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*Published in:*  
Journal of Modern Power Systems and Clean Energy

*DOI (link to publication from Publisher):*  
[10.35833/MPCE.2020.000105](https://doi.org/10.35833/MPCE.2020.000105)

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*Publication date:*  
2022

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Wang, B., Yang, D., Cai, G., Ma, J., Chen, Z., & Wang, L. (2022). Online Inertia Estimation Using Electromechanical Oscillation Modal Extracted from Synchronized Ambient Data. *Journal of Modern Power Systems and Clean Energy*, 10(1), 241-244. [9237073]. <https://doi.org/10.35833/MPCE.2020.000105>

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# Online Inertia Estimation Using Electromechanical Oscillation Modal Extracted from Synchronized Ambient Data

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**Abstract**—An ambient modal framework for inertia estimation using synchrophasor data is proposed in this letter. Specifically, an analytical formulation is developed for the estimation of inertia based on the frequency and damping ratio modes extracted from ambient data. An advantage of the proposed framework is that it can rely on synchronized ambient data under non-disturbed conditions for online estimation and tracking of inertia. Ultimately, numerical simulation studies and physical experiments demonstrate the feasibility of the proposed approach.

**Index Terms**—Inertia, electromechanical oscillation, ambient modal.

## I. INTRODUCTION

THE increasing penetration rate of renewable power generation units has been a key factor for the concept of power system inertia estimation to garner significant attention [1]. In the case of transmission system operators (TSOs), the estimation of inertia is critical for the accurate assessment of system security, and an appropriate controller can be designed.

The common deployment of a phasor measurement unit (PMU) in the power system enables the inertia to be estimated from the measurement data. The strong correlation between the frequency change rate at the disturbance moment and the inertia, i.e., the swing equation without the damping coefficient, is utilized to estimate the inertia by means of the preset sudden power shock and measured frequency deviation [2].

To avoid the operation point deviation caused by sudden shock power, inertia estimation methods have been proposed to use the electromechanical signal oscillating around the operation point as the input [3], [4]. These methods determine the inherent relationship between the electromechanical modes and the inertia using the second-order oscillator model. Based on the developed relationship, the inertia can be estimated through the identification of electromechanical oscillation parameters [3] and the extraction of modes and mode shapes [4]. However, it is difficult to obtain an online rolling estimation of inertia using the existing methods wherein the oscillation modes are extracted from historical PMU data relying on a disturbance. In addition, the oscillation modes can be extracted from synchronized ambient data, which are always present in the power system without disturbance. Consequently, the online estimation of inertia using ambient modal data has become an active research topic.

The main contributions of this letter are as follows:

- 1) According to the power system dynamic equations under ambient excitation, a mathematical relationship between the inertia and ambient modal is established.
- 2) An analytical formulation for the estimation of inertia is developed.
- 3) Numerical simulations and physical experiments are conducted to test and validate the proposed framework.

## II. AMBIENT MODAL FRAMEWORK FOR INERTIA ESTIMATION

### A. Ambient Modal

Natural excitation exists in a real power system. The typical causes of this excitation are the electrical loads, which vary randomly by nature. Under natural excitation circumstances, the oscillatory behavior of a generator can be described by the transfer function shown in Fig. 1. The corresponding dynamic equation is as follows:

$$T_j \Delta \ddot{\delta}(t) + D \Delta \dot{\delta}(t) + K_s \Delta \delta(t) = F(t) \quad (1)$$

where  $\Delta \delta(t)$  is the rotor angle deviation;  $T_j$  is the inertia constant;  $D$  is the damping coefficient;  $K_s$  is the synchronizing power coefficient; and  $F(t)$  is the natural excitation of the system. In Fig. 1,  $\Delta \omega$  is the rotor speed deviation;  $H$  is the

Manuscript received: February 25, 2020; revised: June 2, 2020; accepted: July 23, 2020. Date of CrossCheck: July 23, 2020. Date of online publication: October 22, 2020.

This work was supported in part by the National Science Foundation of China (No. 51877032), and in part by the State Grid Corporation of China (No. 2018GWJLDKY06).

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DOI: 10.35833/MPCE.2020.000105



inertia constant; and  $\Delta P_e$  is the power deviation.

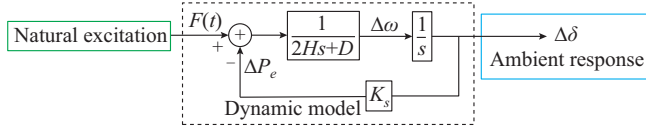


Fig. 1. Dynamic behavior of a generator under natural excitation.

Let  $f_n$  ( $\omega_n = 2\pi f_n$ ) and  $\zeta$  be the natural frequency and damping ratio, respectively. We can then obtain:

$$\Delta\ddot{\delta}(t) + 2\zeta\omega_n\Delta\dot{\delta}(t) + \omega_n^2\Delta\delta(t) = F(t) \quad (2)$$

Based on the random vibration theory [5], the time-domain response of the linear system, described by (2) under the action of the natural excitation  $F(t)$ , is given by:

$$\Delta\delta(t) = \int_{-\infty}^{+\infty} F(t-\tau)h(\tau)d\tau \quad (3)$$

where  $h(t)$  is the unit pulse response of the linear system described in (2). The power spectral density of  $h(t)$  can be described as:

$$H_u(\omega) = \int_{-\infty}^{+\infty} h(\tau)d\tau = \frac{1}{\omega_n^2 - \omega^2 + j(2\zeta\omega_n)\omega} \quad (4)$$

Suppose that the natural excitation  $F(t)$  is approximately a stationary Gaussian process, and that the power spectral density of  $F(t)$  is constant, i.e.,  $S_F(\omega) = C$ .

Furthermore, the autocorrelation power spectral density of the response  $\Delta\delta(t)$  can be obtained as follows:

$$S_\delta(\omega) = |H_u(\omega)|^2 S_F(\omega) = \frac{C}{(\omega_n^2 - \omega^2)^2 + 4(\zeta\omega_n)^2\omega^2} \quad (5)$$

Therefore, when  $\omega = \pm\omega_n\sqrt{1-2\zeta^2} \approx \pm\omega_n\hat{H}$  (as the damping ratio  $\zeta$  is much smaller than 1), there exist two stress peaks in the autocorrelation power spectral density of the ambient response, as shown in Fig. 1. The presence of the two peaks indicates that the oscillation modes are contained in the ambient responses under natural excitation circumstances. However, the ambient response is still characterized as random Gaussian white noise. The oscillation modes (frequency  $f_d$  and damping ratio  $\zeta$ ) can be extracted from synchronized ambient data measured by a PMU via appropriate methods [6] such as stochastic subspace identification (SSI) and frequency domain decomposition (FDD).

### B. Inertia Estimation

The inertia is strongly coupled with the electromechanical dynamic behavior of a power system, which affects not only the transient response but also the electromechanical oscillation by the oscillation frequency  $f_d$  and damping ratio  $\zeta$ . According to the modal analysis theory [7], the oscillation frequency and damping ratio of the dynamic system in (1) can be expressed as:

$$2\pi f_d = \frac{\sqrt{8\omega_0 H P_{e0} \cot \delta_0 - D^2}}{4H} \quad (6)$$

$$\zeta = \frac{D}{2\sqrt{P_{e0} \cot \delta_0 \cdot 2H\omega_0}} \quad (7)$$

where  $P_{e0}$  is the steady-state electrical power;  $\omega_0$  is the rated rotor speed; and  $\delta_0$  is the steady-state rotor angle.

Based on (6) and (7), the inertia constant expressed by the oscillation modes is derived as:

$$H = k_0 (1 - \zeta^2) f_d^{-2} \quad (8)$$

where  $k_0 = \omega_0 P_{e0} \cot \delta_0 / (8\pi^2)$  is the steady-state coefficient.

As the steady-state variables  $P_{e0}$  and  $\delta_0$  can be measured or calculated directly by a PMU, the inertia can be estimated once the oscillation modes are extracted from the synchronized ambient data.

### III. SIMULATION

In this letter, numerical simulations of a single-generator infinite bus system are conducted to demonstrate the performance of the proposed method using the Power System Toolbox (PST) [8]. The PST can simulate the ambient response of a power system by adding a random load following an independent Gaussian distribution with a determined amplitude. In the simulations, the amplitude is set to be 2% of the normal value. Moreover, a more mature SSI technology is adopted to extract the electromechanical oscillation modes from the ambient data. The accuracy of the proposed method is verified through 500 Monte-Carlo simulations.

The extracted electromechanical oscillation frequency and damping ratio are shown in Fig. 2. In addition, Table I lists the statistical results, that is, the mean value  $\mu$  and standard deviation  $Std$  of the extracted results. A comparison of the results shows that the mean value of the extracted modes is close to that of the small-signal stability analysis (SSSA) with a small  $Std$ .

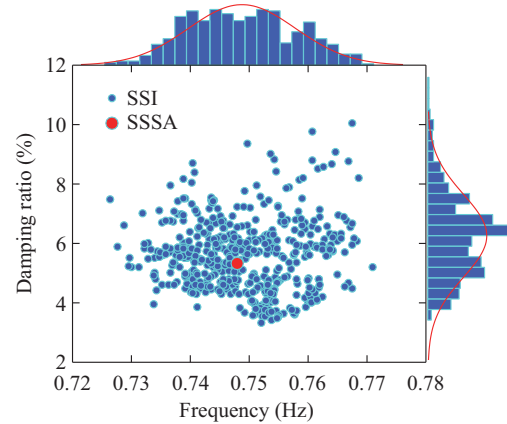


Fig. 2. Electromechanical modes extracted by SSI.

TABLE I  
COMPARISON OF SSI EXTRACTION MODE AND SSSA CALCULATION MODE

Method	Frequency (Hz)	Damping ratio (%)
SSSA	0.748	5.333
SSI	$\mu = 0.759, Std = 0.009$	$\mu = 5.596, Std = 1.171$

In the proposed inertia estimation method, the electrical power and rotor angle should be determined using the measured data, while the modes are to be extracted. To avoid er-

rors caused by random fluctuations, the mean values of the measured power and rotor angle are used in each simulation. Meanwhile, two calculation approaches are considered. The first approach, denoted by C-1, is to calculate the inertia based on the extracted modes and steady-state variables for each simulation so that the results contain 500 inertia estimation results. The second approach, denoted by C-2, is to calculate the inertia based on the means of the extracted modes and steady-state variables in 500 simulations to obtain only one inertia estimation result.

The inertia estimations obtained via C-1 are shown in Fig. 3. The estimation results randomly fluctuate around the real value. However, the range of fluctuation is small. A histogram of the inertia estimation results shows that the estimation results are highly concentrated close to the true value. In addition, as the deviation between the estimation and reality values increases, the counts gradually decrease. The distribution fitting curve of the inertia estimation has a peak point close to the real inertia. The data listed in Table II show that the estimation results follow a distribution with  $\mu$  of 2.859 s and *Std* of 0.0388.

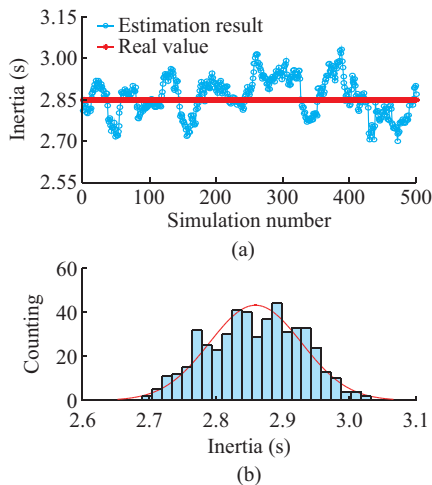


Fig. 3. Inertia estimation results obtained via C-1. (a) Inertia estimation results. (b) Counting results.

TABLE II  
ESTIMATION RESULTS BASED ON DIFFERENT MODELS

Model order	Reality	C-1	C-2	
			$\mu$	<i>Std</i>
2 <sup>nd</sup> -order	2.85	2.855	2.859	0.0388
6 <sup>th</sup> -order	2.85	2.787	2.794	0.0382
6 <sup>th</sup> with exciter	2.85	2.916	2.921	0.0367

The inertia obtained via the C-2 approach is 2.855 s, which is close to the real value. The inertia estimations obtained via the C-1 and C-2 approaches are both close to the real value, which indicates the accuracy of the proposed method.

Table II shows the estimation results considering the 2<sup>nd</sup>- and 6<sup>th</sup>-order with the generator exciter models. The small deviations between the estimation results based on the high-

order models and the real value show the high robustness of the proposed inertia estimation method for different models.

#### IV. PHYSICAL EXPERIMENT

To verify the effectiveness of the proposed method further, a physical experiment is performed and the system parameters are given in Appendix A. The system configuration is shown in Fig. 4, wherein the load variation is controllable by a program to excite the ambient response of the system, and  $X_l$  is the line reactance. The PMU-measured electrical power signal is shown in Fig. 5.

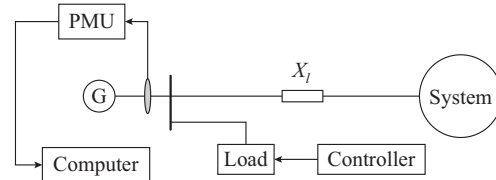


Fig. 4. System configuration of physical experiment.

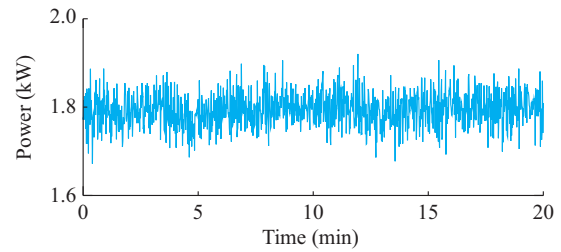


Fig. 5. PMU-measured electrical power signal.

As presented in Table III, the estimation result obtained using the mean values of the oscillation parameters is 1.768 s, which deviates from the reality by 0.183 s. The  $\mu$  and *Std* values of the estimation results obtained using the time-evolution oscillation parameters are 1.717 and 0.0478, respectively. The mean value is close to the real value, and the small standard deviation shows the effectiveness of the proposed method.

TABLE III  
ESTIMATION RESULTS FROM PHYSICAL EXPERIMENT

Reality	C-1	C-2	
		$\mu$	<i>Std</i>
1.585	1.768	1.717	0.0478

#### V. CONCLUSION

In this letter, an ambient modal based framework is proposed for power system inertia estimation using synchronized data. The results of a numerical test system simulation and a physical experiment show that the proposed method achieves high computation efficiency and exhibits robust performance under ambient excitation conditions. These advantages render the proposed method an appropriate and promising approach for effective inertia estimation in online rolling applications.

## APPENDIX A

The system parameters for the physical experiment are as follows: the rated capacity  $S_n = 5$  kVA, the rated voltage  $V_n = 400$  V, the line reactance  $X_l = 0.125$  p.u., the synchronous reactance  $X_d = 0.96$  p.u., the transient reactance  $X'_d = 0.14$  p.u., the sub-transient reactance  $X''_d = 0.07$  p.u., the inertia constant  $H = 1.585$  s, and the open circuit time constants  $T'_{d0} = 2.01$  s,  $T''_{d0} = 0.15$  s.

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