintenance* as all planned maintenance actions performed at a working turbine to prevent future failures. This is opposed to *corrective maintenance*, which is applied after a failure occurs. Further, *preventive maintenance*, *condition-based maintenance* and *predictive maintenance* are defined as subcategories of the term proactive maintenance. A slightly different categorisation is presented by Rausand and Høyland (2004), where *preventive maintenance* signifies all maintenance actions performed to pre-

vent future failures. *Time-based preventive maintenance*, *opportunistic maintenance* and *predictive maintenance* are in their turn presented as subcategories of the term preventive maintenance.

Despite some variation in the terminology, there is a broad agreement (Tusar and Sarker, 2021; Ren et al., 2021; Rausand and Høyland, 2004) that the following two policies in maintenance categories must be considered. Firstly,

1. planned maintenance, performed on a functioning turbine to prevent future failures, and secondly
2. unplanned corrective maintenance, performed at a turbine after a failure occurs.

Moreover, some policies consider a combination of the two, as applies for *opportunistic maintenance*. The latter policy takes advantage of the opportunities that arise when a visit to the OWF is required. An opportunistic maintenance policy favours that additional preventive maintenance of functioning turbines are performed when the failure of a turbine enforce a visit to the OWF (Rausand and Høyland, 2004).

To avoid confusion, we use the categorisation described in Figure 4.3 for maintenance terminology in the current work.

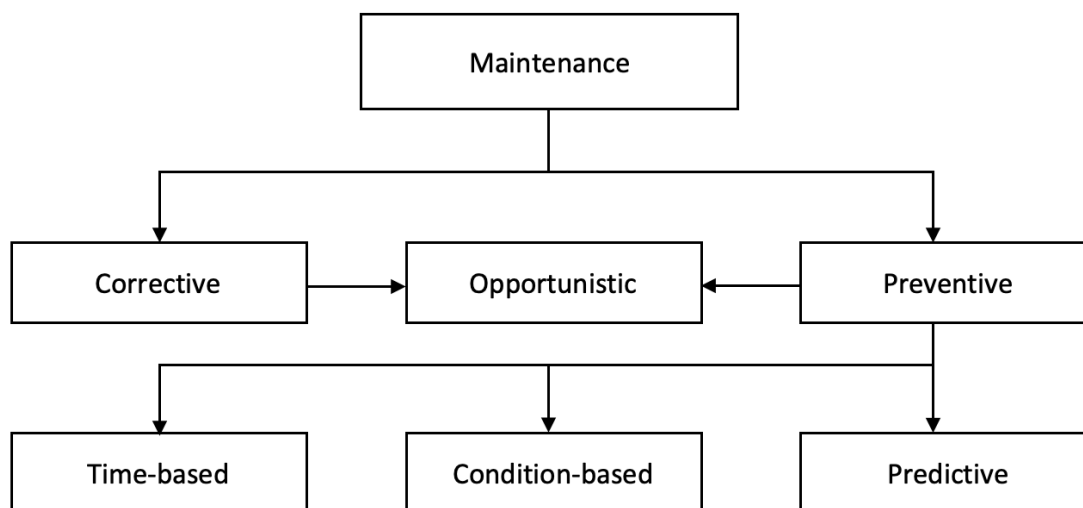


Figure 4.3: The chart describes the chosen maintenance categorisation, modified based on the chart by Ren et al. (2021).

Under the term maintenance we introduce planned *preventive maintenance* (PM), performed to avoid future failures, and *corrective maintenance* (CM) performed to correct a failure. The two policies meet in *opportunistic maintenance*. Moreover, we introduce *time-based maintenance*, *condition-based maintenance* and *predictive maintenance* as examples of preventive maintenance. A more detailed definition of each term follows.

4.2.1 Corrective maintenance

Corrective maintenance is defined by [IEEE \(2000\)](#), as

The maintenance carried out after a failure has occurred and intended to restore an item to a state in which it can perform its required function.

As breakdown events are often unforeseen, the corresponding maintenance tasks is scheduled subsequently. Such events have a tendency to be particularly costly. The OWF operators experience running production losses from breakdown to repair. In addition, the cost of spare parts, the cost of equipment needed, and the transportation costs are incurred.

4.2.2 Preventive maintenance

At an OWF, a breakdown event of a major component can account for several months of downtime ([Yu, 2021](#)). As stressed by [Tusar and Sarker \(2021\)](#), finding the right balance between preventive and corrective actions are of crucial importance. It is of interest to decrease the probability of a breakdown event, by applying some type of preventive maintenance policy. Preventive maintenance is performed at an operating turbine or component with the goal of preventing future failures.

The following two examples of time-based preventive policies are based on the book by [Rausand and Høyland \(2004\)](#). It is important to be especially aware of how time is defined with the following policies. Time could be measured as the conventional calendar-time, or in terms of operation time, or a completely different unit, like units of power produced.

The first preventive policy we mention is *Age replacement* ([Rausand and Høyland, 2004](#)), where a component is replaced at a specified operational age, or in an event of failure. This means that, the system is either replaced after a specific time period has elapsed since last renewal, or it fails before the planned renewal time, and thus is replaced due to failure. The policy requires that the operating age of each component is monitored, and maintenance will be required at different times for different components.

The *Block replacement policy* ([Rausand and Høyland, 2004](#)) is based on calendar time intervals, regardless of the component's age. The advantage of such an approach is that there is no need to monitor age, as preventive maintenance is done simultaneously at all components. Nevertheless, the approach is often wasteful. The events where components fail before the scheduled maintenance action, and therefore are correctively replaced in-between of calendar intervals, will result in replacement of brand new components at the next scheduled replacement time.

Figure 4.4 illustrates age replacement (1), and block replacement (2). Performed corrective maintenance is marked by CM, and performed preventive maintenance is marked by PM on the timelines. PM tasks are scheduled periodically, with period length T . In both cases, CM is required before the scheduled PM task. For the age based approach, the component's age is set to zero after the event, and the next PM task is scheduled after an additional period T . In contrast, the block approach carries out the PM task as planned, shortly after the CM task. The latter results in a replacement of an almost new component.

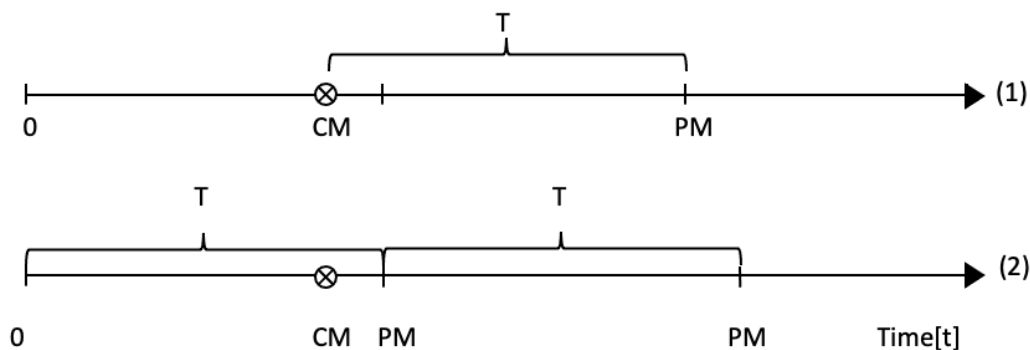


Figure 4.4: The illustration is modified based on the figure and explanation by [Rausand and Høyland \(2004\)](#). The upper timeline (1) illustrates age replacement, while the lower timeline (2) illustrates block replacement. Performed corrective and preventive tasks are marked by CM and PM, respectively. PM tasks are planned for intervals of lengths T .

Condition-Based maintenance

For condition-based maintenance policies, maintenance is decided based on the measured data describing the *health* of the component ([Rausand and Høyland, 2004](#); [Yu, 2021](#)). By health, we refer to the performance of the component, or the degree of degradation. Different parameters can be used to describe the health of a component. The measured parameters can be, either

- physical measurements, such as temperature and pressure, or
- measurement of performance and the quality of products ([Rausand and Høyland, 2004](#)).

Using data from sensors to perform condition-based maintenance may decrease total maintenance cost, by avoiding unnecessary trips to each turbine. The approach can also prevent minor faults from evolving into larger breakdowns. However, the additional cost of equipment and false alarms will affect the performance of this policy ([Ren et al., 2021](#)).

Predictive maintenance

Combining measurement data and parameter analysis to predict the remaining lifetime of a component, for then to schedule maintenance before the predicted breakdown, is referred to as a *predictive maintenance policy* by [Ren et al. \(2021\)](#). The policy is closely related to condition-based maintenance, as the predictions may depend on measured data. It is for this reason sometimes listed as a type of condition-based maintenance ([Rausand and Høyland, 2004](#)).

4.2.3 Opportunistic Maintenance

When a scheduled maintenance task needs attending at a wind turbine, this opens up an opportunity for further unscheduled maintenance during the same visit or time period. The method combines preventive and corrective maintenance actions, and can be very beneficial in offshore wind, due to large setup costs. Such a policy has the potential to lower costs by reducing the number of visits to each turbine, and is called *opportunistic scheduling* (OM) in the literature ([Rausand and Høyland, 2004](#)).

Chapter 5

Introduction to optimization and the Weibull distribution function

In this chapter, we introduce the theory that is considered relevant for the optimization model that we propose in this Master's Thesis. Mathematical optimization models are a widely used approach to solve large-scale problems. In their book, [Korte and Vygen \(2018\)](#) state that thousands of problems from real life can be re-written into *combinatorial optimization* problems. One definition of combinatorial optimization is given by [SINTEF \(2022\)](#):

"Within the field of mathematical optimization, combinatorial optimization represents a sub topic with several techniques for finding the optimal solution from a finite (and huge) set of discrete candidate solutions (SINTEF, 2022)."

We argue that constructing a combinatorial optimization model is a good approach to optimize OM of an offshore wind farm. We see in [Table 6.1](#) that several papers apply optimization models for this purpose.

We briefly introduce ([Section 5.1](#)) different approaches that may be used to solve combinatorial optimization problems. In particular, *Linear Programming* (LP), *Integer Programming* (IP), *Mixed Integer Programming* (MIP) and *Meta-Heuristics/Heuristics* are mentioned. Moreover ([Section 5.2](#)), we introduce the Weibull distribution function. The distribution plays an important role in our optimization model. In which, the distribution is used to calculate the probability that a component of a certain age is operating.

5.1 Solving combinatorial optimization problems

We mention a few methods that can be applied to solve combinatorial optimization problems.

Linear programming

Ever since the term *Linear Program* (LP) was introduced in the 1950s, the method has been applied for planning purposes within a wide field range (Matousek and Gartner, 2007). We mention planning of work schedules, transportation of equipment or planning of maintenance activities. In the book by Matousek and Gartner (2007), "planning with linear constraints" is suggested as a more describing phrase to capture the concept of linear programming. We refer the reader to the books by Matousek and Gartner (2007) and Vanderbei (2020) for a thorough introduction to linear programming. The following brief description of a LP is based in the book by Vanderbei (2020). To obtain the objective value Ψ we maximize some linear objective function,

$$\Psi = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n,$$

by assigning best possible values to *decision variables*

$$x_i, \forall i \in 1, 2, \dots, n,$$

subject to a set of linear *constraints*,

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leq b_2, \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq b_m, \\ x_i &\geq 0 \quad \forall i \in \{1, \dots, n\} \text{ Vanderbei (2020)}. \end{aligned}$$

Constraints define the *feasible region*. A solution which satisfies all constraints is called a *feasible solution* (Vanderbei, 2020). Further, the feasible solution is *optimal* if the corresponding objective value is the maximum (Vanderbei, 2020).

Integer programming

The variables are often constrained to integrality in practical problems. This is the case for problems requiring scheduling of routes and hiring of personnel (Matousek and Gartner, 2007). We refer to such a problem as an Integer Linear Program, or *Integer Program* (IP) for short.

Mixed integer programming

A linear program, in which only a subset of the variables are constrained to be integers, is referred to as a *Mixed Integer Program* (MIP) (Korte and Vygen, 2018). Although MIPs are considered hard to solve, the speed up from both solvers and of computers makes it possible to solve a MIP that would have needed 16 days in 1991, within a second today (Bertsimas, 2020).

Quadratic constraints

However, it is not unexpected to encounter non-linearities when setting up a problem instance for the optimization problem at hand. When the constraints follow quadratic functions, we refer to them as *quadratic constraints*.

Heuristics and meta-heuristics

Heuristics and meta-heuristics are possibly non-deterministic approaches that can be applied to find near to optimal solutions. In the book by Gendreau and Potvin (2018), meta-heuristics are defined as

"solution methods that orchestrate an interaction between local improvement procedures and higher level strategies to create a process capable of escaping from local optima and performing a robust search of a solution space (Gendreau and Potvin, 2018)."

Improvement in a heuristic search In a heuristic search, one typically starts with a feasible initial solution. Given the corresponding objective value Ψ_{initial} , we define

$$imp = \frac{\Psi^* - \Psi_{\text{initial}}}{\Psi^*}, \quad (5.1)$$

as the fraction denoting the *improvement* from the initial objective, to the new obtained objective value, Ψ^* .

Evaluating a feasible solution

A objective value, Ψ , from a feasible solution, gives a *lower bound* to the the optimal objective value (Vanderbei, 2020). We wish to determine how far this solution is from the optimal solution (Vanderbei, 2020). If we can obtain a *upper bound*, $\Psi_{\text{upper bound}}$, for the objective value, then we can determine the gap in which the optimal solution can be found (Vanderbei, 2020). If the solver, Gurobi Optimization, LLC (2022), finds an upper bound to the optimization problem, the fraction

$$gap = \frac{\Psi_{\text{upper bound}} - \Psi^*}{\Psi^*}, \quad (5.2)$$

is calculated, telling us how far we at most are from the optimal solution.

5.2 Probabilistic failures with Weibull distribution

In the following section, we introduce the Weibull distribution, and its role in failure prediction and maintenance scheduling.

5.2.1 Usage

The Weibull distribution is widely used to model life length distributions for systems and components. Its flexible nature makes it possible to model different types of failure rates, solely by choosing the right parameter values (Rausand and Høyland, 2004). The distribution function comes with different notations in the literature. When applied to the likelihood of breakdowns, the *scaling* parameter η reflects the likelihood of breakdowns caused by random events, while the *shaping* parameter β expresses the modelled component's sensitivity to wear and tear. The component's age is denoted x .

5.2.2 Cumulative distribution and reliability function

The two following equations, with derivations, are found in the work of McCool (2012). The two factor cumulative Weibull distribution function is written as

$$F(L \leq x) = 1 - \exp \left[- \left(\frac{x}{\eta} \right)^\beta \right], x > 0,$$

where $F(L \leq x)$ is the probability that the life length L is less than or equal to a positive value x . That is, the probability that the component fails before time x is reached. Moreover, we are often interested in the probability of the converse, i.e., that a component is still operating in time x . The reliability function, often referred to as the *survivorship* (McCool, 2012) function, is written as,

$$P(L > x) = \exp \left[- \left(\frac{x}{\eta} \right)^\beta \right], x > 0, \quad (5.3)$$

where $P(L > x)$ denotes the probability that the life length L of the component exceeds a given value x .

Chapter 6

Review of existing models for maintenance scheduling at OWFs

This chapter provides a literature review on existing models for optimizing maintenance and operations at OWFs. We apply the terminology that is defined in the previous chapter. There are too many models on the subject to include all. Therefore, we introduce a selection of maintenance models in this chapter. As a group, the models cover aspects that are especially relevant for the model we propose in Chapter 7. More specifically, this applies to

- probabilistic failure-times,
- multiple components per wind turbine,
- imperfect preventive maintenance, and
- decisions regarding maintenance routes.

For further reading, we refer to the review by [Tusar and Sarker \(2021\)](#), a study of 190 papers related to maintenance at OWFs.

We start (Section 6.1) by shortly introducing a selection of optimization models that aim to minimize the cost of maintenance for OWFs. Further (Section 6.2), we mention a few models that address the choice of vessels and routes needed to perform maintenance at an OWF. Moreover (Section 6.3), we shortly introduce our contribution to the literature with this Master's Thesis. Finally (Section 6.4), we compare the reviewed work, and the current work of this Master's Thesis.

6.1 Models optimizing maintenance scheduling

An integer linear optimization model is presented by [Besnard et al. \(2009\)](#). The model determines the optimal plan of preventive and corrective maintenance tasks at an offshore wind farm for the current work day. The decisions are based on information about required corrective maintenance for that day, and forecasts of power productions. That is, information that the authors assume is known. Moreover, the model assumes that a set of preventive maintenance tasks has to be performed by the end of the planning horizon.

A model by [Tian et al. \(2011\)](#) schedules condition-based maintenance at a wind farm. The authors schedule maintenance actions based on the calculated probability that components and turbines are operating. The solution gives the wind farm operators information on whether to send a team to the wind farm, and which components and turbines that should be maintained. The authors include economical dependencies favouring additional maintenance when a turbine is stopped due to maintenance. Hence, we argue that the model apply opportunistic maintenance.

Moreover, [Besnard et al. \(2011\)](#) present a stochastic program. Every day, present knowledge of corrective maintenance that needs attending, as well as a forecast of power production, is used to decide on additional maintenance tasks to be performed during the same shift. The program solution is an opportunistic maintenance plan, where preventive maintenance is performed at low power production levels, and if corrective maintenance is required.

The maintenance simulation model by [Ding and Tian \(2012\)](#), minimize expected maintenance costs. The gearbox, the generator, the rotor and the main bearing are the considered components for each turbine at the wind farm. The decision of which components to maintain throughout the simulation is based on whether or not a component's age, at a considered simulation step, exceeds an age threshold. The age threshold is different for components belonging to a turbine where a failure is present, and for the components in fully operating turbines. The objective of the simulation is to find optimal values for the age thresholds, that minimize the expected cost of maintenance for the wind farm. The authors apply both perfect replacement, and imperfect maintenance.

Another work by [Lu et al. \(2018\)](#), combine concepts from predictive, condition-based and opportunistic maintenance. The authors assume continuous condition monitoring of four main components at each wind turbine. Moreover, the model apply a perfect maintenance policy. The authors calculate the probabilities that components and turbines operate, with an artificial neural network. The model schedule maintenance actions based on those probabilities, and aim to minimize long-term costs.

In their paper, [Sarker and Faiz \(2016\)](#) introduce an opportunistic maintenance model for off-

shore wind turbines. The model include both perfect and imperfect preventive maintenance for multiple components. In this model, corrective maintenance creates opportunities to perform preventive maintenance.

Moreover, [Zhu et al. \(2017\)](#) presents a simulation method. The authors do not specify whether the method can apply for offshore wind farms. Nevertheless, we observe that the model apply similar maintenance concepts as the previously mentioned models. The model monitor one critical component's condition continuously, while the rest of the components are not monitored. Instead, the authors assume that the rest of the components follow a pre-defined maintenance schedule. Such a schedule can be a time-based preventive maintenance, in addition to corrective maintenance where needed. Scheduled maintenance events and unscheduled breakdowns for the non-monitored components, create opportunities to perform additional preventive maintenance work at the critical component.

A binary linear optimization model, for deciding the next preventive maintenance activity at an OWF makes the first paper of the doctoral dissertation by [Yu \(2021\)](#). The turbines at the OWF consist of multiple components. Each component has a random life length according to a Weibull distribution. The solution to this model specify the next component to maintain, as well as the optimal time to maintain this component. The model schedule the next preventive maintenance task based on the expected benefit of performing the task, relative to a purely corrective maintenance plan.

6.2 Models that consider route decisions

The solutions from the previously mentioned optimization models supply wind farm operators with decisions regarding the need of maintenance at the OWE. However, the models have in common that they ignore route considerations. We refer to route considerations to decisions regarding the travel to the wind farm. For example, there can be limitations on available vessels or technicians. *Fleet composition* is a term that appear in the literature. It refers to the selection of vessels needed to perform maintenance of an OWF ([Gutierrez-Alcoba et al., 2019](#)). The following models address the fleet composition and route decisions that is required to perform maintenance at an OWF ([Gutierrez-Alcoba et al., 2019](#); [Stålhane et al., 2020](#); [Stålhane et al., 2015](#)).

A paper by [Stålhane et al. \(2015\)](#) present two optimization models. Both aim to find the optimal maintenance route and schedules for a fleet of vessels that perform maintenance at an offshore wind farm. We refer to the first model, formulated as a MIP, in [Table 6.1](#). The authors use a heuristic approach for the second model, to find solutions that are close to optimal with significantly less computing time.

To find an optimal fleet composition and maintenance schedule for an offshore wind farm, [Gutierrez-Alcoba et al. \(2019\)](#) present a MIP. The authors address the drawback of making decisions based on predefined weather conditions and breakdown scenarios. To overcome this limitation, the authors present an additional model with a heuristic approach. The latter model schedule maintenance solely based on available information.

Finally, [Stålhane et al. \(2020\)](#) give a mathematical formulation to find the optimal vessel fleet composition to support the required maintenance of an offshore wind farm. The model assumes that the offshore wind farms apply a time-based preventive maintenance strategy, and that corrective maintenance is scheduled when a failure occurs.

The three latter references have in common that required maintenance is assumed to be known a priori. According to [Gutierrez-Alcoba et al. \(2019\)](#), this assumption could result in an underestimate of maintenance cost given the unrealistic advantage of known failure times. Nevertheless, the models give valuable information to the wind farm operators about required vessels and optimal routes. That is, decisions not addressed by the previous authors.

6.3 Introducing the current work

In this Master's Thesis, we introduce a mixed integer program with both quadratic and linear constraints to find an optimal maintenance schedule at an OWE. We refer to the model as a Mixed Integer Quadratic and Linear Program (MIQLP) in the following sections. We base maintenance actions on the calculated probabilities of future failures. Hence, we avoid the drawback of using pre-defined failure times. Each turbine consist of multiple components. Moreover, we allow for each component to be either perfectly, or imperfectly maintained. Additionally, we include route considerations in the maintenance scheduling model. We assume that the wind farm operators have some pre-defined routes to choose from. Each route has limitations on which turbines they visit, which components they can maintain, and to what extent the route can restore a component. A maintenance action requires a route that visits the turbine in question, and that has the capability to maintain the target component to the required extent. The latter maintenance model is thoroughly introduced in Chapter 7.

6.4 Comparing the reviewed models

We compare the reviewed models, in addition to the current work, in Table 6.1. In this table, we emphasise the model type used to find a problem solution. Further, we state what we find to be the author's main choice of maintenance policy, although the models are often based on a combination of several concepts. Moreover, we specify whether or not the authors allow for probabilistic failures, multiple components, imperfect maintenance, and consideration of maintenance routes. In the column concerning probabilistic failures, all works that somehow include uncertain failure times are checked, whereas those that rely on a predefined set of failure times are left unchecked.

Authors	Model type	Strategy	Probabilistic failures	Multiple components	Imperfect maintenance	Route decisions
Besnard et al. (2009)	LP	Opportunistic	✓	✓	-	-
Besnard et al. (2011)	MIP	Opportunistic	-	-	-	-
Tian et al. (2011)	simulation	Opportunistic & CBM	✓	✓	-	-
Ding and Tian (2012)	simulation	Opportunistic	✓	✓	✓	-
Stålhane et al. (2015)	MIP	CBM	-	-	-	✓
Sarker and Faiz (2016)	simulation	Opportunistic	✓	✓	✓	-
Zhu et al. (2017)	simulation	Opportunistic & CBM	-	-	-	-
Lu et al. (2018)	simulation	Predictive & CBM	✓	✓	-	-
Gutierrez-Alcoba et al. (2019)	MIP	-	-	-	-	✓
Gutierrez-Alcoba et al. (2019)	heuristic	-	-	-	-	✓
Stålhane et al. (2020)	MIP	time-based	-	-	-	✓
Yu (2021)	LP	CBM	-	✓	-	-
Current work	MIQLP	Predictive & CBM	✓	✓	✓	✓

Table 6.1: Comparison of models for OM optimization from literature. The we refer to the model introduce in this Master's as *Current work*.

Chapter 7

Model description of the MIQLP

We present the MIQLP formulation in this chapter. The model maximize the expected net profit from turbine power production over a planning horizon at an OWE. In similarity to models proposed by [Ding and Tian \(2012\)](#) and [Yu \(2021\)](#), we find the probability of future turbine failures from a two-factor Weibull distribution function. We schedule maintenance to increase the probability that each turbine is operating, and therefore producing power from wind.

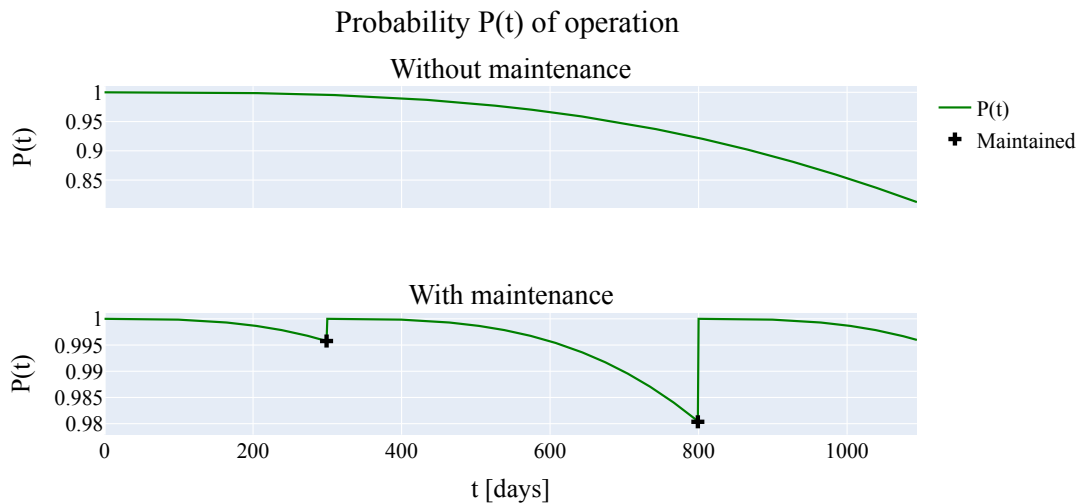


Figure 7.1: The probability $P(t)$, that a turbine is operating, decreases with time elapsed since last repair. The upper plot shows a case where the turbine is not maintained. The turbine in the lower plot is maintained at two occasions throughout the planning horizon. The horizontal axis shows time measured in days.

Figure 7.1 illustrate how the probability, $P(t)$, that a turbine is operating, changes with time. The upper graph shows how $P(t)$ decreases in a scenario where no maintenance is performed. Further, the lower graph show $P(t)$ for a scenario where the turbine is perfectly

maintained at two occasions. For the latter case, we observe that $P(t)$ returns to its initial value, 1, when the turbine is perfectly maintained. The probability, $P(t)$, follows by a two-factor Weibull distribution with an increasing failure rate in both scenarios. The scaling parameter is set to 1847 days (Lu et al., 2018), and the value of the shaping parameter is 3 (Lu et al., 2018). If a turbine were to be imperfectly maintained, which is not shown in this example, this would improve the value of $P(t)$ without fully restoring it to $P(t) = 1$.

We have previously established that corrective maintenance at an OWF is considered costly due to the long downtime, and the related production losses (Ren et al., 2021). For this reason, some models distinguish between corrective and preventive maintenance (Gutierrez-Alcoba et al., 2019; Besnard et al., 2011). However, we argue that such a major difference in cost of corrective and preventive maintenance is unnecessary in the following optimization model. We propose an objective function where expected income from turbine power production is proportional to the turbine's probability of operation. Consequently, the probability of downtime losses are already accounted for. To further distinguish between corrective and preventive maintenance would be superfluous.

Moreover, we propose a model where in addition to perfect maintenance, we allow for *imperfect maintenance*, improving the component to some extent, but not necessarily fully renewed it by a maintenance action. There exist some models that consider imperfect maintenance of OWFs (Ding and Tian, 2012; Sarker and Faiz, 2016). It may not be necessary, or even possible to fully restore the operating condition of a component. In our model, a component is either perfectly or imperfectly maintained, depending on the assigned maintenance strategy.

In the proposed model, we consider multiple major components at each turbine. We assume that their contributions to power production are vital, and together they define the operating state of each turbine. Analogously to the practise in power electronics, we assume that the components are connected in *series*, where the failure of one of these components leads to a malfunction of the entire turbine (Tian et al., 2011). Moreover, we assume that a wind farm operator wishes to repair each component individually, based on its respective *age*. Following, the concept of age is used in a sense that should not be understood literally. Rather than the conventional understanding, time elapsed since creation, age henceforth reflects the state of a component. That is, a component is said to have age a if its probability to be operative is identical to an equivalent, unmaintained component that was new a time periods ago.

We divide the planning horizon into discrete time periods. Furthermore, we assume that the wind farm operators have a set of pre-defined maintenance routes to choose from. Each route visits a set of turbines, and has limitations on which components the route can maintain, and

to which extent the wind farm operators can restore a component with this route. To illustrate, an example route visits turbine nr 1, 4 and 5. Although each turbine consists of several components, this route is restricted to perfect replacement of the gearboxes. However, the gearbox can be replaced all the visited turbines if the wind farm operators chose to do so. For each time period, the wind farm operator may choose one or several routes, in order to visit the desired turbines. Moreover, we assume the wind farm operators can perform the chosen maintenance routes within one time period. Maintenance of components can exclusively be performed at the turbines that are covered by the selected routes. The cost of performing each maintenance action appear in addition to the cost of performing a route. We assume there exists a route such that, by selecting a subset of routes, the wind farm operators are able to maintain every component at every turbine with every maintenance strategy.

The solution to this optimization model provides the wind farm operators with information about which turbines to visit, which components to maintain and to what extent each component should be improved, for every time period in the planning horizon.

7.1 Deriving the model formulation

In the current section, we introduce the sets, the parameters and the variables for the model formulation. Secondly, we discuss the objective function of our problem. We thereafter introduce each constraint, and discuss its contribution. The full model description is attached at the end of this chapter.

7.1.1 Sets

We represent an OWF by a set, \mathcal{K} , of turbines. The wind farm operators have some pre-defined maintenance routes to choose from, denoted by the set \mathcal{R} . Furthermore, \mathcal{K}_r is the set of turbines that are visited if route $r \in \mathcal{R}$ is chosen, and is a subset of \mathcal{K} . Different maintenance strategies can be used to improve the operating state of a component to a certain extent. The set, \mathcal{F} , holds all maintenance strategies. Moreover, the set \mathcal{O}_j denotes the set of components that make up turbine $j \in \mathcal{K}$. We assume that factors such as choice of vessel, the technical level of the technicians, the equipment available, and spare parts constrain which components, and which maintenance strategies that are possible to perform with a selected route. Therefore, we introduce, \mathcal{R}_{fij} , as the subset of routes that are able to maintain component $i \in \mathcal{O}_j$ at turbine j with maintenance strategy $f \in \mathcal{F}$. Table 7.1 contains the sets used to describe our model.

\mathcal{K}	Set of turbines
\mathcal{R}	Set of routes
\mathcal{K}_r	Set of turbines covered by route $r \in \mathcal{R}$, $\mathcal{K}_r \subseteq \mathcal{K}$
\mathcal{O}_j	Set of components belonging to turbine $j \in \mathcal{K}$
\mathcal{F}	Set of maintenance strategies
\mathcal{R}_{fij}	Set of routes covering component $i \in \mathcal{O}_j$ at turbine $j \in \mathcal{K}$ with maintenance strategy $f \in \mathcal{F}$, $\mathcal{R}_{fij} \subseteq \mathcal{R}$

Table 7.1: The table contains the sets that describe the MIQLP.

7.1.2 Parameters

We introduce T as the number of time periods in the planning horizon. Income from turbine production vary with uncertainties like electricity price and weather conditions. Therefore, we assume that the expected value E_{jt} is taken over the uncertain parameters and holds the expected monetary value of power production from turbine $j \in \mathcal{K}$ in time period $t \in \{0, \dots, T\}$, given that turbine j is operative in time period t . Moreover, the loss in income due to downtime during a maintenance operation is ignored, as we expect the loss to be insignificant relative to the income of one time period.

Further, S_{rt} , holds the cost of route $r \in \mathcal{R}$ in period t . This route cost includes travel cost, access cost, and the equipment setup cost at the relevant turbines, but excludes the cost of the maintenance actions itself. The parameter, M_{fijt} holds the cost of maintaining component $i \in \mathcal{O}_j$ at turbine j during time period t using maintenance strategy $f \in \mathcal{F}$. Moreover, it is not assumed that all components are brand new at the beginning of the planning horizon. The component's initial ages are given by the parameters $a_{0ij} \in \mathbb{Z}^+$.

The probability P_{aij} that the life length of a component i at a turbine j exceeds the age $a \in \{0, \dots, T + a_{0ij}\}$, is sampled from the two-factor Weibull survivorship function, (5.3), as

$$P_{aij} = \exp \left[- \left(\frac{a}{\eta_{ij}} \right)^{\beta_{ij}} \right], \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \quad \forall a \in \{0, \dots, T + a_{0ij}\},$$

with corresponding Weibull scaling parameter η_{ij} and shaping parameter β_{ij} (Section 5.2).

In similarity to [Ding and Tian \(2012\)](#), we introduce a factor $Q_f \in [0, 1]$, describing the degree of rejuvenation related to each maintenance strategy $f \in \mathcal{F}$. Henceforth, we refer to Q_f as the *rejuvenation factor*. Executing a maintenance action with $Q_f = 1$ returns the component "as good as new", whereas if $Q_f = 0$ it does not improve the component's performance at all. For example, a component with an initial age $a = 8$ periods is assigned the new age $a = 4$ periods after a maintenance event with a rejuvenation factor $Q_f = 0.5$. Moreover, all maintenance strategies with a rejuvenation factor smaller than 1, are what we refer to as

imperfect maintenance strategies.

Throughout this chapter, we will continue to denote components by index i , turbines by index j , routes by index r , ages by index a and maintenance strategies by index f . All parameters can be found in Table 7.2.

T	Number of periods in the planning horizon
E_{jt}	Expected monetary value of production from turbine j in period t
S_{rt}	Cost of route r , chosen for period t
M_{fijt}	Cost of maintenance of component i , at turbine j in period t with maintenance strategy f
Q_f	Rejuvenation factor of maintenance strategy $f \in \mathcal{F}$
a_{0ij}	Initial age of component i at turbine j
P_{aij}	Probability that component i at turbine j with age a is still operating
η_{ij}	Weibull scaling parameter for component i at turbine j
β_{ij}	Weibull shaping parameter for component i at turbine j

Table 7.2: The table contains the parameters that describe the MIQLP.

7.1.3 Variables

The binary variable x_{fijt} equals 1 if a maintenance task f is used to maintain component i at turbine j during time period t , and zero otherwise. We choose to track the age of each component with a binary variable. By making this choice, most of our constraints can be presented as linear constraints. We introduce the binary variable b_{ijta} to keep track of the age of each component. The value of b_{ijta} is 1 if component i belonging to turbine j during time period t has age a , and 0 otherwise. Further, y_{rt} takes the value 1 if the wind farm operators choose to perform route r during period t , and 0 otherwise. We emphasise that the probability that a component is operating, depends on the maintenance actions performed in the previous time period. For this reason, no benefit can be obtained from performing maintenance actions in the final period. Therefore, we only schedule maintenance and routes for the first $T - 1$ time periods. Finally, we introduce two continuous variables. The variable $Z_{jt} \in [0, 1]$ holds the probability that turbine j operates in period t , while $w_{ijt} \in [0, 1]$ holds the probability that component i at turbine j operates during time period t .

7.1.4 Objective function

The objective function,

$$\text{Maximize } \sum_{j \in \mathcal{K}} \sum_{t=0}^T E_{jt} Z_{jt} - \sum_{f \in \mathcal{F}} \sum_{i \in \mathcal{O}_j} \sum_{j \in \mathcal{K}} \sum_{t=0}^{T-1} M_{fijt} x_{fijt} - \sum_{r \in \mathcal{R}} \sum_{t=0}^{T-1} S_{rt} y_{rt}, \quad (7.1)$$

maximizes expected net profit from the total turbine electricity production over the planning horizon, while subtracting expenses related to operating and maintaining the OWE. Looking closer, the first term in the objective equals the expected monetary value of power production from the turbines at the OWE. That is, a value proportional to the probability Z_{jt} of task for each turbine. The sum of the expenses M_{fijt} that constitute the maintenance strategy $f \in \mathcal{F}$ that is performed at components $i \in \mathcal{O}_j$ at turbines $j \in \mathcal{K}$ during the time periods $t \in \{0, \dots, T-1\}$ is then subtracted. Lastly, the costs S_{rt} corresponding to the routes the wind farm operators choose to perform, are subtracted in the final term.

7.1.5 Constraints

We introduce the constraints for our model description in this section. The first constraint assigns at most one maintenance strategy f to each component i at turbine j during time period t as follows,

$$\sum_{f \in \mathcal{F}} x_{fijt} \leq 1 \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T-1\}.$$

Furthermore, the constraint,

$$x_{fijt} \leq \sum_{r \in \mathcal{R}_{fij}} y_{rt} \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T-1\},$$

induces at least one of the maintenance routes, $r \in \mathcal{R}_{fij}$ that are able to maintain component i at turbine j with a strategy f during time period t , if the strategy f is assigned to component i at turbine j during time period t .

Moreover, an essential part of our model description is to update the age of each component according to performed maintenance, or the lack of maintenance. Firstly, we initialise b_{ijta} with the initial age for each component,

$$b_{i,j,0,a_{0ij}} = 1 \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}.$$

To every component at every turbine, there must be assigned exactly one age for every time

period throughout the planning horizon. It is ensured by the equation,

$$\sum_{a=0}^{T+a_{0ij}} b_{ijta} = 1 \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{1, \dots, T\},$$

where the upper limit of the sum reflects that a component i at a turbine j could at most have age $a = T + a_{0ij}$ at the end of the planning horizon, due to the component's initial age a_{0ij} .

The two following inequalities update the age of a component for each time period, with respect to the actions made in the previous time period. Recall that the binary variable, b_{ijta} , takes the value 1 if component i at turbine j during time period t has age a . Firstly,

$$b_{ijta} \geq b_{i,j,t-1,a-1} - \sum_{f \in \mathcal{F}} x_{f,i,j,t-1} \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{1, \dots, T\}, \forall a \in \{1, \dots, T + a_{0ij}\},$$

force component i at turbine j during time period t to have age a if **no maintenance is done** at that component in the previous time period. On the contrary, if the component is **maintained** during the previous time period, the latter inequality is still valid, but the new age of the component is constrained by,

$$b_{i,j,t,[(1-Q_f)a]} \geq b_{i,j,t-1,a} + x_{f,i,j,t-1} - 1 \quad \forall f \in \mathcal{F}, i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{1, \dots, T\}, a \in \{0, \dots, T + a_{0ij}\}. \quad (7.2)$$

We justify the inequality as follows. For the case when a is the age of component i at turbine j during time period $t - 1$, hence,

$$b_{i,j,t-1,a} = 1,$$

and maintenance is done at the component during time period $t - 1$, meaning,

$$x_{f,i,j,t-1} = 1,$$

the left hand side of (7.2) equals 1. We obtain the new age for this component i at turbine j during time period t by rounding up $(1 - Q_f)a$ to the nearest integer value

$$\lceil (1 - Q_f)a \rceil,$$

where Q_f is the rejuvenation factor of the applied maintenance strategy f , and a is the correct age of the component during time period $t - 1$. We further argue that rounding $(1 - Q_f)a$ up to the closest integer is sufficient, considering that the duration of one time period is

typically much smaller than the life length of a component.

The final state condition,

$$w_{ijT} \geq w_{ij0} \quad \forall j \in \mathcal{K}, i \in \mathcal{O}_j,$$

force the probability that every component is operating in the final time period to be at least as great as the components's initial probability of operation. By applying this condition we avoid leaving the OWF at a poor operating level at the end of our planning horizon. Moreover, we find the probability w_{ijt} that component i at turbine j operates during period t by summing the probabilities sampled from the Weibull distribution, multiplied with their corresponding binary variables for all ages,

$$w_{ijt} = \sum_{a=0}^{T+a_{0ij}} P_{ija} b_{ijta} \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T\}.$$

Finally, the product,

$$Z_{jt} = \prod_{i \in \mathcal{O}_j} w_{ijt} \quad \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T\}, \quad (7.3)$$

of the probabilities w_{ijt} , that the components $i \in \mathcal{O}_j$ at turbine j are operative in period t , equals the probability of operation of the entire turbine j in time period t .

7.1.6 Quadratic constraints

For model instances where the number of components exceeds two per turbine, we rewrite the constraint that determines the probability Z_{jt} that turbine j is operating in time period t . The continuous variables,

$$h_{jt}^i \in [0, 1] \quad \forall i \in \{0, \dots, |\mathcal{O}_j| - 2\}, j \in \mathcal{K}, t \in \{0, \dots, T\},$$

are introduced in order to rewrite constraint (7.3) into the following quadratic equations,

$$h_{jt}^0 = w_{ijt} w_{i+1,j,t}, \quad \forall i = 0, j \in \mathcal{K}, t \in \{0, \dots, T\},$$

$$h_{jt}^i = h_{jt}^{i-1} w_{i+1,j,t}, \quad \forall 1 \leq i < |\mathcal{O}_j| - 2, j \in \mathcal{K}, t \in \{0, \dots, T\},$$

$$Z_{jt} = h_{jt}^{i-1} w_{i+1,j,t}, \quad \forall i = |\mathcal{O}_j| - 2, j \in \mathcal{K}, t \in \{0, \dots, T\}.$$

We manually decide whether to include the latter set of constraints, or constraint (7.3) in the model formulation based on the number of components per turbine in the problem at hand.

7.2 Full model formulation of the MIQLP

The full model description is attached below.

$$\text{Maximize } \sum_{j \in \mathcal{K}} \sum_{t=0}^T E_{jt} Z_{jt} - \sum_{f \in \mathcal{F}} \sum_{i \in \mathcal{O}_j} \sum_{j \in \mathcal{K}} \sum_{t=0}^{T-1} M_{fijt} x_{fijt} - \sum_{r \in \mathcal{R}} \sum_{t=0}^{T-1} S_{rt} y_{rt} \quad (7.4)$$

Subject to:

$$\sum_{f \in \mathcal{F}} x_{fijt} \leq 1 \quad \forall i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{0, \dots, T-1\}$$

$$x_{fijt} \leq \sum_{r \in \mathcal{R}_{fij}} y_{rt} \quad \forall f \in \mathcal{F}, i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{0, \dots, T-1\}$$

$$b_{i,j,0,a_{0ij}} = 1 \quad \forall i \in \mathcal{O}_j, j \in \mathcal{K}$$

$$\sum_{a=0}^{T+a_{0ij}} b_{ijta} = 1 \quad \forall i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{1, \dots, T\}$$

$$b_{ijta} \geq b_{i,j,t-1,a-1} - \sum_{f \in \mathcal{F}} x_{f,i,j,t-1} \quad \forall i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{1, \dots, T\}, a \in \{1, \dots, T+a_{0ij}\}$$

$$b_{i,j,t,[(1-Q_f)a]} \geq x_{f,i,j,t-1} + b_{i,j,t-1,a-1} \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{1, \dots, T\}, a \in \{0, \dots, T+a_{0ij}\}$$

$$w_{ijT} \geq w_{ij0} \quad \forall j \in \mathcal{K}, i \in \mathcal{O}_j$$

$$w_{ijt} = \sum_{a=0}^{T+a_{0ij}} P_a b_{ijta} \quad \forall i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{0, \dots, T\}$$

Only for data instances where $|\mathcal{O}_j| \leq 2$,

$$Z_{jt} = \prod_{i \in \mathcal{O}_j} w_{ijt} \quad \forall j \in \mathcal{K}, t \in \{0, \dots, T\}$$

Only for data instances where $|\mathcal{O}_j| > 2$,

$$h_{jt}^i = w_{ijt}w_{i+1,j,t}, \quad \forall i = 0, j \in \mathcal{K}, t \in \{0, \dots, T\}$$

$$h_{jt}^i = h_{jt}^{i-1}w_{i+1,j,t}, \quad \forall 1 \leq i < |\mathcal{O}_j| - 2, j \in \mathcal{K}, t \in \{0, \dots, T\}$$

$$Z_{jt} = h_{jt}^{i-1}w_{i+1,j,t}, \quad \forall i = |\mathcal{O}_j| - 2, j \in \mathcal{K}, t \in \{0, \dots, T\}$$

Binary variables:

$$b_{ijta} \in \{0, 1\} \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T\}, a \in \{0, \dots, T + a_{0ij}\}$$

$$x_{fij t} \in \{0, 1\} \quad \forall f \in i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T - 1\}$$

$$y_{rt} \in \{0, 1\}, \quad \forall r \in \mathcal{R}, t \in \{0, \dots, T - 1\}$$

Continuous variables:

$$w_{ijt} \in [0, 1] \quad \forall i \in \mathcal{O}_j, j \in \mathcal{K}, \forall t \in \{0, \dots, T\}$$

$$Z_{jt} \in [0, 1] \quad \forall j \in \mathcal{K}, t \in \{0, \dots, T\}$$

$$h_{jt}^i \in [0, 1] \quad \forall i \in \{0, \dots, |\mathcal{O}_j| - 2\}, j \in \mathcal{K}, t \in \{0, \dots, T\}$$

Chapter 8

Heuristic algorithm

We introduce a heuristic algorithm in the current chapter. The goal is to find *sufficiently good* solutions to the previously described maintenance optimization model. The heuristic is proposed as a starting point to further development. The algorithm improves a feasible solution inside a loop, by making slight changes in each iteration. A solution that is both feasible and better than the current best solution is brought to the next iteration.

We initialise (Section 8.1) the heuristic algorithm with a feasible, but costly solution. That is, performing every maintenance route, and maintaining every component at every turbine, for each time period in the planning horizon. We emphasise (Section 8.3) the conditions that must hold for the model solution to be feasible. Further (Section 8.4), we improve a solution by making one of the following changes,

1. remove one route,
2. remove one maintenance action, or
3. reduce the extent of one maintenance strategy.

Lastly (Section 8.5), we describe the heuristic search, in which the feasible solution is improved inside a loop by performing one of the three latter actions in each iteration.

8.1 Solution representation and initial solution

We pursue the notation from the MIQLP model in Chapter 7. Any solution to the heuristic can be represented using the independent variables,

$$x_{fijt} \in \{0, 1\} \quad \forall f \in \mathcal{F} \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T-1\}, \text{ and}$$

$$y_{rt} \in \{0, 1\}, \quad \forall r \in \mathcal{R}, \forall t \in \{0, \dots, T-1\}.$$

Moreover, we introduce the vector \mathbf{x} which consists of the variables $x_{fijt} \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T-1\}$, and the vector \mathbf{y} which consists of the variables $y_{rt} \forall r \in \mathcal{R}, \forall t \in \{0, \dots, T-1\}$. Hence, each solution can be represented as (\mathbf{x}, \mathbf{y}) .

Further, we initialise the search with the feasible solution where all routes

$$y_{rt} = 1, \forall r \in \mathcal{R}, \forall t \in \{0, \dots, T-1\},$$

are performed and all perfect maintenance actions

$$x_{f_{\text{perfect}}, i, j, t} = 1, \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T-1\},$$

are executed. The maintenance strategy $f_{\text{perfect}} \in \mathcal{F}$ is such that $Q_{f_{\text{perfect}}} = 1$, hence returns component i at turbine j in perfect operating condition in the following time period, $t+1$. Similarly, the maintenance strategy $f_{\text{min}} \in \mathcal{F}$ denotes the maintenance strategy such that the corresponding rejuvenation factor is the smallest possible,

$$Q_{f_{\text{min}}} = \text{minimum}(Q_f \forall f \in \mathcal{F}).$$

Henceforth, the indices denoting each maintenance strategy $f \in \mathcal{F}$ are ordered increasingly,

$$[f_{\text{min}}, f_{\text{min}} + 1, \dots, f_{\text{perfect}}] \in \mathcal{F},$$

in respect to the corresponding rejuvenation factor, Q_f .

8.2 Calculating the objective

We keep the objective function 7.1 from the MIQLP model formulation in Chapter 7. However, the following additions are introduced to the notation, for simplicity. Henceforth, the total income from turbine production is kept in the variable $totalE$. We name the variable corresponding to the total maintenance cost, $totalM$. Finally, the expences related to routes are kept in the variable $totalS$. For each solution (\mathbf{x}, \mathbf{y}) , we calculate the objective,

$$Obj = totalE - totalM - totalS.$$

In order to calculate $totalE$, we require the additional step of calculating the operation probabilities of each turbine during each time period. In similarity to the model description in

Chapter 7, we keep track of the age of component i at turbine j during time period t . Instead of the previous notation of using a binary variable to express the age, we introduce the new variables,

$$a_{ijt} \in \mathbb{Z}^+ \quad \forall j \in \mathcal{K}, \quad i \in \mathcal{O}_j, \quad t \in \{0, \dots, T\}.$$

The initial age, a_{0ij} of component i at turbine j is used to sample the initial operation probability,

$$w_{ij0} = P_{a_{ij0}, i, j} \quad \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K},$$

from the Weibull distribution. Thereafter, the age of the component is computed for every time period in the planning horizon. For time periods t in which the component is maintained, that is if $x_{fijt} = 1$, the age is updated by

$$a_{ijt+1} = \lceil (1 - Q_f)a_{ijt} \rceil.$$

Whereas if the component is not maintained, if $x_{fijt} = 0$, we let

$$a_{ijt+1} = a_{ijt} + 1.$$

The corresponding operational probabilities for component i at turbine j are sampled from the Weibull distribution,

$$w_{ijt} = P_{a_{ijt}, i, j}.$$

Finally, the operational probabilities for each turbine is calculated,

$$Z_{jt} = \prod_{i \in \mathcal{O}_j} w_{ijt}.$$

We further introduce the vector \mathbf{Z} which holds the variables $Z_{jt} \forall j \in \mathcal{O}_j, \forall t \in \{0, \dots, T\}$, and the vector \mathbf{w} which holds the variables $w_{ijt} \forall i \in \mathcal{O}_j, \forall j \in \mathcal{K}, \forall t \in \{0, \dots, T\}$. The above approach for calculating \mathbf{Z} and \mathbf{w} is described with pseudo code in Algorithm 1.

Algorithm 1 prob(x, y)

```

1: INPUT (x, y)
2: SET  $w_{ij0} \leftarrow P_{a_{0ij}, i, j}$  initial probability of operation for component  $i$  at turbine  $j$ 
3: SET  $Z_{j0} \leftarrow \prod_{i \in \mathcal{O}_j} w_{i, j, 0}$  initial probability of operation for turbine  $j$ 
4:
5: FOR each time period,  $t \in \{0, \dots, T-1\}$ 
6:   FOR each turbine,  $j \in \mathcal{K}$ 
7:     FOR each component,  $i \in \mathcal{O}_j$ 
8:       maintained  $\leftarrow$  False
9:       FOR each maintenance strategy,  $f \in \mathcal{F}$ 
10:        IF  $x_{fijt} = 1$  ▷ maximum true for one f
11:          THEN maintained  $\leftarrow$  True
12:          update age  $a_{i, j, t+1} \leftarrow \lceil (1 - Q_f) a_{ijt} \rceil$ 
13:        END IF
14:        IF maintained is False
15:          THEN update age  $a_{i, j, t+1} \leftarrow a_{ijt} + 1$ 
16:        END IF
17:      END FOR
18:       $w_{i, j, t+1} \leftarrow P_{a_{ijt+1}, i, j}$ 
19:    END FOR
20:     $Z_{j, t+1} \leftarrow \prod_{i \in \mathcal{O}_j} w_{i, j, t+1}$ 
21:  END FOR
22: END FOR
23: RETURN Z, w

```

We can now obtain $totalE$ as the sum, $\sum_{j \in \mathcal{K}} \sum_{t=0}^T E_{jt} Z_{jt}$. Moreover, the variables $totalM$ and $totalS$ can be calculated directly from the sum of performed maintenance actions and routes.

Firstly, $totalM$ is calculated as $\sum_{f \in \mathcal{F}} \sum_{i \in \mathcal{O}_j} \sum_{j \in \mathcal{K}} \sum_{t=0}^{T-1} M_{fijt} x_{fijt}$. Similarly, $totalS$ is computed as

$\sum_{r \in R} \sum_{t=0}^{T-1} S_{rt} y_{rt}$. The approach of calculating the objective value is described with pseudo code in Algorithm 2.

Algorithm 2 calculateObjective(\mathbf{x}, \mathbf{y})

```

1: INPUT ( $\mathbf{x}, \mathbf{y}$ )
2: CALL  $\mathbf{Z}, \mathbf{w} \leftarrow \text{prob}(\mathbf{x}, \mathbf{y})$ 
3: SET  $totalM, totalS, totalE \leftarrow 0$ 
4:
5: FOR each time period,  $t \in \{0, \dots, T\}$ 
6:   FOR each turbine,  $j \in \mathcal{K}$ 
7:      $totalE \leftarrow totalE + (E_{jt} * Z_{jt})$ 
8:   END FOR
9: END FOR
10:
11: FOR each time period,  $t \in \{0, \dots, T - 1\}$ 
12:   FOR each turbine,  $j \in \mathcal{K}$ 
13:     FOR each component,  $i \in \mathcal{O}_j$ 
14:       FOR each maintenance strategy,  $f \in \mathcal{F}$ 
15:         IF action  $x_{fijt} = 1$  ▷ maximum true for one f
16:           THEN  $totalM \leftarrow totalM + M_{fijt}$ 
17:         END IF
18:       END FOR
19:     END FOR
20:   END FOR
21: END FOR
22:
23: FOR each time period,  $t \in \{0, \dots, T - 1\}$ 
24:   FOR each route,  $r \in \mathcal{R}$ 
25:     IF  $y_{rt} = 1$ 
26:       THEN  $totalS \leftarrow totalS + S_{rt}$ 
27:     END IF
28:   END FOR
29: END FOR
30: SET  $objective \leftarrow totalE - totalS - totalM$ 
31: RETURN objective

```

8.3 Feasibility conditions

A special attention is given to the two conditions that must be held for a feasible solution. For all maintenance variables x_{fijt} assigned the value 1, there must be a corresponding route r , that is able to use maintenance strategy f at component i at turbine j in time period t , for which $y_{rt} = 1$ in the solution. Additionally, the final probability of operation for each component at each turbine, w_{ijT} , must be greater than or equal to its initial probability corresponding, w_{ij0} .

8.4 Improving a solution

Given a feasible solution (\mathbf{x}, \mathbf{y}) , we perform one of the functions *removeRoute* (Section 8.4.1), *removeAction* (Section 8.4.2), or *reduceAction* (Section 8.4.3) in order to improve the solution.

8.4.1 Remove random route

The function *removeRoute*, selects a random route $r^* \in \mathcal{R}$ and a random time period $t^* \in \{0, \dots, T-1\}$, for which $y_{r^*, t^*} = 1$, from the current solution (\mathbf{x}, \mathbf{y}) . We copy the current solution, and obtain $(\mathbf{x}', \mathbf{y}')$. Thereafter, we set $y'_{r^*, t^*} = 0$ in the new solution. A special attention is given to the maintenance actions in the new solution. We replace each variable x_{f, i, j, t^*} where $x_{f, i, j, t^*} = 1$ in the old solution, by the variable $x'_{f, i, j, t^*} = 0$ in the new solution, for cases where there no longer exist a route r such that $y'_{r, t^*} = 1$ in the new solution, that is able to apply maintenance strategy f to component i at turbine j during time period t^* . In other words, we remove the maintenance actions from time period t^* that only can be performed with the route r^* . The corresponding pseudo code is described in Algorithm 3. The objective value is calculated for the new solution. The new solution is returned if it is feasible, and better than the current best solution.

Algorithm 3 *removeRoute(x, y, bestObjective, notImproved)*

```

1: INPUT  $(x, y, bestObjective, notImproved)$ 
2: SET  $\mathbf{x}', \mathbf{y}' \leftarrow \mathbf{x}, \mathbf{y}$  ▷ copy the solution
3: SET select random  $r^* \in \mathcal{R}$  and  $t^* \in \{0, \dots, T-1\}$  such that  $y'_{r^*, t^*} = 1$ 
4: SET  $y'_{r^*, t^*} \leftarrow 0$  ▷ set the random variable to 0 in the solution
5: FOR each maintenance strategy,  $f \in \mathcal{F}$ 
6:   FOR each component,  $i \in \mathcal{O}_j$ 
7:     FOR each turbine,  $j \in \mathcal{K}$ 
8:       IF  $x_{f, j, i, t^*} = 1$ 
9:         IF  $y'_{r, t^*} = 0 \ \forall r \in \mathcal{R}_{fij} \setminus \{r^*\}$ 
10:        THEN  $x'_{f, i, j, t^*} \leftarrow 0$  ▷ remove the infeasible maintenance action
11:        END IF
12:      END IF
13:    END FOR
14:  END FOR
15: END FOR
16:
17: CALL  $\mathbf{Z}, \mathbf{w} \leftarrow \text{prob}(\mathbf{x}', \mathbf{y}')$ 
18: SET feasible  $\leftarrow$  True
19: FOR each component,  $i \in \mathcal{O}_j$ 
20:   FOR each turbine,  $j \in \mathcal{K}$ 
21:     IF final state condition,  $w_{ijT} < w_{ij0}$ , is violated
22:       THEN feasible  $\leftarrow$  False
23:       notImproved = notImproved + 1
24:       RETURN  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$  ▷ return old solution
25:     END IF
26:   END FOR
27: END FOR
28:
29: CALL currentObjective  $\leftarrow$  calculateObjective( $\mathbf{x}', \mathbf{y}'$ )
30: IF feasible = True and currentObjective > bestObjective
31:   THEN bestObjective  $\leftarrow$  currentObjective
32:    $\mathbf{x}, \mathbf{y} \leftarrow \mathbf{x}', \mathbf{y}'$  ▷ update solution
33: ELSE notImproved = notImproved + 1
34: END IF
35: RETURN  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$ 

```

8.4.2 Remove random maintenance action

The second function, *removeAction*, is described by pseudo code in Algorithm 4. We again introduce $(\mathbf{x}', \mathbf{y}')$ as a copy of the current (\mathbf{x}, \mathbf{y}) solution. Thereafter, a strategy $f^* \in \mathcal{F}$, a turbine $j^* \in \mathcal{K}$, a component $i^* \in \mathcal{O}_j$, and a time period $t^* \in \{0, \dots, T\}$, for which $x_{f^*, i^*, j^*, t^*} = 1$, are randomly selected. We set $x'_{f^*, i^*, j^*, t^*} = 0$ in the new $(\mathbf{x}', \mathbf{y}')$ solution. We keep the new solution if the final state condition still holds, and the new objective is better than the current

objective.

Algorithm 4 $\text{removeAction}(\mathbf{x}, \mathbf{y}, \text{bestObjective}, \text{notImproved})$

```

1: INPUT  $\mathbf{x}, \mathbf{y}, \text{bestObjective}, \text{notImproved}$ 
2: SET  $\mathbf{x}', \mathbf{y}' \leftarrow \mathbf{x}, \mathbf{y}$  ▷ copy the solution
3: SET select random  $f^* \in \mathcal{F}, i^* \in \mathcal{O}_j, j^* \in \mathcal{K}, t^* \in \{0, \dots, T-1\}$  such that  $x'_{f^*, i^*, j^*, t^*} = 1$ 
4: SET  $x'_{f^*, i^*, j^*, t^*} \leftarrow 0$  ▷ remove action
5:
6: CALL  $\mathbf{Z}, \mathbf{w} \leftarrow \text{prob}(\mathbf{x}', \mathbf{y}')$ 
7: SET  $\text{feasible} \leftarrow \text{True}$ 
8: IF final state condition,  $w_{i^*, j^*, T} < w_{i^*, j^*, 0}$ , is violated
9:   THEN  $\text{feasible} \leftarrow \text{False}$ 
10:    $\text{notImproved} = \text{notImproved} + 1$ 
11:   RETURN  $\mathbf{x}, \mathbf{y}, \text{bestObjective}, \text{notImproved}$  ▷ return old solution
12:   END IF
13:
14: CALL  $\text{currentObjective} \leftarrow \text{calculateObjective}(\mathbf{x}', \mathbf{y}')$ 
15: IF  $\text{feasible} = \text{True}$  and  $\text{currentObjective} > \text{bestObjective}$ 
16:   THEN  $\text{bestObjective} \leftarrow \text{currentObjective}$ 
17:    $\mathbf{x}, \mathbf{y} \leftarrow \mathbf{x}', \mathbf{y}'$  ▷ update solution
18: ELSE  $\text{notImproved} = \text{notImproved} + 1$ 
19: END IF
20: RETURN  $\mathbf{x}, \mathbf{y}, \text{bestObjective}, \text{notImproved}$ 

```

8.4.3 Reduce the extent of a maintenance strategy

A third function, reduceAction , is introduced in Algorithm 5 to swap the maintenance strategy that is applied to one component i , at one turbine j , during one time period t . Recall that we initialise the model with a pure replacement strategy. We attempt to reduce the current applied strategy f to a less perfect strategy $f - 1$. For example, a strategy f that reduces a component's age by 100% can be replaced by a strategy that reduce the component's age by 50%.

We randomly pick a strategy $f^* \in \mathcal{F}$, a turbine $j^* \in \mathcal{K}$, a component $i^* \in \mathcal{O}_j$, and a time period $t^* \in \{0, \dots, T\}$, for which $x_{f^*, i^*, j^*, t^*} = 1$ in the solution (\mathbf{x}, \mathbf{y}) . If the applied maintenance strategy is the minimal maintenance strategy, $f^* = f_{\min}$, then we pick another set (f^*, i^*, j^*, t^*) . However, if we find a variable $x_{f^*, i^*, j^*, t^*} = 1$ where $f^* > f_{\min}$, then set,

$$x_{f^*, i^*, j^*, t^*} = 0$$

and set

$$x_{f-1^*, i^*, j^*, t^*} = 1$$

in the new solution $(\mathbf{x}', \mathbf{y}')$.

We return the old solution if no strategy f^* , for a component i^* , at a turbine j^* , during a time period t^* such that $x_{f^*, i^*, j^*, t^*} = 1$ and $f^* > f_{\min}$ are found within the maximum number $maxDraw$ of tries is reached. Similarly, the old solution is returned if either the new solution is infeasible or not better than the old solution.

Algorithm 5 *reduceAction(x, y, bestObjective, notImproved)*

```

1: INPUT  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$ 
2: SET  $\mathbf{x}', \mathbf{y}' \leftarrow \mathbf{x}, \mathbf{y}$  ▷ copy the solution
3: SET  $foundAction \leftarrow \text{False}$ 
4: SET  $f^* \leftarrow f_{\min}$ 
5: SET  $count \leftarrow 0$ 
6:
7: While  $count < maxDraw$ 
8:   IF the strategy  $f^* = f_{\min}$  is minimal
9:     THEN select random  $f^* \in \mathcal{F}, i^* \in \mathcal{O}_j, j^* \in \mathcal{K}, t^* \in \{0, \dots, T-1\}$  s.t.  $x'_{f^*, i^*, j^*, t^*} = 1$ 
10:     $count \leftarrow count + 1$ 
11:   ELSE  $count \leftarrow maxDraw$ 
12:    $foundAction \leftarrow \text{True}$ 
13:   END IF
14: END WHILE
15:
16: IF  $foundAction$  is True
17:   THEN  $x'_{f^*, i^*, j^*, t^*} \leftarrow 0$  ▷ remove old action
18:    $x'_{f^*-1, i^*, j^*, t^*} \leftarrow 1$  ▷ add new action
19:   IF  $y'_{r, t^*} = 0 \forall r \in \mathcal{R}_{f^*, i^*, j^*}$ 
20:     THEN  $notImproved \leftarrow notImproved + 1$ 
21:     RETURN  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$  ▷ return old solution
22:   END IF
23: ELSE RETURN  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$  ▷ return old solution
24: END IF
25:
26: CALL  $\mathbf{Z}, \mathbf{w} \leftarrow \text{prob}(\mathbf{x}', \mathbf{y}')$ 
27: SET  $feasible \leftarrow \text{True}$ 
28: IF final state condition,  $w_{i^*, j^*, T} < w_{i^*, j^*, 0}$ , is violated
29:   THEN  $feasible \leftarrow \text{False}$ 
30:    $notImproved = notImproved + 1$ 
31:   RETURN  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$  ▷ return old solution
32: END IF
33:
34: CALL  $currentObjective \leftarrow \text{calculateObjective}(\mathbf{x}', \mathbf{y}')$ 
35: IF  $feasible = \text{True}$  and  $currentObjective > bestObjective$ 
36:   THEN  $bestObjective \leftarrow currentObjective$ 
37:    $\mathbf{x}, \mathbf{y} \leftarrow \mathbf{x}', \mathbf{y}'$  ▷ update solution
38: ELSE  $notImproved = notImproved + 1$ 
39: END IF
40: RETURN  $\mathbf{x}, \mathbf{y}, bestObjective, notImproved$ 

```

8.5 Heuristic function

The heuristic function is described in Algorithm 6. We improve the heuristic solution inside a loop consisting of $N \in \mathbb{Z}^+$ iterations. In each iteration, we apply one of the functions `removeRoute` (Algorithm 3), `removeAction` (Algorithm 4), or `reduceAction` (Algorithm 5), with the overall goal of improving the solution from one iteration to the next.

Algorithm 6 `heuristic(N, maxnotImproved)`

```

1: SET  $(\mathbf{x}, \mathbf{y}) \leftarrow$  initial solution
2: SET  $bestObjective \leftarrow$  initial objective
3: SET  $n \leftarrow 0$ 
4: SET  $notImproved \leftarrow 0$  ▷ count the number of nonimproving iterations
5: SET  $mode \leftarrow 0$ 
6: FOR  $n < N$ 
7:   IF  $mode$  is 0
8:     THEN  $\mathbf{x}, \mathbf{y}, notImproved, bestObjective \leftarrow$  removeRoute( $\mathbf{x}, \mathbf{y}, notImproved$ )
9:   ELIF  $mode$  is 1
10:    THEN  $\mathbf{x}, \mathbf{y}, notImproved, bestObjective \leftarrow$  removeAction( $\mathbf{x}, \mathbf{y}, notImproved$ )
11:   ELIF  $mode$  is 2
12:    THEN  $\mathbf{x}, \mathbf{y}, notImproved, bestObjective \leftarrow$  reduceAction( $\mathbf{x}, \mathbf{y}, notImproved$ )
13:   END IF
14:   IF  $notImproved > maxnotImproved$ 
15:     THEN  $count \leftarrow 0$ 
16:      $mode \leftarrow mode + 1$ 
17:   END IF
18:   IF  $mode > 2$ 
19:     THEN  $mode \leftarrow 0$ 
20:   END IF
21: END FOR
22: RETURN  $bestObjective$ 

```

We start the search by applying the function, `removeRoute`, until the number of not improving iterations reached `maxnotImproved`. Thereafter, we apply the function, `removeAction`, until `maxnotImproved` is reached. Similarly, `reduceAction`, is applied until `maxnotImproved` once again is reached. We continue looping between the three functions until the maximum number of iterations, N is reached.

Chapter 9

Experimental study

We present the input that is given to both the MIQLP, and the heuristic algorithm in this chapter. Moreover, we introduce the numerical experiments that are performed. As addressed by [Sørensen \(2009\)](#), models based on relative costs should be based on real data, obtained from practice. Unfortunately, information on maintenance and repair costs are difficult to obtain ([Sørensen, 2009](#)). Such data tends to be confidential and the following experimental cases are therefore mostly based on data obtained from the literature and logical reasoning. However, we apply turbine capacities found for the turbines by [Vestas \(2022\)](#), which are implemented at Horns Rev 3 by [Vattenfall \(2022a\)](#). The reasoning behind each parameter choice will be made clear throughout the chapter.

We start (Section [9.1](#)) this chapter by defining the planning horizon. Moreover, we justify our choice of the expected monetary value of power production, the Weibull parameters, the routes and route costs, and the maintenance costs. Thereafter (Section [9.2](#)), we introduce three numerical experiments and emphasise their purposes. The number of turbines, the number of components, and the applied maintenance strategies are specified for the individual experiments.

9.1 Generating model instances

We start by introducing the parameters that are held constant for all experiments. That is, the planning horizon (Section [9.1.1](#)), the monetary value of power production (Section [9.1.2](#)) and the Weibull parameters (Section [9.1.3](#)). The solution sensitivity to route cost (Section [9.1.4](#)), and maintenance cost (Section [9.1.5](#)) is explored, as we generate them for different scenarios.

9.1.1 Planning horizon

We set the planning horizon T to 24 time periods in all instances. Although having the length of one period equal to the shift length of a maintenance crew ([Gutierrez-Alcoba et al., 2019](#)) seems like a reasonable approach, we observe little to no difference in the operational probabilities for such small time intervals. We therefore set the length of each time period to 28 days.

9.1.2 Expected monetary value of power production

Recall that the expected monetary value of power production, E_{jt} , gives the total income from a fully operating turbine j in time period t . This parameter is calculated as follows,

$$\begin{aligned} E_{jt} &= 24[\text{h/days}] \times \text{length of one time period}[\text{days}] \\ &\quad \times \text{Expected electricity sale price}[\text{euro/MWh}] \\ &\quad \times \text{Turbine Capacity}[\text{MW}] \quad \forall j \in \mathcal{K}, t \in \{0, \dots, T\}. \end{aligned}$$

Moreover, we assume the wind farms consist of identical Vestas V164 turbines ([Vestas, 2022](#)), with corresponding capacities of 8.3MW. Vattenfall state that they will produce energy with the electricity price of DKK 0.77 per kilowatt hour ([Vattenfall, 2022a](#)) in their new offshore wind farm consisting of these turbines. Simple unit calculation based on the exchange rates given by [Norges Bank \(The central bank of Norway\) \(2022\)](#) (1 DKK = 1.29 NOK, 1 Euro = 9.63 NOK), verify that this results in approximately 100 euro per MWh. We obtain the following value for E_{jt} ,

$$\begin{aligned} E_{jt} &= 24[\text{h/days}] \times 28[\text{days}] \\ &\quad \times 100[\text{euro/MWh}] \\ &\quad \times 8.3[\text{MW}] \quad \forall j \in \mathcal{K}, t \in \{0, \dots, T\}, \\ &= 557760[\text{euro}] \quad \forall j \in \mathcal{K}, t \in \{0, \dots, T\}. \end{aligned}$$

9.1.3 Weibull parameters

The generator, the pitch system, the rotor and the gearbox are mentioned as critical components by [Lu et al. \(2018\)](#). We use the Weibull parameters and replacement costs described by the authors.

Component	Shaping β	Scaling η [days]	Replacement cost[euro]
Rotor	3	1847	185000
Gearbox	3	1477	230000
Generator	2	1594	60000
Pitch	3	1144	14000

Table 9.1: The table shows the shaping parameter, the scaling parameter and the replacement cost for four critical components ([Lu et al., 2018](#)).

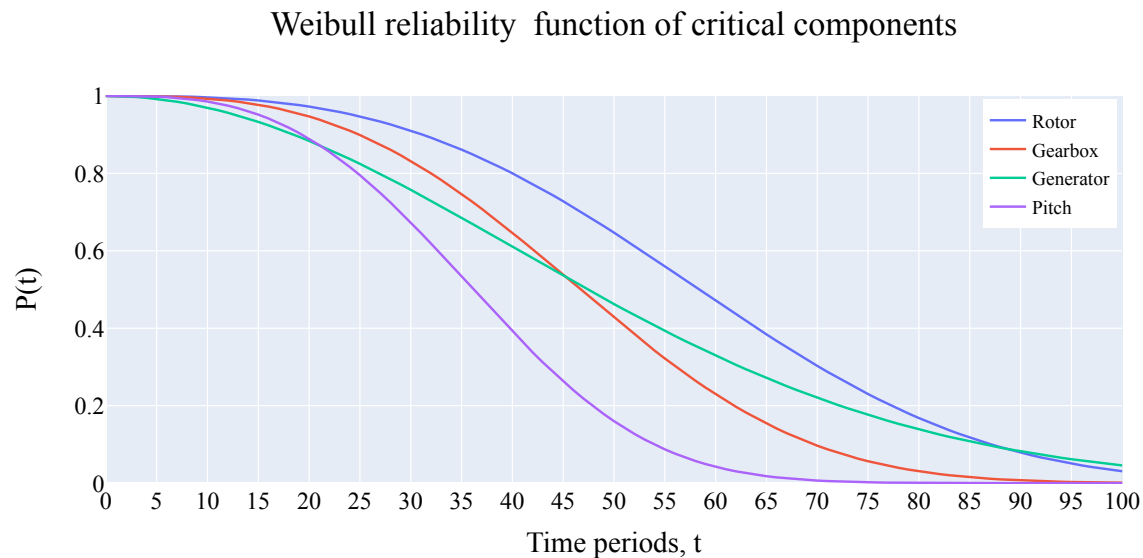


Figure 9.1: The figure shows the Weibull survivorship function for the four critical components. The distributions are based on information given by [Lu et al. \(2018\)](#).

The survivorship function 5.3 is adjusted for a time period of length of 28 days to scale the Weibull parameters. The probability P_{aij} of operation for component i , with the corresponding age a time periods, belonging to turbine j gets,

$$P_{aij} = \exp \left[- \left(\frac{a * 28[days]}{\eta} \right)^\beta \right], a > 0.$$

9.1.4 Routes and route cost

We generate routes by randomly selecting turbines from the set of turbines. We assume that the cost of a maintenance route is independent of the time period it is performed in. The

OWF operators can perform all routes during all time periods. We assign each turbine to at least one route, and all routes visit fairly the same number of turbines.

Turbines are usually spaced with 5-9 rotor diameters distance in the prevailing wind direction, and with 3-5 rotor diameters distance in the perpendicular direction (Pardalos et al., 2013). We calculate the approximate travel distance for each route. The turbines are placed in a two dimensional grid with 7 times the rotor dimensions distance in x-direction, and y-direction. The corresponding rotor diameters are 164m (Vestas, 2022). We assume there exist a base at a specified distance to the wind farm. Each maintenance route starts and ends at this base. The coordinates of the base varies with the size of the farm. The parameter C_{dist} holds the price of a route per meter of travel.

It is reasonable to assume that the cost of each route is determined by more than just travel distance. Therefore, we assume that the cost of one route additionally depends on factors as the vessel type, the available equipment and spare parts, and the crew of technicians. These factors constrain the type of maintenance strategy that can be performed, and the type of component that can be maintained. Therefore, we assume that the cost of a maintenance route also is dependent on the number of different maintenance strategies it can perform, and the number of different component types it can maintain. We introduce two new parameters. Henceforth, the cost $C_{\text{extra},f}$ holds the cost of being compatible with a maintenance strategy, and arise for each maintenance strategy that is compatible with the route in question. Similarly, $C_{\text{extra},i}$ holds the cost for that route to be able to maintain a component. For example, Route X1 visits turbine 1, 5 and 7, with a total travel distance of 5000m. The route is only able to perform one maintenance strategy: full replacement. The route brings extra equipment to maintain only one component: the gearbox. Nevertheless, the gearbox can be replaced at all the visited turbines in route X1. The cost of this route is,

$$\begin{aligned} S_{rt} &= 5000m \times C_{\text{dist}} \\ &+ 1 \text{ compatible strategy} \times C_{\text{extra},f} \\ &+ 1 \text{ compatible component} \times C_{\text{extra},i} \end{aligned}$$

On the other hand, Route X2 visits the same turbines, but is able to perform both full replacement, and minor maintenance to with a rejuvenation factor of 20%. All four components at

each turbine can be maintained. The cost of this route is therefore,

$$\begin{aligned}
 S_{rt} &= 5000m \times C_{\text{dist}} \\
 &\quad + 2 \text{ compatible strategies} \times C_{\text{extra},f} \\
 &\quad + 3 \text{ compatible components} \times C_{\text{extra},i}
 \end{aligned}$$

Moreover, we make sure to generate the routes in such a way that all components can be maintained to all strategies $f \in \mathcal{F}$, as we wish to capture the behaviour of imperfect vs perfect maintenance. Therefore, the model should have the choice between maintaining one or several components per turbine. We therefore generate several routes that seem identical at first glance. Nevertheless, they differ in allowed maintenance strategies and the compatible components. Suppose an offshore wind farm where two components are considered for each turbine, and three different maintenance strategies can be applied. We make the following duplications for each route, each with a unique pair of compatible maintenance strategies and components. The route is able to maintain

1. both components, with all strategies
2. component 0, with all strategies, or
3. component 1 with all strategies,

and

4. both components, with the first strategy
5. component 0, with the first strategy, or
6. component 1, with the first strategy,

and

7. both components, with the second strategy
8. component 0, with the second strategy, or
9. component 1, with the second strategy,

and

10. both components, with the third strategy
11. component 0, with the third strategy, or

12. component 1, with the third strategy.

As demonstrated, the number of maintenance routes increases rapidly for instances where several components and strategies are considered.

9.1.5 Maintenance cost

The cost of full replacement for each component is given in Table 9.1. However, we expect the choice of maintenance strategy to depend on the cost of applying each maintenance strategy. We consider two scenarios. Firstly, an exponential relation between the cost of a maintenance action and the strategy of choice,

$$M_{fijt} = \left(Q_f * \sqrt{\text{replacement cost}} \right)^2 \quad \forall f \in \mathcal{F}, i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{0, \dots, T-1\},$$

or a linear relation

$$M_{fijt} = Q_f * \text{replacement cost} \quad \forall f \in \mathcal{F}, i \in \mathcal{O}_j, j \in \mathcal{K}, t \in \{0, \dots, T-1\}.$$

We illustrate the two alternatives for a gearbox with total replacement cost of 230000 euro in Figure 9.2.

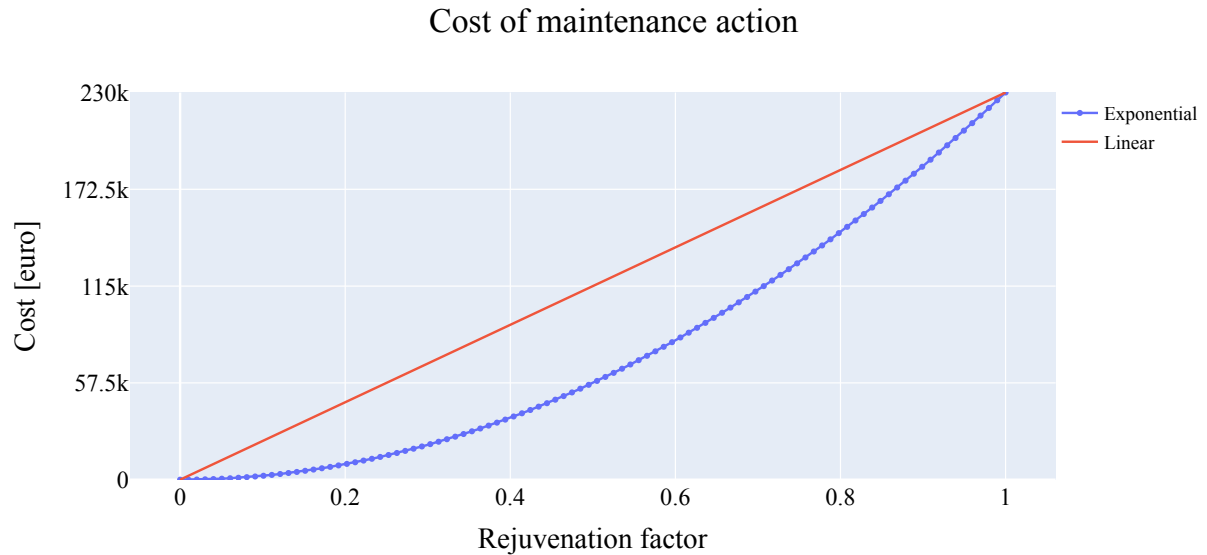


Figure 9.2: The figure shows the maintenance cost M_{fijt} for the gearbox for a linear (solid red), and an exponential (dotted blue) relation to the rejuvenation factor Q_f .

9.2 Introducing the experiments

Recall the research questions we present in Chapter 1. We perform several numerical experiments answer whether,

1. we can solve the MIQLP for realistic wind farm sizes, in reasonable time, and if
2. OWF operators should perform imperfect preventive maintenance tasks, as opposed to a preventive replacement strategy.

We implement both the MIQLP and the heuristic algorithm in Python 3. We use the commercial solver, Gurobi ([Gurobi Optimization, LLC, 2022](#)), with an academic license to solve the MIQLP ([Gurobi Optimization, LLC, 2022](#)). At response to feedback from the terminal output, we set the parameter *NonConvex* to 2, and *NumericFocus* to 3. A MacBook Air with 1,6 GHz Dual-Core Intel Core i5 Processor, and memory 8 GB 1600 MHz DDR3 is used.

9.2.1 Experiment I: Increasing input sizes

We wish to answer whether the model can be used to solve realistic wind farm sizes. Therefore, we use the commercial solver, Gurobi ([Gurobi Optimization, LLC, 2022](#)), to solve the MIQLP for increasing input sizes. We terminate the MIQLP solver when an optimality gap of minimum 5% is found. While keeping the other sets constant, we increase either the number, $|\mathcal{K}|$, of turbines, the number, $|\mathcal{O}_j|$, of considered components, or the number, $|\mathcal{F}|$, of strategies. The number, $|\mathcal{R}|$, of routes is a result of the number of components and the number of strategies used, as described in Section 9.1.4. The length of the planning horizon remains constant at $T = 24$ time periods. The number of applied maintenance strategies are more relevant than the maintenance strategies itself in this experiment. This experiment consists of 12 runs which input is described in Table 9.2.

Input sizes for experiment I

Nr	File	$ \mathcal{K} $	$ \mathcal{O}_j $	$ \mathcal{F} $	$ \mathcal{R} $
(1)	K20_Oj1_F1	20	1	1	2
— " —	K20_Oj2_F1	20	2	1	6
— " —	K20_Oj3_F1	20	3	1	8
— " —	K20_Oj4_F1	20	4	1	10
(2)	K20_Oj1_F1	20	1	1	2
— " —	K50_Oj1_F1	50	1	1	6
— " —	K100_Oj1_F1	100	1	1	12
— " —	K200_Oj1_F1	200	1	1	25
(3)	K20_Oj1_F1	20	1	1	2
— " —	K20_Oj1_F2	20	1	2	6
— " —	K20_Oj1_F3	20	1	3	8
	K100_Oj4_F3	100	4	3	240
	K200_Oj4_F3	200	4	3	500

Table 9.2: The table shows the input sizes of each file.

9.2.2 Experiment II: Different cost scenarios

With this numerical experiment, we explore if OWF operators should perform imperfect preventive maintenance tasks, as opposed to a preventive replacement strategy. As mentioned in the hypothesis, we expect that the answer to this question vary with the model instance. Therefore, we introduce different scenarios to explore typical behaviour for different scenarios.

We focus on one single component, the rotor, throughout this experiment. The Weibull parameters, and replacement cost for the rotor can be found in Table 9.1. Moreover, the initial age for each rotor, the visited turbines K_r for each route $r \in \mathcal{R}$, the route costs S_{rt}

We generate route costs In the first cost scenario, which applies to cases 1.1, 1.2 and 1.3 in Table 9.3, we assume that C_{dist} is low while $C_{\text{extra},f}$ and $C_{\text{extra},i}$ are higher, relative to the second scenario. The second cost scenario is applied to cases 2.1, 2.2 and 2.3 in Table 9.3. Moreover, the choice of maintenance strategy, $f \in \mathcal{F}$, depend on the relationship between the cost of each maintenance action, M_{fijt} , and the corresponding rejuvenation factor. For this reason, we explore both an exponential (case X.2) and a linear (case X.3) relation for the cases where imperfect maintenance is allowed.

To summarise, we obtain the scenarios, (case 1.1) cost scenario 1 with perfect maintenance, (case 1.2) cost scenario 1 with imperfect maintenance with an exponential M_{fijt} relation, (case 1.3) cost scenario 1 with imperfect maintenance with a linear M_{fijt} relation, (case 2.1) cost scenario 2 with perfect maintenance,

(case 2.2) cost scenario 2 with imperfect maintenance with an exponential M_{fijt} relation, and

(case 2.3) cost scenario 2 with imperfect maintenance with a linear M_{fijt} relation.

The number $|\mathcal{K}|$ of turbines, the number $|\mathcal{O}_j|$ of components, the number $|\mathcal{F}|$ of strategies, the corresponding rejuvenation factors Q_f , the number $|\mathcal{R}|$ of generated routes, the values for C_{dist} , $C_{\text{extra},f}$, $C_{\text{extra},i}$ and the choice of relation between M_{fijt} and Q_f are given in Table 9.3 for each file. We choose values for the three latter values so that the corresponding route cost S_{rt} varies between 22100 euro and 26800 euro for the first scenario and 53000 euro and 70120 euro for the second euro. In comparison, the paper by Tian et al. (2011) model a trip to the wind farm with a fixed access cost of 50 000\$.

The coordinates of the turbines, and the coordinates for the base are constant in the six cases for Experiment II, and are attached in Section A.1 of the Appendix. Additional data for cases 1.1, 1.2, 1.3, 2.1, 2.2, and 2.3 are given in the Appendix in Sections A.2, A.3, A.4, A.5, A.6, A.7. The latter information include the visited turbines \mathcal{K}_r for each route $r \in \mathcal{R}$, the initial age for each gearbox, the compatible maintenance strategies for each route, and the compatible components for each route. Moreover, we terminate the MIQLP solver when an optimality gap of 2% is found.

Input files for experiment II

File	$ \mathcal{K} $	$ \mathcal{F} $	Q_f	$ \mathcal{R} $	C_{dist} [euro]	$C_{\text{extra},f}$ [euro]	$C_{\text{extra},i}$ [euro]	relation
case1.1	30	1	1	3	0.2	200	200	-
case1.2	30	3	0.2,0.6,1.0	12	0.2	200	200	exp
case1.3	30	3	0.2,0.6,1.0	12	0.2	200	200	linear
case2.1	30	1	1	3	0.5	100	100	-
case2.2	30	3	0.2,0.6,1.0	12	0.5	100	100	exp
case2.3	30	3	0.2,0.6,1.0	12	0.5	100	100	linear

Table 9.3: The table shows the characteristics for each input each file for experiment 2.

9.2.3 Experiment III: Evaluation of the heuristic algorithm

We apply the heuristic algorithm (Chapter 8) for the files in experiment II (Table 9.3) to evaluate its performance. The specifications for each file are given in Table 9.3, and in Appendix A.2 to A.7. We use $N = 5000$ iterations to improve the solution, and perform the search in a loop 10 times. The average best objective, and the global best objective, and the time of solving the algorithm is reported. We use $\text{maxnotImproved} = 100$ and $\text{maxDraw} = 20$.

We additionally solve the heuristic for file K100_Oj4_Qf3 and K200_Oj4_Qf3 in Table 9.2. We increase the number of iterations to $N = 10000$, and perform the search 2 times per file. We

set $maxnotImproved = 100$ and $maxDraw = 20$. We report the average best objective, and the global best objective, and the time of solving the algorithm.

We report gaps and improvement based on the best values values objective function. Recall the equations (5.2) and (5.1) for calculation of gaps and improvement, respectively.

Chapter 10

Experimental results

The results from the numerical experiments are presented in this chapter. Firstly (Section 10.1), we present the results from Experiment I. Moreover (Section 10.2), we give the results that we obtain from Experiment II. Finally (Section 10.3), we present the results from Experiment III.

10.1 Results I: Increasing input sizes

We report the objective function values, upper bounds, gaps and time of solving each data instance for Experiment I in Table 10.1. We give the objective functions values and upper bounds in euros. Further, we give gaps in relative terms, as percentages. We report time of solving each instance in seconds. Gurobi obtain an upper bound for the the objective function value from the instance file 'K100_Oj4_Qf3' within 6 hours. However, the instance file 'K200_Oj4_Qf3' could not be solved as the PC run out of application memory. The lack of results is denoted by '-' in Table 10.1.

Solving the MIQLP for experiment I

Nr	File	Objective [euro]	Upper bound[euro]	Gap [%]	Time [s]
(1)	K20_Oj1_F1	259802365	263296092	1	91
— " —	K20_Oj2_F1	251055492	257092642	2	970
— " —	K20_Oj3_F1	243870433	254038074	4	5098
— " —	K20_Oj4_F1	242133385	252537280	4	15791
(2)	K20_Oj1_F1	259802365	263296092	1	91
— " —	K50_Oj1_F1	632682995	657998084	4	33
— " —	K100_Oj1_F1	1263386409	1314978173	4	125
— " —	K200_Oj1_F1	2519761003	2628667538	4	607
(3)	K20_Oj1_F1	259802365	263296092	1	91
— " —	K20_Oj1_F2	259608184	267152993	3	485
— " —	K20_Oj1_F3	260434457	267176900	3	571
	K100_Oj4_Qf3	-	1305800000	-	ca 21000
	K200_Oj4_Qf3	-	-	-	-

Table 10.1: Results for the MIQLP in experiment I.

Moreover, three subplots are given in Figure 10.1 to show the running time of the MIQLP for the files marked with number (1), (2), and (3) in Table 9.2. The subplots marked (1), (2) and (3) show the time of solving the MIQLP when we increase the number of components, turbines and strategies, respectively. We terminate the MIQLP solver at 5 % optimality gap for the instances in the latter figure.

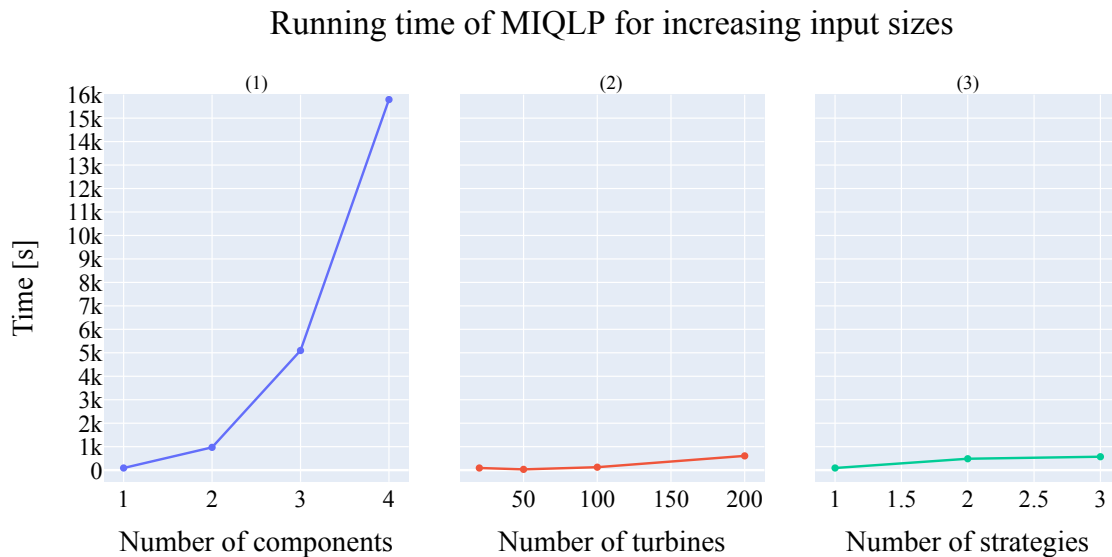


Figure 10.1: Running time for the instances in Experiment I. Time of solving the model is given in seconds at the shared vertical axis. The subplots marked (1), (2) and (3) show increase in components, turbines and strategies, respectively.

10.2 Results II: Different cost scenarios

We give the results from Experiment II in the current section. The MIQLP solver is terminated at 2% optimality gap in each case for Experiment II. We aim to present the results in a logical order, such that findings easily can be discussed in the next chapter. Rather than presenting the results from each case separately, we introduce similar findings alongside each other. Particularly, all results regarding maintenance intensity are presented in Section 10.2.1, and all results showing the probability of operation of each OWF are presented in Section 10.2.2. Before finally, the objective function values are presented in Section 10.2.3.

10.2.1 Maintenance intensity

The Figures 10.2 to 10.7 show the optimal maintenance intensity over the time horizon for each case of Experiment II. In each figure, maintenance actions with strategy $Q_f = 1.0$ are presented in green, the maintenance actions with strategy $Q_f = 0.6$ are presented in red, while the maintenance actions performed with $Q_f = 0.2$ are presented in blue. We give the time periods along the horizontal axis, and the number of executed maintenance actions along the vertical axis.

Figure 10.2 shows the maintenance actions that are performed during each time period for cost scenario 1. A perfect maintenance strategy ($Q_f = 1$) is applied.

MIQLP 1.1: Number of maintenance actions performed each time period

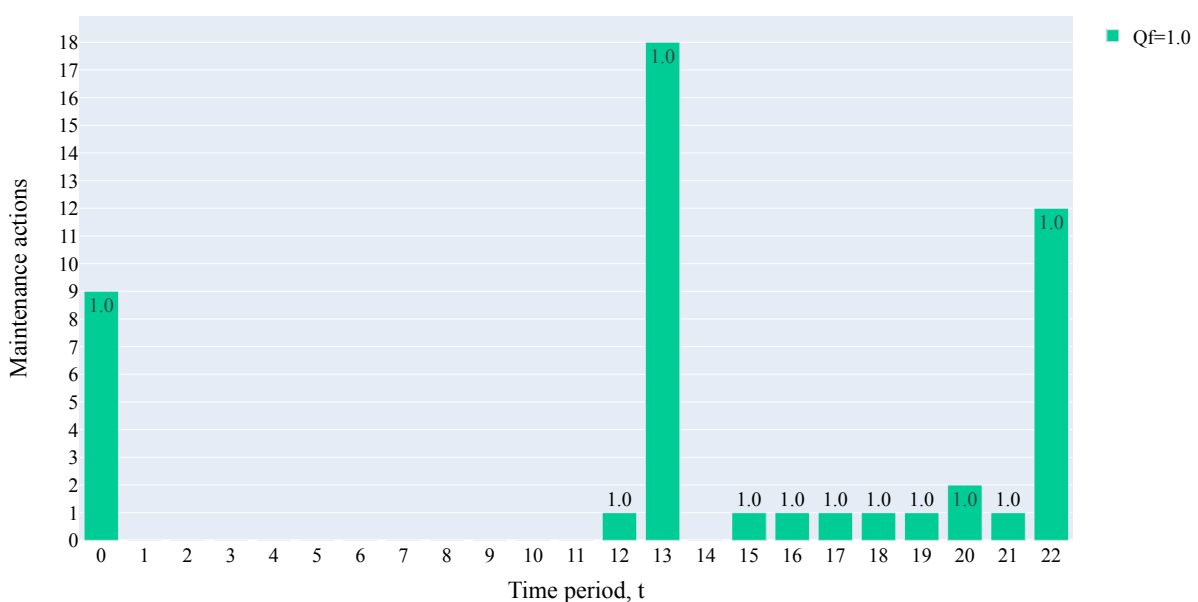


Figure 10.2: The number of executed maintenance actions performed each time period in case 1.1.

Similarly, Figures 10.3 and 10.4 show maintenance intensity for cost 1 with imperfect maintenance for an exponential (case 1.2) and a linear (case 1.3) relation, respectively.

MIQLP 1.2: Number of maintenance actions performed each time period

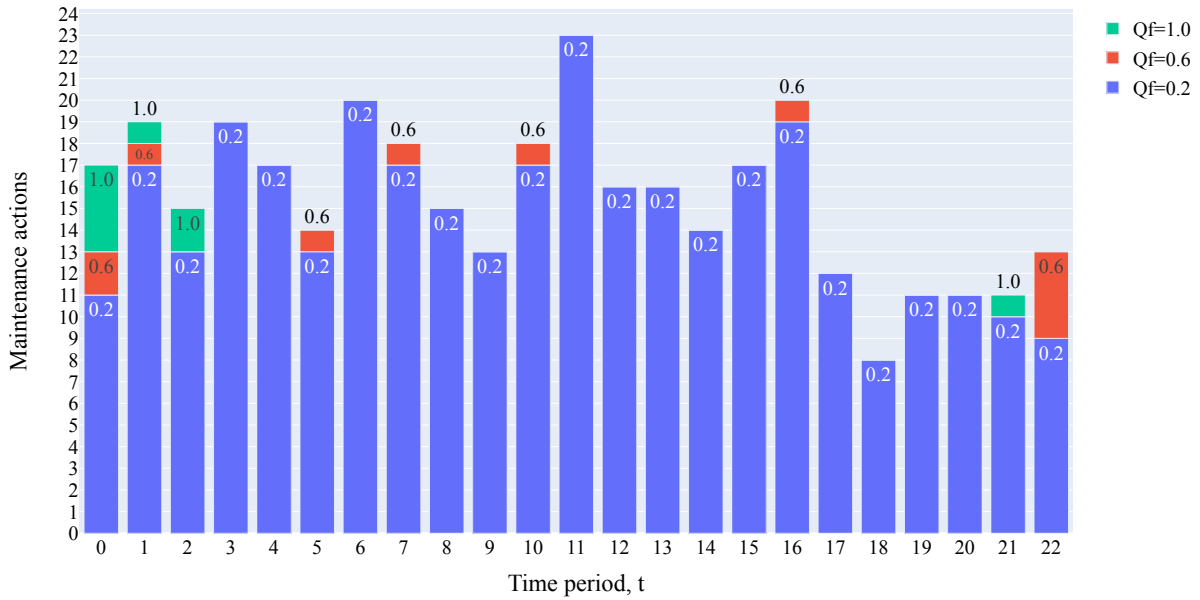


Figure 10.3: The number of executed maintenance actions performed during each time period in case 1.2.

MIQLP 1.3: Number of maintenance actions performed each time period

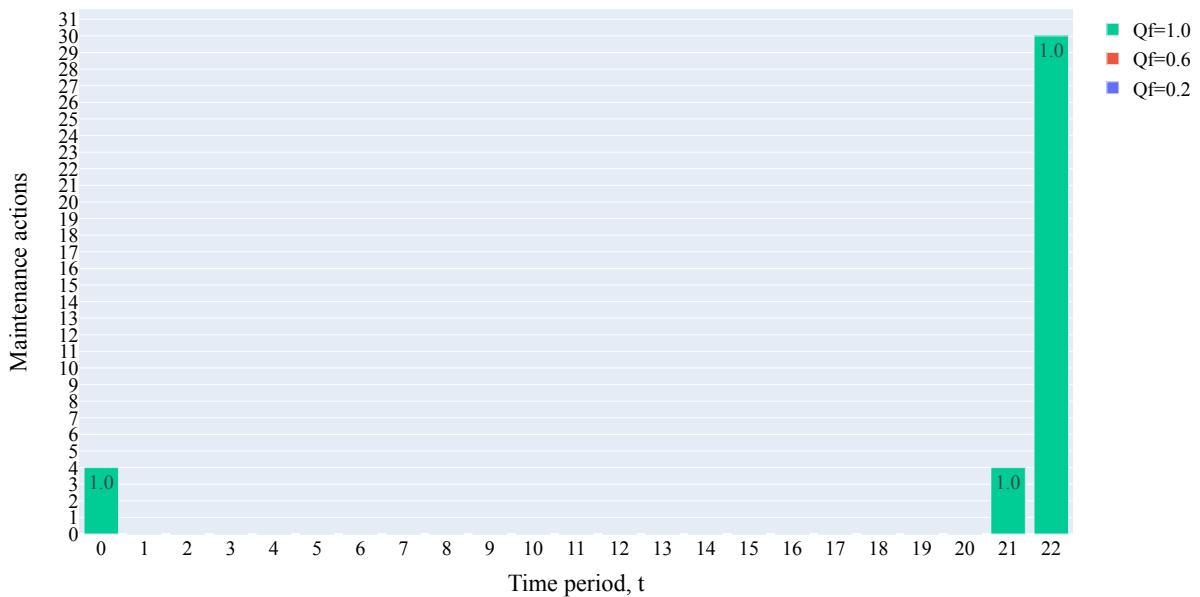


Figure 10.4: The number of executed maintenance actions performed during each time period in case 1.3.

Figure 10.5 shows the number of executed maintenance actions performed during each time period for cost scenario 2, when a perfect maintenance strategy is applied.

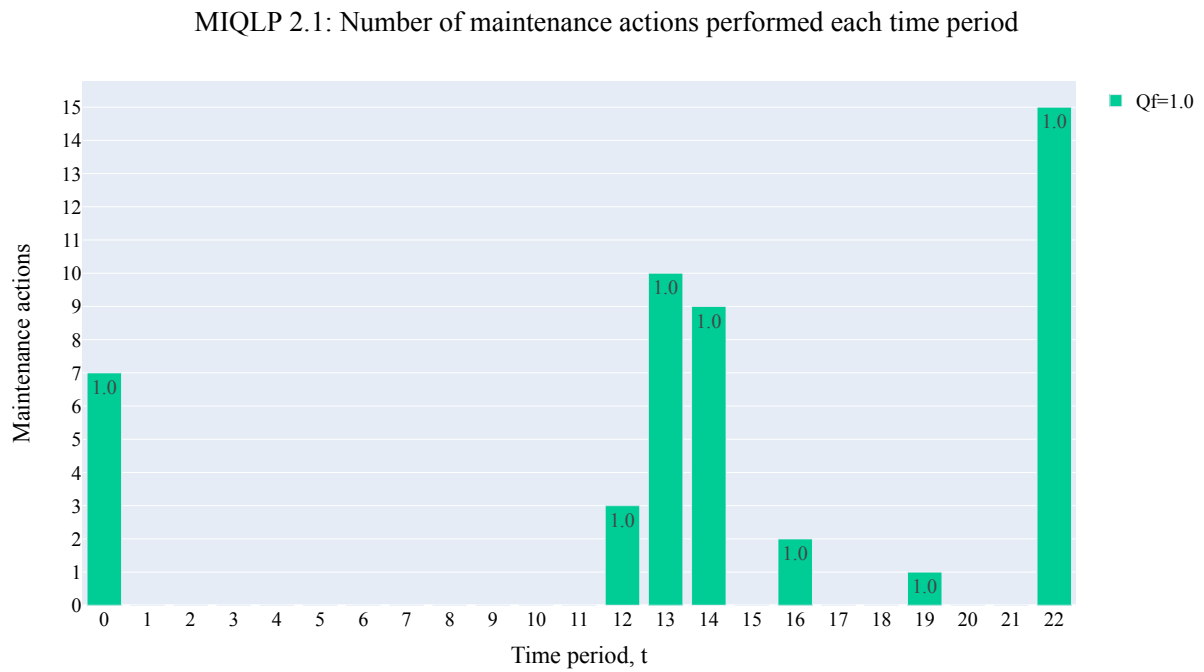


Figure 10.5: The number of executed maintenance actions performed during each time period in case 2.1.

Moreover, the Figures 10.3 and 10.4 show maintenance intensity for cost 2 with imperfect maintenance for an exponential (1.2) and a linear (1.3) relation, respectively.

MIQLP 2.2: Number of maintenance actions performed each time period

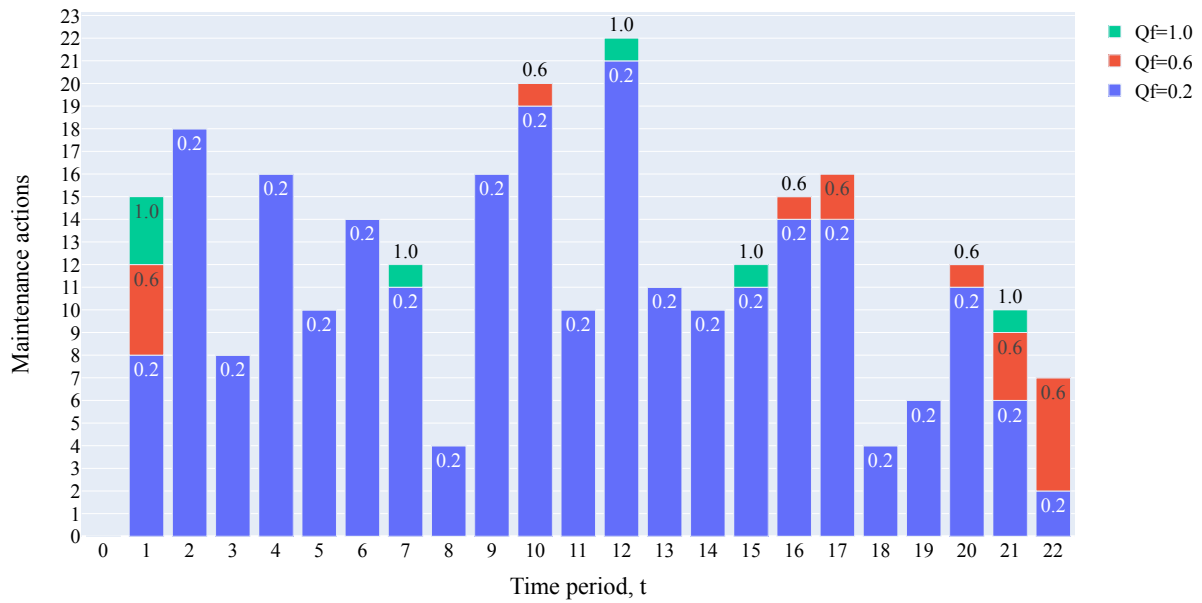


Figure 10.6: The number of maintenance executed maintenance actions performed during each time period in case 2.2.

MIQLP 2.3: Number of maintenance actions performed each time period

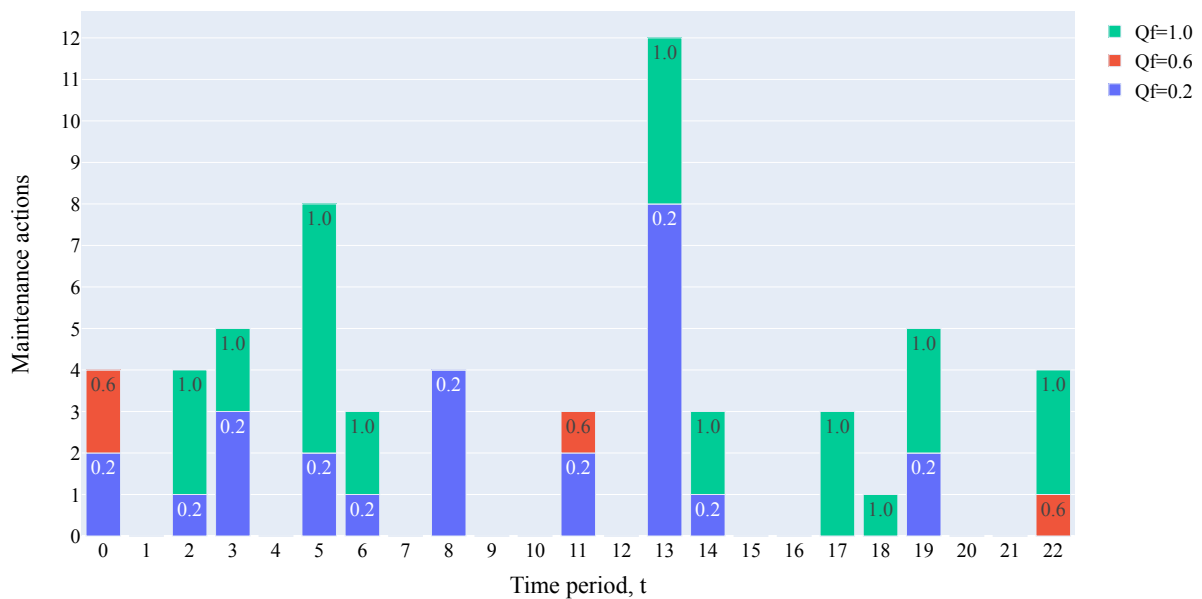


Figure 10.7: The number of executed maintenance actions performed during each time period in case 2.3.

10.2.2 Operational probabilities

In this section, we present the probabilities Z_{jt} that selected turbines $j \in \mathcal{K}$ are operating in time period $t \in \{0, \dots, T\}$ in each case for Experiment II. Those selected turbines are the turbine with the highest initial age, the turbine with the lowest initial age, and the turbine with the median initial age. IN each figure, we present the turbines with the lowest, median and highest initial age by a blue, red and green line, respectively. The average operation probability of all turbines for each period is given by a purple line.

Figure 10.8 shows the operational probabilities in the first cost scenario, when a perfect maintenance strategy is applied. Moreover, Figure 10.9 and Figure 10.10 are also obtained for cost scenario 1. However, an imperfect maintenance strategy is applied in the cases reported in the two latter figures. An exponential relation is applied for Figure 10.9, while a linear relation applies case reported in Figure 10.10.

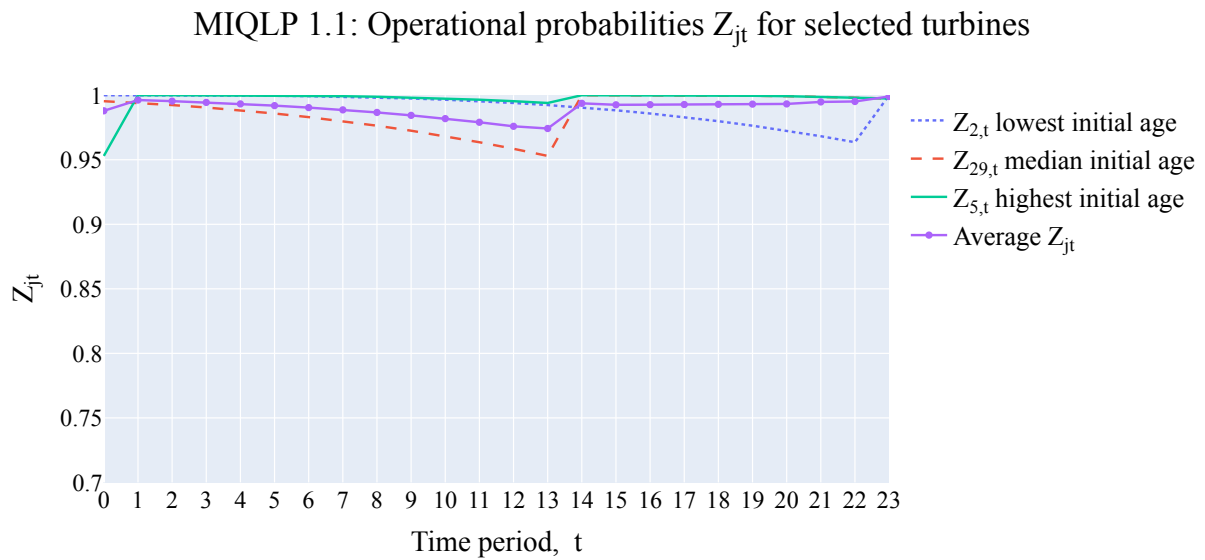


Figure 10.8: The probability of operation over the planning horizon in case 1.1.

MIQLP 1.2: Operational probabilities Z_{jt} for selected turbines

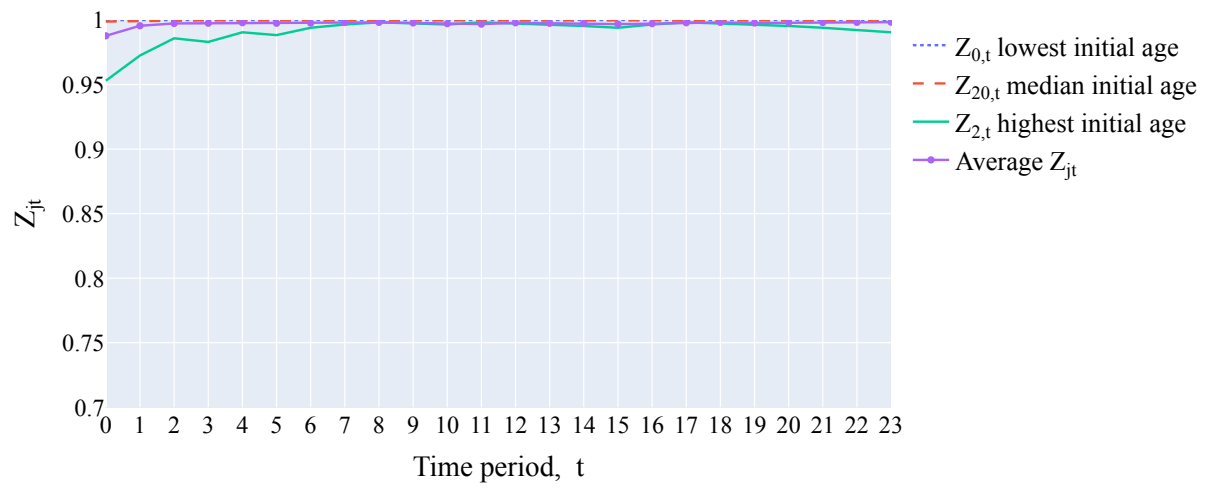


Figure 10.9: The probability of operation over the planning horizon in case 1.2.

MIQLP 1.3: Operational probabilities Z_{jt} for selected turbines

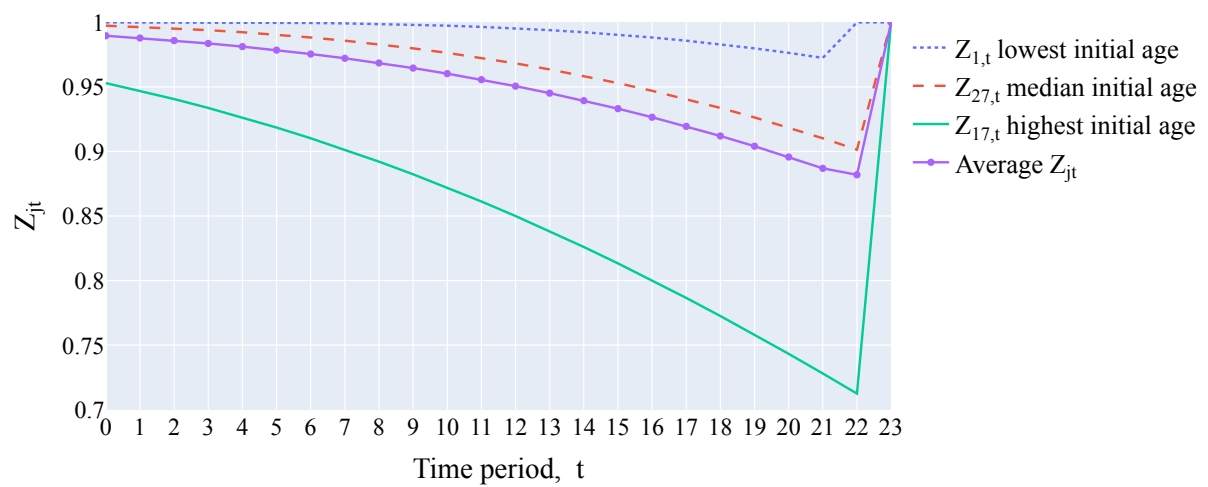


Figure 10.10: The probability of operation over the planning horizon in case 1.3.

Similarly, Figures 10.11, 10.12 and 10.13 show the operational probabilities in the second cost scenario. We apply a perfect maintenance strategy for the case in Figure 10.11. Moreover, an imperfect maintenance strategy with an exponential relation applies in for Figure 10.12, as opposed to the imperfect maintenance strategy with a linear relation for Figure 10.13.

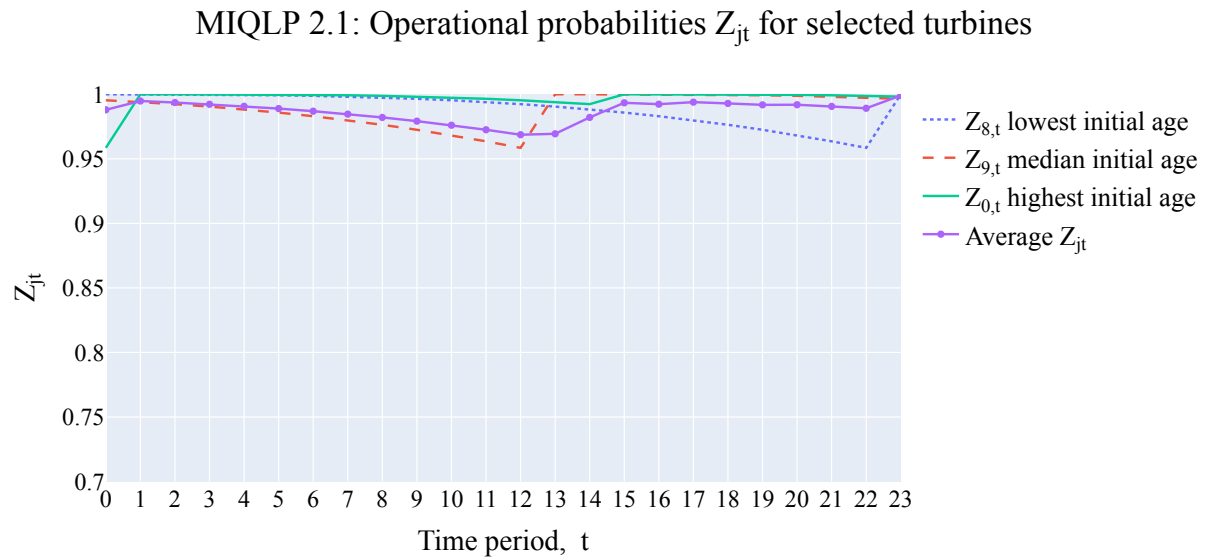


Figure 10.11: The probability of operation over the planning horizon in case 2.1.

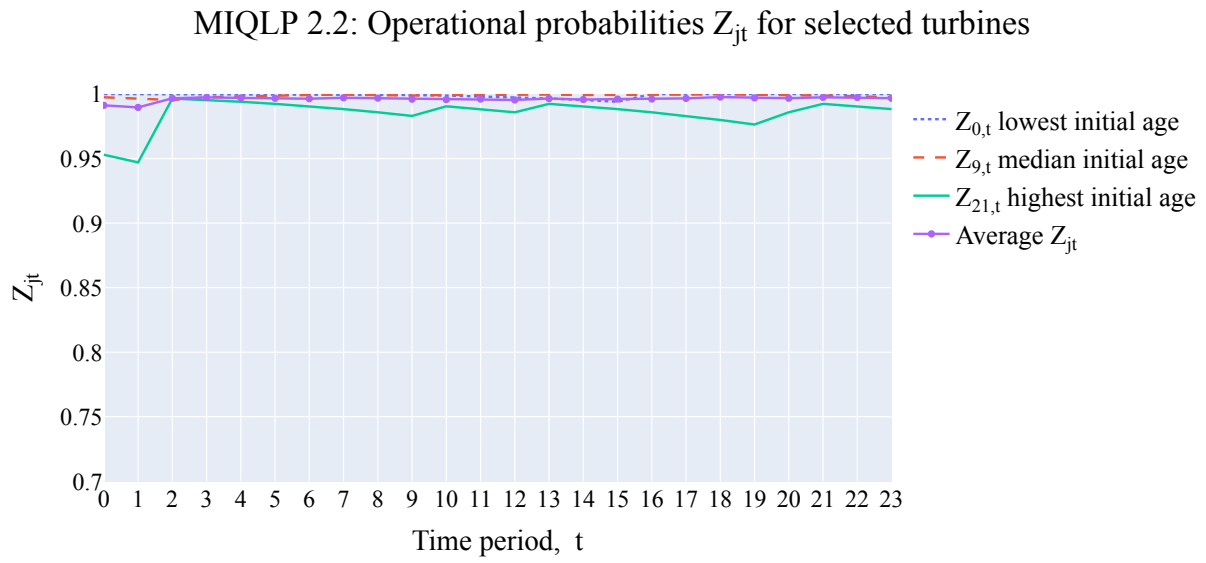


Figure 10.12: The probability of operation over the planning horizon in case 2.2.

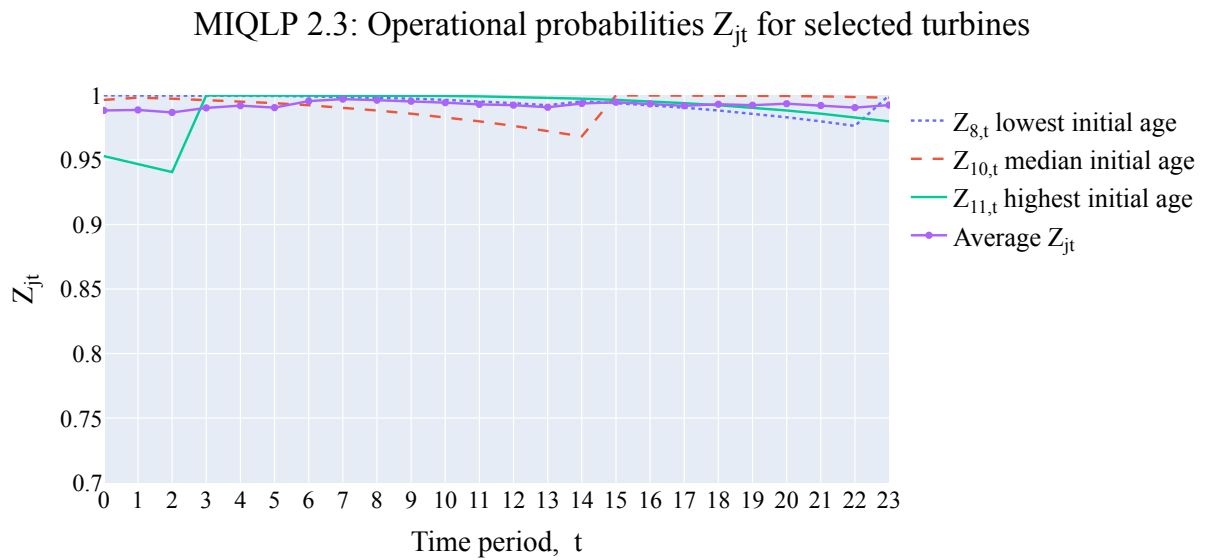


Figure 10.13: The probability of operation over the planning horizon in case 2.3.

10.2.3 Objective function values

The objective function values, upper bounds, gaps and time of solving the MIQLP for Experiment II are given in Table 10.2.

MIQLP solutions

File	Best obj. [euro]	Upper bound [euro]	Gap [%]	Time [s]
case1.1	388294215	394758373	2	284
case1.2	394173041	400496368	2	2847
case1.3	391303965	399109163	2	3703
case2.1	386937388	394605391	2	429
case2.2	392717865	400437717	2	3797
case2.3	390645538	398446723	2	11982

Table 10.2: Results for the MIQLP in Experiment II.

10.3 Results III: Evaluating the heuristic algorithm

This section give the results obtained with the heuristic algorithm. We give the best values for the objective function in the column "Best obj.". Moreover, we obtain the average objective function value for each instance by solving the heuristic multiple times. The average values are given in the column "Avg obj.". Moreover, we give the gaps, improvement (Imp.), and running time of the heuristic algorithm for Experiment II in the first six rows of Table 10.2. Additionally, we report the results found by the heuristic algorithm for file 'K100_Oj4_Qf3' and 'K200_Oj4_Qf3' in the two final rows in Table 10.2. We emphasise that the upper bounds are carried from the MIQLP solutions, and used to calculate gaps.

Heuristic solutions

File	Best obj. [euro]	Avg. obj. [euro]	Upper bound [euro]	Gap [%]	Imp. [%]	Time [s]
case1.1	388916798	387817895	394758373	2	43	35
case1.2	389679402	388317548	400496368	3	46	67
case1.3	389754392	388461553	399109163	2	46	68
case2.1	388866601	388145461	394605391	1	44	34
case2.2	389322488	388683872	400437717	3	51	66
case2.3	388706197	387217639	398446723	3	51	71
K100_Oj4_Qf3	1106663879	1105212843	1305800000	18	243	1670
K200_Oj4_Qf3	1426316459	1416735977	-	-	221	7635

Table 10.3: Results for the heuristic algorithm.

Chapter 11

Discussion

We analyse findings in the current chapter. Firstly, findings from the numerical experiments are discussed. We discuss (Section 11.1, Section 11.2 and Section 11.3) the findings for Experiment I, Experiment II and Experiment III. Thereafter (Section 11.4), we consider the strengths of the model, and give suggestions for further work.

11.1 Discussion of Experiment I

We solve the MIQLP for the instances in Table 9.2, and report the time of solving the model to a minimum of 5 % from optimality in Table 10.1. More specifically, while holding the other sets constant, we increase either the number of components, the number of turbines, or the number of strategies. The corresponding running times are shown in Figure 10.1. Recall that the four first instances in Table 9.2 model an OWF with 20 turbines, and a perfect maintenance strategy ($Q_f = 1$). We observe that by increasing the number of components from one per turbine, to four per turbine, the corresponding time of solving the MIQLP model increases from 91s to 15791s. However, we are able to solve an instance with 200 turbines in 607s if we consider only one component per turbine. Moreover, in the rows marked (3) in Table 9.2, we increase the number of maintenance strategies $|\mathcal{F}|$. For an instance with 20 turbines and one component per turbine, we observe that the running time of solving the MIQLP increase from 91s for an instance with $|\mathcal{F}| = 1$ to 571s for an instance with $|\mathcal{F}| = 3$. Additionally, we know that the number of routes increase rapidly when we increase the number of strategies and the number of turbines. The way routes are generated results in many routes for a case with several components or several strategies, which may be a contributing factor to the running time. An interesting observation is that the running time grows more rapidly when we increase the number of components, than when we increase the number of turbines or maintenance strategies. The two final instances in Table 9.2 challenge the

MIQLP solver. The instance with 200 turbines, 4 components and 3 maintenance strategies could not be solved as the computer ran out of application memory. However, we obtain the upper bound of 1305800000 euro for the instance with 100 turbines, 4 components and 3 strategies in about 6 hours.

11.2 Discussion of Experiment II

11.2.1 The intensity of maintenance

In experiment II, we solve the MIQLP model to 2% optimality gap for six different input sets, to explore the value of allowing for imperfect preventive maintenance. While the cases 1.1 and 2.1 allow for solely perfect preventive maintenance, imperfect preventive maintenance is allowed in the cases 1.2, 1.3, 2.2, and case 2.3. We give the optimal objective function values in Table 10.2. Moreover, we observe that profits are higher for all cases where imperfect preventive is an option, as opposed to the cases where a perfect strategy is applied.

Another observation is that maintenance is typically performed in the beginning and the end of the planning horizon. This trend is especially easy to observe for the two cases with perfect strategies, shown in the Figures 10.2 and 10.5. The trend is also easily verified for case 1.3, shown in Figure 10.4, where despite the option of imperfectly maintaining components, a pure perfect strategy is applied. Additionally, we observe that maintenance strategies with higher rejuvenation factors $Q_f = 1.0$ and $Q_f = 0.6$ are more often applied in the beginning and end of the planning horizons, for cases 1.2 and 2.2. The latter statement is verified by the Figures 10.3 and 10.6, respectively. The trend of more frequent maintenance in the beginning and end of the planning horizon is justifiable. As we do not expect the turbines to be new at the beginning of the planning horizon, maintenance is performed to increase the operating state of the elder turbines at the OWE. Similarly, the increase in maintenance intensity towards the end of the planning horizon can be justified by the final state condition. The condition forces the final operating state of each turbine to be greater than or equal to its initial operating state. Hence, it is reasonable that more maintenance is applied towards the end of planning horizon to fulfil this constraint. Nevertheless, case 2.3 break the trend, as maintenance is scheduled throughout the planning horizon with no clear indicator of an increase in maintenance in the beginning or end of the planning horizon.

A linear maintenance cost relation is the characteristic that connects case 1.3 and case 2.3. Additionally, lower value of C_{travel} , and higher values of $C_{\text{extra},i}$ and $C_{\text{extra},f}$, applies to case 1.3 when compared with case 2.3. A quick look at the Figures 10.4 and 10.4 tells that less frequent maintenance is performed in case 1.3 than in case 2.3. The solution in case 1.3 shows that

perfect maintenance is optimal in this instance, as no imperfect actions are performed. The high cost of performing several maintenance actions along each route supports the solution of only applying one maintenance strategy in case 1.3.

A general observation is that applying imperfect maintenance favours more frequent visits to the wind farm than applying perfect maintenance. Further, we observe that a maintenance strategy with rejuvenation factor of 0.2 is clearly the favourable option for the scenarios with imperfect maintenance and exponential cost relations (Figure 10.3 and Figure 10.6). It can be observed in Figure 9.2 illustrating the cost of a maintenance action for a linear vs when M_{fijt} increase exponentially with Qf , that the latter is the cheaper option of the two, for all actions that are not 100% renewal. This fact supports the observation that less imperfect maintenance actions are performed for cases 1.3 and 2.3, as opposed to cases 1.2 and 2.2 where the exponential dependency is applied. A paper by [Ding and Tian \(2012\)](#) finds that show that a maintenance strategy that reduce the age of a component by 50% is a more cost-effective imperfect maintenance strategy than 25 % and 75%. Their result emphasises that the relative costs affect the optimal maintenance strategy.

11.2.2 The probability of operation

We give the optimal operational probabilities Z_{jt} for selected turbines in Figures 10.8, 10.9, 10.10, 10.11, 10.12 and 10.13. As there are too many turbines to individually examine, only the turbine with the highest initial age, the lowest initial age and the median initial age are shown. In addition, we give the average value for the optimal Z_{jt} . That is, the average over all turbines $j \in \mathcal{K}$ for each time period $t \in \{0, \dots, T\}$ in the planning horizon. We argue that this selection of operational probabilities is sufficient to show typical behaviour of the OWE.

Turbine 5 has the highest initial age in case 1.1. We observe in Figure 10.8 that at least one component belonging to the turbine is maintained in time period 0. As a result, the the probability of operation increase to the next time period. Moreover, turbine 2 with the lowest initial age in this case, is not significantly maintained until the second to last period, leaving it fully operating in the final time period. We interpret the rest of the Figures 10.9, 10.10, 10.11, 10.12 and 10.13 similarly.

From visual inspection we find that the optimal value Z_{jt} averaged over turbines $j \in \mathcal{K}$ is typically high in each time period $t \in \{0, \dots, T\}$. More specifically, the average values are above 95% for all cases except for case 1.3. The high probabilities of operation evidence that the loss of income is more critical than the cost of maintenance in cases 1.1, 1.2, 2.1, 2.2 and 2.3. Another observation is that Z_{jt} is more stable for the cases where imperfect maintenance is used (Figures 10.9, 10.12 and 10.13). We justify the latter observation with

the frequent maintenance that occur in the cases where imperfect maintenance applies in the optimal solution (Figures 10.3, 10.6, and 10.7). Moreover, we note that despite imperfect strategies are an option in case 1.3, only a perfect preventive strategies are performed in the optimal solution in this case (Figure 10.4).

11.2.3 Objective function values

We give the objective function values, best upper bounds and the running time of each instance for experiment II in Table 10.2. We observe that an imperfect maintenance strategy is optimal in several scenarios. The only exception is case 1.3. It looks like a linear relation between maintenance cost and the degree of rejuvenation, in addition to low value of C_{travel} and high values of $C_{\text{extra},i}$ and $C_{\text{extra},f}$ could favour perfect replacement. However, the latter statement requires further experiments to be verified.

11.3 Discussion of Experiment III

We give the heuristic algorithm as an alternative to the MIQLP, and apply the algorithm to the same problem instances as described in Section 9.2.2. We observe from Table 10.3 that the heuristic finds close to optimal solutions for all instances in Experiment II (cases 1.1, 1.2, 1.3, 2.1, 2.2, 2.3). We obtain optimality gaps from 1% to 3% for the heuristic, as opposed to the 2% optimality gap with the MIQLP solver. For case 2.1, we observe that the heuristic algorithm provides an even better solution than the MIQLP, which results in a tighter optimality gap of 1%. The heuristic algorithm improves the initial solution by 43, 46, 46, 44, 51 and 51 % for cases 1.1, 1.2, 1.3, 2.1, 2.2 and 2.3, respectively. The running time of the heuristic algorithm is significantly lower than for the MIQLP, with over one hour difference for case 2.2. Ten runs of the heuristic algorithm show that the average objective function values are close to the best objective function value for each instance.

The two latter instances in Table 10.3 show that the heuristic algorithm obtain solutions to instances of larger size than what are found by the MIQLP solver. We carry the upper bound (Table 10.1) from the MIQLP solver, and find a optimality gap of 18% for instance *K100_Oj4_Qf3*. The heuristic is meant to be a starting point for further development of a smarter heuristic solution. However, it yields good solutions.

11.4 Model strengths and suggestions for further work

This study aims to find the optimal maintenance plan of an OWF of a given size. We do not assume that information about future failure times are pre-defined. Additionally, the optimal number of preventive maintenance actions are calculated for each specific case. As we observe in Section 11.2.1, the optimal number of preventive maintenance actions varies with the problem instance. The MIQLP address the choice of maintenance routes, multiple components per turbine, probabilistic failure times and both perfect and imperfect preventive maintenance. That is, aspects that to the best of our knowledge, are not considered simultaneously in other maintenance optimization models. We argue that the latter is a strength of this optimization model.

Moreover, the relationship between C_{travel} , C_{extra} , M_{fijt} and E_{jt} can be adjusted to reflect reality. Nevertheless, we can not be sure that the results we report from the numerical experiments reflect real scenarios, as actual data is challenging to obtain. Weaknesses with the proposed maintenance model include that good solutions require good pre-defined routes to choose from. A solution to the model supply the OWF operators with the turbines to be visited and maintenance actions to be performed for each period. Nevertheless, as a time period is modelled as 28 days, decisions regarding when to perform each action will likely have to be made additionally. While decreasing the length of each time period is an option, this will decrease the planning horizon that we are able to model. We propose the following as further work for the maintenance model MIQLP:

1. Generate routes smarter for the pre-defined set of routes,
2. including downtime costs from performing maintenance actions,
3. consider variations in weather conditions and electricity prices,
4. combine the operational probabilities based on age, with condition monitor data from sensors.

Moreover, we propose the following improvements for further work of the heuristic algorithm:

1. A solution should be able to add maintenance actions and routes, as opposed to only removing,
2. an escape algorithm could be applied to prevent the search of to be stuck in local maximum,

3. a less random and more greedy approach could be implemented to remove the costliest routes and actions from the solution, and
4. an acceptance criterion could be applied to save the diversity in the solution.

Chapter 12

Conclusion

UN Sustainable development goal 7 is within reach if tripling wind capacity is tripled by 2030 ([UN Summary of the Secretariat, 2021](#)). It is crucial to minimize the cost of OM at OWFs, so offshore wind energy can take its planned part in reaching the UN sustainability goals and the Paris Agreement. This Master's Thesis present a mixed integer optimization model, with both quadratic and linear constraints (MIQLP). The goal of the model is to find the optimal schedule for maintaining and operating offshore wind farms. We perform several experiments to explore the time of solving problem instances of different input sizes. The results show that the number of components per turbine is the parameter increasing the time of solving the model the most. We want to determine whether the find the optimal maintenance schedule for actual wind farm sizes. In Section 3.1, we find that new additions include the OWF "Hornsea One" consisting of 174 turbines. In this Master's Thesis, we are able to schedule the optimal maintenance plan for an OWF with 200 turbines. However, we can only consider one single component per turbine, and only allow for perfect maintenance for OWFs of these sizes. The instance with 200 turbines with four components each, while allowing both perfect and imperfect maintenance is too large for the MIQLP in this experiment. Nevertheless, we develop a heuristic algorithm to find near-optimal solutions, with shorter running time. The heuristic are able to obtain a solution for larger instances than we obtain this the MIQLP solver. Moreover, we introduce different cost scenarios to answer whether imperfect maintenance is a cost reducing maintenance strategy. The experimental results verify that an imperfect preventive maintenance plan is favourable in several scenarios. Moreover, the optimal number of preventive maintenance action vary with the scenario and should not be pre-determined. We argue that although the model is instance sensitive, it is a good foundation for further development. We conclude that the heuristic find good solutions, as small optimality gaps are obtained within significantly shorter time than from solving the MIQLP.

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Appendix A

Instance files for experiment II

We supply additional data for Experiment II in Appendices [A.1](#) to [A.7](#). Firstly, the x - and y -coordinates of the turbines and the base remain constant for each case. We give their values in Section [A.1](#). For each case, we give the initial ages for the component at each turbine, the route costs, the turbines visited by each route, the compatible components to each route and the compatible strategies to each route (Appendices [A.2](#) to [A.7](#)).

A.1 Coordinates of the turbines and the base

The coordinates of the turbines and the base remain the same for the following six cases.

x-, y- coordinates of turbines [m]

0.0,0.0

0.0,1148.0

0.0,2296.0

0.0,3444.0

0.0,4592.0

0.0,5740.0

1148.0,0.0

1148.0,1148.0

1148.0,2296.0

1148.0,3444.0

1148.0,4592.0

1148.0,5740.0

2296.0,0.0

2296.0,1148.0

2296.0,2296.0

2296.0,3444.0

2296.0,4592.0

2296.0,5740.0

3444.0,0.0

3444.0,1148.0

3444.0,2296.0

3444.0,3444.0

3444.0,4592.0

3444.0,5740.0

4592.0,0.0

4592.0,1148.0

4592.0,2296.0

4592.0,3444.0

4592.0,4592.0

4592.0,5740.0

#x-, y- coordinates of base [m]

34450,3445

A.2 Case 1.1

Initial age of components

23,13,19,6,10,13,22,5,1,11,16,2,11,9,3,
14,19,14,17,6,22,8,19,5,15,5,22,10,22,8

#Route Cost [euro]

59641.2,
54496.0,
69112.7

Routes: row index = route index

26,2,11,7,29,15,0,6
16,23,8,1,17,27,21,9
10,14,13,19,5,24,20,4,28,12,3,22,18,25

Components that can be maintained by route index nr

0
0
0

Strategies that can be used by each route, row = route

1.0
1.0
1.0

A.3 Case 1.2

```
# Initial age of components
0,19,24,4,14,10,23,17,1,7,18,12,7,4,
6,9,4,10,24,22,7,23,4,7,0,16,4,24,2,1
#Route Cost [euro]
24262.5
23862.5
23862.5
23862.5
23134.3
22734.3
22734.3
22734.3
26867.5
26467.5
26467.5
26467.5
# Routes: row index = route index
6,2,17,10,28,5,29,20
6,2,17,10,28,5,29,20
6,2,17,10,28,5,29,20
6,2,17,10,28,5,29,20
3,7,27,4,15,8,16,26
3,7,27,4,15,8,16,26
3,7,27,4,15,8,16,26
3,7,27,4,15,8,16,26
24,18,11,1,22,23,9,14,21,0,25,13,12,19
24,18,11,1,22,23,9,14,21,0,25,13,12,19
24,18,11,1,22,23,9,14,21,0,25,13,12,19
24,18,11,1,22,23,9,14,21,0,25,13,12,19
# Components that can be maintained by route index nr
0
0
0
0
0
```

0

0

0

0

0

0

0

Qf that can be used by route index nr

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

A.4 Case 1.3

```
# Initial age of components
17,0,8,8,5,18,0,1,20,1,16,2,5,21,0,13,
23,24,10,22,11,14,11,1,19,0,2,9,16,2
#Route Cost: row index = route index
23741.3
23341.3
23341.3
23341.3
22300.8
21900.8
21900.8
21900.8
27595.0
27195.0
27195.0
27195.0
# Routes: row index = route index
20,19,10,3,16,12,22,7
20,19,10,3,16,12,22,7
20,19,10,3,16,12,22,7
20,19,10,3,16,12,22,7
9,13,14,8,4,2,6,11
9,13,14,8,4,2,6,11
9,13,14,8,4,2,6,11
9,13,14,8,4,2,6,11
28,24,0,15,17,21,27,29,23,18,26,1,25,5
28,24,0,15,17,21,27,29,23,18,26,1,25,5
28,24,0,15,17,21,27,29,23,18,26,1,25,5
28,24,0,15,17,21,27,29,23,18,26,1,25,5
# Components that can be maintained by route index nr
0
0
0
0
0
```

0

0

0

0

0

0

0

Qf that can be used by a route: row index = route index

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

A.5 Case 2.1

Initial age of components

23,13,19,6,10,13,22,5,1,11,16,2,11,9,3,

14,19,14,17,6,22,8,19,5,15,5,22,10,22,8

#Route Cost [euro]: row index = route index

59641.2

54496.0

69112.7

Routes: row index = route index

26,2,11,7,29,15,0,6

16,23,8,1,17,27,21,9

10,14,13,19,5,24,20,4,28,12,3,22,18,25

Components that can be maintained by each route : row index = route index

0

0

0

Qf that can be used by each route: row index = route index

1.0

1.0

1.0

A.6 Case 2.2

```
# Initial age of components
1,12,21,1,7,10,3,10,16,9,2,1,2,13,9,4,
5,14,1,14,13,24,11,19,8,15,6,21,23,3
#Route Cost [euro] : row index = route index
53442.3
53242.3
53242.3
53242.3
56867.6
56667.6
56667.6
56667.6
56667.6
70319.8
70119.8
70119.8
70119.8
# Routes: row index = route index
22,0,2,3,26,21,16,10
22,0,2,3,26,21,16,10
22,0,2,3,26,21,16,10
22,0,2,3,26,21,16,10
13,15,9,7,24,5,20,29
13,15,9,7,24,5,20,29
13,15,9,7,24,5,20,29
13,15,9,7,24,5,20,29
18,8,6,17,11,19,1,28,12,25,4,27,14,23
18,8,6,17,11,19,1,28,12,25,4,27,14,23
18,8,6,17,11,19,1,28,12,25,4,27,14,23
18,8,6,17,11,19,1,28,12,25,4,27,14,23
# Components that can be maintained by route index nr
0
0
0
0
0
```

0

0

0

0

0

0

0

Qf that can be used each route: row index = route index

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

A.7 Case 2.3

Initial age of components

4,6,22,3,12,3,4,17,0,4,10,24,24,20,16,

15,18,1,0,9,23,14,6,9,19,12,19,9,6,11

#Route Cost [euro] : row index = route index

56413.7

56213.7

56213.7

56213.7

56078.1

55878.1

55878.1

55878.1

69539.4

69339.4

69339.4

69339.4

Routes: row index = route index

12,1,3,25,9,10,28,16

12,1,3,25,9,10,28,16

12,1,3,25,9,10,28,16

12,1,3,25,9,10,28,16

18,7,27,23,22,14,0,29

18,7,27,23,22,14,0,29

18,7,27,23,22,14,0,29

18,7,27,23,22,14,0,29

2,5,6,19,15,21,13,24,4,26,8,17,20,11

2,5,6,19,15,21,13,24,4,26,8,17,20,11

2,5,6,19,15,21,13,24,4,26,8,17,20,11

2,5,6,19,15,21,13,24,4,26,8,17,20,11

Components that can be maintained by each route: row index = route index

0

0

0

0

0

0

0

0

0

0

0

0

Qf that can be used by each route: row index = route index

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0

0.2,0.6,1.0

0.2

0.6

1.0