A Systematic Literature Review of Quantum Computing for Routing Problems

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ABSTRACT Quantum Computing is drawing a significant attention from the current scientific community. The potential advantages offered by this revolutionary paradigm has led to an upsurge of scientific production in different fields such as economics, industry, or logistics. The main purpose of this paper is to collect, organize and systematically examine the literature published so far on the application of Quantum Computing to routing problems. To do this, we embrace the well-established procedure named as Systematic Literature Review. Specifically, we provide a unified, self-contained, and end-to-end review of 18 years of research (from 2004 to 2021) in the intersection of Quantum Computing and routing problems through the analysis of 53 different papers. Several interesting conclusions have been drawn from this analysis, which has been formulated to give a comprehensive summary of the current state of the art by providing answers related to the most recurrent type of study (practical or theoretical), preferred solving approaches (dedicated or hybrid), detected open challenges or most used Quantum Computing device, among others.

INDEX TERMS Quantum computing, quantum annealer, quantum gates, IBM, DWAVE, traveling salesman problem, vehicle routing problem, routing problems.

I. INTRODUCTION

In the optimization and transportation communities, routing problems are intensively studied [1]–[3]. Two are the main reasons behind the interest in this specific topic: *i*) their high computational complexity, making them difficult to tackle even in medium-sized instances [4]; and *ii*) their proven applicability on business and logistics real-world scenarios, as well as on leisure and tourism-based problems [5]. In other words, advances in the design of efficient routing problem algorithms derive into business and social benefits. This situation is the reason for the upsurge of intelligent methods designed for dealing with routing problems.

Regarding the complexity of routing problems, it is important to highlight that most solving algorithms demand notable computational resources. In fact, some modern computers are completely unable to embrace brute force techniques for even relatively small instances. In consequence, a plethora of time-efficient solving schemes have been developed along the

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last decades, being heuristic and metaheuristic techniques the most resorted approaches.

Up to now, the vast majority of intelligent solvers have been conceived for their development and execution in classical computation resources. However, as an alternative to these classical devices, Quantum Computing (QC, [6]) has emerged as a promising paradigm for tackling optimization and routing problems. Deemed as the next frontier in computation, QC is a hot topic in the current scientific community. This is so because of its potential power in terms of performance, bringing a remarkable advantage in complex optimization problems [7].

In a nutshell, a quantum computer consists of a device which performs computation by leveraging quantum mechanical phenomena. Unlike classical computers, quantum devices work with an information unit coined as qubit [8], which can hold much more information than a classical bit. In particular, a qubit can be both 1 and 0 at the same time, overcoming limitations imposed by the classical binary representation.

Today, two QC paradigms coexist: the *annealing-based* and the *gate-model* quantum computers [9], [10]. The former

ones are characterized by performing a process known as *quantum annealing*, which returns the state of minimum energy of a given quantum Hamiltonian (i.e., energy function). In optimization problems, the quantum Hamiltonian is defined according to the fitness function of the problem, so that the lower energy state represents the solution to the problem at hand [11]. *Quantum-gate* devices in contrast are characterized by employing quantum circuits (i.e. operations) named as quantum gates, which can be compared to the classical logic gates in traditional circuits. Thus, quantum-gates are sequentially applied to qubit states manipulating them up to reaching the final solution of the problem [12].

Having said this, a closer examination in the most renowned databases unveils that scientific studies revolving around QC applied to routing problems are continuously growing. The activity is significant and steady, hence perfectly illustrating the current condition of the community. This accumulation of material establishes an ideal breeding ground for the conduction of a reference manuscript that systematically summarizes all the works already done. This situation has fueled the performance of this paper, motivating its main goal: to provide a unified, self-contained and end-to-end review of the research done in QC focused on routing problems.

An efficient way to properly synthesize the research that has recently been done is through a Systematic Literature Review (SLR, [13], [14]). Embracing the definition provided in [13], a SLR is a procedure for evaluating and interpreting relevant works related to a specific topic, phenomenon, or research question. For this reason, a SLR can be deemed as a mean for aggregating the experience obtained from a significant collection of studies [15]. Thus, a SLR constitutes a rigorous and fair procedure for selecting and analyzing all the scientific material that answers a group of well-defined and specific research questions. In this kind of studies, an special analysis is carried out on the meta-data of each selected paper, in order to obtain findings such as the most cited works, the geographical situation of the research community, or the type of papers preferred by practitioners. As can be read in works such as [16], one of the advantages of SLRs is its replicability, as well as its adequacy for depicting the past and current status of a certain research field. Another advantage of SLRs is their inherent facility for being extended with additional studies.

With all this, we adopt the SLR mechanism for analyzing the work already done in the confluence of QC and routing problems fields, leading to a comprehensive survey carried out through a meta-analytic approach. Thus, we systematically organize all the studies conducted up to date on this specific topic, with the main intention of *i*) summarizing their features, *ii*) understanding their contributions and *iii*) highlighting principal authors and relevant institutions. This kind of paper is also valuable for identifying diverse domains that are likely to receive further attention in the near-term future.

In general, the present review covers a lapse of 18 years. To perform this work, we have considered four of the most reputed scientific databases (Google Scholar, Scopus, IEEE and Web Of Science), resulting in a collection of 53 documents published in form of journal or conference papers, book chapters, PhD thesis, Master thesis and reports.

The remainder of this manuscript is structured as follows: in Section II we deeply describe the research protocol followed for the conduction of this SLR, contributing in this way to its replicability and extension. Furthermore, we highlight the main research question of our SLR, as well as the secondary questions that will be answered along the whole paper. Section III is the central part of this work, outlining the published works in this research area under six different criteria. After this section, a critical analysis of published milestones and influential papers is provided in Section IV. Finally, Section V concludes our survey with a summary of the main conclusions with views to the future of this exciting field.

II. RESEARCH PROTOCOL

In order to build a valuable SLR, some formal steps must be necessarily followed, conceiving what is known as *research protocol*. Specifically, this procedure is composed of the following phases: *i*) establish the main and secondary research questions, *ii*) formulate the search key, *iii*) select the most appropriate scientific databases, *iv*) set the inclusion/exclusion rules, and *v*) report the phases of the survey.

Thus, sticking to the aforementioned protocol, the first step is to clearly define the scope of this review: to thoroughly examine the work conducted on QC applied to routing problems in the last two decades. Additionally, we are interested on characterizing the types of problems that are most frequently addressed, as well as the algorithms and quantum processors employed in its resolution. Furthermore, we also intend to discern those most recurring types of study in the literature (in both theoretical or experimental approaches), and which are the institutions and countries paying more attention to this novel paradigm.

Therefore, our main research question (**MQ**) is formulated as follows:

MQ: "*In the whole twenty-first century, what are the principal research works conducted around quantum computing applied to routing problems?*"

Starting from **MQ**, several secondary questions arise (**SQ**). These **SQ** are essential to nourish the review with valuable information. Furthermore, **SQ**s are also necessary to guide the whole research procedure, offering the guiding light for finding and selecting relevant documents. In the SLR presented in this manuscript, we have defined the following 8 secondary questions:

SQ1: "*Which is the temporal progression of the field under study?*"

SQ2: "*Which are the main routing problems faced by the researchers?*"

SQ3: "*What kind of studies (theoretical or practical) are the most preferred ones by authors?*"

TABLE 1. Scientific databases considered in this SLR.

Database	URL
Google Schoolar	https://scholar.google.es/
Scopus	https://www.scopus.com/search/form.uri?display=basic#basic
IEEE Xplore Digital Library	https://ieeexplore.ieee.org/
Clarivate Analytics - Web of Science	https://www.webofscience.com/wos/alldb/basic-search

SQ4: "*Which type of algorithmic schemes are the most used ones?*"

SQ5: "*What is the most employed quantum computer?*"

SQ6: "*Which are the most cited papers of the field?*"

SQ7: "*Which authors, institutes and regions are the most prolific ones?*"

SQ8: "*Which are the main future challenges detected by the community?*"

For building a rigorous and comprehensive SLR, appropriate search keys (**SK**) must be constructed for being used in the engines of the digital scientific databases.

Inclusive **SK**s are required to write a comprehensive SLRs. For this purpose, an empirical experimentation has been implemented by testing different configurations, far beyond the simple combination of the logical operators along with the topic keywords. The resulting **SK** is the following

SK: ("Quantum Computation" **OR** "Quantum Computing" **OR** "Quantum Annealing" **OR** "DWAVE" **OR** "D-WAVE" **OR** "Quantum Gate" **OR** "Qiskit") **AND** ("Routing Problem" **OR** "Traveling Salesman Problem" **OR** "Vehicle Routing Problem" **OR** "TSP" **OR** "VRP").

Thanks to this **SK**, we can reach studies exploring both quantum paradigms (*annealing-based* and *gate-model* quantum computers), and also a wide variety of routing problems, with the Traveling Salesman Problem (TSP, [17], [18]) and Vehicle Routing Problem (VRP, [19], [20]) as the most representative ones.

The next step of the SLR after the definition of the **MQ**, **SQ**s and the **SK** is the selection of the scientific databases. These databases must meet two crucial requisites: *i*) they need to be exhaustive enough to incorporate studies coming from different sources, such as conferences, books, journals or reports; and *ii*) they must provide open and configurable search engines that admit the input of the established **SK**. We summarize the four databases considered in this SLR in Table 1. Furthermore, it is important to highlight here that ACM Digital Library[1] has not been deemed because of the same reasons depicted in previous studies such as [16]. In a few words, this search engine does not efficiently deal with the principal components of the search key, providing less satisfactory results in comparison to the other four scientific databases.

In any case, despite the efforts made to define an accurate and restrictive **SK**, the amount of works returned are usually immense. To fix possible inaccuracies, once the search has been performed over the scientific databases, additional exclusion/inclusion rules are applied in order to clearly define which works fall under the specific umbrella of the SLR. In this review, we have used the following criteria:

1) Papers must be clearly fall within the field of quantum computing, excluding works framed into the knowledge branch known as *quantum-inspired evolutionary algorithms* [21].
2) Selected works must be completely focused on routing problems, either from a theoretical or an applied perspective. Articles that deal with these types of problems in a tangential way are discarded.
3) Chosen manuscripts must be written in English (at least, the title and abstract).
4) All the papers must be available for its consultation on any of the selected databases. Furthermore, at least, the title, authors information and abstract must be openly accessible.

It is interesting to pause briefly on the first adopted criteria. In the last two decades, a research stream known as *quantum-inspired evolutionary algorithms* has been made its way into the optimization community. In a nutshell, quantum-inspired techniques are a specific class of evolutionary algorithms which base their operation on concepts and principles of quantum computing such as interference, coherence, and the qubit unit of information. Anyway, these methods are conceived "*for a classical computer rather than for quantum mechanical hardware*", as explained in [22]. That is, despite quantum-inspired evolutionary algorithms are based on quantum computation, they cannot be run on any quantum machine. In fact, a recent paper published by Gammanpila and Fernando [23] demonstrates that these algorithms adopt certain assumptions that make impossible to being directly handled by any quantum computer. Consequently, papers modelling quantum-inspired evolutionary algorithms are excluded from this SLR.

Finally, and based on the protocol described in this section, we have performed the following steps for fairly conduct our SLR: *i*) *search in selected scientific databases*, in which we have applied our defined **SK** and gathered all the resulting papers for further analysis; *ii*) *filtering process*, which objective is to discard all papers that are not compliant with our inclusion/exclusion criteria; and *iii*) *manual search*, in which new works coming from the bibliographic references of the already selected papers are either included or discarded.

III. ANALYSIS OF THE CONTRIBUTIONS GATHERED

This section is devoted to the presentation and analysis of the collection of papers contributing to this SLR. This section is, in turn, divided into different subsections and sorted to answer independently the **SQ**s specified above. Thus, Section III-A is associated with **SQ1**. Additionally, Section III-B is related to **SQ2**, while **SQ3**, and **SQ4**, and **SQ5** are tackled in Section III-C. Furthermore, Section III-D addresses **SQ6**, and

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[1]https://dl.acm.org/

the **SQ7** is answered in III-E. The last secondary question **SQ8** is tackled in III-F. Finally, we conclude this section focusing our attention on our main research question **MQ** in Section III-G.

A. COLLECTION AND PUBLICATION TIMELINE

Adopting the research protocol above described, first step was the collection of papers. Here are the results retrieved in November 2021, which total these quantities of papers:

- Google Scholar: 975 papers.
- Scopus: 764 papers.
- IEEE Xplore Digital Library: 20 papers.
- Clarivate Analytics - Web of Science: 50 papers.

From this initial collection, we proceeded to eliminate duplicates and works not meeting the 3rd and 4th exclusion/ inclusion clauses. After that, we discarded works unrelated to routing problems and quantum computing (2nd exclusion/inclusion clause). From this process, a collection of 95 papers was selected. From this filtering process, we strictly applied our defined 1st inclusion/exclusion criterion. The application of this step led us to the elimination of 42 papers, yet working on routing problems, implement quantum-inspired evolutionary algorithms. For this reason, the total amount of works contributing to this SLR was reduced to 53.

Table 2 shows the whole collection of works remaining after the complete procedure. These works are listed chronologically, using the publishing date (in case of journal papers, pre-prints, PhD Thesis and Master Thesis) and event celebration date (in case of conferences) as reference. For each work, identification, reference, year and type of paper are added. In this last category, we consider journals (Jour), conferences (Conf), PhD Thesis (PhD), Master Thesis (MsT) and pre-prints (Prepr) published in reputed repositories, such as `arXiv`.[2] Additional aspects of these papers will be further analyzed in the following sections.

By analyzing Table 2 one can detect the date of the very first published study (in this case, a journal paper from 2003) which originated the subsequent flow of studies. Nevertheless, the impact was low during the next years, supported by the absence of works published in 2010, 2014, 2015 and 2016. After the three-year hiatus among 2014 and 2016 (both included), the field experienced a remarkable upsurge with a growth maintained up to the present days. The main reasons of this resurgence are the substantial technological advances implemented on hardware by companies such as IBM or DWAVE in the building of quantum computers, and the democratization of these resources to any researcher and developer (under certain conditions). In fact, the most used developing platforms, Qiskit of IBM and Leap from DWAVE, were launched in March 2017 and October 2018, respectively, coinciding with the increase of works focused on QC and routing problems (see Fig. 1). Additionally, note that 25 papers (47,16%) have been published in scientific
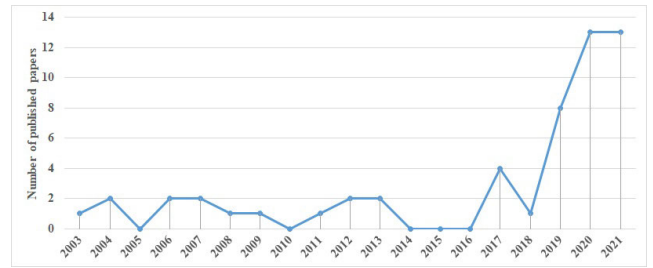
[2]https://arxiv.org/

FIGURE 1. Timeline of the collected papers.

journals, 17 (32,07%) in international conferences, and 9 (16,98%) in reputed scientific repositories, something understandable considering the immaturity of the field of study. Finally, and aiming at answering the first of the secondary research questions (**SQ1**), we depict in Fig. 1 the timeline of the works under study. This figure shows that the most prominent interval is that between the 2019 and 2021, comprising the 64,15% of the complete collection.

B. TYPES OF PROBLEMS STUDIED

Historically, two of the most studied routing problems are the aforementioned TSP and VRP. These two problems, and all their variants, are the focus of a significant number of studies in the literature. Their solving complexity, along with their easy formulation, flexibility to add mathematical constraints, and wide applicability to real-world problems have led both TPS and VRP to receive a constant and moderate growth for the last 5 decades.

This situation is also evident in the work proposed in this survey, in which the vast majority of the papers hybridizing QC and routing problems deal with the TSP, VRP or any of their many variants. Table 3 summarized all gathered papers sorted according to the problem they solve.

As can be deduced from Table 3, 32 (60,37%) of the identified papers are built for the TSP problem. Most of these works make use of the canonical version of the problem, whereas some papers tackle any variant such as the Asymmetric TSP [70] or the TSP with Time Windows [75]. Regarding the topic of these papers, 29 are used with benchmaking purposes, while the rest are conceived to deal with real-world applications. We can see examples in industrial robotics [65], [69], Smart Cities [72] and UAV planning [33].

In turn, VRP-related works make up 24,52% of the total compilation (13 out of 53), being the canonical formulation the most recurrent one. Other interesting problem versions are also posed, such as the formulation known as Social Worker in [41] and [46], or the VRP with Balanced Pick-up in [60]. Further studied variants are the large scale VRP in [57] and the VRP with time restrictions, state, and capacity in [71].

Furthermore, we have found 11 other studies revolving around routing problems that fall outside the umbrella of TSP and VRP. Here, we can distinguish optimization problems

TABLE 2. Details of the whole collection of papers gathered. Jour: journal; Conf: conference; Prepr: preprint; PhD: PhD Thesis; MsT: Master Thesis.

ID	Reference	Year	Type	ID	Reference	Year	Type
1	Moser [24]	2003	Jour	28	Ruan et. al [25]	2020	Jour
2	Goswami et. al [26]	2004	Prepr	29	Gui et. al [27]	2020	Prepr
3	Martonak et. al [28]	2004	Jour	30	Behera & Panigrahi [29]	2020	Prepr
4	Raj & Shivakumar [30]	2006	Conf	31	Harikrishnakumar et. al [31]	2020	Prepr
5	Chen & Zhang [32]	2006	Jour	32	Narottama et. al [33]	2020	Conf
6	Dörn [34]	2007	Conf	33	Borowski et. al [35]	2020	Conf
7	Dörn [36]	2007	Conf	34	Dan et. al [37]	2020	Conf
8	Nie et. al [38]	2008	Jour	35	Krauss & McCollum [39]	2020	Jour
9	Roy [40]	2009	Jour	36	Adelomou et. al [41]	2020	Conf
10	Chen et. al [42]	2011	Jour	37	Dong & Huang [43]	2020	Jour
11	Bang [44]	2012	Jour	38	Gammanpila & Fernando [23]	2020	Conf
12	Fujimura [45]	2012	Jour	39	Adelomou et. al [46]	2020	Jour
13	Warren [47]	2013	Jour	40	Warren [48]	2020	Jour
14	Crispin & Syrichas [49]	2013	Conf	41	Harwood et. al [50]	2021	Jour
15	Heim et. al [51]	2017	Prepr	42	Siloi et. al [52]	2021	Jour
16	Moylet et. al [53]	2017	Jour	43	Botez et. al [54]	2021	Conf
17	Warren [55]	2017	Jour	44	Paler et.al [56]	2021	Jour
18	Syrichas & Crispin [57]	2017	Jour	45	McCollum & Krauss [58]	2021	Jour
19	Srinivasan et. al [59]	2018	Prepr	46	Bao et. al [60]	2021	Conf
20	Mahasinghe et al. [61]	2019	Conf	47	Atchade-Adelomou et. al [62]	2021	Prepr
21	Syrichas [63]	2019	PhD	48	Warren [64]	2021	Prepr
22	Clark et. al [65]	2019	Conf	49	Salehi et.al [66]	2021	Prepr
23	Kieu [67]	2019	Jour	50	Osaba et. al [68]	2021	Conf
24	Mehta et. al [69]	2019	Conf	51	Osaba et. al [70]	2021	Conf
25	Irie et. al [71]	2019	Conf	52	Lopes [72]	2021	MsT
26	Feld et. al [73]	2019	Jour	53	Warren [74]	2021	Jour
27	Papalistsas et. al [75]	2019	Jour				

TABLE 3. Gathered papers organized by the problem the solve.

Problem	Papers ID	Total
TSP	1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 15, 16, 17, 19, 24, 26, 27, 28, 29, 32, 34, 37, 38, 40, 44, 48, 49, 50, 51, 52, 53	32
VRP	14, 18, 21, 23, 25, 26, 30, 31, 33, 36, 39, 41, 46	13
Others	5, 6, 7, 8, 20, 22, 35, 42, 43, 45, 47	11

in graphs, such as the Hamiltonian Cycle[3] in [30], [61] or Graph Traversal problems (such as eulerian tours or optimal postman tours) in [34], [36]. Other works encompass studies dealing with the well-known shortest path problem in [39], [54] or the less studied longest path problem [58].

Finally, it is interesting to highlight those works which combine several techniques or even present the problem from a mixed technical perspective. In [34] and [36], for example, in addition to the graph traversal problems above mentioned, the TSP is also considered. Another example can be found in [73], in which the Capacitated VRP is solved by decomposing it into a TSP and a clustering problem.

It is interesting to point here that some of the works above mentioned represent the first efforts made by the community to apply the advances of quantum computing in real environments. In [69], for example, authors propose a preliminary method for the optimization of robotic assembly lines in automotive industrial plants. Authors in [65] present an approach

for optimizing non-colliding multi-robot routes, building use cases composed by two robots performing straight lines between two points. Another example can be found in [72], in which the author contextualizes its work on the routing of garbage trucks on smart cities. In any case, due to the embryonic state of the field, the adoption of quantum technology in realistic environments is still an open challenge, as it is discussed in the upcoming Section III-F. It is for this reason that the works focused on the resolution of real-world routing problems employ reduced and controlled use cases.

Lastly, and intending to answer the secondary question **SQ2** as thoroughly as possible, a timeline distributing the amount of works per problem paradigm is depicted in Fig. 2. This figure demonstrates the predominance of the TSP line of research over the years, despite VRP and other approaches are gaining attention due to the evolution of the QC resources and their open access.

C. TYPES OF STUDIES, ALGORITHMS AND QUANTUM PROCESSOR EMPLOYED

In this section, we analyze the collection of papers from three different perspectives. First, aiming to answer the above scheduled **SQ3**, we conduct a brief study in Table 4 in order to determine which kind of research work (theoretical or practical) is the most prolific in the current related literature. A deeper examination is performed in this aspect, taking advantage of the data laid out in this Table 4 along with a characterization of the publications formats employed by researchers (journal, conference, report...). Centering ourselves on the practical approaches, we later give the reader
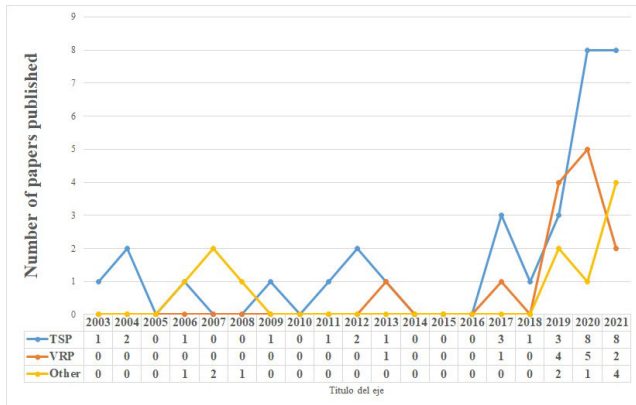
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[3]Despite the TSP can be seen as a specific formulation of the Hamiltonian Cycle problem, we have decided to preserve routing problems and application-agnostic formulations as independent entities

FIGURE 2. Timeline of the collected papers divided by the problems they solve.

TABLE 4. Gathered papers organized by the type of paper and publication sources.

	Journal	Conference	Report	PhD Thesis	Ms Thesis	Total
Theoretical	17	5	3	1	0	26
Practical	8	12	6	0	1	27
Total	**25**	**17**	**9**	**1**	**1**	**53**

insights into some figures of interest related to the type of algorithms (*Pure Quantum* and *Hybrid* solvers) and the quantum processors employed (IBM quantum-gate model, DWAVE quantum annealer and Fujitsu Digital Annealer).

Several conclusions can be drawn from Table 4. At a first glance, it is worth-mentioning the equity in the number of theoretical and practical papers. As for the publication source, conferences are preferred for practical papers whereas journals have emerged as more appropriate forum for theoretical papers.

Another interesting aspects can be inferred from Fig. 3, where practical and theoretical papers are counted on a time axis. On the one hand, and coherently with the conclusions mentioned in III-A, an extraordinary upsurge of practical papers has been witnessed during the last 3 years on account of the unleashing of open platforms such as Qiskit of IBM and Leap from DWAVE. Until the unveiling of these kind of platforms, the access to QC resources was very limited, hindering the conduction of studies in this field. On the other hand, theoretical papers have settled into a regular pace along the years.

Having answered the secondary question **SQ3**, we proceed now with a thorough analysis of the 27 practical papers, giving answer to **SQ4** and **SQ5**. For this purpose, we firstly introduce a few brief remarks about two important concepts.

On the one hand, it should be highlighted that, despite it is deemed as the next frontier in computation, QC is still in an incipient stage of development and current available computers suffer from remarkable limitations in terms of capability and performance. In this context, two types of solvers prevail in the current literature: i) *purely quantum* approaches, which aim to solve a problem using only QC
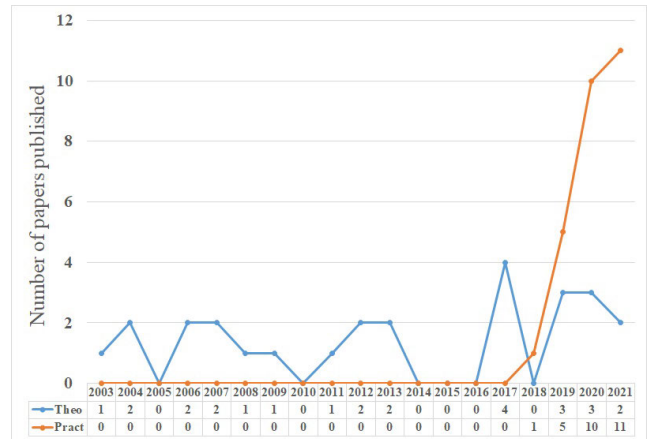

FIGURE 3. Timeline of the collected works divided by the type of paper.

resources, and *quantum-classical hybrid* algorithms, which are conceived to overcome current QC limitations.

On the other hand, as mentioned in the introduction, two QC architectures coexist in the current literature: the *annealing-based* and the *gate-model* quantum computers. The leading provider of annealers is D-Wave Systems. This company launched the *D-Wave Advantage_system1.1* device in 2020, which is accessible via D-Wave's cloud interface, coined as Leap. This device counts with a working graph of 5436 qubits and its Hamiltonian is expressed as an Ising model. Another option is the Fujitsu Digital Annealer.[4] This device, which is presented as an alternative to quantum computing technology, uses a digital circuit design inspired by quantum phenomena. Regarding gate-model quantum computers, current commercial devices have from 10 to 50 qubits in place, being IBM their most renowned provider.

Having said this, Table 5 shows the results of the second study belonging to this subsection. In this table, we list the complete collection of 27 practical papers, along with the type of solver (*Pure Quantum* and *Hybrid* solvers) and QC device picked out (IBM quantum-gate model, DWAVE quantum annealer and Fujitsu Digital Annealer).

Answering **SQ4**, an equilibrium exists between the type of solvers used by researchers, with 13 papers presenting purely quantum approaches, and 14 proposing an hybrid formulation. By taking a closer look, one can observe that researchers using the IBM machine tend to come up with purely quantum approaches (9 out of 11), while practitioners on annealers develop hybrid methods in a more frequent way (12 out of 16).

With reference to **SQ5** secondary question, DWAVE quantum annealer has been the most exploited device up to now by the community, appearing in 16 out of 27 practical papers. In addition, researchers have picked the IBM computer in 11 papers, while the Fujitsu Digital Annealer has been resorted to one single paper.

[4] https://www.fujitsu.com/es/digitalannealer/
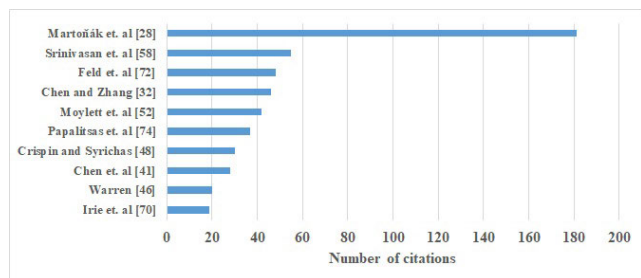
FIGURE 4. Most cited papers according to Google Scholar digital library (data gathered in January 2022).

D. NUMBER OF CITATIONS

In this section, we evaluate the impact, in the form of amount of citations, of the papers to reply to **SQ6**. Fig. 4 arranges the ten most cited works given by Google Scholar digital library as reference. We find [28] as the most cited paper with 181 citations, as a result of being the first quantum annealing approach for the TSP. The legacy of this paper has become as inspiration for many subsequent studies.

Also groundbreaking is the work presented by Srinivasan *et al.* [59], proposing the first solver for the TSP based on an IBM approach. This paper has accumulated 55 citations in approximately 3 years.

The total number of citations for the whole collection of papers is 1042, which makes an average of 19,66 cites per paper. Given the immaturity of the field (proved in Fig. 4) and taking into account that almost the half of the papers (26 out of 53) have been published in the last two years, this value is an indication of the interest that this field is bringing to the scientific community. More precisely, the most cited paper [28] was published in 2004, although recent papers are also ranked in the top, such as [59], published in 2018, or [71], [73] and [75], issued in 2019.

E. RESEARCHERS AND AFFILIATIONS

Regarding the nationality of the collected works and aiming to answer **SQ7**, we have analyzed the information in two different ways in order to gain a deeper understanding about the real mass of research groups and institutions:

- With focus on researchers' country of affiliation, extracting the number of researchers working on this field (Fig. 5.a).
- With focus on institutions themselves, calculating the number of distinct-geographically-located centers involved in the research and/or development of this technology (Fig. 5.b).

As a result, we have identified 21 distinct nationalities, which have been summarized in Table 6. In this table, and also in the depicted Fig. 5, we can clearly distinguish that the most representative countries are United States, China and India with 30, 18 and 16 researchers distributed in 15, 10 and 8 institutions, respectively.

Disposing the information in these columns, let us gain specific insights about the level of collaboration among the institutions and how the research community is integrating knowledge or expertise from various groups and even disciplines. Therefore, by comparing the count of papers and institutions, it is remarkable how countries like China or India are creating strong networks to conduct their works.

F. OPEN CHALLENGES POSED BY THE COMMUNITY

Considering the review of the research activity discussed in preceding sections, there is no doubt that QC has brought a fresh breeze to the community. Advances made in the joint area of QC applied to routing problems is growing at a remarkable pace, exposing the potential benefits of using quantum technologies in this logistics-related optimization problems. In any case, the incipient stage of maturity of QC has brought to the fore several challenges and research niches that shall mark the path of future studies.

It should be noted that the QC research community holds complementary interests, presumably as a result of individual areas of knowledge/interest:

- *Practitioners coming from industrial and applied research groups*, mostly concerned about QC-based formulations and experiments over more realistic scenarios [69], [73].
- *Researchers coming from quantum physics*, usually interested in analyzing the hardware performance and reliability [42], [53].
- *Researchers with background in traditional Artificial Intelligence*, involved in testing the limits of QC, comparing results and leveraging fundamentals, heuristics or shareable-across-platforms knowledge [29], [71].

The research activity produced by this heterogeneous community has led to a series of open research questions, which can be grouped into the following three blocks to give a clear-cut answer to **SQ8**:

- Up to now, several canonical problems have been formulated and addressed by researchers, such as the TSP [24], VRP [49] or the Hamiltonian Cycle Problem [36]. More sophisticated variants of these problems have also been formulated, considering features such as time-windows [75] or capacity constraints [71]. However, researchers are forced to devise problems tailored to hardware capacity, which often fall short from advanced or realistic formulations [62], [64], [71]. Furthermore, the necessity of taking a step forward and advance in the formulation of more realistic problems is a widely accepted open challenge [31], [50], [66], [68]. Within this frame of reference, the following research lines have been identified by the community as future work: *i*) the efficient modeling of advanced routing problems, focusing on building mathematical formulations which optimize the problem embedding [61], [66], *ii*) the solving of real-world problems oriented to realistic applications (such as traffic optimization or detecting routing anomalies), even if the size of these problems is restricted [50], [69], [70] and *iii*) the development of

TABLE 5. Practical papers characterized by type of algorithm and used device. Pure Q: *Pure Quantum* solver. Hybrid: *hybrid quantum-classical* approach.

ID	Reference	Type of Solver	Used Device	Problem	ID	Reference	Type of Solver	Used Device	Problem
19	Srinivasan et. al [59]	Hybrid	IBM	TSP	39	Adelomou et. al [46]	Pure Q	IBM	VRP
20	Mahasinghe et al. [61]	Pure Q	DWAVE	Other	40	Warren [48]	Hybrid	DWAVE	TSP
22	Clark et. al [65]	Hybrid	DWAVE	Other	41	Harwood et. al [50]	Pure Q	IBM	VRP
24	Mehta et. al [69]	Hybrid	DWAVE	TSP	42	Siloi et. al [52]	Pure Q	DWAVE	Other
25	Irie et. al [71]	Pure Q	DWAVE	VRP	43	Botez et. al [54]	Pure Q	IBM	Other
26	Feld et. al [73]	Hybrid	DWAVE	TSP - VRP	44	Paler et.al [56]	Pure Q	IBM	TSP
28	Ruan et. al [25]	Pure Q	IBM	TSP	46	Bao et. al [60]	Hybrid	Fujitsu	VRP
30	Behera & Panigrahi [29]	Pure Q	IBM	VRP	47	Atchade-Adelomou et. al [62]	Hybrid	IBM - DWAVE	Other
31	Harikrishnakumar et. al [31]	Hybrid	DWAVE	VRP	48	Warren [64]	Hybrid	DWAVE	TSP
32	Narottama et. al [33]	Pure Q	IBM	TSP	49	Salehi et.al [66]	Pure Q	DWAVE	TSP
33	Borowski et. al [35]	Hybrid	DWAVE	VRP	50	Osaba et. al [68]	Hybrid	DWAVE	TSP
35	Krauss & McCollum [39]	Hybrid	DWAVE	Other	51	Osaba et. al [70]	Hybrid	DWAVE	TSP
36	Adelomou et. al [41]	Pure Q	IBM	VRP	52	Lopes [72]	Hybrid	DWAVE	TSP
38	Gammanpila & Fernando [23]	Pure Q	IBM	TSP					

(a) Distribution by Authors

(b) Distribution by Affiliation
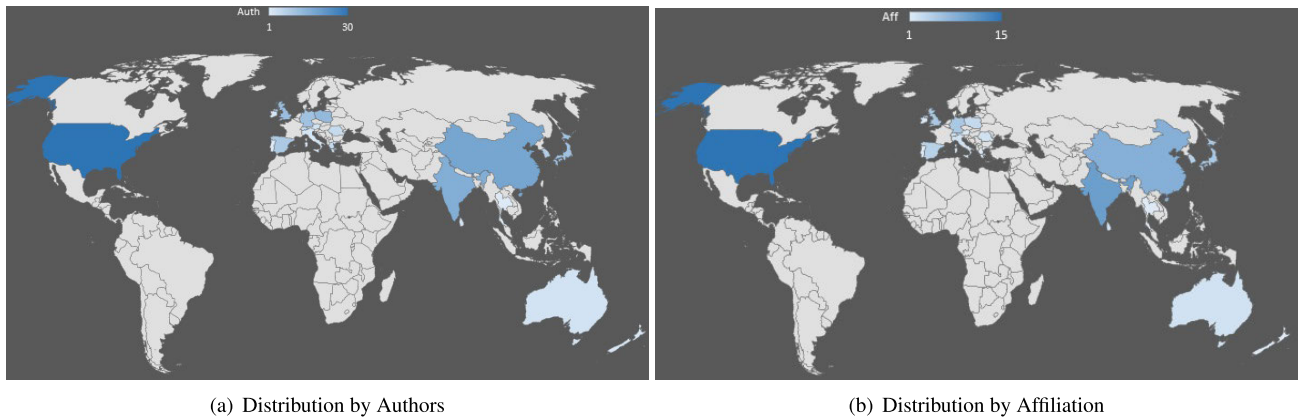
FIGURE 5. Geographical distribution of authors (a) and institutions (b) working on QC and Routing problems among the world.

TABLE 6. Collected papers distributed by nationality.

Country	Papers	Institutions	Researchers
U.S.A	14	15	30
China	5	8	18
India	6	10	16
Poland	3	3	13
United Kingdom	6	5	12
Germany	5	5	11
Japan	4	5	11
Spain	5	4	9
South Korea	2	4	8
Greece	1	1	5
Switzerland	3	2	5
Romania	2	2	5
Australia	2	2	3
Austria	1	1	3
Sri Lanka	2	2	3
Italy	1	2	2
Ireland	1	1	2
New Zealand	1	1	2
Singapore	1	1	1
Thailand	1	1	1
Portugal	1	1	1

hybrid solvers using classical optimization mechanisms to tackle complex problems [33], [72].

- Current commercial quantum devices impose some inherent limitations, such as noise or decoherence [7]. To overcome this latent problem, practitioners have elaborated on the formulation of error correction and mitigation strategies, as can be seen in [27]. In any case, the amount of studies working on this specific aspect is still low. Some researcher have actively shown and proved the effect of this phenomena in the performance of their algorithmic schemes [23], [66], [72], [73], whereas others just add some remarks in this respect in the future work section [41]. On this grounds, some authors have spotlighted the importance of conducting studies analyzing the effect of the quantum hardware noise [23], [29], since some error mitigation or correction mechanisms can be addressed in the problem formulation itself, giving rise to more resilient algorithms

- Quantum devices have demonstrated to be very sensitive to parameterization [69]. On this regard, these parameterizations apply to multiple stages and elements on QC: *i*) parameters in the problem formulation (such as penalties), *ii*) parameters for algorithm tuning (commonly referred as hyperparameters), and *iii*) parameters targeted to quantum hardware (specifications for configurable behaviors of the quantum process). Despite this, up to now, the great majority of practical papers are focused on solving a specific problem or testing the adequacy of a formulation given an intuitively suitable parameterization. In fact, the literature lack profound research on the fine-tuning of these routing problems,

TABLE 7. Characteristics of the most cited papers in the current literature.

Ref.	Title	Source of publication	Year	Problem	QC resource
[28]	Quantum annealing of the traveling-salesman problem	Physical Review E	2004	TSP	Theoretical
[59]	Efficient quantum algorithm for solving travelling salesman problem: An IBM quantum experience	arXiv	2018	TSP	IBM
[73]	A hybrid solution method for the capacitated vehicle routing problem using a quantum annealer	Frontiers on ICT	2019	VRP-TSP	DWAVE
[32]	Optimized annealing of traveling salesman problem from the nth-nearest-neighbor distribution	Physica A: Statistical Mechanics and its Applications	2006	TSP	Theoretical
[53]	Quantum speedup of the traveling-salesman problem for bounded-degree graphs	Physical Review A	2017	TSP	Theoretical
[75]	A QUBO model for the traveling salesman problem with time windows	Algorithms	2019	TSP	Theoretical
[49]	Quantum annealing algorithm for vehicle scheduling	IEEE International Conference on Systems, Man, and Cybernetics	2013	VRP	Theoretical
[42]	Experimental demonstration of a quantum annealing algorithm for the traveling salesman problem in a nuclear-magnetic-resonance quantum simulator	Physical Review A	2011	TSP	Theoretical
[47]	Adapting the traveling salesman problem to an adiabatic quantum computer	Quantum information processing	2013	TSP	DWAVE
[71]	Quantum annealing of vehicle routing problem with time, state and capacity	International Workshop on Quantum Technology and Optimization Problems	2019	VRP	DWAVE

algorithms and hardware, as stated by [66]. In this concrete paper, authors express that one of the most difficult challenge is the proper choice of penalty values in problem formulation, and they theorize some alternatives to assist in the proper selection of these parameters. Other works also underline the importance of parameterization in algorithms and quantum hardware, calling for the development of more sophisticated solving models and suggesting the adoption of classical fine-tuning heuristics [69]. In short, practiotioners are nowadays recognizing the necessity of better parameterization procedures in order to fairly analyze the impact of these configurations on the quality of results, robustness or scalability of the approach. [31], [73].

G. SUMMARY OF THE SLR

As a summary of this Section III, we should first say that the meta-analysis carried out on the gathered collection of papers have helped us to answer eight different secondary questions (listed in Section II). After addressing these **SQ**s, we are now in position to give a response to the key question of the SLR (**MQ**), which is the main objective of this study.

We outline the ten most cited papers in Table 7, highlighting some important information that has been discussed along this whole Section III: year, problem solved, type of study, source of publication and quantum resource used. These ten papers are quite representative of the current state of the field, in which the immaturity of quantum resources makes theoretical papers more present in the literature. Furthermore, the TSP has been the most studied routing problem during the period, with the VRP in the second position. Lastly, it is worth mentioning that 7 of these papers have been published in scientific journals, while 2 were presented in international conferences and 1 was uploaded as a pre-print in the reputed *arXiv* repository.

IV. CRITICAL ANALYSIS OF MILESTONE AND INFLUENTIAL PAPERS

Lastly, we complement our SLR providing a critical analysis of some selected milestone and influential papers. Considering the immature stage of this knowledge field, and taking into account that this manuscript represents the first review paper conducted on the conjunction of routing problems and quantum computation, we have found interesting to give some technical specifications aiming at helping readers to find a guide for their future works on this field. To do so, we include Table 8 gathering both the most cited papers (deemed as influential works) and those milestone papers that developed routing problems algorithms for the first time or introduced advanced technological strategies. For each selected work, we incorporate a brief description of its main contribution as well as the problem complexity (for applied studies).

As can be noted in this table, first influential studies were focused on problem formulations at a time when quantum computers were not accessible. Examples of this statement include formulations for the TSP [28], [32], VRP [49] and other routing related problems [36]. These works encouraged the community to evolve new strategies and complement these theoretical studies with experimentation.

In this context, Moser introduced the first theoretical paper aiming to combine both routing problems and QC [24]. Furthermore, Martoňák et. al proposed the first ever formulation of the TSP for a quantum annealer in [28]. That work has gained a lot of attention in the last two decades becoming the most cited one in this specific field. More specifically, authors endorsed their formulation through an experimentation carried out with classical techniques using TSP instances of a size up to 1002 nodes. Two years later, authors in [32] continued on the road taken by [28], presenting an alternative TSP formulation for a quantum annealer to improve efficiency in escaping from local optima. Also significant is the research conducted in [42], which introduces one of

TABLE 8. Characteristics of milestone papers published in the field.

Ref.	Title	Year	Main Contribution
[24]	The quantum mechanical solution of the traveling salesman problem	2003	First theoretical paper focused on QC and routing problems (TSP)
[28]	Quantum annealing of the traveling-salesman problem	2004	First formulation of the TSP for a Quantum Annealer. Classical simulations are conducted using this formulation with instances up to 1002 nodes
[32]	Optimized annealing of traveling salesman problem from the nth-nearest-neighbor distribution	2006	An alternative formulation of the TSP for being solved by a quantum annealer. Proposed model proved effective for escaping from the local energy minima. An experimentation is carried out with classical solvers with instances up to 532 nodes.
[36]	Quantum algorithms for graph traversals and related problems	2007	First formulation of some important graph traversal problems for a Quantum Annealer. In this work, problems such as the eulerian graph, hamiltonian cycle, TSP and the project scheduling are faced.
[42]	Experimental demonstration of a quantum annealing algorithm for the traveling salesman problem in a nuclear-magnetic-resonance quantum simulator	2011	A step forward in the experimentation of quantum annealers to the TSP, by experimentally demonstrating the application of a simulated quantum annealing method to a simplified version of the TSP. The framework selected is a nuclear-magnetic-resonance, which is a quantum simulator executed in a classical computer. The experimentation conducted is carried out on a single instance composed of 4 nodes.
[47]	Adapting the traveling salesman problem to an adiabatic quantum computer	2013	Recommended entry point for beginners and new practitioners to a Quantum-based TSP solver: understandable explanation which has served as an inspiration for many subsequent studies.
[49]	Quantum annealing algorithm for vehicle scheduling	2013	First formulation of the VRP for a Quantum Annealer. Aiming to verify the effectiveness of the proposed formulation, instances from 40 to 135 nodes are solved using a classical computing algorithm.
[53]	Quantum speedup of the traveling-salesman problem for bounded-degree graphs	2017	This paper proves a quadratic quantum speedup in TSP instances with a maximum degree of 3, by applying a quantum backtracking algorithm.
[59]	Efficient quantum algorithm for solving travelling salesman problem: An IBM quantum experience	2018	First QC solver (IBM) for the TSP, using an instance composed of 4 nodes.
[61]	Solving the Hamiltonian Cycle Problem using a Quantum Computer	2019	First solving scheme proposed for the Hamiltonian cycle Problem through a DWAVE quantum device. Graphs of size 4 are used in the experimentation.
[69]	Quantum annealing based optimization of robotic movement in manufacturing	2019	First DWAVE solver for the TSP. 20 instances with sizes ranging from 2 to 5 nodes are considered.
[71]	Quantum annealing of vehicle routing problem with time, state and capacity	2019	First QC based solver (DWAVE) for the VRP. A single use case consisting of 6 customers and 3 vehicles is solved in this research.
[73]	A hybrid solution method for the capacitated vehicle routing problem using a quantum annealer	2019	A Novel Capacitated VRP solver scheme. Decomposition of the problem into two procedures: a clustering problem + a TSP. Influential paper with a clear representation of the QUBO formulation used for solving the TSP, which has been referred as inspiration in many subsequent studies. Tackling of TSP instances composed of 14 to 38 nodes, and CVRP instances from 22 to 101 clients.
[75]	A QUBO model for the traveling salesman problem with time windows	2019	Fist QUBO formulation for a complex TSP which accommodates the most frequently applied restriction: time-windows.
[29]	Solving vehicle routing problem using quantum approximate optimization algorithm	2020	First IBM-based solver for the VRP, with instances composed of 4 and 5 nodes.
[60]	An Approach to the Vehicle Routing Problem with Balanced Pick-up Using Ising Machines	2021	First Fujitsu solver for a routing problem (VRP). In this paper, a complex variant of the VRP is solved, with instances up to 8 nodes.

the fist experiments on a simulated quantum hardware. More concretely, authors developed a classically simulated nuclear-magnetic-resonance, which was applied to a single small instance of the TSP consisting of 4 nodes.

Furthermore, a remarkable milestone was the research conducted in [47] in 2013, which gave birth to a guide for beginners offering very understandable explanations of TSP formulations translated to a real adiabatic computer. Another influential work (ranked among the most cited ones in the field) is [53], which introduces a quantum backtracking algorithm for dealing with TSP instances with a maximum degree of 3. Finally, it is worth-mentioning the recent theoretical work carried out by Papalitsas *et al.* in [75], in which the first

QUBO representation of a complex TSP is given. That TSP variant deals with time-windows, which is arguably the most frequently imposed restriction on routing problems.

Turning our attention to the VRP, the first theoretical groundbreaking paper can be found in [49]. This paper, which is also among the most cited ones in the field, introduces the first ever formulation of the VRP for being solved in a quantum annealer. Lastly, also significant was the theoretical work developed by Dörn in [36]. In that paper, unveiled in 2007, the author introduced the first quantum annealing formulation of some relevant routing problems, such as the *eulerian graph problem*, *hamiltonian cycle problem*, or the *project scheduling problem*. In that work, also the TSP is

considered, strengthening the conclusions drawn in previous related studies.

Focusing on papers elaborating on practical experiments on real QC devices, the first crucial milestone was achieved in 2017 through the work proposed by Srinivasan *et al.* in [59]. Authors presented the first solving scheme for a routing problem using a real quantum computer. Specifically, that revolutionary paper tackled the TSP using the IBM quantum hardware, addressing a single TSP instance of 4 nodes. On another vein, authors in [69] achieved a further remarkable milestone, proposing the first DWAVE-technology-based solver for the TSP, facing instances with a maximum of 5 nodes. Despite the publication of these pioneering works, arguably, the most influential research on this context is the one carried out by Feld *et al.* in [73]. In that study, authors faced the well-known Capacitated VRP decomposing it into a clustering problem and a TSP. While the first of these problems is dealt using classical techniques, the TSP is tackled using the DWAVE quantum annealer. The technical and informative nature of this paper, giving clear details about the QUBO formulation, has made it an influential work in the community. Using a hybrid quantum-classical approach, TSP instances composed of 14 to 38 nodes are faced, while the overall algorithm addresses CVRP instances from 22 to 101 clients. Lastly, it is interesting to highlight the study proposed in [60], in which the first approach based on the Fujitsu quantum inspired device is presented, able to solve small instances of the TSP.

Regarding pure VRP formulations, the first milestone belongs to [71], whose authors employed the DWAVE quantum annealer for instances up to 6 nodes, being the first VRP solver run on a real quantum hardware. Additionally, Behera et. al introduced the first approach based on IBM universal quantum device in [29]. That work, published in 2020, tackles small VRP instances composed of 4 and 5 nodes. Finally, it is also interesting to mention the research conducted by Mahasinghe *et al.* in [61] where authors solved the Hamiltonian Cycle problem using the DWAVE quantum hardware, analyzing results on graphs of size 4.

V. CONCLUSION AND FURTHER WORK
The main purpose of this manuscript has been the conduction of a systematic literature review (SLR) to analyze the research carried out in the confluence of Quantum Computing and routing problems topics. The time span analyzed in this paper covers 18 years of research, which have witnessed 53 research papers published in form of journal or conference contributions, book chapters, PhD thesis, Master thesis and reports.

From this analysis, it is noticeable that the TSP engages most of the researchers (60,37% - 32 out of 53 papers), while the VRP amounts to the 25,52% of the contributions (13 out of 53). The rest of the papers deal with other routing problems, such as the Shortest Path Problem or the Hamiltonian Cycle.

Regarding the type of papers, an equality has been found in the literature, with 26 theoretical works and 27 practical contributions. In any case, and thanks to the advance of the technology, a clear inclination towards practical articles come into sight in the last three years. Further equality can be also detected in the kind of solvers preferred by practitioners, with 13 papers presenting purely quantum approaches and 14 studies proposing hybrid techniques. Regarding QC devices, DWAVE beats IBM by 16 to 11.

Furthermore, we have complemented our analysis by sharing the envisioned status of this area, pointing out some challenges detected by researchers and practitioners working on this field. These open opportunities should stimulate research efforts in years to come. We have identified three different main challenges: *i*) the need of dealing with more sophisticated and real-world oriented problems, *ii*) the value that will provide the proper understanding of the quantum hardware limitations, and *iii*) the necessity of properly parameterize problem formulations, algorithms and quantum devices.

As a final note, we can arguably affirm that the application of quantum computing to vehicle routing problems counts with an exciting and prolific future. Continuous advances made on quantum technology will encourage the community to implement more realistic routing problems (increasing complexity and adding new constraints), taking future studies to unprecedented challenges not even imagined today.

REFERENCES
[1] T. Erdelić and T. Carić, "A survey on the electric vehicle routing problem: Variants and solution approaches," *J. Adv. Transp.*, vol. 2019, pp. 1–48, May 2019.
[2] M. Gansterer and R. F. Hartl, "Collaborative vehicle routing: A survey," *Eur. J. Oper. Res.*, vol. 268, no. 1, pp. 1–12, Jul. 2018.
[3] A. Mor and M. G. Speranza, "Vehicle routing problems over time: A survey," *4OR*, vol. 18, no. 2, pp. 129–149, Jun. 2020.
[4] J. K. Lenstra and A. H. G. R. Kan, "Complexity of vehicle routing and scheduling problems," *Networks*, vol. 11, no. 2, pp. 221–227, 1981.
[5] J. Caceres-Cruz, P. Arias, D. Guimarans, D. Riera, and A. A. Juan, "Rich vehicle routing problem: Survey," *ACM Comput. Surv.*, vol. 47, no. 2, pp. 1–28, 2014.
[6] A. Steane, "Quantum computing," *Rep. Prog. Phys.*, vol. 61, no. 2, p. 117, 1998.
[7] A. Ajagekar, T. Humble, and F. You, "Quantum computing based hybrid solution strategies for large-scale discrete-continuous optimization problems," *Comput. Chem. Eng.*, vol. 132, Jan. 2020, Art. no. 106630.
[8] T. Hey, "Quantum computing: An introduction," *Comput. Control Eng. J.*, vol. 10, no. 3, pp. 105–112, Jun.1999.
[9] Y. Wang, Y. Li, Z.-Q. Yin, and B. Zeng, "16-Qubit IBM universal quantum computer can be fully entangled," *npj Quantum Inf.*, vol. 4, no. 1, pp. 1–6, Dec. 2018.
[10] P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, and W. D. Oliver, "Perspectives of quantum annealing: Methods and implementations," *Rep. Prog. Phys.*, vol. 83, no. 5, May 2020, Art. no. 054401.
[11] G. E. Santoro and E. Tosatti, "Optimization using quantum mechanics: Quantum annealing through adiabatic evolution," *J. Phys. A, Math. Gen.*, vol. 39, no. 36, pp. R393–R431, Sep. 2006.
[12] L. Gyongyosi and S. Imre, "Quantum circuit design for objective function maximization in gate-model quantum computers," 2018, *arXiv:1803.02460*.
[13] B. Kitchenham, "Procedures for performing systematic reviews," Keele Univ., Keele, U.K., Tech. Rep. TR/SE-0401, 2004, pp. 1–26, vol. 33.
[14] Y. Levy and T. J. Ellis, "A systems approach to conduct an effective literature review in support of information systems research," *Informing Sci., Int. J. Emerg. Transdiscipline*, vol. 9, pp. 181–213, Jan. 2006.
[15] D. Budgen and P. Brereton, "Performing systematic literature reviews in software engineering," in *Proc. 28th Int. Conf. Softw. Eng.*, 2006, pp. 1051–1052.

[16] E. C. da Silva and P. H. R. Gabriel, "A comprehensive review of evolutionary algorithms for multiprocessor DAG scheduling," *Computation*, vol. 8, no. 2, p. 26, Apr. 2020.

[17] M. M. Flood, "The traveling-salesman problem," *Oper. Res.*, vol. 4, no. 1, pp. 61–75, Feb. 1956.

[18] E. Osaba, X.-S. Yang, and J. Del Ser, "Traveling salesman problem: A perspective review of recent research and new results with bio-inspired metaheuristics," in *Nature-Inspired Computation and Swarm Intelligence*. London, U.K.: Elsevier, 2020, pp. 135–164.

[19] P. Toth and D. Vigo, *The Vehicle Routing Problem*. Philadelphia, PA, USA: SIAM, 2002.

[20] E. Osaba, X.-S. Yang, and J. Del Ser, "Is the vehicle routing problem dead? An overview through bioinspired perspective and a prospect of opportunities," in *Nature-Inspired Computation in Navigation and Routing Problems*. Singapore: Springer, 2020, pp. 57–84.

[21] K.-H. Han and J.-H. Kim, "Quantum-inspired evolutionary algorithm for a class of combinatorial optimization," *IEEE Trans. Evol. Comput.*, vol. 6, no. 6, pp. 580–593, Dec. 2002.

[22] G. Zhang, "Quantum-inspired evolutionary algorithms: A survey and empirical study," *J. Heuristics*, vol. 17, no. 3, pp. 303–351, Jun. 2011.

[23] A. Gammanpila and T. Fernando, "Implementation of modified Talbi's quantum inspired genetic algorithm for travelling salesman problem on an IBM quantum computer," in *Proc. Future Technol. Conf.* Cham, Switzerland: Springer, 2020, pp. 70–88.

[24] H. R. Moser, "The quantum mechanical solution of the traveling salesman problem," *Phys. E, Low-Dimensional Syst. Nanostruct.*, vol. 16, no. 2, pp. 280–285, Feb. 2003.

[25] Y. Ruan, S. Marsh, X. Xue, Z. Liu, and J. Wang, "The quantum approximate algorithm for solving traveling salesman problem," *Comput., Mater. Continua*, vol. 63, no. 3, pp. 1237–1247, 2020.

[26] D. Goswami, H. Karnick, P. Jain, and H. K. Maji, "Towards efficiently solving quantum traveling salesman problem," 2004, *arXiv:quant-ph/0411013*.

[27] K. Gui, T. Tomesh, P. Gokhale, Y. Shi, F. T. Chong, M. Martonosi, and M. Suchara, "Term grouping and travelling salesperson for digital quantum simulation," 2020, *arXiv:2001.05983*.

[28] R. Martoňák, G. E. Santoro, and E. Tosatti, "Quantum annealing of the traveling-salesman problem," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 70, no. 5, Nov. 2004, Art. no. 057701.

[29] Utkarsh, B. K. Behera, and P. K. Panigrahi, "Solving vehicle routing problem using quantum approximate optimization algorithm," 2020, *arXiv:2002.01351*.

[30] C. V. Raj and M. Shivakumar, "Applying quantum algorithm to speed up the solution of Hamiltonian cycle problems," in *Proc. Int. Conf. Intell. Inf. Process.* Boston, MA, USA: Springer, 2006, pp. 53–61.

[31] R. Harikrishnakumar, S. Nannapaneni, N. H. Nguyen, J. E. Steck, and E. C. Behrman, "A quantum annealing approach for dynamic multi-depot capacitated vehicle routing problem," 2020, *arXiv:2005.12478*.

[32] Y. Chen and P. Zhang, "Optimized annealing of traveling salesman problem from the nth-nearest-neighbor distribution," *Phys. A, Stat. Mech. Appl.*, vol. 371, no. 2, pp. 627–632, Nov. 2006.

[33] B. Narottama, D. K. Hendraningrat, and S. Y. Shin, "Quantum optimization for UAV: An initial work," in *Proc. Korea Telecommun. Soc. Conf.*, 2020, pp. 1262–1263.

[34] S. Dörn, "Quantum algorithms for optimal graph traversal problems," *Proc. SPIE*, vol. 6573, Apr. 2007, Art. no. 65730D.

[35] M. Borowski, P. Gora, K. Karnas, M. Błajda, K. Król, A. Matyjasek, D. Burczyk, M. Szewczyk, and M. Kutwin, "New hybrid quantum annealing algorithms for solving vehicle routing problem," in *Proc. Int. Conf. Comput. Sci.* Cham, Switzerland: Springer, 2020, pp. 546–561.

[36] S. Dörn, "Quantum algorithms for graph traversals and related problems," in *Proc. CIE*, vol. 7, 2007, pp. 123–131.

[37] A. Dan, R. Shimizu, T. Nishikawa, S. Bian, and T. Sato, "Clustering approach for solving traveling salesman problems via ising model based solver," in *Proc. 57th ACM/IEEE Design Automat. Conf. (DAC)*, Jul. 2020, pp. 1–6.

[38] Z. Nie, P. Wang, J.-P. Li, and G. Yin, "An algorithm for routing based on quantum superposition states," in *Information Computing and Automation*. Singapore: World Scientific, 2008, pp. 231–234.

[39] T. Krauss and J. McCollum, "Solving the network shortest path problem on a quantum annealer," *IEEE Trans. Quantum Eng.*, vol. 1, pp. 1–12, 2020.

[40] U. Roy, S. P. Chawdhury, and S. Nayek, "Quantum computer and quantum algorithm for travelling salesman problem," *Nat. J. Syst. Inf. Technol.*, vol. 2, no. 1, p. 54, 2009.

[41] A. P. Adelomou, E. G. Ribe, and X. V. Cardona, "Using the parameterized quantum circuit combined with variational-quantum-eigensolver (VQE) to create an intelligent social workers' schedule problem solver," 2020, *arXiv:2010.05863*.

[42] H. Chen, X. Kong, B. Chong, G. Qin, X. Zhou, X. Peng, and J. Du, "Experimental demonstration of a quantum annealing algorithm for the traveling salesman problem in a nuclear-magnetic-resonance quantum simulator," *Phys. Rev. A, Gen. Phys.*, vol. 83, no. 3, Mar. 2011, Art. no. 032314.

[43] Y. Dong and Z. Huang, "An improved noise quantum annealing method for TSP," *Int. J. Theor. Phys.*, vol. 59, no. 12, pp. 3737–3755, Dec. 2020.

[44] J. Bang, J. Ryu, C. Lee, S. Yoo, J. Lim, and J. Lee, "A quantum heuristic algorithm for the traveling salesman problem," *J. Korean Phys. Soc.*, vol. 61, no. 12, pp. 1944–1949, 2012.

[45] T. Fujimura, "Quantum algorithm for traveling salesman problem by quarter method," *Global J. Pure Appl. Math.*, vol. 16, no. 5, pp. 711–721, 2020.

[46] A. P. Adelomou, E. G. Ribé, and X. V. Cardona, "Formulation of the social Workers' problem in quadratic unconstrained binary optimization form and solve it on a quantum computer," *J. Comput. Commun.*, vol. 8, no. 11, pp. 44–68, 2020.

[47] R. H. Warren, "Adapting the traveling salesman problem to an adiabatic quantum computer," *Quantum Inf. Process.*, vol. 12, no. 4, pp. 1781–1785, Apr. 2013.

[48] R. H. Warren, "Solving the traveling salesman problem on a quantum annealer," *Social Netw. Appl. Sci.*, vol. 2, no. 1, pp. 1–5, Jan. 2020.

[49] A. Crispin and A. Syrichas, "Quantum annealing algorithm for vehicle scheduling," in *Proc. IEEE Int. Conf. Syst., Man, Cybern.*, Oct. 2013, pp. 3523–3528.

[50] S. Harwood, C. Gambella, D. Trenev, A. Simonetto, D. Bernal, and D. Greenberg, "Formulating and solving routing problems on quantum computers," *IEEE Trans. Quantum Eng.*, vol. 2, pp. 1–17, 2021.

[51] B. Heim, E. W. Brown, D. Wecker, and M. Troyer, "Designing adiabatic quantum optimization: A case study for the traveling salesman problem," 2017, *arXiv:1702.06248*.

[52] I. Siloi, V. Carnevali, B. Pokharel, M. Fornari, and R. Di Felice, "Investigating the Chinese postman problem on a quantum annealer," *Quantum Mach. Intell.*, vol. 3, no. 1, pp. 1–10, Jun. 2021.

[53] D. J. Moylett, N. Linden, and A. Montanaro, "Quantum speedup of the traveling-salesman problem for bounded-degree graphs," *Phys. Rev. A, Gen. Phys.*, vol. 95, no. 3, Mar. 2017, Art. no. 032323.

[54] R. Botez, I.-A. Ivanciu, C.-M. Iurian, and V. Dobrota, "Quantum implementation of the modified dijkstra's routing algorithm," *Proc. Romanian Acad. A, Math. Phys. Tech. Sci. Inf. Sci.*, vol. 22, no. 1, pp. 91–98, 2021.

[55] R. H. Warren, "Small traveling salesman problems," *J. Adv. Appl. Math.*, vol. 2, no. 2, pp. 1–7, Apr. 2017.

[56] A. Paler, A. Zulehner, and R. Wille, "NISQ circuit compilation is the travelling salesman problem on a torus," *Quantum Sci. Technol.*, vol. 6, no. 2, Apr. 2021, Art. no. 025016.

[57] A. Syrichas and A. Crispin, "Large-scale vehicle routing problems: Quantum annealing, tunings and results," *Comput. Oper. Res.*, vol. 87, pp. 52–62, Nov. 2017.

[58] J. McCollum and T. Krauss, "QUBO formulations of the longest path problem," *Theor. Comput. Sci.*, vol. 863, pp. 86–101, Apr. 2021.

[59] K. Srinivasan, S. Satyajit, B. K. Behera, and P. K. Panigrahi, "Efficient quantum algorithm for solving travelling salesman problem: An IBM quantum experience," 2018, *arXiv:1805.10928*.

[60] S. Bao, M. Tawada, S. Tanaka, and N. Togawa, "An approach to the vehicle routing problem with balanced pick-up using ising machines," in *Proc. Int. Symp. VLSI Design, Automat. Test (VLSI-DAT)*, Apr. 2021, pp. 1–4.

[61] A. Mahasinghe, R. Hua, M. J. Dinneen, and R. Goyal, "Solving the Hamiltonian cycle problem using a quantum computer," in *Proc. Australas. Comput. Sci. Week Multiconference*, Jan. 2019, pp. 1–9.

[62] P. Atchade-Adelomou, G. Alonso-Linaje, J. Albo-Canals, and D. Casado-Fauli, "QRobot: A quantum computing approach in mobile robot order picking and batching problem solver optimization," *Algorithms*, vol. 14, no. 7, p. 194, Jun. 2021.

[63] A. Syrichas, "Quantum annealing for vehicle routing and scheduling problems," Ph.D. dissertation, Manchester Metropolitan Univ., Manchester, U.K., 2019.

[64] R. H. Warren, "Solving combinatorial problems by two D_Wave hybrid solvers: A case study of traveling salesman problems in the TSP library," 2021, *arXiv:2106.05948*.

[65] J. Clark, T. West, J. Zammit, X. Guo, L. Mason, and D. Russell, "Towards real time multi-robot routing using quantum computing technologies," in *Proc. Int. Conf. High Perform. Comput. Asia–Pacific Region*, Jan. 2019, pp. 111–119.

[66] O. Salehi, A. Glos, and J. A. Miszczak, "Unconstrained binary models of the travelling salesman problem variants for quantum optimization," 2021, *arXiv:2106.09056*.

[67] T. D. Kieu, "The travelling salesman problem and adiabatic quantum computation: An algorithm," *Quantum Inf. Process.*, vol. 18, no. 3, p. 90, Mar. 2019.

[68] E. Osaba, E. Villar-Rodriguez, I. Oregi, and A. Moreno-Fernandez-de-Leceta, "Hybrid quantum computing–tabu search algorithm for partitioning problems: Preliminary study on the traveling salesman problem," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2021, pp. 351–358.

[69] A. Mehta, M. Muradi, and S. Woldetsadick, "Quantum annealing based optimization of robotic movement in manufacturing," in *Proc. Int. Workshop Quantum Technol. Optim. Problems*. Cham, Switzerland: Springer, 2019, pp. 136–144.

[70] E. Osaba, E. Villar-Rodriguez, I. Oregi, and A. Moreno-Fernandez-de-Leceta, "Focusing on the hybrid quantum computing–Tabu search algorithm: New results on the asymmetric salesman problem," in *Proc. Genet. Evol. Comput. Conf. Companion*, Jul. 2021, pp. 1476–1482.

[71] H. Irie, G. Wongpaisarnsin, M. Terabe, A. Miki, and S. Taguchi, "Quantum annealing of vehicle routing problem with time, state and capacity," in *Proc. Int. Workshop Quantum Technol. Optim. Problems*. Cham, Switzerland: Springer, 2019, pp. 145–156.

[72] A. F. D. S. T. Lopes, "Quantum computing for optimizing routes in smart cities," M.S. thesis, Faculdade de Engenharia, Universidade do Porto, Porto, Portugal, 2021.

[73] S. Feld, C. Roch, T. Gabor, C. Seidel, F. Neukart, I. Galter, W. Mauerer, and C. Linnhoff-Popien, "A hybrid solution method for the capacitated vehicle routing problem using a quantum annealer," *Frontiers ICT*, vol. 6, p. 13, Jun. 2019.

[74] R. H. Warren, "A benchmark for quantum optimization: The traveling salesman problem," *Quantum Inf. Comput.*, vol. 21, nos. 7–8, pp. 557–562, 2021.

[75] C. Papalitsas, T. Andronikos, K. Giannakis, G. Theocharopoulou, and S. Fanarioti, "A QUBO model for the traveling salesman problem with time windows," *Algorithms*, vol. 12, no. 11, p. 224, Oct. 2019.

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