Modeling and simulation of frictional contacts in multi-rigid-body systems

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Abstract. Fictional contacts occur in many mechanical systems, and often affect their dynamic response, since the collisions cause a significant change the systems' characteristics, namely in terms of velocities. This work describes and compared different formulations to handle frictional contacts in multi-rigid-body dynamics. For that, regularized and non-smooth techniques are revisited. In a simple manner, the regularized methods describe the contact forces as a continuous function of the indentation, while the non-smooth formulations use unilateral constraints to model the contact problems, which prevent the indentation from occurring. The main motivation for the performing this study came from the permanent interest in developing computational models for the dynamic modeling of contact-impact events under the framework of multibody systems methodologies. The problem of modeling and simulating contacts with friction in multibody systems includes several steps, the definition of the contact geometry; the determination of the contact points; the resolution of the contact itself; and the evaluation of the transitions between different contact regimens. The last two aspects are investigated in this work within the context of contact dynamics. In the sequel of this process, an application example is utilized to show the effectiveness of the modelling process of contact problems in multibody systems. Finally, future developments and new perspectives for further developments related to contact-impact problems are presented and discussed in this study.

Key words: Frictional Contacts, Contact Dynamics, Contact Detection, Contact Resolution, Regularized Methods, Non-smooth Techniques, Linear Complementarity Problem, Multibody Dynamics.

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1 Introduction

By and large, frictional contacts involves the problem of the modeling the interaction of colliding bodies in the presence of frictional phenomena. This discipline is often named as contact dynamics and deals with the motion of multibody systems subjected to contact-impact forces/impulses. Contact dynamics is omnipresent in many multibody applications, and in most of the cases the function of the systems depends on contact modeling process. Over the last decades, contact dynamics has been one of the most challenging and demanding areas of research in engineering that play a crucial role in vehicle systems, robotics, railway models, mechanisms and machinery, biomechanics, granular systems, toys, just to mention some examples under the framework of multibody systems[1-4].

Contact-impact events are complex phenomena characterized by short duration and high forces that cause rapid changes or discontinuities in systems' velocities and eventually energy dissipation [5]. The key ingredients of the modeling process of a frictional contact problem include the several issues that can be condensed in two independent steps, chiefly the determination of the contact points and the evaluation of the contact forces/impulses [6]. The determination of the contact points focuses on the resolution of two main issues: the geometric description of the contacting surfaces, and the detection of the potential contact points. In turn, the resolution task can be performed used two main approaches in dynamical systems: the regularized approaches [7], and the non-smooth models [8].

This work aims at analyzing the main aspects related to the modeling problem of frictional contacts in multibody dynamics. The emphasis of the study in on the regularized approaches and non-smooth formulations, where the fundamental issues associated with each technique are highlighted when treating collisions. Discussion of the extensive literature on computational schemes for contact detection is beyond the scope of this study. Anyway, the evaluation of the geometry of contact and search for contact (contact detection) is the same regardless of the choice of the technique utilized to handle the contact interaction between the colliding bodies (contact resolution), being based on regularized or non-smooth methods.

2 Techniques to model frictional contacts

There are two main techniques to solve problems, namely the regularized approaches (continuous methods) and the non-smooth formulations (piecewise methods). In the former techniques, also known as compliance or elastic methods, the contacting bodies are considered to be deformable at the contact zone, and the contact forces can be expressed as a continuous function of the local deformation between the contacting surfaces. In the non-smooth formulations, also called instantaneous or rigid methods, the contacting bodies are assumed to be truly rigid, and the contact dynamics is resolved by applying unilateral constraints in order to avoid the penetration from occurring.

The regularized approaches are quite important in the context of multibody dynamics because of their good efficiency and extreme simplicity to be implemented. In some circumstances, numerical problems can arise, resulting from bad conditioned system matrices [9]. The transition between contact and non-contact situations can easily be handled from the system configuration and contact kinematics. With the regularized methods, the contact forces include spring-damper elements to prevent interpenetration from occurring. In the regularized approaches, the location of the contact point does not coincide in the contacting bodies, and a large number of potential contact points exists, being the actual contact point the one associated with the maximum indentation. The pseudo-penetration plays a key role as it is utilized to calculate the contact forces according to an appropriate constitutive law [10]. The existence of friction in the continuous methods can easily be incorporated by considering any regularized friction model [11].

Assuming that the contacting bodies are absolutely rigid, as opposed to locally deformable bodies as in the regularized approaches, the non-smooth formulations resolve the contact problems using unilateral constraints to determine impulses to avoid penetration from occurring. The central idea of the non-smooth formulations is the non-penetration condition that prevents bodies from moving toward each other and not apart [12]. A complementarity formulation is used to describe the relation between the contact force and gap distance at the contact point. Such unilateral constraint does not permit the interpenetration of the two colliding bodies, and ensures that either contact force or gap distance is null. When the gap distance is positive (inactive contact), the corresponding contact force is null. Conversely, when the contact force is positive (active contact), the gap distance is null [13]. This formulation leads to a complementarity problem, which constitutes the rule that permits to treat multibody systems with unilateral constraints [14].

Figure 1 shows the graphical representation of the normal and tangential contact forces for the regularized approaches and non-smooth formulations. The regularized approaches and the non-smooth methods, utilized to handle contact-impact events under the framework of multibody dynamics, have inevitably advantages and disadvantages. None of these techniques can be identified as superior. In fact, a particular multibody system with collisions might be easily described by one method, nevertheless, this does not automatically implies a general predominance of that formulation in all multibody applications.

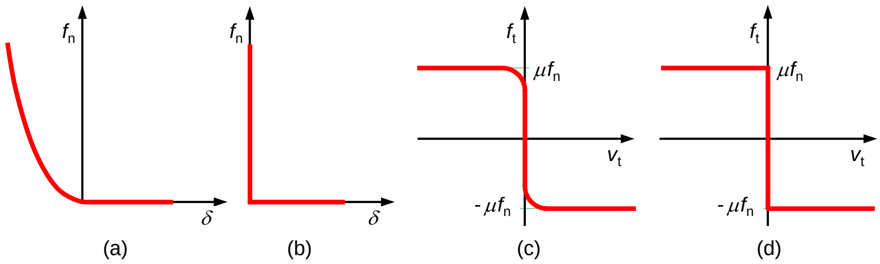

Fig. 1 **(a)** Regularized normal contact force model; **(b)** Non-smooth normal contact force model; **(c)** Regularized tangential contact force model; **(d)** Non-smooth tangential contact force model

3 Regularized methods for contact dynamics

The oldest contact force model is the one associated with Hooke's theory, which can be used when a contact is active. This regularized force model considers a linear spring to mimic the contact interaction, and can be expressed as [10]

$$f_{\mathrm{n}} = k\delta \tag{1}$$

where k represents the spring stiffness related to the contact materials, and δ is the penetration between the contacting surfaces.

A more advanced contact force model was developed by Hertz, which considers a nonlinear relation between force and penetration as [10]

$$f_{\mathrm{n}} = K\delta^{n} \tag{2}$$

where the nonlinear exponent, n, is typically equal to 3/2. The contact stiffness, K, can be determined analytically as function of the material properties and geometry of the contacting surfaces.

Hunt and Crossley presented a contact force model that associates as nonlinear spring with a nonlinear damper in parallel to mimic the contact interaction. This force model can be expressed as [15]

$$f_{\mathrm{n}} = K\delta^{n}\left[1+\frac{3(1-c_{\mathrm{r}})}{2}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{3}$$

where the first term is the nonlinear elastic Hertz's law, and the second term is the dissipative parcel, being c_{r} the coefficient of restitution, $\dot{\delta}$ represents the contact velocity, and $\dot{\delta}^{(-)}$ is the contact normal velocity at the initial instant of impact.

The most popular contact force model in the multibody dynamics community is the one proposed by Lankarani and Nikravesh [5], which was developed with basis on the hertzian contact theory and on the damping approach by Hunt and Crossley, and can be is written as

$$f_{\mathrm{n}} = K\delta^{n}\left[1+\frac{3(1-c_{\mathrm{r}}^{2})}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{4}$$

in which is valid for collisions with high values of the coefficient of restitution, that is, this model is applicable to elastic impacts.

More recently, Flores et al. [16] described a contact force model applicable to the entire domain of the possible values for the coefficient of restitution, which is given by

$$f_{\mathrm{n}} = K\delta^{n}\left[1+\frac{8(1-c_{\mathrm{r}})}{5c_{\mathrm{r}}}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{5}$$

Over the last decades, a good number of contact force models have been presented in the literature, being the interested reader in specific information referred to the following references [10].

The most well-known friction force model is undoubtedly the one represented by Coulomb's law, which can be expressed as [17]

$$f_{\mathrm{t}} = \begin{cases} \leq \mu_{\mathrm{s}} f_{\mathrm{n}} & \text{if } v_{\mathrm{t}} = 0 \\ \mu_{\mathrm{d}} f_{\mathrm{n}} \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} \neq 0 \end{cases} \tag{6}$$

$$\operatorname{sgn}(v_{\mathrm{t}}) = \begin{cases} 0 & \text{if } \|\mathbf{v}_{\mathrm{t}}\| = 0 \\ \dfrac{v_{\mathrm{t}}}{\|\mathbf{v}_{\mathrm{t}}\|} & \text{if } \|\mathbf{v}_{\mathrm{t}}\| \neq 0 \end{cases} \tag{7}$$

in which μ_{s} and μ_{d} represent the static and dynamic coefficient of friction, respectively, f_{n} denotes the normal contact force, and v_{t} is the relative contact tangential velocity of the contacting elements.

Threlfall [18] proposed a regularized friction force model that does not present discontinuities, which can be written as

$$f_{\mathrm{t}} = \begin{cases} \mu_{\mathrm{d}} f_{\mathrm{n}} \left(1 - \mathrm{e}^{-\frac{3v_{\mathrm{t}}}{v_0}}\right) \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} \leq v_0 \\ 0.95 \mu_{\mathrm{d}} f_{\mathrm{n}} & \text{if } v_{\mathrm{t}} > v_0 \end{cases} \tag{8}$$

where v_0 is the threshold velocity.

Bengisu and Akay [19] presented an alternative friction force model as

$$f_{\mathrm{t}} = \begin{cases} \left[-\dfrac{\mu_{\mathrm{s}} f_{\mathrm{n}}}{v_0} \left(\|\mathbf{v}_{\mathrm{t}}\| - v_0 \right)^2 + \mu_{\mathrm{s}} f_{\mathrm{n}} \right] \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} \leq v_0 \\ \left[\mu_{\mathrm{d}} f_{\mathrm{n}} + \left(\mu_{\mathrm{s}} f_{\mathrm{n}} - \mu_{\mathrm{d}} f_{\mathrm{n}} \right) \mathrm{e}^{-\kappa\left(\|\mathbf{v}_{\mathrm{t}}\| - v_0\right)} \right] \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} > v_0 \end{cases} \tag{9}$$

where κ is a positive parameter representing the negative slope of the sliding state.

Ambrósio [20] proposed another regularized approach for the Coulomb's law that includes a ramp to avoid the numerical difficulties, which can be expressed as

$$f_{\mathrm{t}} = c_{\mathrm{d}} \mu_{\mathrm{d}} f_{\mathrm{n}} \operatorname{sgn}(v_{\mathrm{t}}) \tag{10}$$

$$c_{\mathrm{d}} = \begin{cases} 0 & \text{if } v_{\mathrm{t}} < v_0 \\ \dfrac{v_{\mathrm{t}} - v_0}{v_1 - v_0} & \text{if } v_0 \leq v_{\mathrm{t}} \leq v_1 \\ 1 & \text{if } v_{\mathrm{t}} > v_1 \end{cases} \tag{11}$$

in which the dynamic correction factor, c_{d}, prevents that the friction force changes direction for almost null values of the relative tangential velocity.

The use of the friction force models given by Eqs. (8), (9) and (10) has the advantage of allowing the numerical stabilization of the integration algorithm used during the resolution of the equations of motion for multibody systems. It must be noticed that several alternative friction force models have been proposed over last decades, being the interested readers referred to the following references [11].

4 Non-smooth formulations for contact dynamics

The equations of motion suitable appropriate to describe multibody systems involving impacts can be expressed at the velocity level as [8]

$$\mathbf{M}\mathrm{d}\mathbf{u} - \mathbf{h}\mathrm{d}t - \mathbf{w}_{\mathrm{n}}\mathrm{d}\mathbf{P}_{\mathrm{n}} - \mathbf{w}_{\mathrm{t}}\mathrm{d}\mathbf{P}_{\mathrm{t}} = \mathbf{0} \tag{12}$$

where $\mathbf{M}$ is the positive definite and symmetric mass matrix, $\mathbf{h}$ represents the vector of all external and gyroscopic forces acting on the system, $\mathbf{w}_{\mathrm{n}}$ and $\mathbf{w}_{\mathrm{t}}$ are the generalized normal and tangential force directions. The measure for the velocities $\mathrm{d}\mathbf{u} = \dot{\mathbf{u}}\mathrm{d}t + (\mathbf{u}^{+} - \mathbf{u}^{-})\mathrm{d}\eta$ is split in Lebesgue measurable part $\dot{\mathbf{u}}\mathrm{d}t$, that is continuous, and the atomic parts, which occur at the discontinuity points with the left and right limits $\mathbf{u}^{-}$ and $\mathbf{u}^{+}$ and the Dirac point measure $\mathrm{d}\eta$. For impact free motion it holds that $\mathrm{d}\mathbf{u} = \dot{\mathbf{u}}\mathrm{d}t$. Similarly, the measure for the so-called percussions corresponds to a Lagrangian multiplier which gathers both finite contact forces, $\boldsymbol{\lambda}$, and impulsive contact forces, $\boldsymbol{\Lambda}$, that is, $\mathrm{d}\mathbf{P}=\boldsymbol{\lambda}\mathrm{d}t+\boldsymbol{\Lambda}\mathrm{d}\eta$ [21].

Let consider a multibody system with a total f of unilateral constraints, which can be represented by f inequalities as

$$g_{\mathrm{n}_i}\left(\mathbf{q},t\right) \geq 0 \qquad (i = 1, \ldots, f) \tag{13}$$

in which the quantities $g_{\mathrm{n}i}$ are the normal gap functions of the contacts. They are formulated such that, $g_{\mathrm{n}i}>0$ indicates an inactive, $g_{\mathrm{n}i}=0$ corresponds to an active contact, and $g_{\mathrm{n}i}<0$ indicates the forbidden interpenetration between rigid bodies.

The normal and tangential relative velocities at the contacts as [13]

$$\gamma_{\mathrm{n}i} = \mathbf{w}_{\mathrm{n}i}^{\mathrm{T}}\mathbf{u} + \tilde{w}_{\mathrm{n}i} \tag{14}$$

$$\gamma_{\mathrm{t}i} = \mathbf{w}_{\mathrm{t}i}^{\mathrm{T}}\mathbf{u} + \tilde{w}_{\mathrm{t}i} \tag{15}$$

The equations of motion (12) can be complemented with constitutive laws for normal and tangential contact-impact forces, for that, a unilateral version of the Newton's impact law is considered for the normal direction with local coefficient of restitution $\varepsilon_{\mathrm{n}i}$. The Coulomb's friction law is used for the tangential direction with coefficient of friction μ_i, which is complemented by a tangential coefficient of restitution $\varepsilon_{\mathrm{t}i}$. Normal and tangential impact laws can be stated as inclusions

$$-\mathrm{d}\mathbf{P}_{\mathrm{n}i} \in \mathrm{Upr}\left(\xi_{\mathrm{n}i}\right) \tag{16}$$

$$-\mathrm{d}\mathbf{P}_{\mathrm{t}i} \in \mu_i \mathrm{d}\mathbf{P}_{\mathrm{n}i}\mathrm{Sgn}\left(\xi_{\mathrm{t}i}\right) \tag{17}$$

with

$$\xi_{\mathrm{n}i} := \gamma_{\mathrm{n}i}^{+} + \varepsilon_{\mathrm{n}i}\gamma_{\mathrm{n}i}^{-} \tag{18}$$

$$\xi_{\mathrm{t}i} := \gamma_{\mathrm{t}i}^{+} + \varepsilon_{\mathrm{t}i}\gamma_{\mathrm{t}i}^{-} \tag{19}$$

The complete description of the dynamics of non-smooth system, which accounts for both impact and impact-free phases, is given by Eqs. (12)-(19). This problem can be solved by using the Moreau's time-stepping method [13].

5 Demonstrative example of application

Figure 2 shows a hexapod system, which consists of one rigid, load carrying mainframe with six legs, similar and symmetrically distributed. Each leg is composed by four links, interconnected by four revolute joints and attached to the main body by means of a fifth revolute joint. Revolute motors and linear actuators accomplish traction movement and elevation, respectively. Two representative virtual simulations are presented in order to study the behavior of the movement characteristics of the proposed legged robot model. In the first simulation, a straight path on a planar, horizontal and non-rough surface is considered. The second one deals with climbing a standard set of stairs (height of 170 mm, deep of 280 mm). In the former simulation it was considered both static and dynamic stability, while in the later only static stability was used to simulate the motion. Figure 2 shows an animation sequence of the virtual simulation that corresponds to the second situation.

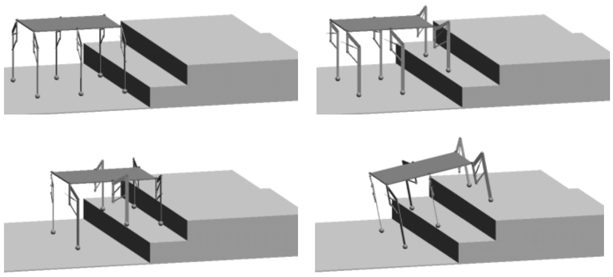

Fig. 2 Snapshots of the hexapod robot system climbing a set of stairs

One crucial issue deals with the selection of the time step used to perform the computational analysis, since it plays a key role in the contact detection and, hence, on the contact-impact forces that can be artificially large and affect the outcomes. Secondly, the identification of the contact parameters, namely in terms of restitution and friction coefficients is also of paramount importance in order to properly handle the different contact regimens and the transition between them.

6 Concluding remarks

In this work, the problem of modeling frictional contact problems in dynamical systems was revisited. The regularized and non-smooth approaches were considered. A hexapod system was used as a demonstrative example of application. It was clear the contact dynamics is complex problem, requiring more research to reach better models and approaches. Future research can include the development of new algorithms to deal with contact analysis and systems with contact transition regimens. Investigation on parameters identification and estimation, using the input date from physical experiments to drive the simulation of uncertain model parameters will be also a potential future direction for further research.

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