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Comparing aggregation methods in large-scale group AHP: Time for the shift to distance-based aggregation

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ABSTRACT

This paper aims to compare the efficiency of the conventional aggregation methods and the new, distance-based aggregation techniques in simulated and real-world group AHP cases. For the comparison, we not only applied rank correlation methods, but also examined the compatibility among the individual priority vectors of the group and the created common priority vector in the different consensus creation approaches. Results have shown that in small dimensions, both Euclidean Distance-Based Aggregation Method (EDBAM) and Aitchison Distance-Based Aggregation Method (ADBAM) outperform significantly the conventional techniques. In large dimensions, the dominance of EDBAM remains. Since the computational time of the proposed methods (especially EDBAM) is low and EDBAM maintains its efficiency in large-scale group AHP (proven by 96.000 simulation cases) in every possible dimension within the AHP domain, we can state in case of high number of evaluators, distance-based aggregation is a better approach than the conventional methods.

1. Introduction

Multi-criteria decision-making (MCDM) techniques were originally created to support the complex decisions of individuals, a few experts, or a couple of stakeholders in specific problems. As the range of applications extended, the possible number of decision-makers increased and the group approach emerged in not only theoretical, but also practical participatory problems. Recently, the number of participants in some MCDM surveys not seldom reaches thousand (Wu & Xu, 2018), mainly due to the significant role of e-democracy and social networks (Palomares et al., 2014) or public involvement in public service development (Duleba & Moslem, 2019). Chen & Liu (2006) created the first, widely accepted definition of a large-scale group: if more than 20 decision-makers are involved, the decision process is considered as Large-scale Group Decision-Making (LSGDM). Other researchers argued, however (Huang et al., 2009), that in the case of five participants or over, the nature of preference aggregation transforms, thus the characteristics of LSGDM emerge. Nowadays it is a rapidly growing and very promising topic within the decision sciences but up to now, studies on LSGDM are still in its inceptive stage (Xu et al., 2018) and the available literature is very scarce in this domain. The most recent advancements can be connected to Liu et al., 2019; Song & Li, 2019; Ren et al., 2020, Chai & Ngai, 2020.

Analytic Hierarchy Process (AHP) is undoubtedly one of the most popular MCDM methodologies. Like the other techniques, it is also capable of handling multiple decision-makers, Group AHP (GAHP) models are applied for problems with several participants. Although solving GAHP is almost as old as AHP itself (Aczél & Saaty, 1983), the topic is still in the focus of researchers (Marcarelli & Squillante, 2020; Amenta et al., 2021; Faizi et al., 2020). Owing to large-scale GAHP, most studies deal with finding the best solution to manage the transitivity and consistency of individual preferences in the aggregation procedure (Wu & Tu, 2021) or to approximate the evaluators' intention in scoring as much as possible to reach accurate evaluation (Du & Shan, 2020).

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A cardinal issue in GAHP is how to aggregate individual preferences and create a group consensus, which reflects the opinion of the set of participants most. The reigning techniques of aggregation are: Aggregation of Individual Judgements (AIJ) (Aczél & Alsina, 1986), and Aggregation of Individual Preferences (AIP) (Basak & Saaty, 1993, Keeney, 2009). Both techniques apply the weighted arithmetic or weighted geometric mean of individual values, however, AIJ creates first a group matrix from the same entries of individual pairwise comparison matrices and uses the eigenvector method of AHP for this matrix afterward to derive weight vectors, while AIP computes first the individual weight vectors and then aggregates them by one of the mean calculations. Based on their comparative analysis, Ossadnik et al. (2016) stated that AIP has dominance over the AIJ technique from the aspect of preference aggregation efficiency and from the practical point of view in handling a large number of participants (which is highly relevant for the objective of our paper). Within the AIP method, there is an ongoing debate on the application of weighted arithmetic mean (AIP WAMM) and weighted geometric mean (AIP WGMM). The only existing evidence for the primacy of AIP WAMM is the remarkable work of Ishizaka & Labib (2011), in which they claimed that using the arithmetic mean provides more appropriate consensus creation than the geometric mean, however, they did not demonstrate this statement by simulation cases (our paper aims to contribute to this debate, please see Sub-section 3.1.). The main criticism against AIP (both forms, WAMM and WGMM) is that it could lead to a consensual priority vector which does not reflect the majority of priorities because it is subject to the influence of extreme opinions (Amenta et al., 2020). As a solution, Amenta et al. (2020) proposed a new procedure to create a common priority vector (CPVP) by a loss function minimization which is based on Euclidean norm and Euclidean distance measure. In CPVP, the saliences from the created consensual vector are minimized and thus, the distance between individual priority vectors and the common priority vector is mitigated. The authors demonstrated the new method on a real-world case study, however, simulation evidence was just remarked as further research.

Distance-based consensus creation indeed seems very promising in substituting the conventional preference aggregation techniques. The logic of finding a common priority vector in the decision space which is capable of representing all individual preference vectors by its minimum proximity to the others is reasonable. However, it is possible that other distance measures provide better consensus than the Euclidean metrics.

One of the most relevant of such different distance measures due to its widespread application in statistics, is the Aitchison distance (Aitchison et al., 2000, Stewart, 2017, Feng et al., 2020). Since the Aitchison measure is highly applicable in multi-dimensional scaling and non-Euclidean vector spaces (Quinn et al., 2018, Martín-Fernández et al., 2019), for determining a consensual priority vector in an n-dimensional decision space in case of n decision criteria, this metric worth investigating. Another strong argument for examining the Aitchison distance as a possible basis for preference vector aggregation is that this measure is generally used for simplexes for relative scale property (Hron et al., 2010), which is similar to the group AHP approach. Furthermore, we preliminary examined other metrics, e.g. Chebishev and Manhattan distances, and from a compatibility point of view (between the individual vectors and the compromise vector) they significantly underperformed the Euclidean and Aitchison metrics.

The objective of our paper is to test the efficiency of different preference aggregation methods from the aspect of concordance between the individual rankings and the created consensual ranking in simulated large-scale decision-making cases. As listed above, the two techniques of the AIP method, WAMM and WGMM, furthermore two distance-based aggregations: the Euclidean Distance-Based Aggregation Method (EDBAM) and the Aitchison Distance-Based Aggregation Method (ADBAM) have been investigated. For concordance measure, first the modified form of the Kendall W calculation (Kendall, 1938) has been selected to detect the strength of the rank correlations between individual and consensual priorities. Furthermore, we applied Spearman's rank correlation coefficient (Kumar & Abirami, 2018), as another ordinal correlation method that deals with larger interval ([-1,1]) than the Kendall W to detect the possible negative correlation of the rankings. Moreover, we utilized Garuti's compatibility index (Garuti, 2017) for cardinal comparison, considering not only the similarity of the orders but also the weight scores themselves. Note that the G index was compared to Saaty's compatibility index (based on Hadamard calculations, Saaty, 2005), and found to be more general with higher performance (Garuti, 2020). On top of that, the G index could outperform Jaccard's index, Hilbert's index, and the inner-vector product (IVP), thus can be considered as one of the most relevant compatibility measures between vectors.

Due to the high difficulty of generating matrices with acceptable consistency ratio and the complexity of conducting a large number of simulations in the case of Spearman and Garuti calculations, we examined a smaller number of simulated matrices in this phase. In the large-scale procedure based on Kendall W, we randomly generated normalized vectors in different dimensions (following the AHP characteristics and the rule of Saaty from two dimensions to nine dimensions - because of Saaty's concept (Saaty, 1994) on the maximum size of pairwise comparison matrices 9×9 to keep the consistency of evaluations) and examined the cases of five, ten, 100, 200,…, 1000 decision-makers. Afterwards, we computed the consensual group priority vector by the four different aggregation techniques and calculated the Kendall W value to measure each rank correlation. We repeated it 1000 times for each combination of dimensions and number of decision-makers. Altogether 8x12.000 = 96.000 simulation cases were examined as we tested the eight different dimensions for 12 different evaluator numbers with the repetition of 1000 times, and for each case, four different Kendall W is computed by AIP WAMM, AIP WGMM, EDBAM and ADBAM aggregation methods. The whole analysis is demonstrated in Section 3.

In the next phase of simulations, we followed the original approach of AHP, and generated random pairwise comparison matrices in different dimensions using merely the values of the Saaty-scale and considering the consistency threshold of 10% for the Consistency Ratio in case of at most 6 alternatives (criteria) and 20% for even larger instances. From 2×2 up to 9×9 sized matrices, 10 participants were assumed, their evaluations were derived by the eigenvector method and these weight vectors were aggregated by EDBAM, ADBAM, AIP WAMM, AIP WGMM and CPVP methods. The results were compared by both Spearman's rank correlation coefficient and G compatibility index.

Thus, for the first time in the literature of decision science and multi-criteria decision-making, we can compare the efficiency of the examined different aggregation approaches and recommend the most appropriate for large-scale group AHP decision-making. Moreover, we apply all examined aggregation methods on real-world data gained from a survey conducted in a Turkish city, Mersin, on citizen preferences towards public transport development issues. The preferences of ten evaluators are highlighted on five attributes and we demonstrate the difference in determining the common priority vectors and the different concordances of the common priority vectors respect to the individual preference rankings. Different Kendall W values are computed from the case study which supports the simulation results and the initial idea: it is time for shift to distance-based methods in aggregating individual preferences in group AHP.

The remainder of the paper is organized as follows. We introduce the AHP methodology in detail, along with the studied different aggregation methods and the measurements used in the comparisons in Section 2. The large-scale simulation results and the analysis of the gained information can be found in Sub-section 3.1. More specific simulated group AHP examples are presented in Sub-section 3.2. using both ordinal and cardinal indicators. We present the above-mentioned real-world numerical example in detail, based on the previously conducted group AHP survey in Sub-section 3.3. Finally, we draw some conclusions and make suggestions to the future appliers of the aggregation methods in Section 4.

Input: (n, m) pair of parameters, $w^{(k)}$ individual preference vectors for $k = 1, \ldots, m$, $x \in \mathbb{R}^n$ initial test point (in our case $x = (1/n, 1/n, \ldots, 1/n)$), $\alpha = 1$, $\beta = 0.5$, $\gamma = 2$ and $\delta = 0.5$ parameters and the $f(.)$ function (the sum of weighted distances).
Output: w, the aggregated priority vector

1. Create $n+1$ test points, the original x, and n other with a fixed step along each dimension
2. Order the current test points: $f(x_1) \le f(x_2) \le \cdots \le f(x_n) \le f(x_{n+1})$
   Set the iterative counter l to 1
   Compute the sample standard deviation of $f(x_1), f(x_2), \ldots, f(x_{n+1})$ denoted by σ_l
3. Calculate the centroid of $x_1, x_2, \ldots, x_n$ denoted by $\bar{x}$
4. Compute the reflected point: $x_r = \bar{x} + \alpha(\bar{x} - x_{n+1})$
5. **If** $f(x_1) \le f(x_r) < f(x_n)$ **then**
   replace test point x_{n+1} by x_r
   go to step 9
6. **If** $f(x_r) < f(x_1)$ **then**
   Calculate the expanded point: $x_e = \bar{x} + \gamma(x_r - \bar{x})$
7. **If** $f(x_e) < f(x_r)$ **then**
   replace test point x_{n+1} by x_e
   go to step 9
   else
   replace test point x_{n+1} by x_r
   go to step 9
8. Here certainly $f(x_r) \ge f(x_n)$
   Compute the contracted point: $x_c = \bar{x} + \beta(x_{n+1} - \bar{x})$
   If $f(x_c) < f(x_{n+1})$ **then**
   replace test point x_{n+1} by x_c
   go to step 9
   else
   replace all test points, except the best, x_1 as follows
   $x_i = x_1 + \delta(x_i - x_1)$ for $i = 2, 3, \ldots, n+1$
   go to step 9
9. Order the current (new) test points: $f(x_1) \le f(x_2) \le \cdots \le f(x_n) \le f(x_{n+1})$
   Increase the iterative counter: $l = l + 1$
   Compute σ_l, the sample standard deviation of $f(x_1), f(x_2), \ldots, f(x_{n+1})$
   If $\sigma_l < 0.00001$ **or** $l = 500$ **then**
   go to step 10
   else
   go to step 3
10. Save $w = x_1$ and Exit.

Return: w

2. Methodology

As we mentioned in the Introduction, the application of the Analytic Hierarchy Process is indeed widespread. It is based on the Pairwise Comparison Matrices (PCMs), which can be used both for determining the weights of the different criteria and for the rating of alternatives according to a criterion. The $n \times n$ matrix $A = [a_{ij}]$ is a PCM if it is positive ($a_{ij} > 0$ for $\forall i$ and j) and reciprocal ($a_{ji} = 1/a_{ij}$ for $\forall i$ and j). Its general element a_{ij} shows, how many times item i is better/larger/more important than element j. A PCM is said to be consistent if and only if $a_{ik} = a_{ij}a_{jk}$ for $\forall i,j,k, 1 \le i,j,k \le n$, and it is inconsistent in all other cases. In a practical decision problem the matrix is most likely to be inconsistent, however there can be significant differences in the degree of consistency that can be measured by the Consistency Ratio (CR):

$$CR = \frac{CI}{RI},$$

where RI is the average CI value of randomly generated PCMs of the same size, while:

$$CI = \frac{\lambda_{\max} - n}{n - 1},$$

where CI denotes the Consistency Index and $\lambda_{\max}$ is the largest eigenvalue of the examined PCM. In AHP the acceptable consistency degree is generally indicated by $CR < 0.1$.

Let us denote the number of evaluators in a group decision-making

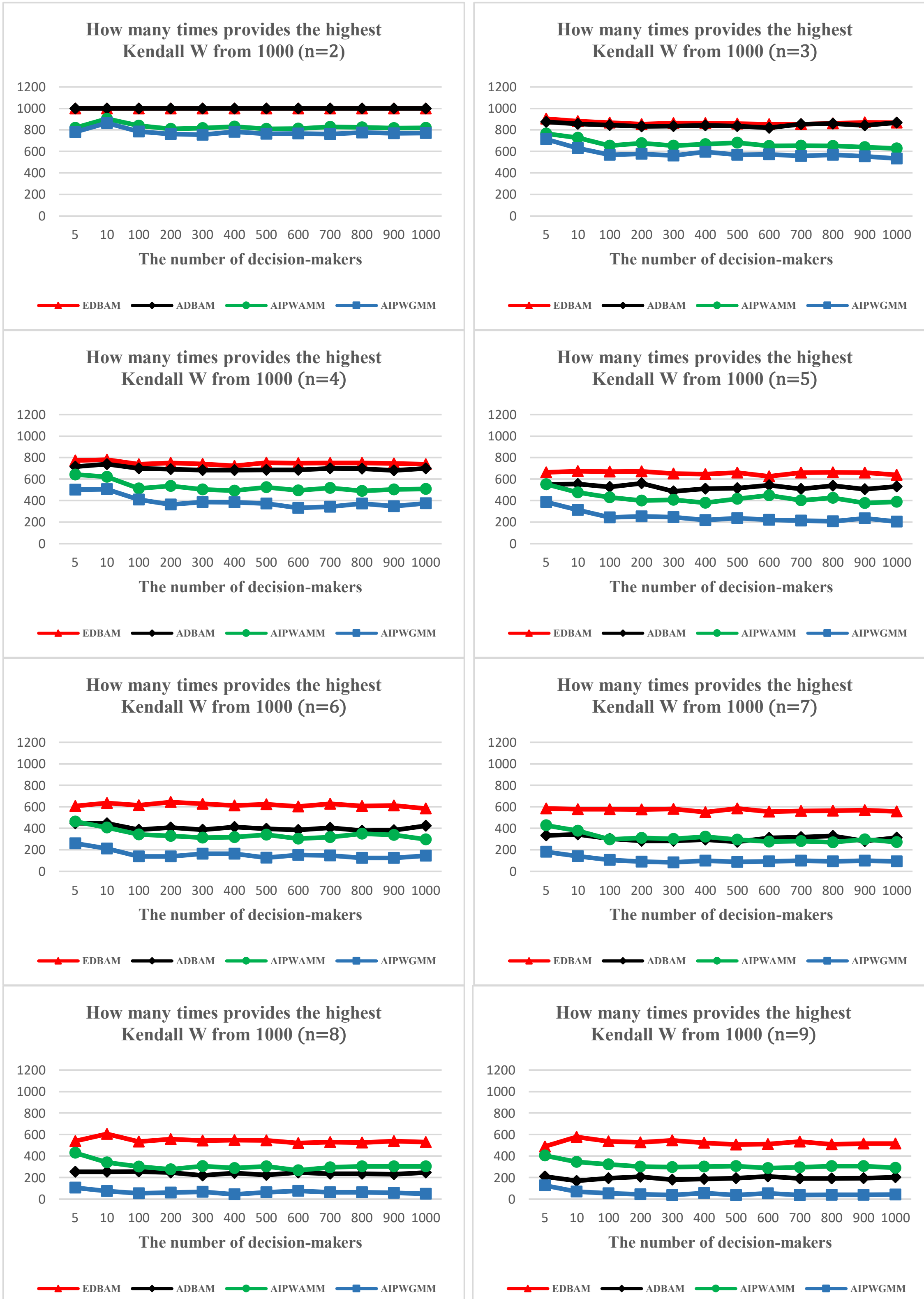

Fig. 1. Large-scale simulation results for the different aggregation methods.

problem by m, let $w^{(k)} = \left(w_1^{(k)}, w_2^{(k)}, \cdots, w_n^{(k)}\right)^T$ be the individual priority vector for decision-maker k ($w_i^{(k)} > 0$ for $\forall i = 1, 2, \cdots, n$ and $\sum_{i=1}^{n} w_i^{(k)} = 1$ for $\forall k = 1, 2, \cdots, m$). Besides the two proposed distance-based preference aggregation techniques, we also include two variants of the well-known Aggregation of Individual Preferences (AIP) as a benchmark in this paper.

As for the AIP Weighted Arithmetic Mean Method (WAMM), the consensual preference vector $w^{(A)}$ is computed as the weighted arithmetic mean of the individual priorities:

Table 1
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 2$.

	w_1	w_2
DM1	0.100	0.900
DM2	0.889	0.111
DM3	0.125	0.875
DM4	0.667	0.333
DM5	0.111	0.889
DM6	0.750	0.250
DM7	0.667	0.333
DM8	0.100	0.900
DM9	0.833	0.167
DM10	0.667	0.333
EDBAM	0.667	0.333
ADBAM	0.667	0.333
AIPWAMM	0.491	0.509
AIPWGMM	0.461	0.539
CPVP	0.199	0.801

Table 2
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 2$. Please see Fig. 2 for the graphical presentation.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.640	0.640	0.551	0.542	0.518
Average Spearman rho	0.2	0.2	−0.2	−0.2	−0.2
Average Kendall W	0.6	0.6	0.4	0.4	0.4
St. dev. G index	0.320	0.320	0.111	0.089	0.106
St. dev. Spearman rho	1.033	1.033	1.033	1.033	1.033
St. dev. Kendall W	0.516	0.516	0.516	0.516	0.516

$$w_i^{(A)} = \sum_{k=1}^{m} a_k w_i^{(k)}, i = 1, 2, \cdots, n.$$

where a_k is the weight of evaluator k and $\sum_{k=1}^{m} a_k = 1$. In case of the AIP Weighted Geometric Mean Method (WGMM), the common preference vector $w^{(G)}$ is obtained by calculating the weighted geometric mean of the individual priorities:

$$w_i^{(G)} = \frac{\prod_{k=1}^{m}\left(w_i^{(k)}\right)^{a_k}}{\sum_{i=1}^{n}\prod_{k=1}^{m}\left(w_i^{(k)}\right)^{a_k}}, i = 1, 2, \cdots, n.$$

where $\sum_{k=1}^{m} a_k = 1$ as before.

Later, in the presentation of more specific examples, we also include the CPVP method (Amenta et al., 2020) in our examinations, which is based on the optimization of a loss function defined as follows.

$$\min_{\lambda_k, q} L(q, \lambda_k) = \sum_{k=1}^{m} a_k \| W_k - \lambda_k q q^T \|_F^2$$

Where $\|.\|_F$ denotes the Frobenius norm (that is basically the matrix version of the Euclidean distance), $\sum_{k=1}^{m} a_k = 1$ as before and W_k is defined by:

$$W_k = \lambda_k q_k q_k^T$$

where λ_k is the principal eigenvalue of the PCM related to evaluator k, while q_k is the eigenvector connected to the principal eigenvalue. In order to obtain a numerical solution, one should use Algorithm 1 of Amenta et al., 2020, as the optimization is needed in two variables, which affect each other. This method is the closest to our proposals in its logic, thus it is important to include it in the paper, however, in the first,

Table 3
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 3$. Please see Fig.3 for the graphical presentation.

	w_1	w_2	w_3
DM1	0.073	0.671	0.256
DM2	0.705	0.211	0.084
DM3	0.674	0.226	0.101
DM4	0.117	0.806	0.077
DM5	0.489	0.067	0.444
DM6	0.537	0.364	0.099
DM7	0.078	0.750	0.171
DM8	0.167	0.094	0.740
DM9	0.600	0.300	0.100
DM10	0.063	0.458	0.479
EDBAM	0.446	0.359	0.195
ADBAM	0.429	0.392	0.179
AIPWAMM	0.350	0.395	0.255
AIPWGMM	0.327	0.418	0.255
CPVP	0.227	0.580	0.192

Table 4
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 3$. Please see Fig. 3 for graphical presentation.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.555	0.552	0.533	0.530	0.511
Average Spearman rho	0.25	0.25	0.2	0.2	0.2
Average Kendall W	0.625	0.625	0.6	0.6	0.6
St. dev. G index	0.161	0.146	0.104	0.100	0.163
St. dev. Spearman rho	0.791	0.791	0.632	0.632	0.632
St. dev. Kendall W	0.395	0.395	0.316	0.316	0.316

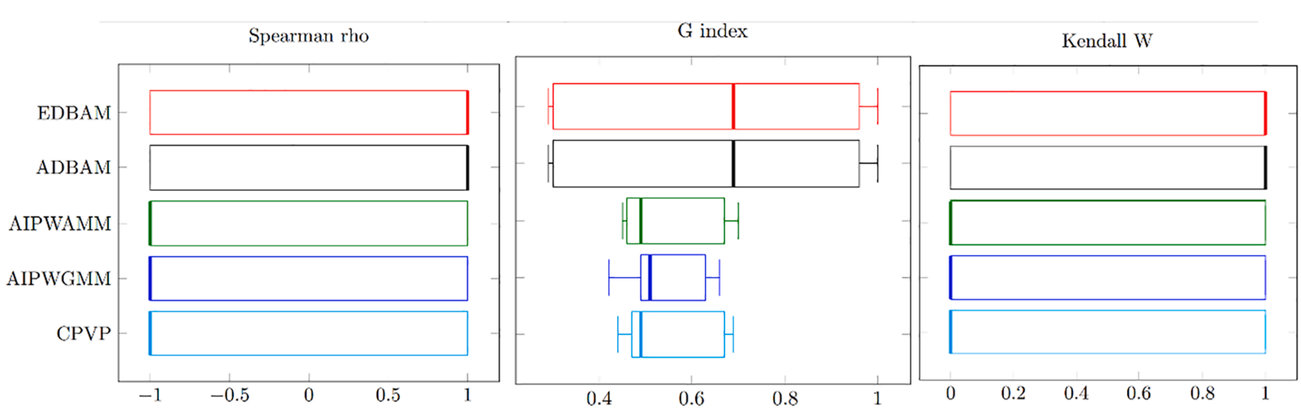

Fig. 2. Box-plots of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 2$.

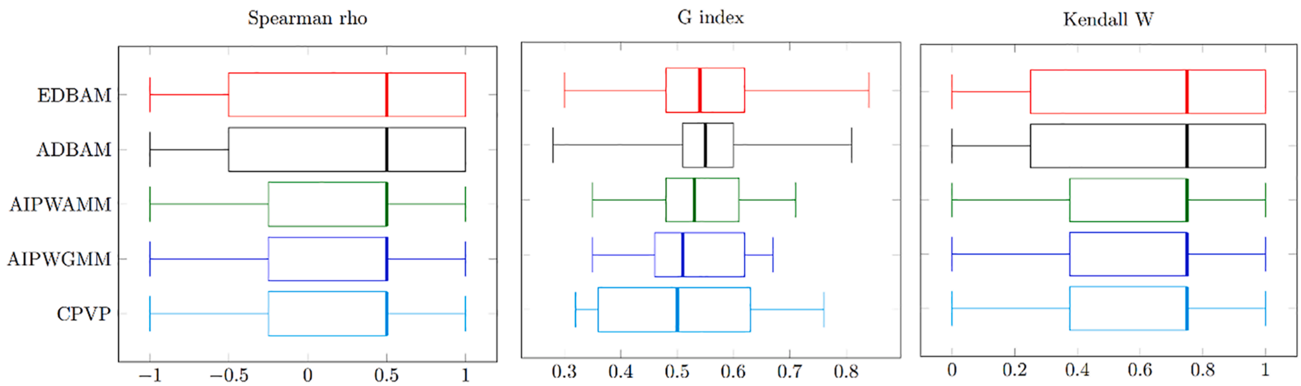

Fig. 3. Box-plots of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 3$.

Table 5
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 4$.

	w_1	w_2	w_3	w_4
DM1	0.081	0.074	0.397	0.448
DM2	0.174	0.051	0.520	0.255
DM3	0.108	0.095	0.704	0.093
DM4	0.259	0.508	0.056	0.177
DM5	0.061	0.535	0.302	0.102
DM6	0.472	0.339	0.106	0.082
DM7	0.178	0.400	0.086	0.336
DM8	0.249	0.203	0.051	0.497
DM9	0.531	0.090	0.094	0.285
DM10	0.043	0.699	0.174	0.084
EDBAM	0.220	0.330	0.203	0.248
ADBAM	0.231	0.313	0.189	0.268
AIPWAMM	0.216	0.299	0.249	0.236
AIPWGMM	0.220	0.288	0.231	0.261
CPVP	0.178	0.432	0.202	0.188

Table 6
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 4$. Please see Fig. 4 for graphical presentation.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.551	0.551	0.542	0.544	0.547
Average Spearman rho	0.06	0.06	0.02	0.04	0.02
Average Kendall W	0.53	0.53	0.51	0.52	0.51
St. dev. G index	0.114	0.117	0.091	0.097	0.154
St. dev. Spearman rho	0.640	0.640	0.649	0.617	0.649
St. dev. Kendall W	0.320	0.320	0.325	0.308	0.325

large-scale phase of the simulations we only concentrate on the benchmark techniques (AIP WAMM and AIP WGMM) in the comparisons.

2.1. Distance-based preference aggregation techniques

The main reason for the preference aggregation methods that we propose is to look for the closest vector to the individual priority vectors according to a certain metric. Regarding the Euclidean Distance-Based Aggregation Method (EDBAM), the group preference vector $w^{(E)}$ is the solution of the following formula normalized to one:

$$argmin f(x)$$

Where $x \in \mathbb{R}^n$ and $f(x)$ is defined as follows:

$$f(x) = \sum_{k=1}^{m} (a_k \bullet d_E(w^{(k)}, x))$$

Where $d_E(w^{(k)}, x) = \sqrt{\sum_{i=1}^{n} \left(w_i^{(k)} - x_i\right)^2}$ is the Euclidean distance, and $\sum_{k=1}^{m} a_k = 1$ as before. Thus, we determine the vector in $\mathbb{R}^n$ that is the nearest one to the individual priority vectors and then normalize it to one.

In case of the Aitchison Distance-Based Aggregation Method (ADBAM) we follow the same steps except that we are minimizing according to the Aitchison distance, $d_A(.,.)$:

$$d_A\left(w^{(k)}, x\right) = \sqrt{\sum_{i=1}^{n} \left[\log\left(\frac{w_i^{(k)}}{g(w^{(k)})}\right) - \log\left(\frac{x_i}{g(x)}\right)\right]^2}$$

Where $g(w^{(k)})$ and $g(x)$ denote the geometric mean of the respective vectors and $\log(.)$ is the function of natural logarithm.

In order to find the solution of the optimization problems connected to the distance-based methods, we use the method of Nelder & Mead (1965), which is a robust technique and uses only function values. In our simulations we apply the R implementation of this method, which can be seen as an algorithm below.

2.2. The comparison of the techniques

As for the comparison of the different preference aggregation methods, from the several techniques that can measure the degree of consensus, we examine various indicators. The first one is the Kendall coefficient of concordance (Kendall W) calculation, which provides an overall measure of agreement in ranking, as the prioritization of the alternatives or criteria is our major goal. In our case, the appropriate procedure is to supplement the rankings provided by the individual decision-makers with the consensual priority ranking calculated with the help of one of the aforementioned methods, and examine the strength of concordance. Kendall W has the common $[0, 1]$ range, and the higher its value is, the stronger the correlation in ranking. Thus, the technique that provides the highest Kendall coefficient of concordance for a given dataset has the strongest correlation in ranking with the preferences of the evaluators, accordingly, it can be considered as the best in that case. This way it is a suitable tool to compare the different preference aggregation methods with each other. However, keep in mind that not the value of the Kendall W itself, but the differences between the provided measures is the key, as the coefficient quantifies the concordance between the different individual evaluators as well. The Kendall W is a non-parametric statistic that can be calculated as follows.

Let $r_{i,k}$ be the rank given to item i by decision-maker k, and R_i be the aggregated ranking of element i:

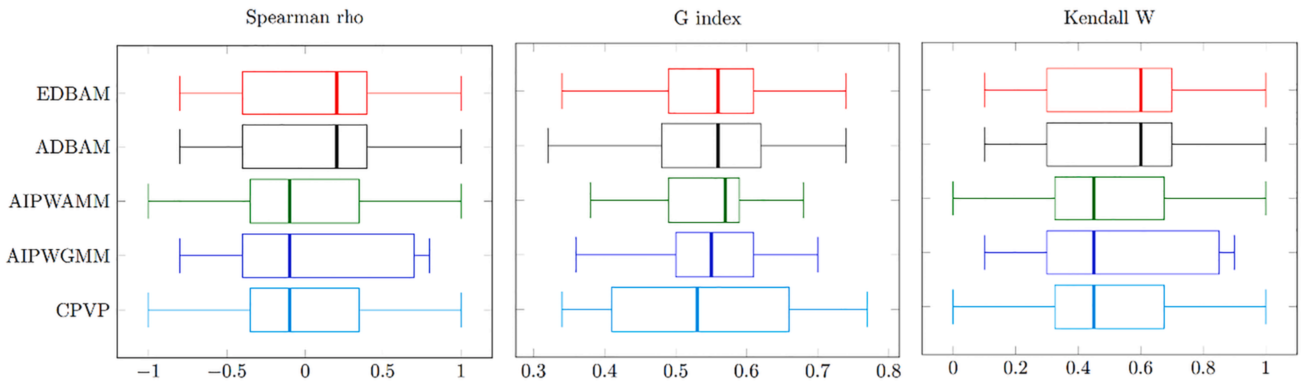

Fig. 4. Box-plots of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 4$.

Table 7
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 5$.

	w_1	w_2	w_3	w_4	w_5
DM1	0.429	0.073	0.102	0.341	0.055
DM2	0.152	0.038	0.127	0.100	0.582
DM3	0.040	0.081	0.088	0.456	0.336
DM4	0.317	0.171	0.372	0.042	0.097
DM5	0.038	0.067	0.134	0.549	0.211
DM6	0.034	0.135	0.199	0.048	0.583
DM7	0.500	0.309	0.044	0.082	0.065
DM8	0.066	0.515	0.052	0.255	0.112
DM9	0.389	0.224	0.272	0.065	0.050
DM10	0.200	0.404	0.211	0.043	0.142
EDBAM	0.162	0.329	0.127	0.145	0.238
ADBAM	0.152	0.271	0.127	0.145	0.305
AIPWAMM	0.157	0.279	0.153	0.151	0.259
AIPWGMM	0.162	0.244	0.145	0.149	0.300
CPVP	0.144	0.352	0.117	0.132	0.255

Table 8
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 5$.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.514	0.516	0.512	0.514	0.499
Average Spearman rho	0.01	0.02	−0.02	−0.01	−0.06
Average Kendall W	0.490	0.485	0.500	0.485	0.490
St. dev. G index	0.088	0.072	0.069	0.067	0.094
St. dev. Spearman rho	0.402	0.380	0.419	0.380	0.402
St. dev. Kendall W	0.201	0.190	0.209	0.190	0.201

$$R_i = \sum_{k=1}^{m} r_{i,k}, i = 1, 2, \cdots, n$$

Let us denote the mean of the aggregated ranking by R:

$$R = \frac{m(n+1)}{2}$$

The sum of squares deviation statistic of the aggregated rankings defined by the following:

$$S = \sum_{i=1}^{n} (R_i - R)^2$$

Finally, the Kendall coefficient of concordance can be obtained from the formula below:

Table 9
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 6$.

	w_1	w_2	w_3	w_4	w_5	w_6
DM1	0.029	0.452	0.042	0.101	0.136	0.241
DM2	0.100	0.316	0.065	0.042	0.027	0.450
DM3	0.506	0.104	0.054	0.202	0.092	0.042
DM4	0.086	0.034	0.295	0.050	0.086	0.449
DM5	0.034	0.095	0.275	0.335	0.161	0.100
DM6	0.384	0.025	0.329	0.112	0.054	0.096
DM7	0.092	0.061	0.045	0.285	0.218	0.299
DM8	0.062	0.096	0.124	0.555	0.133	0.030
DM9	0.328	0.044	0.083	0.047	0.224	0.274
DM10	0.413	0.253	0.028	0.058	0.192	0.057
EDBAM	0.157	0.156	0.124	0.275	0.131	0.158
ADBAM	0.116	0.161	0.134	0.304	0.114	0.172
AIPWAMM	0.146	0.165	0.145	0.254	0.126	0.164
AIPWGMM	0.115	0.163	0.144	0.286	0.115	0.176
CPVP	0.144	0.157	0.138	0.286	0.119	0.156

Table 10
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 6$.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.508	0.508	0.504	0.505	0.507
Average Spearman rho	0.189	0.194	0.149	0.189	0.171
Average Kendall W	0.580	0.523	0.523	0.509	0.523
St. dev. G index	0.098	0.109	0.082	0.104	0.101
St. dev. Spearman rho	0.250	0.313	0.311	0.416	0.311
St. dev. Kendall W	0.125	0.157	0.155	0.208	0.155

$$W = \frac{12S}{m^2(n^3 - n)}$$

When ties occur, W has to be modified by the following correction factor:

$$T_k = \sum_{i=1}^{g_k} (t_i^3 - t_i)$$

where g_k is the number of groups of ties for decision-maker k and t_i is the number of tied ranks in the i th group of tied ranks. In this case the corrected Kendall coefficient of concordance can be calculated as follows.

$$W = \frac{12\sum_{i=1}^{n}(R_i)^2 - 3m^2n(n+1)^2}{m^2(n^3 - n) - m\sum_{k=1}^{m} T_k}$$

Table 11
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 7$.

	w_1	w_2	w_3	w_4	w_5	w_6	w_7
DM1	0.497	0.255	0.051	0.043	0.083	0.041	0.030
DM2	0.023	0.034	0.237	0.052	0.509	0.078	0.066
DM3	0.024	0.244	0.056	0.262	0.140	0.055	0.219
DM4	0.223	0.065	0.057	0.214	0.063	0.053	0.326
DM5	0.064	0.036	0.303	0.071	0.360	0.081	0.084
DM6	0.231	0.052	0.060	0.398	0.027	0.069	0.163
DM7	0.062	0.273	0.076	0.299	0.022	0.114	0.153
DM8	0.054	0.163	0.212	0.051	0.415	0.042	0.062
DM9	0.052	0.213	0.099	0.064	0.127	0.136	0.309
DM10	0.414	0.034	0.163	0.057	0.042	0.097	0.193
EDBAM	0.211	0.096	0.116	0.120	0.227	0.147	0.083
ADBAM	0.217	0.103	0.098	0.125	0.238	0.140	0.081
AIPWAMM	0.203	0.101	0.129	0.123	0.213	0.135	0.096
AIPWGMM	0.208	0.108	0.112	0.128	0.217	0.132	0.095
CPVP	0.228	0.089	0.105	0.109	0.255	0.136	0.078

Table 12
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 7$.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.523	0.522	0.517	0.520	0.495
Average Spearman rho	0.196	0.232	0.096	0.232	0.157
Average Kendall W	0.430	0.427	0.429	0.430	0.430
St. dev. G index	0.051	0.049	0.051	0.049	0.057
St. dev. Spearman rho	0.377	0.314	0.460	0.377	0.377
St. dev. Kendall W	0.188	0.157	0.230	0.188	0.188

We also use the well-known Spearman's rank correlation coefficient (Spearman rho) that is applicable for two vectors, given by the formula below:

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n^3 - n}$$

where d_i is the difference of the rank of element i for the examined vectors. When there are ties in the ranks the equation is modified as follows:

$$\rho = \frac{cov(r_1, r_2)}{\sigma_{r_1}\sigma_{r_2}}$$

where r_1 and r_2 denote the rankings defined by the first and second examined vectors, $cov(.,.)$ is the covariance of two variables, while σ is the standard deviation of a variable. This indicator's value is in the range $[-1, 1]$, thus it can also show the complete disagreement between two preference vectors, compared to the Kendall W measurement.

As for the cardinal indicators, we use Garuti's compatibility index (G index) (Garuti, 2020) that is also applicable for two preference vectors and defined as follows.

$$G = \frac{1}{2}\sum_{i=1}^{n} \frac{\min\left(w_i^{(1)}, w_i^{(2)}\right)}{\max\left(w_i^{(1)}, w_i^{(2)}\right)} \left(w_i^{(1)} + w_i^{(2)}\right)$$

Where $w_i^{(1)}$ and $w_i^{(2)}$ denote the i th element of the first and second examined vectors. The higher the G index, the more compatible the two respective vectors are. It is important to focus on both ordinal and cardinal measurements as there can be large contrasts between the results according to these indicators, because their calculation method and logic is different, indeed.

In Sub-section 3.2. we compute the average Spearman rho-s and G indices of the individual preference vectors respect to the given aggregated vectors and compare these averages for the aggregation methods to find the best techniques.

3. Results

3.1. Large-scale preference vector simulations

In order to compare the performance of the different preference aggregation techniques in Large-scale Group Decision-Making problems, we completed a wide range of numerical simulations. We examined two to nine dimensional preference vectors (n), while the studied number of evaluators (m) were 5, 10, 100, 200, …, 1000. The simulation for a given

Table 14
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 8$.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.534	0.535	0.531	0.533	0.534
Average Spearman rho	0.248	0.245	0.240	0.240	0.233
Average Kendall W	0.442	0.449	0.468	0.461	0.442
St. dev. G index	0.057	0.064	0.053	0.060	0.059
St. dev. Spearman rho	0.433	0.366	0.445	0.334	0.433
St. dev. Kendall W	0.216	0.183	0.222	0.167	0.216

Table 13
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 8$.

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
DM1	0.088	0.080	0.063	0.024	0.143	0.351	0.051	0.199
DM2	0.090	0.024	0.016	0.230	0.071	0.038	0.224	0.307
DM3	0.191	0.082	0.024	0.149	0.030	0.317	0.044	0.163
DM4	0.033	0.319	0.051	0.103	0.291	0.141	0.039	0.023
DM5	0.403	0.099	0.031	0.091	0.018	0.237	0.079	0.042
DM6	0.167	0.051	0.184	0.298	0.018	0.078	0.018	0.186
DM7	0.237	0.318	0.059	0.090	0.023	0.129	0.016	0.129
DM8	0.030	0.179	0.154	0.357	0.038	0.036	0.176	0.029
DM9	0.097	0.028	0.167	0.024	0.066	0.136	0.197	0.286
DM10	0.045	0.072	0.108	0.019	0.368	0.085	0.176	0.127
EDBAM	0.141	0.120	0.085	0.136	0.096	0.164	0.099	0.159
ADBAM	0.157	0.123	0.083	0.120	0.078	0.189	0.089	0.161
AIPWAMM	0.138	0.125	0.086	0.138	0.107	0.155	0.102	0.149
AIPWGMM	0.144	0.126	0.091	0.127	0.086	0.170	0.099	0.156
CPVP	0.166	0.122	0.077	0.134	0.084	0.178	0.089	0.150

Table 15
The ten normalized simulated preference vectors and the related aggregated priority vectors for $n = 9$.

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
DM1	0.045	0.072	0.052	0.043	0.046	0.392	0.154	0.023	0.173
DM2	0.041	0.187	0.051	0.404	0.071	0.028	0.127	0.076	0.016
DM3	0.094	0.053	0.226	0.315	0.029	0.016	0.026	0.118	0.123
DM4	0.135	0.177	0.020	0.070	0.023	0.030	0.148	0.325	0.071
DM5	0.225	0.235	0.135	0.158	0.028	0.088	0.076	0.035	0.019
DM6	0.233	0.046	0.165	0.172	0.052	0.015	0.055	0.094	0.168
DM7	0.333	0.036	0.127	0.020	0.098	0.045	0.023	0.269	0.050
DM8	0.225	0.235	0.135	0.158	0.028	0.088	0.076	0.035	0.019
DM9	0.233	0.046	0.165	0.172	0.052	0.015	0.055	0.094	0.168
DM10	0.333	0.036	0.127	0.020	0.098	0.045	0.023	0.269	0.050
EDBAM	0.207	0.107	0.136	0.158	0.050	0.056	0.067	0.122	0.096
ADBAM	0.215	0.105	0.145	0.148	0.059	0.045	0.075	0.122	0.086
AIPWAMM	0.190	0.112	0.120	0.153	0.052	0.076	0.076	0.134	0.086
AIPWGMM	0.206	0.115	0.134	0.136	0.062	0.057	0.082	0.127	0.081
CPVP	0.219	0.103	0.130	0.144	0.057	0.048	0.063	0.155	0.080

Table 16
Basic statistics of the G index, Spearman's rank correlation coefficient and Kendall W of the aggregated priority vectors respect to the individual preference vectors for $n = 9$.

	EDBAM	ADBAM	AIPWAMM	AIPWGMM	CPVP
Average G index	0.599	0.599	0.582	0.588	0.593
Average Spearman rho	0.380	0.382	0.377	0.375	0.375
Average Kendall W	0.690	0.691	0.688	0.688	0.688
St. dev. G index	0.147	0.143	0.125	0.126	0.129
St. dev. Spearman rho	0.421	0.445	0.420	0.436	0.420
St. dev. Kendall W	0.210	0.222	0.210	0.218	0.210

(n, m) pair consists of the steps detailed below.

Algorithm 2.: Process of the simulation

Input: (n, m) pair of parameters.
Output: The number of times when a method provided the highest Kendall W of 1000 cases for AIP WAMM, AIP WGMM, EDBAM, and ADBAM.

1. Generate m different n-dimensional random vectors, which are normalized to one (individual preference vectors).
2. Apply the AIP WAMM, AIP WGMM, EDBAM and ADBAM methods on the individual vectors (detailed in Section 2) resulting in the following priority vectors: $w^{AIPWAMM}$, $w^{AIPWGMM}$, w^{EDBAM} and w^{ADBAM}.
3. Collect the rankings provided by the individual decision-makers and by the consensual priority ranking calculated according to the appropriate method in the following matrices: $M^{AIPWAMM}$, $M^{AIPWGMM}$, M^{EDBAM} and M^{ADBAM}.
4. The tie-corrected Kendall coefficient of concordance measure is calculated for the four matrices and those methods that provide the highest Kendall W are saved.
5. Steps 1–4 are repeated 1000 times.

Return: A (4-dimensional) vector contains the number of times when AIP WAMM, AIP WGMM, EDBAM, and ADBAM provided the highest Kendall W of 1000 cases.

Based on this procedure, we examined a total of $8 \times 12000 = 96000$ cases, and the efficiency of the preference aggregation techniques according to the dimensions of the priority vectors and the number of

Table 18
The normalized consensual priority vector of EDBAM.

EDBAM	
Approachability	0.2276
Directness	0.2234
Time availability	0.1792
Speed	0.1776
Reliability	0.1922
Kendall W	**0.1521**

Table 19
The calculation of Kendall W in case of EDBAM.

Criteria	Rank of Evaluator1	…	Rank of EDBAM	$\mathbf{R_i}$	$(\mathbf{R_i} - \mathbf{R})^2$
Approachability	2	…	1	23	100
Directness	1	…	2	30	9
Time availability	4	…	4	38	25
Speed	5	…	5	40	49
Reliability	3	…	3	34	1
n = 5	m = 11	S = 184	**R** = 33	**W** = 0.1521	

Table 20
The normalized consensual priority vector of ADBAM.

ADBAM	
Approachability	0.2269
Directness	0.2215
Time availability	0.1764
Speed	0.1761
Reliability	0.1990
Kendall W	**0.1521**

Table 17
The normalized AHP scores of ten individual respondents in the public transport development survey.

	Approachability	Directness	Time availability	Speed	Reliability
Evaluator1	0.2406	0.2940	0.1408	0.1388	0.1858
Evaluator2	0.2575	0.1448	0.2244	0.1246	0.2488
Evaluator3	0.2886	0.0717	0.1257	0.2535	0.2604
Evaluator4	0.1853	0.2881	0.1707	0.1332	0.2227
Evaluator5	0.2160	0.4059	0.0609	0.2129	0.1043
Evaluator6	0.0524	0.3127	0.2296	0.3135	0.0918
Evaluator7	0.0615	0.1586	0.2587	0.3635	0.1577
Evaluator8	0.2930	0.0721	0.2771	0.1625	0.1953
Evaluator9	0.2608	0.2143	0.1311	0.1546	0.2393
Evaluator10	0.2708	0.2189	0.2510	0.1739	0.0854

Table 21
The calculation of Kendall W in case of ADBAM.

Criteria	Rank of Evaluator1	…	Rank of ADBAM	$\mathbf{R_i}$	$(\mathbf{R_i} - \mathbf{R})^2$
Approachability	2	…	1	23	100
Directness	1	…	2	30	9
Time availability	4	…	4	38	25
Speed	5	…	5	40	49
Reliability	3	…	3	34	1
n = 5	m = 11	S = 184	**R** = 33	**W** = 0.1521	

Table 22
The normalized consensual priority vector of AIP WAMM.

AIPWAMM	
Approachability	0.2126
Directness	0.2181
Time availability	0.1870
Speed	0.2031
Reliability	0.1792
Kendall W	**0.1289**

Table 23
The calculation of Kendall W in case of AIP WAMM.

Criteria	Rank of Evaluator1	…	Rank of AIP WAMM	$\mathbf{R_i}$	$(\mathbf{R_i} - \mathbf{R})^2$
Approachability	2	…	2	24	81
Directness	1	…	1	29	16
Time availability	4	…	4	38	16
Speed	5	…	3	38	36
Reliability	3	…	5	36	9
n = 5	m = 11	S = 156	**R** = 33	**W** = 0.1289	

Table 24
The normalized consensual priority vector of AIP WGMM.

AIPWGMM	
Approachability	0.2052
Directness	0.2099
Time availability	0.1902
Speed	0.2106
Reliability	0.1841
Kendall W	**0.0959**

Table 25
The calculation of Kendall W in case of AIP WGMM.

Criteria	Rank of Evaluator1	…	Rank of AIP WGMM	$\mathbf{R_i}$	$(\mathbf{R_i} - \mathbf{R})^2$
Approachability	2	…	3	26	49
Directness	1	…	2	30	9
Time availability	4	…	4	37	16
Speed	5	…	1	36	9
Reliability	3	…	5	36	9
n = 5	m = 11	S = 92	**R** = 33	**W** = 0.0959	

evaluators became comparable. As the different methods can provide exactly the same ranking of the items, it is important to note that ties might occur between them according to the Kendall W measure. It is also notable that in the simulation process we used equal weights for every decision-maker. Fig. 1 highlights the results of the simulations, one can see the number of times, when the different techniques provided the highest Kendall W for different preference vector dimensions (2 to 9) depending on the number of decision-makers. The entire data of the simulations are presented in Appendix A

Because of the ties, it is visible that there are basically always more than 1000 first places for a given (n, m) pair in our outcomes, however the larger the priority vectors are, the lower the number of ties. Probably the most remarkable point is that EDBAM provides the best aggregation efficiency in every examined case, thus its dominance is indeed strong, and for higher n parameters the difference between the methods is larger as well. It is also interesting that for $n = 2$ EDBAM and ADBAM even provide the best coefficient of concordance 1000 times out of 1000 possible ones for all examined m. For smaller and mid-sized $(2 \leq n \leq 6)$ priority vectors ADBAM also produces better efficiency compared to the AIP based methods, moreover for small dimensions it has approximately the same results as EDBAM. However, its efficiency is decreasing in the number of dimensions, and for large priority vectors $(7 \leq n \leq 9)$ even AIP WAMM tends to overtake it. This is probably due to the higher unknowns in finding the nearest consensus vector in higher dimensions of preference vectors. It is also clear that AIP WGMM provides the least favorable efficiency in every studied case, thus the Arithmetic Mean variant definitely outperforms the Geometric one among the AIP methods.

Based on all the simulations, it is important to emphasize that the computational process of the consensual vector for EDBAM on an average PC remained very low, around one second even in higher dimensional cases and for higher number of decision-makers. On the other hand, ADBAM turned out to be a computationally more difficult problem, although its running time also remained around one minute for larger priority vectors even in the case of 1000 decision-makers.

Thus, we must highlight the fact that according to our results EDBAM is indeed dominant in its efficiency compared to the other methods, it is also fast and easy to implement, therefore it seems to be an excellent choice to apply in case of Large-scale Group Decision-Making problems.

3.2. Specific simulated Group-AHP examples

In this sub-section, we present the results of PCM-examples in different dimensions (2 to 9) relevant in AHP methodology (see Saaty's maximum 9x9 rule, Saaty, 1994) considering relevant thresholds for the consistency: CR < 0.1 for at most 6 dimensions and CR < 0.2 from 7 until 9 alternatives (criteria). In each case, we generate ten different random matrices, derive their priority vectors (by the eigenvector method), and then aggregate them by five different methods: EDBAM, ADBAM, AIP WAMM, AIP WGMM, and the CPVP method (Amenta et al., 2020) assuming equal weights of the decision-makers. Owing to the condition of applying different comparison methods and overcome the pitfalls of Kendall W (only positive interval and ordinal aspect), we utilize Spearman's rank correlation coefficient (its related interval is [-1,1]) and Garuti's index (for measuring cardinal differences) to detect the performance of the possible aggregation methods. Regarding the difficulty of PCM generations with the proper consistency ratio, we refer to Bozoki and Rapcsak, 2008. The authors declared that for PCMs larger than 6×6, it is very difficult to find a randomly generated matrix that meets the CR < 0.1 criterion, in case of the largest matrix sizes 8×8 and 9×9 no appropriate example was found out of 10 million simulated cases.

For this reason, we generated ten appropriate PCMs that meet with the aforementioned thresholds for each dimension. The related results are presented in Tables 1–16. The hypothetic decision-makers are denoted by DM, while the elements of the preference vectors are referred to as w_i $(i = 1, \cdots, 9)$. It is important to emphasize that the differences between the provided averages of G indices and Spearman's rank correlation coefficients (Spearman rho-s) show the performance of the aggregation methods, not the indicators themselves, as the randomly generated individual preferences are colorful enough to decrease the average of these measurements. The related randomly generated PCMs can be found in Appendix B.

The derived weight vectors are heterogeneous in the two-dimensional simulation example for the ten hypothetical evaluators, and we can detect a rank conflict between the aggregation of the proposed distance-based methods and the two benchmark and CPVP techniques (Table 1). Table 2 shows how the compatibility index and the Spearman rank correlation index indicate the performance of aggregation techniques. For convenience, we also attach the calculated Kendall W values of the specific simulation results. Moreover, standard deviations of all three indices are presented by values and exhibited by box- plots along with the mean values of G-index, Spearman rho and Kendall W.

Thick lines represent the median values and the size of the horizontal column the interquartile-range. As could be expected based on the large-scale simulations, both EDBAM and ADBAM outperform the other three methods not only in ordinal (indicated by Spearman rho, and Kendall W), but also in cardinal (G index) compatibility. The gap of performance is also significant, the G index measures from 0 to 1, the Spearman rho from −1 to 1. We note that the threshold of compatibility in the case of G index is 0.9 so better performance is imaginable but for the known aggregation methods, the two proposed techniques provide the best compromise vector.

In three dimensions, the supremacy of our proposed techniques remained but the gap between the performance of EDBAM and ADBAM, and the other three methods had decreased (Table 3 and Table 4). This phenomenon also corresponds to the results of the large-scale simulations (see Fig. 1). Even though the differences are smaller, the distance-based aggregation approach indicates better results than the other well-proven approaches.

Interestingly, in the fourth dimension, the relative gap between the distance-based methods and the other techniques increased in the case of the Spearman index and Kendall W (the absolute gap remained small), while the G index indicated a slightly smaller difference (Table 5 and Table 6). The CPVP method performed somewhat better than the two benchmark AIP WAMM and AIP WGMM techniques considering the G-index but for ranking approaches (Spearman and Kendall) it is still in the last position. The dominance of our approach remained also in this dimension. In the followings, the difference among the aggregation techniques is smaller, so we only present the values and not the box plots in the cases from five to nine dimensions.

The fifth dimension (Table 7) reduced the differences both for the G index and Spearman rho. As Table 8 indicates, AIP WGMM tied with EDBAM and ADBAM became just slightly better than the other competitors by the G index. For the Spearman's rank correlation coefficient, the proposed aggregation techniques kept their primacy, while Kendall W indicated a very slight primacy of the AIP WAMM.

In the sixth dimension (Table 9), the difference in performance was further mitigated (Table 10). However, a slight advantage of the distance-based methods could still be detected, especially ADBAM performed well, which slightly contradicts the large-scale simulation results. Considering Kendall W, EDBAM has kept the strong primacy as could be expected by the vector simulations. In the following, we introduce the results of the large dimensional (7,8,9) simulation cases (Tables 1–16).

In large dimensions, the alterations of rank correlation and compatibility indices are very similar in all aggregation types. Some slight advantage of the distance-based methods remained but we cannot exclude that further simulation cases might bring a different ranking in performance. As one can see, although the differences are not huge, the distance-based aggregation methods provide the best G index and Spearman rho for every dimension. Even though in the eighth dimension the slight advantage of AIP WAMM occurred, the results of Kendall W calculations also supported our assumption of better performance of the EDBAM and ADBAM. This supports the former results and also extends the findings to the cardinal indicators. We should highlight the fact that these techniques tend to perform better, when there are serious contrasts in the individual preferences, while in case of large agreement between the individuals, all the aggregation methods provide indeed similar results. These outcomes also suggest that our findings do not depend on the applied comparison methods. Thus, based on these simulated examples, in the case of AHP-specific circumstances (using the Saaty-scale, and thresholds for the CR) the distance-based methods outperform not only the AIP techniques, but the CPVP method as well.

In the next sub-section, we utilize the proposed aggregation methods on real-world data, gained by a group AHP survey on public transport development in Turkey. In the comparison of performance, we apply the two benchmark methods, AIP WAMM and AIP WGMM to examine, whether the outcomes of the real-world decision analysis correspond the large-scale simulation results. Consequently, we used the Kendall W measure in the next phase to reflect the success of the aggregation techniques.

3.3. A real-world application

In December 2017, a group AHP survey was conducted in the Turkish big city, Mersin with the participation of citizens (Duleba & Moslem, 2018). All attributes of public transport supply quality were mapped, and the created questionnaire followed strictly the rules of AHP, it contained a hierarchical criteria structure, and the participants compared the criteria pair-wisely, according to the branches of the decision tree. Altogether 97 evaluators participated, the computed Consistency Ratio was below 0.1, and thus, based on Saaty's rule, all evaluations could be considered tolerably inconsistent.

For the demonstration of the relation of distance-based aggregation and the conventional AIP WAMM and AIP WGMM, we selected one branch of the decision tree of transport supply quality. It includes five criteria: Approachability, Directness, Time availability, Speed, and Reliability. Approachability stands for bus line access and connected services; Directness means the simplicity of reaching the destination without shifting vehicles; Time availability represents the time frame when using a certain vehicle; Speed means the speed of the whole travel process, while Reliability represents the on-time arrivals and keeping the schedule. For a more detailed description of the attributes, please see Lakatos & Mandoki (2021).

We selected ten respondents out of the citizen pattern to make it visible, how the two distance-based methods, the EDBAM and the ADBAM perform in aggregating the individual preferences compared to the two conventional aggregation approaches, the AIP WAMM and AIP WGMM. Following the characteristics of group AHP, the priority vector coordinates of the individual evaluators are normalized to one. It is worth emphasizing that all participants had equal weights related to the final decision, so we did not differentiate the evaluators at all in the citizen pattern neither by age nor by other demographical or social characteristics.

Table 17 demonstrates the calculated normalized AHP scores of the ten individual evaluators.

We can observe that the respondents had a very different image of the need for improvement of specific public transport supply quality attributes in the conducted survey. For instance, Evaluator 6 allocated merely 0.0524 to the Approachability criterion and 0.3135 to Speed (which means that Speed has much more importance in system development than ameliorating the Approachability issue of transport service), while Evaluator 10 judged Approachability 0.2708 and the Speed only 0.1739. Since the normalized scores and the priority rankings differ to a large extent, it is difficult to obtain a consensual priority vector and ranking. Also, the value of Kendall concordance coefficient is expected to be very low due to the high diversity of the respondent opinions. In the following, we present the calculated common priority vector based on the four examined techniques. Furthermore, we calculate the Kendall W value between the common priority vector and the ten individual priority vectors and compare the results. Evidently, those techniques that reach a higher Kendall W value can be considered more efficient due to the higher concordance of individual and group rankings. As no ties occur in our problem, we can use the uncorrected version of the Kendall W.

First, in Table 18 we calculate the common priority vector by the Euclidean distance measure based on Algorithm 1.

Consequently, by the Euclidean distance minimization, Approachability has reached the highest importance score in the common priority vector with 0.2276 and Speed has been ranked last. The normalized scores are relatively close to each other, which can be explained by the diverse preference evaluations of the ten individuals, however, the ranking is quite clear. Following the rank correlation formula of Kendall W, the calculation in Table 19 can be exhibited.

Now, in Table 20 we present the common priority vector calculation by the Aitchison distance.

Using the Aitchison distance minimization for creating the common priority vector, we obtain exactly the same ranking as in the Euclidean case (EDBAM). Note that Approachability has been ranked to the first place again by a slightly lower normalized score (0.2269), the order is the same and Speed gained a little less importance value than in the previous case. Next in Table 21 we demonstrate the Kendall W calculation for this case.

For the comparison, we calculate the consensual vector by the conventional AIP Weighted Arithmetic Mean Method in Table 22.

AIP WAMM has produced totally different ranking of the criteria while keeping relatively close values of the normalized coordinates. Directness has taken over Approachability and Speed has been ranked higher than Reliability and Time availability. The total difference of the most prioritized attribute (0.2181) and the least prioritized one (0.1792) has remained almost the same but the order has changed significantly. This has had an impact on the Kendall W value, which is calculated in many details in Table 23.

Finally, in Table 24 we do the same for AIP Weighted Geometric Mean Method.

The coordinates of the common priority vector gained by AIP WGMM are still relatively close, but the ranking has been modified again, mainly compared to the distance-based rankings. Speed has taken the first place in the order of attribute importance, while Approachability (0.2052), Time availability (0.1902), and Reliability (0.1841) has been seeded 3rd, 4th, and 5th. The Kendall W value, which is calculated in Table 25, reflects the similarity of the criteria order of the calculated common priority vector and the rankings of individual priority vectors in WGMM aggregation technique.

4. Conclusions

In our paper, we demonstrated the primacy of distance-based aggregation methods EDBAM and ADBAM in small and mid-dimensional ($2 \leq n \leq 6$) priority vectors and the dominancy of EDBAM for large dimensional ($7 \leq n \leq 9$) cases compared to the two most popular aggregation methods; AIP WAMM and AIP WGMM in group AHP. As evidence, we provided 96.000 simulation cases on normalized randomly generated vectors and also tested the efficiency of the proposed approaches on a real-world group AHP example.

Furthermore, we conducted a second simulation phase, in which pairwise comparison matrices were generated from two to nine dimensions (altogether 80) with acceptable consistency degree and the derived priority vectors were aggregated by the examined methods amended by the CPVP approach, moreover, their performance was tested by the Spearman rho, Kendall W, and the Garuti index. In smaller dimensions, the distance-based methods significantly outperformed the competitors, while in larger cases (7, 8, 9) the difference was mitigated but still palpable.

Results have shown that it is worth searching for a common priority vector, which is situated by the minimum (Euclidean or Aitchison) total distance from the individual priority vectors, and thus, we gain higher rank correlation and compatibility than applying the conventional aggregation methods or the recently emerged CPVP aggregation. This statement is even more relevant for large-scale group decision-making, because most aggregation techniques lose their efficiency if the number of evaluators (the number of individual priority vectors) grow.

In final conclusion, we emphasize that Euclidean Distance-Based Aggregation Method or Aitchison Distance-Based Aggregation Method dominate all other examined techniques in terms of efficiency measured by correlation and compatibility indices, especially in smaller dimensions (2 to 6). Moreover, their computational time is very low, and the methods are applicable for 1000 participants and most likely for even bigger patterns of decision-makers. We also note that in real decision problems, large pairwise comparison matrices are very seldom, thus our findings are even more relevant in practical decision support. According to our results, ADBAM can be especially recommended for those decision problems in which just a few alternatives exist, and the computational time can be larger, while EDBAM for more alternatives (up to nine) and quicker decisions.

Owing to the limitation of our research, we note that there are other possible aggregation techniques in the scientific literature besides AIP WAMM, AIP WGMM, or CPVP. We did not test the Aggregation of Individual Judgements (AIJ) method cases, mainly due to its complicated use in large-scale decision-making, and did not examine other solutions, e.g. consistency-based aggregations either. Also, for complete assurance, large-scale matrix simulations will be necessary to conduct to analyze the nature of EDBAM and ADBAM aggregation techniques. However, we emphasize that this type of simulation is very difficult to execute, e.g. for analyzing the 5-dimensional case (5x5 matrices) merely one sufficiently consistent PCM ($CR < 0.1$) can be found by the generation of 500 (see Bozoki and Rapcsak, 2008). Consequently, if we examine 1000 cases, for

1000 simulated participants, the approximate number of the necessary pairwise comparisons matrices is 500.000.000.

As a remark for further research, the group of the tested aggregation techniques can be extended. Moreover, it is worth investigating models outside the AHP methodology, for instance, TOPSIS and VIKOR, which are also distance-based methods themselves. The case of different weights of individual decision-makers is to be further investigated, as well.

However, based on our simulation results and case study, we can state that a step forward has been taken towards the shift from the conventional aggregation methods to distance-based aggregation in large-scale group decision-making.

CRediT authorship contribution statement

Szabolcs Duleba: Conceptualization, Methodology, Validation, Writing – original draft, Writing – review & editing. **Zsombor Szádoczki:** Methodology, Software, Data curation, Writing – original draft, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Table 26
The data of the simulations in details for small ($n = 2, 3$) dimensional preference vectors.

	n = 2				n = 3			
m	EDBAM	ADBAM	AIPWAMM	AIPWGMM	EDBAM	ADBAM	AIPWAMM	AIPWGMM
5	1000	1000	820	781	904	875	766	714
10	1000	1000	904	865	881	856	728	632
100	1000	1000	841	785	867	844	655	570
200	1000	1000	810	761	854	834	677	577
300	1000	1000	817	756	863	836	654	563
400	1000	1000	830	782	864	843	667	596
500	1000	1000	810	765	858	836	682	570
600	1000	1000	813	766	853	820	652	573
700	1000	1000	828	763	853	854	655	557
800	1000	1000	824	776	862	859	652	569
900	1000	1000	818	768	869	842	641	555
1000	1000	1000	819	772	867	868	628	534

Table 27
The data of the simulations in details for mid-sized ($n = 4, 5$) dimensional preference vectors.

	n = 4				n = 5			
m	EDBAM	ADBAM	AIPWAMM	AIPWGMM	EDBAM	ADBAM	AIPWAMM	AIPWGMM
5	774	716	643	502	663	550	554	388
10	781	740	622	506	674	555	478	313
100	738	700	513	411	670	527	432	244
200	751	692	536	365	672	561	402	255
300	742	684	505	388	652	487	409	248
400	724	684	493	386	647	512	381	220
500	753	686	525	374	661	516	417	237
600	749	686	495	333	626	544	449	221
700	751	701	519	344	661	509	404	215
800	751	698	491	373	663	539	427	208
900	745	681	504	349	662	507	378	235
1000	738	701	510	376	641	532	389	206

Table 28
The data of the simulations in details for mid-sized and larger ($n = 6,7$) dimensional preference vectors.

	n = 6				n = 7			
m	EDBAM	ADBAM	AIPWAMM	AIPWGMM	EDBAM	ADBAM	AIPWAMM	AIPWGMM
5	608	448	464	261	585	334	428	182
10	636	447	409	212	578	344	377	141
100	616	388	344	140	578	304	297	107
200	644	408	331	140	575	284	312	91
300	628	387	314	165	580	284	303	84
400	612	413	319	164	551	296	322	100
500	624	396	341	127	584	275	295	89
600	603	385	304	152	555	312	277	93
700	628	405	319	148	561	318	281	100
800	609	379	350	125	565	331	271	93
900	612	383	338	125	570	282	298	99
1000	584	425	298	146	557	314	273	93

Table 29
The data of the simulations in details for large ($n = 8,9$) dimensional preference vectors.

	n = 8				n = 9			
m	EDBAM	ADBAM	AIPWAMM	AIPWGMM	EDBAM	ADBAM	AIPWAMM	AIPWGMM
5	538	253	430	106	489	210	405	127
10	606	253	342	74	578	170	347	70
100	535	256	302	55	537	195	322	53
200	557	246	276	61	528	207	302	44
300	544	220	307	67	547	181	298	38
400	549	242	288	44	522	187	302	56
500	545	224	304	63	508	195	306	39
600	521	246	267	78	512	211	288	53
700	531	235	296	63	534	191	296	38
800	525	236	304	64	509	193	306	41
900	540	230	304	59	516	194	306	40
1000	531	246	305	49	516	204	291	43

Appendix B

Table 30
The generated random PCMs using the Saaty-scale for $n = 2$.

1	0.111	1	8	1	0.143	1	2	1	0.125
9	1	0.125	1	7	1	0.5	1	8	1
1	3	1	2	1	0.111	1	5	1	2
0.333	1	0.5	1	9	1	0.2	1	0.5	1

Table 31
The generated random PCMs using the Saaty-scale for $n = 3$.

1	0.125	0.25	1	4	7	1	4	5	1	0.111	2	1	8	1
8	1	3	0.25	1	3	0.25	1	3	9	1	8	0.125	1	0.167
4	0.333	1	0.143	0.333	1	0.2	0.333	1	0.5	0.125	1	1	6	1
1	2	4	1	0.143	0.333	1	2	0.2	1	2	6	1	0.143	0.125
0.5	1	5	7	1	6	0.5	1	0.143	0.5	1	3	7	1	1
0.25	0.2	1	3	0.167	1	5	7	1	0.167	0.333	1	8	1	1

Table 32
The generated random PCMs using the Saaty-scale for $n = 4$.

1	1	0.167	0.25	1	3	0.25	1	1	1	0.2	1	1	0.333	5	2	1	0.2	0.125	0.5
1	1	0.25	0.111	0.333	1	0.125	0.143	1	1	0.125	1	3	1	8	2	5	1	3	5
6	4	1	1	4	8	1	2	5	8	1	9	0.2	0.125	1	0.333	8	0.333	1	3
4	9	1	1	1	7	0.5	1	1	1	0.111	1	0.5	0.5	3	1	2	0.2	0.333	1
1	2	5	4	1	0.5	3	0.333	1	1	6	0.5	1	4	8	2	1	0.111	0.2	0.333
0.5	1	2	8	2	1	3	2	1	1	4	0.333	0.25	1	1	0.2	9	1	7	8
0.2	0.5	1	1	0.333	0.333	1	0.25	0.167	0.25	1	0.125	0.125	1	1	0.5	5	0.143	1	3
0.25	0.125	1	1	3	0.5	4	1	2	3	8	1	0.5	5	2	1	3	0.125	0.333	1

Table 33
The generated random PCMs using the Saaty-scale for $n = 5$.

1	3	9	1	7	1	3	2	2	0.167	1	0	0.5	0.167	0.111	1	2	0.5	7	6
0.333	1	0.5	0.2	1	0.333	1	0.143	0.25	0.125	4	1	1	0.111	0.167	0.5	1	0.333	9	1
0.111	2	1	0.25	3	0.5	7	1	1	0.2	2	1	1	0.125	0.5	2	3	1	5	3
1	5	4	1	5	0.5	4	1	1	0.143	6	9	8	1	1	0.143	0.111	0.2	1	0.5
0.143	1	0.333	0.2	1	6	8	5	7	1	9	6	2	1	1	0.167	1	0.333	2	1
1	0.333	0.2	0.111	0.25	1	0.25	0.125	0.5	0.111	1	3	7	8	5	1	0	1	0.5	0.333
3	1	0.5	0.111	0.2	4	1	0.5	6	0.125	0.333	1	5	8	5	7	1	7	3	6
5	2	1	0.25	0.5	8	2	1	4	0.25	0.143	0.2	1	0.333	0.5	1	0.143	1	0.2	0.333
9	9	4	1	4	2	0.167	0.25	1	0.125	0.125	0.125	3	1	2	2	0.333	5	1	5
4	5	2	0.25	1	9	8	4	8	1	0.2	0.2	2	0.5	1	3	0.167	3	0.2	1
					1	2	1	9	6	1	1	1	3	1					
					0.5	1	1	4	4	1	1	3	9	4					
					1	1	1	4	5	1	0.333	1	4	3					
					0.111	0.25	0.25	1	2	0.333	0.111	0.25	1	0.167					
					0.167	0.25	0.2	0.5	1	1	0.25	0.333	6	1					

Table 34
The generated random PCMs using the Saaty-scale for $n = 6$.

1	0	0.5	0.125	0.167	0.2	1	0.2	2	3	7	0.111	1	8	3	5	5	9
8	1	7	7	6	2	5	1	8	8	7	0.5	0.125	1	3	0.500	1	4
2	0.143	1	0.5	0.25	0.125	0.5	0.125	1	2	5	0.125	0.333	0.333	1	0.143	0.333	1
8	0.143	2	1	1	0.2	0.333	0.125	0.5	1	2	0.167	0.2	2	7	1	3	4
6	0.167	4	1	1	1	0.143	0.143	0.2	0.5	1	0.125	0.2	1	3	0.333	1	2
5	0.5	8	5	1	1	9	2	8	6	8	1	0.111	0.25	1	0.25	0.5	1
1	2	0.333	4	0.5	0.143	1	0.2	0.143	0.111	0.5	0.143	1	9	2	2	7	6
0.5	1	0.111	0.5	0.25	0.143	5	1	0.5	0.25	0.25	1	0.111	1	0.111	0.2	0.333	0.143
3	9	1	8	5	0.5	7	2	1	1	3	3	0.5	9	1	4	6	6
0.25	2	0.125	1	1	0.143	9	4	1	1	2	6	0.5	5	0.25	1	2	1
2	4	0.2	1	1	0.125	2	4	0.333	0.5	1	2	0.143	3	0.167	1	1	0.5
7	7	2	7	8	1	7	1	0.333	0.167	0.5	1	0.167	7	0.167	1	2	1
1	2	5	0.125	0.333	0.2	1	1	0.25	0.143	0.5	4	1	7	7	4	2	1
0.5	1	1	0.5	0.333	0.125	2	1	0.5	0.111	1	5	0.143	1	1	1	0.143	0.125
0.2	1	1	0.2	0.25	0.2	4	2	1	0.167	0.5	3	0.143	1	1	5	0.333	0.2
8	2	5	1	1	1	7	9	6	1	6	8	0.25	1	0.2	1	0.2	0.25
3	3	4	1	1	1	2	1	2	0.167	1	6	0.5	7	3	5	1	1
5	8	5	1	1	1	0.25	0.2	0.333	0.125	0.167	1	1	8	5	4	1	1
						1	2	8	7	4	7						
						0.5	1	8	3	3	4						
						0.125	0.125	1	0.333	0.143	0.5						
						0.143	0.333	3	1	0.333	0.5						
						0.25	0.333	7	3	1	9						
						0.143	0.25	2	2	0.111	1						

Table 35
The generated random PCMs using the Saaty-scale for $n = 7$.

1	8	4	7	8	8	8	1	0.5	0.143	1	0.111	0.125	0.111	1	0.167	0.2	0.2	0.125	0.111	0.25
0.125	1	6	8	7	9	9	2	1	0.2	0.333	0.143	0.5	0.5	6	1	9	1	1	6	2
0.250	0.167	1	0.5	0.5	1	4	7	5	1	9	0.111	6	7	5	0.111	1	0.125	0.25	3	0.143
0.143	0.125	2	1	0.25	0.333	3	1	3	0.111	1	0.167	0.333	2	5	1	8	1	5	6	0.5
0.125	0.143	2	4	1	4	3	9	7	9	6	1	7	6	8	1	4	0.2	1	4	0.5
0.125	0.111	1	3	0.25	1	0.5	8	2	0.167	3	0.143	1	1	9	0.167	0.333	0.167	0.25	1	0.25
0.125	0.111	0.25	0.333	0.333	2	1	9	2	0.143	0.5	0.167	1	1	4	0.5	7	2	2	4	1
1	8	3	2	2	7	0.25	1	1	0.2	3	0.143	1	0.5	1	5	9	0.333	5	5	2
0.125	1	3	0.333	2	1	0.125	1	1	0.111	0.333	0.143	1	0.167	0.2	1	0.5	0.2	2	0.25	1
0.333	0.333	1	0.143	2	0.333	0.5	5	9	1	9	0.5	7	3	0.111	2	1	0.2	3	2	0.111
0.5	3	7	1	7	6	0.5	0.333	3	0.111	1	0.125	3	1	3	5	5	1	5	9	6
1	0.5	0.5	0.143	1	4	0.2	7	7	2	8	1	1	9	0.2	0.5	0.333	0.2	1	0.2	0.111
0.143	1	3	0.167	0.25	1	0.25	1	1	0.143	0.333	1	1	1	0.2	4	0.5	0	5	1	0.333
4	8	2	2	5	4	1	2	6	0.333	1	0.111	1	1	0.5	1	9	0.167	9	3	1
1	0.167	0.333	0.143	4	2	0.25	1	0.333	0.143	1	0.2	3	1	1	0.2	1	0.25	0.167	1	0.2
6	1	3	2	5	4	2	3	1	0.5	5	0.333	3	7	5	1	3	4	4	0.5	0.5
3	0.333	1	0.25	4	1	0.167	7	2	1	3	0.167	7	6	1	0.333	1	4	1	1	0.25
7	0.5	4	1	8	8	2	1	0.2	0.333	1	0.111	4	0.2	4	0.25	0.25	1	0.25	0.5	0.25
0.25	0.2	0.25	0.125	1	0.111	0.167	5	3	6	9	1	3	8	6	0.25	1	4	1	1	0.2
0.5	0.25	1	0	9	1	3	0.333	0.333	0.143	0.25	0.333	1	1	1	2	1	2	1	1	0.5
4	0.5	6	0.5	6	0.333	1	1	0.143	0.167	5	0.125	1	1	5	2	4	4	5	2	1
							1	3	7	8	9	3	4							
							0.333	1	0.143	0.2	0.25	0.333	0.2							
							0.143	7	1	4	4	5	0.333							
							0.125	5	0.25	1	2	0.2	0.333							
							0.111	4	0.25	0.5	1	0.5	0.125							
							0.333	3	0.2	5	2	1	0.5							
							0.25	5	3	3	8	2	1							

Table 36
The generated random PCMs using the Saaty-scale for $n = 8$.

1	3	0.5	4	0.5	0.111	2	1	1	2	9	0.2	4	5	0.125	0.143
0.333	1	1	4	0.333	1	0.5	0.5	1	1	1	0.111	0.167	1	0.143	0.125
2	1	1	3	0.333	0.143	2	0.167	0.111	1	1	0.111	0.125	0.111	0.125	0.111
0.25	0.25	0.333	1	0.111	0.25	0.333	0.125	5	9	9	1	2	7	3	0.333
2	3	3	9	1	0.111	6	0.5	0.25	6	8	0.5	1	2	0.143	0.333
9	1	7	4	9	1	4	1	0.2	1	9	0.143	0.5	1	0.2	0.111
0.5	2	0.5	3	0.167	0.25	1	0.167	8	7	8	0.333	7	5	1	0.5
1	2	6	8	2	1	6	1	7	8	9	3	3	9	2	1
1	1	4	3	7	0.2	9	2	1	0.333	0.333	0.167	0.111	0.333	1	1
1	1	4	0.2	2	0.333	4	0.2	3	1	9	2	1	9	5	8
0.25	0.25	1	0.2	0.5	0.2	0.25	0.111	3	0.111	1	1	0.125	0.333	0.5	4
0.333	5	5	1	5	0.2	3	2	6	0.5	1	1	0.333	0.333	3	9
0.143	0.5	2	0.2	1	0.2	0.5	0.167	9	1	8	3	1	5	8	5
5	3	5	5	5	1	7	1	3	0.111	3	3	0.2	1	9	8
0.111	0.25	4	0.333	2	0.143	1	0.5	1	0.2	2	0.333	0.125	0.111	1	2
0.5	5	9	0.5	6	1	2	1	1	0.125	0.25	0.111	0.2	0.125	0.5	1
1	3	9	4	9	8	3	8	1	9	1	0.25	9	3	9	1
0.333	1	8	0.5	5	0.125	1	6	0.111	1	0.333	0.25	7	0.5	6	0.125
0.111	0.125	1	0.25	4	0.167	1	0.167	1	3	1	0.2	7	3	7	4
0.25	2	4	1	7	0.2	1	3	4	4	5	1	6	6	8	0.5
0.111	0.2	0.25	0.143	1	0.111	0.333	0.5	0.111	0.143	0.143	0.167	1	0.111	0.5	0.2
0.125	8	6	5	9	1	2	7	0.333	2	0.333	0.167	9	1	9	0.5
0.333	1	1	1	3	0.5	1	4	0.111	0.167	0.143	0.125	2	0.111	1	0.111
0.125	0.167	6	0.333	2	0.143	0.25	1	1	8	0.25	2	5	2	9	1
1	0.333	8	5	8	2	7	4	1	0	0.167	0.2	0.3	0.333	0.25	2
3	1	4	1	9	4	9	7	4	1	4	0.143	9	7	0.25	6
0.125	0.25	1	0.25	7	0.5	8	0.25	6	0.25	1	1	6	2	2	5
0.2	1	4	1	3	0.2	4	0.25	5	7	1	1	7	8	5	7
0.125	0.111	0.143	0.333	1	0.2	4	0.125	3	0.111	0.167	0.143	1	3	0.2	0.5
0.5	0.25	2	5	5	1	8	1	3	0.143	0.5	0.125	0.333	1	0.25	1
0.143	0.111	0.125	0.25	0.25	0.125	1	0.125	4	4	0.5	0.2	5	4	1	9
0.25	0.143	4	4	8	1	8	1	0.5	0.167	0.2	0.143	2	1	0.111	1
1	5	0.5	7	1	1	0.5	0.5	1	0.2	0.333	5	0.125	2	0.167	0.111
0.2	1	0.333	2	0.167	0.167	0.167	0.2	5	1	0.333	4	0.167	2	0.167	0.333
2	3	1	3	9	0.5	2	0.143	3	3	1	4	0.5	0.5	1	1
0.143	0.5	0.333	1	0.333	0.143	0.2	0.143	0.2	0.25	0.25	1	0.111	0.167	0.2	0.167
1	6	0.111	3	1	0.333	0.111	0.5	8	6	2	9	1	3	6	6
1	6	2	7	3	1	0.333	0.5	0.5	0.5	2	6	0.333	1	1	0.5
2	6	0.5	5	9	3	1	0.5	6	6	1	5	0.167	1	1	4
2	5	7	7	2	2	2	1	9	3	1	6	0.167	2	0.25	1

Table 37
The generated random PCMs using the Saaty-scale for $n = 9$.

1	0.5	2	1	0.167	0.111	0.333	5	0.333	1	0.111	1	0.333	0.3	1	0.2	0.333	6
2	1	4	1	2	0.111	0.5	6	0.2	9	1	9	0.125	5	8	1	2	9
0.5	0.25	1	5	2	0.125	0.167	2	0.333	1	0.111	1	0.5	0.5	2	0.333	1	2
1	1	0.2	1	3	0.125	0.167	3	0.143	3	8	2	1	8	6	9	4	8
6	0.5	0.5	0.333	1	0.125	0.2	1	0.111	3	0.2	2	0.125	1	5	0.333	2	4
9	9	8	8	8	1	2	5	8	1	0.125	0.5	0.167	0.2	1	0.5	0.143	2
3	2	6	6	5	0.5	1	5	1	5	1	3	0.111	3	2	1	4	7
0.2	0.167	0.5	0.333	1	0.2	0.2	1	0.125	3	0.5	1	0.25	0.5	7	0.25	1	8
3	5	3	7	9	0.125	1	8	1	0.167	0.111	0.5	0.125	0.25	0.5	0.143	0.125	1
1	7	0.167	0.333	4	5	6	0.5	0.333	1	1	3	8	6	2	1	0.25	2
0.143	1	0.167	0.333	3	6	5	0.25	0.333	1	1	9	6	7	8	1	0.25	6
6	6	1	0.333	7	8	9	6	0.5	0.333	0.111	1	0.5	1	0.5	0.125	0.111	0.167
3	3	3	1	9	9	5	4	9	0.125	0.167	2	1	4	8	1	0.333	0.5
0.25	0.333	0.143	0.111	1	6	1	0.167	0.333	0.167	0.143	1	0.25	1	0.333	0.333	0.25	0.143
0.2	0.167	0.125	0.111	0.167	1	0.333	0.167	0.333	0.5	0.125	2	0.125	3	1	0.167	0.167	0.2
0.167	0.2	0.111	0.2	1	3	1	0.5	0.167	1	1	8	1	3	6	1	0.2	9
2	4	0.167	0.25	6	6	2	1	3	4	4	9	3	4	6	5	1	9
3	3	2	0.111	3	3	6	0.333	1	0.5	0.167	6	2	7	5	0.111	0.111	1
1	1	6	1	8	4	1	8	5	1	2	2	2	5	7	5	7	2
1	1	3	2	7	5	5	8	5	0.5	1	0.5	0.125	2	2	1	0.333	0.2
0.167	0.333	1	1	2	2	5	8	9	0.5	2	1	2	6	9	5	1	2
1	0.5	1	1	5	2	6	7	4	0.5	8	0.5	1	9	7	5	2	1
0.1	0.143	0.5	0.2	1	0.333	0.125	1	2	0.2	0.5	0.167	0.111	1	5	5	0.125	0.333
0.25	0.2	0.5	0.5	3	1	3	5	7	0.143	0.5	0.111	0.143	0.2	1	0.2	0.167	0.111
1	0.2	0.2	0.167	8	0.333	1	2	6	0.2	1	0.2	0.2	0.2	5	1	3	0.2
0.125	0.125	0.125	0.143	1	0.2	0.5	1	9	0.143	3	1	0.5	8	6	0.333	1	0.167
0.2	0.2	0.111	0.25	0.5	0.143	0.167	0.111	1	0.5	5	0.5	1	3	9	5	6	1
1	9	9	8	7	5	7	1	8	1	0.5	0.333	0.25	0.143	0.2	0.125	0.333	0.125
0.111	1	0.143	1	0.143	0.5	3	0.333	2	2	1	2	2	0.111	0.111	9	1	0.5
0.11	7	1	8	2	6	6	0.125	4	3	1	1	1	0.333	0.143	4	0.5	0.2
0.125	1	0.125	1	0.25	0.5	0.5	0.143	0.143	4	0.5	1	1	0.333	0.143	2	0.25	1
0.143	7	0.5	4	1	5	9	0.333	0.5	7	9	3	3	1	4	7	6	1
0.2	2	0.167	2	0.2	1	4	0.167	2	5	9	7	7	0.25	1	2	1	1
0.143	0.333	0.167	2	0.111	0.25	1	0.143	1	8	0.111	0.25	0.5	0.143	0.5	1	0.111	0.111
1	3	8	7	3	6	7	1	8	3	1	2	4	0.167	1	9	1	0.5
0.125	0.5	0.25	7	2	0.5	1	0.125	1	8	2	5	1	1	1	9	2	1
1	0.2	0.333	2	0.2	0.125	0.2	0.333	0.25	1	9	3	6	1	5	6	6	8
5	1	0.143	9	1	1	9	4	2	0.111	1	0.167	5	0.111	6	3	4	2
3	7	1	7	1	6	7	6	5	0.333	6	1	3	0.143	4	6	2	5
0.5	0.111	0.143	1	0.125	0.143	0.111	0.25	0.333	0.17	0.2	0.333	1	0.2	0.333	7	3	0.333
5	1	1	8	1	0.333	2	1	0.5	1	9	7	5	1	7	5	9	7
8	1	0.167	7	3	1	1	1	0.2	0.2	0.167	0.25	3	0.143	1	8	2	1
5	0.111	0.143	9	0.5	1	1	1	1	0.167	0.333	0.167	0.143	0.2	0.125	1	1	0.25
3	0.25	0.167	4	1	1	1	1	2	0.167	0.25	0.5	0.333	0.111	0.5	1	1	0.111
4	0.5	0.2	3	2	5	1	0.5	1	0.125	0.5	0.2	3	0.143	1	4	9	1

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