# Radiotherapy Patient Scheduling During Pandemics

by

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## Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

## Abstract

With the Covid-19 outbreak happening worldwide, clinically vulnerable people should be of concern, as they are more likely to be exposed to the virus. Cancer patients with weak immune systems are a group of aforementioned people that often have to undergo radiotherapy treatment sessions every day for several weeks. Therefore, special measures are to take place for more protection. During the treatment process, they will be assigned to Linear Accelerator (LINAC) machines that are located in separate rooms of the radiotherapy center. During each visit, they are in close contact with other patients that are assigned to the same LINAC, but for different time slots. Our research focuses on scheduling radiotherapy patients, using two mixed-integer linear programming models, to minimize the total number of potential interactions between patients. A secondary objective is then proposed to choose among the set of optimal solutions, and the models' complexity growth is discussed. Then, we introduce a heuristic algorithm to increase the efficiency of the proposed model for large instances and use a visual step-by-step example to further elaborate the algorithm details. Finally, small numerical examples are used to demonstrate the effectiveness of the models, followed by larger instances from our partner clinic, the Grand River Regional Cancer Center (GRRCC). The results show that implementing the proposed model and the heuristic will decrease the number of interactions up to 75%, compared to the center's original schedule.

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# Chapter 1

# Introduction

Radiation is the energy of a stream of particles that is directed to the DNA of cancer cells in a specific area of the body to prevent them from growing, which leads to the death of such cells over time. The type of radiation and the cell growth speed affect the time needed to stop cancer cells from dividing. The total amount of radiation needed for a specific type of cancer is divided into fractions that might take up to several weeks. According to the guidelines of the American Cancer Society, treatments are often given 5 days a week, for about 5 to 8 weeks.

Radiation therapy can be delivered in several ways, but external beam radiation is the most widely used around the world. It can be given in a treatment center so that the patient does not have to stay in the hospital. It can also be used in treating more than one area, such as the lymph nodes nearby the main tumor. In the most common form of radiotherapy, the high-energy photon beams are delivered by a linear accelerator machine (LINAC).

Treatment planning for external beam radiation therapy is the first step in the process. The radiation team will be designing a personalized treatment for each patient, deciding on the dose of radiation, and generating a care path. The care path starts with a simulation session, in which the patient is asked to lie on a bed so that the staff members decide on the most efficient treatment position, and the exact place on the body for the radiation to be aimed will be marked. In all the remaining sessions, the patient will have to stay still during their treatment to ensure that the right amount of radiation is directed to the right cells.

With the growing number of cancer patients, there will be an average of 16% increase in demand for radiotherapy until 2025 [Lievens et al., 2015]. Therefore, radiotherapy centers have to manage their limited resources most efficiently. With the COVID-19 outbreak, given the increasing number of people undergoing treatments in radiotherapy centers, and the weak immune system of radiotherapy patients, the problem has become even more challenging. There must be special measures in place to ensure the safety of cancer patients during their visit to a radiotherapy center.

According to the World Health Organization (WHO) guidelines, the risk of transmission is higher in closed spaces where people are in contact for a while. Consequently, a crucial measure that was vastly practiced, avoiding close contact, had an important role in controlling the pandemic. This measure resulted in a reduced capacity in most public places, and longer wait times to get service. For a radiation therapy center, the same measure would not be ideal. It has been shown that longer wait times for treatment sessions will result in psychological distress in patients undergoing radiotherapy treatment [Mackillop, 2007], hence, it has to be ensured that the capacity is not decreased so that the patients receive their treatment timely.

Recent findings in [Johansson et al., 2021] have shown that more than 50% of new COVID-19 infections arise from exposure to asymptomatic patients, which indicates that a regular COVID-19 symptom screening strategy would not be an ideal solution to minimize transmission as well. An alternative approach to decrease transmission in a radiotherapy center is to decrease the number of close contacts between patients when they are present in the radiotherapy center. Interaction is going to happen between a patient leaving a specific treatment room, and another patient who is booked for the same room right after the previous patient. They will be present in the same waiting area, which is a closed space, in between their sessions.

In this thesis, we study the radiotherapy scheduling problem (RTSP), which consists of finding the treatment schedule for a set of patients over a given horizon, discretized in time slots, given a set of LINACs, considering side constraints and preferences. The main goal of this research is to schedule radiotherapy patients and to reduce the transmission of airborne diseases, by minimizing the number of interactions between radiotherapy patients. The problem specifications are going to be elaborated in detail in Chapter 3.

The remainder of this thesis is organized as follows: Chapter 2 reviews similar studies in this field in three different categories. Chapter 3 defines the radiotherapy patient scheduling and the problem specifications, and introducing mixed-integer linear programming models, followed by the heuristic method proposed in Chapter 4. The results obtained from various instances are discussed in Chapter 5, and finally, Chapter 6 includes the conclusions.

# Chapter 2

# Literature Review

Scheduling is an extensively studied topic in various fields. Schmidt [2000] reviews papers that address the scheduling problem, from operating systems to healthcare applications. However, among the vast literature on scheduling, we focus on the most related papers to this study. With the increasing global longevity, healthcare scheduling problems have attracted more interest [Rais and Viana, 2010]. It has been considered a challenging problem, given the resource limitations and several constraints and preferences. Nevertheless, optimizing the scheduling of healthcare resources results in improving service quality provided for patients, as well as increasing benefits of hospitals [Gupta and Denton, 2008]. Analyzing about 200 articles in healthcare scheduling, Abdalkareem et al. [2021] reports that patient scheduling, nurse scheduling, and operation room scheduling problems are the most common studies in the literature.

Over the past years, the number of cancer patients has been continuously increasing, resulting in a growing demand for Radiotherapy (RT). Expensive resources, limited capacity, and the restricted time availability of specialists make the planning for RT centers complex. Vieira et al. [2016] reviews studies where operations research has been implemented to help decision-making in RT. But, with surging COVID-19 cases around the world, the infrastructure of the healthcare systems has been challenged. It has affected scheduling, staffing, and surgical procedures, and resulted in emerging studies on its impact [Soltany et al., 2020a]. In this chapter, first, we review papers that discuss different aspects of the patient scheduling problem, followed by the sub-problem of radiotherapy patient scheduling studies with different objectives, and will conclude with recent studies in response to the COVID-19 pandemic. Lastly, the contributions of this study are going to be highlighted.

## 2.1 Patient Scheduling

The patient scheduling problem is an optimization problem, in which the assignment of patients to rooms in the healthcare organization, over a given time horizon is studied. It supports decision-makers at strategic, tactical, and operational level [Lusby et al., 2016]. Although receiving immediate service in a healthcare organization plays a vital role in patient satisfaction [Naidu, 2009], not all studies in the literature are aiming to maximize patient satisfaction by minimizing the completion time. Another objective function type is becoming more cost-efficient and maximizing profit by minimizing the idle time in resources or maximizing the number of scheduled patients. Both groups are valid, and it is up to the healthcare organization to decide which objective group is preferred. Marynissen and Demeulemeester [2019] proposed a classification of patient scheduling research papers based on the objective function that is being optimized. The most popular group of goals and the respective literature are discussed in the following sections.

### 2.1.1 Minimizing Total Completion Time

Patients often complain about the long delays within the treatment process. Zhao et al. [2018] aims to enhance the service quality by reducing waiting time for the rehabilitation scheduling problem, using a heuristic genetic problem. Suss et al. [2018] propose and test a novel simulation-based novel algorithm to reduce waiting times in an oncology clinic. Saadani et al. [2014] study the problem of assigning patients to different hospital resources during their stay. They propose a mixed integer model to minimize the total duration of stay of all patients, and the experiments have shown that with the growing size of the instances, the run-time rises exponentially. Munavalli et al. [2020] proposed a patient scheduling model for scheduling walk-in patients in outpatient clinics. Their main focus is to find the optimal pathways to direct patients, decreasing waiting time and cycle time for them using a hybrid ant agent algorithm. Hachicha and Mansour [2018] propose two Mixed Integer Linear Programs (MILP) to model the surgery scheduling problem, minimizing both the average length of stay and the number of overnight stays. The models are then evaluated by real-world data of a private clinic.

### 2.1.2 Maximizing Number of Scheduled Patients

To restrict cost increases in healthcare organizations, they need to operate efficiently with the resources already available. Burdett and Kozan [2018] schedule an entire hospital using a flexible job-shop scheduling model, and solve the problem using a meta-heuristic algorithm. Conforti et al. [2011] tackle the problem of scheduling Week Hospital patients, using an innovative integer programming model to maximize the patient flow. Week Hospital is a healthcare organization, in which services are planned in advance and delivered weekly. Kortbeek et al. [2017] proposed an integer linear program for a children's medical center, which is then used as an input to a simulation study, followed by a queuing model to maximize the number of scheduled patients.

### 2.1.3 Minimizing Wait Time

The waiting time to start treatment plays a vital role in patients' satisfaction in healthcare organizations, hence, optimization methods are used to minimize the wait times for patients and improve services. Mahmoudzadeh et al. [2020] schedule patients using an optimization approach considering the wait time target for the acuity level of each patient. The study presents a MILP, followed by the numerical results to show that the proposed approach increases solution qualities in terms of service levels and wait times. Daldoul et al. [2018] addresses the waiting time problem in the emergency department (ED), focusing on the staff members and beds available in the ED. Proposing a mixed-integer programming model, followed by the experimental study, it is shown that the average total waiting time can be improved up to 23.24%. The impact of operations, resource allocation, and scheduling on wait times have been addressed in various studies, but Santibáñez et al. [2009] analyzes their simultaneous impact using simulation in a cancer agency. It points out that the best outcomes heavily rely on the on-time clinic start and the need for effective patient scheduling in radiotherapy centers.

Various other objectives have been studied in the literature as well. Ruiz-Hernández et al. [2020] aim to maximize the health center's revenue, using the estimated probability of no-show for each appointment and associating the socio-demographic characteristics of each patient with behavioral issues that affect the probability. The proposed model results in about a 5% increase in revenue and a 13% decrease in waiting list length, compared to a first-come-first-serve approach. However, the objective of minimizing the number of interactions between patients has not been addressed to the best of our knowledge. As mentioned, the general patient scheduling problem has been studied for different healthcare organizations, including radiotherapy patient scheduling. However, the main difference between radiotherapy patient scheduling and general patient scheduling is that the radiotherapy treatments are often divided into fractions, which are delivered once a day until the planned dosage is reached. Therefore it has been extensively studied separately as well.

## 2.2 Radiotherapy Patient Scheduling

Radiotherapy patient scheduling is the problem of assigning cancer patients to the resources available in a radiotherapy center. Radiotherapy is considered to be at least as cost-effective as other cancer treatments, justifying the continuous growth of demand in radiotherapy centers [Shukla et al., 2015]. The radiotherapy process can be affected by technological constraints, like a limited number of staff members that are trained to operate a subset of linear accelerator machines, or medical constraints, like when the treatment depends on the surgery. The variety of these constraints leads to the development of ad-hoc approaches with specific characteristics [Vieira et al., 2016]. While different problem types have been tackled in this field, including strategic decision making, patient prioritizing, resource planning, and scheduling, we will focus on the scheduling models, which are both the most studied, according to Vieira et al. [2016], and the most relevant to this thesis.

### 2.2.1 Mathematical Programming Approach

Mathematical programming techniques have been the most common approaches in the radiotherapy patient scheduling studies that have been reviewed in Vieira et al. [2016]. Frimodig and Schulte [2019] introduce and compare two Constraint Programming (CP) and one Integer Programming (IP) models for the radiotherapy patient scheduling problem. They consider various scenarios based on the patient arrival rate and backlog, as well as LINAC availability and the number of time windows in a day. Although the IP model finds the optimal solution faster, the CP models perform better in finding feasible solutions. Similarly, Pham et al. [2022] introduces a two-phase method for the radiotherapy scheduling problem. First, an Integer Linear Programming (IP) model is used to assign the sessions to LINACs and days. In the second phase, a Constraint Programming (CP) model and a Mixed Integer Linear Programming (MILP) model are used for sequencing patients on each LINAC and time. Comparing the CP and MILP using a real-world dataset, it has been shown that although CP is faster in finding good solutions, MILP outperforms CP in reaching optimality with more run time. In another study, Conforti et al. [2010] aims to minimize the mean waiting time or maximize the number of scheduled new patients, using an integer linear optimization model. The effectiveness of the proposed model is then demonstrated by conducting numerical experiments with some use-case scenarios.

### 2.2.2 Heuristic Approach

The next most common approaches are heuristic and metaheuristic algorithms in the radiotherapy scheduling literature. Vogl et al. [2019] aims to minimize both the penalties corresponding to the violation of given time window constraints and the operation time of the particle beam as the bottleneck resource in Ion beam radiotherapy. A local search and a genetic algorithm are introduced as meta-heuristic solution approaches, and for real-world instances, a combination of both algorithms is shown to have the best results. Braune et al. [2021] addresses the problem of appointment planning with uncertain treatment duration and a single device. First, they propose a model, introducing a buffer based on duration distributions, and a procedure to adapt a pre-determined schedule to the patient flow. A real-world dataset is used to compare the experimental results of the different versions of the problem, implementing a heuristic based on a combination of Monte Carlo simulation and Genetic Algorithm. Petrovic et al. [2011] present an optimization model for categorized cancer patients, to minimize both the average length of breaches of wait times and the average of waiting time for patients. Three genetic algorithms are implemented and compared using statistical analysis. Vieira et al. [2020] propose a MILP considering time window preferences given by patients to solve the scheduling and sequencing problems for radiotherapy sessions. Although the proposed model can solve small instances in a reasonable time, for larger instances a heuristic method is proposed, which first assigns patients to LINACs and then solves the sub-problem for sequential clusters of LINACs.

### 2.2.3 Other Approaches

Although mathematical programming and (meta)heuristics are the most popular approaches, other approaches have been used as well. Sauré et al. [2012] seek to find policies for assigning incoming patients to the available radiotherapy units, while minimizing wait times, by solving a discounted infinite-horizon Markov decision process. Then, an approximate optimal policy is found by using column generation for solving the equivalent LP model. Finally, the performance of the proposed method is evaluated, using data from a cancer agency.

The literature is mainly focused on the objective of improving efficiency for radiotherapy centers, using mathematical programming and (meta)heuristics. With that being said, it is important to note that the objective of this research has not been addressed in the literature, but the used method is a combination of the two most common approaches in this field.

## 2.3 Scheduling During Pandemics

With the rising number of infected patients during the COVID-19 pandemic, the medical priority was redirected towards COVID-19 patients, challenging the healthcare systems' infrastructure. Several studies have been in progress since then, to adapt the healthcare organizations to the new guidelines recommended by authorities. Soltany et al. [2020b] have reviewed the impact of the COVID-19 pandemic on staffing, scheduling, and almost all aspects of surgical practices, including cancer surgeries, highlighting the precautions to take and the new treatment modalities. Neethirajan and Manickam [2020] have mentioned strategies to overcome the challenges during the pandemic in surgery facilities, including minimizing the chance of exposure. To reduce the risk of transmission, the mandatory wearing of masks, designated areas for infected patients, and social distancing for staff and patients should be in place. However, these studies heavily rely on delaying services until the pandemic is controlled, which will have a negative impact on cancer patients' treatment outcomes.

Liu et al. [2022] model the weekly physician scheduling problem during COVID-19. The waiting patients' queue has been increasing, due to special measures during COVID-

19. A mathematical model is proposed, which is challenging to solve for the real-world instances, provided by the collaborating hospitals. Therefore, a two-phase approach is introduced, including a staffing model, and a branch-and-price algorithm to solve the problem. The performance of the proposed model and the two-phase approach is also discussed. Moosavi et al. [2022] aims to minimize the total cost of part-time staff members' salaries, the violation of service over time, and the waiting time for residential care. In order to adapt the basic problem to the COVID-19 conditions, a new objective is introduced to minimize the number of distinct cohorts assigned to each staff member, and the number of staff members assigned to each room. A population-based heuristic approach is then introduced to solve the problem, which is shown to outperform two benchmark solution approaches. Zucchi et al. [2021] conduct a study on the personnel scheduling problem during the COVID-19 pandemic. The proposed MILP aims to minimize the risk of contagion by grouping employees mutually exclusive. Experiments using an open-source solver indicate that the solution improves using the proposed model. Ghatnekar et al. [2021] use simulation modeling to find out how to vary resources at the Department of Dermatology, to minimize patient contact times during the pandemic. They measured average wait time (AWT) and the percentage of patients in contact with 1 or more patients in the waiting area (PTPC), aiming to decrease AWT from 29 minutes to less than 15 minutes, based on the guidelines. The best outcome was achieved with 5 rooms, 2 residents per physician, and

2 medical assistants. However, this approach can not be implemented in a radiotherapy center. In addition to various constraints for radiotherapy scheduling, there are specific LINACs in each room that are used for a specific subset of cancer types, each being booked for a specific group of patients. Furthermore, LINACs are expensive resources and the number of LINACs in radiotherapy centers can not be easily increased. In this thesis, we consider operational constraints for a radiotherapy center, as well as assigning patients to the available resources.

## 2.4 Contributions

Radiotherapy patient scheduling and scheduling during the COVID-19 pandemic in healthcare centers have been extensively studied in the past years. In the radiotherapy patient scheduling literature, the impact of the COVID-19 pandemic has been studied, focusing only on the staff scheduling problem, whereas cancer patients also need special attention to be protected from infection. On the other hand, in the studies addressing the challenges arising from the COVID-19 pandemic, the main focus is on increasing the quality of service, and just a few studies have addressed the minimization of transmission within this context. This thesis is aiming to bridge the gap between the two fields, by scheduling radiotherapy patients and minimizing the chance of transmission in radiotherapy centers between cancer patients. Our contributions are as follows:

- Proposing a Mixed-Integer Linear Programming model for radiotherapy patient scheduling to minimize the number of patient-patient interactions
- Developing a heuristic algorithm to improve efficiency
- Evaluating the model and the heuristic using real-world data obtained from the Grand River Regional Cancer Center (GRRCC)

# Chapter 3

# Methodology

In Radiation Therapy Patient Scheduling (RTPS), the main goal is to schedule recurring sessions for different patients over the time horizon. Consider a set of treatment sessions for a set of patients $\mathcal{P}$ over a planning horizon $\mathcal{D}$ discretized down to a set of time slots $\mathcal{T}$ of the same length. Each patient $p \in \mathcal{P}$ will need to undergo a total of $r_p$ radiotherapy sessions, using linear accelerator (LINAC) machines located in one of the rooms of the set $\mathcal{F}$. All the required sessions for patient $p$ should take place on consecutive days to maximize effectiveness. In this setting, each room only has one LINAC operating, which is usually used for a specific type of treatment and a specific set of staff members are assigned to each LINAC for specific shifts. Therefore, patients will be assigned to the same LINAC throughout their treatment. Also, there is a deadline assigned to each patient, $m_p$ which indicates the due date to start the treatment for each patient.

## 3.1 Problem Definition

The goal of this study is to reduce airborne disease transmission during a pandemic, in a radiotherapy center by reducing interactions or contacts between patients. In order to reach this goal, we minimize the number of interactions between patients as the objective of the optimization model. Each patient will be in contact with the same staff members, and there can not be any improvements in the number of interactions between patients and staff members. However, the interaction between patients may happen in different scenarios, including the interaction between patients in common areas, or between the patients entering and leaving the treatment room at the same time. Given the fact that special measures and specific guidelines are considered to clean entirely and disinfect the rooms thoroughly after each treatment session, the possibility of transmission while inside the treatment room is very low, hence the aforementioned scenario is not considered in the optimization model.

At Grand River Regional Cancer Center (GRRCC), the duration of treatment sessions is always multiples of 15 minutes, so we discretize $\mathcal{T}$ into time slots of 15 minutes. For instance, the first radiotherapy session for brain cancer is always 60 minutes, so it will be represented as 4 time slots. The length of treatment for each patient $p$ in terms of the number of time slots is denoted by $n_p$. To calculate the number of interactions between patients, $x_{ij}$ is introduced as in Table 3.1, which is calculated for each pair of distinct patients, $i$ and $j$. It will be equal to 1 if patient $j$ is assigned to the time slot right after patient $i$ on the same day $d$ and in the same room $f$. Minimizing the sum of $x_{ij}$ over all pairs of patients will result in the minimum number of interactions.

To ensure that each patient will start their treatment before the indicated deadline, $v_p^d$ is introduced as indicated in Table 3.1 to represent the first day of treatment for each patient $p$. The first day of treatment for patient $p$ is the first day $d$ that the patient is booked for a treatment session, but because all the sessions are on consecutive days for each patient, there will not be a session for patient $p$ in the previous day, i.e. $d - 1$.

In this study, we consider the preference of the GRRCC to find a weekly schedule for patients, which can be updated regularly with incoming new patients. This way the optimization model can be running right after the end of a working week, to have the next week's schedule ready on time. Therefore, the planning horizon is set to be equal to 5 days, from Monday to Friday each week. Moving forward in this study, we will be referring to the planning horizon as 1 week, instead of 1 working week, for simplicity.

Usually, radiotherapy patients need several treatment sessions and after scheduling patients for the planning horizon of one week, there will be remaining sessions for some patients. From a patient perspective, it is more desirable to be booked on the same day of the week and the same time slot, so that they can work with their schedules with maximum consistency. Furthermore, it is a requierement for GRRCC to have the same time slots of each day booked for the same patient, until their treatment is completed. Therefore, we aim to book the remaining sessions on the same day of the week, in the same room, and in the same time slot of the day. In order to calculate the remaining sessions, we will deduct the number of treatment sessions that patient $p$ has had during the planning horizon (1 week) from the total sessions needed at the beginning of the horizon and update $r_p$, the remaining number of sessions for patient $p$, at the end of each planning horizon.

To keep track of the number of sessions that patient $p$ has had during the planning horizon, the assignment variable, $\omega_{pf}^{dt}$, is going to be used. As mentioned in Table 3.1, $\omega_{pf}^{dt}$ is going to be equal to 1 if patient $p$ is assigned to a session on day $d$, time slot $t$, and in room $f$, hence, the sum of $\omega_{pf}^{dt}$ over the days in the planning horizon will be equal to the number of treatment sessions for patient $p$. Then, using the updated $r_p$, we will pre-book the same days and time slots of the same room for patient $p$, as the first $r_p$ sessions of the previous planning horizon, marking them as unavailable. Availability of room $f$ on day $d$ and in time slot $t$ is given at the start of each planning horizon, using $a_f^{dt}$ as shown in Table 3.1. To pre-book the remaining sessions for patient $p$, setting $a_f^{dt}$ to be 1 for the same $d$, $t$, and $f$ of the first $r_p$ sessions of the previous horizon will be enough.

As an example, assume that the patient $p$ needs 10 sessions in total, and is booked on the first time slot of each day for a week in the room $f$. At the end of the first planning horizon, the remaining number of sessions, $r_p$, will be equal to 3 and the first time slot of days 1, 2, and 3 in the room $f$ will be marked as unavailable, setting the respective $a_f^{dt}$ equal to 1.

In the next sections, we will propose MILP models for the RTPS problem with the objective of minimizing the number of interactions between patients, for specific problem settings. These settings require including multiple binary variables, which will be a challenge when solving the models using a solver. Based on the main objective, a heuristic approach is going to be proposed, in order to overcome the complexity of the models, and will be assessed during the computational experiments.

<table>
<tr><th rowspan="2"></th><th colspan="3">Room 1</th><th colspan="3">Room 2</th></tr>
<tr><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th></tr>
<tr><td>Day 1</td><td>Patient 2</td><td>Patient 1</td><td>Patient 5</td><td></td><td colspan="2">Patient 6</td></tr>
<tr><td>Day 2</td><td>Patient 2</td><td>Patient 1</td><td>Patient 5</td><td></td><td colspan="2">Patient 6</td></tr>
<tr><td>Day 3</td><td>Patient 4</td><td>Patient 1</td><td>Patient 5</td><td colspan="3">Patient 7</td></tr>
<tr><td>Day 4</td><td>Patient 4</td><td>Patient 3</td><td>Patient 5</td><td colspan="3">Patient 7</td></tr>
</table>

Figure 3.1: Example of a 4-day schedule

| Sets | Description |
|---|---|
| $\mathcal{P}$ | Set of Patients |
| $\mathcal{D}$ | Set of days available for booking |
| $\mathcal{T}$ | Set of Time slots |
| $\mathcal{F}$ | Set of Rooms |
| Parameters | |
| $r_p$ | Remaining number of days for patient $p \in \mathcal{P}$ |
| $m_p$ | Deadline to start the treatment for patient $p \in \mathcal{P}$ |
| $n_p$ | The length of treatment for patient $p \in \mathcal{P}$ |
| $a_f^{dt}$ | 1 if room $f \in \mathcal{F}$ is not available on day $d \in \mathcal{D}$ and time slot $t \in \mathcal{T}$, 0 otherwise |
| Variables | |
| $x_{ij}$ | 1 if patients $i, j \in \mathcal{P}$ interact, 0 otherwise |
| $\omega_{pf}^{dt}$ | 1 if patient $p \in \mathcal{P}$ has a session on day $d \in \mathcal{D}$ and time slot $t \in \mathcal{T}$ in room $f \in \mathcal{F}$, 0 otherwise |
| $v_p^d$ | 1 if patient $p \in \mathcal{P}$ starts the first session of treatment on day $d \in \mathcal{D}$, 0 otherwise |
| $z_p^{dt}$ | 1 if the treatment session for patient $p \in \mathcal{P}$ on day $d \in \mathcal{D}$, starts in time slot $t \in \mathcal{T}$, 0 otherwise |

Table 3.1: Notations used in this study

We propose two different models, based on the length of the treatment sessions for each patient. In the first version shown in (3.1), we assume that all the sessions have a fixed treatment duration (FTD) of 1 time slot, whereas in the second version shown in (3.2) it is assumed that sessions have various treatment duration (VTD), denoted by $n_p$ time slots. The FTD model is a simplified method that has fewer variables and constraints and is easier to solve. According to GRRCC documents, more than 75% of treatments take 1 time slot to complete, therefore, FTD will be adequate to schedule these sessions. Using FTD, multiple optimal schedules might be generated, especially if the problem size is small. To choose the most desired solution among the set of all candidate solutions, we introduced a secondary objective function in (3.3), which aims to choose the solution that starts the treatments as soon as possible, decreasing the patients' wait times.

Although the FTD model is useful for most treatment scenarios, there can be different types of cancer treatments that are assigned to the same machine while having different treatment duration per session. These treatments are not necessarily of the same length, but they should be assigned to the same room $f$. To schedule the patients that have to undergo these types of treatments a more complex model that can incorporate variable treatment duration is needed, therefore, we introduce VTD. All the notations used throughout the mathematical models are listed in Table 3.1.

## 3.2 Fixed Treatment Duration Model

The fixed Treatment Duration (FTD) model is a radiotherapy patient scheduling problem in which patient $i$ interacts with patient $j$ if they are assigned to consecutive time slots on the same day and room. There is 1 specific LINAC operating in each room and 1 particular team of staff members working with each LINAC. Assigning patient $p$ to the room $f$ means that the patient is going to have all their treatment sessions in the same room and with the same LINAC throughout their treatment duration. Some LINAC machines are being used for a specific type of cancer, and all the sessions booked with them are of the same length. This specification allows us to reduce the overall complexity of the model, by setting sessions to be of the same length (1 time slot). Although this model is a simplified method, it does not eliminate crucial constraints of the patient scheduling problem. Therefore, it can be used for similar types of cancer treatment that have the same duration.

### 3.2.1 Mathematical Model

$$(\mathbf{FTD}): \quad \text{Min} \quad \sum_{i,j \in \mathcal{P}, i \neq j} x_{ij} \tag{3.1a}$$

$$\text{s.t.} \quad x_{ij} \geq \omega_{if}^{dt} + \omega_{jf}^{d(t+1)} - 1 \qquad \forall d \in \mathcal{D}, \forall t \in \mathcal{T}, \forall f \in \mathcal{F}, \forall i, j \in \mathcal{P}, i \neq j \tag{3.1b}$$

$$v_p^d = \omega_{pf}^{dt} \qquad \forall p \in \mathcal{P}, \forall t \in \mathcal{T}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D}, d = 1 \tag{3.1c}$$

$$1 + v_p^d \geq \omega_{pf}^{dt} + (1 - \omega_{pf}^{(d-1)t}) \qquad \forall p \in \mathcal{P}, \forall t \in \mathcal{T}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D}, d \geq 2 \tag{3.1d}$$

$$v_p^d \leq \omega_{pf}^{dt} + (1 - \omega_{pf}^{(d-1)t}) \qquad \forall p \in \mathcal{P}, \forall t \in \mathcal{T}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D}, d \geq 2 \tag{3.1e}$$

$$\sum_{d \in \mathcal{D}} v_p^d = 1 \qquad \forall p \in \mathcal{P} \tag{3.1f}$$

$$v_p^d = 0 \qquad \forall p \in \mathcal{P}, \forall d \in \mathcal{D}, d > m_p \tag{3.1g}$$

$$\omega_{pf}^{dt} + a_f^{dt} \leq 1 \qquad \forall p \in \mathcal{P}, \forall t \in \mathcal{T}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D} \tag{3.1h}$$

$$\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} \sum_{f \in \mathcal{F}} \omega_{pf}^{dt} = min[|\mathcal{D}|, r_p] \qquad \forall p \in \mathcal{P} \tag{3.1i}$$

$$\sum_{p \in \mathcal{P}} \omega_{pf}^{dt} \leq 1 \qquad \forall t \in \mathcal{T}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D} \tag{3.1j}$$

$$\sum_{t \in \mathcal{T}} \sum_{f \in \mathcal{F}} \omega_{pf}^{dt} \leq 1 \qquad \forall p \in \mathcal{P}, \forall d \in \mathcal{D} \tag{3.1k}$$

Constraint (3.1b) indicates that patient $i$ has an interaction with patient $j$ if they are in consecutive sessions in the same room and day. Constraints (3.1c), (3.1d) and (3.1e) define and set the starting day of treatments, $v_p^d$, for each patient. If day $d$ is the first session for patient $p$, there should be at least 1 treatment session booked on that day for that patient. Also, if day $d$ is the first session for patient $p$, there should be no sessions booked on the day $d-1$. Constraint (3.1f) indicates that each patient must have exactly one start day. Constraint (3.1g) prevents booking the first treatment after $m_p$, the deadline. Constraint (3.1h) is used to make sure that a new session for patient $p$ is booked, only if the corresponding room $f$ is available on day $d$ and time slot $t$. Constraint (3.1i) limits the total booked sessions for each patient to the total remaining sessions for them, or the end of the planning horizon, whichever comes first. Constraint (3.1j) ensures that at most one patient is assigned to a specific day, time slot, and room. Similarly, constraint (3.1k) ensures that each patient is scheduled at most once a day.

In order to consider different lengths for time slots, we need to introduce the second model which is more general than the first one but has an additional set of binary variables that represent the first time slot of the day on which patient $p$ is booked for a treatment session. Introducing this new variable results in more complexity, hence being harder to solve.

## 3.3 Variable Treatment Duration Model

For the variable Treatment Duration (VTD) model, the definition of the patient-patient interaction stays the same. There is 1 specific LINAC operating in each room and 1 particular team of staff members working with each LINAC. Assigning patient $p$ to the room $f$ means that the patient is going to have all their treatment sessions in the same room and with the same LINAC throughout their treatment duration. As previously mentioned, some LINAC machines can be used for different types of cancer treatments, with various session duration. The corresponding model must allow the sessions to be of different lengths, i.e. $n_p$ time slots. In order to allow different treatment duration, a new set of variables are introduced in addition to all the variables used in FTD, to help make sure that the desired treatment duration has been met.

### 3.3.1 Mathematical Model

$$(\textbf{VTD}): \quad \text{Min} \quad \sum_{i,j \in \mathcal{P}, i \neq j} x_{ij} \tag{3.2a}$$

$$\text{s.t.} \quad (3.1b) - (3.1h) \tag{3.2b}$$

$$1 + z_p^{dt} \geq \omega_{pf}^{dt} + (1 - \omega_{pf}^{d(t-1)}) \qquad \forall p \in \mathcal{P}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D}, \forall t \in \mathcal{T}, t \geq 2 \tag{3.2c}$$

$$z_p^{dt} \leq \omega_{pf}^{dt} + (1 - \omega_{pf}^{d(t-1)}) \qquad \forall p \in \mathcal{P}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D}, \forall t \in \mathcal{T}, t \geq 2 \tag{3.2d}$$

$$z_p^{dt} = \omega_{pf}^{dt} \qquad \forall p \in \mathcal{P}, \forall f \in \mathcal{F}, \forall d \in \mathcal{D}, \forall t \in \mathcal{T}, t = 1 \tag{3.2e}$$

$$\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} z_p^{dt} = min[|\mathcal{D}|, r_p] \qquad \forall p \in \mathcal{P} \tag{3.2f}$$

$$\sum_{t\in\mathcal{T}}\sum_{f\in\mathcal{F}}\omega_{pf}^{dt} \le n_p \qquad \forall p \in \mathcal{P}, \forall d \in \mathcal{D} \tag{3.2g}$$

$$\sum_{t\in\mathcal{T}}\sum_{d\in\mathcal{D}}\sum_{f\in\mathcal{F}}\omega_{pf}^{dt} = n_p \times min[|\mathcal{D}|, r_p] \qquad \forall p \in \mathcal{P} \tag{3.2h}$$

Constraints (3.1b)-(3.1h) are included in this model as well, as they capture the basic specifications of the problem. As mentioned in table (3.1), $z_p^{dt}$ is defined as the first time slot booked in day $d$ for patient $p$. Constraints (3.2c), (3.2d), (3.2e) ensure that $z$ is equal to 1, if and only if no prior time slots are booked for patient $p$ on day $d$. Constraint (3.2f) limits the number of treatments for each patient to $r_p$, the total remaining sessions for them, or the end of the planning horizon, whichever comes first. Constraint (3.2g) prevents the session duration on day $d$ for patient $p$ from exceeding $n_p$, the intended treatment duration for patient $p$. Constraint (3.2h) limits the number of time slots for each patient to total remaining sessions ($r_p$) times the duration of each session ($n_p$), or the length of the planning horizon times the duration of each session, whichever comes first.

## 3.4 Secondary Objective

There can be multiple optimal schedules for RTPS problem, especially when the schedule is not packed with patients, therefore, it can be beneficial to introduce a secondary objective that can help us select the most appropriate solution among the set of multiple optimal solutions. From a radiotherapy center's perspective, it is desirable to schedule sessions as soon as possible and to avoid unnecessary empty time slots during a work day. As an example, Figure 5.1 demonstrates an optimal solution generated by the FTD model. It can be seen that the FTD solution waits until day 2 to start the treatment for patient 11, which is the latest day to start the treatment for that patient, as indicated in Table 5.1. This leaves the first time slot of the first day of the planning horizon in Room 3 empty, while it could have been avoided.

In order to prevent this from happening, we propose a secondary objective to start booking sessions as soon as possible, and not wait until the deadline approaches to start the treatment sessions for patients. The secondary objective function shown in equation (3.3a) aims to minimize the difference between the first day of treatment for patient $p$ (i.e., when $v_p^d = 1$) and the deadline for patient $p$ (i.e., $m_p$). Furthermore, we need an additional constraint as in (3.3c) to force the first objective function value to be equal to the optimal value obtained from FTD, denoted by $x_{ij}^*$, to make sure that we are searching among all the optimal solutions for the original problem. All other constraints are going to be the same as in (3.1).

$$\text{Min} \quad \sum_{p\in\mathcal{P}} \sum_{d\in\mathcal{D}, v_p^d=1} d - m_p \tag{3.3a}$$

$$\text{s.t.} \quad (3.1b) - (3.1k) \tag{3.3b}$$

$$\sum_{i,j\in\mathcal{P}, i\neq j} x_{ij} = x^*_{ij} \tag{3.3c}$$

## 3.5 Model complexity

To discuss the complexity levels of FTD and VTD models, we are going to compare the number of variables and the number of constraints for these two. In FTD, three variables are used, $\omega_{pf}^{dt}$, $v_p^d$, and $x_{ij}$, the first having 4 indices for the number of the day, the number of the time slot, the number of the patient, and the number of the room, the second having 2 indices for the number of the day and the number of the patient, and the last one having 2 indices for the number of patients. The total number of $\omega_{pf}^{dt}$ variables will be equal to $|\mathcal{D}| \times |\mathcal{T}| \times |\mathcal{P}| \times |\mathcal{F}|$. Likewise, the total number of $v_p^d$ will be equal to $|\mathcal{D}| \times |\mathcal{P}|$. For $x_{ij}$, the total number will be equal to $|\mathcal{P}|!$, because it will be distinguishing between patients when calculating the objective value, and not when making the variables. For example, if there are 3 patients, 7 days, 1 room, and 1 time slot, there will be 21 variables generated by $\omega_{pf}^{dt}$, 21 variables generated by $v_p^d$, and 6 variables by $x_{ij}$, summing up to 46 variables in total.

In VTD, a new set of binary variables are introduced, $z_p^{dt}$, which generates $|\mathcal{D}|\times|\mathcal{T}|\times|\mathcal{P}|$ new variables, similar to the other variables discussed. Therefore, with increasing the size of the input, VTD will be growing quicker than FTD, in terms of the number of variables. However, the formula to calculate the number of constraints is not as straightforward as the number of variables, but it is important to note that all constraints for both problem types are binary constraints, and there are no integer or continuous constraints. In the next chapter, we will demonstrate the trend for complexity growth in terms of the number of constraints and the number of variables, expressing the need for a heuristic method for solving larger instances. As previously discussed, the heuristic proposed and the real-world large instances are going to be based on VTD, as it considers all types of treatment styles.

# Chapter 4

# Heuristic Solution Algorithm

In this section, we discuss the need for a heuristic approach for the proposed model, introduce a heuristic approach to generate a feasible solution for the problem of minimizing the number of interactions when scheduling radiotherapy patients and conclude with a visual step-by-step example.

## 4.1 Growth Rates

Optimization problems can take much time to solve, especially when the size of the input increases. The model introduced in section 3 is a mixed-integer linear programming model, and the complexity of the model increases with increasing the size of the input. Figure

4.1 illustrates the growth rate for both the number of binary variables and the number of constraints, with respect to increasing the number of patients, while all other parameters are fixed as 5 days, 5 time slots, and 1 room. As shown, the complexity of the model grows quickly, especially due to the growth of the number of constraints. Similarly, figure 4.2 illustrates the growth rate for the number of binary variables and the number of constraints, with respect to increasing the number of time slots, while all other parameters are fixed as 5 days, 10 patients, and 1 room. As shown, the number of constraints grows exponentially, resulting in quick growth in the complexity of the model. Therefore, the model is going to be hard to solve for real-world instances, where there are plenty of incoming patients waiting to be scheduled, and the run-time is going to increase exponentially. The solver will take a very long time to search the solution space and reach a feasible solution for the problem, hence, a good way to reduce the run-time is to help the algorithm find a feasible solution faster.

Initial solutions are always feasible, and ideally near-optimal solutions. They can be used to cut the feasible space for a complex model by providing an upper bound or a lower bound on the objective, as well as creating a warm start for a MILP model. They can be obtained by solving another mathematical model problem, or by using a heuristic approach.

In scheduling problems, one strategy that is often used by hospitals, clinics, or radio-

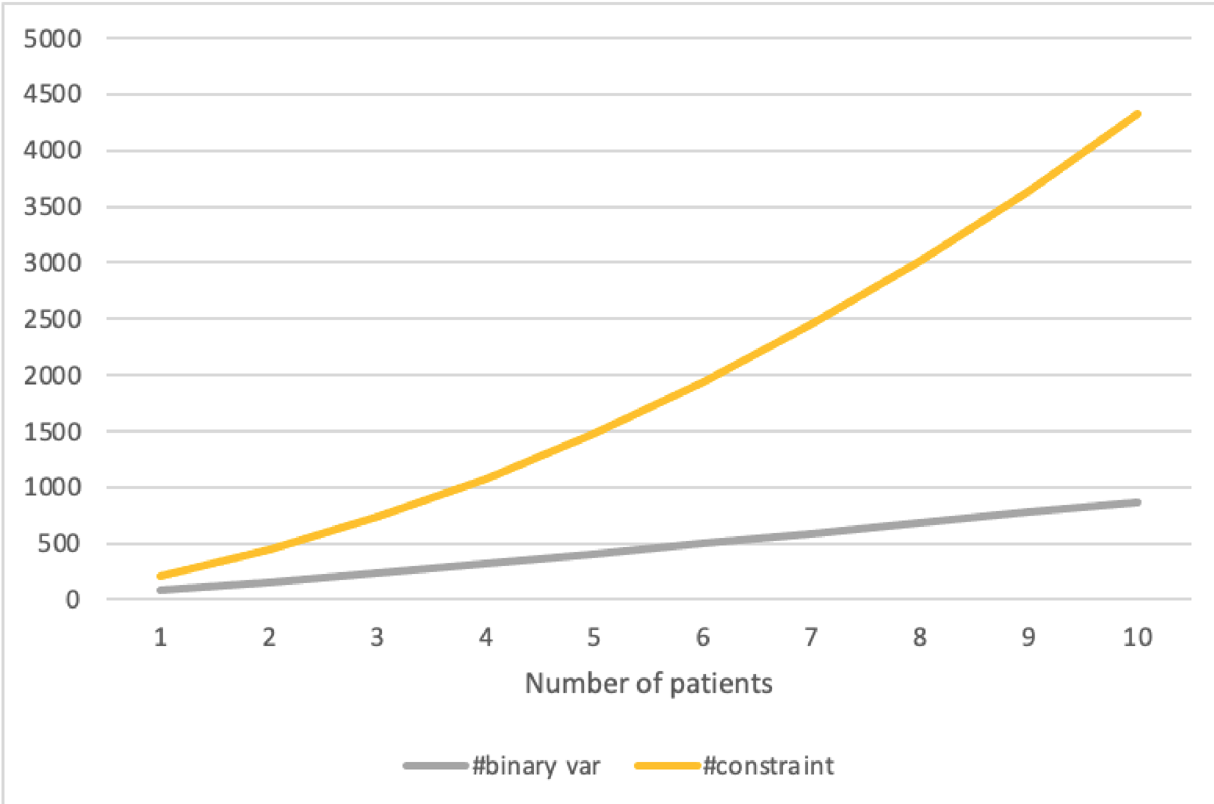


Figure 4.1: Complexity growth rate with respect to the number of patients

therapy centers is the first-come first-serve (FCFS) approach. They assign patients to the first available day and time slot as they come, to reduce their waiting times, but this approach would not always produce the best results, because it does not consider any other constraints. However, it can easily be used to obtain a feasible solution as an initial solution. The usual FCFS algorithms assign patients to the first available day and time slot, but they do not take consecutive sessions into account. Therefore, there can be unexpected

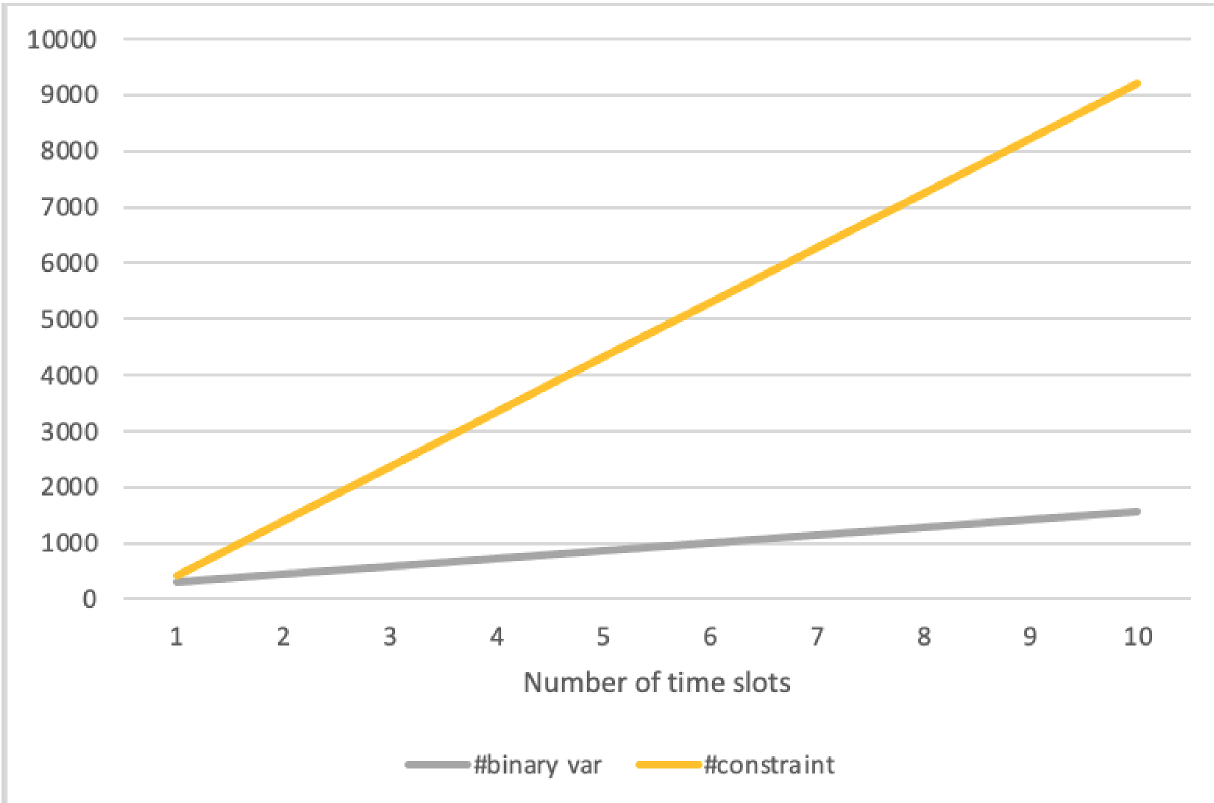


Figure 4.2: Complexity growth rate with respect to the number of time slots

delays between sessions of a patient, which will have a negative impact on their treatment effectiveness. Furthermore, it does not consider patient interactions as they just select the first available session, which will result in a higher number of interactions between patients. This would increase airborne disease transmission among patients, including COVID-19.

## 4.2 Proposed Algorithm

In this study, we introduce a heuristic to generate a feasible solution but consider the number of interactions and more constraints than a FCFS approach. The goal is to minimize the number of interactions between patients, which happens when patients are booked in consecutive time slots on a specific day in a specific room. A good feasible solution for this objective will have to ensure that the same patients are booked with each other so that they don't interact with many other patients during their treatment. To find such a solution, it is helpful to start booking patients with the most number of sessions in consecutive time slots to ensure that they will interact with as few patients as possible. Therefore, we start by sorting patients based on their remaining number of treatments in decreasing order and start assigning them based on the new order. It starts from the first available room and calculates the number of time slots that are not available or busy for each day and stores it as $b_d$. Then it continues with the least busy day with the most available time slots and chooses the first available time slot. Then it checks to see if the next $r_p$ days are available, and whether we reach the end of the planning horizon or not. It does the same process with $t$ to see if the next $n_p$ time slots are available and whether we reach the end of the day. If so, it books those sessions for the patient and marks them as unavailable, and updates the number of busy time slots, but if one of the conditions is not met, then it moves to the next available time slot or the next available day and repeats the process until it books all the patients. The pseudo-code of the heuristic used can be found in Algorithm 1.

| Patients | Total Sessions | Duration |
|---|---|---|
| Patient 1 | 2 | 3 |
| Patient 2 | 1 | 2 |
| Patient 3 | 4 | 1 |
| Patient 4 | 5 | 1 |
| Patient 5 | 3 | 1 |

Table 4.1: The set of patients for the heuristic example

## 4.3 Visual Example

To further elaborate the steps of the algorithm, a small visual example is used. The input used for this example is shown in Table 4.1. Assume that there are 5 days on the planning horizon and there are 5 time slots on each day.

**Algorithm 1** Pseudo-code of the heuristic used to generate the initial feasible solution for VTD

Sort patients based on $r_p$ in decreasing order

**for** $p \in \mathcal{P}$ **do**

  **for** $f \in \mathcal{F}$ **do**

    $b_d \leftarrow \sum_{t\in\mathcal{T}} a_f^{dt} \qquad \forall d \in \mathcal{D}$

    Choose $d^*$ such that $b_{d^*} = min\{b_d | d \in \mathcal{D}\}$

    Choose $t^*$ such that $t^* = min\{t | t \in \mathcal{T}, a_f^{d^*t^*} = 0\}$

    **if** $d^* + r_p \leq |\mathcal{D}|$ and $a_f^{(d^*+r_p)t^*} = 0$ **then**

      **if** $t^* + n_p \leq |\mathcal{T}|$ and $a_f^{d^*(t^*+n_p)} = 0$ **then**

        $b_{d^*} \leftarrow b_{d^*} + r_p$

        $a_f^{d^*t^*}, ..., a_f^{(d^*+r_p)t^*} \leftarrow 1$

        $a_f^{d^*t^*}, ..., a_f^{d^*(t^*+n_p)} \leftarrow 1$

      **else** $t^* \leftarrow$ next available time slot

      **end if**

    **else** $d^* \leftarrow$ next least busy day

    **end if**

### 4.3.1 Step 1

Sorting the patients in the decreasing order of total sessions, the first patient to be scheduled is patient 4, which needs 5 sessions with the duration of 1 time slot. At first, the schedule is empty and all time slots are available, therefore we choose the first available time slot of the first day, and book needed sessions for patient 4, updating the availability parameters $a_1^{11}$,...,$a_1^{51}$ to 1. Values of $b_1$,...,$b_5$ are changed from 0 to 1 as well. Figure 4.3 shows the schedule at the end of this step.

<table>
<tr><th rowspan="2"></th><th colspan="5">Room 1</th></tr>
<tr><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 4</th><th>Slot 5</th></tr>
<tr><td>Day 1</td><td rowspan="5">P4</td><td colspan="4" rowspan="5"></td></tr>
<tr><td>Day 2</td></tr>
<tr><td>Day 3</td></tr>
<tr><td>Day 4</td></tr>
<tr><td>Day 5</td></tr>
</table>

Figure 4.3: The schedule after Step 1

### 4.3.2 Step 2

The next patient in the order is patient 3 that needs 4 sessions of 1 time slot. As all the $b_d$ values are equal, again we choose the first available $t$ in the first available $d$, and update $a_1^{12}$,...,$a_1^{42}$ to 1. The value of $b_5$ remains unchanged, but $b_1$,...$b_4$ increase from 1 to 2. Figure 4.4 shows the schedule at the end of this step.

<table>
<tr><th rowspan="2"></th><th colspan="5">Room 1</th></tr>
<tr><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 4</th><th>Slot 5</th></tr>
<tr><td>Day 1</td><td rowspan="5">P4</td><td rowspan="4">P3</td><td rowspan="5" colspan="3"></td></tr>
<tr><td>Day 2</td></tr>
<tr><td>Day 3</td></tr>
<tr><td>Day 4</td></tr>
<tr><td>Day 5</td><td></td></tr>
</table>

Figure 4.4: The schedule after Step 2

### 4.3.3 Step 3

The next patient in line is patient 5 with 3 sessions, 1 time slot each. $b_5$ is the smallest, therefore, the second time slot of day 5 will be chosen. However, it will reach the end of the planning horizon before booking all sessions for this patient, hence, it moves to the next least busy day. All the other days are equally busy, so it again chooses the first available sessions and update $a_1^{13}$, $a_1^{14}$, $a_1^{15}$ to 1. $b_5$ and $b_4$ remain unchanged, but $b_1$, $b_2$ and $b_3$ increase to 3. Figure 4.5 shows the schedule at the end of this step.

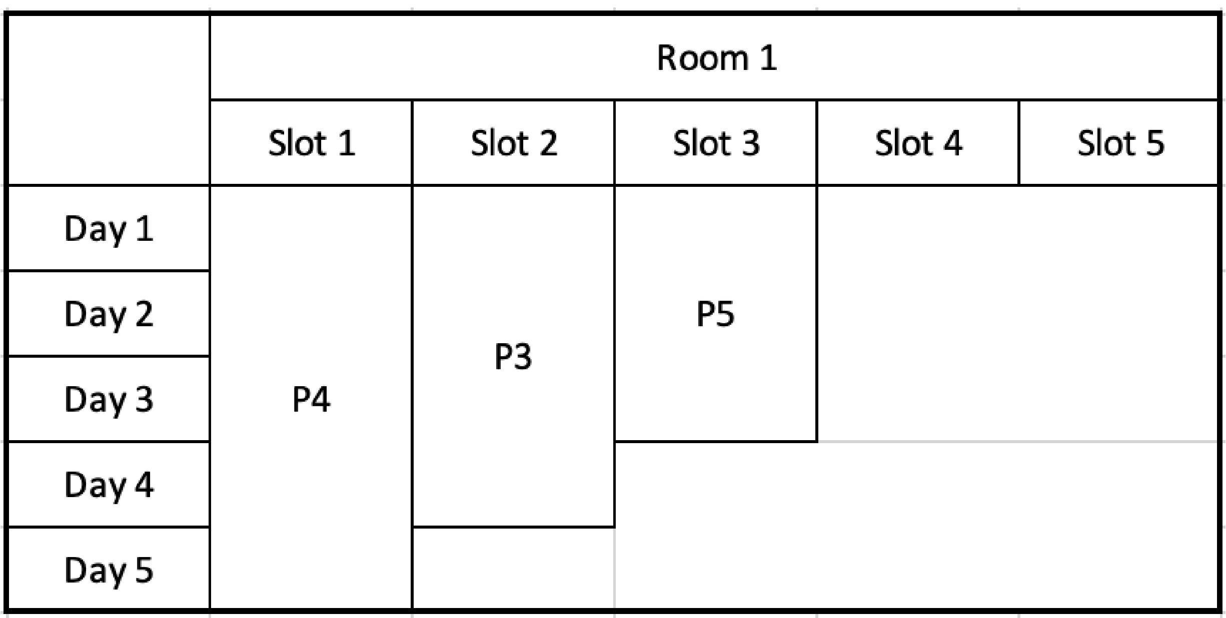


Figure 4.5: The schedule after Step 3

### 4.3.4 Step 4

To schedule patient 1, the algorithm first chooses day 5, because it is the least busy day, but it can not schedule all the sessions needed for patient 1 starting from day 5. Hence, the next least busy day will be chosen, which is day 4. For all the booked time slots, the availability parameter will be changed to 1, i.e. $a_1^{43}$,...,$a_1^{55}$. The updated values for $b_1$, $b_2$ and $b_3$ equal 2, whereas $b_4$ is equal to 0, and $b_5$ is equal to 1. Figure 4.6 shows the schedule at the end of this step.

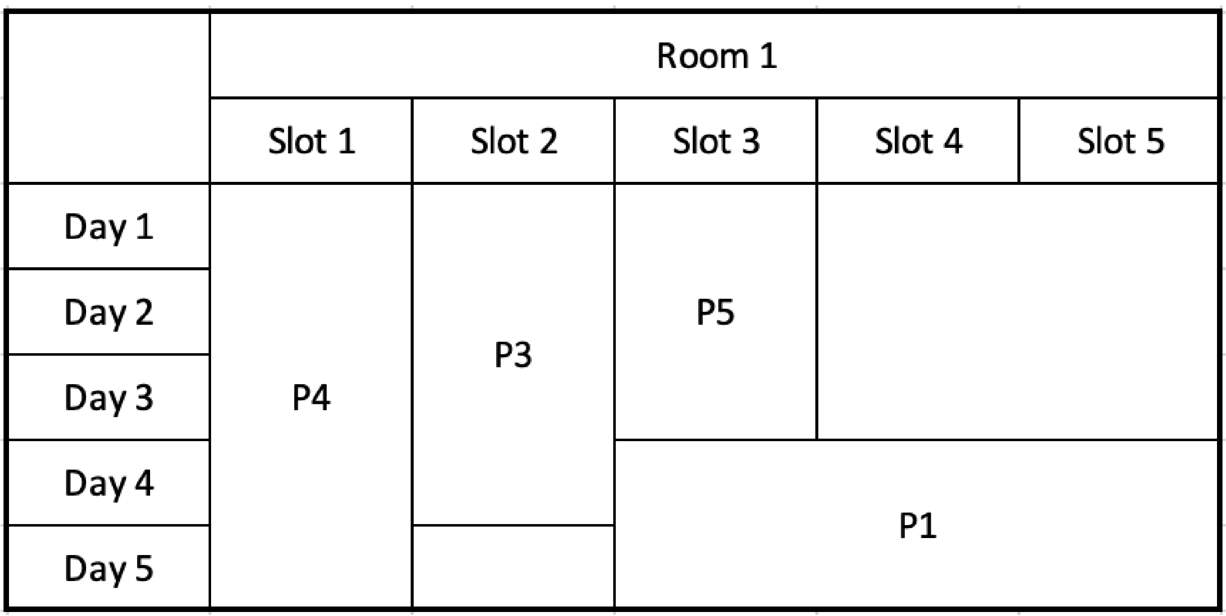


Figure 4.6: The schedule after Step 4

### 4.3.5 Step 5

In the final step, the last patient will be scheduled. Day 1, 2, and 3 are the least busy days, therefore patient 2 will be assigned to the last 2 time slots of day 1, and will result in a change in the values of $a_1^{14}$ and $a_1^{15}$. The final schedule is shown in Figure 4.7.

<table>
<tr><th rowspan="2"></th><th colspan="5">Room 1</th></tr>
<tr><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 4</th><th>Slot 5</th></tr>
<tr><td>Day 1</td><td rowspan="5">P4</td><td rowspan="4">P3</td><td rowspan="3">P5</td><td colspan="2">P2</td></tr>
<tr><td>Day 2</td><td colspan="2" rowspan="2"></td></tr>
<tr><td>Day 3</td></tr>
<tr><td>Day 4</td><td colspan="3" rowspan="2">P1</td></tr>
<tr><td>Day 5</td><td></td></tr>
</table>

Figure 4.7: The schedule after Step 5

# Chapter 5

# Results

In this chapter, illustrative small instances are used to evaluate the effectiveness of the proposed models, followed by a case study, using real-world larger instances obtained from Grand River Regional Cancer Center (GRRCC). The solution quality and the computational experiments are discussed, and the results have been compared using the proposed heuristic. Finally, some insights will be provided in the discussion section.

## 5.1 Illustrative Numerical Examples

### 5.1.1 FTD Numerical Example

In this experiment, a small instance is used to compare the results for a first-come first-serve (FCFS) approach with the proposed model. The instance used consists of 12 patients that need to be scheduled over a 7 days horizon, and there are 3 rooms available, each operating for 3 time slots each day. The number of sessions needed for each patient and the deadline to start the treatment are shown in Table 5.1. The solution for this set of entries with the first-come first-serve approach, and with the proposed model is shown in Figures 5.2 and 5.1, respectively. In the first-come first-serve solution, as the name indicates, each patient is scheduled in the first available time slot of the first available room, and waits to start the treatment until the given deadline if it is possible i.e. if it can schedule all the needed sessions within the given horizon. For example, patient 2 has a deadline of 3, which means their treatment has to start before or on the third day of the horizon. Therefore, in the first-come first-serve schedule the first session for this patient is scheduled on day 3. They need 4 sessions in total which can be scheduled within the horizon, as well as meeting the indicated deadline. On the other hand, patient 4 cannot be starting their treatment on the deadline which is day 4, because in that case, they would not be undergoing all their treatment sessions within the 7 days horizon. Hence, they start the treatment on day 2, which is the latest they can wait to start their treatment and still meet the requirements. In this approach, there are 10 interactions between patients, 4 happening in room 1, 4 happening in room 2, and 2 happening in room 3. Solving the same problem with the proposed model will result in the schedule shown in Figure 5.1, where the number of interactions is 4, 2 interactions happening in room 1 between patient 10 and 9, and between patient 9 and 3, and 2 interactions happening in room 2 between patient 1 and 4, and between patient 4 and 7.

<table>
<tr><th></th><th colspan="3">Room 1</th><th colspan="3">Room 2</th><th colspan="3">Room 3</th></tr>
<tr><th></th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th></tr>
<tr><td>Day 1</td><td rowspan="7">P1</td><td rowspan="2"></td><td rowspan="5">P3</td><td></td><td rowspan="2"></td><td rowspan="7">P7</td><td rowspan="2"></td><td></td><td></td></tr>
<tr><td>Day 2</td><td rowspan="6">P4</td><td rowspan="6">P10</td><td rowspan="4">P11</td></tr>
<tr><td>Day 3</td><td rowspan="4">P2</td><td rowspan="2">P12</td><td rowspan="5">P9</td></tr>
<tr><td>Day 4</td></tr>
<tr><td>Day 5</td><td rowspan="3">P6</td></tr>
<tr><td>Day 6</td><td rowspan="2">P5</td><td rowspan="2"></td></tr>
<tr><td>Day 7</td><td>P8</td></tr>
</table>

Figure 5.1: FCFS Solution Example for FTD

| Patients | Total Sessions | Deadline |
|---|---|---|
| Patient 1 | 7 | 2 |
| Patient 2 | 4 | 3 |
| Patient 3 | 5 | 1 |
| Patient 4 | 6 | 4 |
| Patient 5 | 2 | 6 |
| Patient 6 | 3 | 5 |
| Patient 7 | 7 | 7 |
| Patient 8 | 1 | 7 |
| Patient 9 | 5 | 5 |
| Patient 10 | 6 | 2 |
| Patient 11 | 4 | 2 |
| Patient 12 | 2 | 6 |

Table 5.1: The set of patients for the FTD experiment

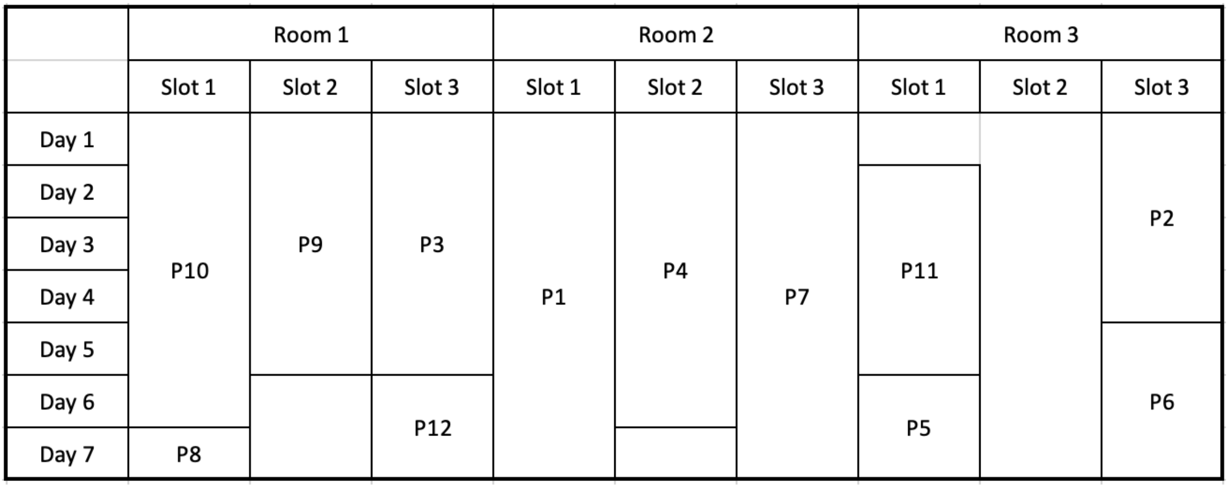


Figure 5.2: Proposed Solution Example for FTD

### 5.1.2 VTD Numerical Example

In this experiment, another small instance is used to compare the results for a first-come first-serve approach with the proposed model. The instance used consists of 10 patients that need to be scheduled over a 7 days horizon, and there are 2 rooms available, each operating for 5 time slots a day. The number of sessions needed for each patient and the deadline to start the treatment, as well as the treatment duration for each patient is shown in Table 5.2. The solution for this set of entries with the first-come first-serve approach, and with the proposed model is shown in Figures 5.3 and 5.4, respectively. In the first-come first-serve solution, as previously mentioned, each patient is scheduled in the first available time slot of the first available room, and waits to start the treatment until the given deadline if it is possible. In this approach, there are 7 interactions between patients, 2 happening in room 1, and 5 happening in room 2. Solving the same problem with the proposed model will result in the schedule shown in Figure 5.4, where the number of interactions is 3, 1 interaction happening in room 1 between patient 1 and 4, and 2 interactions happening in room 2 between patient 2 and 3, and between patient 9 and 6.

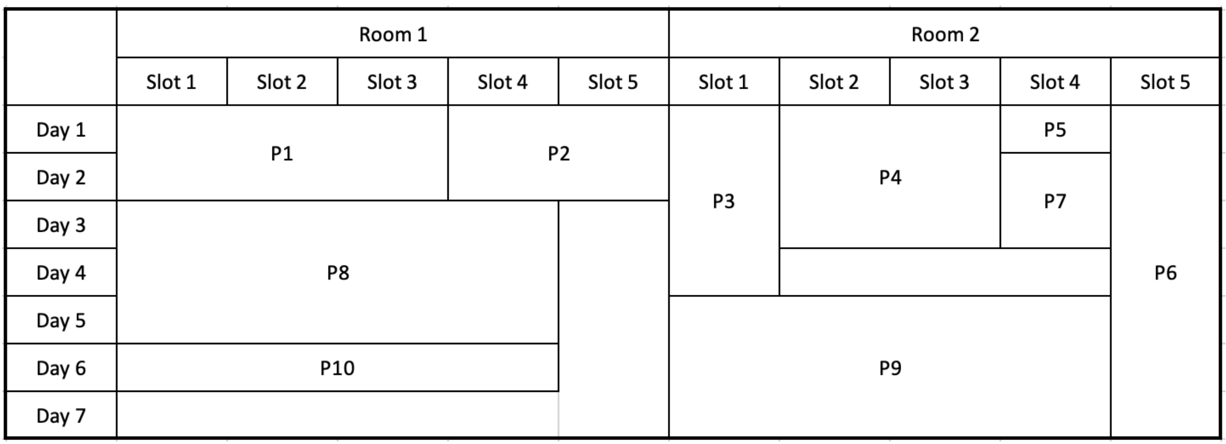


Figure 5.3: FCFS Solution Example for VTD

### 5.1.3 Secondary Objective Numerical Example

This example is to demonstrate the result of using the secondary objective on an optimal solution found by the proposed MILP in Chapter 3. To compare the outcome before and after using this objective function, we will use the same input as in the numerical example used for the FTD model for simplicity, shown in Table 5.1. An optimal solution

| Patients | Total Sessions | Deadline | Duration |
|---|---|---|---|
| Patient 1 | 2 | 5 | 3 |
| Patient 2 | 2 | 3 | 3 |
| Patient 3 | 4 | 2 | 1 |
| Patient 4 | 3 | 1 | 2 |
| Patient 5 | 1 | 4 | 3 |
| Patient 6 | 7 | 1 | 1 |
| Patient 7 | 2 | 6 | 2 |
| Patient 8 | 3 | 7 | 5 |
| Patient 9 | 3 | 5 | 6 |
| Patient 10 | 1 | 6 | 7 |

Table 5.2: The set of patients for the VTD experiment

<table>
<tr><th rowspan="2"></th><th colspan="5">Room 1</th><th colspan="5">Room 2</th></tr>
<tr><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 4</th><th>Slot 5</th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 4</th><th>Slot 5</th></tr>
<tr><td>Day 1</td><td colspan="3" rowspan="2">P1</td><td colspan="2" rowspan="3">P4</td><td rowspan="2">P7</td><td rowspan="2"></td><td rowspan="4">P3</td><td rowspan="4"></td><td rowspan="7">P6</td></tr>
<tr><td>Day 2</td></tr>
<tr><td>Day 3</td><td>P5</td><td colspan="2"></td><td colspan="2" rowspan="2">P2</td></tr>
<tr><td>Day 4</td><td colspan="4">P10</td><td></td></tr>
<tr><td>Day 5</td><td rowspan="3"></td><td colspan="4" rowspan="3">P8</td><td colspan="4" rowspan="3">P9</td></tr>
<tr><td>Day 6</td></tr>
<tr><td>Day 7</td></tr>
</table>

Figure 5.4: Proposed Solution Example for VTD

for this input without including the secondary objective is presented in Figure 5.1. The other optimal solution after using the secondary objective is presented in Figure 5.5. The number of interactions is the same for both solutions as they both are optimal solutions, but in Figure 5.1, the solver waits until day 2 to start the treatment for patient 11, which is the deadline to start their treatment. On the other hand, in Figure 5.5, the solver starts the treatment for the same patient as soon as possible and does not necessarily wait until the deadline. Therefore, the same patient starts the treatment on the first day of the horizon.

<table>
<tr><th></th><th colspan="3">Room 1</th><th colspan="3">Room 2</th><th colspan="3">Room 3</th></tr>
<tr><th></th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th><th>Slot 1</th><th>Slot 2</th><th>Slot 3</th></tr>
<tr><td>Day 1</td><td rowspan="5">P3</td><td rowspan="7"></td><td rowspan="4">P2</td><td rowspan="6">P4</td><td rowspan="4">P11</td><td rowspan="5">P9</td><td rowspan="7">P1</td><td rowspan="6">P10</td><td rowspan="7">P7</td></tr>
<tr><td>Day 2</td></tr>
<tr><td>Day 3</td></tr>
<tr><td>Day 4</td></tr>
<tr><td>Day 5</td><td rowspan="3">P6</td><td rowspan="3"></td></tr>
<tr><td>Day 6</td><td rowspan="2">P5</td><td rowspan="2">P12</td></tr>
<tr><td>Day 7</td><td>P8</td><td></td></tr>
</table>

Figure 5.5: Proposed Solution Example for the secondary objective

## 5.2 GRRCC Case Study

### 5.2.1 Real-life Data

According to the patient clinical path documents provided by our collaborators from the Grand River Regional Cancer Centre (GRRCC), cancer patients that are going to start their radiation therapy treatment have to undergo a few initial sessions. The initial sessions differ based on the cancer type and severity they are dealing with and include Consultation, Teaching, Injection, Surgery, and Planning sessions. Each pre-treatment session varies for different patients based on characteristics of patients e.g. age, sex, diagnosis stage, etc. Therefore, it is not computationally efficient to model these types of sessions. Moreover, these sessions usually need specific accommodations, and there are not many options to decide from. Another point to mention is that they are not recurring sessions, and happen only once during the pre-treatment period, which is after diagnosis and before radiotherapy sessions start. Hence, we will be using real-life data for radiotherapy treatment sessions, collected from Grand River Regional Cancer Center (GRRCC). There are 5 different radiotherapy rooms, and there is only 1 LINAC in each room. The time frame for the data extracted is the start of January to the end of April 2020, and around 700 patients did undergo their radiotherapy treatment in this period. The operating hours of LINACs are from 8:00 AM to 4:30 PM. However, these times may vary to accommodate the patient load, staff schedules, and other unexpected incidentals. It is also important to mention that all appointments provided are booked sessions for treatment, and may have been changed due to unforeseen circumstances in the actual schedule.

### 5.2.2 Computational Experiments

In this section, we start with solving the problem for the GRRCC data. As discussed in Chapter 3, the planning horizon is set to be 1 work week (5 days) from Monday to Friday. Given the fact that each room has a LINAC which is used for a specific type of cancer, the test data will include patients undergoing treatment using a pre-determined LINAC, which means the problem will be solved for each room $f$ separately. Therefore, the number of rooms and the number of days are going to be fixed for all the experiments with GRRCC instances. The CPLEX solver is used throughout the experiments, with 32 GB of memory and 4 CPUs, using the Graham cluster located at the University of Waterloo, and all instances are extracted from the original data set provided by GRRCC.

Table 5.3 includes 5 instances that were used to conduct experiments, as well as the specifications of each instance, i.e. the number of patients, the number of time slots, and the number of binary variables and linear constraints.

| Instance Num. | #Patients | #Time slots | #Binary var. | #Constraints |
|---|---|---|---|---|
| 1 | 11 | 10 | 1276 | 7673 |
| 2 | 18 | 18 | 3654 | 33930 |
| 3 | 21 | 20 | 4746 | 50143 |
| 4 | 34 | 34 | 12886 | 213146 |
| 5 | 38 | 34 | 14554 | 263282 |

Table 5.3: Model specification for various input sizes

As shown in Table 5.3, increasing the size of input will result in an increasing number of variables and constraints, hence the run time grows rapidly. All the instances in Table 5.3 except for the first two instances originally require more than 2 days to find an optimal solution, which is not efficient for a radiotherapy center, as they need to be able to solve the problem in the weekend for the upcoming week. The complexity level of the model for instances 4 and 5 results in failure in even finding an incumbent solution in this period, therefore there is a need to include a warm start using an incumbent solution, which will be providing an upper bound for the objective value. In order to do so, we will use the heuristic approach proposed in Chapter 4.

### 5.2.3 Heuristic Solution

| Instances | FCFS Solution | Heuristic Solution | Decrease Compared to FCFS | MILP + Heuristic Solution | Decrease Compared to FCFS | Optimal Solution |
|---|---|---|---|---|---|---|
| 1 | 11 | 11 | 0% | 3 | 73% | 3 |
| 2 | 19 | 18 | 5% | 7 | 63% | 7 |
| 3 | 42 | 21 | 50% | 11 | 75% | - |
| 4 | 62 | 34 | 45% | 17 | 73% | - |
| 5 | 64 | 37 | 42% | 18 | 72% | - |

Table 5.4: Model results for various input sizes

Table 5.4 includes the results for the same set of instances as Table 5.3. The original number of interactions in the GRRCC schedule using the first-come first-serve approach and the incumbent solution provided by the heuristic approach for each instance are mentioned. The solution provided by the heuristic results in up to 50% decrease in the number of interactions. The time limit for all the instances is set to be 48 hours, and the best objective value found in this period using the proposed MILP and heuristic is included, as well as the optimal solution if the model was solved to optimality within 48 hours.

For the first instance in Table 5.4, the heuristic provides an incumbent solution with the objective value of 11, and the model is able to reach the objective value of 3 within less than 15 minutes. To test the quality of the incumbent solution provided by the heuristic approach and to compare it with the original number of interactions in GRRCC schedule, we can use the optimality gap measure as well. Optimality gap measures the gap between the objective of the incumbent solution and the optimal value, if the optimal solution is available. It is calculated as follows:

$$\frac{|\text{incumbent solution} - \text{optimal solution}|}{|\text{optimal solution}| + 1e - 10}$$

In this case, the incumbent solution is equal to the original number of interactions, hence they both have the same optimality gap of 260%. The optimality gap decreases in every iteration until it reaches the optimal solution, where the optimality gap is 0. The trend of the objective values for each iteration is shown in Figure 5.6.

For the second instance, the incumbent objective value is 18, whereas the original number of interactions is 19. The model was able to reach the objective value of 7 within

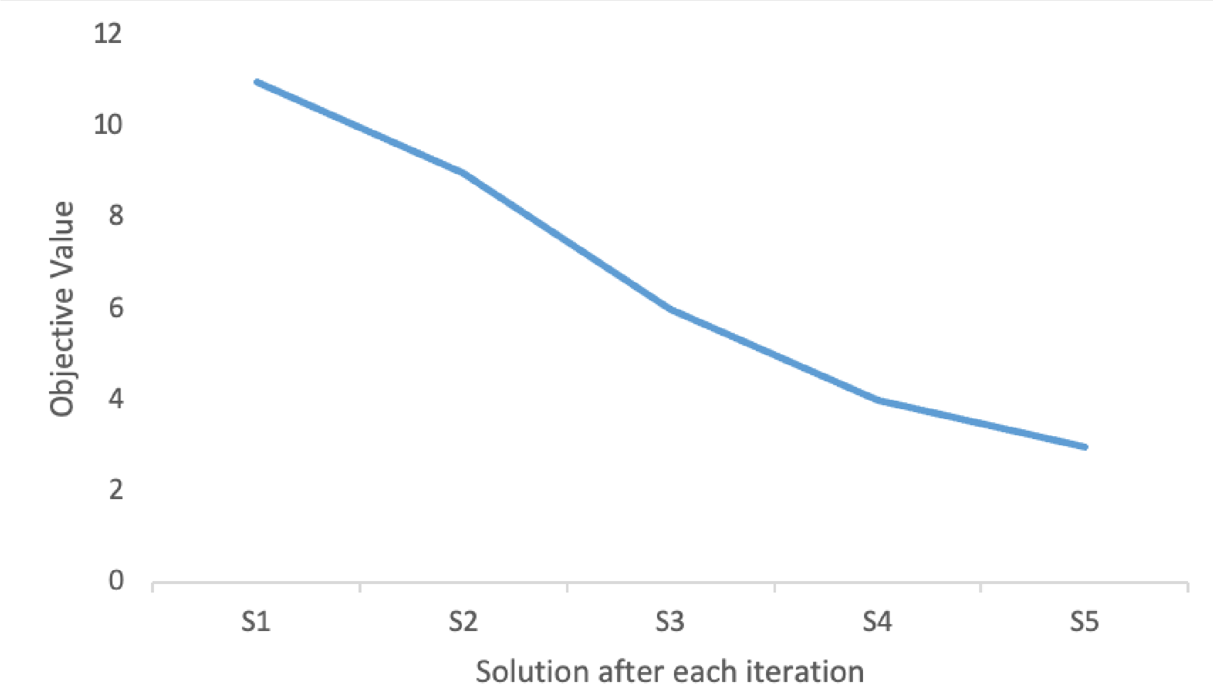


Figure 5.6: Objective value trend for Instance 1

48 hours, starting from 18, with an optimality gap of 157%, whereas the optimality gap using the FCFS approach is 171%. The trend of the objective values for solutions found at each step is shown in Figure 5.7.

For the third instance, the objective value of the incumbent solution has a meaningful difference from the original number of interactions, even before solving the optimization model. Starting from the objective of 21 as the warm start, the trend of the objective values for each iteration is shown in Figure 5.8.

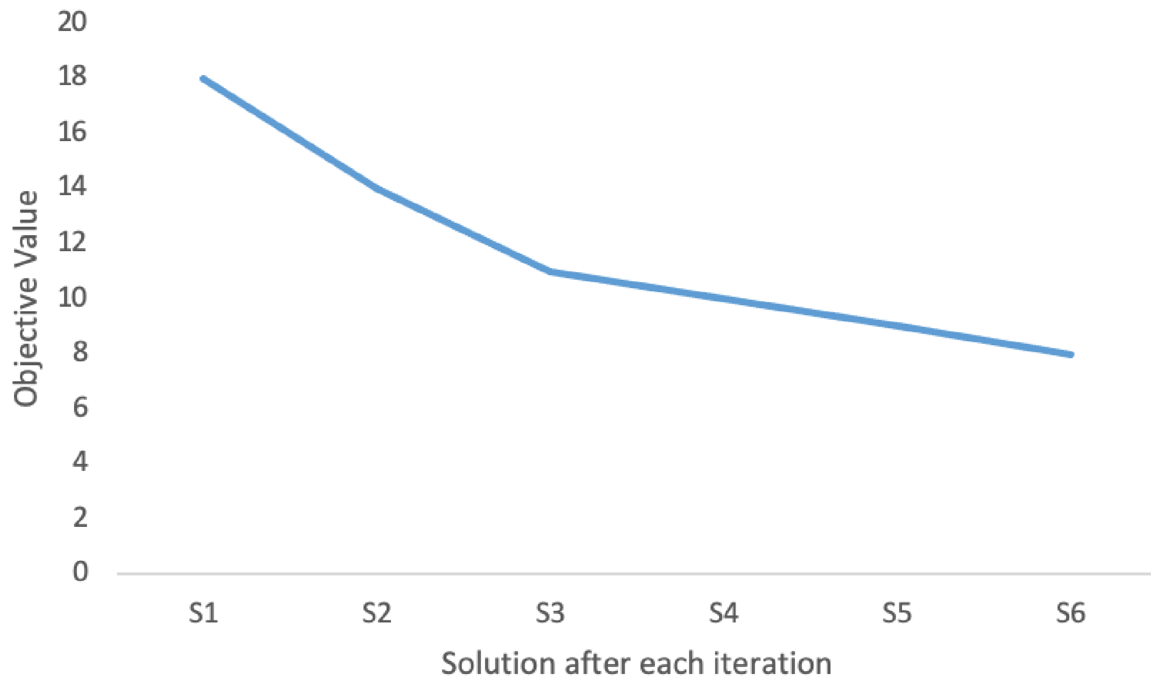


Figure 5.7: Objective value trend for Instance 2

Similarly, the objective values trends for instances 4 and 5 are shown in 5.9 and 5.10 respectively.

### 5.2.4 Solution Quality

With the increasing size of the instances, the difference between the objective of the incumbent solution and the original objective increases, and the heuristic approach provides solutions that are of higher quality. Table 5.4 compares the quality of the incumbent solu-

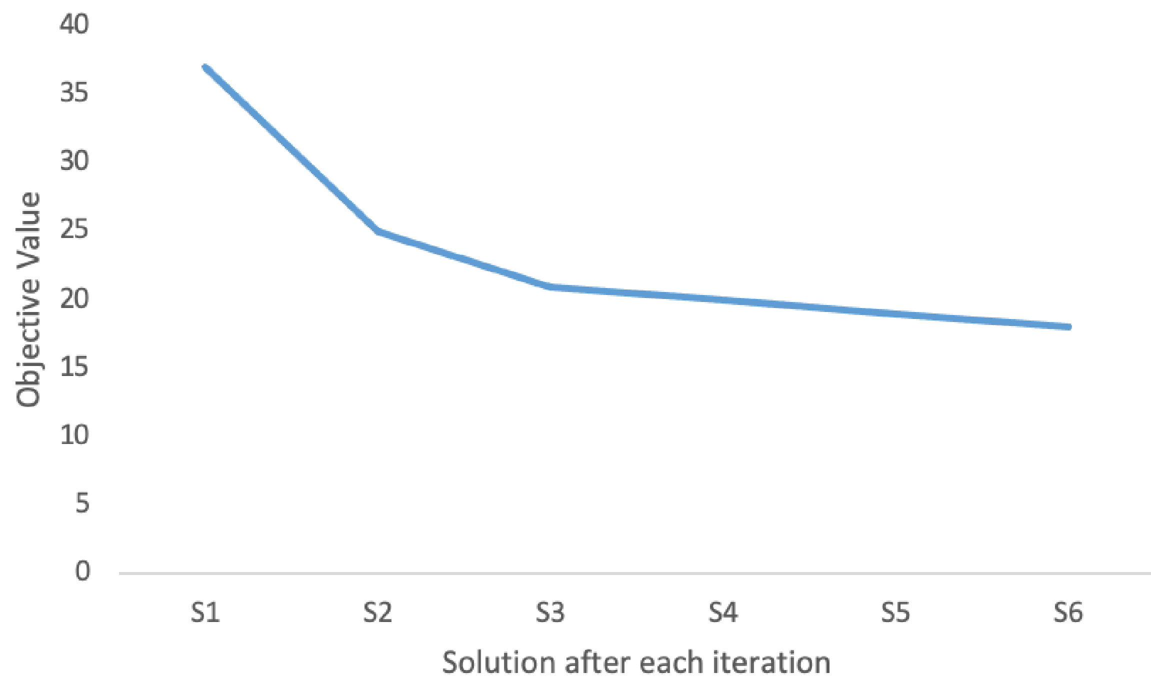


Figure 5.8: Objective value trend for Instance 3

tion with the original schedule, showing that using the heuristic approach provides a better warm start solution for this optimization problem, instead of using the original schedule as an incumbent solution.

## 5.3 Discussions

It has been demonstrated that the proposed heuristic provides high-quality incumbent solutions that can help the proposed model to find good solutions faster when the size

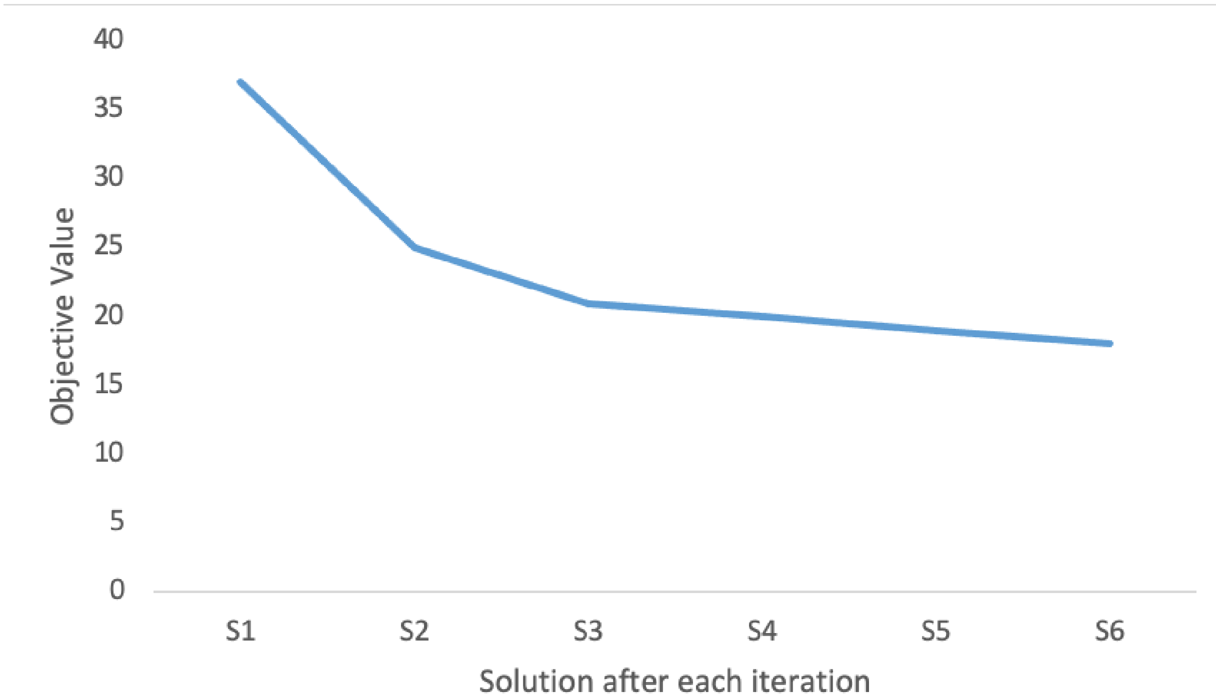


Figure 5.9: Objective value trend for Instance 4

of input increases. The combination of these approaches results in up to 75% decrease in the number of patient-patient interactions in GRRCC schedule. This goal has been achieved, considering the operational constraints of a radiotherapy center and the timeliness of the treatment process. Unlike most studies on overcoming the challenges of the COVID-19 pandemic, treatments have not been postponed or delayed to reduce the risk of transmission.

The proposed methodology can be easily implemented in the radiotherapy admissions

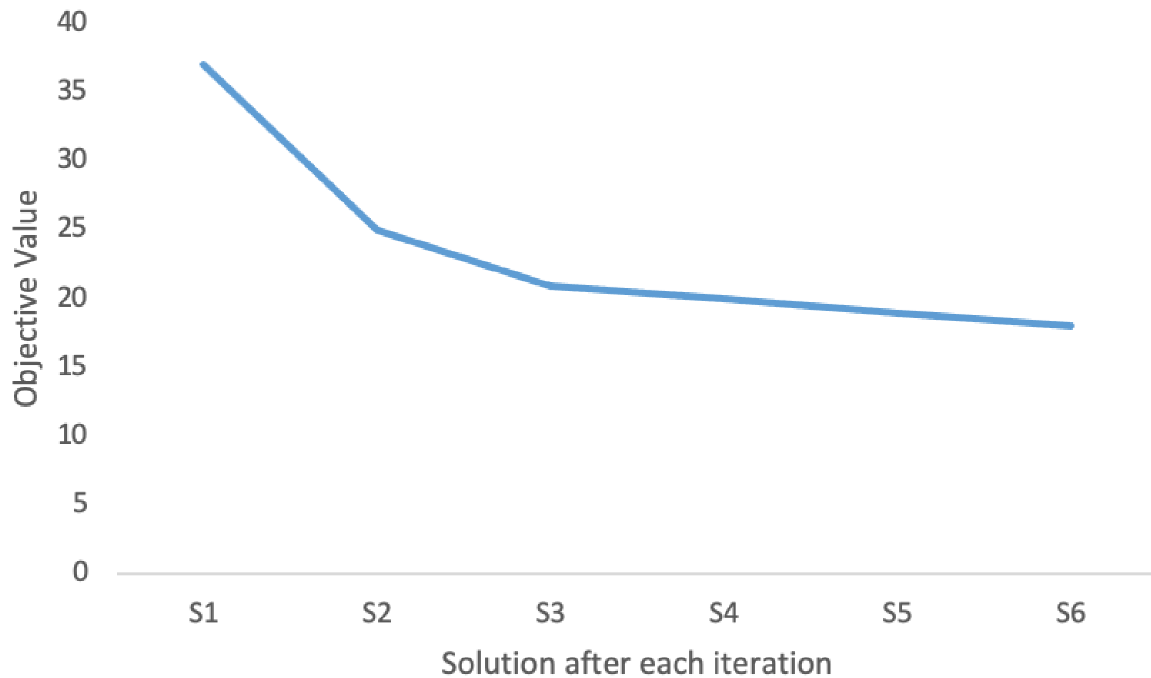


Figure 5.10: Objective value trend for Instance 5

department. At the end of each week, patients that are going to be under treatment for the next week can be assigned to specific rooms with specific LINACs based on their cancer type. When the list of upcoming patients is complete, the proposed model combined with the heuristic will generate the schedule for the next week during the weekend. However, if the radiotherapy center decides to consider a longer planning horizon, it will take more time to generate the desired solution.

It is important to note that assigning patients to the same room and the same time slot during their treatment process plays an important role in reducing the number of interactions, and it helps patients to better organize their personal schedules. However, if it is not a requirement for a radiotherapy center, the problem will be considered a bin-packing problem and the proposed MILP will be less complex and easier to solve. Furthermore, the outcome of this approach enables radiotherapy clinics to have organized and consecutive free time slots, so that they can admit new urgent patients that need to start their treatment immediately and can not wait until the beginning of the next planning horizon.

# Chapter 6

# Conclusions

The present study draws attention to the importance of implementing special measures in scheduling cancer patients in radiotherapy centers during the outbreak of airborne diseases, like COVID-19. Due to their weaker immune system, cancer patients are more likely to be easily affected by COVID-19 and similar diseases. In order to prevent that from happening, we aim to minimize the interaction between patients, using MILP models for scheduling radiotherapy patients. The first proposed model, FTD, is a simplified version of the RTPS problem, assuming that all sessions are of the same length, but the second proposed model, VTD, is a more complicated version, assuming that sessions are of different lengths. A secondary objective function is then proposed to choose a more suitable optimal solution from the set of multiple optimal solutions generated by two models.

The performance of the proposed models is first evaluated using small instances, but it is shown that as the instances are growing, the model is quickly getting more complex. This makes the rest of the experiments challenging, as the run time starts to become inefficient for real-world and larger instances. Hence, a heuristic solution algorithm is introduced, which helps the solver by providing an initial solution to use as a warm start for the main MILP model. The results have demonstrated both the performance of the heuristic approach in providing high-quality incumbent solutions, compared to the original schedule provided by GRRCC, and the effectiveness of the proposed models in terms of the number of interactions, resulting in up to 75% decrease in the number of patient-patient interactions.

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