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Distribution planning with random demand and recourse in a transshipment network

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ABSTRACT

In this paper we consider a distribution planning problem in a transshipment network under stochastic customer demand, to account for uncertainty faced in real-life applications when planning distribution activities. To date, considerations of randomness in distribution planning networks with intermediate facilities have received very little attention in the literature. We address this gap by modeling uncertainty in a distribution network with an intermediate facility, and providing insight on the benefit of accounting for randomness at the distribution planning phase. The problem is studied from the perspective of a third-party logistics provider (3PL) that is outsourced to handle the logistics needs of its customers; the 3PL uses a consolidation center to achieve transportation cost savings. We formulate a two-stage stochastic programming model with recourse that aims to minimize the sum of transportation cost, expected inventory holding cost and expected outsourcing cost. The recourse variables ensure that the problem is feasible regardless of the realization of demand, by allowing the option of using a spot market carrier if demand exceeds capacity. We propose a flow-based formulation with a nonlinear holding cost component in the objective function. We then develop an alternative linear path-based formulation that models the movement of freight in the network as path variables. We apply Sample Average Approximation (SAA) to solve the problem, and show that it results in reasonable optimality gaps for problem instances of different sizes. We conduct extensive testing to evaluate the benefits of our proposed stochastic model compared to its deterministic counterpart. Our computational experiments provide managerial insight into the robustness and cost-efficiency of the distribution plans of our proposed stochastic model, and the conditions under which our model achieves significant distribution cost savings.

1. Introduction

In many supply chain networks, third party logistics providers (3PLs) are employed to handle the distribution needs within the supply chain. The 3PL faces the challenging task of coordinating these distribution activities between suppliers and customers, possibly through the use of intermediate facilities, so as to create a lean cost-efficient supply chain, while ensuring timely customer deliveries. Third party logistics is a fast growing market; in 2016, it had an estimated worldwide market size of 802.2 billion US dollars, 38% of which is in the Asia Pacific region, 25% in North America, 21.5% in Europe (Langley, 2017). With this growth comes increased competition which further necessitates that the 3PL create leaner logistics solutions, in order to survive in a growingly contested market. In recent years, there has been an increasing trend in businesses outsourcing their transportation needs to 3PL's to focus on their core business competencies. The various players within the supply chain expect the 3PL to accommodate shipping quantities that may fluctuate depending on customer demand. This creates a compelling need for a 3PL to operate more efficiently, with imperfect information, to secure profitability, while providing competitive shipping rates for clients and building customer loyalty.

Many variations of freight distribution coordination with intermediate facilities have been investigated by researchers. However, very limited work addresses such problems with stochastic customer demand. In their literature surveys, both SteadieSeifi et al. (2014) and Guastaroba et al. (2016) acknowledge the need for more research that considers stochasticity in freight transportation planning. In addition, from an industrially-practical point of view, when customer demand arrives in real-time, accounting for demand variation at the distribution planning phase will enable the creation of efficient distribution plans that more accurately anticipate actual distribution costs.

We study the problem of a 3PL that is coordinating transportation

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needs between suppliers and customers when customer demand is stochastic. That coordination considers the release time of shipments from suppliers, the delivery due dates of customers, the different transportation options that could be used, as well as the holding cost at the consolidation center. In our problem setting, the 3PL does not operate its own fleet, but rather chooses the best available multi-modal transportation services for its clients. The 3PL determines a suitable shipping schedule, arranging for the pickup at suppliers when shipments are ready, i.e., after their release time.

For a given supplier, orders of multiple customers are consolidated in fewer high-volume loads and sent to the consolidation center, operated by the 3PL, through one or more transportation options. A transportation option between a supplier and the consolidation center is referred to as an *inbound transportation option*. We define an inbound transportation option as a combination of a transportation mode (or multiple modes), a capacity, an arrival time at the consolidation center, and a cost associated with the service. At the consolidation center, the 3PL combines orders from multiple suppliers to the same customer and delivers them through one or more transportation options, such that customer delivery deadlines are satisfied. A transportation option from the consolidation center to a customer is referred to as an *outbound transportation option*, and is defined as a combination of transportation mode, capacity, dispatch time from the consolidation center, and cost.

Most transportation service prices are not simply based on the weight and volume of the shipment. Prices also depend on when the service is taking place (e.g., peak seasons, holidays), the mode of transportation used, as well as other factors such as the particular route taken. Thus, inbound and outbound transportation cost may not be monotonically increasing or decreasing with lead time. We adopt this general definition, where inbound and outbound transportation options have nonlinear discrete cost functions.

The distribution service provided by the 3PL is for a predefined number of periods, rather than a one-time service. However, the choice of transportation options, for inbound and outbound shipments, is contractual, and is kept for the full length of the planning horizon. Once customer demand is known, if the actual demand cannot be fulfilled with the particular choice of inbound and outbound options made at the beginning of the planning horizon, a spot market carrier may be used to ship the additional demand at a higher cost. The goal of the 3PL is to select transportation options that minimize the expected transportation cost of the network plus the expected holding cost at the consolidation center, while ensuring that customer demand is fulfilled by the due date.

One motivating example of the problem comes from a 3PL that manages the distribution planning of an e-retailer. The latter operates multiple distribution centers and orders its products from a number of global suppliers. Each supplier provides different types of products that the e-retailer sells. To manage their inventory, each distribution center periodically places a replenishment order, which varies depending on end-customer's demand. In fulfilling those orders, the 3PL uses a consolidation center to save on transportation cost between suppliers and the e-retailer's distribution centers. The 3PL needs to choose a minimal-cost transportation plan with specific transportation modes, capacity and arrival/dispatch times at the consolidation center, for inbound and outbound shipments, respectively, to carry on the regular transportation needs between suppliers and distribution centers. For simplicity and to make our problem applicable to other application areas, we will refer to the third-tier of the supply chain (which are the distribution centers in this example) simply as customers.

The main contributions of this paper are threefold. Firstly, we address the need for considering randomness in freight distribution planning with intermediate facilities by proposing a two-stage stochastic programming model that accounts for stochastic customer demand at the planning phase. Our proposed model addresses tactical decisions, i.e., the choice of transportation options, and minimizes the sum of transportation-choice costs plus expected operational costs. Secondly, modeling this problem from the perspective of a third party logistics provider, even without demand uncertainty, has received very little attention. This paper aims to fill that gap. Thirdly, we conduct a thorough analysis on the benefits and limitations of our proposed model and present managerial insights on the conditions under which our model achieves significant distribution cost savings.

The rest of this paper is arranged as follows. In Section 2 we provide a review of relevant literature. In Section 3, we detail the problem setting and assumptions, and formulate the proposed stochastic model. We discuss the solution methodology used in solving the problem in Section 4. We then discuss our numerical testing and analysis, and compare the performance of our stochastic model to its deterministic counterpart in Section 5. Finally, we outline some concluding remarks and future directions in Section 6.

2. Literature review

References on freight consolidation have considered distinct goals and the viewpoints of different decision makers. Relevant literature is in three main categories: freight/shipment consolidation, freight transportation with intermediate facilities, and freight forwarder/3PL operations. We also discuss important publications that explicitly incorporate stochasticity.

In the past three decades, considerable research has been done on shipment consolidation (SCL). This classical problem mainly aims to find the optimal dispatch policy, from the perspective of a shipper, that determines for how long to consolidate shipments, and when to dispatch the aggregate load. Early research laid the foundation of this topic (Masters, 1980). Later, Higginson and Bookbinder (1994), Çetinkaya and Bookbinder (2003), Mutlu et al. (2010), and Bookbinder et al. (2011) used simulation and stochastic modeling to compare different dispatch policies and determine optimal ones under various settings and considering additional costs, such as inventory cost.

The preceding references explicitly analyze SCL policies, but other researchers have integrated those decisions within wider-scope supply chain network decisions. Freight transportation problems with intermediate facilities were reviewed by Guastaroba et al. (2016). The authors suggested three classes of such problems, the second of which: intermediate facilities in transshipment problems, is the closest to our problem setting, since the consolidation center acts as a transshipment node. Our problem extends the cited references in Guastaroba et al. (2016) by considering a stochastic model rather than a deterministic one. Another article by SteadieSeifi et al. (2014) surveys the literature on multi-modal freight transportation planning. Our proposed model fits under their category of tactical planning, i.e., choice of transport services, associated modes and capacities, and allocating customer orders to the services selected.

Croxton et al. (2003), Berman and Wang (2006) and Song et al. (2008) each studied distribution coordination with consolidation center(s) or merge-in-transit centers. Each paper developed different models to determine the best distribution plan that minimizes transportation plus inventory costs. Croxton et al. (2003) assume that suppliers provide components, which are shipped to a merge-in-transit center, assembled, and dispatched to the customer as a finished product. Song et al. (2008) suppose that suppliers also furnish components, but the customer assembles the product after receiving all parts as one consolidated load. Berman and Wang (2006) assume that each supplier provides a number of products, which are sent to customers via a cross-dock.

Both Croxton et al. (2003) and Berman and Wang (2006) assume that freight is moved via a pre-determined transportation arrangement, so the choice of carriers is not studied. Song et al. (2008), however, assume that the decision maker (a 3PL) selects from a large number of possible carriers, each with a given dispatch time and cost. We adopt this latter assumption: a typical 3PL chooses the modes and capacities from a number of potential transportation service providers.

The three aforementioned papers had nonlinear cost functions. Transportation costs follow a nonlinear discrete cost function in Song et al. (2008). Similar to those authors, we adopt a general cost function that can capture the various factors affecting transportation cost.

In the context of freight forwarding, most publications assume the relevant company operates its own fleet, proposing different models that extend the classical vehicle routing problem, or the pickup and delivery problem with time windows (Krajewska and Kopfer, 2009; Wang et al., 2014; Bock, 2010). Models that study 3PL coordination issues are closely related to freight forwarders problems; transportation in supply chains is typically outsourced to both 3PLs and freight forwarders. However, 3PLs may coordinate additional distribution activities, like warehousing and managing inventory. Song et al. (2008) study the scheduling problem faced by a 3PL who is arranging shipments between suppliers and customers in an international distribution network through the use of a consolidation center. Cai et al. (2013) analyze the outsourcing of fresh products to a 3PL, where the products could deteriorate during the transportation process, and derive the optimal decisions for supply chain members. Qin et al. (2014) consider the freight consolidation and containerization problem from the perspective of a 3PL that wants to determine the optimal allocation of shipments to international shipping containers and the routing of those containers.

Of some relevance to our work is the extensive family of problems on service network design (SND), as surveyed by Crainic (2000) and Wieberneit (2008). SND decisions relate to the network structure, i.e., selection of routes where service is conducted, and also the movement of freight on the network. Our problem, however, assumes an already-established network, where only the modal choice and scheduling of the freight movements, on predefined routes, is of interest. Furthermore, SND problems often take the carrier's perspective, whereas our view is that of a 3PL that also manages a consolidation center, hence inventory holding cost need be included. Guastaroba et al. (2016) argue that most papers on SND with intermediate facilities concern applications at a national or regional level with a single transport mode. Contrarily, our problem is applicable to global distribution networks, with multiple transportation modes.

All previously reviewed papers assume deterministic customer demand. Limited work addresses similar stochastic demand problems. Guastaroba et al. (2016) recognize that intermediate facilities in stochastic transshipment problems have received no attention, and highlight this for future research. To the best of our knowledge, the only related papers that consider randomness, but without transshipment, are Kılıç and Tuzkaya (2015) and some stochastic service network design papers, surveyed below.

Kılıç and Tuzkaya (2015) investigate a two-echelon distribution network design problem between distribution centers and wholesalers when demand is uncertain. The authors use two-stage stochastic mixed integer programming, where the first stage selects location of distribution centers; the second stage addresses transportation and inventory decisions, as well as unmet demand. In contrast to that article, our work addresses transportation needs in an already-established network.

Several papers have examined the benefit of considering demand randomness in designing service networks. Lium et al. (2009) study demand stochasticity in SND by formulating a two-stage stochastic programming model that chooses the routes and frequency of service in the first stage, and decides on the allocation of commodities to established routes or outsourcing a portion of demand in the second stage. Bai et al. (2014) later extend this model to allow possible rerouting of vehicles, to reduce the amount of outsourcing needed when demand is high. Both our research and Bai et al. (2014) consider outsourcing demand when it exceeds available first-stage capacity. However, since the 3PL in our case does not operate its own vehicle fleet, rerouting is not an option. Moreover, we examine the trade-off between choice of first-stage transportation options and inventory holding cost, a dimension not studied in stochastic network design problems.

Other publications (Hoff et al., 2010; Crainic et al., 2014) focused on creating efficient solution methodologies for solving realistic instances of stochastic SND problems. Furthermore, more recent work by Wang et al. (2016) examined the value of deterministic solutions, in terms of their quality and upgradeability, in a stochastic environment. Another publication by Wang and Wallace (2016) studied the effect of considering spot markets at the design stage of creating a transportation plan under uncertain demand. The article showed that in most situations, accounting for spot markets when designing a service network reduces total cost.

In the following section, we describe our problem setting, assumptions and formulation.

3. Problem description and formulation

We propose a two-stage stochastic programming model with recourse, to formulate the Stochastic Distribution Planning with Consolidation (SDPC) problem faced by a 3PL that is coordinating shipments between suppliers, $i \in I$, and customers, $j \in J$, whose demands, d_{ij}, are uncertain. Given customers' demand distributions, delivery due dates and supplier release times, the 3PL needs to select the transportation options for shipments inbound to and outbound from the consolidation center, at the beginning of the planning horizon. Similar to Song et al. (2008), we adopt general, possibly nonlinear, cost functions for inbound and outbound transportation options, $f(x_{iq})$ and $g(y_{jl})$, respectively. Note that these cost functions may differ for distinct inbound and outbound transportation options, $q \in Q_i$ and $l \in L_j$, respectively. Exploiting a general cost function enables consideration of different transportation modes or multi-modal transportation options with varying capacity levels, with those differences reflected in the cost structure.

In our problem setting, the chosen transportation options and their associated capacities are fixed for the whole planning horizon. Once demand is realized, if total demand from a supplier (to a customer) exceeds the capacity of inbound (outbound) transportation option(s) reserved for that supplier (customer), a spot market carrier is used. A spot market may also be used if inventory cost savings outweigh the increased spot market cost. We note that the preceding additional cost of utilizing a spot market carrier is incurred by the 3PL. This cost is composed of two parts, (a) the estimated spot market per unit price premium between the origin and destination, and (b) a disutility factor that represents the 3PL's disutility to transport shipments through a spot market carrier. Such a disutility factor can be set to zero, if shipping price is the only consideration for the 3PL to make shipments through a spot market carrier. However, the 3PL may not favor shipping through a spot market carrier, e.g., due to unpredictable price fluctuations in that market, or limited availability of spot market carriers in peak seasons. Thus, the disutility factor is meant to adjust the level of favorability in using a spot market carrier by the specific 3PL.

The first stage of the two-stage stochastic program tackles the selection of inbound and outbound transportation options to be reserved for the duration of the planning period. The second stage allocates orders to chosen first-stage options, or to spot market carriers. The two stages are optimized simultaneously so as to minimize the sum of transportation cost, expected inventory holding cost and expected spot market carrier shipping cost.

Let x_{iq} be a binary variable indicating whether inbound transportation option $q \in Q_i$ is reserved for supplier $i \in I$. Similarly, the binary variable y_{jl} shows whether outbound transportation option $l \in L_j$ is reserved for customer $j \in J$. Let S be the set of possible scenarios or demand realizations. u^s_{ijq} and w^s_{ijl} are binary variables that express whether shipment (i,j) is shipped through reserved inbound and outbound transportation options q and l, respectively, in scenario $s \in S$. Similarly, μ^s_{ij} and λ^s_{ij} are binary variables that indicate whether shipment (i,j) is

Table 1
List of notations used in SDPC-FF.

Notation	Meaning
Parameters	
I	The set of suppliers
J	The set of customers
S	The set of demand realizations or scenarios
$I(j)$	The set of suppliers from which customer j orders
$J(i)$	The set of customers that order from supplier i
Q_i	The set of inbound transportation options for supplier i
L_j	The set of outbound transportation options for customer j
C_{iq}	The capacity of inbound transportation option q of supplier i
C_{jl}	The capacity of outbound transportation option l of customer j
τ_{iq}	The arrival time of inbound transportation option q of supplier i at the CC
X_i	The set of all arrival times of inbound transportation options of supplier $i \in I$
τ_{jl}	The dispatch time of outbound transportation option l of customer j from the CC
Y_j	The set of all dispatch times of outbound transportation options of customer $j \in J$
$\overline{\tau}_{iq}$	The latest arrival time of all inbound transportation options $q \in Q_i$ of a given supplier i, i.e., $\max_{q \in Q_i}\{\tau_{iq}\}$
d_{ij}^s	The demand of customer j from supplier i in scenario s
h_i	The holding cost of supplier i's shipment at the CC
π_i	Inbound spot market carrier cost per unit from supplier i to CC
π_j	Outbound spot market carrier cost per unit from CC to customer j
$f(x_{iq})$	Inbound transportation cost of transportation option q for supplier i
$g(y_{jl})$	Outbound transportation cost of transportation option l for customer j
Decision Variables	
x_{iq}	Binary variable, equals 1 if inbound transportation option q for supplier i is reserved
y_{jl}	Binary variable, equals 1 if outbound transportation option l for customer j is reserved
u_{ijq}^s	Binary variable, equals 1 if shipment (i,j) is transported from supplier i to CC through option q in scenario s
w_{ijl}^s	Binary variable, equals 1 if shipment (i,j) is transported from CC to customer j through option l in scenario s
μ_{ij}^s	Binary variable, equals 1 if shipment (i,j) is transported from supplier i to CC by a spot market carrier, in scenario s
λ_{ij}^s	Binary variable, equals 1 if shipment (i,j) is transported from CC to customer j by a spot market carrier, in scenario s

moved via an inbound and outbound spot market carrier, respectively, in scenario $s \in S$. Table 1 outlines the complete list of notation. We formulate the problem as shown below and refer to the model as the *stochastic distribution planning with consolidation - flow based formulation* (SDPC-FF).

Note that we assume that if shipment (i,j) is transported by a spot market carrier, no holding cost is incurred at the consolidation center, since the shipping time will be chosen such that the interval the load is held at the consolidation center is negligible. Therefore, the holding cost is incurred only if a shipment (i,j) is shipped through reserved inbound and outbound transportation options, i.e., for a specific shipment (i,j), if both variables u_{ijq}^s and w_{ijl}^s have a value of 1. This requirement results in the nonlinearity of the holding cost component of objective function (4).

$$[\textbf{SDPC}-\textbf{FF}] \quad \min \sum_{i\in I}\sum_{q\in Q} f\left(x_{iq}\right) + \sum_{j\in J}\sum_{l\in L} g\left(y_{jl}\right) + \zeta\left(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}}\right) \tag{1}$$

$$\text{subject to } x_{iq}, y_{jl} \in \{0,1\}, \quad i\in I, q\in Q, j\in J, l\in L \tag{2}$$

where

$$\zeta\left(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}}\right) = \mathbb{E}_{\xi}\zeta_s\left(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}},\boldsymbol{\xi}\right) \tag{3}$$

$$\begin{aligned}\zeta_s\left(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}},\boldsymbol{d}^s\right) = \min\Bigg\{ & \sum_{i\in I}\sum_{j\in J(i)} d_{ij}^s h_i\Bigg(\sum_{l\in L}\tau_{jl}w_{ijl}^s\left(1-\mu_{ij}^s\right) \\ & -\sum_{q\in Q}\tau_{iq}u_{ijq}^s\left(1-\lambda_{ij}^s\right)\Bigg) + \sum_{i\in I}\sum_{j\in J(i)} d_{ij}^s\left(\pi_i\mu_{ij}^s + \pi_j\lambda_{ij}^s\right)\Bigg\}\end{aligned} \tag{4}$$

subject to

$$\sum_{q\in Q} u_{ijq}^s + \mu_{ij}^s = 1, \qquad i\in I, j\in J(i) \tag{5}$$

$$\sum_{l\in L} w_{ijl}^s + \lambda_{ij}^s = 1, \qquad \in J, i\in I(j) \tag{6}$$

$$\sum_{q\in Q}\tau_{iq}u_{ijq}^s \le \sum_{l\in L}\tau_{jl}w_{ijl}^s + \overline{\tau}_{iq}\lambda_{ij}^s, \qquad j\in J, i\in I(j) \tag{7}$$

$$\sum_{j\in J(i)} d_{ij}^s u_{ijq}^s \le C_{iq}x_{iq}, i\in I, q\in Q \tag{8}$$

$$\sum_{i\in I(j)} d_{ij}^s w_{ijl}^s \le C_{jl}y_{jl}, j\in J, l\in L \tag{9}$$

$$\begin{aligned} & u_{ijq}^s, \mu_{ij}^s \in \{0,1\}, i\in I, q\in Q, j\in J(i) \\ & w_{ijl}^s, \lambda_{ij}^s \in \{0,1\}, j\in J, i\in I(j), l\in L \end{aligned} \tag{10}$$

The objective function (1) minimizes the total inbound and outbound transportation cost, $f(x_{iq})$ and $g(y_{jl})$, respectively, plus the recourse function: the expected value of the second-stage problem. For a particular demand realization $\boldsymbol{d}^s$ of the random vector $\boldsymbol{\xi}$, objective (4) minimizes the sum of the holding cost and the cost of shipping through a spot market carrier. Constraints (5) and (6) guarantee that the model allocates each order to exactly one inbound shipment, and exactly one outbound shipment, respectively, whether the shipment is through a reserved first-stage transportation option or a spot market carrier. Constraints (7) make sure that the outbound dispatch time of an order is greater than its inbound arrival time. Constraints (8) and (9) ensure that the total demand allocated to a transportation option, for a given supplier and customer, respectively, does not exceed the capacity of that option. Finally, Constraints (10) impose the binary requirement on all variables.

In order to avoid the nonlinearity in objective function (4), we propose an equivalent linear path-based formulation that replaces the flow variables with path variables. We refer to this formulation as the *stochastic distribution planning with consolidation - path based formulation* (SDPC - PF), and we detail it next.

Linear path formulation

We define a set of feasible paths for shipment (i,j) from supplier i to customer j through the consolidation center as P_{ij}, where a feasible path $p_{ijql} \in P_{ij}$ represents a pair of inbound and outbound transportation options (q,l) that is feasible with regard to arrival/dispatch times for shipment (i,j). In other words, shipment (i,j) has a feasible path p_{ijql} if inbound option q arrives at the consolidation center before outbound option l is dispatched. Shipment (i,j) also has feasible paths through each of its inbound options q and through an outbound spot market, and similarly, there are feasible paths along each outbound option l and an inbound spot market. For a shipment (i,j), inbound and outbound shipping via a spot market carrier is also a feasible path. For brevity, we refer to a feasible path as $p \in P_{ij}$. We define the following additional notation. a_{iqp} and b_{jlp} are binary parameters indicating if options q and l are on path $p \in P_{ij}$. c_{ijp}^s is the cost of sending shipment (i,j) on path p in scenario s, where

$$c_{ijp}^{s}=\begin{cases} d_{ij}^{s}h_i\left(\tau_{jl}-\tau_{iq}\right), & \text{if no spot market carrier is used on path } p\in P_{ij} \\ \pi_i d_{ij}^{s}, & \text{if a spot market carrier is used only for inbound shipping on path } p\in P_{ij} \\ \pi_j d_{ij}^{s}, & \text{if a spot market carrier is used only for outbound shipping on path } p\in P_{ij} \\ (\pi_i+\pi_j)d_{ij}^{s}, & \text{if a spot market carrier is used for both inbound and outbound shipping on path } p\in P_{ij} \end{cases}$$

We also define decision variables β_{ijp}^{s} as binary variables that indicate whether or not shipment (i,j) traverses path $p\in P_{ij}$ in scenario s. The path based formulation, SDPC-PF, can then be expressed as follows:

$$[\mathbf{SDPC-PF}]\ \min\sum_{i\in I}\sum_{q\in Q}f(x_{iq})+\sum_{j\in J}\sum_{l\in L}g(y_{jl})+\zeta(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}}) \tag{11}$$

$$\text{subject to } x_{iq},y_{jl}\in\{0,1\},\quad i\in I,q\in Q,j\in J,l\in L \tag{12}$$

where

$$\zeta(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}})=\mathbb{E}_{\xi}\zeta_s(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}},\boldsymbol{\xi}) \tag{13}$$

$$\zeta_s(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}},\boldsymbol{d^s})=\min\sum_{i\in I}\sum_{j\in J(i)}\sum_{p\in P_{ij}}c_{ijp}^{s}\beta_{ijp}^{s} \tag{14}$$

subject to

$$\sum_{p\in P_{ij}}\beta_{ijp}^{s}=1\qquad \forall i\in I,j\in J(i) \tag{15}$$

$$\sum_{p\in P_{ij}}a_{iqp}\beta_{ijp}^{s}\le x_{iq}\qquad \forall i\in I,j\in J(i),q\in Q \tag{16}$$

$$\sum_{p\in P_{ij}}b_{jlp}\beta_{ijp}^{s}\le y_{jl}\qquad \forall j\in J,i\in I(j),l\in L \tag{17}$$

$$\sum_{p\in P_{ij}}\sum_{j\in J(i)}a_{iqp}d_{ij}^{s}\beta_{ijp}^{s}\le C_{iq}\qquad \forall i\in I,q\in Q \tag{18}$$

$$\sum_{p\in P_{ij}}\sum_{i\in I(j)}b_{jlp}d_{ij}^{s}\beta_{ijp}^{s}\le C_{jl}\qquad \forall j\in J,l\in L \tag{19}$$

$$\beta_{ijp}^{s}\in\{0,1\},\qquad i\in I,j\in J(i),p\in P_{ij} \tag{20}$$

The objective function (11) minimizes the total transportation cost plus the expected value of the second-stage problem. For a specific realization $\boldsymbol{d^s}$ of the random vector $\boldsymbol{\xi}$, objective (14) minimizes the total allocation cost of shipments to feasible paths. Constraints (15) ensure that exactly one path is chosen for each shipment (i,j) in the network. Constraints (16) and (17) guarantee that shipment (i,j) traverses a path only if both the inbound transportation option of supplier i (x_{iq}) and the outbound transportation option of customer j (y_{jl}) have a value of 1. Constraints (18) and (19) require that the total demand that traverses a given path does not exceed the capacity of the inbound or outbound transportation options of that path. Finally, Constraints (20) impose the binary requirement on the variables.

Note that Constraints (18) and (19) may be modified by multiplying their left-hand-side by x_{iq} and y_{jl}, respectively. However, empirical testing showed that such modification caused a slight increase in computational time for some instances. Therefore, we refrain from using this adjustment in our computational testing.

We use Sample Average Approximation (SAA) to solve SDPC-PF. The main advantage of this technique is that it provides a statistical estimate of the optimality gap of the *true* stochastic optimization problem, which is discretized by a very large scenario tree. In contrast, solving the problem directly with a commercial solver with 50 or more scenarios is computationally intensive for reasonable size problems, as it results in a large number of path variables. Solving the problem directly also gives little information on the quality of the solutions obtained, relative to the *true* stochastic problem. We therefore use SAA to measure the quality of the resulting distribution plans by utilizing the optimality gap estimate as a quality metric, and also to keep the problem size manageable and obtain good solutions in a reasonable amount of time.

4. Solution methodology: Sample Average Approximation

Sample Average Approximation is a Monte Carlo simulation-based solution technique for solving two-stage or multi-stage stochastic optimization problems (Mak et al., 1999; Kleywegt et al., 2002). In this technique, the objective function of the stochastic model is approximated by a sample average estimate obtained from a random finite set of samples. The problem is then solved, with the approximate objective function and a set of scenarios S^N, as a deterministic optimization problem either directly or using other solution techniques. The process is repeated M times with different samples, and each time results in a candidate solution. To assess the quality of the candidate solutions, statistical estimates of their optimality gaps can be obtained.

SAA solves the *true* problem with a reasonable level of accuracy provided some conditions are met (Kleywegt et al., 2002; Shapiro and Philpott, 2007). Those conditions, and justifications on how SDPC-PF meets them, are as follows:

i. It is possible to generate a sample realization of the random vector ζ. For our proposed problem, this can be done by sampling from each (i,j) demand distribution.
ii. The SAA problem can be solved efficiently with a moderate sample size. We will show in Section 5 that we can solve SDPC-PF in a reasonable amount of time, for most test instances, with a sample size of $N=10$.
iii. The function $\zeta_s(\boldsymbol{x_{iq}},\boldsymbol{y_{jl}},\boldsymbol{d^s})$ can be easily computed for given $\boldsymbol{x_{iq}},\boldsymbol{y_{jl}}$ and $\boldsymbol{d^s}$. That is, for a given first stage solution and a given realization of demand, the optimal objective function (14) can be easily evaluated by solving the model in Equations (14)–(20).
iv. The true problem has relatively complete recourse, i.e., any solution to the first stage problem is feasible to the second stage because it can be *corrected*. In SDPC-PF, this is done through the assumption that a spot market carrier is always available when demand cannot be fulfilled with reserved first stage variables. Thus, any choice of transportation plan would result in a feasible second stage problem, because shipping via the spot market is a feasible path for all shipments (i,j).

We now detail how SAA is used to solve SDPC-PF. Applying SAA, the objective function of the second stage problem of SDPC-PF is approximated as:

$$\zeta(\boldsymbol{x_{iq}}, \boldsymbol{y_{jl}}) = \frac{1}{N}\sum_{s\in S^N}\sum_{i\in I}\sum_{j\in J(i)}\sum_{p\in P_{ij}} c^s_{ijp}\beta^s_{ijp} \tag{21}$$

where S^N is the set of scenarios of size N sampled in a given SAA problem. The full SAA model is expressed as follows:

$$[\textbf{SAAModel}] \ \min \sum_{i\in I}\sum_{q\in Q} f(x_{iq}) + \sum_{j\in J}\sum_{l\in L} g(y_{jl}) + \zeta(\boldsymbol{x_{iq}}, \boldsymbol{y_{jl}}) \tag{22}$$

subject to

$$\text{constraints} \ \ (15)-(20), (21) \qquad \forall s \in S^N$$

To assess the quality of the SAA solution, statistical estimates of lower and upper bounds on the objective function value of the original stochastic problem may be obtained, as well as estimates of the variances of these bounds. This is achieved by solving the SAA model M times, where each time a set of independent samples of size N is generated. This results in M candidate solutions, $\boldsymbol{x}^1,\ldots,\boldsymbol{x}^M$, where $\boldsymbol{x}^m$ is the vector notation of the solution of the first stage variables, $\bar{\boldsymbol{x}}_{iq}, \bar{\boldsymbol{y}}_{jl}$ for candidate solution $m \in \{1,\ldots,M\}$, with objective function values $\eta^1,\ldots,\eta^M$.

To estimate the lower bound of the true objective function value, we first compute the mean ($\bar{\eta}$) and the variance ($\hat{\sigma}^2_{N,M}$) of the objective function values $\eta^1,\ldots,\eta^M$ as:

$$\bar{\eta} = \frac{1}{M}\sum_{m=1}\eta^m \tag{23}$$

$$\hat{\sigma}^2_{N,M} = \frac{1}{M(M-1)}\sum_{m=1}(\eta^m - \bar{\eta})^2 \tag{24}$$

The lower bound is expressed as:

$$LB = \bar{\eta} - t_{\alpha,v}\hat{\sigma}_{N,M} \tag{25}$$

where $t_{\alpha,v}$ is the α-critical value of the t-distribution with v degrees of freedom, $v = M-1$.

Kleywegt et al. (2002) note there is a trade-off between SAA solution quality and computational requirements as the size N changes. With a larger N, the objective function value of the SAA problem gets closer to the true objective value, but the computational requirement increases significantly. Similarly, as the number of replications M increases, a better lower bound can be obtained with a smaller standard deviation $\hat{\sigma}^2_{N,M}$. However, the algorithm may become computationally inefficient. The exact values of N and M used in our computational testing are explained in Section 5.

The upper bound on the true objective function value of each candidate solution is obtained by evaluating the solution with a very large scenario tree of size N' that is assumed to represent the true distribution of demand. Since each scenario $s \in \{1,\ldots,N'\}$ is an i.i.d. random sample, the problem of evaluating a candidate solution decomposes into N' subproblems. The size of the scenario tree N' is much larger than the size of the scenario tree maintained in each SAA run, N. We denote the objective function value of a given subproblem s as $\varphi(\boldsymbol{x}^m, s)$, which is computed as shown in Equation (26). Note that because each subproblem is solved separately, N' can be very large without causing a significant computational burden. The estimate of the true objective value of the second stage problem, denoted as $\varphi(\boldsymbol{x}^m)$, is computed as shown in (27).

$$\varphi_s(\boldsymbol{x}^m, s) = \sum_{i\in I}\sum_{j\in J(i)}\sum_{p\in P_{ij}} c_{ijp}\beta_{ijp} \qquad \forall s \in \{1,\ldots,N'\} \tag{26}$$

$$\varphi(\boldsymbol{x}^m) = \frac{1}{N'}\sum_{s=1}\varphi_s(\boldsymbol{x}^m, s) \tag{27}$$

The value of the true objective function, $\bar{\eta}^m$, for candidate solution $\boldsymbol{x}^m$, and its variance, $\sigma^2_{N'}(\boldsymbol{x}^m)$, are computed as shown in Equations (28) and (29), respectively,

$$\bar{\eta}^m = \sum_{i\in I}\sum_{q\in Q} f(\bar{x}_{iq}) + \sum_{j\in J}\sum_{l\in L} g(\bar{y}_{jl}) + \varphi(\boldsymbol{x}^m), \tag{28}$$

$$\sigma^2_{N'}(\boldsymbol{x}^m) = \frac{1}{N'(N'-1)}\sum_{s=1}[\varphi_s(\boldsymbol{x}^m, s) - \varphi(\boldsymbol{x}^m)]^2. \tag{29}$$

Finally, the upper bound of a candidate solution, z^m_U is computed as:

$$\eta^m_U = \bar{\eta}^m + z_\alpha\sigma_{N'}(\boldsymbol{x}^m) \tag{30}$$

where z_α is the α-critical value of the standard normal distribution. The upper bound of the algorithm is the smallest η^m_U, $\forall m \in \{1,\ldots,M\}$, as shown in Equation (31). The final solution of SAA, $\boldsymbol{x}^*$, is the candidate solution that results in the smallest optimality gap ($\eta^m_U - \eta_L$) for all candidate solutions $m \in \{1,\ldots,M\}$, which corresponds to the solution with the smallest upper bound η^m_U, as shown in (32).

$$UB = \min_{m\in\{1,\ldots,M\}} \eta^m_U \tag{31}$$

$$\boldsymbol{x}^* = argmin_{m\in\{1,\ldots,M\}}(\eta^m_U) \tag{32}$$

5. Computational experiments and analysis

In this section, we conduct extensive computational testing to assess the effectiveness of SAA in solving the SDPC-PF and to evaluate the benefit of accounting for uncertainty in modeling SDPC. We solve problem instances of various sizes and different experimental settings using the SAA algorithm. We then compare the solution of SDPC to its deterministic counterpart with average demand values. We refer to the deterministic problem as the *deterministic distribution planning with consolidation* (DDPC) and we describe its formulation in Appendix B. We compare the stochastic and deterministic solutions and objective values to evaluate the benefit of accounting for uncertainty, through computing the *value of stochastic solution*.

We briefly describe the data generation method used and discuss the different data sets used in testing and analysis, and how they compare and contrast, in Section 5.1. Detailed explanation of data generation is provided in Appendix A. We further elaborate on our computational testing by detailing the setting used for the SAA algorithm as well as some key performance measures in Section 5.2. Finally, we report and discuss the results of our computational testing in Section 5.3. The SAA algorithm was implemented in Python 2.7 on an Intel(R) Core(TM) i7 CPU, 2.90 GHz, 16.00 GB of RAM. The optimization problems were solved by CPLEX 12.8.

5.1. Data generation and data sets

We generate the parameters of the test instances partly following the method outlined by Song et al. (2008), since their proposed model also

Table 2
Sizes of the data sets used in the computational experiments.

Data Set No.	No. of Suppliers	No. of Customers	No. of Shipments
Set 1	5	5	20
Set 2	5	10	20
Set 3	5	10	40
Set 4	10	10	50
Set 5	10	20	50
Set 6	10	20	100
Set 7	10	30	100
Set 8	10	30	200
Set 9	20	20	100
Set 10	20	20	200

studies a distribution planning problem from the perspective of a 3PL. We randomly generate the additional parameters used in our SDPC-PF. More particularly, we use Song et al.'s method to generate the network of suppliers and customers and the sets of arrival times and dispatch times of inbound and outbound options, X_i, Y_j, for suppliers and customers, respectively. We modify their proposed cost function of inbound and outbound options $f(x_{iq}), g(y_{jl})$, by incorporating capacity. We also scale down their holding cost h_i to make it a cost per unit, rather than per shipment. We then generate the following additional parameters: capacities C_{iq} and C_{jl} for inbound and outbound options, demand distributions for each (i,j) shipment, and spot market inbound and outbound transportation cost, π_i and π_j, respectively. The detailed data generation method is outlined in Appendix A.

Based on this method, we generate 10 data sets of different sizes. Each set is composed of 5 instances that share the same network data but differ in the demand distributions and the transportation options available for suppliers and customers. Table 2 outlines the problem size of each data set in terms of the numbers of suppliers and customers, and number of (i,j) shipments. The disutility factor of the spot market carrier is set to r = 4. Recall that transportation cost through a spot market carrier is a variable per unit cost. The 3PL thus pays for exactly the shipping amount needed and has more flexibility in shipping time, as opposed to the reserved transportation options, which justifies the cost difference. The effect of the disutility factor on the expected outsourcing amount and the benefit of using SDPC-PF are analyzed in Section 5.2.

We now better examine how inventory holding times at the consolidation center and the number of inbound and outbound transportation options may influence the benefit of using the SDPC-PF. For each set outlined above, we develop four experimental settings. Each setting considers three arrival/dispatch times for each supplier i and customer j. We assume that each supplier and customer have a slow option, an average-speed option and a fast option. The arrival times $\tau_{iq} \in X_i$ of an inbound fast option, average-speed option, and slow option for a given supplier i are generated uniformly in the ranges U[100,235], U [235,370], and U[370,500], respectively. Similarly, the dispatch times $\tau_{jl} \in Y_j$ of outbound options that are fast, of average speed option, and slow for a given customer j are generated uniformly in the respective ranges U[370,500], U[235,370], and U[100,235].

The four experimental settings differ in the following way:

- **(A)** For each of the three arrival and dispatch times of inbound and outbound options, two levels of capacity are considered, creating a total of six transportation options per supplier and customer. The two capacity levels are $\gamma = 1.00$ and $\gamma = 1.15$. In this experimental setting, arrival/dispatch times of inbound and outbound options are generated independently. For example, an average-speed option of supplier i does not necessarily arrive before the dispatch time of the average-speed option of customer j, given that $i \in I(j)$. This results in higher average wait times at the consolidation center, and therefore greater inventory cost.
- **(B)** For each of the three arrival and dispatch times of inbound and outbound options, the same two levels of capacity are considered $\gamma = 1.00$ and $\gamma = 1.15$, creating six transportation options per supplier and customer. However, under this setting, arrival times $\tau_{iq} \in X_i$ of suppliers $i \in I$ are synchronized with the dispatch times $\tau_{jl} \in Y_j$ of customers $j \in J(i)$ for specific speed levels. That is, for a given supplier i and customer $j \in J(i)$, supplier i's fast transportation option is guaranteed to arrive before customer j's slow option is dispatched. The same synchronization is done for different speed levels, such that average-speed and slow supplier options arrive before the dispatch time of average-speed and fast customer options, respectively. This creates instances of lower average holding times at the consolidation center.
- **(C)** Arrival and dispatch times are generated independently, similar to **(A)**, but three capacity levels ($\gamma = 1.00, \gamma = 1.15$, and $\gamma = 1.3$) are considered for each time, thus creating a total of nine options per supplier and customer. We are interested to know how having an additional capacity level may change the solution, and also how increasing the number of transportation options may affect the efficiency of the SAA algorithm, when average holding times at the consolidation center are high.
- **(D)** Arrival and dispatch times are synchronized, similar to **(B)**, and three capacity levels are considered for each time, $\gamma = 1.00, \gamma = 1.15$, and $\gamma = 1.3$, creating a total of nine options per supplier and customer. Similar to **(C)**, we wish to understand the impact of an additional capacity level on the solution of SDPC, and the efficiency of SAA with the increased number of transportation options, when average holding times are low.

5.2. *Experiments*

5.2.1. *SAA settings*

We use SAA to solve the SDPC-PF, as outlined in Section 4. We define $N = 10$ scenarios to estimate the expected second stage cost. This is then repeated for $M = 10$ SAA problems so as to estimate a lower bound on the *true* expected cost.

Each scenario includes a realization of demand for each shipment (i, j). We sample these realizations from the demand distribution data (Section A.2). Each SAA problem is solved using CPLEX 12.8, with a maximum time limit of 1200 s (20 min). The lower bound is computed as in Equation (25), with $t_{\alpha=5,\nu=9} = 1.833$, for 95% confidence interval and 9 (N-1) degrees of freedom.

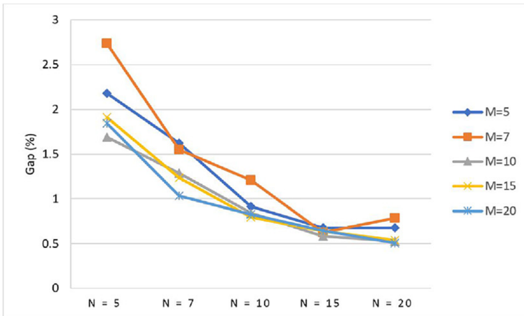
Our choice of N and M was based on empirical results so as to achieve a reasonable trade-off between gap and computational time. Fig. 1 shows the gap and the computational time of two instance sets, 6A and 8A. Though the computational time when $N = 10$ and $M = 10$ is higher than when both or either N and M take lower values, the benefit is apparent in the reduced estimated optimality gap. Set 8A is more computationally demanding; notice how increasing the value of N and M actually causes an increase in the estimated gap since the optimality of each SAA run is not achieved in the imposed time limit.

To obtain the upper bound on expected cost of the *true* problem, we consider all individual solutions, M, of the M runs, and evaluate them using a scenario tree of $N' = 1000$ scenarios. For each of the x^m solutions, we calculate the expected second stage cost of the solution and compute $\overline{\eta}^m$ as shown in Equation (28). We then compute the upper bound η_U^m as in Equation (30), with $z_{\alpha=5} = 1.64$, for a 95% confidence interval. The estimated upper bound of the algorithm is $\min_{m=\{1,\ldots,M\}} \eta_U^m$, as shown in Equation (31).

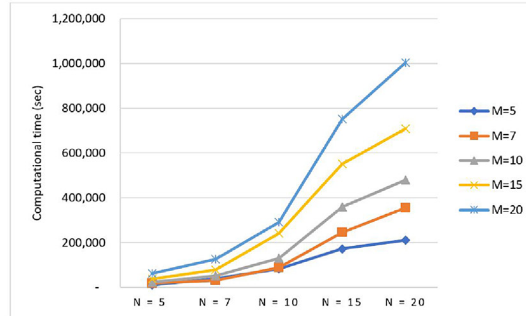
5.2.2. *Performance measures*

To assess the advantage of taking demand uncertainty into account at the modeling phase, we compare the SAA solution for each problem instance to the solution of its deterministic counterpart DDPC-PF, shown in Section B. Particularly, we solve the deterministic problem to obtain the distribution plan when the mean demand is used for each shipment (i, j). We then evaluate the deterministic distribution plan using the same 1000 scenario tree used to obtain the upper bounds of the SAA solutions. This shows the expected cost savings achieved when distribution plans are constructed using SDPC as opposed to DDPC. This value is referred to in the literature as *the value of stochastic solution* (Birge and Louveaux, 2011). We report this value as $\frac{(\overline{\eta}_{det}-\overline{\eta}_{stoch})}{\overline{\eta}_{stoch}}$, where $\overline{\eta}_{det}$ and $\overline{\eta}_{stoch}$ are the objective values of the deterministic and the stochastic solutions, respectively, when the second stage problem is assessed on the full $N' = 1000$ scenario tree, computed as shown in Equation (28). Note that $\overline{\eta}_{stoch}$ is the objective value of the best SAA run, with the lowest optimality gap.

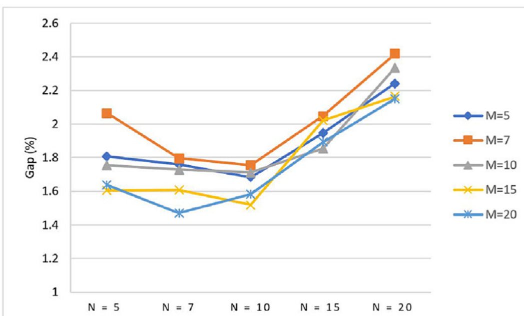
To further highlight the potential benefits of our model, we report the *expected outsourcing* that each distribution plan requires. That is, the expected shipment amount, as a percentage of total expected demand, that travels via spot market carriers rather than a reserved first stage



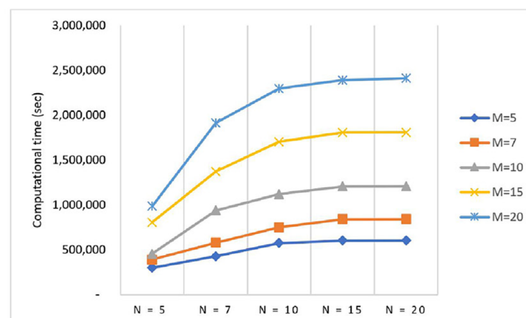
(a) Gap estimate for different values of N and M - Set 6A.



(b) Total computational time of the M SAA runs, each with N scenarios - Set 6A.



(c) Gap estimate for different values of N and M - Set 6A.



(d) Total computational time of the M SAA runs, each with N scenarios - Set 8A.

Fig. 1. Trade-off between gap and computational time for different values of *N* and *M*.

Table 3
Results of computational experiments Sets A and B with $r = 4$.

Set No.	Gap (%)	No. of Paths	Value of Stochastic Solution(%)	Expected Outsourcing SDPC(%)	Expected Outsourcing DDPC(%)	Expected Utilization SDPC(%)	Expected Utilization DDPC(%)	LB std. dev.(%)	UB std. dev.(%)	Total Time (sec)	SAA Time (sec)	UB Time (sec)	Det. Time (sec)
1A	0.30	606	23.58	0.59	8.85	80.03	82.78	0.09	0.00	37	7	29	0.31
2A	0.25	622	39.11	0.78	13.46	69.47	65.90	0.08	0.01	46	7	38	0.36
3A	0.96	1227	11.79	0.44	7.31	87.38	91.25	0.18	0.04	220	90	125	0.58
4A	0.76	1512	13.38	1.02	7.30	82.57	87.11	0.18	0.03	323	138	177	0.73
5A	0.39	1502	27.54	0.15	11.85	81.89	78.14	0.13	0.03	191	44	141	0.82
6A	1.10	3009	9.25	1.19	5.82	85.45	89.59	0.19	0.05	1558	1312	231	1.38
7A	0.94	2984	18.42	1.79	7.94	79.67	80.68	0.20	0.05	987	710	261	1.52
8A	1.71	6077	4.45	1.63	3.75	87.65	93.55	0.08	0.05	11572	11198	344	2.96
9A	0.64	2995	16.57	0.99	7.24	78.14	83.88	0.11	0.02	998	574	408	1.47
10A	2.66	6003	3.00	2.46	3.74	87.63	90.83	0.10	0.07	12502	12019	453	2.71
Avg	0.97	2654	16.71	1.10	7.73	81.99	84.37	0.13	0.04	2843	2610	221	1.28
1B	0.46	740	24.41	0.00	9.27	86.39	88.68	0.13	0.00	56	8	46	0.33
2B	0.31	740	42.79	0.00	13.46	76.01	72.55	0.09	0.00	40	8	31	0.37
3B	1.28	1480	13.27	1.01	7.00	88.50	91.43	0.29	0.10	217	90	121	0.58
4B	0.54	1850	16.50	0.00	7.80	85.78	90.68	0.12	0.00	245	80	158	0.74
5B	0.88	1850	29.49	0.23	12.07	86.46	83.23	0.20	0.04	250	76	167	0.83
6B	1.06	3700	10.91	0.61	5.74	87.92	92.22	0.19	0.06	2691	2439	237	1.40
7B	0.69	3700	20.22	0.26	7.41	83.49	87.35	0.13	0.04	799	524	259	1.51
8B	2.38	7400	5.78	0.97	3.72	91.56	95.72	0.09	0.05	12392	12022	340	2.92
9B	0.68	3700	17.93	0.12	7.36	86.56	90.34	0.11	0.02	983	582	385	1.48
10B	3.12	7400	3.95	1.29	3.46	89.34	94.88	0.06	0.07	12508	12022	456	2.97
Avg	1.14	3256	18.52	0.45	7.73	86.20	88.71	0.14	0.04	3018	2785	220	1.31

transportation option in the 1000-scenario tree demand distribution. This provides a measure of risk associated with the transportation plan of a given solution by emphasizing the extent to which first stage reserved transportation options $\bar{x}_{iq}, \bar{y}_{jl}$ are capable of satisfying demand. We also report the *expected utilization* of reserved transportation options for each instance, seeking a possible relationship between expected utilization of options and expected outsourcing.

5.3. Computational results

5.3.1. SAA results

The SAA algorithm was applied to the different data sets of Section 5.1 for experimental settings A, B, C and D. Results are shown in Tables 3 and 4. Note that each row reports the average over 5 instances of the specified size and setting. For example, row 1A in Table 3 shows the

Table 4
Results of computational experiments Sets C and D with $r = 4$.

Set No.	Gap (%)	No. of Paths	Value of Stochastic Solution(%)	Expected Outsourcing SDPC(%)	Expected Outsourcing DDPC(%)	Expected Utilization SDPC(%)	Expected Utilization DDPC(%)	LB std. dev.(%)	UB std. dev.(%)	Total Time (sec)	SAA Time (sec)	UB Time (sec)	Det. Time (sec)
1C	0.41	1147	22.75	1.55	10.00	80.57	83.63	0.12	0.01	59	13	45	0.35
2C	0.15	1182	40.67	1.50	13.97	66.16	62.47	0.05	0.00	47	10	35	0.44
3C	1.06	2313	11.92	1.02	7.41	85.48	91.21	0.22	0.06	266	125	134	0.66
4C	0.94	2874	13.73	0.48	7.51	84.09	88.27	0.20	0.02	306	129	170	0.83
5C	1.01	2889	26.11	0.90	11.90	82.38	79.22	0.17	0.04	291	94	188	0.99
6C	0.94	5781	9.69	1.17	5.76	86.76	91.39	0.22	0.05	4428	4164	247	1.67
7C	0.65	5754	17.20	1.63	7.47	78.87	80.84	0.14	0.06	2647	2349	280	1.78
8C	2.29	11494	3.85	1.30	3.63	88.49	93.51	0.10	0.05	12441	12028	378	3.28
9C	0.81	5754	16.32	0.81	7.28	78.67	83.83	0.15	0.03	1779	1342	419	1.74
10C	3.52	11531	2.75	2.72	3.65	86.98	91.82	0.06	0.07	12559	12029	495	3.29
Avg	1.18	5072	16.50	1.31	7.86	81.85	84.62	0.14	0.04	3482	3228	239	1.50
1D	0.34	1320	27.19	0.00	9.54	85.24	88.14	0.11	0.00	49	13	34	0.36
2D	0.87	1350	40.59	0.00	12.91	77.34	72.77	0.26	0.00	58	12	44	0.42
3D	1.10	2771	12.39	0.64	6.76	87.20	91.30	0.26	0.07	301	149	145	0.68
4D	1.07	3394	14.64	0.07	7.79	85.70	90.28	0.21	0.01	376	169	198	0.87
5D	0.77	3447	28.57	0.22	11.79	85.55	83.14	0.17	0.03	241	63	170	1.00
6D	1.04	6812	10.27	0.88	5.19	87.91	92.00	0.18	0.07	3824	3565	241	1.68
7D	0.78	6706	19.20	1.04	6.89	84.89	86.55	0.19	0.07	1418	1139	262	1.90
8D	2.12	13498	5.89	1.16	3.50	90.78	95.18	0.07	0.06	12392	11974	382	3.50
9D	0.60	6854	18.30	0.15	7.38	85.81	89.98	0.09	0.02	786	377	391	1.77
10D	3.74	13590	3.07	1.87	3.50	89.17	94.73	0.08	0.07	12571	12031	504	3.41
Avg	1.24	5974	18.01	0.60	7.53	85.96	88.41	0.16	0.04	3202	2949	237	1.56

average results of 5 instances of Set 1 (5 suppliers, 5 customers, 20 shipments), experimental setting A. For additional clarity, we also provide detailed results for one problem instance in Appendix C.

The first column of Tables 3 and 4 specifies the data set number. The relative gap of the algorithm is reported in the second column and the number of feasible paths used to form the model is shown in the third, to demonstrate the problem size. Column 4 exhibits the *value of stochastic solution*, a measure of the benefit of using our proposed stochastic model over its deterministic counterpart. Columns 5 and 6 report the *expected outsourcing* for solutions of the stochastic and the deterministic models, respectively. Columns 7 and 8 show the *expected utilization* of reserved inbound and outbound transportation options for the stochastic and deterministic models. For insight on the fluctuation of cost over different scenarios, we report the relative standard deviation of the lower and upper bounds, as a percentage of their respective means, in columns 9 and 10. Finally, columns 11 to 14 respectively report the computational time (in seconds) of the full algorithm, the time to solve the 10 SAA runs, the time to compute upper bounds, and the time to solve the deterministic counterpart (DDPC).

5.3.2. SAA performance

The results clearly demonstrate that the optimal distribution plans of the SDPC are more cost efficient than the plans of DDPC, once the actual demand is realized. Instances across the different data sets and experimental settings show that the SDPC yields significant expected cost savings, as outlined by the *value of stochastic solution*. Observe, however, that the advantage of the SDPC, compared to DDPC with nominal values, is less notable for denser problem instances, when a greater number of customer orders are consolidated in a single load. This can be observed in the average *value of stochastic solution* of Sets 2 and 3, 5 and 6, 7 and 8, 9 and 10 across all experimental settings. For each of these pairs of sets, the network size is the same, i.e., the same number of suppliers and customers, but the number of shipments doubles. For example, the average value of stochastic solution drops from 39.11% in Set 2A to 11.79% in Set

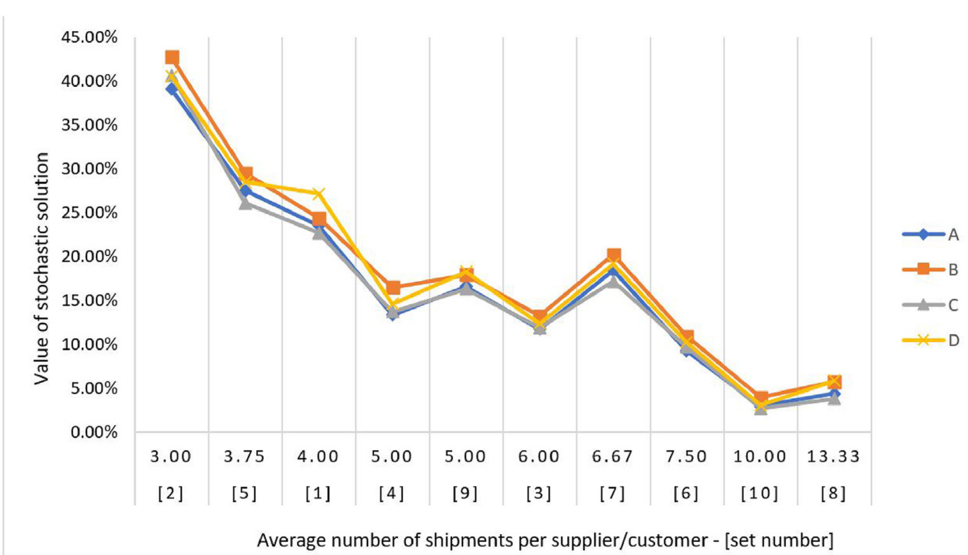


Fig. 2. Relationship between value of stochastic solution and number of shipments per supplier and customer.

3A. The same trend can be observed when comparing Sets 9A and 10A; the value of stochastic solution drops from 16.57% to 3.00%. This implies that the SDPC problem is more beneficial in sparse networks as opposed to denser ones. This observation can be explained by the fact that as the number of combined (i,j) shipments in an inbound or outbound transportation option increases, the mean of the consolidated shipment approaches the *true* mean, and therefore the deterministic solution, with mean demand, becomes comparable to the stochastic one.

To show the relationship between the value of stochastic solution and the number of shipments in an instance, we calculate the average ratio of number of shipments per supplier and per customer as $\left(\frac{no.shipments}{|I|} + \frac{no.shipments}{|J|}\right) \Big/ 2$. Then, for all instances shown in Tables 3 and 4, we graph the ratio of the average number of shipments per supplier and customer versus the value of stochastic solution in Fig. 2. The x-axis shows the ratio in an ascending order and its corresponding set number. We see in the figure that as the number of shipments per supplier and customer increases, the value of stochastic solution decreases. We also observe that the different experimental settings show very similar trends, with a slightly higher value of stochastic solution for settings B and D, with the lower average holding time.

Note from the tabular results that denser problem instances have much higher computational burden than sparser ones. This is seen in the SAA time reported in column 12 of Tables 3 and 4. Sets 7 and 8, for example, both have 10 suppliers and 30 customers. However, Set 8 has double the number of shipments of Set 7, i.e., 200 and 100 shipments, respectively. The average computational time of Set 8B for solving the 10 SAA runs is 12022 s, while that of Set 7B is only 524 s. Nonetheless, the average computational time of the upper bound calculation is slightly higher for Set 8B, but somewhat comparable, respectively 340 and 259 s for 8B and 7B.

Tables 3 and 4 also suggest that the difference in *expected outsourcing* percentage between solutions of SDPC and DDPC is more significant for sparser problems. This difference decreases for denser problem instances. In other words, for denser instances, the expected outsourcing percentage of DDPC is low (compared to sparser instances) and is closer in value to that of SDPC. Fig. 3 plots the expected outsourcing for solutions of both SDPC and DDPC versus the ratio of number of shipments to suppliers and customers and highlights such an observation; the difference between the expected outsourcing percentage of SDPC and DDPC decreases for denser problem instances. For the expected utilization of reserved options, we observe that denser problem instances have slightly higher expected utilization than sparser instances in the solutions of both SDPC and DDPC.

The average optimality gap is at most 1.28% for instances of all data sets except Sets 8 and 10, with 200 shipments. For those two sets, the average optimality gap is at most 3.74%. We note, however, that in those sets, the maximum time limit of each SAA run (1200 s) is reached, which implies that some or all of the SAA runs may have not been solved to optimality, negatively affecting the quality of the candidate solutions and, in turn, the optimality gap estimate. Because of the low optimality gap across different instances under the current SAA setting outlined in Section 5.2.1, there is no motive to increase the number of scenarios N maintained in the SAA problem.

We notice that the results of Set B, with the lower holding time at the consolidation center, are comparable to those of Set A, implying that holding time does not have a major impact on the benefit of SDPC. Nonetheless, the value of stochastic solution is slightly higher for Set B compared to Set A, and the expected outsourcing of Set B is a bit lower than that of Set A. This indicates that the reduced wait time in Set B marginally decreases the need for outsourcing done to reduce holding cost, thus improving the value of stochastic solution.

Sets C and D, with the greater number of options, show similar trends to Sets A and B, in terms of the value of stochastic solution, the percentage of outsourcing and utilization for both SDPC and DDPC. However, Sets C and D have higher average computational time of SAA runs, for most instances, compared to Sets A and B. This increase in computational time is at most double for most instances, with a few exceptions, e.g., the SAA time of 6C is about 3 times that of 6A.

5.3.3. Effect of the spot market disutility factor and the holding cost on the benefit of SDPC

We conduct analysis on experimental settings A and B, for all datasets, when the disutility factor changes from $r = 4$ to $r = 2$ and the holding cost decreases by 50%. Since experimental settings C and D show similar trends to A and B in the computational results in Tables 3 and 4 but have higher computational time, we focus on settings A and B only. We compare the optimality gap estimate, the value of stochastic solution, the expected outsourcing and expected utilization for different combinations of r and h. Results are shown in Table 5, where each row displays the average of 5 instances of the specified size and setting.

We observe that the change in optimality gap as the disutility factor and holding cost decrease is very slight for both experimental settings. We also note that for a given disutility level, reducing the holding cost provides minor improvements in the value of stochastic solution. In other words, reduction in holding cost only provides a very small amount of additional cost savings for the solutions of SDPC compared to DDPC, even for instances with higher average holding time. The value of stochastic

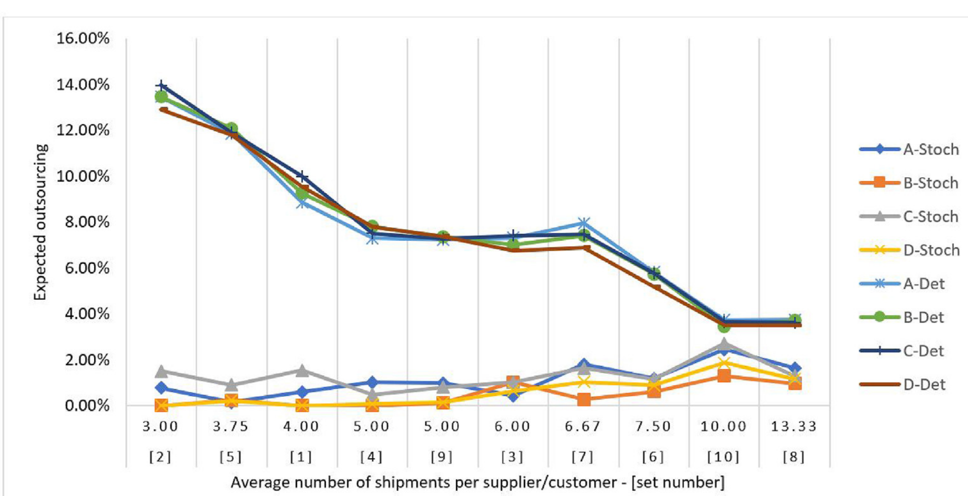


Fig. 3. Expected outsourcing vs. number of shipments per supplier and customer for SPDC and DDPC.

Table 5
Analysis on the effect of changing r and h for experimental settings A and B.

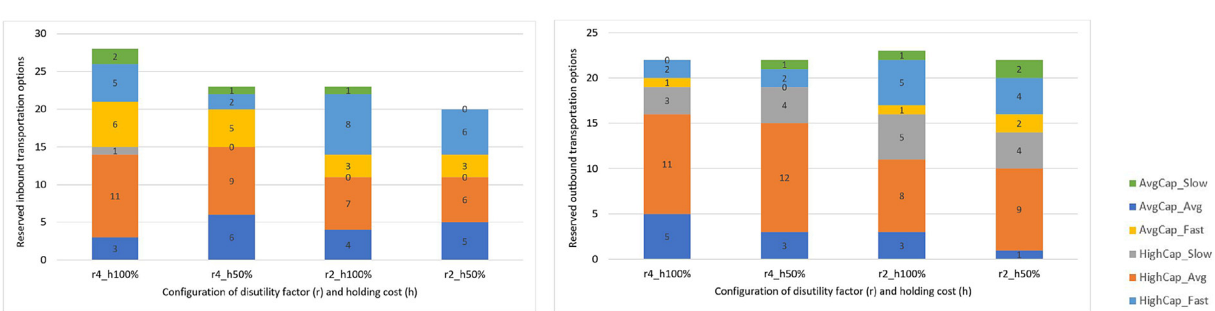
Set No.	Gap				Value of Stochastic Solution				Expected Outsourcing (%)				Expected Utilization (%)			
	$r = 4$		$r = 2$		$r = 4$		$r = 2$		$r = 4$		$r = 2$		$r = 4$		$r = 2$	
	$h =$ 100%	50%	$h =$ 100%	50%	$h =$ 100%	50%	$h =$ 100%	50%	$h =$ 100%	50%	$h =$ 100%	50%	$h =$ 100%	50%	$h =$ 100%	50%
1A	0.30	0.58	1.43	1.31	23.58	24.38	9.29	11.18	0.59	0.00	9.15	7.36	80.03	82.51	82.71	83.33
2A	0.25	0.29	0.62	0.88	39.11	41.18	21.42	22.30	0.78	0.00	10.77	5.13	69.47	73.25	66.58	70.36
3A	0.96	0.75	0.96	0.85	11.79	12.28	3.39	3.40	0.44	0.36	6.23	5.92	87.38	87.23	88.51	88.66
4A	0.76	0.85	1.01	1.16	13.38	13.80	2.88	3.44	1.02	0.77	5.70	5.95	82.57	84.56	84.32	85.92
5A	0.39	0.51	0.80	0.89	27.54	28.61	12.84	13.68	0.15	0.15	5.25	4.47	81.89	83.25	83.14	84.10
6A	1.10	1.23	0.75	0.85	9.25	9.72	2.79	2.81	1.19	1.06	5.95	4.92	85.45	86.96	86.93	89.24
7A	0.94	0.93	0.59	0.60	18.42	19.24	7.19	8.13	1.79	1.35	8.02	6.41	79.67	80.74	78.42	79.98
8A	1.71	1.71	0.54	0.47	4.45	4.64	1.06	1.08	1.63	1.45	5.31	4.68	87.65	89.52	90.17	92.04
9A	0.64	0.79	0.84	0.88	16.57	17.45	4.79	6.11	0.99	0.97	10.24	8.98	78.14	80.41	76.84	81.09
10A	2.66	2.99	0.38	0.47	3.00	3.02	1.04	0.84	2.46	2.52	6.11	5.97	87.63	90.08	87.43	88.37
Avg	0.97	1.06	0.79	0.84	16.71	17.43	6.67	7.30	1.10	0.86	7.27	5.98	81.99	83.85	82.51	84.31
1B	0.46	0.76	1.40	1.78	24.41	25.00	8.93	9.32	0.00	0.00	3.53	1.76	86.39	86.39	86.92	86.95
2B	0.31	0.14	0.96	0.96	42.79	43.50	23.48	24.23	0.00	0.00	6.75	3.30	76.01	77.44	73.78	76.63
3B	1.28	1.10	1.10	1.05	13.27	13.48	4.11	4.20	1.01	0.83	2.35	2.26	88.50	88.15	90.00	89.87
4B	0.54	0.93	1.04	1.28	16.50	16.83	5.00	5.05	0.00	0.00	4.55	3.57	85.78	85.78	85.54	86.37
5B	0.88	0.65	0.86	0.87	29.49	30.05	14.02	14.42	0.23	0.23	1.92	1.76	86.46	86.46	86.96	87.55
6B	1.06	1.15	1.16	1.26	10.91	11.13	2.60	2.58	0.61	0.53	3.89	3.53	87.92	87.79	90.05	89.44
7B	0.69	0.82	0.72	0.77	20.22	20.62	7.96	8.26	0.26	0.38	2.65	2.45	83.49	83.77	85.12	84.93
8B	2.38	2.63	0.80	0.57	5.78	5.75	0.83	0.91	0.97	0.97	2.41	2.34	91.56	91.19	93.91	93.70
9B	0.68	0.81	1.28	1.27	17.93	18.37	5.65	5.84	0.12	0.38	5.29	3.99	86.56	86.83	85.38	86.60
10B	3.12	3.33	0.61	0.73	3.95	3.97	0.77	0.80	1.29	1.38	2.33	2.25	89.34	89.95	92.44	92.36
Avg	1.14	1.23	0.99	1.05	18.53	18.87	7.33	7.56	0.45	0.47	3.57	2.72	86.20	86.37	87.01	87.44

solutions improves an average of 0.67% and 0.29% for settings A and B, respectively, with the reduction in holding cost. This result is explained by the fact that the lower holding cost is reflected in both SDPC and DDPC, and the cost savings between the two problems are mainly achieved through reserving higher transportation option capacity to reduce the need to outsource to the spot market when demand is high. Nevertheless, we note that the reduction in holding cost does slightly reduce the amount of expected outsourcing, and this decrease is more notable for setting A instances as compared to B. We also observe that the expected utilization is only marginally affected by the changes in holding cost. Settings A and B have an average increase of utilization of 1.83% and 0.30% as holding cost decreases.

Results also suggest that the disutility factor has the most impact on both the value of stochastic solution and expected outsourcing percentage. This is anticipated, since the disutility cost is a variable per unit cost and the model assumes that shipping through a spot market carrier results in no holding cost. Lowering the disutility factor to $r = 2$ therefore reduces the cost difference between first stage options and spot market. Particularly, we notice that the benefit of incorporating randomness in the model is positively correlated to the value of the disutility factor. That is, as the spot market shipping gets closer to that of reserving a transportation option ahead of time, and the 3PL is indifferent to spot market shipping, considering customer demand stochasticity in the planning phase does not result in remarkable cost savings. Thus, any chosen first-stage transportation plan can easily be adjusted when actual demand is realized, at only a small cost.

5.3.4. Structural differences between solutions of the different configurations of disutility factor and holding cost

The previous section examines the impact of changes in the disutility factor and holding cost on the benefit of SDPC. Here we analyze how the structure of the distribution plans obtained from solution of SDPC differs as the configuration of disutility factor and holding cost changes. To do so, we focus on instance 9A3, discussed in Appendix C, and see how the transportation plan changes for distinct capacity and speed levels. Fig. 4 shows a breakdown of inbound and outbound transportation options for each of the four combinations of disutility and holding cost considered in Section 5.3.3. The x-axis refers to the instance name by the specific values of r and h in the instance. For example, r4_h50% shows the results of the instance 9A3 when we solve it with $r = 4$ and 50% of the holding cost. Each column in Figures Fig. 4a and Fig. 4b shows the breakdown of all reserved inbound and outbound transportation options, respectively, based on their capacity and speed levels, under each parameter setting. The different parts of a given column show the number of options with a given capacity and speed level, where HighCap and AvgCap refer to options with $\gamma = 1.15$ and $\gamma = 1.00$, respectively, and Fast, Avg, Slow,



(a) Inbound reserved options (b) Outbound reserved options

Fig. 4. Breakdown of the transportation plan of the best SAA run of different configurations of r and h.

refer to the three speed levels considered in the instance.

Fig. 4 suggests that both the holding cost and the disutility factor impact the actual distribution plan of SDPC. A decrease of 50% in holding cost, when $r = 4$, reduces the number of inbound options by 5, but keeps the number of outbound options unchanged. This implies that with a high holding cost, and when average holding time is high, reserving additional capacity may diminish the total network cost by cutting down on holding cost. We note, however, that the reduction in holding cost has a greater impact on the choice of speed levels of reserved options more than their capacity levels. For example, for a given value of r, when h decreases by 50%, approximately the same number of high capacity options is reserved as when h is kept at 100%. However, the change in the breakdown of the reserved options based on speed is more apparent. This is intuitive since as holding cost decreases, the trade-off between transportation and holding cost becomes less important. Therefore, the solution may choose plans that result in higher wait times in an effort to reduce transportation cost and in turn, minimize total cost.

As opposed to reduction in holding cost, we note from Fig. 4 that the reduction in the disutility factor affects both the choice of the reserved transportation options' capacity and speed levels. For example, when h is at 100%, the number of high capacity inbound options decreases from 15 to 11, and the number of high capacity outbound options decreases from 19 to 16, when r changes from 4 to 2. We also notice an increase in the number of average-capacity average-speed options, both inbound and outbound. This reinforces the results of the analysis in Section 5.3.3; as the value of the disutility factor decreases and the spot market cost decreases, there is less need to develop robust distribution plans, since adjusting plans after demand is realized is not costly.

6. Conclusion and future research

In this paper, we studied the stochastic distribution planning with consolidation problem from the viewpoint of a 3PL in a three-echelon supply chain network. We proposed a two-stage stochastic program with recourse to model the problem of selecting inbound and outbound transportation options for 3PL distribution planning, subject to stochastic customer demand. To date, the literature on distribution planning in transshipment networks does not consider uncertainty faced in practical applications. This study offers an extension of previous work by considering probabilistic demand in tactical decisions faced by a 3PL that is handling the distribution needs of its clients.

Because of the nonlinearity in the objective function of our proposed *stochastic distribution planning with consolidation - flow based formulation* (SDPC-FF) model, we suggested an alternative linear formulation, the *stochastic distribution planning with consolidation - path based formulation* (SDPC-PF). The latter generates all feasible paths for shipments in the network, and decides on the transportation options to reserve and the allocation of shipments to paths. We applied Sample Average Approximation (SAA) to solve the SDPC-PF, and tested it extensively to evaluate its benefits and limitations. We also compared the solutions obtained by the SDPC-PF to its deterministic counterpart, the *deterministic distribution planning with consolidation* (DDPC), with mean demand values, to assess the advantage of incorporating stochasticity in the modeling phase.

Our computational testing suggests that significant cost savings can be achieved when generating distribution plans using the SDPC rather than DDPC. The results also demonstrate that the stochastic model greatly reduces the amount of outsourcing needed in the second stage problem, compared to the deterministic case. We notice, however, that the benefit of SDPC is less notable for denser problem instances, where large numbers of shipments are consolidated in a single load. We also observe that changes in second-stage cost may affect the benefit of SDPC or the structure of the choice of first-stage transportation options, or both. For instance, reduction in holding cost does not affect the benefit of SDPC, but changes the choice of transportation options. On the other hand, the spot market cost plays a major role in how beneficial the SDPC problem is, and in the choice of transportation options. This finding suggests that if the cost of shipping through a spot market carrier is not much greater than the cost of reserving transportation options, and the 3PL's disutility of shipping through the spot market is low, there is less need to establish a robust distribution plan ahead of time. That follows since correcting the initial plan, once actual demand is realized, would not result in remarkable additional costs.

Future research could extend SDPC to also incorporate stochasticity in the arrival/dispatch times of transportation options and study how the solution would change compared to the current model as well as the deterministic case. Another possible extension is to consider the decision variables of transportation options as integer rather than binary, with the 3PL having to choose how many vehicles, of a particular level of capacity and a certain arrival/dispatch time, to reserve for inbound and outbound shipping for the duration of the planning horizon.

Studying multiple consolidation centers, but choosing to send each shipment through exactly one, is another interesting direction. A related suggestion is a model with two consolidation centers, one closer to suppliers and the other closer to customers. This is a more representative model of global distribution planning; what is the benefit of accounting for uncertainty under that setting? Other directions include considering different cost functions for the spot market, and the possibility of consolidating inbound and outbound spot market shipments, to achieve economies of scale.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendices.

A. Data generation

We explain below the specific procedure used to generate data for our computational experiments.

A.1. The network of suppliers and customers

Locations of suppliers and customers are randomly generated within a radius of 1000 miles from the consolidation center. We locate suppliers and customers on opposite sides of the 1000-mile-radius circle, such that the consolidation center is a natural middle point. We do that since, from a practical point of view, suppliers in some long-haul freight transportation applications are clustered in a different geographical area, which may be overseas. We conducted some experiments with suppliers and customers randomly located throughout the 1000-mile-radius circle; we observed very similar results to when they are located on opposite sides. This is attributed to the fact that our network is a pure transshipment network where direct deliveries between suppliers and customers are not possible. In order to justify the need for consolidation, supplier-customer shipments (i, j) are selected on the network such that the distance between supplier i and the consolidation center plus the distance from the consolidation center to customer j is at most 1.25 times the direct distance from i to j.

A.2. Distribution of demand, holding cost, and transportation option capacity

Each shipment (i, j) has a uniform demand distribution; the lower bound of the distribution is generated in the range U[300,450] and the width of the distribution is set at 30%. For example, if shipment (i, j) has a lower bound of 350, with a 30% width the demand follows a uniform distribution U [350,455]. The holding cost of shipments from a given supplier i is a variable cost per volumetric unit, per time unit, generated uniformly as $h_i =$ U [0.005,0.01].

The capacity of inbound and outbound transportation options is determined as follows. For a given inbound transportation option q, the average demand of all customer orders for which option q is feasible, denoted as $\overline{d}_{iq}$, is calculated. Option $q \in Q_i$ is feasible for customer j if at least one outbound option $l \in L_j$ for customer j leaves the consolidation center after inbound option q arrives there. Capacity C_{iq} of option q is then generated as $C_{iq} = \gamma \overline{d}_{iq}$, where γ is in between $[1.0, 1.3]$; the exact value of γ is specified when generating data sets in Section 5.1. This capacity is then rounded up to be in multiples of 10 units. The capacity of an outbound option is generated in a similar manner, $C_{jl} = \gamma \overline{d}_{jl}$.

A.3. Supplier and Customer Data

For supplier $i \in I$, a release time υ_i is generated in the range [0,100]. A supplier has a number of inbound transportation options $q \in Q_i$ with arrival times $\tau_{iq} \in X_i$ between [100,500]. The number of options and their arrival times are specified in the data sets in Section 5.1.

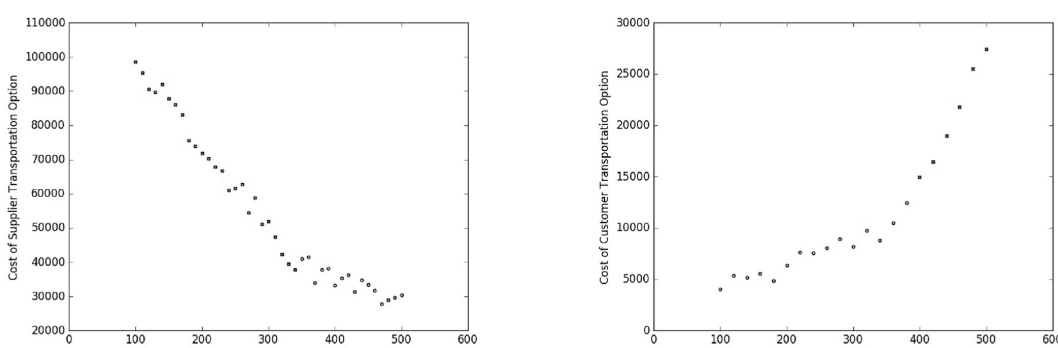
For option q with arrival time τ_{iq}, the transportation cost is expressed as $f_i(\tau_{iq}) = \theta_i \rho_i(\tau_{iq}) \frac{C_{iq}}{\xi}$, where θ_i is the scale factor of supplier i and is randomly generated in U[0.5,1.5]. $\rho_i(\tau_{iq})$ is the transportation rate corresponding to arrival time τ_{iq}, C_{iq} is the capacity of option q, and ξ is the baseline capacity that is assumed to be 4000 units. This capacity level corresponds to approximately an average consolidated demand of 8 customers, following the demand distributions outlined above.

We generate $\rho_i(\tau_{iq})$ as follows. First, a baseline transportation rate $\overline{\rho}_i$ is generated as $\overline{\rho}_i = \delta_i \cdot \rho$, where δ_i is the distance and ρ is the average unit rate that is uniformly distributed in the range U[30,50]. A baseline transportation time ι_i is then generated as $\iota_i = \delta_i / \nu$, where ν is the average speed per time unit, generated in the range U[2,3]. If the transportation time is shorter than the baseline, i.e., $\tau_{iq} - \upsilon_i \leq \iota_i$, the transportation rate is higher than the baseline rate; $\rho_i(\tau_{iq}) = \overline{\rho}_i + 2 \cdot \overline{\rho}_i \cdot [1 - ((\tau_{iq} - \upsilon_i) / \iota_i)] + \overline{\rho}_i \cdot \varepsilon$, where the first term denotes the baseline rate, the second represents the extra cost to make the transportation time shorter than the baseline time, and the third term is some random perturbation in which ε is generated in the range U[-0.1,0.1]. On the other hand, if the transportation time is longer than the baseline, i.e., $\tau_{iq} - \upsilon_i > \iota_i$, we set $\rho_i(\tau_{iq}) = \overline{\rho}_i - 0.1 \cdot \overline{\rho}_i \cdot [1 - (\iota_i / (\tau_{iq} - \upsilon_i))] + \overline{\rho}_i \cdot \varepsilon$. So, transportation options that need less time to reach the consolidation center, once the consolidated shipment is released, are faster options, and therefore have higher rates. The supplier cost function for the *baseline* capacity ξ is plotted in Fig. 5a.

Finally, we generate the cost of shipping through a spot-market carrier π_i, for each supplier $i \in I$. As mentioned earlier, this is a per unit cost composed of two elements: (a) the expected inbound spot market rate per unit and (b) the 3PL's disutility to ship through a spot market carrier. For each supplier i the cost is generated as $\pi_i = \frac{f_i(0.5\iota_i)}{\xi} r$, where a spot market carrier is assumed to be a fast option with 0.5 the baseline speed, and r is the 3PL's disutility factor of using a spot market carrier. The exact value of r is specified when generating data sets in Section 5.1.

Random customer data is obtained in a similar manner as supplier data. For customer $j \in J$, we generate due date κ_j uniformly in range [500,600]. Each customer has a number of outbound transportation options $l \in L_j$, with dispatch times $\tau_{jl} \in Y_j$ between [100,500]. The number of options and their dispatch times are specified in Section 5.1.

Outbound transportation cost is expressed as $g_j(\tau_{jl}) = \theta_j \rho_j(\tau_{jl}) \frac{C_{jl}}{\xi}$, where θ_j and ξ are generated in the same ranges as in supplier data. For $\rho_j(\tau_{jl})$, a baseline transportation rate $\overline{\rho}_j$ and a baseline transportation time ι_j are generated. The expressions are $\overline{\rho}_j = \delta_j \cdot \rho$, and $\iota_j = \delta_j / \nu$, where ρ and ν are generated in the same uniform ranges as in supplier data. If the transportation time is shorter than the baseline, i.e., $\kappa_j - \tau_{jl} < \iota_j$, the transportation price $\rho_j(\tau_{jl})$ is calculated as $\rho_j(\tau_{jl}) = \overline{\rho}_j + 2 \cdot \overline{\rho}_j \cdot [1 - ((\kappa_j - \tau_{jl}) / \iota_i)] + \overline{\rho}_i \cdot \varepsilon$. However, if transportation time is longer than the baseline, i.e., $\kappa_j - \tau_{jl} \geq \iota_j$, the price equals $\rho_j(\tau_{jl}) = \overline{\rho}_j - 0.1 \cdot \overline{\rho}_j \cdot [1 - (\iota_j / (\kappa_j - \tau_{jl}))] + \overline{\rho}_j \cdot \varepsilon$. Likewise, faster transportation options, i.e., ones that require less time to reach the customer from the time they depart the consolidation center, cost more than slower ones. The customer cost function for the *baseline* capacity ξ is plotted in Fig. 5b.



(a) Supplier cost function (b) Customer cost function

Fig. 5. Transportation options cost function for suppliers and customers.

The cost of spot-market carrier shipping π_j, for each customer $j \in J$, is generated as $\pi_j = \frac{f_j(0.5\iota_j)}{\xi} r$, where a spot market carrier is assumed to be a fast option with 0.5 the baseline speed.

B. Deterministic distribution planning with consolidation (DDPC)

The *deterministic distribution planning with consolidation - path based formulation* (DDPC-PF) is modeled below, using the same notation and decision variables defined in Section 3. We use the path formulation as opposed to the flow formulation, to avoid the nonlinearity in the objective function and since the number of paths is small when there is only a single scenario. In this model, the mean demand of each shipment (i,j), which we denote as $\overline{d}_{ij}$, is used instead of samples from the demand distribution.

$$[\mathbf{DDPC-PF}]\ \min\sum_{i\in I}\sum_{q\in Q}f(x_{iq})+\sum_{j\in J}\sum_{l\in L}g(y_{jl})+\sum_{i\in I}\sum_{j\in J(i)}\sum_{p\in P_{ij}}c_p\beta_{ijp} \tag{33}$$

$$\text{subject to}\ \sum_{p\in P_{ij}}\beta_{ijp}=1 \qquad \forall i\in I, j\in J(i) \tag{34}$$

$$\sum_{p\in P_{ij}}a_{iqp}\beta_{ijp}\le x_{iq} \qquad \forall i\in I, j\in J(i), q\in Q \tag{35}$$

$$\sum_{p\in P_{ij}}b_{jlp}\beta_{ijp}\le y_{jl} \qquad \forall j\in J, i\in I(j), l\in L \tag{36}$$

$$\sum_{p\in P_{ij}}\sum_{j\in J(i)}a_{iqp}\overline{d}_{ij}\beta_{ijp}\le C_{iq} \qquad \forall i\in I, q\in Q \tag{37}$$

$$\sum_{p\in P_{ij}}\sum_{i\in I(j)}b_{jlp}\overline{d}_{ij}\beta_{ijp}\le C_{jl} \qquad \forall j\in J, l\in L \tag{38}$$

$$\begin{array}{ll} x_{iq}, y_{jl}\in\{0,1\}, & i\in I, q\in Q, j\in J, l\in L \\ \beta_{ijp}\in\{0,1\}, & i\in I, j\in J(i), p\in P_{ij} \end{array} \tag{39}$$

Similar to SDPC-PF, the objective function (33) minimizes the total transportation cost and the cost of allocating shipments to paths. Constraints (34) ensure that exactly one path is chosen for each shipment (i,j) in the network. Constraints (35) and (36) guarantee that shipment (i,j) traverses a path only if both the inbound transportation option q of supplier i and the outbound transportation option l of customer j are open. Constraints (37) and (38) ensure that the total demand that traverses a given path does not exceed the capacity of the inbound and outbound transportation options of that path. Finally, Constraints (39) impose the binary requirement on the variables.

C. Detailed SAA results example

In this section we provide and analyze the detailed result of one problem instance; instance 3 from set 9A, with 20 suppliers, 20 customers and 100 shipments. We exhibit the results of the 10 SAA runs in Table 6. The first column shows the run number and the second shows the objective value of the run, which is the expected transportation and holding cost based on the 10 scenarios within the run as defined by objective function (22). This objective value is the sum of the first stage and the second stage costs, shown in columns 3 and 4, respectively. We evaluate each of those runs on a 1000-scenario tree and report the upper bound estimate and standard deviation in columns 5 and 6. We also report the *expected outsourcing* and *expected utilization* for each run in columns 7 and 8, respectively.

In Table 7, we show statistics of the instance. The upper bound is the minimum of the upper bound estimates reported in column 5 of Table 6, which is the upper bound of run 5 in this instance. We also report the lower bound and its standard deviation; the lower bound is the mean of the objectives in column 2 of Table 6, minus $t_{\alpha=5,\nu=9}\widehat{\sigma}_{N,M}$, as shown in Equation (25). The absolute gap of the SAA algorithm, i.e., the difference between upper and lower bounds, as well as the relative gap are also shown. We compute the relative gap as $\frac{UB-LB}{UB}$, since our goal is to evaluate the quality of the UB; the expected cost of the *true* problem. The cost values reported in columns 2 to 6 are all in units of 1000s of dollars.

We note that the SAA run with the optimal transportation plan, which is run 5 in this instance, resulted in relatively low *expected outsourcing* and *expected utilization* values, compared to other runs. This suggests that this plan reserved a higher level of capacity compared to plans of other SAA runs.

To further analyze the results, we provide a summary of the distribution plan of each SAA run in Table 8. That table summarizes, for each run, the total number of inbound and outbound reserved options with different speed and capacity levels. The breakdown of the optimal transportation plan, run 5, is also plotted in Fig. 6. We note that the different runs result in somewhat similar transportation plans. This low variability in the solutions of the SAA runs indicates the stability of sampling among the different runs. We also notice that for some suppliers and customers, more than one transportation option is reserved. Run 5, for instance, has a total of 28 inbound reserved options for 20 suppliers, and 22 outbound options for 20 customers. Since this instance is from experimental setting A, with the high expected wait time at the consolidation center, the results suggest that in some cases, overbooking capacity is justifiable to reduce expected holding cost.

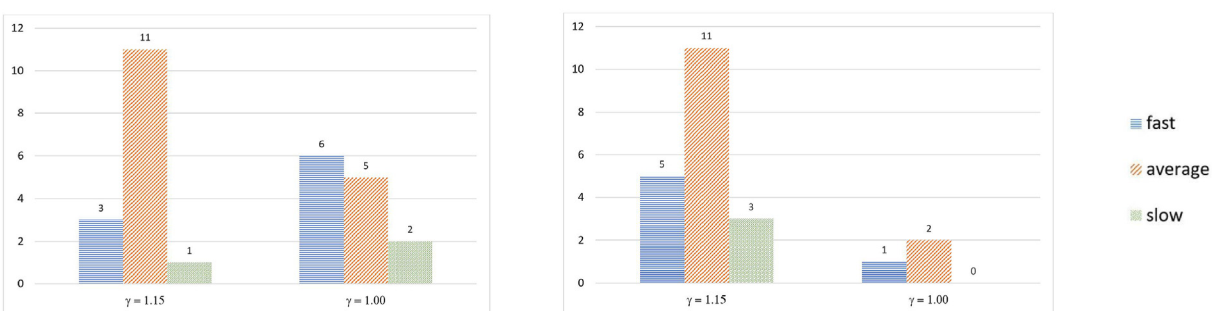
Table 8 also shows that average-speed options are the most reserved, especially with higher capacity level. This is explained by the fact that the arrival and dispatch times for inbound and outbound options in this instance are independent, meaning that slow inbound options do not necessarily create feasible paths with fast outbound options, making reserving fast options with higher costs less justifiable.

Table 6
Detailed SAA solution of Instance 3 - Set 9A.

SAA run	Objective	First stage cost	Second stage cost	Upper bound (UB)	UB standard deviation	Expected outsourcing (%)	Expected utilization (%)
1	515.087	463.319	51.767	525.381	0.344	1.51	72.30
2	520.095	470.902	49.193	523.708	0.233	1.22	70.38
3	517.050	467.325	49.725	525.101	0.283	1.41	71.52
4	518.735	469.991	48.744	521.468	0.171	1.21	71.95
5	519.759	473.242	46.516	**520.006**	0.039	1.03	71.85
6	518.214	471.784	46.430	522.902	0.171	1.25	70.24
7	516.361	461.241	55.119	531.877	0.444	1.85	72.21
8	519.626	472.740	46.886	522.398	0.102	1.10	72.94
9	516.499	470.524	45.975	522.383	0.216	1.58	76.76
10	521.323	473.636	47.687	520.549	0.041	0.92	72.79

Table 7
SAA solution statistics of Instance 3 - Set 9A.

Upper bound	520.006
Std. dev. upper bound	0.039
Lower bound	517.126
Std. dev. lower bound	0.626
Absolute Gap	2.880
Relative Gap	0.55%



(a) Inbound reserved options (b) Outbound reserved options

Fig. 6. Breakdown of the transportation plan of the best SAA run (run 5).

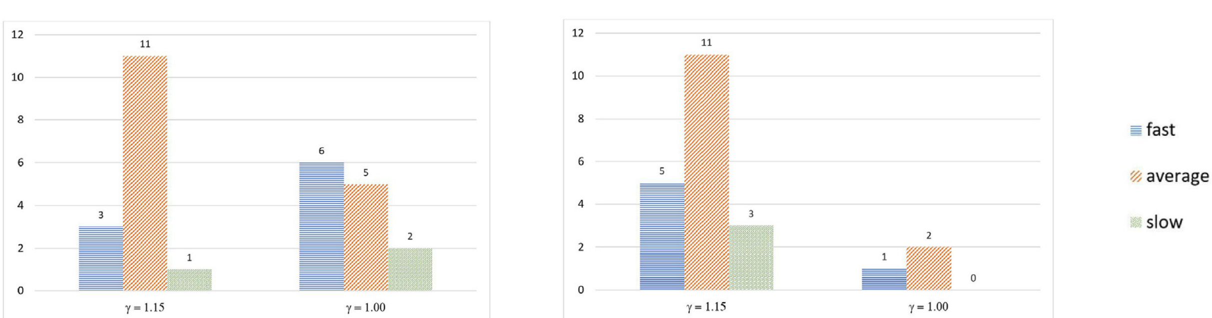

(a) Inbound reserved options (b) Outbound reserved options

Fig. 6. Breakdown of the transportation plan of the best SAA run (run 5).

Table 8
Transportation plans for each SAA run, Instance 3 - Set 9A.

SAA run	Inbound Options						Outbound Options					
	fast		average		slow		fast		average		slow	
	$\gamma =$		$\gamma =$		$\gamma =$		$\gamma =$		$\gamma =$		$\gamma =$	
	1.00	1.15	1.00	1.15	1.00	1.15	1.00	1.15	1.00	1.15	1.00	1.15
1	6	3	5	11	2	1	3	3	3	10	0	3
2	6	3	6	10	2	1	1	5	2	11	1	3
3	6	3	5	11	1	1	1	5	3	10	1	3
4	7	2	5	11	2	1	1	5	2	11	0	3
5	**6**	**3**	**5**	**11**	**2**	**1**	**1**	**5**	**2**	**11**	**0**	**3**
6	7	2	5	11	2	1	1	5	2	11	1	3
7	7	2	6	10	2	1	2	4	3	10	0	3
8	5	1	6	13	1	1	1	6	2	11	0	2
9	7	4	3	9	1	0	0	3	2	12	1	4
10	5	1	5	14	1	1	1	6	2	11	0	2

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