Numerical Investigations on Detectability of Crack by Contactless j_C-Measurement Method*)

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The inductive method for measuring the critical current density j_C and detecting the crack in a high-temperature superconducting (HTS) film have been investigated numerically. To this end, a numerical code has been developed for analyzing the shielding current density in the film with a crack. The results of computations show that the accuracy of the inductive method is degraded remarkably due to the crack. Specifically, it is found that, if the orthographic projection of the coil overlaps with the crack, the value of measured j_C decreases. This is mainly because the spatial distribution of the shielding current density becomes asymmetry. In conclusion, the crack size and position can be accurately detected by measuring the j_C-distribution in the HTS film.

Keywords: critical current density, crack detection, high-temperature superconductor, inductive method, numerical simulation

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1. Introduction

As is well known, a high-temperature superconductors (HTSs) are used for developing various devices and systems such as nuclear fusion reactor, flywheel, and MRI, and they are characterized by some parameters. In particular, a critical current density j_C is one of the most important parameters, and it is important to measure the value of j_C accurately.

The standard four-probe method [1] has been generally used for measuring j_C. In the method, electrodes are deposited with a silver paste to decrease the measurement error. After a large current source flows in an HTS sample, j_C can be evaluated accurately from nonlinear V-I characteristics. However, HTS characteristics may be degraded because of a heat generated between the electrodes and the sample. As a result, the process may lead to the destruction of a sample surface or to the degradation of superconducting characteristics. For this reason, contactless methods have been so far desired for measuring j_C.

As a contactless method for measuring j_C, Claassen *et al.* have proposed the inductive method [2]. By applying an ac current to a small coil placed just above an HTS film, they monitored a harmonic voltage induced in the coil. They found that, only when a coil current exceeds a threshold current I_T, the third-harmonic voltage develops suddenly. They conclude that j_C can be evaluated from the threshold current. In contrast to this, Mawatari *et al.* elucidated the inductive method on the basis of the critical state model [3]. From their results, they derived a theoretical formula between j_C and I_T. In addition, they found that the theoretical curve shows an excellent agreement with the experimental one. In this way, the scientific basis of the inductive method was theoretically justified. Incidentally, this method has been successfully employed as the measurement of the j_C-distributions the detection of a crack [4].

In the previous study, a numerical code was developed for analyzing the time evolution of a shielding current density in an HTS film with crack [5]. By using the code, the inductive method was reproduced numerically. The results of computations showed that, although the threshold current I_T decreases due to the crack, its value does not necessarily decrease for the case with a small crack size. In fact, the accuracy is not degraded when the inner radius of the coil contains the crack of the film. For this reason, we conclude that the smallest possible inner radius is preferable to detect the crack.

The purpose of the present study is to simulate the inductive method for the case with crack containing in the HTS film, and we investigate the applicability of the detection of the crack size and position.

2. Governing Equation

In Fig. 1, we show a schematic view of an inductive method. A small M-turn coil is placed just above a rectangle-shaped HTS film of the length a and the thickness b, and an ac current $I(t) = I_0 \sin 2\pi f t$ flows in it. Furthermore, the square cross-section of the film is denoted by Ω, and an outer boundary of Ω is expressed by C_0. Furthermore, we assume that a crack exists in Ω and its shape is given by an inner boundary C_1.

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Throughout the present study, we adopt the Cartesian coordinate system $\langle O : \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z \rangle$, where z-axis is parallel to the thickness direction. The origin O is chosen at the center of an HTS upper surface. In addition, the unit vectors along the x-, y- and z-directions are denoted by $\boldsymbol{e}_x$, $\boldsymbol{e}_y$ and $\boldsymbol{e}_z$, respectively. We assume that a vertical section of the coil is given by $r_1 \le r \le r_2$ and $z_1 \le z \le z_2$ with the cylindrical coordinates (r, θ, z). In order to determine the coil position, the symmetrical axis of the coil is shown by $(x, y) = (x^*, y^*)$.

According to the experimental results, the YBCO superconductors have a strong crystallographic anisotropy: the current flow in the c-axis direction differs from that in the ab-plane, and the flow along c-axis is almost negligible. Here, the c-axis is the direction along z, and it is perpendicular to the ab-plane. On the basis of the fact, we assume the thin-layer approximation [6]: the thickness of the HTS film is sufficiency thin that the shielding current density $\boldsymbol{j}$ can hardly flow in the thickness direction.

A shielding current density $\boldsymbol{j}$ is closely related to an electric field $\boldsymbol{E}$. The relation can be written as

$$\boldsymbol{E} = E(|\boldsymbol{j}|)\left(\frac{\boldsymbol{j}}{|\boldsymbol{j}|}\right), \tag{1}$$

where a function $E(j)$ is given by the power law:

$$E(j) = E_{\mathrm{C}}\left(\frac{j}{j_{\mathrm{C}}}\right)^N. \tag{2}$$

Here, E_{C} is the critical electric field, and N is a constant.

Under the above assumptions, the shielding current density $\boldsymbol{j}$ can be written as $\boldsymbol{j} = (2/b)\nabla S \times \boldsymbol{e}_z$, and the time evolution of the scalar function $S(\boldsymbol{x}, t)$ is governed by the following integro-differential equation [6]:

$$\mu_0 \frac{\partial}{\partial t}(\hat{W}S) + \frac{\partial}{\partial t}\langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = 0, \tag{3}$$

where a bracket $\langle\ \rangle$ denotes an average operator over the thickness of the film, and $\boldsymbol{E}$ is an electro magnetic field. In addition, $\hat{W}S$ is defined by

$$\hat{W}S \equiv \iint_{\Omega} Q(|\boldsymbol{x} - \boldsymbol{x}'|)S(\boldsymbol{x}', t)d^2\boldsymbol{x}' + (2/b)S(\boldsymbol{x}, t). \tag{4}$$

Here, both $\boldsymbol{x}$ and $\boldsymbol{x}'$ are position vectors in the xy-plane. The explicit form of $Q(r)$ is described in [6].

$$Q(r) = -\frac{1}{\pi b^2}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + b^2}}\right). \tag{5}$$

Note that $Q(r)$ becomes singular at $r = 0$. Therefore, it is clearly necessary to evaluate the first right-hand side of (2) as an improper integral. Incidentally, the high-performance method of this integral is described in [7].

The initial and boundary conditions to (3) are assumed as follows:

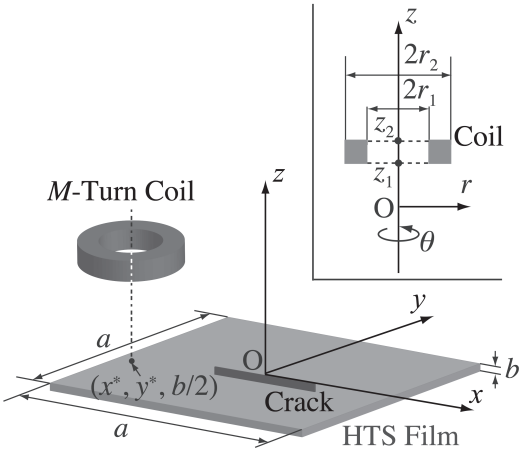

Fig. 1 A schematic view of an inductive method.

$$S = 0 \text{ at } t = 0, \tag{6}$$

$$S = 0 \text{ on } C_0, \tag{7}$$

$$\frac{\partial S}{\partial s} = 0 \text{ on } C_1, \tag{8}$$

$$\oint_{C_1} \boldsymbol{E} \cdot \boldsymbol{t} ds = 0. \tag{9}$$

Here, s and $\boldsymbol{t}$ denote an arclength along C_1 and the tangential unit vector on C_1, respectively. By solving the initial-boundary-value problem of (3), we can determine the time evolution of the shielding current density in an HTS film.

Discretized with the backward Euler method, the initial-boundary-value problem of (3) is reduced to the nonlinear boundary-value problem. In order to solve the problem, we adopt the Newton method and the finite element method. On the basis of the numerical method, a numerical code has been developed for analyzing the shielding current density in an HTS film with a crack.

3. Simulation of Inductive Method

3.1 Mawatari's theory

According to Mawatari's theory [3], a critical current density j_{C} can be calculated from

$$j_{\mathrm{C}}^{\mathrm{N}} = \frac{2F(r_{\max})I_{\mathrm{T}}^*}{b}, \tag{10}$$

where $j_{\mathrm{C}}^{\mathrm{N}}$ denotes an estimated value of j_{C}, and $F(r_{\max})$ is the maximum value of a primary coil-factor function $F(x)$ [3] determined from the configuration of the coil and the HTS . I_{T}^* is a lower limit of the coil current I_0 when a third-harmonic voltage V_3 begins to develop in the coil. An important point is that (10) is also applicable only to an HTS film without any cracks.

In order to simulate the inductive method, calculating a third-harmonic voltage V_3 from time evolution of the shielding current density $\boldsymbol{j}$, we must determine an V_3-I_0 curve. For evaluating the value of I_{T}, we use the conven-

Table 1 The Geometrical and the physical parameters.

HTS Film	Length	a	20 mm
	Thickness	b	600 nm
	Critical current density	j_C	1 MA/cm^2
	Critical electric field	E_C	1 mV/m
	N-value of power law	N	20
Coil	Number of turns	M	400
	Frequency	f	1 kHz
	Bottom	z_1	0.2 mm
	Top	z_2	1.2 mm
	Inner radius	$2r_1$	2 mm
	Outer radius	$2r_2$	4 mm
	Maximum value of $F(x)$	F_{max}	7.28×10 m^{-1}

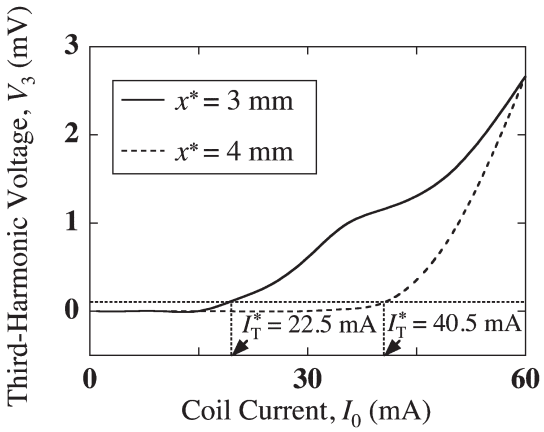

Fig. 2 V_3-I_0 curves for the case with $L_c = 4$ mm and $y^* = 0$ mm.

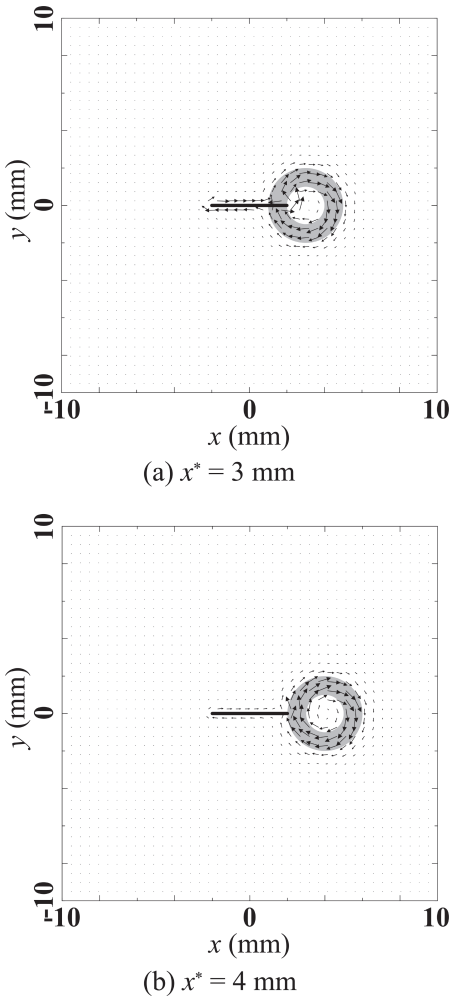

Fig. 3 Spatial distributions of the shielding current density $\boldsymbol{j}$ at time $ft = 1.2$. Here, $L_c = 4$ mm and $y^* = 0$ mm.

tional voltage criterion $V_3 = 0.1$ mV $\Leftrightarrow I_0 = I_T^*$ [3]. The numerical method of V_3 is described in [7].

3.2 Detection of crack size and position

In this subsection, we investigate the applicability of the detection of a crack size and position by simulating the inductive method. Hereafter, we assume that a crack shape is a line segment parallel to x-axis. In addition, the center of crack position is denoted by $(x, y) = (x_c, y_c)$ in the xy-plane, and the crack size is expressed as L_c. Values of the geometrical and the physical parameters used in the present study are tabulated in Table 1. In this study, the critical current density j_C is assumed to be homogeneous.

Firstly, we investigate the influence of the crack on a third-harmonic voltage V_3. In Fig. 2, we show the V_3-I_0 curves for various coil position L_c's. We see from this figure that, beginning to develops drastically from a certain value of I_0, the third-harmonic voltage V_3 increases with I_0. By applying the voltage criterion to the V_3-I_0 curve for $x^* = 3$ mm, we get $I_T^* = 22.5$ mA (see Fig. 2). This value is remarkably different from the analytic value $I_T^A = 41.2$ mA of I_T. Incidentally, I_T^A can be calculated from the formula $I_T^A = j_C b/[2F(r_{max})]$ derived from (10). On the other hand, the value of I_T^* for $x^* = 4$ mm is $I_T^* = 40.5$ mA by using the voltage criterion.

Let us investigate the behavior of the shielding current density $\boldsymbol{j}$ for Fig. 2. In Figs. 3 (a) and (b), we show the spatial distributions of the shielding current density $\boldsymbol{j}$. We see from this figure that, for $x^* = 3$ mm, the spatial distribution of the shielding current density $\boldsymbol{j}$ is drastically disorder due to the crack. As a result, the distribution becomes asymmetry. On the other hand, for $x^* = 4$ mm, the spatial distribution of $\boldsymbol{j}$ is almost symmetric about the symmetry axis of the coil because $\boldsymbol{j}$ hardly affects the crack. From these results, we imply that the crack size may be detected by using the inductive method.

Let us investigate the applicability of the detection of the crack size L_c. To this end, we define the relative error:

$$\varepsilon \equiv \frac{|j_C^N - j_C|}{j_C}. \tag{11}$$

Moreover, the symmetry axis of the coil is moved to x- and y-directions, and we examine the spatial distribution of the relative error ε. In Fig. 4, we show the dependence of the relative error ε on the coil position x^* for the case with $L_c = 4$ mm and $L_c = 8$ mm. This figure indicates that, for

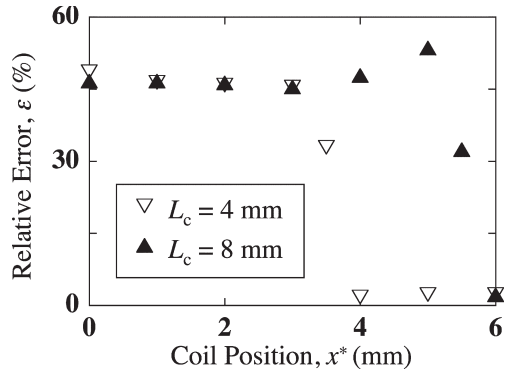

Fig. 4 Dependence of the relative error ε on the coil position x^* for the case with $y^* = 0$ mm.

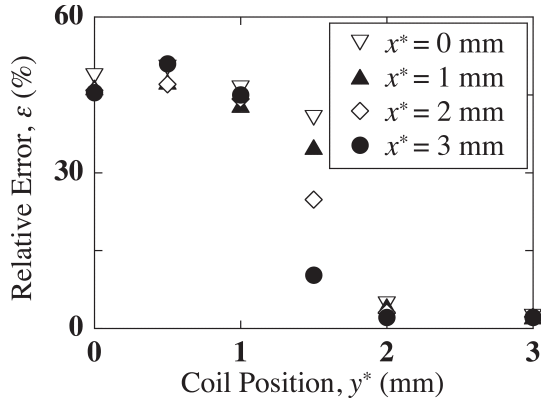

Fig. 5 Dependence of the relative error ε on the coil position y^* for the case with $L_c = 4$ mm.

$L_c = 4$ mm, the relative error ε becomes remarkably large for $x^* < 4$ mm. This case means that the orthographic projection of the coil overlaps with the crack (e.g. Fig. 3 (a)). In contrast, it is found that, for $x^* > 4$ mm, the value of ε is almost constant, and the crack and the coil have in contact with each other at the point $(x, y) = (4\,\text{mm}, 0\,\text{mm})$ (see Fig. 3 (b)). For $L_c = 8$ mm, the value of ε is not less than 30% except for $x^* = 6$ mm. This result shows a similar tendency to $L_c = 4$ mm. Therefore, the orthographic projection of the coil significantly affects the accuracy of the inductive method.

Finally, the relative error ε is calculated as a function of the coil position y^* and is depicted in Fig. 5. We see from this figure that, for $y^* < 2$ mm, the value of the relative error ε is not less than 10% regardless x^*, whereas ε is about 5% or less for $y^* \geq 2$ mm. In Fig. 6, we show the spatial distribution of the shielding current density $\boldsymbol{j}$ for $x^* = y^* = 2$ mm. From this figure, although $\boldsymbol{j}$ slightly flow along the crack, the spatial distribution of $\boldsymbol{j}$ becomes roughly symmetry.

From these results, we conclude that if the orthographic projection of the coil overlaps with the crack, the accuracy of the inductive method is hardly degraded. Therefore, by measuring the spatial distribution of the critical current density j_C, the crack size and position can be detected accurately.

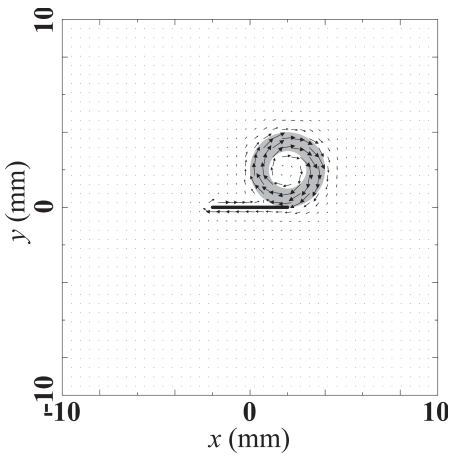

Fig. 6 Spatial distribution of the shielding current density $\boldsymbol{j}$ at time $ft = 1.2$. Here, $L_c = 4$ mm and $x^* = y^* = 2$ mm.

4. Conclusion

We have developed a numerical code analyzing the time evolution of the shielding current density in an HTS film containing a crack. By using the code, reproducing the inductive method numerically, we assess the applicability of the detection of the crack size and position. Conclusions obtained in the present study are summarized as follows: if the orthographic projection of the coil overlaps with the crack, the accuracy of the inductive method is hardly degraded. This is mainly because the shielding current density flow on the region of the orthographic projection of the coil. Therefore, by measuring the spatial distribution of the critical current density, the crack size and position can be detected accurately.

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