Electric-field-dependent variable-range hopping conductance in quasi-two-dimensional systems: Application to $PrBa_2Cu_3O_{7-y}$-based superconductor–normal-metal–superconductor junctions

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(Received 19 July 1995)

We have calculated the hopping conductivity of quasi-two-dimensional systems by using the variable-range hopping conduction mechanism in the presence of an electric field. We considered that the localized states are randomly distributed both in energy and space coordinates. Localized carriers hop from one state to another in both coordinates. We also considered that at a particular temperature the localized carriers are distributed according to the Fermi distribution function both below and above the Fermi level. The expression of the conductivity for the constant density of states was calculated. After some approximations, the expression of the conductivity was shown to reduce to expressions found in the literature. We also compared our theory with experimental results of $PrBa_2Cu_3O_{7-y}$ based $S/N/S$ junctions. Good agreement between theory and experiment was found.

I. INTRODUCTION

Recently, there has been a considerable interest in the study of the variable-range hopping (VRH) conduction in thin films of oxide materials such as superconductor–normal-metal (S/N) and superconductor–normal-metal–superconductor ($S/N/S$) junctions.[1–4] Here S and N stand for high-temperature superconductors [i.e. $HoBa_2Cu_3O_{7-y}$ (HBCO)] and normal-metal or oxide superconductors [i.e., $PrBa_2Cu_3O_{7-y}$ (PBCO)] respectively. Most workers have studied the effect of temperature on the electronic conductivity of different oxide compounds, but recently Kabasawa *et al.*[1] and Tarutani *et al.*[1] have studied the electric-field-dependent VRH hopping conductivity of PBCO based $S/N/S$ junctions. Electric-field-dependent VRH hopping conductivity has also been studied by Liu and Soonpaa[5] in quasi-two-dimensional systems such as $Bi_{10}Te_{11}S_{10}$. The study of VRH mechanisms in bulk materials has been the subject of a number of investigations,[6–8] but not much theoretical work has been done in quasi-two-dimensional (QTD) systems such as oxide thin films and oxide based S/N and $S/N/S$ junctions. Recently, Singh *et al.*[2] have performed a theoretical calculation for variable range hopping in QTD systems, including the electric-field and temperature dependencies. Their expression for electrical-field-dependent conductivity is valid for β less than one, where $\beta = qE/2\alpha k_B T$. Here q is the charge of the carrier, E is the electric field, α is the inverse of localization length, and T is the temperature. Their expression cannot be applied to explain the above experiments, since data in these experiments vary from β less than one to β greater than one.

In the present work, we have derived an expression for electric-field-dependent conductivity for a general value of β by using the method developed in our previous papers.[2] In this method, it is assumed that the localized states are distributed randomly in both space and energy coordinates. The states occupied both above and below the Fermi level have been included in the calculations. It is assumed that states are occupied according to Fermi-Dirac statistics. For $\beta \gg 1$, our expression of logarithmic conductivity is proportional to $(\beta)^{(-1/3)}$. If we assume that all states above and below the Fermi level are empty and occupied respectively, then for the constant density of states (DOS), our expression of the conductivity reduces to that of Mott.[6] The present theoretical calculations are applied to explain recent experiments showing electric-field dependence of resistivity in PBCO based $S/N/S$ junctions.[1] A good agreement between theory and experiment is observed.

II. THEORY

We considered that localized states are randomly distributed in energy and space coordinates and they form a discrete array of sites. The probability of a charge-carrier hopping from an initial state to a final state in this space is therefore given by[8,2]

$$W(R) = W_0 \exp(-R), \tag{1}$$

where W_0 is a constant and R is the distance between two states in the energy-space coordinates, and is called the range. In presence of an electric field , the range is given as[2]

$$R = x(1+\beta\cos\theta) + \omega - \varepsilon \quad \varepsilon < (\omega + x\beta\cos\theta)$$

$$R = x \quad \varepsilon > (\omega + x\beta\cos\theta). \tag{2}$$

Here x is a distance between two sites in the space coordinates. ε and ω are the energy variables of the initial and the final sites, respectively. θ is an angle between the space variable x and the electric field E. Note that the space and energy variables are presented in reduced coordinates.[2] The reduced coordinate x should be multiplied by $1/2\alpha$ to express it in distance units. Similarly, ε and ω should be multiplied by $k_B T$ to write them in energy units.

According to Mott, the main contribution to conductivity comes from electrons located at the Fermi level and the expression of conductivity[6] can be obtained with the help of Eq. (1) as

$$\sigma = \sigma_0 \exp(-R_{nn}^0), \tag{3}$$

where σ_0 and R_{nn}^0 represent a constant conductivity and the critical hopping distance at the Fermi level, respectively. To find the hopping range, we used the method developed in Ref. 2. To find the critical hopping distance, one has to calculate first the number of unoccupied states within a range R in the hopping space as a function of temperature and electric field E. For a given initial site of energy ε, the number of vacant sites within range R is given by[2]

$$N = \frac{k_B T}{2\alpha^2} \int_0^{2\pi} d\theta \int_0^R x\, dx \int_{-\infty}^{\omega_{\max}} D(\omega)[1 - f(\omega)] d\omega, \tag{4}$$

where $\omega_{\max}$ is the maximum value of ω that can be obtained from Eq. (2). For a given x and R, $\omega_{\max} = R + \varepsilon - x(1 + \beta \cos\theta)$. Here $D(\omega)$ is the density of states (DOS) of localized carriers in two dimensional systems. Mott and others[6,2] used the constant DOS, i.e., $D(\omega) = D_0$ to calculate the temperature-dependent VRH conductivity. Using the constant DOS, we found the following analytic expression for N

$$N = \frac{T}{2T_0^M}[R^3 + 4\beta R^3/\pi + 2\eta(R + \varepsilon)^3 + 3R^2\varepsilon], \tag{5}$$

where η is given as

$$\eta = \frac{7}{15\pi}, \quad \beta = 1,$$

$$\eta = \frac{\beta(\beta^2 - 4)}{2\pi(\beta^2 - 1)^2} + \frac{(\beta^2 + 2)}{\pi(1 - \beta^2)^{5/2}} \times \tan^{-1}\sqrt{(1 - \beta)/(1 + \beta)}, \quad \beta < 1, \tag{6}$$

$$\eta = \frac{\beta(\beta^2 - 4)}{2\pi(\beta^2 - 1)^2} + \frac{(\beta^2 + 2)}{\pi(\beta^2 - 1)^{5/2}} \times \tanh^{-1}\sqrt{(\beta - 1)/(\beta + 1)}, \quad \beta > 1.$$

Here $T_0^M = (12\alpha^2/\pi D_0 k_B)$. To get the above expression, we replaced the energy derivative of the Fermi distribution by the δ function. The critical hopping distance, R_{nn}, can be obtained from N by putting the total number of the accessible states equal to one. After some mathematical manipulations, we get the value of the critical hopping distance as

$$R_{nn} = R_{nn}^0 \left(1 + \frac{(3\pi/2 + 3\pi\eta)\varepsilon}{R_{nn}^0(2\beta + \pi/2 + \pi\eta)}\right)^{-1/3}, \tag{7}$$

where R_{nn}^0 is given by

$$R_{nn}^0 = \left(\frac{T}{T_0^M}\right)^{-1/3} \left(\frac{1}{2} + \frac{2\beta}{\pi} + \eta\right)^{-1/3}. \tag{8}$$

Note that our expression of the critical hopping distance R_{nn} depends on the initial energy of the electrons. At the Fermi level R_{nn} reduces to R_{nn}^0. Putting the value of R_{nn}^0 into Eq. (3), we obtain the following analytical expression of conductivity:

$$\sigma = \sigma_0 \exp\left[-\left(\frac{T}{T_0^M}\right)^{-\nu} \left(\frac{1}{2} + \frac{2\beta}{\pi} + \eta\right)^{-\nu}\right], \tag{9}$$

with $\nu = 1/3$. At zero electric field, $\eta = 1/2$ and the above expression reduces to that given in the literature.[2,5,6] For $\beta \gg 1$, Eq. (9) reduces to $\ln(\sigma/\sigma_0) \propto (T/T_0^M)^{-1/3}(\beta)^{-1/3}$.

Let us define the electric field scaling function of Davis *et al.*[8] as

$$\beta_E = -\frac{\partial \lng}{\partial \ln E}, \tag{10}$$

where g is the dimensionless conductance defined as $g = \sigma(e^2/h)^{-1}$. From Eq. (9), we get the value of β_E as

$$\beta_E = \nu(\lng - \lng_0)\left(\frac{1 + \frac{\Lambda}{2}}{1 + \pi/4\beta + \pi\eta/2\beta}\right), \tag{11}$$

where Λ is given as

$$\Lambda = 0, \quad \beta = 1,$$

$$\Lambda = \frac{a_+(24\beta + 30\beta^3 + 6\beta^5) + b_+(4 + 13\beta^2 + 8\beta^4 - \beta^6) - (4 - 2\beta^2 - 2\beta^4)}{2(1 + \beta^2)(1 - \beta^2)^{7/2}}, \quad \beta < 1,$$

$$\Lambda = \frac{a_-(24\beta + 30\beta^3 + 6\beta^5) + b_-(4 + 13\beta^2 + 8\beta^4 - \beta^6) - (4 - 2\beta^2 - 2\beta^4)}{2(1 + \beta^2)(-1 + \beta^2)^{7/2}}, \quad \beta > 1, \tag{12}$$

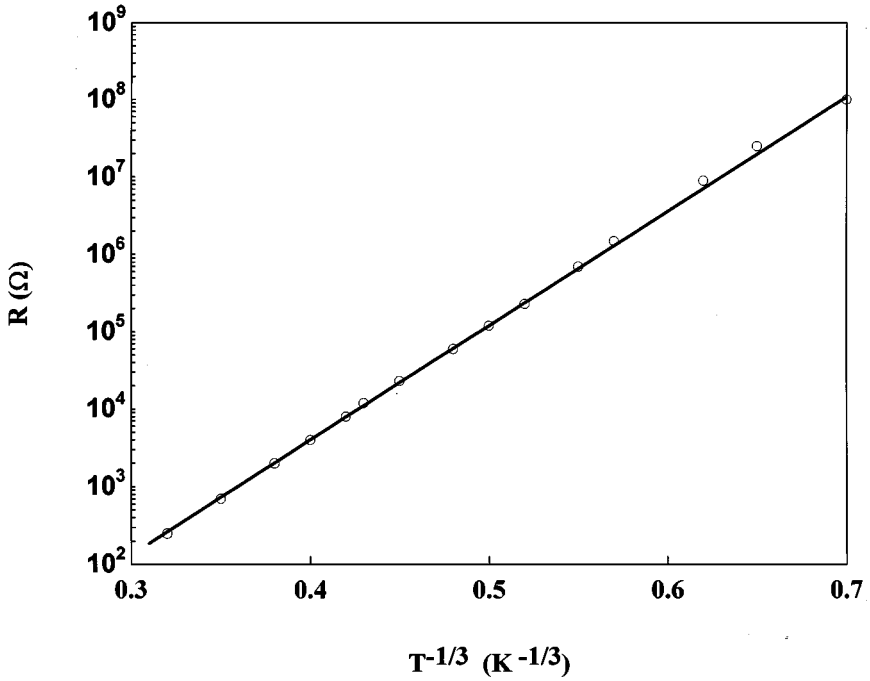

FIG. 1. Plot of resistance versus temperature for PBCO-based $S/N/S$ junctions for sample $L=0.9$ μm. Symbol ○ denotes the experimental data and the solid curve represents the theoretical results.

and where $a_+=\tan^{-1}\sqrt{(1-\beta/1+\beta)}$, $a_-=\tanh^{-1}\sqrt{(\beta-1/\beta+1)}$ and $b_\pm=\sqrt{\pm(1-\beta^2)}$. For $\beta\gg 1$, the second term in the numerator and the second and third terms in the denominator in Eq. (11) are small compared to one. Neglecting these terms with respect to one, we get

$$\beta_E=\nu(\ln g-\ln g_0), \tag{13}$$

where $g_0=\sigma_0(e^2/h)^{-1}$. This expression agrees with results of Davis *et al.*[8] and Liu and Soonpaa.[5] It is interesting to note that the electric-field scaling function β_E is related to the critical hopping distance, since $(\ln g-\ln g_0)$ is directly proportional to the critical hopping distance.

III. RESULTS AND DISCUSSION

In this section, we compare our theoretical calculations with the resistivity experiments of oxide based $S/N/S$ junctions. Recently, Kabasawa *et al.*[1] have measured the resistivity as a function of temperature and electric field in $S/N/S$ junctions where they have taken S as HBCO and N as PBCO. The thickness of PBCO was 80 nm. The current path at the junction area is parallel to the CuO plane and perpendicular to the c axis. Because of this, the PBCO films act like QTD systems. Singh *et al.*[2] were able to explain the temperature-dependent resistivity results of PBCO *thin films* by using their theory of variable-range hopping developed for quasi-two-dimensional systems. They did not explain the

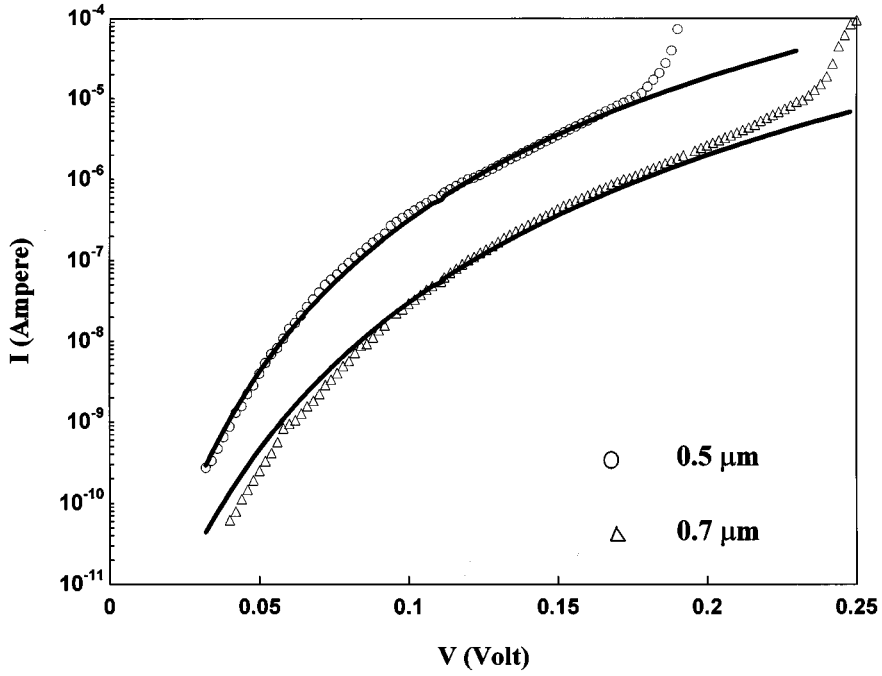

FIG. 2. Plot of log$I$ versus V for PBCO-based $S/N/S$ junctions. Symbols ○ and △ denote the experimental data for samples $L=0.7$ μm and $L=0.5$ μm respectively. Solid curves represent the theoretical results for both samples.

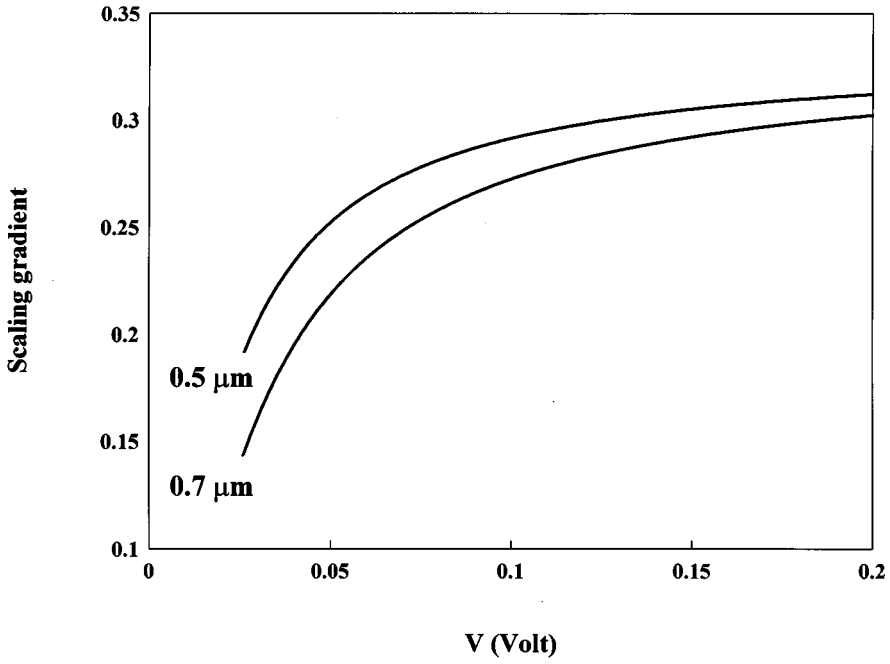

FIG. 3. Plot of the scaling gradient as a function of voltage for PBCO-based $S/N/S$ junctions. Solid and dotted curves represent the theoretical results for samples $L=0.7$ μm and $L=0.5$ μm respectively.

electric-field-dependent data since their expression was not valid for these experiments. Kabasawa *et al.*[1] and Tarutani *et al.*[1] have plotted their electric-field-dependent conductivity experiments in the form of current versus voltage curves. Therefore, we use the expression $I=I_0\exp(-R_{nn}^0)$ to explain the above experimental results where R_{nn}^0 is given by Eq. (8). This expression has been used by many authors in the literature.[1,5,6,9,10]

By using a suitable value of α and T_0^M we compared our theory with the temperature- and electric-field-dependent conductivity experiments. The results are presented in Figs. 1 and 2. A good agreement between theory and experiment is obtained for $\alpha^{-1}=8.5$ nm and $T_0^M=4.3\times10^4$ K. The temperature versus resistivity data was taken from the work of Kabasawa *et al.*[1] The current versus voltage data for samples $L=0.7$ μm and $L=0.5$ μm was taken from the work of Kabasawa *et al.*[1] and Tarutani *et al.*[1] respectively. By using the above values of localization length and T_0^M, we obtained the constant density of states at the Fermi level as $D_0=9.1\times10^{34}$ J^{-1} m^{-2}.

Kabasawa *et al.*[1] have reported in their paper a value of α^{-1} equal to 8.5 nm. This value has also been used by Singh *et al.*[2] to explain the temperature-dependent conductivity of PBCO film. Kastener *et al.*[4] have reported $\alpha^{-1}=1.0$ nm for Li-doped single crystals and ceramics of $La_{2-y}Sr_yCuO_4$. In conventional insulating barrier and amorphous semiconductors, the value of α^{-1} is on the order of a few tenths of a nm and about one nm respectively. Our value of α^{-1} is the same as that of Kabasawa *et al.*[1] and Singh *et al.*[2] Recently Singh *et al.*[2] have reported that in PBCO *thin films* the value of T_0^M is equal to 2.7×10^4 K. On other hand Kabasawa *et al.*[2] have reported the value of T_0^M to be 7.6×10^4 K in PBCO-based $S/N/S$ junctions. Our value of T_0^M lies between these two values and therefore does not agree with that of Kabasawa *et al.*[1] It may be that they have used a three-dimensional VRH expression of conductivity with some modifications to fit their data. It is worth noting that there is deviation of the calculated curve from the experimental curve for large $V>0.2$ V. This may be due to the neglect of the effect of electron-electron interaction in our theory. It is known that electron-electron interaction plays an important role in the PBCO compounds at low temperatures.[2] It has been shown by Shklovskii that at low temperatures and at high electric fields the electron-electron interaction plays an important role in three-dimensional systems. In the future we are planning to study the effect of electron-electron interaction on the electric-field-dependent variable-range hopping in PBCO based $S/N/S$ junctions.

Let us define a function $\partial\beta_E/\partial \lg$ and call it the *scaling gradient*. As one can see from Eq. (13) for $\beta\gg1$, the scaling gradient is constant and equal to ν. This agrees with the results of Liu and Soonpaa and others.[5] On the other hand, for the general value of β, the scaling gradient can be obtained from Eq. (11) and it is written as

$$\frac{\partial\beta_E}{\partial \lg}=\nu\left(\frac{1+\frac{\Lambda}{2}}{1+\frac{\pi}{4\beta}+\frac{\pi\eta}{2\beta}}\right). \tag{14}$$

It is clear that the scaling gradient is not constant but rather depends on the electric field. To find the value of ν, experimentalists generally try to report their data in the form of β_E versus lng curves.

In Fig. 3, we plotted the scaling gradient versus V for samples of $L=0.7$ μm and $L=0.5$ μm PBCO-based $S/N/S$ junctions. We used the values $T_0^M=4.3\times10^4$ K and $\alpha^{-1}=8.5$ nm which are given above. The scaling gradient for the junction varies between 0.15 and 0.3 in the experimental range of voltage. One can see from the figure that as V (i.e., β) increases the value of the scaling gradient also increases. It is also interesting to see that as V (i.e., β) increases the difference between the two curves decreases and

at very large values of V both curves will reach the highest value of scaling gradient. In other words, at very large values of V (i.e., β) the scaling gradient becomes independent of electric field.

ACKNOWLEDGMENT

One of the authors (M.S.) is thankful to NSERC of Canada for financial support in the form of a research grant.

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