ELSEVIER

Insurance: Mathematics and Economics 30 (2002) 219–230

INSURANCE
MATHEMATICS & ECONOMICS

www.elsevier.com/locate/econbase

Recursive calculation of time to ruin distributions

Rui M.R. Cardoso [a,b,1], Alfredo D. Egidio dos Reis [c,*,2]

[a] *CMA and Departamento de Matemática, FCT, Universidade Nova de Lisboa, Lisbon, Portugal*
[b] *Department of Actuarial Mathematics & Statistics, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS, UK*
[c] *Departamento de Matemática, CEMAPRE and ISEG, Universidade Técnica de Lisboa, Rua do Quelhas 6, 1200-781 Lisbon, Portugal*

Received 1 August 2001; received in revised form 1 December 2001; accepted 14 February 2002

Abstract

In this paper we present a different approach on Dickson and Waters [Astin Bulletin 21 (1991) 199] and De Vylder and Goovaerts [Insurance: Mathematics and Economics 7 (1988) 1] methods to approximate time to ruin probabilities. By means of Markov chain application we focus on the direct calculation of the distribution of time to ruin, and we find that the above recursions appear to be less efficient, although giving the same approximation figures. We show some graphs of the time to ruin distribution for some examples, comparing the different shapes of the densities for different values of the initial surplus. Furthermore, we consider the presence of an upper absorbing barrier and apply the proposed recursion to find ruin probabilities in this case.

Keywords: Probability of ruin; Time to ruin; Barrier problems; Finite time; Discrete time model; Recursive calculation; Markov chain

1. Introduction

In this draft we present a different approach on Dickson and Waters (1991) method to approximate finite time ruin probabilities. Their paper uses a discrete time compound Poisson model, an appropriate parameter choice and a recursion on the initial reserve to approximate finite time and ultimate ruin probabilities in the classical risk model. We use this approach to study the distribution of the time to ruin random variable, in particular the different shapes of its density function in the classical model. We first consider the method by De Vylder and Goovaerts (1988) and then its modification by Dickson and Waters (1991) giving way to a different algorithm. For their purpose the method by Dickson and Waters (1991) is shown to be less time consuming than the one by De Vylder and Goovaerts (1988). This is not the case for our purpose. Besides, the algorithm by the former authors has the disadvantage to be unstable. Our model will be presented as a discrete time Markov chain, which with the appropriate parameter choice by the former authors will be used to approximate the classical model. The quality of the approximations is of course dependent of the discretising unit. It will be easy to understand that we get the same approximations obtained by these authors for the finite time ruin probabilities, considering the same discretising unit. This proposed approach appears to be a more efficient way to calculate an approximation to the distribution of the time to ruin random variable when compared to the algorithms cited above. Furthermore, the algorithm is stable.

* Corresponding author. Tel.: +351-21-392-5867; fax: +351-21-392-2781.
E-mail addresses: r.cardoso@ma.hw.ac.uk (R.M.R. Cardoso), alfredo@iseg.utl.pt (A.D. Egidio dos Reis).
[1] Support from Fundação para a Ciência e a Tecnologia—FCT/POCTI is gratefully acknowledged.
[2] Support from Fundação para a Ciência e a Tecnologia—FCT/POCTI is gratefully acknowledged. URL: http://www.iseg.utl.pt/∼alfredo.

PII: S0167-6687(02)00102-6

Because the approximations we get are the same as those by Dickson and Waters (1991), for the same discretising unit, we will not produce any figures. This has been widely discussed by several authors. Instead, we will produce some graphs for some selected examples, with the approximations to the density functions of the time to ruin random variable in the classical model and compare the different shapes of the functions for different values of the initial surplus.

One could argue about the use of a discrete approximation. Of course there is a clear price in accuracy to pay for the use of a discrete approximation. This can be compensated by the flexibility of the proposed method. For instance, the Markov chain approach that will be developed in Section 3 has clear advantages over other methods, not when the claim size distribution is not parametric, but, e.g., when it is empirical or not *well behaved*. By contrast, other numerical methods may be superior for *well behaved* claim size distributions. However, it is known that the method gives good approximations, besides, it fits well our purpose. We introduce a different approach over a discrete model known to give good approximations to finite time ruin probabilities in the classical model.

In the next section we introduce the basic continuous time surplus model as well as the discrete model that approximates the basic model, including definitions and notation. In Section 3 we present the proposed recursion in the discrete model, based on the homogeneous and discrete Markov chain, that will allow to compute approximations for ruin probabilities. In Section 4 we work out some examples building approximate values for the density of the time to ruin random variable, comparing the different shapes of the densities according to different values of the initial surplus for the classical model, by showing some graphs for the cases worked out. In Section 5 we discuss some features for recursions presented by De Vylder and Goovaerts (1988) and Dickson and Waters (1991), including the one introduced in Section 3. Finally, in the last section we set in the surplus model an upper absorbing barrier and show how we can calculate ruin probabilities by using the recursion in Section 3, and show some graphs for the density of time to ruin in this case.

2. Models and notation

Let $\{U(t)\}_{t\geq 0}$ be a classic continuous time surplus process so that

$$U(t) = u + ct - S(t),$$

where $u(\geq 0)$ is the insurer's initial surplus, c the insurer's rate of premium income per unit time, $S(t) = \sum_{i=1}^{N(t)} Y_i$ the aggregate claim amount up to time t, $N(t)$ the number of claims in the same time interval having a Poisson (λt) distribution, and $\{Y_i\}_{i=1}^{\infty}$ a sequence of i.i.d. random variables representing the individual claim amounts. We denote by $P(x)$ the distribution of Y_i with $P(0) = 0$, and by $F(x, t)$ the distribution of $S(t)$. If $P(x)$ is absolutely continuous we will denote by $p(x)$ its density function and $f(x, t) = (\mathrm{d}/\mathrm{d}x)F(x, t)$ for $x > 0$. We note that the density $f(x, t)$ has a *spike* at $x = 0$. We also assume that the mean of Y_i, which we denote by p_1, is finite. We set $c = (1 + \theta)\lambda p_1$, with $\theta > 0$, so that θ is the insurer's loading factor.

Without loss of generality, we make the two following assumptions: $c = p_1 = 1$.

Time to ruin is denoted by T and defined as

$$T = \begin{cases} \inf\{t : U(t) < 0\}, \\ \infty \text{ if } U(t) \geq 0 \text{ for all } t > 0, \end{cases}$$

finite time ruin probability from some initial surplus $u \geq 0$ is defined as

$$\psi(u, t) = \Pr[T \leq t | U(0) = u]$$

and ultimate ruin probability $\psi(u) = \Pr[T < \infty | U(0) = u]$. We denote by $\phi(u, t)$ as the (defective) density function of T and let $\tilde{\phi}(u, t) = \phi(u, t)/\psi(u)$.

We are interested in computing approximations for finite time ruin probabilities, i.e., in finding an approximation for the distribution of the random variable T. For this purpose we consider Dickson and Waters (1991) approach considering a discrete time compound Poisson model

$$U_{\mathrm{d}}(t) = u + t - S_{\mathrm{d}}(t) \tag{1}$$

for $t = 1, 2, \ldots$, for a given initial reserve $U_{\mathrm{d}}(0) = u$ ($u = 0, 1, 2, \ldots$). Also

- $S_{\mathrm{d}}(t)$ is the aggregate claim amount up to time t with distribution and probability function $F_{\mathrm{d}}(x, t)$ and $f_{\mathrm{d}}(x, t)$, respectively.
- Individual claims are i.i.d. random variables on the non-negative integers with mean $\beta > 1$, where β is an integer.
- Premium income per unit time is 1.
- Expected number of claims per unit time is $1/(1 + \theta)\beta$.

For simplicity we state $f_j = f_{\mathrm{d}}(j, 1)$ and $F_j = F_{\mathrm{d}}(j, 1)$ for $j = 0, 1, 2, \ldots$. Notice that $S_{\mathrm{d}}(t)$ is the sum of i.i.d. random variables, each with probability function $\{f_j\}_{j=0}^{\infty}$. If X_n denotes the aggregate claim amount from time $n - 1$ until time n, then $S_{\mathrm{d}}(t) = \sum_{n=1}^{t} X_n$ and f_j is the probability of X_n taking value j. The probability function of X_n, $\{f_j\}_{j=0}^{\infty}$, can be obtained by using Panjer (1981) recursion.

Discrete time to ruin, T_{d}, is defined as

$$T_{\mathrm{d}} = \begin{cases} \min\{n : U_{\mathrm{d}}(n) < 0,\ n = 1, 2, 3, \ldots\}, \\ \infty \text{ if } U_{\mathrm{d}}(n) \geq 0 \text{ for } n = 1, 2, 3, \ldots. \end{cases}$$

Discrete and finite time ruin probability, for a given non-negative integer u is

$$\psi_{\mathrm{d}}(u, t) = \Pr[T_{\mathrm{d}} \leq t | U_{\mathrm{d}}(0) = u] = \sum_{i=1}^{t} \phi_{\mathrm{d}}(u, i),$$

where $\phi_{\mathrm{d}}(u, t) = \Pr[T_{\mathrm{d}} = t | U_{\mathrm{d}}(0) = u]$, $t = 1, 2, \ldots$, is the (defective) probability function of T_{d}. Instead, we will use the *modified* random variable

$$T_{\mathrm{d}}^{*} = \begin{cases} \min\{n : U_{\mathrm{d}}(n) \leq 0,\ n = 1, 2, 3, \ldots\}, \\ \infty \text{ if } U_{\mathrm{d}}(n) > 0 \text{ for } n = 1, 2, 3, \ldots \end{cases}$$

for which

$$\psi_{\mathrm{d}}^{*}(u, t) = \Pr[T_{\mathrm{d}}^{*} \leq t | U_{\mathrm{d}}(0) = u] = \sum_{i=1}^{t} \phi_{\mathrm{d}}^{*}(u, t) \tag{2}$$

with $\phi_{\mathrm{d}}^{*}(u, t) = \Pr[T_{\mathrm{d}}^{*} = t | U_{\mathrm{d}}(0) = u]$, $t = 1, 2, \ldots$. We underline that under this re-definition, ruin does not occur if $U_{\mathrm{d}}(n)$ stays equal or above 1. However, if $u = 0$ ruin does not occur at time 0.

We will use the approximation $\psi_{\mathrm{d}}^{*}(u\beta, (1 + \theta)\beta t)$ for $\psi(u, t)$, where $u\beta$ and $(1 + \theta)\beta t$ are integers. According to Dickson and Waters (1991, Section 8), $\psi_{\mathrm{d}}^{*}(u\beta, (1 + \theta)\beta t)$ is usually a better approximation for $\psi(u, t)$ than is $\psi_{\mathrm{d}}(u\beta, (1 + \theta)\beta t)$.

3. The proposed approach

As said earlier, we propose to approximate the finite time ruin probability $\psi(u, t)$ by $\psi_{\mathrm{d}}^{*}(u\beta, (1 + \theta)\beta t)$, i.e., by using the discrete time model (1), which has shown to give good results. Dickson and Waters (1991) algorithm is based on De Vylder and Goovaerts (1988) formulae:

$$\psi_{\mathrm{d}}^*(u,1)=1-F_u,\qquad \psi_{\mathrm{d}}^*(u,t)=1-F_u+\sum_{j=0}^{u} f_j\psi_{\mathrm{d}}^*(u+1-j,t-1)\quad \text{for } t=2,3,\ldots \tag{3}$$

and $u=0,1,2,\ldots$ proposing instead the following:

$$\psi_{\mathrm{d}}^*(0,t)=\frac{1}{t}\sum_{j=1}^{t}[1-jf_{\mathrm{d}}(t-j,t)]=\frac{1}{t}\sum_{j=0}^{t-1}[1-F_{\mathrm{d}}(j,t)],\qquad \psi_{\mathrm{d}}^*(1,t)=f_0^{-1}[\psi_{\mathrm{d}}^*(0,t+1)-(1-f_0)],$$

$$\psi_{\mathrm{d}}^*(u,t)=f_0^{-1}\left[\psi_{\mathrm{d}}^*(u-1,t+1)-(1-F_{u-1})-\sum_{j=1}^{u-1} f_j\psi_{\mathrm{d}}^*(u-j,t)\right]\quad \text{for } u=2,3,\ldots \tag{4}$$

and $t=1,2,\ldots$. Although unstable this recursion is shown to be less time consuming for their purpose. This is not necessarily the case for our purpose. We want to compute ruin probabilities to compute the distribution of T, i.e., we want to compute ruin probabilities for a fixed initial surplus u and a varying $t=1,2,\ldots$. For illustration we reproduce (from Dickson and Waters (1991)) in Figs. 1 and 2 the combinations of $\psi_{\mathrm{d}}^*(i,j)$ that are necessary in order to calculate $\psi_{\mathrm{d}}^*(u,t)$ using recursions (3) and (4), respectively. Now considering our purpose, the necessary values in Fig. 1 stay unchanged for recursion (3). For recursion (4) it is a different story, we will need all the values showed in Fig. 3.

We propose instead a third way already suggested by Dickson and Gray (1984) in a different perspective. We use the property of the discrete time process $U_{\mathrm{d}}(t)$ being a homogeneous discrete time Markov chain, i.e., the surplus at time t in our model can be written as $U_{\mathrm{d}}(t+1)=U_{\mathrm{d}}(t)+1-X_{t+1}$, where X_{t+1} is distributed as X_i $(i=1,2,\ldots)$. This recursion will be based on the calculation of the transition probabilities, which allow the calculation of the probability $\phi_{\mathrm{d}}^*(u,t)$, $t=1,2,\ldots$ and then summing up for the calculation of $\psi_{\mathrm{d}}^*(u,t)$ [see (2)].

According to the method, we note that ruin occurs at the end of period n, for $2\le n\le t$, if

1. the surplus has been positive up to time $n-1$, where the surplus is j, j being a value between 1 and $u+n-1$, and

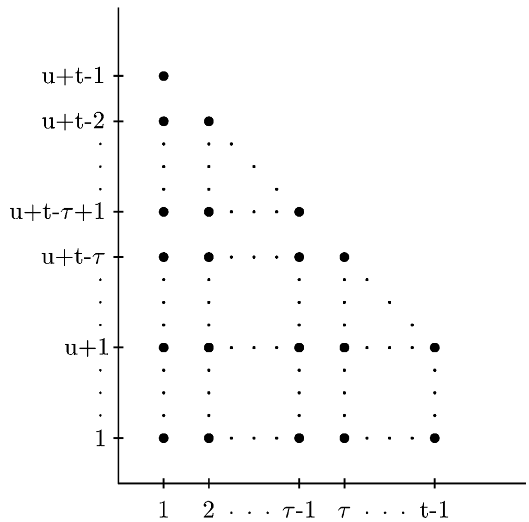

Fig. 1. Combinations of i e j for which values of $\psi_{\mathrm{d}}(i,j)$ are required to calculate $\psi_{\mathrm{d}}^*(u,t)$ using the method of De Vylder and Goovaerts (1988).

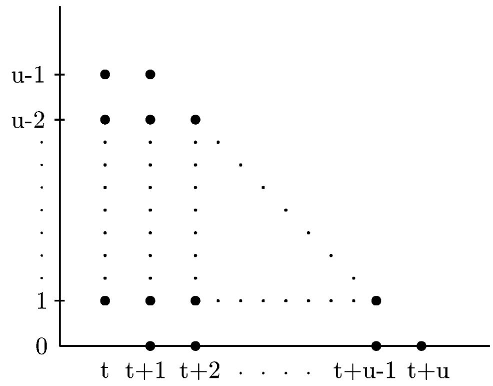

Fig. 2. Combinations of i e j for which values of $\psi_{\mathrm{d}}(i, j)$ are required to calculate $\psi_{\mathrm{d}}^*(u, t)$ using the method of Dickson and Waters (1991).

2. the amount of aggregate claims at the nth period is greater or equal to $j + 1$.

If we denote by $P_{i,j}^{(n)}$ the n-step transition probability

$$P_{i,j}^{(n)} = \Pr[U_{\mathrm{d}}(n) = j | U_{\mathrm{d}}(0) = i], \quad n = 1, 2, \ldots,$$

we can write the probability of ruin at time t as

$$\phi_{\mathrm{d}}^*(u, t) = \Pr[T_{\mathrm{d}}^* = t | U_{\mathrm{d}}(0) = u] = \sum_{j=1}^{u+t-1} P_{u,j}^{(t-1)}(1 - F_j), \quad \text{for } t = 2, 3, \ldots$$

and $\phi_{\mathrm{d}}^*(u, 1) = 1 - F_u$. Note that in the two previous formulae, due to the definition of T_{d}^*, transitions from i to j are only possible through positive intermediate states.

Fig. 4 shows the values of $U_{\mathrm{d}}(t)$ that are required to calculate $\phi_{\mathrm{d}}^*(u, t)$ through transition probabilities, leading then to $\psi_{\mathrm{d}}^*(u, t)$ from a starting and fixed value u. The diagram in this figure is the mirror image of that of Fig. 1. This method allows to calculate the distribution of T_{d}^* *as you go*, while De Vylder and Goovaerts (1988) calculates more in the beginning and less in the end. The other difference is that this method uses transition probabilities instead of $\psi_{\mathrm{d}}^*(u, i)$ for earlier periods.

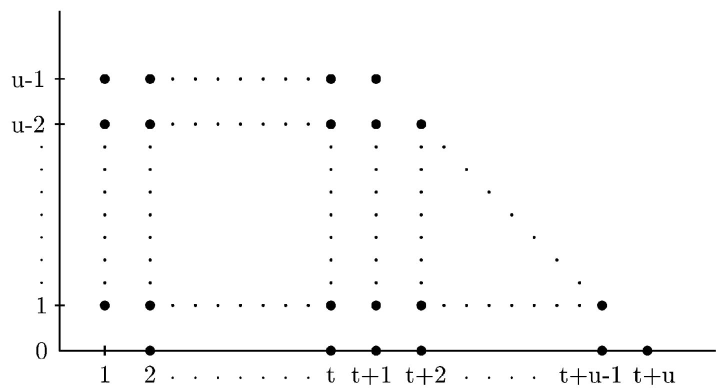

Fig. 3. Combinations of i e j for which values of $\psi_{\mathrm{d}}(i, j)$ are required to calculate $\psi_{\mathrm{d}}^*(u, t)$ using the method of Dickson and Waters (1991) and according to our purpose.

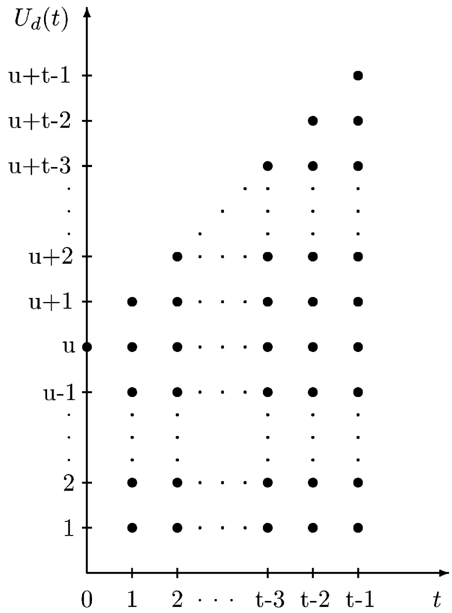

Fig. 4. Values required to compute $\psi(u,t)$ according to the method proposed.

For the recursive calculation we will need the transition probabilities

$$P_{u,j}^{(1)} = \Pr[U_\mathrm{d}(1) = j | U_\mathrm{d}(0) = u] = f_{u+1-j} \tag{5}$$

for $1 \le j \le u + 1$, and

$$P_{r,s}^{(1)} = \Pr[U_\mathrm{d}(n) = s | U_\mathrm{d}(n-1) = r] = f_{r+1-s} \tag{6}$$

for $1 \le r \le u + n - 1$, $1 \le s \le u + n$ and $2 \le n \le t$. We have that $r + 1 \ge s$ because we can only go up one position at the most. For the recursion and for the first step we need only the probabilities described in (5) since we are starting from a fixed value, the initial surplus u. For further steps ($n = 2, 3, \ldots$) we will need all the probabilities in (6).

One-step transition probability matrix for the process is the following:

	$'0'$	1	2	3	4	...	$u-1$	u	$u+1$	...
$'0'$	1	0	0	0	0	...	0	0	0	...
1	$1-F_1$	f_1	f_0	0	0	...	0	0	0	...
2	$1-F_2$	f_2	f_1	f_0	0	...	0	0	0	...
3	$1-F_3$	f_3	f_2	f_1	f_0	...	0	0	0	...
...	...	...	...	...	...	...	...	...	...	...
$u-1$	$1-F_{u-1}$	f_{u-1}	f_{u-2}	f_{u-3}	f_{u-4}	...	f_1	f_0	0	...
u	$1-F_u$	f_u	f_{u-1}	f_{u-2}	f_{u-3}	...	f_2	f_1	f_0	...
$u+1$	$1-F_{u+1}$	f_{u+1}	f_u	f_{u-1}	f_{u-2}	...	f_3	f_2	f_1	...
...	...	...	...	...	...	...	...	...	...	...

where state $'0'$ corresponds to the event *ruin*, which is an absorbing state.

For the computation we do not need to specify (directly) the whole matrix, i.e., we can consider the sub-matrix obtained by taking out the line and column corresponding to state $'0'$. We will denote this sub-matrix of the transition probabilities by P.

To compute $P_{u,j}^{(t-1)}$, and subsequently $\phi_{\mathrm{d}}^{*}(u,t)$, only the line u of the matrix P^{t-1} is necessary, which we denote by $P_u^{(t-1)}$ (and without the corresponding value for state $'0'$). That is, for the first step we have

$$P_u^{(1)} = \left[\, f_u \quad f_{u-1} \quad \dots \quad f_1 \quad f_0 \quad 0 \quad \dots \right],$$

where the vector has only the first $u+1$ non-zero elements. Next, we have $P_u^{(2)} = P_u^{(1)} \times P$, $P_u^{(3)} = P_u^{(2)} \times P$ and so on. Note that to get $P_u^{(2)}$ we need to only multiply by the first $u+2$ columns from P, since the remaining columns have the first $u+1$ elements equal to zero. After this operation $P_u^{(2)}$ will only have the first $u+2$ non-zero elements and the computation of $P_u^{(3)}$ will be done in the same manner, only using the first $u+3$ columns of P, for the same reason above.

4. Numerical examples

We considered three different examples. For the individual claim amounts we considered Exponential with mean 1, Gamma(2, 2) and Pareto(2, 1) distributions. In all cases we set $\theta = 0.1$ and the discretising parameter $\beta = 20$, which we found to be adequate for the purpose.

Figs. 5–10 show approximations to the proper density $\tilde{\phi}(u,t)$, where $\phi(u,t)$ was approximated according to the recursion in the previous section and the ultimate ruin probability $\psi(u)$ was approximated by the algorithm by Dickson et al. (1995). Figs. 5 and 6 relate to the exponential case, Figs. 7 and 8 to the Gamma(2, 2) case and Figs. 9 and 10 to the Pareto(2, 1) case. In all cases we see that for small values of the initial surplus u the shapes of the densities are of the decreasing type, changing shape to have a larger mode for bigger values of u.

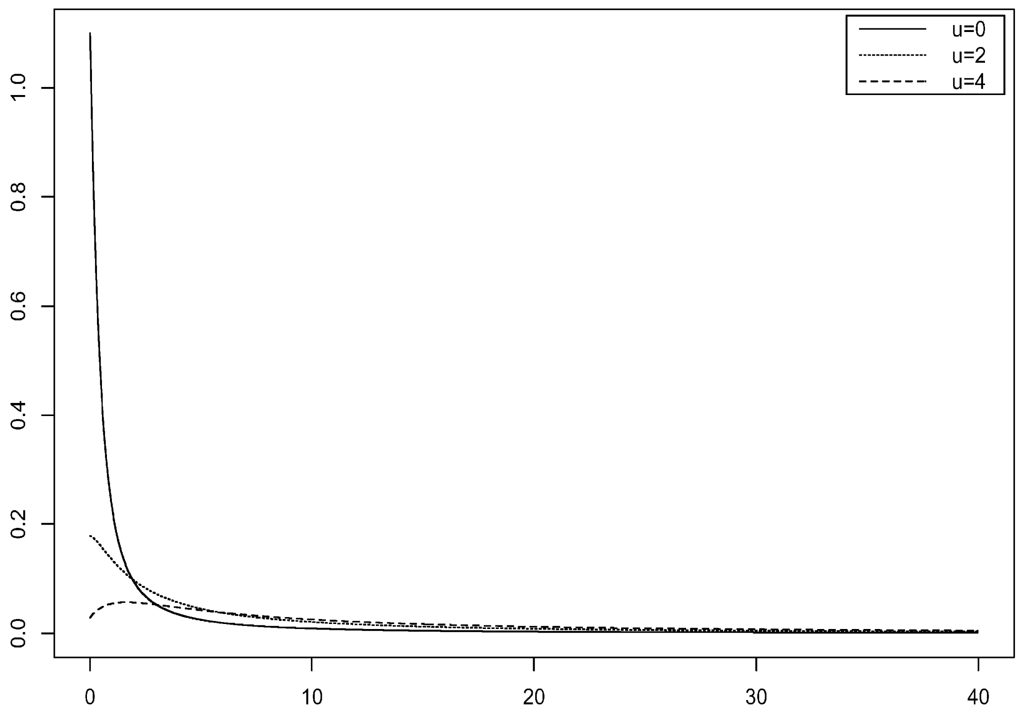

Fig. 5. Exponential (1) for $u = 0, 2$ and 4.

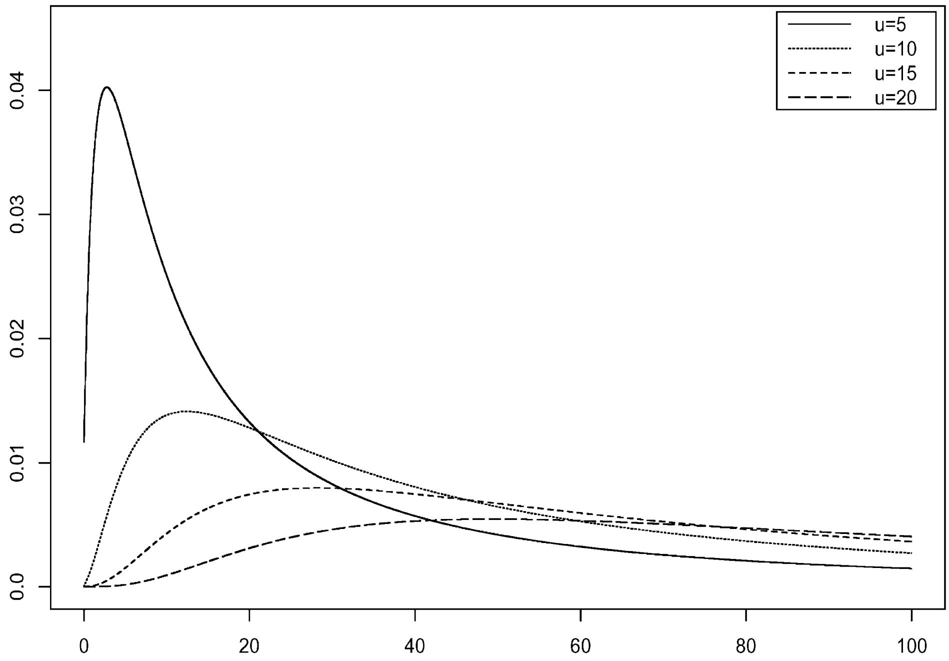

Fig. 6. Exponential (1) for $u = 5$, 10, 15 and 20.

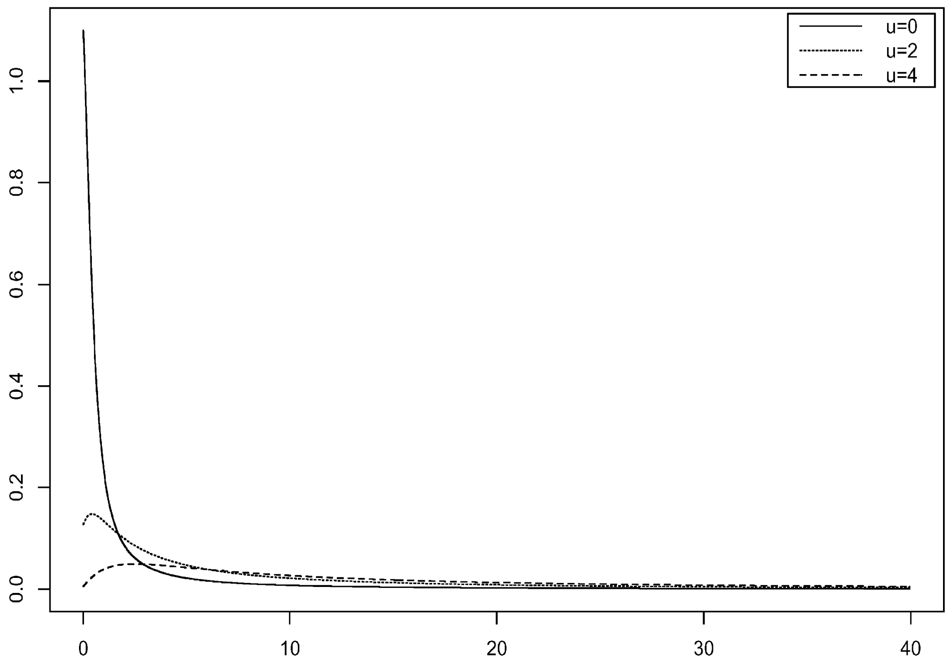

Fig. 7. Gamma(2, 2) for $u = 0$, 2 and 4.

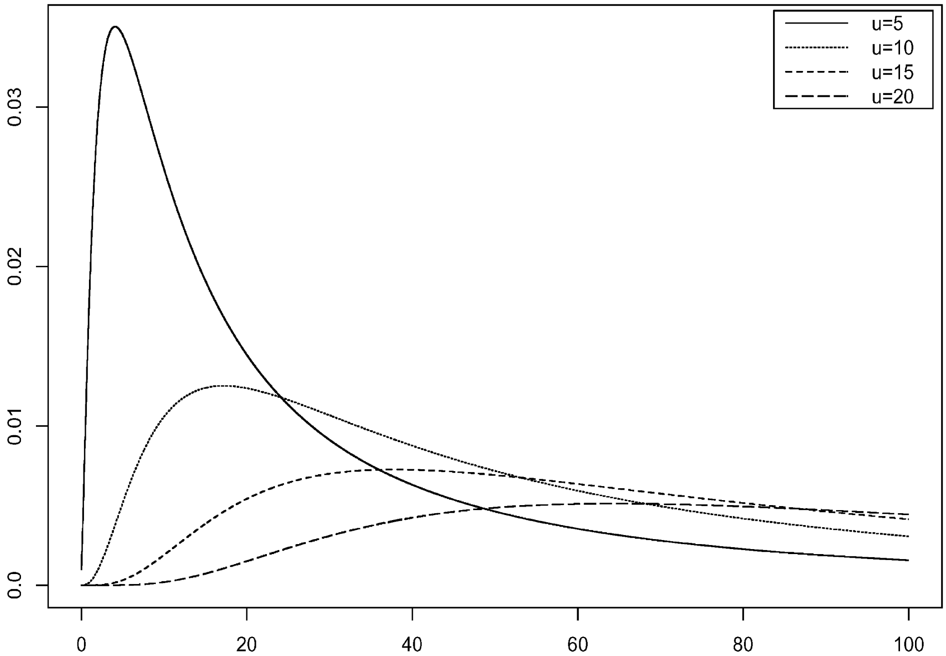

Fig. 8. Gamma(2, 2) for $u = 5$, 10, 15 and 20.

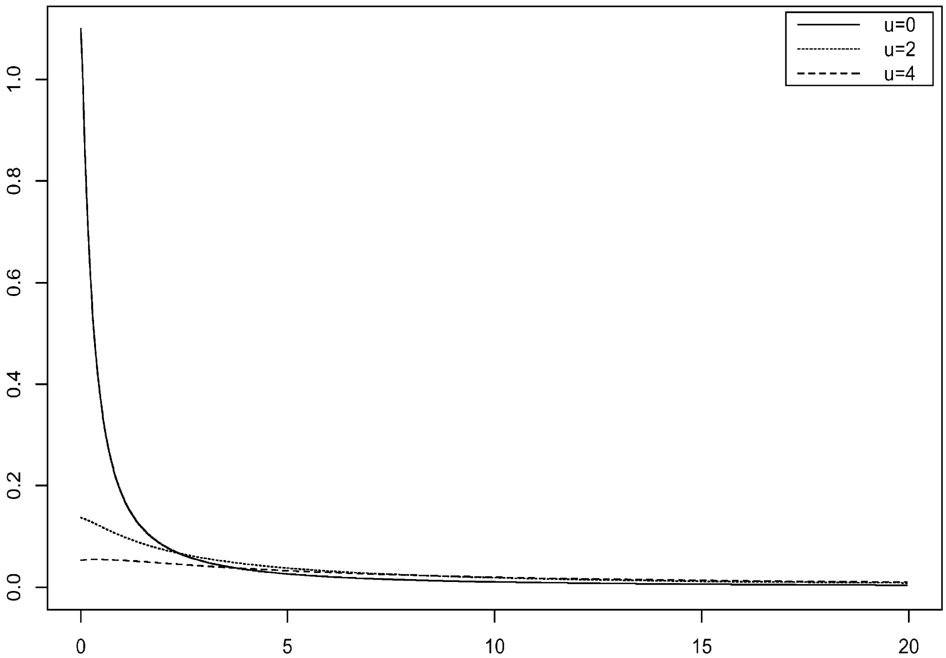

Fig. 9. Pareto(2, 1) for $u = 0$, 2 and 4.

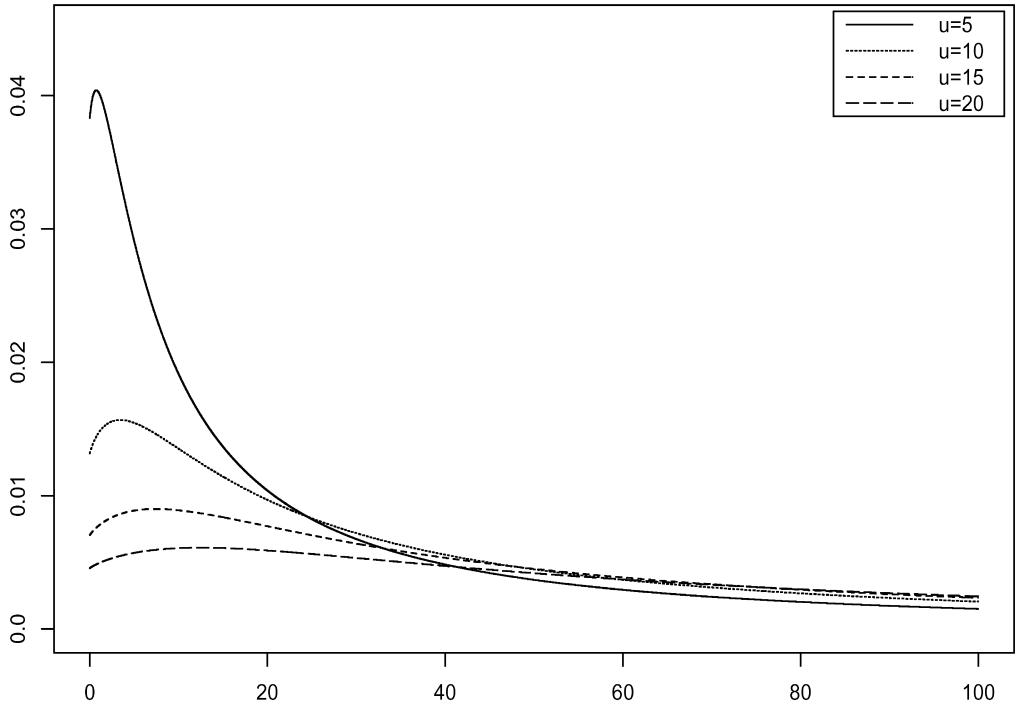

Fig. 10. Pareto(2, 1) for $u = 5$, 10, 15 and 20.

5. Some remarks on the algorithms

We now try to compare the three different algorithms discussed above, which includes the approach in Section 3. Of course they all give the same approximations for the same discretising parameter β, so that is not an issue. The focus is on the number of iterations. Referring to the method by De Vylder and Goovaerts (1988) and Fig. 1 we conclude that we need to compute $(t-1)(2u+t)/2 = (2u(t-1)+t(t-1))/2$ values of $\psi_{\mathrm{d}}^*(i, j)$ to calculate $\psi_{\mathrm{d}}^*(u, t)$. Referring to the approach in Section 3, we need the same amount of iterations, noting that we are dealing with transition probabilities instead. In reference to the algorithm by Dickson and Waters (1991) and Fig. 3, we conclude that we need $(2ut + u(u+1))/2 - 1$ values of $\psi_{\mathrm{d}}^*(i, j)$ required to calculate $\psi_{\mathrm{d}}^*(u, t)$. Based on the number of recursions we cannot conclude much about the efficiency of the algorithms, except that it depends on the combination of (u, t) we set for the recursion. However, for our purpose it appears that Dickson and Waters (1991) algorithm is less efficient. If we look at formulae (4) we see that for this latter algorithm we need to compute frequently the probability function of the aggregate claims amount $f_{\mathrm{d}}(\cdot, t)$, which is done by using Panjer (1981) recursion. We note that this is a heavy procedure. We also note that this approach has instability problems as referred by the authors. For the purpose we set, Dickson and Waters (1991) algorithm should not be the choice mostly.

In fact we tried different combinations of (u, t) for testing, such as (0, 10), (0, 20), (5, 5), (5, 10), (5, 20), (10, 1), (10, 5), (10, 20), (15, 20), (20, 1), (20, 5) and (20, 20), all for the exponential case. In most cases De Vylder and Goovaerts (1988) algorithm revealed to be less time consuming, when compared to Dickson and Waters' (1991). For the same combination of values the approach in Section 3 showed to be even less time consuming, on a smaller scale. This must have to do with the way the recursion is set concerning the computer program.

6. Ruin probabilities in the presence of an upper absorbing barrier

We can use the method presented in Section 3 to compute finite time ruin probabilities in the presence of an upper absorbing barrier $k > u$ as proposed by Dickson and Gray (1984), and build approximations for the distribution of time to ruin with the absorbing barrier k. We will denote by $T(k)$ and $T_{\mathrm{d}}^{*}(k)$ time to ruin in the presence of the barrier k in the classical and in the discrete model (1), respectively. In the latter we consider the *modified* definition of ruin. Respective finite time ruin probabilities will be denoted as $\xi(u,k,t)$ and $\xi_{\mathrm{d}}^{*}(u,k,t)$. The (defective) density of $T(k)$ will be denoted as $\chi(u,k,t)$ and the (defective) probability function of $T_{\mathrm{d}}^{*}(k)$ will be denoted as $\chi_{\mathrm{d}}^{*}(u,k,t)$. Approximation for $\xi(u,k,t)$ will be given by $\xi_{\mathrm{d}}^{*}(u\beta, k\beta, (1+\theta)\beta t)$.

Referring to Section 3, the matrix of the transition probabilities is now truncated due to the finite state space of $k+1$ states, where state k is an absorbing state. Following the method in Section 3 we specify a matrix $\bar{P}$ as a sub-matrix taking out the lines and columns corresponding to states $'0'$ and k, so that

$$\chi_{\mathrm{d}}^{*}(u,k,t) = \Pr[T_{\mathrm{d}}^{*}(k) = t \mid U_{\mathrm{d}}(0) = u] = \sum_{j=1}^{\min\{u+t-1,k-1\}} \bar{P}_{u,j}^{(t-1)}(1-F_j) \quad \text{for } t = 2,3,\ldots$$

and $\chi_{\mathrm{d}}^{*}(u,k,1) = 1 - F_u$.

Figs. 11 and 12 show approximations for $\phi(u,t)$—(1)—and $\chi(u,k,t)$—(2)—for the following combinations of (u,k): (5, 10), and (10, 15), respectively, in the exponential case. Note that the difference between the values of the ruin probabilities with and without the barrier will only be effective for values of t such that $\beta u + \beta(1+\theta)t - 1 > k\beta - 1$, or equivalently for $t > (k-u)/(1+\theta)$.

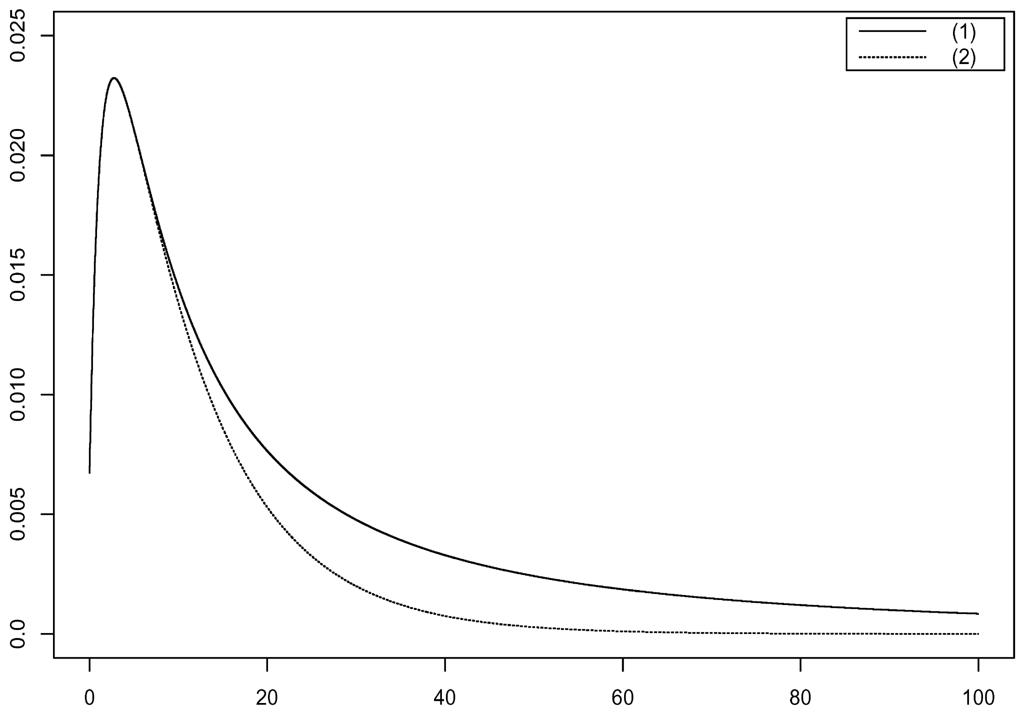

Fig. 11. Exponential (1) for $u = 5$, $k = 10$.

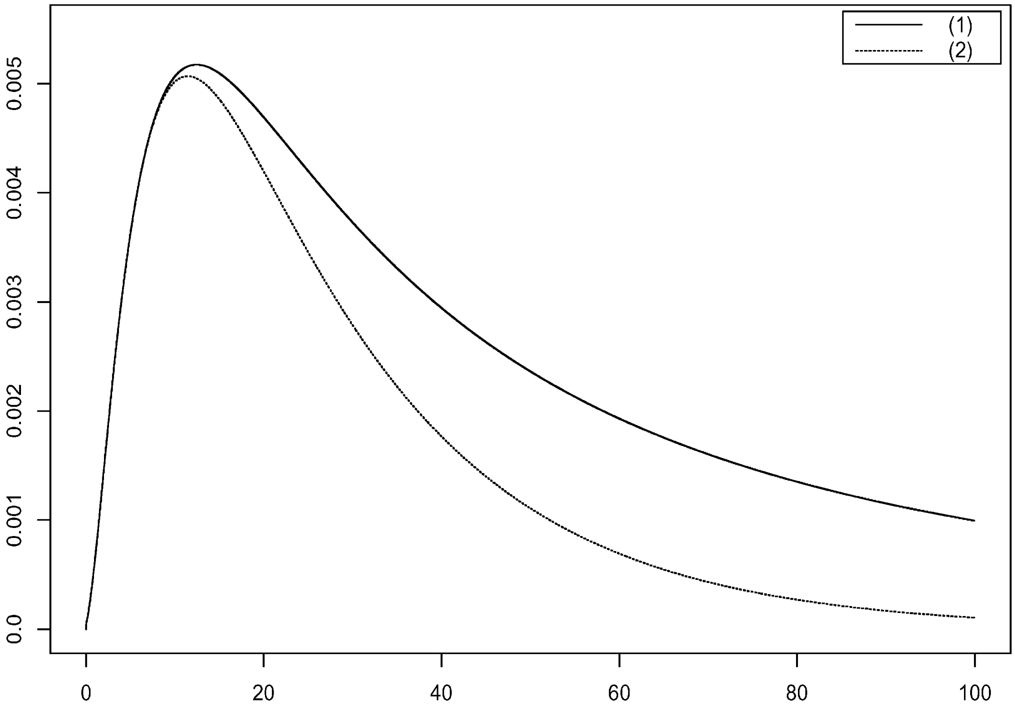

Fig. 12. Exponential (1) for $u = 10$, $k = 15$.

Acknowledgements

We thank an anonymous referee for his useful comments on the manuscript.

References

De Vylder, F., Goovaerts, M.J., 1988. Recursive calculation of finite time ruin probabilities. Insurance: Mathematics and Economics 7, 1–7.

Dickson, D.C.M., Gray, J.R., 1984. Approximations to ruin probability in the presence of an upper absorbing barrier. Scandinavian Actuarial Journal, 105–115.

Dickson, D.C.M., Waters, H.R., 1991. Recursive calculation of survival probabilities. Astin Bulletin 21, 199–221.

Dickson, D.C.M., Egídio dos Reis, A.D., Waters, H.R., 1995. Some stable algorithms in ruin theory and their applications. Astin Bulletin 25, 153–175.

Panjer, H.H., 1981. Recursive evaluation of a family of compound distributions. Astin Bulletin 12, 22–26.