This article has been accepted for publication in 2022 4th Global Power, Energy and Communication Conference (GPECOM). This is the author's version which has not been fully edited and content may change prior to final publication.
Citation information: DOI: 10.1109/GPECOM55404.2022.9815713

Evaluation of Static Network Equivalent Models for N-1 Line Contingency Analysis

M. Ramirez-Gonzalez, M. Bossio, F. R. Segundo Sevilla, P. Korba
Institute of Energy Systems and Fluid Engineering
ZHAW Zurich University of Applied Sciences
Winterthur, Switzerland

Abstract—**Although large scale simulation models may better represent the behavior of practical power systems, they are time-consuming and turn out to be impractical for some desired applications, particularly when the focus of the study is on only a small portion of the entire system and the use of a complete model would dramatically increase the computational effort and time. Therefore, network equivalent models can be used in this case to facilitate and accelerate the completion of the required analysis related to the specific subsystem. In this sense, and considering here small and large sample networks, the evaluation of static equivalents derived from popular Ward and REI reduction methods is presented in this paper. The performance of the reduced networks is assessed under the N-1 contingency criteria, and takes into account not only the base case condition but also post contingency response with the whole set of lines in the particular area of study. Obtained equivalents are compared in simulation in terms of deviations from the original system and average computational time needed to complete the involved power flow calculations.**

Keywords—static network equivalents, line contingency analysis, static security assessment, power system stability

I. Introduction

The complexity and continuously changing dynamics of power systems, as well as the incorporation and massive deployment of new devices and technologies to contribute to safer, reliable, and cleaner electrical networks, is bringing about new challenges in terms of ensuring network security and stability [1], [2]. However, the use of very large simulation models for extensive power system analysis may hinder the effective and practical application of such representations due to the high complexity and significant computational burden involved [3], [4]. In this context, model numerical issues can also become a very time-consuming problem to deal with under various operating conditions. Besides, maintenance of an up-to-date model database may be difficult to accomplish due to multiple system operating entities and data confidentiality restrictions. In addition, depending on the particular and local focus of the analysis, the relative influence of regions far away from the area of interest or internal system may well allow the use of simplified model for the external grid.

In general, according to the intended study and application, the reduction of large power systems can be oriented to the development of static and dynamic network equivalents. In this case, for power flow computations and steady-state security assessment focused on a specific portion of the entire system, the construction of static equivalent models able to reproduce relatively accurate load flow responses as with the full network is of particular interest. Popular approaches widely used in the literature for this purpose are based on the Ward and REI (Radial, Equivalent, Independent) reduction methods [4]-[10]. These alternatives rely on the definition of an internal system (network to be retained), the external system (network to be reduced), and the boundary buses (interface between the internal and external systems). However, while the external network in the Ward methods is basically approximated with equivalent representations that includes power injections and lines at the interface nodes, the REI approach carries out this approximation by separately combining all productions and loads and creating fictitious nodes that are interconnected to the boundary buses through a lossless network.

Since the convenient use of electrical equivalents of major interconnected power systems may significantly reduce computation times and increase the speed of simulations, the derivation of simplified system representations for different study purposes in large power grids continues to be of great value. In this regard, appropriately reduced networks may facilitate and accelerate for example the exploration and validation of innovative analytical approaches, and the design and evaluation of advanced control strategies for addressing different system security and stability challenges brought about by the continuous development of the grid.

Considering that the steady-state response of power systems under a predefined set of credible contingencies is a fundamental part of the security assessment in electrical networks, many research works can be found in the literature about the application of equivalent models in this area using different reduction techniques [4]-[12]. Unfortunately, discussions about the accuracy of the obtained equivalent networks in this regard, as compared to the full systems, are typically related to the results with the base case and only some post contingency condition, which may lead to imprecise and biased conclusions. In addition, relatively small sample networks usually considered for illustration purposes do not give a practical insight about computational speed improvements that network equivalents of large power grid models may achieve in simulations. Based on these issues, commonly used Ward and REI reduction methods are applied in this paper to derive equivalent models for small and relatively large sample power systems, and then an evaluation of the corresponding reduced networks under N-1 line contingency is carried out for the whole set of transmission lines in the retained system. Simulations results in terms of bus voltage and line power flow deviations, as well as average computational times to accomplish power flow calculation tasks, are provided here to compare the effectiveness and efficiency of the obtained equivalents.

II. Ward and REI Static Network Equivalents

A. Ward based approaches

Ward reduction methods are essentially based on the representation of the network to be reduced by appropriate mutual impedances between boundary buses, power injections at boundary nodes, and the inclusion of additional fictitious generators through single fictitious branches. Therefore, according to the model used to represent the less relevant part of the system to be studied, related equivalent types can be mainly classified as Ward Admittance (WA), Ward Injection (WI), and Extended Ward (WX). In this context, while power injections are represented only by equivalent shunt admittances in the WA method, as shown in Fig. 1, constant current injections are additionally included in the WI approach, which is illustrated in Fig. 2. Furthermore, fictitious generator buses (PV nodes) and branches are created in WX reduction, as depicted in Fig. 3, to improve system reactive power response under contingency evaluation [8].

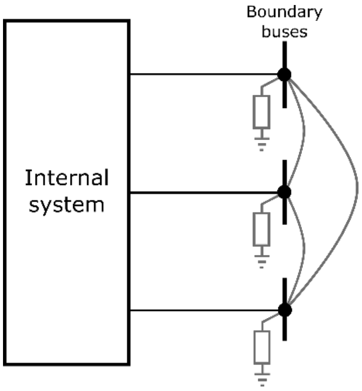

Fig. 1. WA equivalent.

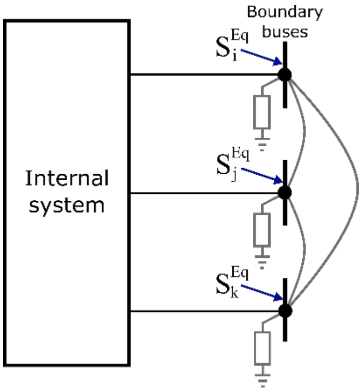

Fig. 2. WI equivalent.

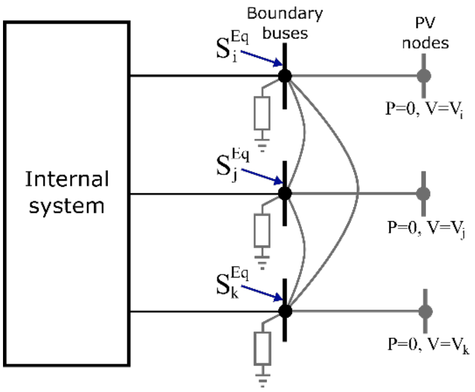

Fig. 3. WX equivalent.

In general, by considering the relationship of system voltages and currents given in expression (1), where boundary buses have been merged with internal ones and the admittance matrix has been partitioned into internal and external networks, Gaussian elimination can be applied to derive an equivalent representation focused on the portion of interest.

$$\begin{bmatrix} I_i \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ii} & Y_{ie} \\ Y_{ei} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_i \\ V_e \end{bmatrix} \quad (1)$$

With I_i and V_i representing respectively current and voltage at the buses to be retained, and I_e and V_e referring correspondingly to the current and voltage at buses to be eliminated, the convenient rearrangement of V_e as indicated in (2), and substitution of it in the first row of (1), will provide the general form of the equivalent given in (3).

$$V_e = Y_{ee}^{-1}(I_e - Y_{ei}V_i) \quad (2)$$

$$I_i = (Y_{ii} - Y_{ie}Y_{ee}^{-1}Y_{ei})\, V_i + Y_{ie}Y_{ee}^{-1}\, I_e \quad (3)$$

If all generation and loads of the external network are transformed first to shunt admittances, then I_e=0 in (1). Therefore, the resultant equivalent in this case will be defined only by the first term of (3), which specifies the corresponding branches and shunt components connecting retained buses. On the other side, by converting the external generation and loads to constant current injections, the obtained equivalent will include in addition the second term of (3), which represents the equivalent currents that should be injected to reproduce the effect of the eliminated part.

B. REI method

The REI approach is basically focused on the aggregation of external network's buses into representative fictitious nodes that conveniently combine all the productions on one side, and all the load on the other side [5]. During this process, a zero power balance network is created between the external system and the fictitious REI nodes, as illustrated in Fig. 4. After that, the non-essential nodes and the lossless fictitious network are reduced together via Gaussian elimination to finally obtain a simplified network interconnecting only the retained boundary buses and the REI generation and load nodes, as represented in Fig. 5.

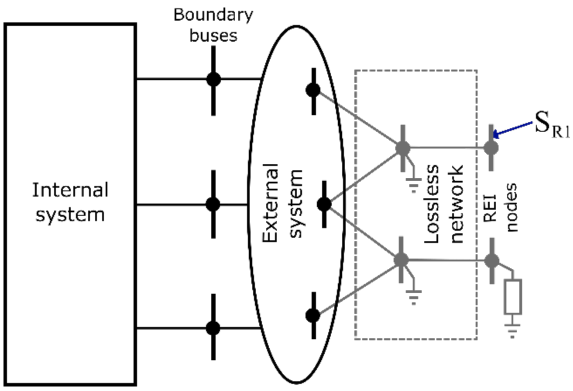

Fig. 4. Inclusion of the lossless network and REI nodes.

As observed from Fig. 5, the REI method will produce an equivalent network comparatively denser than a Ward equivalent. On the other hand, while equivalent current injections are basically considered for every boundary bus in the Ward method, the REI approach offers the capability to define and associate them for example according to production type, which will determine the number of final equivalent injections and related fictious buses.

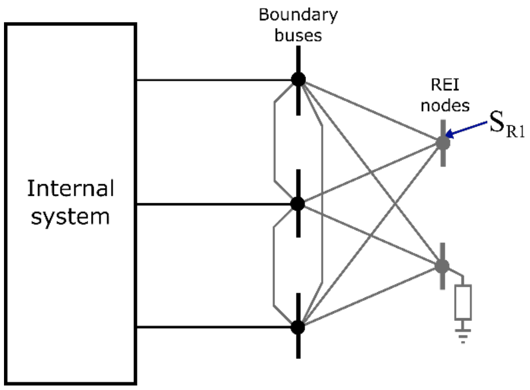

Fig. 5. REI equivalent.

III. Test Systems

A. New England Power System (NEPS)

The 39 Bus New England Benchmark System, which is a simplified model of the high voltage transmission grid in the northeast of the U.S.A., is used here as example of a relatively small system. It consists of 39 buses, 10 synchronous generators, 19 loads, 34 lines, and 12 transformers, and has been extensively used for different research goals in the literature. Figure 6 shows the single line diagram of the system where, for the purpose of the studies here, three different regions are defined and highlighted. Information about the parameters for modeling grid components is taken from [13].

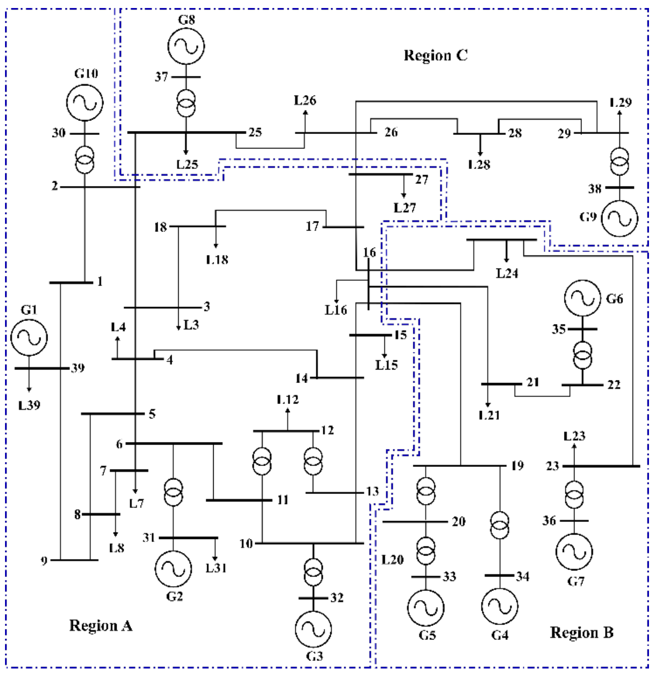

Fig. 6. New England power system.

B. Texas Grid (TXG)

The considered Texas Grid simulation model is a publicly available synthetic representation of the Texas Transmission system [14], [15]. Although it bears no relation to the actual referenced grid in their geographical location, it can be used for research purposes such as the testing and validation of new tools and techniques. The model involves the simulation of 2000 buses, 544 synchronous machines, 1350 loads, 2345 transmission lines, 861 transformers, and 157 static var compensators [13]. An illustration of the system is shown in Fig. 7, which is divided into eight areas: Far West, North, West, South, North Central, South Central, Coast, and East.

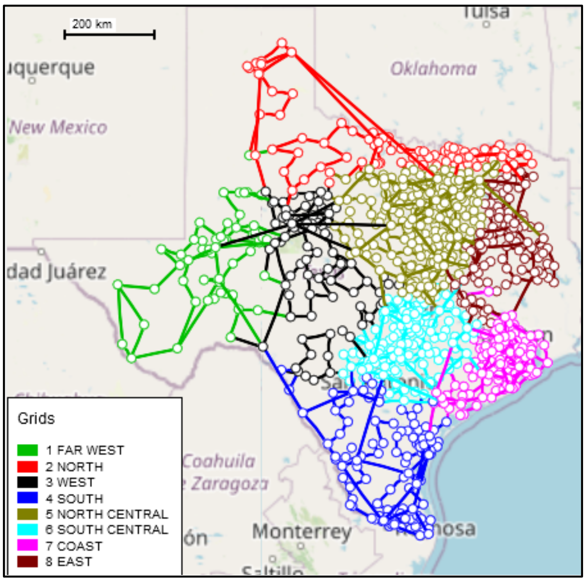

Fig. 7. Texas grid model.

IV. Simulation Results

For illustrative purposes, the methods described in Section II were applied here to derive static equivalents of the sample systems. The whole network reduction process was carried out in DIgSILENT software [13]. Moreover, the simulation of the loss of transmission lines (under N-1 contingency criteria) and the corresponding power flow computations were performed in an automated way through a script developed in Pyhton [16]. Reported computational times in the examples are based on average estimates considering a computer with processor Intel Core i7-8665U, CPU @ 1.90 GHz, and 16.0 RAM.

A. NEPS

Based on Fig. 6, Region A is assumed in this example as the portion of the system to be kept, while Regions B and C represent the external network to be replaced by an equivalent model (interconnected through the region of interest through interface nodes only). A comparison of the number of components involved in the full system against those ones in the region to be retained is provided in Table I.

TABLE I. Comparison of Full and Reduced System

Name	Element	Full system	Reduced system
NEPS	Regions	3	1
	Buses	39	23
	Generators	10	4
	Loads	19	11
	Lines	34	22
	Transformers	12	5

After reducing the system to the region of concern, the performance of the equivalent grids with WA, WI, WX, and REI alternatives is illustrated in Figs. 8, 9, 10, and 11, which respectively show the resultant deviations in voltage magnitude, voltage angle, active power, and reactive power, as compared to the full system. These deviations represent average absolute values, where the average quantities are obtained over the buses involved in Region A for the case of voltage magnitudes and angles, and over the lines contained in this region for the case of active and reactive powers.

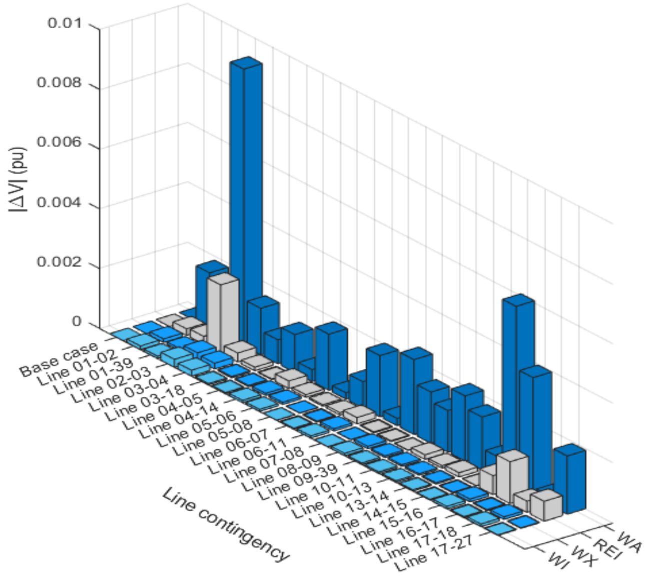

Fig. 8. NEPS: Average absolute value of voltage magnitude deviations.

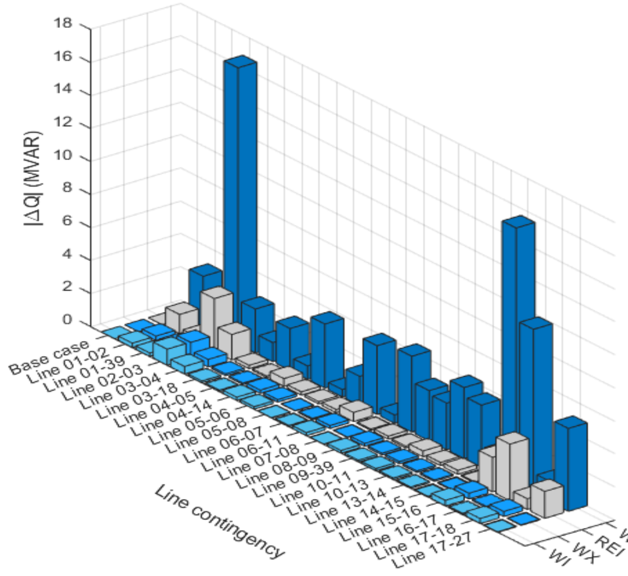

Fig. 11. NES: Average absolute value of reactive power deviations.

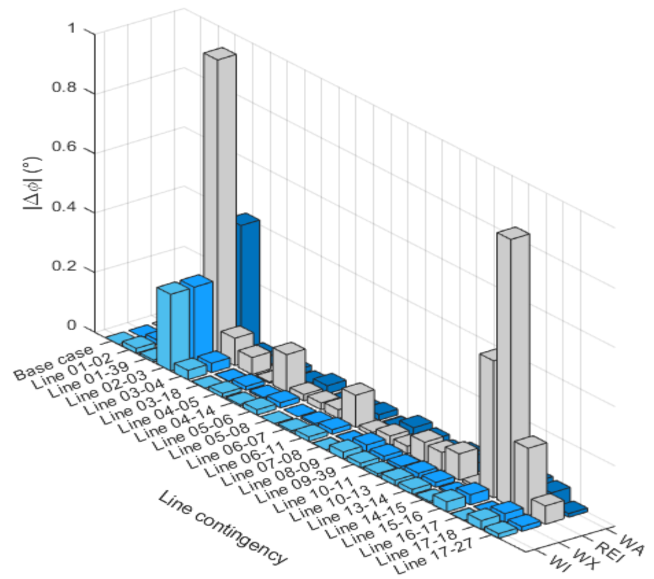

Fig. 9. NEPS: Average absolute value of voltage angle deviations.

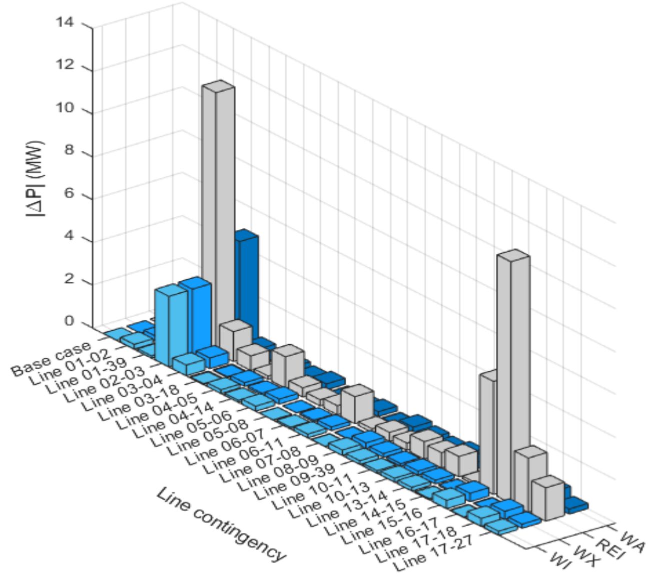

Fig. 10. NEPS: Average absolute value of active power deviations.

Based on the simulation results, the four reduction methods can provide an excellent performance in the base case scenario (according to the considered quantity errors). However, from Figs. 8 through 11, it can be observed that resulting deviations may change for every prescribed line outage, and that a better response is obtained in general with the WI and WX reduced models in this example, as compared to the REI and WA equivalents. The larger deviations individually perceived are predominantly associated with boundary buses and respective interconnecting lines. By computing the total average errors involving base case and line contingency conditions, the effectiveness of the considered reductions methods can be quantified as in Table II. From the values in this table, it is clear that the WI and WX methods were able to provide relatively good and similar results here, as compared to the performance of the REI and WA alternatives.

TABLE II. TOTAL AVERAGE ERRORS

Measure	Reduction method			
	WI	**WX**	**REI**	**WA**
|ΔV| (pu)	6.799E-05	6.768E-05	3.244E-04	1.895E-03
|Δϕ| (°)	2.355E-02	2.356E-02	1.577E-01	4.050E-02
|ΔP| (MW)	3.011E-01	3.013E-01	1.967	4.546E-01
|ΔQ| (MVAR)	1.822E-01	1.821E-01	7.450E-01	3.924

Now, Table III provides the average computational time estimated to complete the power flow calculations under N-1 contingency criteria for all the lines in Region A in Fig. 6. Due to the relatively small size of this test system, only very minor improvements in this sense can be noted in seconds. However, the time accomplished with the WX method for example represents only around 75% of the time with the original system, which can become important when dealing with very large power grids.

TABLE III. COMPARISON OF COMPUTATIONAL TIME

System	Model version				
	Original	**WI**	**WX**	**REI**	**WA**
NES	0.324 s	0.287 s	0.242 s	0.284 s	0.231 s

B. TXG

Different to the NEPS, the sample Texas Grid represents a larger system in this case. Based on Fig. 7, the area denominated as Far West (in green color) is assumed as the part of the system of interest (to be retained). Therefore, the rest of the areas are considered as external and will be substituted by an equivalent network. In this example, the elements of the grid in the full system and in the area to be kept are summarized in Table IV.

TABLE IV. COMPARISON OF FULL AND REDUCED SYSTEM

Name	Element	Full system	Reduced system
TXG	Regions	8	1
	Buses	2000	91
	Generators	544	40
	Loads	1350	54
	Lines	2345	84
	Transformers	861	35

By the application of the WA, WI, WX, and REI reduction techniques, the average deviations (with respect to the original system) of voltage magnitudes and angles, and active and reactive powers, presented in Figs. 12, 13, 14, and 15, are respectively obtained. According to these results, the WI, WX, and REI equivalent models provide a very good performance in this case since absolute deviations remain around very small values in general. On the other hand, it is evident that the WA equivalent is not able to appropriately represent the reactive power response of the system for different line outages, although an acceptable accuracy might be achieved generally for the base case and a few selected N-1 line contingency scenarios. The calculations of the total average errors associated with each equivalent derived for the TXG system are given in Table V, which quantitatively confirms the comparable performance of the WI, WX, and REI models for the particular example under consideration.

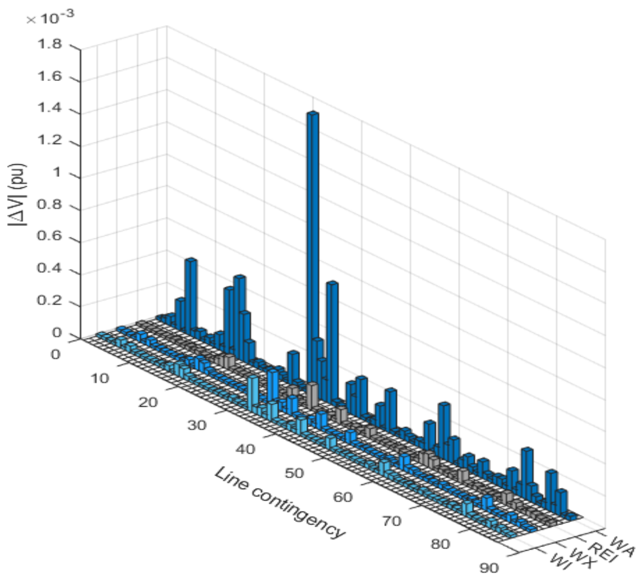

Fig. 12. TXG: Average absolute value of voltage magnitude deviations.

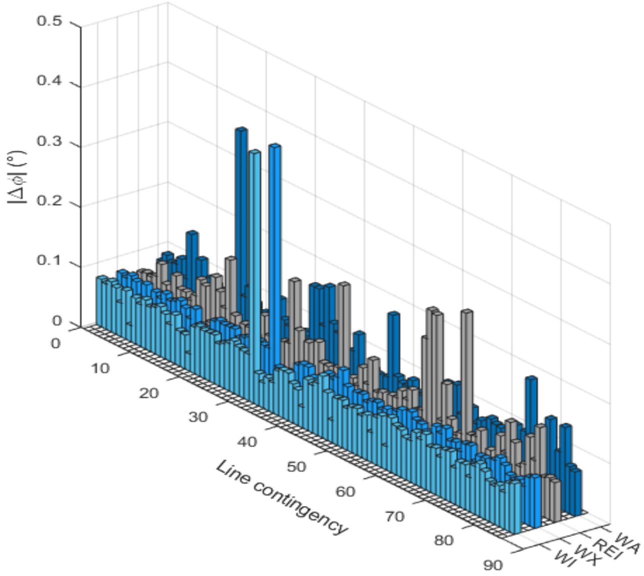

Fig. 13. TXG: Average absolute value of voltage angle deviations.

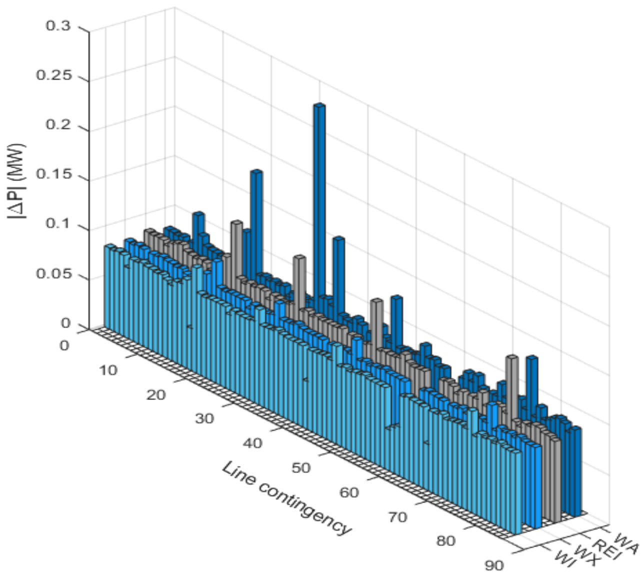

Fig. 14. TXG: Average absolute value of active power deviations.

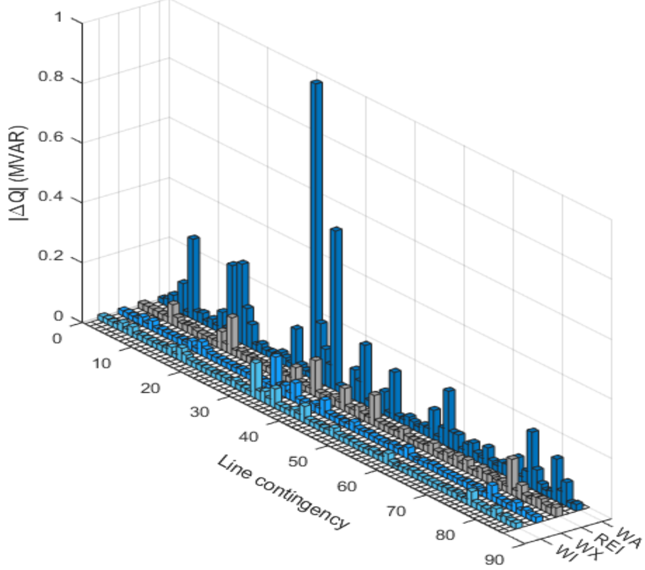

Fig. 15. TXG: Average absolute value of reactive power deviations.

TABLE V. TOTAL AVERAGE ERRORS

Measure	Reduction method			
	WI	WX	REI	WA
$\|\Delta V\|$ (pu)	2.964E-05	2.965E-05	2.834E-05	1.190E-04
$\|\Delta\phi\|$ (°)	7.616E-02	7.616E-02	8.896E-02	9.234E-02
$\|\Delta P\|$ (MW)	8.026E-02	8.026E-02	8.563E-02	8.868E-02
$\|\Delta Q\|$ (MVAR)	2.286E-02	2.286E-02	3.051E-02	7.466E-02

Finally, the computational efficiency of these reduced models, as compared to the original one, is clearly demonstrated in Table VI. It can be deduced from this Table that the estimated computational times to carry out power flows for N-1 contingency analysis, including all the lines in the area of interest, are reduced by almost 60% with the REI equivalent, and by 95% with the WA, WI and WX based representations. In this case, for particular applications of large power grid models, the importance of using network equivalents is exhibited here through the potential and significant reductions in the simulation time to complete specific tasks.

TABLE VI. COMPARISON OF COMPUTATIONAL TIME

System	Model version				
	Original	WI	WX	REI	WA
TXG	51.508 s	2.593 s	2.662 s	21.145 s	2.572 s

V. CONCLUSIONS

An evaluation of the accuracy and computational time involved in the performance of equivalent networks obtained through Ward and REI reduction methods has been presented in this paper. A relatively small benchmark power system such as NEPS, and a significantly large sample grid denominated TXG, were used here for illustrative purposes. According to the results, the WI and WX equivalents were able to offer a superior accuracy in the small sample network, as compared to the REI and WA models. On the other hand, a comparable and good response was provided by the WI, WX, and REI reduction techniques when the large sample grid was considered. Moreover, although the improvements in the computational time achieved with the simplified representations may seem to be irrelevant in the NEPS case, substantial enhancements in this regard were observed with the TXG reduced systems. For the particular application, the evaluation of considered network equivalents with only the base case and a few line outage scenarios may lead to inadequate inferences regarding model performance under N-1 contingencies. Therefore, the complete set of lines of the area of interest was considered in this work. Finally, the poor reactive power response of the WA model was clearly noted with both sample systems.

ACKNOWLEDGMENT

The authors acknowledge the Swiss National Science Foundation (SNSF) under the project number PZENP2_173628 of the program Ambizione Energy Grant (AEG).

REFERENCES

[1] P. D. Lund, J. Byrne, R. Haas, and D. Flynn, Advances in Energy Systems: The Large-scale Renewable Energy Integration Challenge. NJ: John Wiley & Sons Ltd, 2019.

[2] U. Cali, M. Kuzlu, M. Pipattanasomporn, J. Kempf, and L. Bai, Digitalization of Power Markets and Systems Using Energy Informatics. Cham: Springer, 2021.

[3] J. H Chow, Power System Coherency and Model Reduction. NY: Springer, 2013.

[4] A. P. Gupta, A. Mohapatra, and S. N. Singh, “Power System Network Equivalents: Key Issues and Challenges,” in Proc. TENCON-IEEE Region 10 Conference, pp. 1-6, October 2018.

[5] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, “REI method for multi-area modeling of power systems,” Electrical Power and Energy Systems, vol. 60, pp. 283–292, 2014.

[6] S. M. Ashraf, B. Rathore, and S. Chakrabarti, “Performance analysis of static network reduction methods commonly used in power systems,” in Proc. 18th National Power Systems Conference, pp. 1-6, December 2014.

[7] G. Wijeweera, Development of an Equivalent Circuit of a Large Power System for Real-time Security Assessment (Doctoral thesis). University of Manitoba, Winnipeg, MB, Canada, 2016.

[8] M. M. Haji, Methods to Create Equivalent Models for Power System Studies (Doctoral thesis). University of Alberta, Edmonton, AB, Canada, 2017.

[9] F. Karbalaei, M. Jebreilzadeh, and H. Shahbazi, “Reducing power system model dimensions based on linearization for static analysis,” Turkish Journal of Electrical Engineering & Computer Sciences, vol. 26, pp. 2016–2028, 2018.

[10] J. Srivani and K. S. Swarup, “Power system static security assessment and evaluation using external system equivalents,” Electrical Power and Energy Systems, vol. 30, pp. 83–92, 2008.

[11] H. G. Svendsen, “Grid model reduction for large scale renewable energy integration analyses,” Energy Procedia, vol. 80, pp. 349–356, 2015.

[12] B. G. Lee, J. Lee, and S. Kim, “Development of a Static Equivalent Model for Korean Power Systems Using Power Transfer Distribution Factor-Based k-Means++ Algorithm,” Energies, vol. 13 (24), pp. 1–12, 2020.

[13] DIgSILENT PowerFactory 2020, Technical Reference Documentation - Application examples, Gomaringen, Germany.

[14] A. B. Birchfield, T. Xu, K. M. Gegner, K. S. Shetye, and T. J. Overbye, “Grid Structural Characteristics as Validation Criteria for Synthetic Networks,” IEEE Transactions on Power Systems, vol. 32 (4), pp. 3258-3265, 2017.

[15] Texas A&M University, “Electric Grid Test Case Repository.” https://electricgrids.engr.tamu.edu/ (accessed November 17, 2021).

[16] K. D. Lee, Python Programming Fundamentals. London: Springer, 2015.