

〔査読論文〕

Practice of Excel in the Classroom of Operations
Research at Japanese Local Universities

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Practice of Excel in the Classroom of Operations Research at Japanese Local Universities

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Abstract In this article, we report practices of operations research subject which are given in Japanese local universities. The subjects are two. Portfolio optimization and the analytic hierarchy process. They both maintain prominence in Japan because of the academic and educational activities of Professors Konno and Tone. We will see hereafter the compatibility with Microsoft Excel, the most popular spreadsheet application, and add-in tool Solver.

1. Introduction

In Japanese universities, several operations research subjects have been educated for a long time. For example, these are linear programming, nonlinear programming, network optimization, queuing theory, game theory, decision analysis, financial engineering, etc. Because of the outstanding academic and educational activities of Professors Tone and Konno, analytic hierarchy process and portfolio optimization have been maintained their prominence in Japan. In this article, we treat both

subjects. Microsoft Excel can handle them in classrooms of operations research course.

The article is organized as follows. In Section 2, we treat portfolio optimization. Section 2 is divided into two subsections. In Subsection 2.1, we treat the Mean-Variance model of Markowitz, who is the winner of the Nobel Prize. The model is expressed by the quadratic optimization problem with linear equality constraints. In Subsection 2.2, we treat Konno's mean-absolute deviation model, which is equivalent to a linear programming problem. Section 3 is devoted to the analytic hierarchy process. In Subsection 3.1, we treat the geometric mean method to prioritize the weights of the alternatives. Subsection 3.2 is devoted to the Eigenvalue method, which follows the latest result [16][17]. For all models indicated below, we show how to use Excel.

2. Portfolio Optimization Using Excel

In this section, we consider T periods of investments. We allocate the money in n different assets. Let P_{jt} be the price of j -th asset in t -th period and $P_{j,t+1}$ be the price of j -th asset in $(t+1)$ -th period. We calculate the return of j -th asset in t -th period by

$$r_{jt} = \frac{P_{j,t+1} - P_{jt}}{P_{jt}}.$$

We also calculate the average return of j -th asset by

$$r_j = \frac{1}{T} \sum_{t=1}^T r_{jt}.$$

Let σ_{jk} be the covariance between j -th and k -th assets. So we have,

$$\sigma_{jk} = \frac{1}{T} \sum_{t=1}^T (r_{jt} - r_j)(r_{kt} - r_k).$$

The n -vector $x = (x_1, \dots, x_n)$ is called the allocation vector. It represents the proportion of the money allocated to each asset and satisfies the following:

$$\sum_{j=1}^n x_j = 1$$

and $x_j \geq 0, j = 1, \dots, n$.

2. 1. MV Model

In this section, we treat Markowitz's Mean-Variance (MV) model [4][5] [8]. Markowitz proposed the following quadratic optimization problem as a portfolio selection problem.

$$\begin{aligned} (P1) \text{ minimize } & \sum_{j=1}^n \sum_{k=1}^n \sigma_{jk} x_j x_k \\ \text{s. t. } & \sum_{j=1}^n r_j x_j = \rho, \\ & \sum_{j=1}^n x_j = 1, \\ & x_j \geq 0, j = 1, \dots, n. \end{aligned}$$

where the investor requires fixed expected return value ρ . Of course, the fixed return value should satisfy $\min_j r_j \leq \rho \leq \max_j r_j$. To solve the Problem (P1) in our classroom, we can use the Excel add-in tool, so-called, Solverⁱ. In order to express the problem (P1) on Excel, we first use **SUMPRUDUCT**

i It may be necessary to activate this add-in beforehand.

function. Table 1 shows real Dataⁱⁱ and how to input the Excel functions. Here, we have $T=6$ and $n=3$. We easily enter the portfolio value of t -th period of $\sum_{j=1}^n r_{jt}x_j$ by **SUMPRDUCT(\$B\$2:\$D\$2,B3:D3)**. The autofill option enables calculations of the portfolio value of each period. After knowing each period's asset value and portfolio value, we easily obtain the return and the variance of each asset and portfolio by **AVERAGE** and **VARP**, respectively. To express the constraint $\sum_{j=1}^n x_j=1$, we type the formula **SUM(B2:D2)** to the cell **E2** with the beginning '='. The value placed in the cell **F9** represents the expected return value ρ . Here we show steps to use Solverⁱⁱⁱ. Description of how to use Solver relies deeply on [9].

1. **Launching the solver:** Click 'Solver' on 'Data' ribbon on Excel for Windows or choose 'Solver' menu in 'Tools' menu on Excel for Mac.
2. **Select target cell:** We will see a dialog box. The first entry named 'Set Objective:' is the information for the target cell (i.e., objective function). Click cursor into this entry box and click into the cell **E10**.
3. **Select minimization:** Check the radio button named 'Min'.
4. **Select variables:** Click cursor to the 'By Changing Cells:' entry box. Enter the cells references for the variables for the variables by selecting the cells from **B2** to **D2**.
5. **Add constraints:**
 - a. Press the 'Add' button to add the first constraint.
 - b. Click cursor to left entry box and click **E2** containing the formula

ii These are retrieved from <https://finance.yahoo.co.jp> on 5, July, 2021. We select three Japanese assets from the Tokyo stock market: TOYOTA, CANON, and SMBC.

iii We have confirmed the examples in this report by Microsoft Excel Version 2016 on Microsoft Windows 10 Pro Version 20H2 and Microsoft Excel for Mac Version 16.50 on Apple macOS 11.4 Big Sur.

for the first constraint.

- c. The middle entry box should be set to '=' by selecting from the pull-down.
 - d. Click cursor to the right entry box and input 1.
 - e. Click 'Add' button to add the expected return constraint.
 - f. Click cursor to left entry box and click **E9** containing the formula for the expected return constraint.
 - g. The middle entry box should be set to '=' by selecting from the pull-down.
 - h. Click cursor to the right entry box and click into the cell **F9**.
 - i. Click 'OK' button because all constraints have been added.
6. **Add non-negativity condition:** Check 'Make Unconstrained Variables Non-Negative' checkbox.
 7. **Select the solving method:** Select 'GRG Nonlinear' entry from the pull-down for 'Select a Solving Method:'.
 8. **Solve:** Click Solve.

In Table 1, we make initial variable is $(1/3, 1/3, 1/3)$ which means equal allocation. If we set the expected return ρ equals to 0.02 and solve it, then we can find the optimum portfolio, which is (0.0949, 0.8108, 0.0942) for this example, in the cells **B2-D2**.

	A	B	C	D	E	F
1		TOYOTA	CANON	SMBC	Portfolio	
2	weights	0.333	0.333	0.333	=SUM(B2:D2)	
3	Jan, 2021	0.0784	- 0.0008	0.1531	=SUMPRODUCT(\$B\$2:\$D\$2,B3:D3)	
4	Feb, 2021	0.0943	0.0916	0.0708	↓	
5	Mar, 2021	- 0.0567	0.0401	- 0.0511	↓	
6	Apr, 2021	0.1215	- 0.0165	0.0391	↓	
7	May, 2021	0.0652	- 0.0183	- 0.0306	↓	

8	Jun, 2021	0.0113	- 0.0075	0.0143	autofill from E3	
9	mean	=AVERAGE(B3:B8)	→	→	→ autofill from B9	0.02
10	variance	=VARP(B3:B8)	→	→	→ autofill from B10	
11	S.D.	=STDEV(P(B3:B8)	→	→	→ autofill from B11	

Table 1 Data for MV model

If we uncheck the check box of ‘Make Unconstrained Variables Non-Negative’, then we can allow the short-sell position. The problem is formulated by the following.

$$\begin{aligned}
 (P2) \text{ minimize } & \sum_{j=1}^n \sum_{k=1}^n \sigma_{jk} x_j x_k \\
 \text{s. t. } & \sum_{j=1}^n r_j x_j = \rho, \\
 & \sum_{j=1}^n x_j = 1.
 \end{aligned}$$

The writing of undergraduate theses on solving MV models in Excel has been continuously taught in Shiraishi’s and Yokoyama’s seminars at the University of Toyama since 2004. We had instructed the subject in our seminars before [4] was published. In the undergraduate course, Shiraishi gave the lecture entitled ‘Management Science Models’ in the classroom of the University of Toyama from 2007 to 2009, where the scores of the teaching evaluations of students were rather high. In the period of recession after the bubble economy in Japan, we often used the short-sell model.

2. 2. MAD Model

In this section, we treat Konno’s Mean-Absolute Deviation (MAD) model [5] [6]. The MAD model is formulated as follows:

$$\begin{aligned}
(P3) \text{ minimize } & \frac{1}{T} \sum_{t=1}^T |z_t| \\
\text{s. t. } z_t - \sum_{j=1}^n (r_{jt} - r_j) x_j &= 0, t = 1, \dots, T. \\
& \sum_{j=1}^n r_j x_j = \rho, \\
& \sum_{j=1}^n x_j = 1, \\
& x_j \geq 0, j = 1, \dots, n.
\end{aligned}$$

Problem (P3) is equivalent to the following problem (Theorem 3.3 in [5]):

$$\begin{aligned}
(P4) \text{ minimize } & \sum_{t=1}^T y_t \\
\text{s. t. } y_t - \sum_{j=1}^n (r_{jt} - r_j) x_j &\geq 0, t = 1, \dots, T. \\
& \sum_{j=1}^n r_j x_j = \rho, \\
& \sum_{j=1}^n x_j = 1, \\
& x_j \geq 0, j = 1, \dots, n, \\
& y_t \geq 0, t = 1, \dots, T.
\end{aligned}$$

Here we can use the ‘Simplex LP’ method in Solver of Excel because this problem is a linear optimization problem. Table 2 shows an example for applying the MAD model for the data in the previous subsection. In the setting dialog of Solver, we should assign $T+n$ variable cells, **C2:E2,B3:B8**, in ‘By Changing Cells:’ entry box. Do not forget ‘,’ between **C2:E2** and **B3:B8**, where the ranges of variables are discontinuous. We designate $T+2$ constraints by press ‘Add’ button to add the constraints in order to express

$y_{t-} - \sum_{j=1}^n (r_{jt} - r_j)x_j$ term, we can use **SUMPRODUCT** function. We repeat the add operation $T+2$ times to specify the constraints. The investor's fixed expected return value $\rho=0.02$ is designated in the cell **G9**. The target cell to be minimized is **B10**, where we type the objective function value. Do not forget to select the 'Min' radio box and check the 'Make Unconstrained Variables Non-Negative' check-box.

	A	B	C	D	E	F	G
1			TOYOTA	CANON	SMBC	Portfolio	
2		variables	0.333	0.333	0.333	=SUM(C2:E2)	
3	Jan, 2021	0	0.0784	- 0.0008	0.1531	=SUMPRODUCT(\$C\$2:\$E\$2,C3:E3)	=B3-(F3-\$F\$9)
4	Feb, 2021	0	0.0943	0.0916	0.0708	↓	↓
5	Mar, 2021	0	- 0.0567	0.0401	- 0.0511	↓	↓
6	Apr, 2021	0	0.1215	- 0.0165	0.0391	↓	↓
7	May, 2021	0	0.0652	- 0.0183	- 0.0306	↓	↓
8	Jun, 2021	0	0.0113	- 0.0075	0.0143	↓	autofill from G3
9	mean		=AVERAGE(C3:C8)	→	→ autofill from C9	autofill from F3	0.02
10	objective	=SUM(B3:B8)					

Table 2 Data for MAD model

In Table 2, we set initial variables to (1/3,1/3,1/3) which means equal allocation. If we set the expected return ρ equals to 0.02 and solve it, then we have the optimum portfolio, which is (0.0997, 0.8162, 0.0841).

In the undergraduate course, Shiraishi has been teaching the MAD model in the lecture entitled 'Introduction to Finance' in the classroom of the University of Toyama. The writing of undergraduate theses on solving MAD model with LINDO [14] was introduced in Shiraishi's seminar from 1999 to 2001 at the University of Toyama. Now we are ready to introduce the MAD model with Excel for undergraduate theses. We are also planning

to construct a Web Application of MAD model by HTML5^{iv}. Obata already constructed web applications to solve general linear programming problems and to evaluate decision making units using data envelopment analysis [11] and used it in the graduate class at Oita University. These applications can be used not only from personal computers but also from most smartphones and tablet devices—iPhones, iPads, and Androids.

3. Analytic Hierarchy Process Using Excel

The analytic hierarchy process (AHP) is a scaling method for priorities in the hierarchical structure [7][12]. AHP gained and maintained prominence in Japan because of the foresight of Professor Tone, who introduced AHP [18] in Japan in the middle of the 1980s which was 10 years after Saaty’s invention [13].

First, we explain a pairwise comparison matrix, which is the heart of AHP. Decision makers often face multiple alternatives. They should define the priorities of alternatives. In AHP, they do represent priorities by Pairwise Comparison. Let $C_i, i = 1, \dots, n$ be alternatives to be prioritized. One defines the value a_{ij} in pairs between C_i and C_j . The value represents the priority of C_i as compared to C_j . In AHP, we use its value by Saaty’s discrete scale from 1 to 9 and their reciprocals. It is based on the verbal expressions in Table 3 [7].

Verbal expressions	Value of intensity
equal importance	1
moderate importance	3 or 1/3
essential or strong importance	5 or 1/5

iv HTML5 is one of the platforms for web applications. It consists of HTML to describe the structure, CSS to describe the appearance, and JavaScript to describe the behavior of the application.

demonstrated importance	7 or 1/7
absolute importance	9 or 1/9
Intermediate values	2,4,6,8 or their reciprocals

Table 3 Saaty's discrete scale

For instance, suppose that one wishes to determine his/her preference for Japanese foods. In comparison of sushi (C_i) and sukiyaki (C_j), if he/she prefers sushi to sukiyaki moderately, a_{ij} is assigned to 3 and a_{ji} to 1/3. Logically the value a_{ji} in pairs between C_j and C_i should be $1/a_{ij}$. After all pairwise comparisons, one obtains the following so-called *pairwise comparison matrix*:

$$A = \begin{pmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ 1/a_{12} & 1 & a_{23} & \cdots & a_{2n} \\ 1/a_{13} & 1/a_{23} & 1 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & 1/a_{2n} & 1/a_{3n} & \cdots & 1 \end{pmatrix}$$

We can say that AHP can treat subjective judgment. After having the pairwise comparison matrix, we should estimate each priority of the objects w_1, \dots, w_n . In AHP, we usually use the geometric-mean method and the eigenvector method.

3. 1. Geometric Mean Method

The theoretical foundation of the geometric-mean method was established by Crawford and Williams [3]. Let $g = (g_1, \dots, g_n)$ be obtained by the geometric means of rows of the pairwise comparison matrix:

$$g_i = \sqrt[n]{\prod_{j=1}^n a_{ij}}.$$

Weights vector $w = (w_1, \dots, w_n)$ is obtained by a normalization so that component of w add up to 1 [2]:

$$w_i = \frac{g_i}{\sum_{i=1}^n g_i}.$$

Here we use the following data as a well-known example in Japan [18].

	Hokkaido	Honshu	Shikoku	Kyushu	Real Value
Hokkaido	1	1/3	4	2	0.21(78,100km ²)
Honshu	3	1	9	6	0.62(230,700km ²)
Shikoku	1/4	1/9	1	1/2	0.05(18,800km ²)
Kyushu	1/2	1/6	2	1	0.12(44,400km ²)

Table 4 Estimated area of Japan's four main islands

In Excel, we use **GEOMEAN()** and **SUM()** functions to obtain g_i and $\sum_{i=1}^n g_i$ respectively. Here we indicate the Excel expression.

	A	B	C	D	E	F	G
1		Hokkaido	Honshu	Shikoku	Kyushu	geomean	weights
2	Hokkaido	1.00	0.33	4.00	2.00	=GEOMEAN(B2:E2)	=F2/\$F\$6
3	Honshu	3.00	1.00	9.00	6.00	↓	
4	Shikoku	0.25	0.11	1.00	0.50	↓	
5	Kyushu	0.50	0.17	2.00	1.00	autofill from F2	autofill from F2
6						=SUM(F2:F5)	

Table 5 Geometric mean method

The result estimates the real value satisfactorily which is (0.22,0.61,0.06,0.11) \approx (0.21,0.62,0.05,0.12).

The compatibility between AHP and Excel is the main reason that Shiraishi has been teaching as a subject of the writing of undergraduate theses

at the University of Toyama^v. In the undergraduate course, Yokoyama has been teaching AHP in the lecture entitled ‘Operations Research’ in the classroom of the University of Toyama. He instructs to use a hand calculator in the classroom.

3. 2. Eigenvector Method

The theoretical foundation of the eigenvector method was established by Sekitani and Yamaki [15] from the mathematical programming viewpoint. However, calculating the eigenvalue and eigenvector of a matrix is not such an easy task for students without computer programming skills. Obata constructed a web application of eigenvector method by HTML5 [10] so that anyone can easily use the eigenvector method. He uses it in the undergraduate class of Operations Research at Oita University and had opportunities for civilian use of it in conjunction with the open house event at Oita University.

On the other hand, we recap the contents briefly from [16] as utilization of Excel. In AHP, solving the following linear system is called by the eigenvector method:

$$Aw = \lambda_{max}w,$$

where λ_{max} is the maximum eigenvalue and w is the associated eigenvector. If we restrict ourselves to the 3×3 pairwise comparison matrix here, then we can use Newton’s method for the following characteristic

v They have dealt with more than 50 topics, including local government’s decision making, main bank selection, smartphone selection and selecting an on-demand video delivery service, etc.

equation to obtain λ_{max} .

$$P_A(\lambda) = \lambda^3 - 3\lambda^2 - \det A = 0.$$

We generate a sequence by the following iteration.

$$\begin{aligned} \lambda_0 &= 3, \\ \lambda_{k+1} &= \lambda_k - \frac{P_A(\lambda_k)}{P'_A(\lambda_k)}, \text{ for } k \geq 0. \end{aligned}$$

It is shown that this iteration always converges to λ_{max} [17].

Here we treat the following 3×3 pairwise comparison matrix.

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 3 \\ 1/4 & 1/3 & 1 \end{pmatrix}.$$

In Table 6, input the formula

$$=B6-((B6)^3-3*(B6)^2-4)/(3*(B6)^2-6*(B6))$$

which corresponds to $\lambda_0 - P_A(\lambda_0)/P'_A(\lambda_0)$ into the cell **B7** and autofill it to below three cells^{vi} (*1 in Table 6). In **E7-E9** cells, we can use **MMULT()** function to calculate the product of a matrix and a vector. Select a range of cells **E7- E9**, type **=MMULT(A1:C3,D7:D9)**, then press **<CTRL><SHIFT><ENTER>** all at the same time (*2 in Table 6).

To solve the Eigenvector problem, we rely on the least square method[1] . In **G7-G9** cell, we type the square value of the error. **G10** cell is the total value of the errors.

After all input, we launch Solver. The target cell is **G10** to be minimized. Variables are chosen by **D7-D9**. A constraint is such that **D10** equals to

vi In general, iterate appropriate times according to data to be analyzed.

l to be normalize the weight vector w . The solving method should be set to 'GRG Nonlinear'. The theoretical minimum value is 0. The solution is (0.5584,0.3196,0.1220). We will see in Table 6, Newton's method converges after three iterations.

To treat 4×4 and more order dimension pairwise comparison matrices is left to the future research.

	A	B	C	D	E	F	G
1	1.00	2.00	4.00				
2	0.50	1.00	3.00				
3	0.25	0.33	1.00				
4	=MDETERM(A1:C3)						
5	# of iteration						
6	0	3		w	Aw	$\lambda_{max}w$	$\ Aw - \lambda_{max}w\ ^2$
7	1	(*1)		0.33	(*2)	=B\$10*D7	=(E7-F7)^2
8	2	↓		0.33	(*2)	↓	↓
9	3	↓		0.33	(*2)	autofill from F7	autofill from G7
10	4	autofill from B7		=SUM(D7:D9)			=SUM(G7:G9)

Table 6 Realization of the eigenvalue method(*1,*2; see the main text)

4. Conclusion

In this article, we reported practice details given at the classrooms of the University of Toyama and Oita University. Both are the local universities in Japan where students' academic computer skills are standard in Japan. In the report, we expressed the possibility of data science education with Excel. The subjects covered were portfolio optimization and the analytic hierarchy process, which are popular in operations research studies in Japanese universities.

We showed how to use Excel in both subjects. The utilization of Excel and Solver is almost all the same for Excel for Windows and Excel for Mac. We are confident that our proposal will stand up to use under typical PC environments.

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