

Updating Local and Global Probability Events During Maze Navigation

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Our mental models consist of relational knowledge. We apply this knowledge about whether something is near to or far from something else to solve tasks. As a specific example, when we navigate in our environment, we have global (far) location goals that we could navigate to using local (near) landmarks. The question for the present study is whether relational knowledge can be probabilistically and differently represented at global and local levels. To test this, we had participants navigate a maze in which the wall structure was hidden, but in which participants were given global and local cues. We manipulated the reliability of the global and local cues across experimental trials and experiments. Our results demonstrated separable effects for global and local cues. Participants made good estimates of global and local cues' reliability, however, their estimates of global cue reliability were less accurate than their estimates of local ones potentially due to inherent differences in how global and local information is represented. Their use of local cues roughly matched the ground truth local cues reliability whereas their use of global cues did not match the ground truth global cue reliability. In addition, participants relied on both local and global cues when they navigated in the mazes but with local cues dominant possibly because of their confidence in local cue reliability estimates, preference for cues associated with more immediate reward, and feedback proximity. Altogether, this study characterizes the mental representations of uncertain global and local cues and suggests that people negotiate between different probabilistic information when making decisions in maze navigation.

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Dedication

This is dedicated to the project I love.

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Chapter 1

Introduction

To obtain long-term rewards, humans and animals flexibly adjust their behavior according to the environment. Under different situations, we may endure days, weeks, or even months before attaining one reward. Evidence suggests that the brain has evolved multiple solutions to this reinforcement learning (RL) problem: maximizing rewards over a long period time (Daw et al., 2005). We can choose how to act by evaluating actions and reward relationships that have worked in the past, but also based on experiences that are not obviously related (Daw et al., 2011; Kool et al., 2017); we can abstract important features of experiences and generalize the learned rules to new situations (Eckstein et al., 2021; Gazes et al., 2012; Lazareva and Wasserman, 2012).

Since Edward Tolman invented the idea of a cognitive map in the 1940s (Tolman, 1948), the question of how spatial representations support flexible behavior has been a contentious topic. As we navigate our dynamic and complex world, we act differently to reach the next potential states (locations). While one action may lead us to a higher transition probability of one state, another action may lead us to a lower transition probability to the same state. Every action is associated with a cost (biological or economical), and different sequences of actions would have different costs in total. Overall, our purpose is to achieve the goal using the minimum cost in the long run. It is usually challenging to find the path with minimum cost. One solution to finding the goal efficiently is to first encode sensory information into mental representations, then build rich causal models, and at last use them as guidance (Bottini and Doeller, 2020; Stachenfeld et al., 2017). Indeed, there is a converging body of neuroscience research suggesting that the brain learns predictive maps of relational knowledge from sensory information and uses them for fast and adaptable decision-making (Brunec and Momennejad, 2022). These abstract representations can be considered as basis sets for describing relational knowledge (Behrens et al., 2018; Bellmund et al., 2018).

Humans build mental representations of the environment and use them to navigate (Peer et al., 2021). For example, if we want to walk to our favorite restaurant from home, we usually have a map already built in our mind that includes every possible detail of the environment. We can use this map as guidance to the restaurant and we can also easily tell our friends how to get there. However, it is impossible to construct a fully detailed map as big as that area. In fact, the mental representations humans build do not simply reflect every aspect of the world, but rather pick out a manageable subset of details that are relevant to some purpose and/or store useful abstractions of them (Eichenbaum et al., 1989; Ho et al., 2022). Although the input data are sparse, noisy, and ambiguous in every way, we still can construct powerful mental representations (Tenenbaum et al., 2011). The restaurant location might change, the road might be different after construction, the traffic might push us to a never-explored alley. Nevertheless, people in most cases manage to effectively travel from one place to another.

In order to characterize people’s behavior in this complex situation, this thesis examined what are the principles that guided the decision-making during maze navigation and especially when there is uncertainty in maze navigation. Ultimately, this thesis hopes to answer how does abstract knowledge like the cognitive map we mentioned above guide our actions?

I propose that such mental representations can be factored into “local” and “global” probabilistic components each independent of the other and are capable of influencing human choices and behavior.

Here I define them in relation to uncertainty. *Local* uncertainty is the uncertainty related to events that are physically or temporally proximate and whose feedback is immediate and direct; *global* uncertainty is the uncertainty related to events that are associated with general objectives requiring a sequence of actions and whose feedback is distant in space or time.

1.1 Anatomical Justification for Distinct Representations of Local and Global Uncertainty

The anatomical data are consistent with the distinct local and global encoding of uncertainty. Previous research had demonstrated an anatomical disassociation between near and far spatial representations (Shapiro et al., 1997).

The hippocampus and its related brain areas are involved in the learning and representation of temporal statistics (O’Keefe and Nadel, 1978; Stachenfeld et al., 2017). “Place”

cells in rodents' posterior hippocampus restrict their activity to a myopic single location in space and support fine-grained spatial relations (O'Keefe and Nadel, 1978; Poppenk et al., 2013). Moving forward toward the anterior part of the rodents' brain, "grid" cells in the medial entorhinal cortex activate at multiple locations equally spaced on a triangular grid (adjacent edges of excited triangles may also co-opt other cells to complete a large hexagonal synchronous patterns of triangular arrays) (Hafting et al., 2005). The relationships and distances between different spatial locations can be decoded from the population activity of grid cells (Bush et al., 2015; Stemmler et al., 2015). It has also been shown experimentally that entorhinal lesions impair performance on navigation tasks and disrupt the temporal ordering of sequential activation in hippocampus while leaving performance on location-recognition tasks intact (Hales et al., 2014). This suggests that "grid" cells may play a more general role in spatial planning than "place" cells. Anterior to the medial entorhinal cortex, neurons in the rat orbitofrontal cortex (OFC) form spatial representations persistently correlated with the goal destination (Basu et al., 2021).

The idea that the information is encoded in rodent brains at different hierarchical levels along the dorso-ventral axis is in line with the scale increase of mnemonic networks represented along the anterior-posterior axis of the human brain. When participants are asked to form narratives about lifelike events the scale at which these mnemonic networks are represented across the hippocampus differs. The most recently linked pair of events activate posterior parts of hippocampus whereas information about multiple event pairs activates hippocampal mid-portions. Integrated networks for all event conditions in a narrative task were seen in the anterior hippocampus (Collin et al., 2015; Milivojevic and Doeller, 2013). Deuker et al. found that both spatial and temporal distance had a significant effect on pattern similarity across all hippocampal grey-matter voxels when the other factors were regressed out, specifically objects that were close in either space or time shared higher hippocampal pattern similarity (2016).

It has also been suggested that these predictive representations during navigation are organized in the same multi-scale fashion, not only in hippocampus (Momennejad and Howard, 2018; Stachenfeld et al., 2017) but also in prefrontal cortex (Christoff and Gabrieli, 2000; Koechlin and Hyafil, 2007; Momennejad and Haynes, 2013). Using functional magnetic resonance imaging (fMRI) and virtual reality (VR), Brunec and Momennejad found that during virtual navigation, anterior hippocampus would display representational similarity at longer predictive scales than posterior hippocampus (2022). Moreover, the anterior PFC (antPFC) displayed representational similarity to more distant states (location) than posterior PFC (Brunec and Momennejad, 2022). Representing cognitive spaces at different scales allows for the generalization of specific experiences and the formation of contextual features via more global representations. Different scales of information represented at dis-

tinct anatomical locations of the hippocampal formation and prefrontal cortex might serve as a general mechanism across different stimulus domains. Encoding concepts in cognitive spaces for non-spatial abstraction, for example one’s position in a social network, might also benefit from the combination of multiple scales of representation analog to navigable space (Behrens et al., 2018).

Much previous human neuro-psychological research has used maze navigation to study the mental representation of relational knowledge (Brunec and Momennejad, 2022; Deuker et al., 2016; Ho et al., 2022). In this study, we used maze navigation too. Two key features make our maze-navigation paradigm useful for studying the mental representation building process. First, solving mazes is easily self-motivated: participants have a clear goal in their mind throughout the experiment and their progress can be immediately accessible from the visual stimulus. Second, solving mazes is complex enough such that each instance of a maze contains decision-making points from particular compositions of individual elements (for example, the wall, the exit location, the cue). Although those components can be easily accessed by participants, they still need to choose which elements to integrate into an effective mental representation.

1.2 Solving Maze Navigation Using A Successor Representation Model

Much research highlights the computational similarities between RL and maze navigation, which both involve a sequence of state transition and reward maximization or cost minimization (Franklin and Frank, 2018; Liu and Frank, 2021; Stachenfeld et al., 2017) where each state describes the current situation of the agent is in. For a Chess player, the state is the positions of all the pieces on the board; for a robot dog learning to jump, the state is the position of its four legs. Consider the maze navigation task, the state is the position of the player, and it would change after each movement. Overall navigating the maze would produce a sequence of state transition associated with a reward or cost. We can solve this maze navigation problem using a well-studied model called successor representation (SR) (Dayan, 1993). It is a fundamental model that has been used for maze navigation and reinforcement learning (RL). Many current RL methods are built based on it because it tackles a straightforward maze navigation question: finding an exit in the open space with walls (Fujimoto et al., 2021; J. Zhang et al., 2017). It is believed that the SR model can integrate spatial and temporal coding in the hippocampus (Stachenfeld et al., 2017). Thus, the SR model provides one mechanism for how we might reconcile the effects of local and global cues on agent’s behavior both spatially and temporally.

In contrast to the idea of place encoding (like theorized for place cells), the SR theory considers location as a predictive representation for future states given the current state. Maze navigation can be cast in this framework. A maze problem can be considered as a Markov decision process, which is a framework for modeling decision-making in situations where agents (decision makers) will receive outcomes partially due to randomness and partially due to actions under their control (Bellman, 1957). The problem consists of a set of states (spatial locations), a set of actions (e.g move right, follow certain landmark), a transition distribution $P(s'|s, a)$ describing the probability of transitioning to next state s' from state s after taking action a , and a reward obtained from the function $R(s)$ given the state s . There could be no reward at certain state s .

In the simulation of calculating SR, the value of a current state s is defined as the expected sum of the reward at each future state s_t , multiplied by an exponentially decaying discount factor $\gamma \in [0, 1]$ that downweights distal rewards:

$$V(s) = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t) | s_0 = s \right] \quad (1.1)$$

where s_t is the state visited at time t , for example at t second we visited state s . This means when an agent is navigating in a maze that gives rewards at the exit repeatedly, they become more likely to return to this exit. Certain locations near the exit would also become associated with higher reward values several seconds earlier.

Therefore, in terms of state, the value function can be rewritten into an inner production of the reward function and the predictive representation of the state:

$$V(s) = \sum_{s'} M(s, s') R(s') \quad (1.2)$$

Sometimes, an agent would navigate a world where there are a couple states s' that a current state s could lead to with different transition probabilities. A transition probability matrix describes each pair of s and s' 's relationship in terms of their transition probability. When the transition probability matrix is known, we can compute the SR as a discounted sum over transition matrices raised to the exponent t .

In fact, $M(s, s')$ encodes the expected discounted future occupancy of states s' along a trajectory initiated in the state s , so we can also write it as:

$$M = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t \mathbb{I}(s_t = s') | s_0 = s \right] \quad (1.3)$$

Here I is the identity function, if state s at time t is s' then we have $s_t = s'$ and $I(s_t = s') = 1$, otherwise, the value of the identity function will be 0.

According to this model, the states closer to the encoded location of an SR place cell will predict a higher expected discount factor γ of visits to the encoded location and will trigger higher firing of the encoding cell because the close the state to the encoded location the relevant the state. The agent’s mental representation is repeatedly updated (i.e. its SR model). An estimate of the SR can be updated step by step using a form of the temporal difference learning algorithm:

$$\hat{M}_{t+1}(s_t, s') = \hat{M}_t(s_t, s') + \eta[\mathbb{I}(s_t = s') + \gamma\hat{M}_t(s_{t+1}, s') - \hat{M}_t(s_t, s')] \quad (1.4)$$

Eventually, we would have an SR matrix M that predicts not just the next state but a superposition of future states over a possibly infinite horizon.

1.3 Limitations of the Successor Representation Model

However, the SR model has some limitations. I want to argue that, fundamentally, previous researchers did not consider much real-life complexity when they proposed the SR model to solve the navigation tasks.

Firstly, in reality, the structure of the environment and its true underlying transition probabilities from state to state might change (e.g., if turning right causes one to hit a temporary construction wall, the next time turning right in the same location might not lead to the wall again). However, the SR model approach focuses on the learning of a single transition matrix over multiple trials to represent the environment. This might be poorly suited to human experience in real life despite its mathematical elegance. In fact, humans are more likely to assume that the environment would be different from trial to trial.

Previous cognitive science research shows that humans have dynamic beliefs of the environment’s latent structure or mechanisms (Guo and Yu, 2018; S. Zhang and Yu, 2013). For example, learning in a “Bandit task” was well captured by a Bayesian ideal learning model, the Dynamic Belief Model (DBM). In a “Bandit task“, an agent chooses one of many slot machines. Each slot machine provides the agent with a reward randomly generated from a probability distribution specific to that machine. The agent can choose only one machine at a time. The agent may either *explore* a different slot machine from the one just visited or may *exploit* the slot machine it most recently used. The agent does not know the ground truth distribution of each slot machine but needs to maximize its reward over time.

Researchers have found that during this task, humans assume the reward distribution of each slot machine can change over time even though in a particular experiment they are truly stationary (Guo and Yu, 2018). Therefore, it is highly possible that humans are learning how to adapt to dynamic environments via means other than the classical SR model mechanism.

Another way in which our current SR theories potentially oversimplify the human experience is that they do not consider uncertainty as coming from multiple different sources. When arranging dinner with a friend in our favorite restaurant the odds that we meet our friend will be influenced by the reliability of our friend keeping appointments, the accuracy of maps, and the vagaries of public transportation. Our success in meeting the friend in that restaurant is more correctly captured by considering multiple sources of uncertainty that are independent, and not simply one omnibus collapsed probability distribution. This partitioning of uncertainty is valuable if we later plan to meet our friend in a park for a picnic. We can transfer the reliability of our friend keeping appointments to this new scenario, even if the navigation and city specifics change.

The same logic applies to spatial navigation such as Tolman’s cognitive maps. Uncertainty might partition along local and global axes. Imagine we are traveling in a new city, and our favorite restaurant just opened a branch in the north part of the city. It might be hard for us to head to there using the shortest path. We might need to open the map application on our cell phones and search the route. Although we know the likely direction of our destination, there is an embedded uncertainty (e.g., it is probably in the north part of the city, but we do not know whether it is a little bit north-west or north-east). We could use local landmarks to guide us but there may still be uncertainty involved. For example, maybe our cell phones are not correctly oriented in the environment, so the local landmarks in front of us are actually positioned on the cell phone in the other direction. So it is important to keep in mind that the global and local cues are not absolute, but probabilistic.

The real-world problems we face are more dynamic than captured by previous maze navigation tasks. For studying, such tasks are usually constrained, fixed, and only a single source of uncertainty need be learned. Much less research has examined how humans may fare in estimating multiple sources of uncertainty.

The aim of the current study was to explore how our mental representation may encode uncertainties in a dynamic environment where information on different scales may align or contradict each other. Specifically, this thesis asks two questions: How precisely can we encode local uncertainty and global uncertainty? And are we biased to act based on local uncertainty? To answer these questions, I designed a virtual maze navigation game

where participants would navigate in a maze with hidden walls (the wall structures are “masked” to render them invisible, but the walls nonetheless impede passage). Without the information on the location of wall structures, participants need to rely on colored local cues to know which direction they are allowed to move otherwise they would hit a wall and stay at the original position. They could also use the global direction of the exit as a cue to find the shortest path to the exit.

In order to test participants’ separate encoding of uncertain local events and global events, I implemented two tasks using the maze navigation game. One was called the Fixed Global Varied Local task in which I manipulated local uncertainty by manipulating the reliability of certain colored local cues used to indicate the direction of the shortest path to the exit. Another task was called the Varied Global Fixed Local in which I manipulated global uncertainty by varying the exit location. The local uncertainty and global uncertainty were changed from high to low or low to high (counter-balanced) gradually throughout the experiment to avoid the effect produced by a sudden big drop or increase in uncertainty. In both tasks, participants were informed about the local cues and global cues’ reliability through receiving feedback. The feedback about how far (step-wise) they were away from the exit was provided after each movement and the feedback about whether they found the exit or not was provided at the end of maze navigation. Through the use of a maze navigation game, this study demonstrated participants had distinct mental representations of uncertain global and local cues, and they relied on both local and global cues but local cues dominate when making decisions in maze navigation.

Chapter 2

Materials and Methods

2.1 General Participants Information

Participants for all experiments were undergraduates at the University of Waterloo who participated for course credit. 195 participants signed up for the experiments, and 92 completed all trials (the reasons given for dropping out early were length and monotony). Five participants were removed because they either misunderstood the task or were not attentive. They rated as 0% a condition that was in fact 100% on at least 14 of 15 trials. This left us with 88 participants in total.

All participants gave informed consent before completing the experiment that had ethics clearance from the Office of Research Ethics at the University of Waterloo (ORE #43113).

2.2 General Procedures

Maze structures were generated using custom software written in Python and implementing a Growing Tree algorithm (Buck, 2015). The display of the maze and the interface were written in HTML and JavaScript. Analyses were done using Python (Van Rossum and Drake, 2009) and Rstudio (RStudio Team, 2020) in the R statistical analysis environment (R Core Team, 2021) with packages ggplot2 (Wickham, 2016), ggpubr (Kassambara, 2020), plotly (Inc., 2015), moments (Lukasz Komsta, 2022), dplyr (Wickham et al., 2022), tidyverse (Wickham et al., 2019), rstatix (Kassambara, 2021), car (Fox and Weisberg, 2019), ez (Lawrence, 2016), and reshape2 (Wickham, 2007).

Each of our three experiments took about 30 minutes to finish and followed the same basic procedure. Following a general outline of the features common to all experimental versions, we briefly described the distinct manipulations of each particular experiment. We used the words local and global to indicate either the cues available for each directional choices in the maze (local) or to indicate the exit location (global).

Participants were started at a fixed point in the maze (the start) for all mazes (see Figure 2.1). The wall structure of the mazes was hidden from the participants by a gray mask. Participants could only see blue and red colored circles indicating the available directions for movement. Particular colors probabilistically provided local information as to the shortest path to the exit (blue in this experiment). Across trials, the exit location might change (global event). Because the maze walls were invisible to participants, they had to rely on these local cues, their memory of the path traveled, and any ideas about the exit location to successfully navigate the maze.

Before the experiment, participants were instructed to find the exit using as few steps as possible **and** to provide estimates of local and global probabilities. Local probability was how trustworthy the local cues were in pointing to the shortest path direction within a maze, and the global probability was how likely the exit would be in a particular location across all mazes. We also provided various forms of feedback (that differed for individual experiments) to help participants know when they were on the right track.

At the beginning of the experiment, participants were given some practice to familiarize themselves with the interface, how to move in the maze, and how to use the sliders to report their reliability estimates. An example of the maze and the interface can be found at: https://artsresearch.uwaterloo.ca/~brittllab/protocols/MazeGL/Maze_task_cb.php.

After finishing the whole experiment, participants were invited to report strategies and give any additional comments via a post-experiment questionnaire.

2.3 Data Screening

For participants' behavior analysis, only participants' first visits to each decision-making point were considered (some participants would backtrack and face the same decision-making point more than once). This filtration enabled data analysis to be unaffected by participants' behavior after receiving feedback on whether the last movement led to the shortest path or not. Moreover, only participants' first 30 steps in every maze navigation were included in the behavior analysis. This was because the maximum length of the

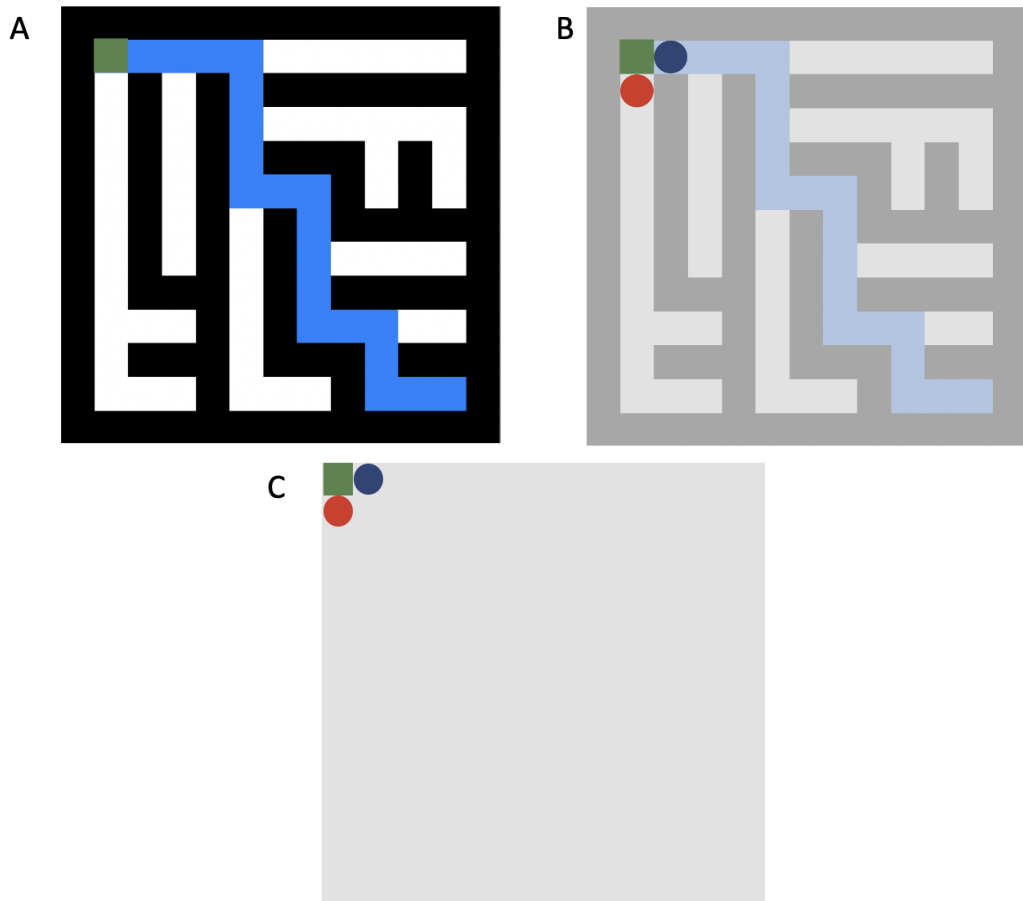


Figure 2.1: Overview of Maze experiment design. A: A typical maze without the gray mask and the shortest path of it is labeled in blue. The player is the green square who always started at the top left. The hidden walls are in black. B: A typical maze along with colorful cues covered by the gray mask. The mask is transparent just for demonstration purpose. C: A typical maze as participants saw it in the experiment.

shortest path was 22 and we did not want to include trials that might reflect distraction, misunderstanding, or fatigue.

Chapter 3

Pilot Experiment

3.1 Participants

22 participants (Female = 16, Male = 4, Right hand-dominant= 17, Left hand-dominant = 3) were enrolled in the pilot experiment.

3.2 Procedure

In the pilot experiment, participants volunteered for the online study and were directed to a website to complete the informed consent. If they consented, they were forwarded to a new web page that provided the structured practice moves. Next, they were placed at the start of a new maze and instructed to locate the exit. They moved within the maze by using the arrow keys on their computer and adjusted the sliders by clicking and dragging with their mouse or equivalent (trackpad, touch screen as this was an online study, we could not enforce a single response method but relied on whatever hardware participants used to run their web browser. We specifically asked participants not to complete the task using smartphones or tablets). After each selected move, the caption below the maze would demonstrate whether they were closer or further away from the exit. This meant that participants were given feedback about whether they were fewer or more steps away from the exit. When they reached the exit, they were informed of their success and a new maze would start. There were 10 mazes per participant, and 20 moves was the typical minimum number of moves for an individual maze.

Participants were informed that the exits were 100% located at the bottom right. In addition, at each decision point of the maze navigation, there were colored circles indicating possible directions to move. Specifically, blue-colored circles (local cues) might indicate the direction that was the shortest path to the exit. There might be multiple red-colored circles at one decision making point to indicate possible directions to move but there will always be just one blue-colored circle (local cue). How trustworthy the local cues were at pointing to the shortest path direction within a maze (local probability) for the first 10 trials was 60%. Participants would familiarize themselves with the practice maze. After that, they would complete 10 trials of randomly generated mazes, each trial contained one maze with $9 * 9$ cells (each cell is one position player can move to). As participants navigated the mazes, they could read the distance from the exit to their current location (displayed at the bottom of the frame, below the maze). By doing this they could know exactly how many steps they were away from the exit. At the same time, they could estimate their euclidean distance to the exit, too. In order to prevent the local cues from providing information about the global direction to the exit we eliminated the reporting of the explicit number of steps to the exit in the feedback of the subsequent experiments. Instead we only provided participants feedback with general feedback (either” You are closer to the exit.” or ”You are further away from the exit”).

3.3 Results

The pilot experiment was done to refine the experimental instructions and to verify appropriate actions in the online environment. Thus, analyses are reported.

Chapter 4

Fixed Global Varied Local Experiment

4.1 Participants

In the Fixed Global Varied Local experiment, another 22 participants (Female = 18, Male = 4, Right hand-dominant = 21, Left hand-dominant = 1) were analyzed.

4.2 Procedure

In the Fixed Global Varied Local experiment participants were informed that the exits were 100% located at the bottom right (see Figure 4.1). After one practice trial, participants would go through a maximum of 50 mazes with 11 * 11 cells (each cell is one position player can move to). This time, the distance from the exit was **not** displayed, but participants were told whether they were closer or farther away (fewer steps) from the exit after each movement. During the experiment, the local reliability of the cues transitioned between 50% to 100%; the global probability was always 100%. The transition in local probability was counterbalanced (low to high and high to low). In the low to high local probability condition, the probability series went from 50%, 60%, 70%, 80%, 90%, 100%, 100%, 90%, 80%, 70%, 60%, 50%. In the high to low local probability condition, the probability series went from 100%, 90%, 80%, 70%, 60%, 50%, 50%, 60%, 70%, 80%, 90%, 100%. 12 participants finished the low to high local probability condition, and 10 participants finished the high to Low local probability condition.

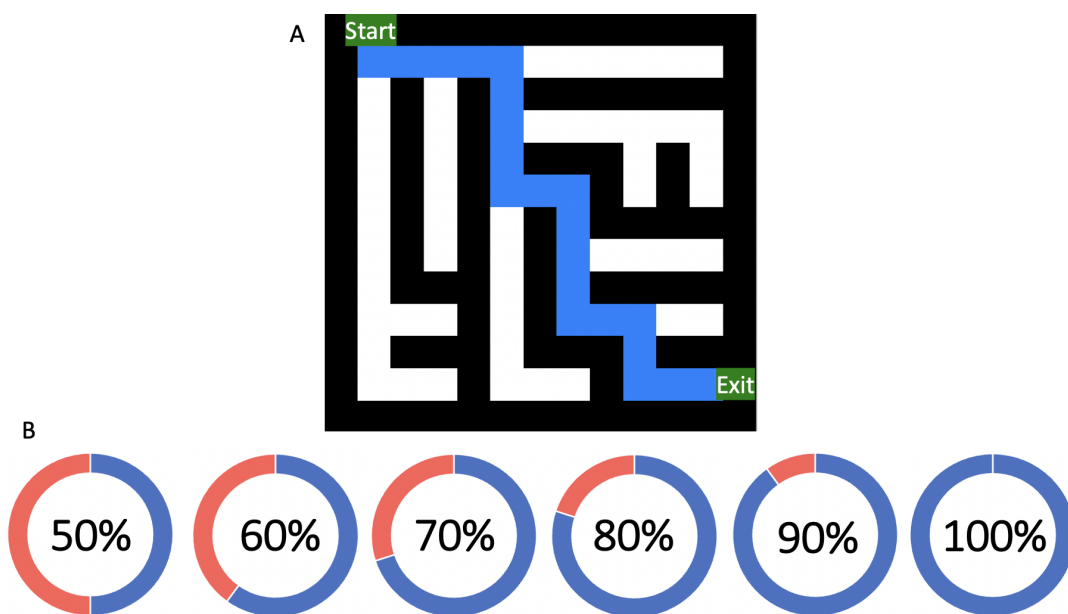


Figure 4.1: Overview of the Fixed Global Varied Local Experiment Design. A: A typical maze with the start point on top left and exit at bottom right. B: The progression of the local cues reliability across the six conditions. For this experiment, the to-be-attended local cues was the blue circle, so the proportion of each ring being blue corresponds to the local probability for that condition.

In addition to making navigational choices, we had participants provide their best estimate of local probability. At the same time, they were instructed to leave the global probability slider set at 100%. After completing each maze participants also received feedback on their local probability estimate. This feedback informed them if the local probability estimate deviated more than $\pm 5\%$ from the ground truth.

Participants knew before the experiment began that if they failed to estimate within this range then the next maze’s local probability would not change, and thus the duration of the experiment would be lengthened. In this way we tried to motivate participants to attend and provide accurate estimates.

4.3 Results

Participant performance was measured using both subjective and objective reports. One example of a subjective response is the records of participants’ estimates of the local probability using a provided ”slider”. An example of an objective response was the participant’s choices for which direction to move in the maze. We used these reports to first confirm that participants understood the task and were generally competent, and then we performed additional analyses comparing how subjective estimates and objective choices tracked changing local cues and global direction reliability.

4.3.1 Participants’ Subjective Estimate of Local Probability

The analysis of participants’ subjective estimate of local cues reliability was used to ensure participants understood the setting of the experiments and what was being asked of them.

Participants were free to adjust the slider whenever they felt the need to update their estimate. Given the heterogeneous timing of updating events we used the average of a participant’s local probability estimates as the principle dependent variable. That is we took the average of participants probability estimates for each of the six ground-truth probability bins.

We checked the sphericity for participants’ local estimates, the Mauchly’s test showed there was no departure from sphericity ($p = .474$). Next, we performed the single factor within-subjects ANOVA and demonstrated that participants’ estimates of local probability were significantly influenced by the ground truth prior local probability (ground-truth probability) of that trial ($F(5, 105) = 144.508, p < .001, \eta^2 = 0.858$).

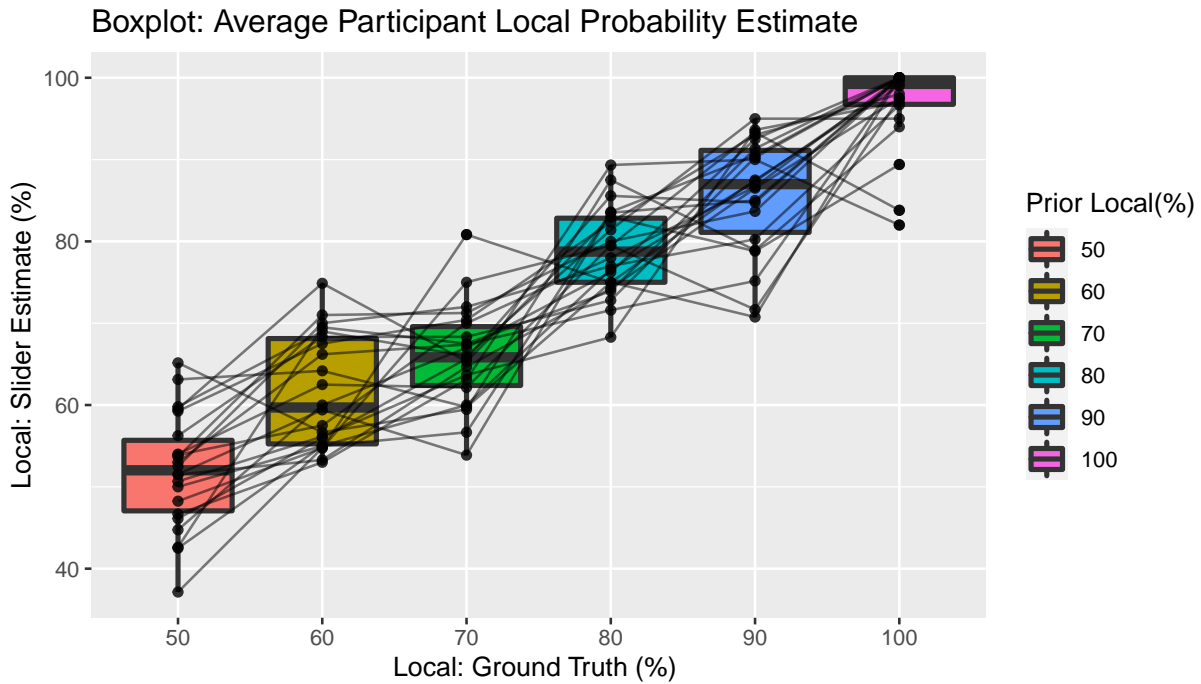


Figure 4.2: Increasing local cues reliability led to higher estimates of local probability. Different colors represent participants' estimates of local probability under different ground truth prior local probability conditions. Each black dot corresponds to one participant's average estimates of local probability given one ground truth prior local probability. The black line connects one participant's estimates of local probability across all prior local probability.

Furthermore, post-hoc paired t-tests (p values adjusted using Bonferroni correction to prevent inflations of family-wise error) revealed that participants' local probability estimates increased as the ground truth prior local probability increased (50% (M = 51.91, SD = 7.01, SEM = 1.49) < 60% (M = 61.57, SD = 6.88, SEM = 1.47, p < .001) = 70% (M = 65.91, SD = 6.15, SEM = 1.31, p = .119) < 80% (M = 78.82, SD = 5.36, SEM = 1.14, p < .001) < 90% (M = 85.75, SD = 7.13, SEM = 1.52, p = .008) < 100% (M = 96.79, SD = 5.21, SEM = 1.11, p < .001). The standard error of the mean (SEM) of participants' subjective estimates of local probability remained similar across different ground truth prior local probability, between 1.11% and 1.52%. The SEM in condition with ground truth prior local probability 100% was smaller due to ceiling effect. (see Figure 4.2)

4.3.2 Objective Measures: Proportion of Choices Following of the Shortest Path Direction

Next we considered the overall performance of participants' maze navigation by looking at the participants' proportions of choices that followed the shortest path direction (i.e., the "correct" direction).

The first analysis was on the probability of choosing the shortest path direction as a function of ground truth prior local probability condition. Mauchly's test showed there was no departure from sphericity ($p = .243$). A single factor within-subjects ANOVA revealed that there was a main effect of the ground truth prior local probability of that trial on participants' proportions of choices following of the shortest path direction ($F(5, 105) = 40.082, p < .001, \eta^2 = 0.578$).

Post-hoc paired t-tests (p values adjusted using Bonferroni correction to prevent inflations of family-wise error) further indicted that participants' following the shortest path direction proportion increased as the ground truth prior local probability increases (50% ($M = 0.57, SD = 0.13, SEM = 0.03$) < 60% ($M = 0.67, SD = 0.13, SEM = 0.03, p = .042$) < 70% ($M = 0.73, SD = 0.15, SEM = 0.03, p = .932$) < 80% ($M = 0.81, SD = 0.11, SEM = 0.02, p = .207$) < 90% ($M = 0.90, SD = 0.08, SEM = 0.02, p = .009$) < 100% ($M = 0.95, SD = 0.07, SEM = 0.02, p = .075$). The standard error of the mean (SEM) of participants' following the shortest path direction proportion remained similar across different ground truth prior local probability, between 0.02 and 0.03.

During the maze navigation, at each decision-making point for a player who would like to move forward, there could be either two directions or three directions for them to choose from. Therefore, the lower bound for participants to randomly choose the shortest path direction was 33.33%, and the higher bound was 50%. We found out that participants always chose the direction leading to shortest path above chance regardless of probability condition. In general participants did not randomly choose the direction but effectively chose the direction to move when navigating the mazes in as few steps as possible. (see Figure 4.3).

4.3.3 Objective Measure: Proportion of Choices Following Local Cues

To examine the influence of local probability changes on objective conduct we analyzed ground truth prior local probability effects on participants' willingness to follow local cues.

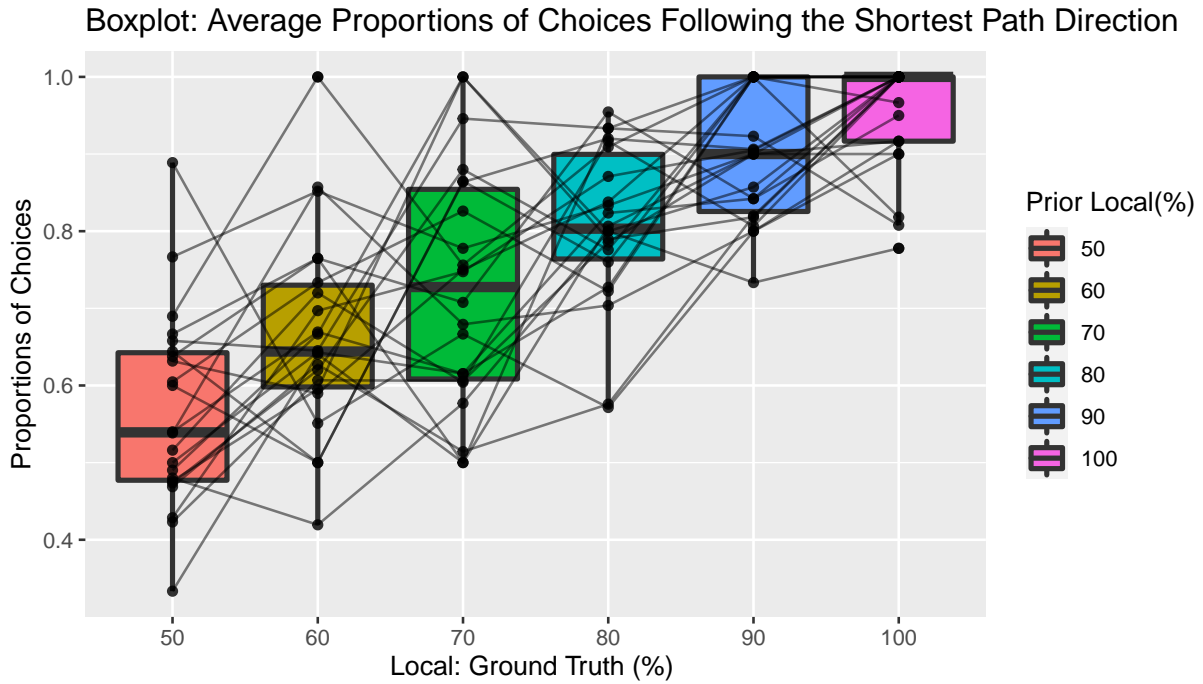


Figure 4.3: Increasing local cues reliability led to a higher proportion of choices following the shortest path direction in the Fixed Global Varied Local Experiment. Average participants' proportions of choices following the shortest path direction proportions were higher than chance in all prior local probabilities. Different colors represent participants' proportions of choices following the shortest path direction under different ground truth prior local probabilities. Each black dot corresponds to one participant's average proportion of choices following the shortest path direction given one ground truth prior local probability. The black line connects one participant's proportion of choices following the shortest path direction across all ground truth prior local probabilities.

If participants managed to successfully encode local cues reliability into a mental representation of local probability, they would be expected to rely more on local cues as prior local cues increase. A single factor within-subjects ANOVA tested was performed to analyze the proportion of choosing the direction indicated by the local cue which we considered as participants local cues following behavior. We checked the sphericity for the proportion of choices following local cues in different ground truth prior local probability condition, the assumption of sphericity for single factor within-subjects ANOVA was met ($p = .319$). Again, the single factor within-subjects ANOVA showed that the proportion of choices following local cues was significantly influenced by the ground truth prior local probability of that trial ($F(5, 105) = 39.126, p < .001, \eta^2 = 0.518$).

In addition, post-hoc paired t-tests (p values adjusted using Bonferroni correction to prevent inflations of family-wise error) revealed that the proportion of participants' following local cues increased as the ground truth prior local probability increased. Statistical tests were typically significant when the difference between global and local reliabilities exceeded 20% (50% ($M = 0.66, SD = 0.12, SEM = 0.03$) = 70% ($M = 0.77, SD = 0.13, SEM = 0.03, p = .014$), 60% ($M = 0.72, SD = 0.09$) < 80% ($M = 0.86, SD = 0.10, SEM = 0.02, p < .001$), 70% ($M = 0.77, SD = 0.13, SEM = 0.03$) < 90% ($M = 0.91, SD = 0.08, SEM = 0.02, p < .001$), 80% ($M = 0.86, SD = 0.10, SEM = 0.02$) < 100% ($M = 0.95, SD = 0.07, SEM = 0.02, p = .005$). Standard errors of the means (SEM) of participants' choices proportions was similar across different ground truth prior local probability conditions (between 0.02 and 0.03; see Figure 4.4).

The impact of ground truth prior local probability on participants' decisions was large. However whether their subjective estimates during each specific decision point predict their actual performance was a question of interest. A logistic regression analysis of how subjective estimates affected objective choices was performed to address this.

The impact of ground truth prior local probability on participants' directional movement decisions was large. What was unknown was the degree to which participants estimates of the ground truth prior local probability impacted these directional movement choices, or in other words, the degree to which their subjective estimates during each specific decision point predicted their objective movement choices. A logistic regression analysis of how subjective estimates of local probability affected objective choices was performed to address this.

It was found that the odds of choosing the movement direction indicated by the local cues increased by 1.82% (95% CI [1.41%, 2.21%]) for every percentage increase in the subjective estimates of local probability (McFadden's Pseudo $R^2 = 0.02$). (see Figure 4.5)

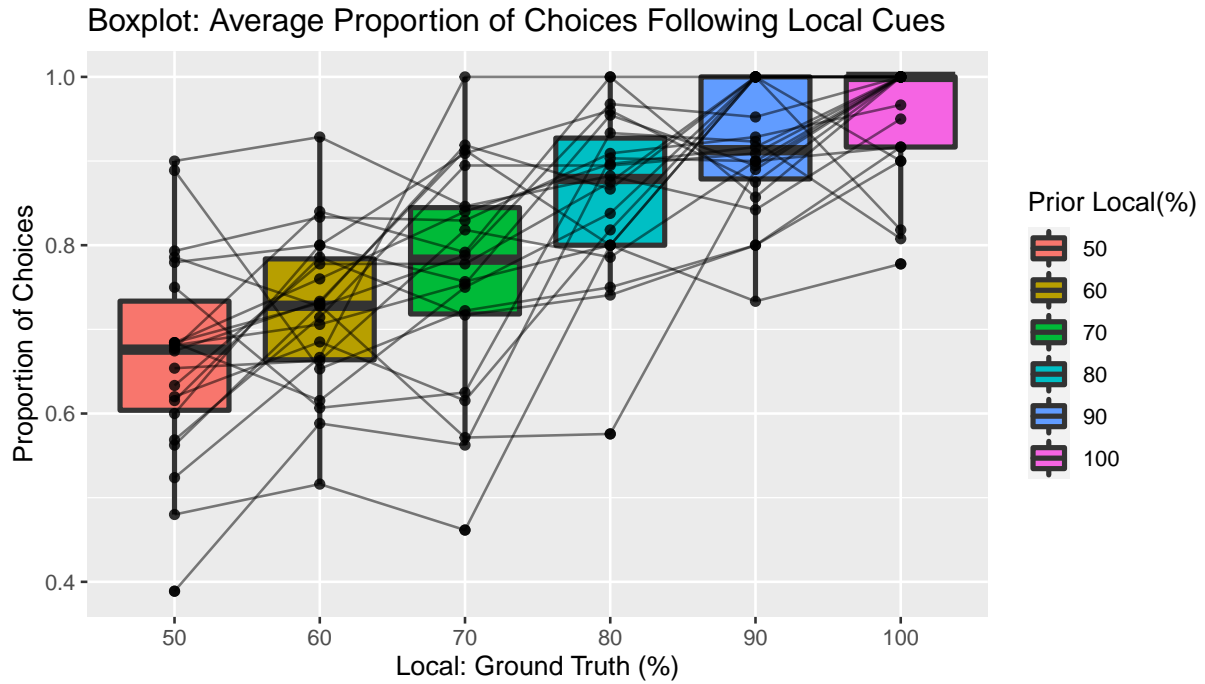


Figure 4.4: Increasing local cues reliability led to a higher proportion following local cues at the decision-making point in the Fixed Global Varied Local Experiment. The average proportion of choices following local cues were higher than the what they would do by chance in all ground truth prior local probabilities. Different colors represent participants' proportions of choices following local cues under different ground truth prior local probabilities. Each black dot corresponds to one participant's average proportion of choices following local cues given one ground truth prior local probability. The black line connects one participant's proportion of choices following local cues across all ground truth prior local probability.

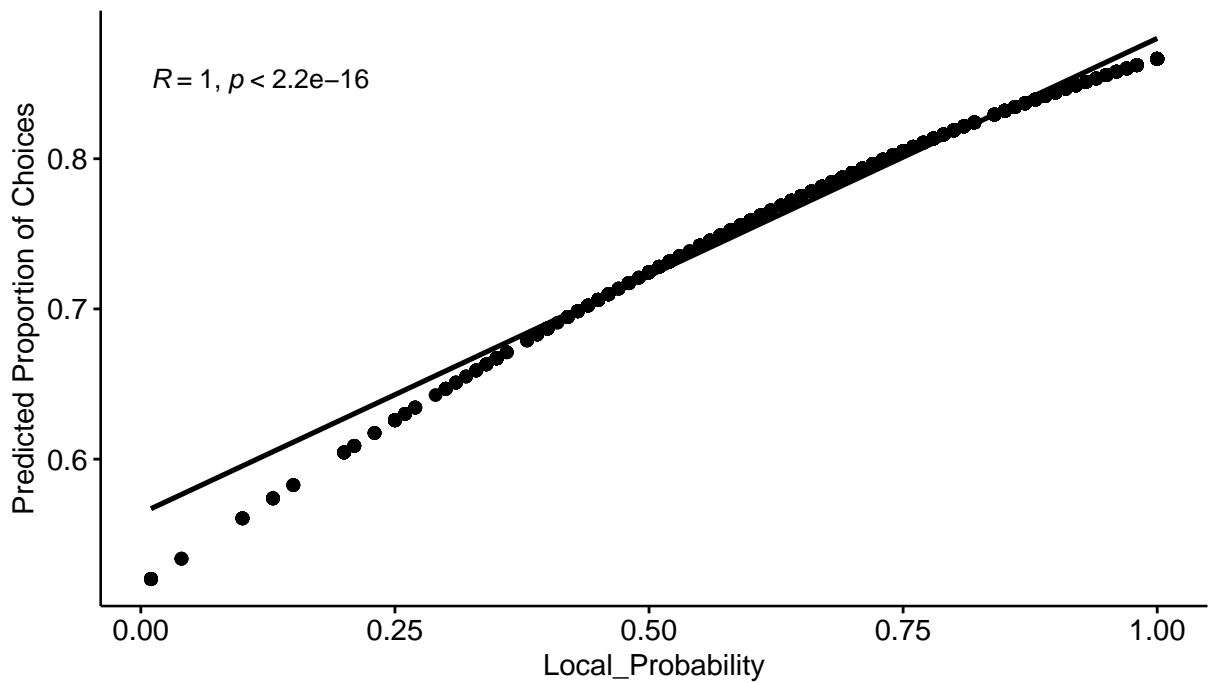


Figure 4.5: Logistic regression showed that the subjective estimates of local probability were a good factor for predicting participants' proportions of choices following local cues. The black line fits the predicted proportion of participants' choices following local cues. The correlation coefficient between participants' subjective estimate of local probability and proportions of choices following local cues is essentially 1 ($p < .001$).

4.3.4 Objective Measure: Proportion of Choices Following the Global Direction

In the Fixed Global Varied Local experiment the exit was always located at the bottom right of the maze. Whether participants were following the global direction was determined from their current coordinates in the maze. At one particular decision-making point, if participants' current x coordinate was greater than their current y coordinate then they were in the right part of the maze, thus the decision of moving down meant they followed the global direction. If the participants' current y coordinate was greater than their current x coordinate then they were in the bottom part of the maze, thus the decision of moving right meant they followed the global direction. If participants current x and y coordinates were equal, then the decision of moving either right or down meant they followed the global direction.

If the consistent global probability was encoded separately from the changing local probability, then no main effect of ground truth prior local probability would be found. The Mauchly's test showed there was no departure from sphericity ($p = .671$). The ANOVA analysis showed that the proportion of choices following the global direction was not significantly influenced by the ground truth prior local probability of that trial ($F(5, 105) = 2.01$, $p = .083$, $\eta^2 = 0.072$). Therefore, no logistic regression was performed. (see Figure 4.6)

4.3.5 Objective Measure: Proportion of Choices Relied More on Local Cues

Previous results showed that participants were using local cues as guidance when the global direction was fixed and 100% reliable. At some decision points participants had two options, they could choose either to follow the global direction or not follow it if the local cues pointed in a different direction. How would they make their choices when the two indicators, local and global, were in conflict?

To see a bigger picture of participants' behavior when local cues were not consistent with the global direction, we compared the overall average proportions of choices still following the global direction ($P(\text{following the global direction}) \mid P(\text{local cues not pointing global direction})$) and still following local cues ($P(\text{following local cues}) \mid P(\text{local cues not pointing the global direction})$) across all six ground truth prior local probability conditions.

Paired t-tests (p values adjusted using Bonferroni correction to prevent inflations of family-wise error) showed that when the local cues were not pointing toward the global exit

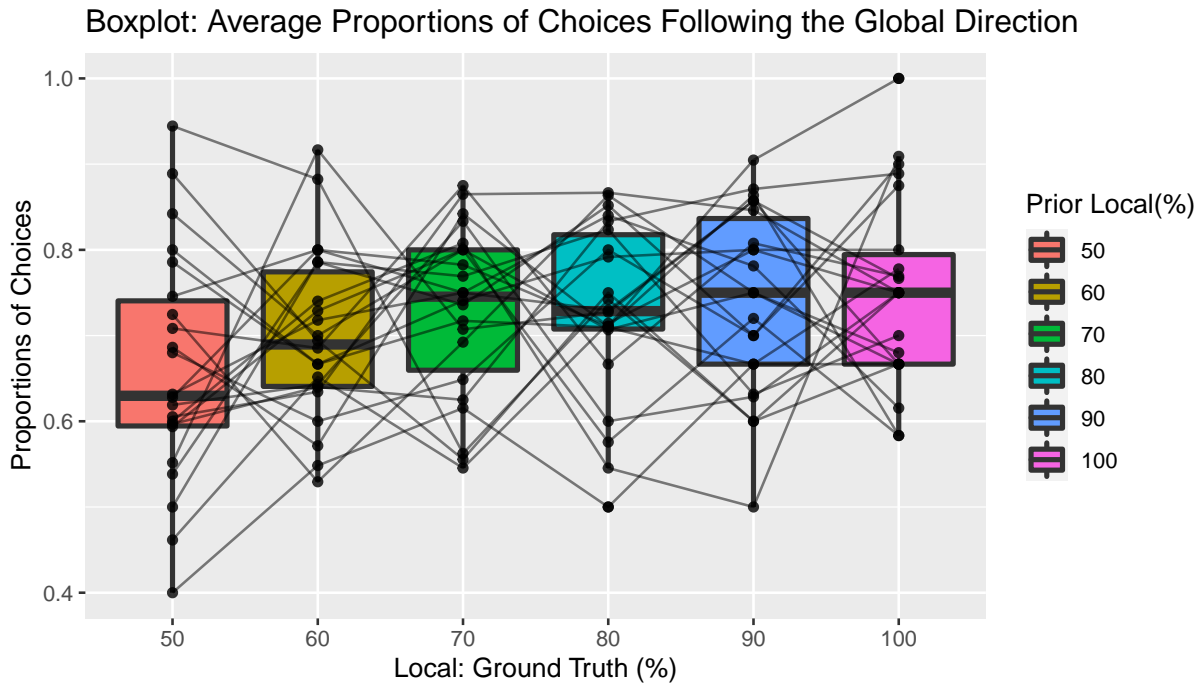


Figure 4.6: The change in local cues reliability did not lead to significant differences in participants' proportions of choices following the global direction. Different colors represent participants' proportions of choices following the global direction under different ground truth prior local probability. Each black dot is the mean of one participant's proportions of choices following the global direction given one ground truth prior local probability. The black line connects one participant proportion of choices following the global directions across all ground truth prior local probability.

direction, the average proportion of choices following local cues did not vary ($P(\text{Following local cues}) \mid P(\text{local cues not pointing the global direction})$) ($M = 0.43$, $SD = 0.07$, $SEM = 0.02$) and was similar with the average proportion of choices following the global direction ($M = 0.49$, $SD = 0.07$, $SEM = 0.01$) although every participant knew the global direction was 100% located at the bottom right, $t(21) = 1.80$, $p = .086$. Therefore, the results suggested participants did not rely more on the global direction even though participants knew the global direction was 100% reliable and the local cued direction were not consistent with that direction. In other words, our analysis indicated that participants relied more on local cues than global cues.

4.3.6 Objective Measure: Proportion of Choices Following Local Cues and the Global Direction Across Prior Local Probability Conditions

Our analyses showed that with a fixed and reliable global exit direction participants still report and use the local cues dominantly for maze navigation on average. Was this true under all ground truth prior local probability conditions? Using paired t-tests, we compared the proportion of choices following local cues and consistent with the global direction across different ground truth prior local probability conditions.

The paired t-tests (p values adjusted using Bonferroni correction to prevent inflations of family-wise error) showed that participants showed no preference for following local cues when it provided no information (50% condition; $p = 1$), or little information (60% ($p = 1$); 70% ($p = 1$)), but showed a strong preference to local cues after ground truth prior local probability reached 80% ($p < .001$), 90% ($p < .001$), and 100% ($p < .001$). Participants only relied on the local cues when its reliability exceeded 80%. (see Figure 4.7)

4.4 Discussion

The main goal of this Fixed Global Varied Local experiment was to explore whether participants had separate mental representation for local cues reliability and global direction reliability when they only needed to consider changing local cues' reliability. We found that people's subjective estimates and objective movements reflected the changing local probabilities in the face of the unchanging global probability. This was consistent with separable representations.

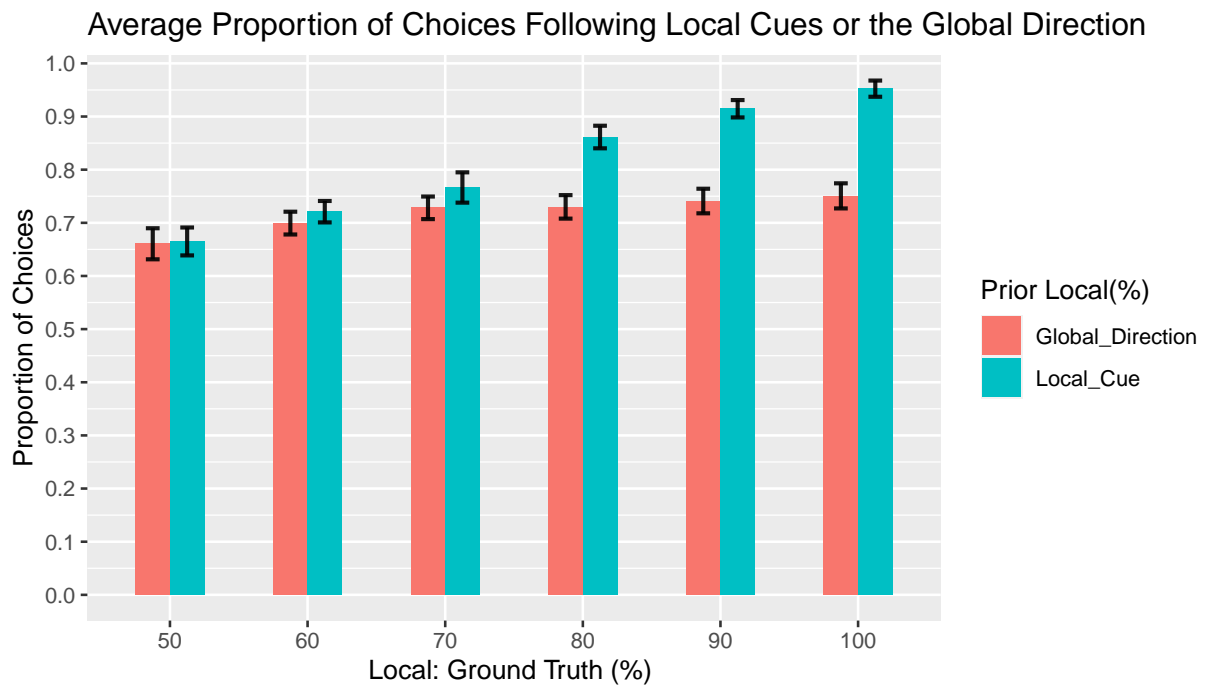


Figure 4.7: Participants only relied more on the local cues in choosing their navigational choices when local cues reliability was higher than 80%. Error bars represent one standard error of the mean.

Additionally, this main effect analysis of ground truth prior local probability on participants' average local estimates suggested that participants' average local estimates approached the ground truth prior local probability when they navigated through the maze. Participants had higher average estimates than ground truth when they were at low ground truth prior local probability levels (the 50% and 60% conditions), but lower average estimates than ground truth when local cues reliability was high (the 70%, 80%, 90%, and 100% conditions). This was consistent with previous uncertainty estimation experiments: people overestimate the low probability events and underestimate the high probability event (Attneave, 1953; Khaw et al., 2021).

Across all conditions participants' always chose the maze direction indicated by the local cue more than chance (50%). They even relied more on the local cues than they relied on the global direction when the ground truth prior local probability was higher than 80%. Because the local cues were typically less than 100% reliable participants could not know before choosing if their choices were "correct". They should understand that choosing the direction indicated by the local cues might not always lead them along the shortest path to the exit.

Given the proportion of participants' choices favoring the local cues directions was higher than following the global direction, we wondered if participants relied solely on local cues. Would changing the global probability change participants' subjective estimates of local probability and their objective behaviors? The Varied Global Fixed Local experiment was designed to address these questions.

Chapter 5

Varied Global Fixed Local Experiment

5.1 Participants

In the Varied Global Fixed Local experiment, a different population of 43 participants (Female = 27, Male = 16, Right hand-dominant = 35, Left hand-dominant = 8) were analyzed.

5.2 Procedure

The Varied Global Fixed Local experiment setting was similar to the Fixed Global Varied Local experiment (see Figure 5.1). After one practice trial, participants needed to finish three blocks of maze navigation with each block containing 15 mazes. Every maze had a fixed local probability of 80% but different global probabilities (the proportion of the times the exit was in a particular corner). The order of different global probability conditions was counterbalanced across participants. In the low to high global probability condition, the global probabilities were 60%, 73%, 80%, while in the high to low global probability condition, the global probabilities were 80%, 73%, 60%. 23 participants finished the low to high local probability condition, and 20 participants finished the high to low local probability condition. Besides the difference in local and global probabilities setting, for this experiment participants were asked to make their best subjective estimates of both global and local probabilities. After each maze navigation, participants would also receive

feedback about whether the global or local probability estimate deviated were more than $\pm 5\%$ from the ground truth.

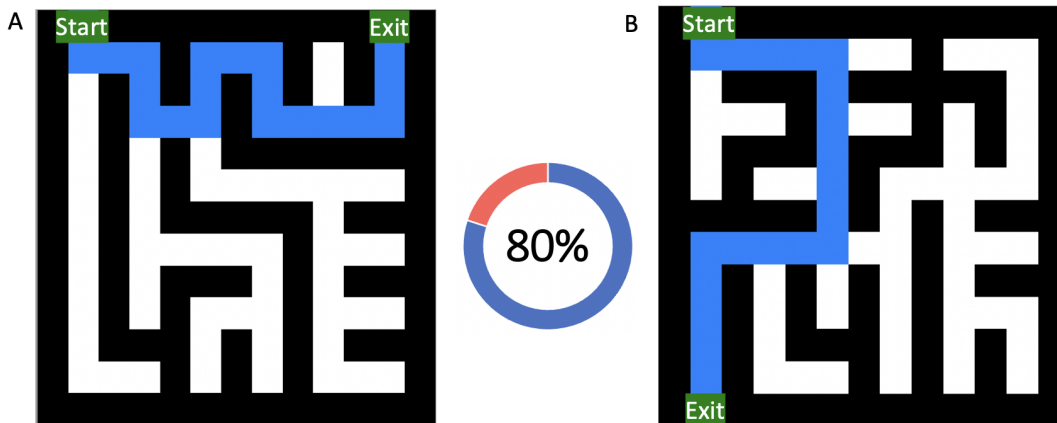


Figure 5.1: Overview of the Varied Global Fixed Local Experiment Design. A: A typical maze with the start point on top left and exit at top right. B: A typical maze with start point on top left and exit at the bottom left.

5.3 Results

5.3.1 Participants' Subjective Estimates of Local Cues Reliability and Global Direction Reliability

Similar to the Fixed Global Varied Local experiment participants could adjust their estimates of local probability and global probability using the sliders whenever they wanted.

Firstly, we analyzed participants' subjective local probability estimates.

The single factor within-subjects ANOVA showed that there were no influences of ground truth prior global probability on participants' estimates of the local probability ($F(2, 84) = 1.201$, $p = .306$, $\eta^2 = 0.007$). Participants' local probability estimates was similar in the blocks with ground truth prior global probability 60% ($M = 74.21$, $SD = 9.45$, $SEM = 1.44$), 73% ($M = 73.82$, $SD = 11.05$, $SEM = 1.69$), and 80% ($M = 72.26$, $SD = 9.01$, $SEM = 1.37$). The standard error of the mean (SEM) of participants' subjective estimates of local probability remained similar across different global probabilities, between

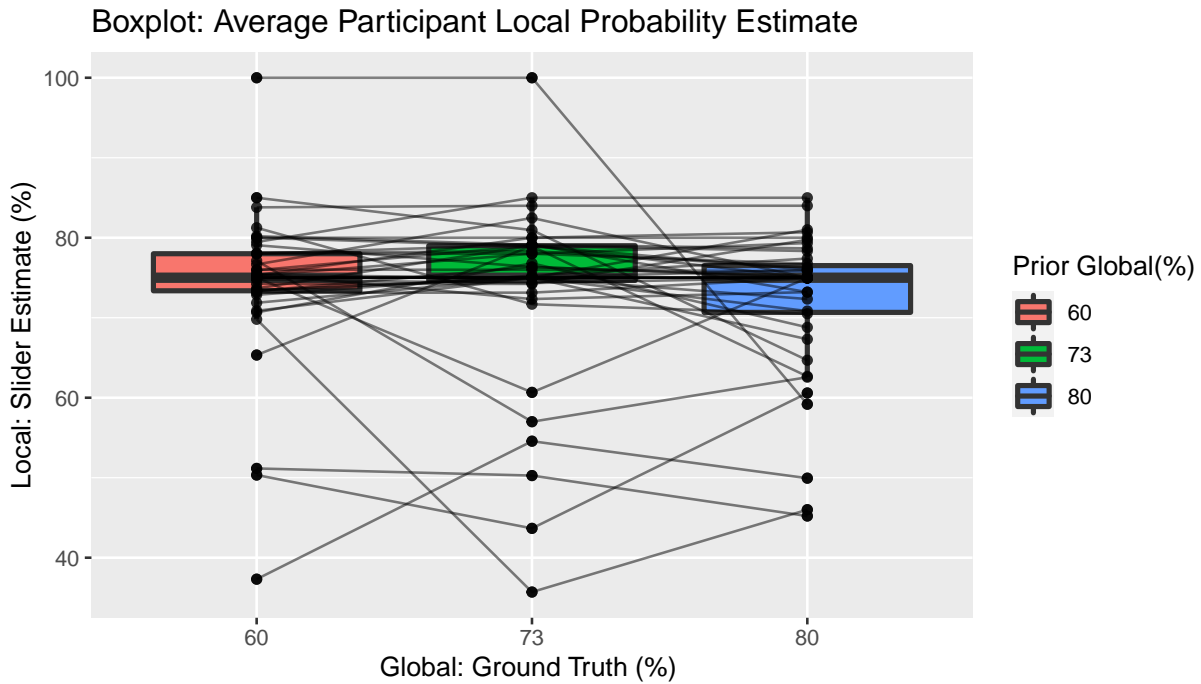


Figure 5.2: The change in ground truth global probabilities did not impact participants' subjective estimate of local probability. Different colors represent participants' estimates of global probability under different ground truth prior global probabilities. Each black dot corresponds to one participant's average estimate of global probability given one ground truth prior global probability. The black line connects one participant's estimates of local probability across all ground truth prior local probabilities.

137.33% and 168.52% (see Figure 5.2) No correction was done to the single factor within-subjects ANOVA, since the Mauchly's test showed the assumption of sphericity for single factor within-subjects ANOVA was met ($p = .344$).

Next, we analyzed participants' subjective global probability estimates. The average global probability estimates for every ground truth prior global probability condition were considered as the dependent variable. We checked the sphericity for participants' global estimates, the Mauchly's test showed there was no violation of the assumption, ($p = .344$). The single factor within-subjects ANOVA showed participants' global estimates in the three blocks was significantly influenced by that block's ground truth prior global probability ($F(1, 42) = 5.557$, $p = .023$, $\eta^2 = 0.117$).

Post-hoc paired t-tests revealed that participants' global probability estimates in the

block with a ground truth prior global probability of 60% ($M = 61.14$, $SD = 13.35$, $SEM = 2.04$) was significantly lower than participants' global probability estimates in the block with a ground truth prior global probability of 73% ($M = 68.66$, $SD = 10.35$, $SEM = 1.58$, $p = .001$) and also for 80% ($M = 67.71$, $SD = 14.97$, $SEM = 2.28$, $p = .042$). The standard error of the mean (SEM) of participants' subjective estimates of global probability remained similar across different prior global probabilities, between 1.58% and 2.28%.

However, the participants' subjective estimates of global probability in the block with ground truth prior global probability 73% was similar to the participants' global probability estimates in the block with ground truth prior global probability of 80% ($p = .655$). This might be due to the small difference (only 7%) between the ground truth prior global probability of the two blocks. The results show that participants are sensitive to the global probability. (see Figure 5.3)

5.3.2 Objective Measure: Proportion of Choices Following the Shortest Path Direction

The influence of ground truth prior global probability on participants' performance following of the shortest path direction was also investigated.

The Mauchly's test showed there was no departure from sphericity ($p = .435$). Participants' proportions of choices following of the shortest path direction was significantly influenced by the ground truth prior global probability of that maze ($F(2, 84) = 3.461$, $p = .036$, $\eta^2 = 0.02$).

The effect of ground truth prior global probability was very marginal as post-hoc paired t-tests demonstrated that participants' proportions of choices following of the shortest path direction increased as the ground truth prior global probability increased only from 60% ($M = 0.71$, $SD = 0.10$, $SEM = 0.02$) to 73% conditions ($M = 0.74$, $SD = 0.09$, $SEM = 0.01$, $p = .009$). But there is no increases in participants' proportions of choices following of the shortest path direction as the ground truth prior global probability increased from 60% to 80% ($M = 0.71$, $SD = 0.10$, $SEM = 0.02$, $p = .057$), and also participants' proportions of choices following correct direction proportion in block with ground truth prior global probability 73% ($M = 0.74$, $SD = 0.09$, $SEM = 0.01$) was similar to behavior in block with ground truth prior global probability 80% ($M = 0.71$, $SD = 0.10$, $SEM = 0.02$, $p = .660$). The standard error of the mean (SEM) of participants' subjective estimates of local probability remained similar across different ground truth prior global probability levels, between 0.02 and 0.03 (see Figure 5.4) This indicates that participants were using the global direction as one guidance, but maybe not as much as they used local cues.

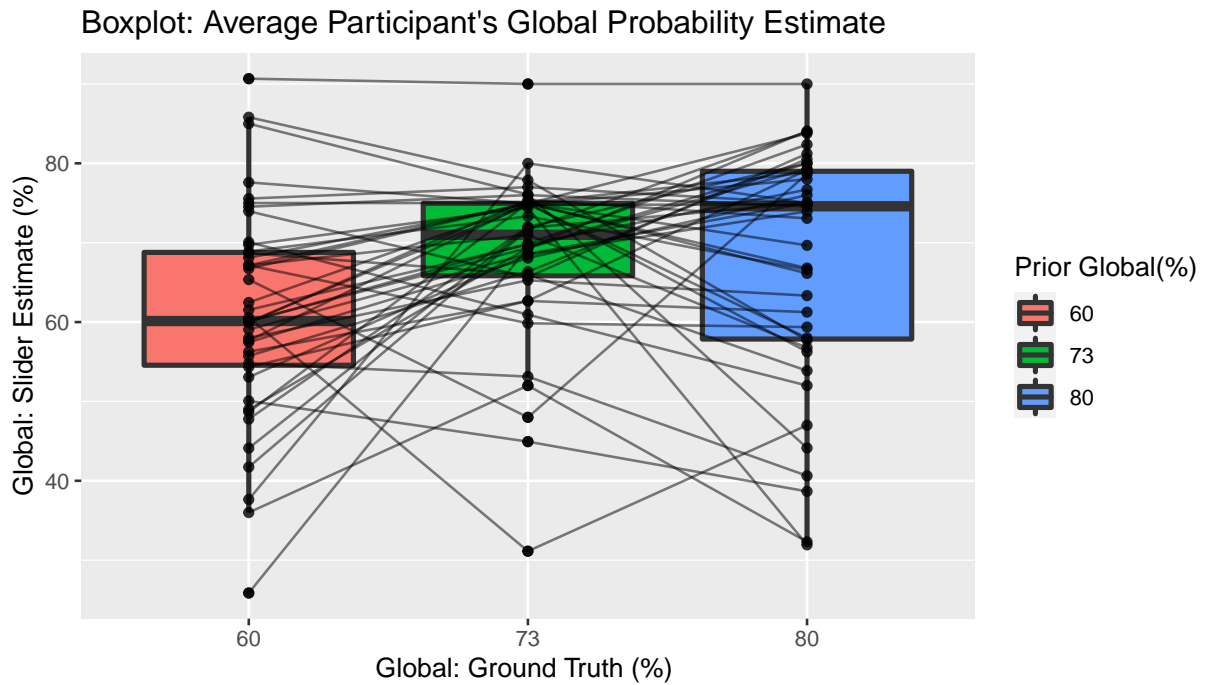


Figure 5.3: Increasing global direction reliability led to higher subjective estimates of global probability. Different colors represent participants' subjective estimates of global probability under different ground truth prior global probability conditions. Each black dot corresponds to one participant's average subjective estimates of global probability given one ground truth prior global probability. The black line connects one participant's average subjective estimates of global probability across all ground truth prior local probabilities.

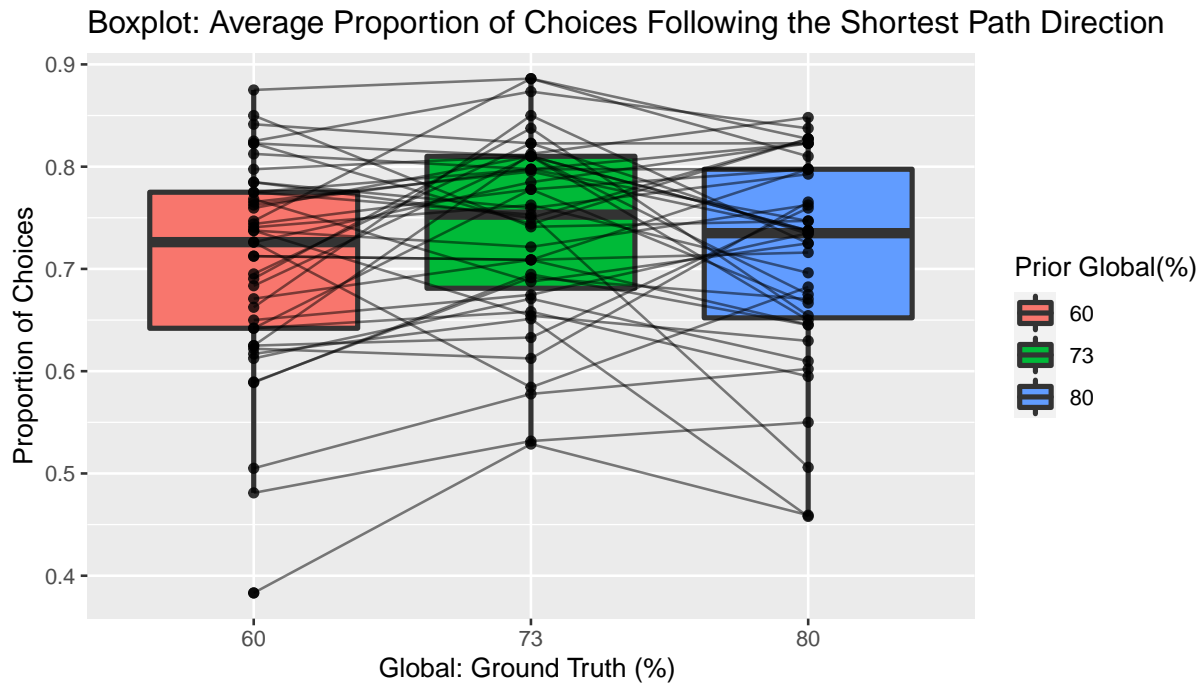


Figure 5.4: Increasing global direction reliability led to higher participants' proportions of choices following of the shortest path direction at the decision-making point. Average participants' proportions of choices following of the shortest path direction proportions were higher than chance in all ground truth prior global probability blocks. Different colors represent participants' proportions of choices following the shortest path direction under different ground truth prior global probability. Each black dot corresponds to one participant's average proportion of choices following the shortest path direction given one ground truth prior global probability. The black line connects one participant's proportion of choices following the shortest path direction across all ground truth prior global probabilities.

5.3.3 Objective Measure: Proportion of Choices Following Local Cues

The ground truth prior local probability was not changed during the whole experiment. If participants had a separate local probability representation from the changing global probability representation, they would be expected to rely on the local cues consistently without too much change. The Mauchly's test showed the sphericity assumption was met for the proportion of choices following local cues in different ground truth prior global probability condition ($p = .046$). As hypothesized, the proportion of choices following local cues was not impacted by the ground truth prior global probability of that trial ($F(2, 84) = 2.043$, $p = .136$, $\eta^2 = 0.011$). Therefore no logistic regression was performed. (see Figure 5.5)

5.3.4 Objective Measure: Proportion of Choices Following the Global Direction

In the Varied Global Fixed Local experiment condition, the exit was either located at the top right or bottom left of the maze. No matter which ground truth prior global probability condition participants experienced, there were always more exits located at the bottom left than exits located at the top right. Participants should consider bottom left as the global direction for every trial. So at one particular decision-making point, the decision of moving either left or down meant the participants followed the global direction.

If participants managed to successfully encode the local cues reliability into a mental representation of local probability, they would be expected to rely more on local cues as prior local cues increased. The Mauchly's test showed the sphericity assumption was not met ($p = .028$). We performed a single factor within-subjects ANOVA test with Huynh-Feldt correction on the participants' proportions of choices following the global direction. As speculated, the single factor within-subjects ANOVA analysis showed that the proportions of choices following the global direction was significantly influenced by the ground truth prior global probability of that trial ($F(2, 84) = 0.896$, $p = .035$, $\eta^2 = 0.055$).

Post-hoc paired t-tests revealed that participants' proportions of choices following the global direction increased as the ground truth prior global probability increased from 60% ($M = 0.61$, $SD = 0.06$, $SEM = 0.01$) to 73% ($M = 0.64$, $SD = 0.05$, $SEM = 0.01$, $p = .006$). But there was no significant difference between from 73% ($M = 0.64$, $SD = 0.05$, $SEM = 0.01$) or 80% ($M = 0.63$, $SD = 0.06$, $SEM = 0.01$, $p = .160$). The standard error of the mean (SEM) of participants' proportions of choices following the global direction remained

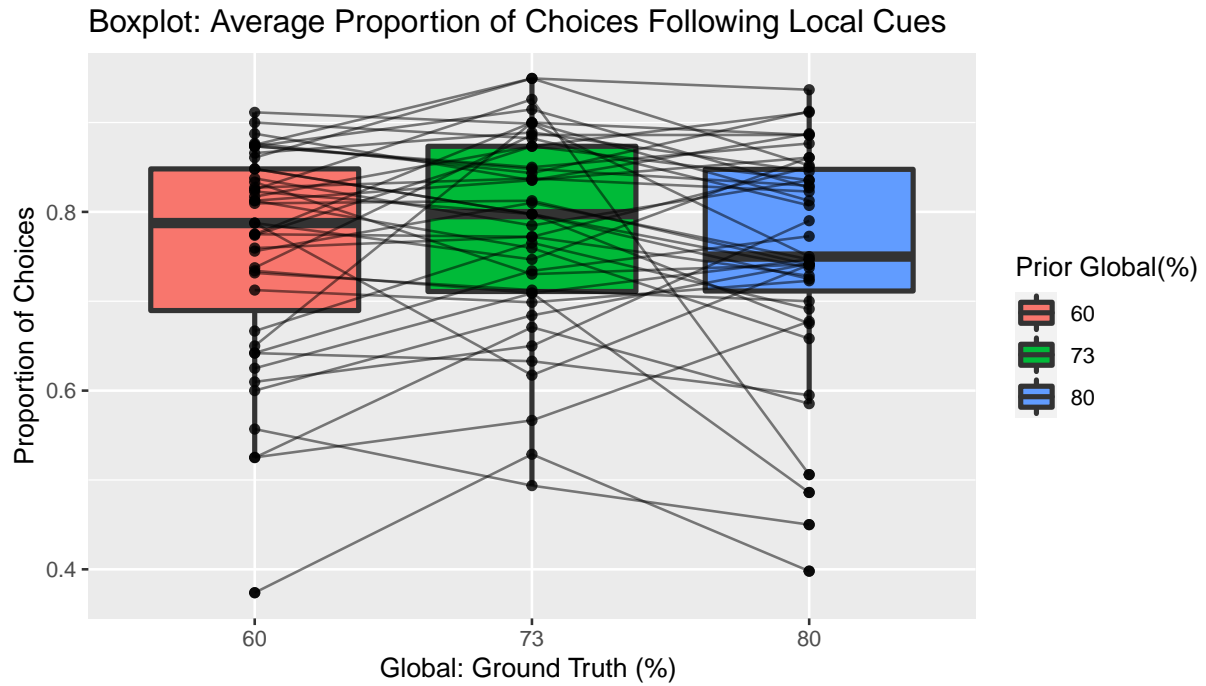


Figure 5.5: Increasing global direction reliability did not lead to a difference in following local cues. Still, the average proportion of choices following local cues was higher than chance. Different colors represent participants' proportions of choices following local cues under different ground truth prior global probability. Each black dot corresponds to one participant's average proportion of choices following local cues given one ground truth prior global probability. The black line connects one participant's proportion of choices following local cues across all ground truth prior global probabilities.

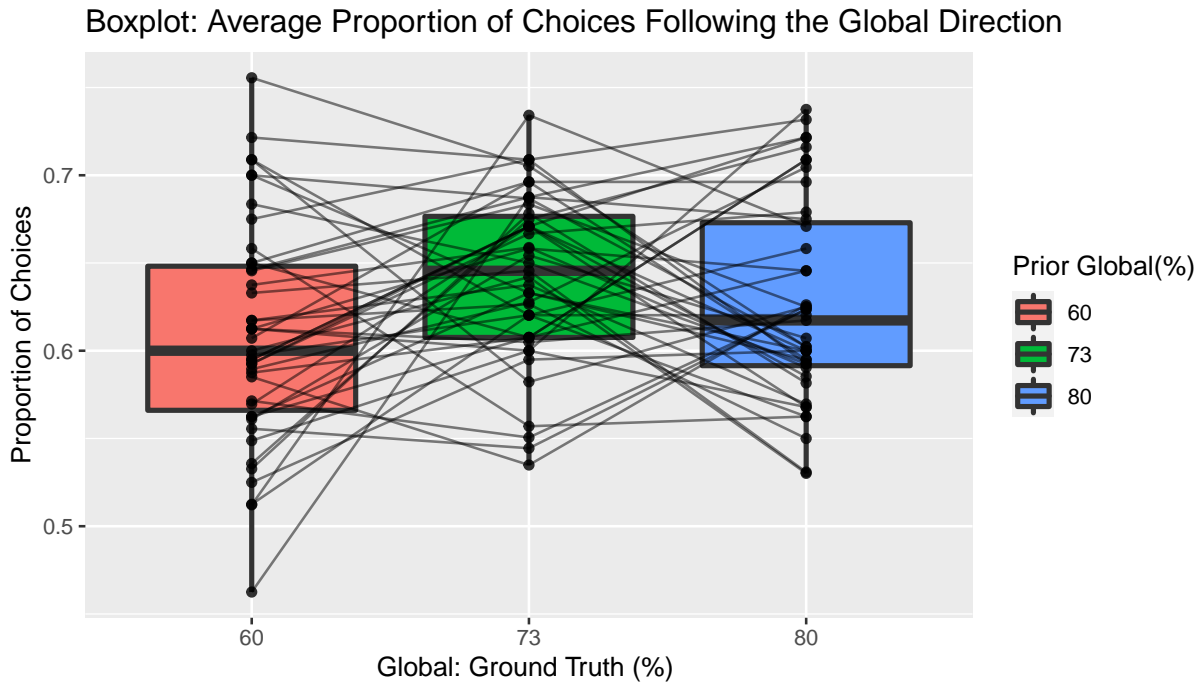


Figure 5.6: Increasing global direction reliability led to a higher proportion of choices following the global direction. Different colors represent participants' proportions of choices following the global direction under different ground truth prior global probability conditions. Each black dot is the mean of one participants proportion of choices following the global direction given one ground truth prior global probability. The black line connects one participant proportion of choices following the global directions across all ground truth prior global probabilities.

similar across different ground truth prior global probability conditions, between 0.01 and 0.01. (see Figure 5.6)

Since participants might not get accurate estimates of prior local and global probability, whether their subjective estimates during each specific decision point predict their performance on following the global direction was further analyzed using logistic regression.

We included both the participant's subjective local probability and global probability estimates in this analysis. The odds of following the global direction decreased by -0.02% (95% CI [-0.29%, 0.26%]) for every percentage point increase in the subjective estimates of local probability (see Figure 5.7), and the odds of following the global direction increased by 0.23% (95% CI [0.06%, 0.40%]) for every percentage increased in the subjective estimate of

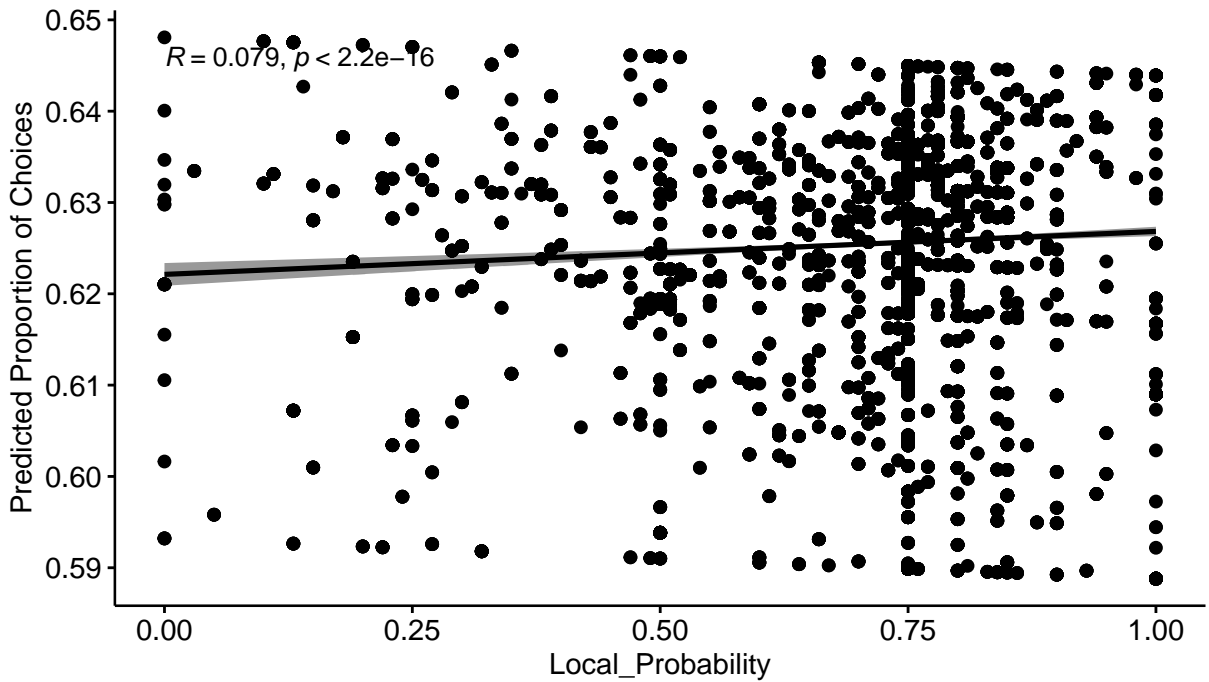


Figure 5.7: Logistic regression showed that the subjective estimates of local probability was a good factor predicting participants’ proportions of choices following the global direction. The black line fits the predicted proportion of participants’ choices following the global direction. The correlation coefficient between participants’ subjective estimate of local probability and proportions of choices following the global direction is 0.079 ($p < .001$).

the subjective estimates of global probability, (McFadden’s Pseudo $R^2 < .001$). (see Figure 5.8) This result suggests that participants rely on both local cues and the global direction when making their decisions, and they were more likely to follow the global direction when their estimates of global probability were high and local probability were low.

5.3.5 Objective Measure: Proportion of Choices Relied More on Local Cues

Next, we explored how participants dealt with conflict. Would participants choose to rely more on the local cues or the global direction when they contradicted each other?

We compared the overall average proportions of choices still following the global direc-

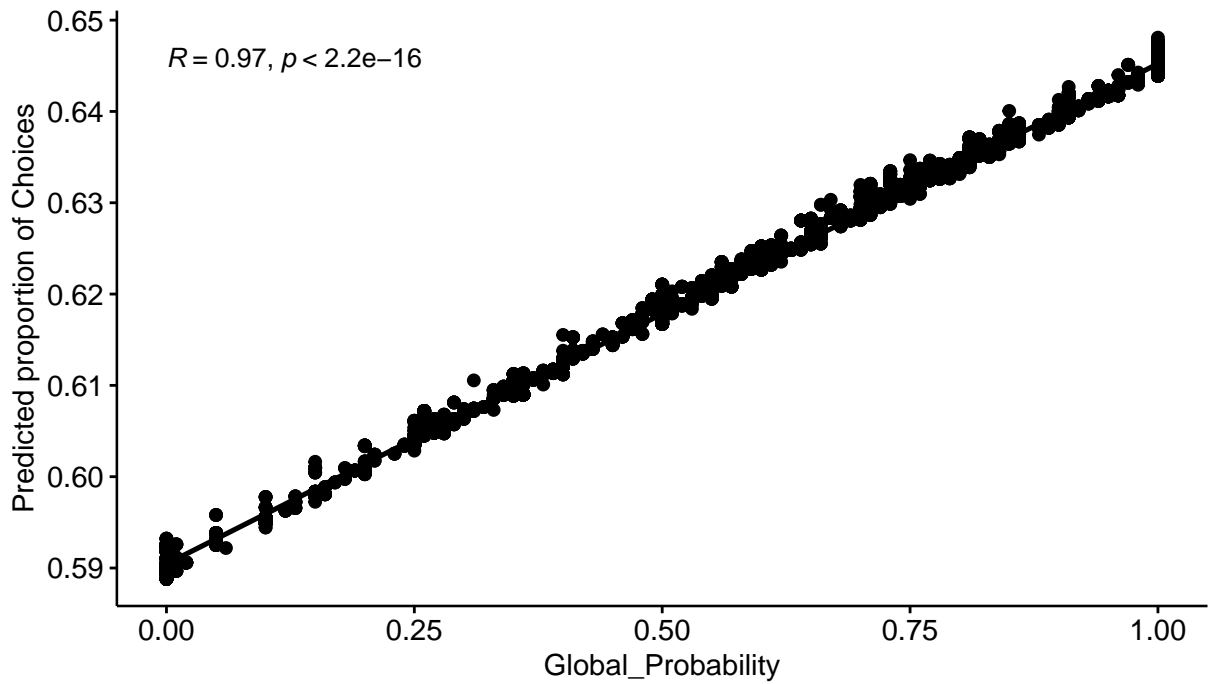


Figure 5.8: Logistic regression showed that the subjective estimates of global probability was a good factor for predicting participants' proportions of choices following the global direction. The black line fits the predicted proportion of participants' choices following the global direction. The correlation coefficient between participants' subjective estimates of global probability and proportions of choices following the global direction is 0.968 ($p < .001$). The correlation coefficient between participants' subjective estimates of global probability and proportions of choices following the global direction is higher than the correlation coefficient between participants' subjective estimates of local probability and proportion of choices following the global direction (correlation coefficient = 0.079).

tion ($P(\text{following the global direction}) \mid P(\text{local cues not pointing the global direction})$) and still following local cues ($P(\text{following local cues}) \mid P(\text{local cues not pointing the global direction})$) across all three ground truth prior global probability blocks.

Paired t-tests showed that when the local cues was not pointing in the global direction, the average proportion of choices still following local cues ($M = 0.71$, $SD = 0.12$, $SEM = 0.02$) was significantly higher than the average proportion of choices still following the global direction ($M = 0.25$, $SD = 0.11$), $t(42) = -13.30$, $SEM = 0.02$, $p < .001$. The result was consistent with our observation in the Fixed Global Varied Local condition, people's choice are influenced more by local cues.

5.3.6 Objective Measure: Proportion of Choices Following Local Cues and the Global Direction Across Prior Global Probability Conditions

When participants were not sure about both the local cues and the global direction, their strategy was to rely on both the local cues and the global direction, but with the local cues dominant. When the global direction became completely uncertain would participants maximize their reliance on the local cues? We compared the proportions of choices following local cues and the global direction across different ground truth prior global probability conditions within Varied Global Fixed Local experiment.

The paired t-test showed that the participants had a strong preference for following local cues, in conditions with ground-truth prior global probabilities, 60% ($p < .001$), 73% ($p < .001$), and 80% ($p < .001$) as shown in Figure 5.9.

5.4 Discussion

Participants had very accurate and precise local probability estimates (SEM around 1.5), and accurate, but less precise, global probability estimates (SEM around 2.0). This result is consistent with participants' encoding local and global probability separably.

We also successfully addressed the question of whether participants would rely solely on local cues when the global direction was uncertain. Surprisingly, as the logistic regressions revealed, objective choices indicate a reliance on subjective estimates of local cues reliability and global direction consistency, not reliance on the local cues alone. This strategy might

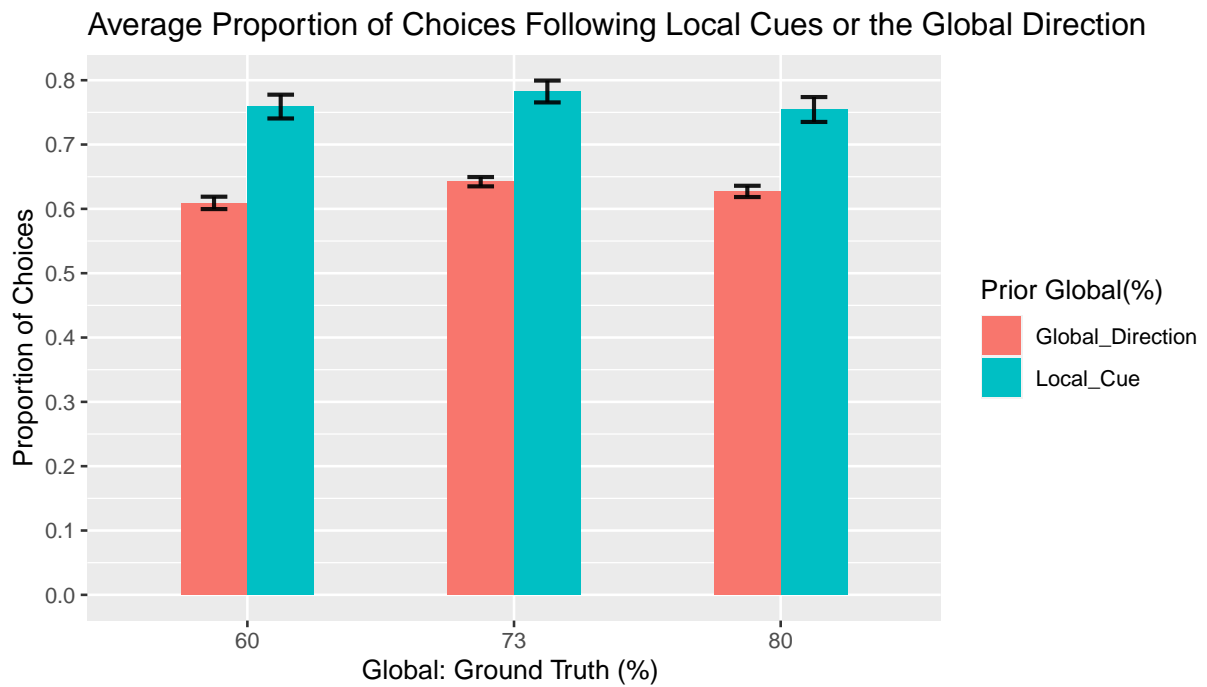


Figure 5.9: Participants showed a strong preference for following local cues in all three prior global conditions (60%, 73%, 80%). Error bars represent one standard error of the mean.

be still beneficial. As the results showed a higher than chance probability of choosing the shortest path across all conditions.

In the Varied Global Fixed Local task, participants always followed the local colored cue at above chance rates. They always relied less on the global direction than the local cues. It is reasonable to conjecture that participants would rely more heavily on local cues than what they would do in the Fixed Global Varied Local task since the global direction was unreliable here. Unexpectedly, their average proportion of choice following local cues was lower than that in the Fixed Global Varied Local task.

One reason for this pattern might be that animals including humans are intrinsically motivated to gain information (Gottlieb et al., 2013). And they have a preference on what kind of information they want to get. People's preferences might be increasingly related to the reward value, or they might ask for information when there is high uncertainty (Gottlieb et al., 2013). Since in the Fixed Global Varied Local task, the global direction was 100% reliable and local cues were usually uncertain, people might pay more attention to the local cues and follow it more to collect information. However, in Varied Global Fixed Local task, people had both unreliable local cues and the global direction, they would split their attention into both sources and collect information, so their average proportion of choice following local cues was lower.

Overall, these results confirm that there are different mental representations for local probability and global probability. People weigh them differently when making choices and those weights reflect their subjective estimates which show clear correlations to environmental conditions.

Chapter 6

General Discussion

The goal of this research was to provide insight into how the brain builds mental representations of local and global uncertainty and uses them for fast and adaptable decision-making. Subjective estimates (slider reports) demonstrated separate encoding for local probability and global probability. Objective behaviors (navigational choices) showed both local cues and the global direction influenced choices. Participants relied more on local cues than the global direction information especially when the local cues' reliability exceeds 80%.

Mental representations during navigation are organized at both local and global scales (Momennejad and Howard, 2018; Stachenfeld et al., 2017). Unlike the well-studied Successor Representation Model (Dayan, 1993; Stachenfeld et al., 2017), our maze navigation task included a dynamic component and cue uncertainty. These procedural extensions echo real-life events frequently occurring in daily life. Just as people have to navigate multiple environments, or the same environment under varying constraints (think navigating during a construction boom), our task changed the maze structure every trial. Yet, people were able to successfully navigate in this dynamic environment and learn which cues to strongly rely on. When either the local cues reliability was fixed (and global reliability varied) or whether the local cues reliability varied (and global reliability was fixed) my results demonstrated that participants performed much better than chance. The average proportion of choosing the shortest path direction was above 70% for all conditions. This is consistent with the idea that we can build powerful mental representations of dynamic noisy inputs and use those representations to make effective decisions. The advantage of multiple scale mental representations, for example, local and global scales, may be understood in terms of usage of memory. Multiple scale representations reduce interference of recalling different memories simultaneously (O'Reilly and Rudy, 2001; Santoro et al., 2016).

My finding of humans' flexibility in extracting the information provided by stored mental representations across the hippocampus and pre-frontal cortex axis is in agreement with hippocampus research. The hippocampus has a close interaction with pre-frontal cortex and related regions, Bostock et al. (1991) and Leutgeb et al. (2004) reported that the same population of neurons in the hippocampus has the capacity to remap between orthogonal representations across behavioral contexts, for example the local and global mental representations. They worked just like the cache in a computer to help us store one mental representation temporarily as a working memory buffer for further computations and change to another mental representation according to the behavioral contexts. In the Varied Global Fixed Local task, participants needed to retain a working representation of their previously received local cues while also adjusting their estimates about global cues' reliability across trials. With varying behavioral contexts, this dynamic remapping system generates a multitude of stable cognitive spaces that span the decision-making hierarchy.

While I found evidence for mental model encoding at different levels, participants generally relied more on the local cues. There are multiple potential reasons for this reliance on local cues. One reason could be that participants were more confident about their local probability estimates, because of differences in the variances associated with their local and global estimates. The variance of their estimates of local probability was generally lower than their variance of estimates of global probability, which could have led to greater confidence. We measured the variance in terms of SEM which considered the number of participants, so although we have different numbers of participants in those two experiment conditions we could still compare their results. The SEM of local probability estimates is around 1.5, and the SEM of global probability estimates is around 2. The SEM of local probability is lower because participants received the feedback of local cues more immediately than the feedback about exit location. It requires a longer term of memory in order to calculate the global probability. Thus, it might be harder for participants to make accurate estimates of global probability. Also, participants received on average 20 instances of feedback for local cues in one maze but only one instance of feedback concerning exit location. The more exposure to local cues could potentially decrease the variance in estimates of local probability. Previous research has demonstrated that probabilistic information guides the computation of one's sense of confidence (Geurts et al., 2022).

Another reason why participants may have preferred the local cues was feedback proximity. The accuracy of the local cues was available at each decision-making point. The accuracy of the global direction was only confirmed at the end of navigating a maze. If participants treat successfully choosing the shortest path direction and finding the exit as rewards, they would receive the local reward on average every 1338.37 milliseconds whereas the global reward they received at each exit on average 29447.78 milliseconds. It has been

known for decades that participants generally prefer rewards now to rewards later. In other words, their preference is negatively related to the reward interval length (Mischel and Metzner, 1962).

However, the difference in the level of difficulty in estimating local and global probability could not entirely account for the pattern of results because even when participants are 100% certain about the global direction, they still relied more on uncertain local cues. To examine those two concerns, in the future study, researchers could only provide one feedback about how many times in total participants made correct movements to the shortest path at the same time when informing participants of finding the exit.

The preference for proximate reward might explain why humans are better at maze navigation tasks than most RL agents (Burda et al., 2018; Mnih et al., 2015; T. Zhang et al., 2020). Humans are more likely to split big tasks into smaller tasks and use intermediate rewards to refine their overall strategy. Even if the sub-tasks are not directly related to the general goal, humans could still benefit from using a divide and conquer strategy.

Despite the impressive results using RL in single tasks, RL agents are less efficient at multiple small tasks (Vithayathil Varghese and Mahmoud, 2020). When Key-to-Door tasks are incorporated into maze navigation, RL performance declines significantly as the number of keys it needs to pick up increases (T. Zhang et al., 2020).

Many RL maze navigation settings including the one that the Successor Representation Model aimed to solve, only provide rewards if the agent successfully finds the exit and no information was provided during the exploration, and their goal is simply to find the exit (Chevalier-Boisvert et al., 2018; Yalnizyan-Carson and Richards, 2022; Zhai et al., 2022). Compared to them, our task also had participants estimate the local probability and global probability. Estimating local probability would encourage orienting to local cues, and this sub-task might create a circumstance with more frequent reward events. Based on the current training method of RL, we proposed that providing intermediate sub-tasks to RL agents will increase their performance.

One limitation of these experiments is that the range of global probability assessed (63% to 80%) was not very large. Thus, participants' subjective estimates of global probability may not have been sufficiently salient. Extending the range of global probability probed (50% to 100%) should be explored in additional research. Additionally, if there were more intervals and a larger range of global probability tested, we could make a more direct assessment of the relative potency of local and global cues. Another direction to be explored in future research is the dependence of our findings on maze size. We used 11*11 cells mazes to make it possible for participants to solve enough mazes in one experimental session to allow for meaningful statistical comparisons, but humans navigate environments

that are much larger physically and take much longer. People might integrate information differently depending on maze geometry.

In all our experiments one cue, either local or global, was fairly reliable. I tested with local cues that were at minimum 50% reliable, but only when the exit location was 100% predictable. I also tested exit location reliability ranging from 63% to 80%, while the local cues were fixed at 80% reliability. However, it remains unknown how preference for local and global signals might vary when both cues have poor reliability. Starting with a lower local probability (e.g below 80%) and ramping up the global direction predictability might elucidate whether preferences are blends or whether there is a discrete switch in cue preference after passing a cue reliability difference threshold.

The findings of this study have broad relevance. We seek probability distributions for our environments that minimize the relative entropy (KL-divergence) representing an error of deviation from the exact Bayes' posterior (Jirsa and Sheheitli, 2022). However, we still do not understand where and how these distinct local and global probability distribution would combine to yield a single decision as required by a Bayesian Inference approach. What are the weights we give local probability and global probability during computation? Where neurally are such assignments made and integrated? Is this process generic for all situations of uncertainty or are they bound to the modalities involved? Functional imaging recordings while participants performed tasks like my maze task under varying conditions of uncertainty and where the relevant modality of uncertainty varied could shed light on the neural mechanisms involved. It might also be useful to consider electroencephalogram (EEG) and event-related potential (ERP) as imaging techniques because they could help to determine which neuron activation comes first, hippocampal or frontal.

In conclusion, we found that people are remarkably accurate at estimating and updating local probability and global probability over time in a series of dynamically changing mazes. Especially, people tend to follow local cues more even when global cues are more reliable. This works potentially sets the stage for a series of future studies probing mental representations of uncertainty about events on a local to global scale as well as human flexibility in a noisy environment.

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