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ABSTRACT

In the estimation of the aerosol single fiber efficiency using fibrous filters, there is a size range, where the particles penetrate most effectively through the fibrous collectors, and corresponding minimum single fiber efficiency. For small particles in which the diffusion mechanism is dominant, the Cunningham slip correction factor (C_c) affects the single fiber efficiency and the most penetrating particle size (MPPS). Therefore, for accurate estimation, C_c is essential to be considered. However, many previous studies have neglected this factor because of its complexity and the associated difficulty in deriving the appropriate parameterization particularly for the MPPS. In this study, the expression for the MPPS, and the corresponding expression for the minimum single fiber efficiency are analytically derived, and the effects of C_c are determined. In order to accommodate the slip factor for all particle-size ranges, C_c is simplified and modified. Overall, the obtained analytical expression for the MPPS is in a good agreement with the exact solution.

Keywords: Aerosol single fiber efficiency, Cunningham slip correction factor, Fibrous collector, Minimum single fiber efficiency, Most penetrating particle size

1. Introduction

The collection efficiency of aerosol particles by fibrous collectors has been investigated by numerous theoretical, numerical, and experimental studies. The filtration mechanisms of fibrous collectors include Brownian diffusion, interception, inertial impaction, and gravitational settling [1-3].

Diffusion and interception (gravitational settling) dominate particle motion at the opposite size regimes. For example, an increase in particle size increases the single fiber efficiency owing to interception and gravitational settling [2-4], whereas a decrease in particle size enhances the single fiber efficiency related to Brownian diffusion. Between diffusion-dominant regime and interception-dominant regime, there exists a size range where none of the diffusion or interception (or gravitational settling) is predominant but they operate simultaneously. The size regime exists where aerosol particles penetrate most effectively through the fibrous collectors [5-7].

For small particle sizes where Brownian diffusion is dominant, Cunningham slip correction factor (C_c) and Knudsen number (Kn) play an important role in estimating accurate filtration efficiency. In cases where the aerosol-particle size in air approaches the mean free path of gas molecules, the discontinuities of the medium should be considered. C_c accounts for these discontinuities and can be useful for describing aerosol properties, such as sedimentation and diffusion [1, 8-10]. For the interaction of gases and particles, Kn, the ratio of mean free path (λ) to particle diameter (d_p), is widely used [11].

Among many filtration studies, Jung and Lee [7] obtained an approximated analytical solution for the most penetrating particle size (MPPS) in a multiple fluid sphere system. In their study, a harmonic-mean type approximation was applied, and the obtained solution was compared with the numerically calculated results. However, C_c was not considered because the derivation of an analytical expression considering this factor was complicated.

Lee and Liu [2] derived equations for predicting the minimum efficiency and the MPPS for fibrous filters, considering the diffusion and interception mechanisms. Their results showed that the MPPS decreased with increases in filtration velocity and fiber volume fraction, and increased with an increase in filter fiber size. In addition, they obtained a corresponding equation for the minimum efficiency, and demonstrated the dependence of the MPPS on the filtration velocity, fiber volume fraction, and fiber size. They approximated C_c for different ranges of Kn, with three different formulas, which exhibited a good correlation with an original equation within specified ranges. However, these approximated equations are limited because they are valid only for a particular size range. However, in the diffusion-mechanism-dominant regime for small particle sizes, aerosol slip effects are non-negligible, and should be corrected using C_c.

In this study, a new expression for the MPPS is analytically derived, and C_c is simplified and modified to accommodate all the particle-size ranges. The newly derived expression includes the slip correction, which is implicitly considered in the diffusion coefficient for aerosol particles with low Kn values.

2. Theoretical Backgrounds

Generally, C_c, can be expressed as follows [8]:

$$C_c = 1 + \frac{\lambda}{d_p}\left(2.514 + 0.8\exp\left(-0.55\frac{d_p}{\lambda}\right)\right) \quad (1)$$

Where λ is the mean free path of air molecules as a constant, and d_p is the particle diameter. However, this expression is complex, and requires simplification for parameterization studies [1, 12]. One of the simplified expressions for C_c, used in many studies, is as follows [2]:

$$C_c = 1 + 3.34\frac{\lambda}{d_p} \quad (2)$$

The simplified expression for C_c (Eq. (2)) is simple and sufficiently accurate compared to original expression (Eq. (1)) [2]. For example, the diffusion coefficient (D) and Peclet number (Pe), considering C_c, can be expressed as follows:

$$D = \frac{kTC_c}{3\pi\mu d_p} \cong \frac{kT}{3\pi\mu d_p}\left(1 + 3.34\frac{\lambda}{d_p}\right) \quad (3)$$

$$Pe^{-2/3} = \left(\frac{2au}{D}\right)^{-2/3} = \left(\frac{D}{D_f u}\right)^{2/3}$$
$$= \left(\frac{1}{D_f u}\right)^{2/3}\left(\frac{kT}{3\pi\mu d_p}\right)^{2/3}\left(1 + 3.34\frac{\lambda}{d_p}\right)^{2/3} \quad (4)$$

where k is the Boltzmann constant, T is the absolute temperature, is the viscosity of air, D_f is the diameter of the filter, and u is the flow velocity. Subsequently, the aerosol single fiber efficiency (η_D) in association with the diffusion mechanism can be expressed as follows:

$$\eta_D = 2.6\left(\frac{1-\alpha}{K}\right)^{1/3}Pe^{-2/3} = 2.6\left(\frac{1-\alpha}{K}\right)^{1/3}\left(\frac{kTC_c}{3\pi\mu d_p u D_f}\right)^{2/3}$$
$$= 2.6\left(\frac{1-\alpha}{K}\right)^{1/3}\left(\frac{kT}{3\pi\mu d_p u D_f}\right)^{2/3}\left(1 + 3.34\frac{\lambda}{d_p}\right)^{2/3} \quad (5)$$

Here, K is the hydrodynamic factor of the Kuwabara flow $\left(= -\frac{1}{2}ln\alpha - \frac{3}{4} + \alpha - \frac{\alpha^2}{4}\right)$, and α is the volume fraction, solidity, or packing density of the filter [13].

Eq. (5) can be further simplified via the approximation of $\left(1 + 3.34\frac{\lambda}{d_p}\right)^{2/3}$. Hence, Lee and Liu [2] approximated C_c for three different size ranges, as follows:

$C_c = 1$, for large particles with a continuum-flow-regime assumption $\left(\frac{\lambda}{d_p} \ll 1\right)$;

$C_c = 3.33\frac{\lambda}{d_p}$, for very small particles $\left(\frac{\lambda}{d_p} \gg 1\right)$;

$C_c = 3.69\left(\frac{\lambda}{d_p}\right)^{1/2}$, for particles with intermediate sizes. (6)

Although these approximations demonstrate good correlations with the original equation, the approximated equations are valid only for the given size conditions and require different approximations in the other size ranges. For example, the single fiber efficiency can be generally expressed using a combination of the single fiber efficiency of diffusion and interception. If these three types of C_c are considered, the filtration single fiber efficiency should be expressed by three different formulas. Moreover, if the minimum single fiber efficiency as well as the corresponding d_p varies for different formulas, the estimation of the filtration-mechanism characteristics considering C_c becomes confusing; therefore, this factor was neglected in several previous studies. Thus, for practical purposes, a more general approximation that covers all size ranges is required, considering the continuous changes in C_c.

3. Results and Discussion

3.1. New Approximated Expression for the Cunningham Slip Correction Factor and Single Fiber Efficiency

According to Lee and Liu [14] and Eq. (5), the single fiber effi-

ciency (η_D) due to diffusion can be expressed in proportion to $C_c^{2/3}$ $(\eta_D \propto C_c^{2/3})$. Thus, for the further evolution of the analytical expression for the MPPS, C_c should be simplified. In this study, we evaluated $C_c^{2/3}$ using the 1st order Taylor series expansion as a function of the d_p:

$$C_c^{2/3} \cong \left(1 + 3.34\frac{\lambda}{d_p}\right)^{\frac{2}{3}} \cong \zeta\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\} \quad (7)$$

Then,

$$C_c \cong \zeta^{3/2}\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\} = \vartheta\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\}^{\frac{3}{2}} \quad (8)$$

Here, we adapted the correlation factors, and , based on the comparison results between the real and approximated formulas:

$$\vartheta = \zeta^{3/2} = 0.6\left(\frac{d_p}{\lambda}\right)^{0.1} \quad (9)$$

Subsequently, D and pe can be obtained using Eq. (3), (5), (8), and (9), as follows:

$$D = \frac{kTC_c}{3\pi\mu d_p} \cong \frac{\vartheta kT}{3\pi\mu d_p}\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\}^{\frac{3}{2}}$$

$$= 0.6\left(\frac{d_p}{\lambda}\right)^{0.1}\frac{kT}{3\pi\mu d_p}\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\}^{\frac{3}{2}}$$

$$D^{2/3} = \left(\frac{kTC_c}{3\pi\mu d_p}\right)^{2/3} \cong \left\{0.6\left(\frac{d_p}{\lambda}\right)^{0.1}\right\}^{2/3}\left(\frac{kT}{3\pi\mu d_p}\right)^{2/3}\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\}$$

$$Pe^{-2/3} = \left(\frac{D_f u}{D}\right)^{-2/3} = \left(\frac{D}{D_f u}\right)^{2/3} = \left\{0.6\left(\frac{d_p}{\lambda}\right)^{0.1}\right\}^{2/3}\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\}$$

$$= (0.6)^{2/3}\left(\frac{d_p}{\lambda}\right)^{1/15}\left(\frac{kT}{3\pi\mu d_p D_f u}\right)^{2/3}\left\{1 + \frac{2}{3}\left(3.34\frac{\lambda}{d_p}\right)\right\} \quad (10)$$
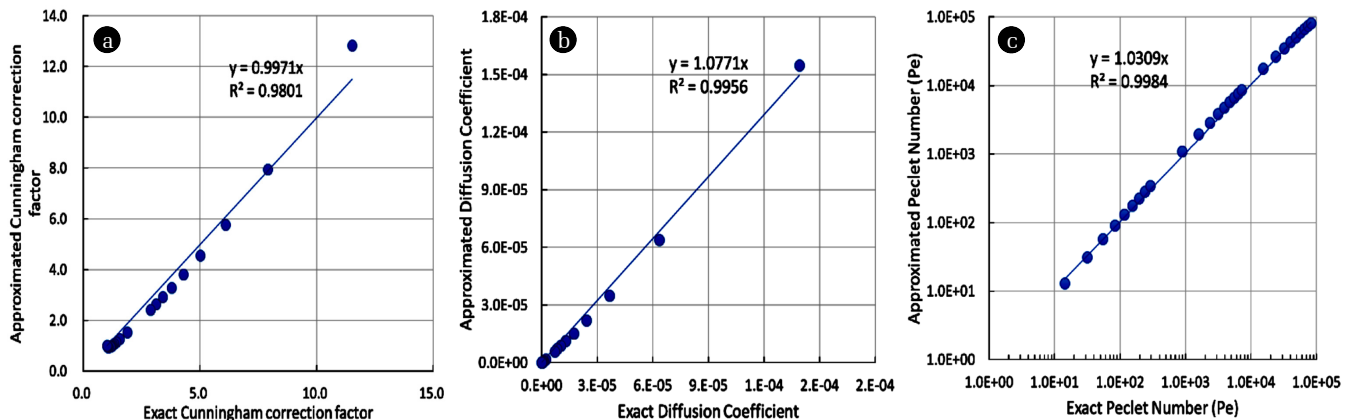
Further, the η_D for diffusion can be expressed as a linear function of the d_p, as in Eq. (5) and (10).

Table 1 depicts the expressions for the single fiber efficiency by diffusion; the previous expression by Lee and Liu [2], and the currently derived expression are presented. As seen in Table 1, the currently derived expression can be used for the entire size range. However, different formulas have to be used for different size ranges when applying Lee and Liu's expression.
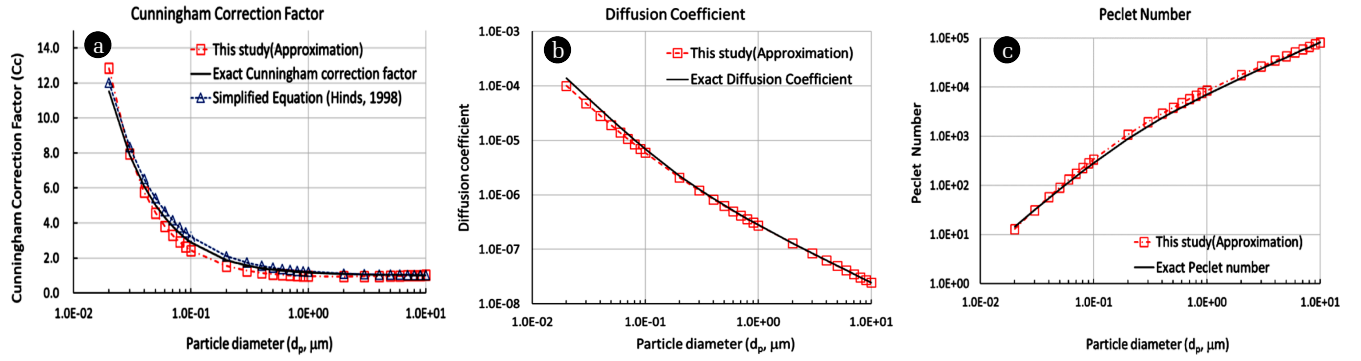
Fig. 1 shows the comparison of the exact values and the newly developed approximated solutions in this study, for C_c, D, and Pe, respectively; the results are in good agreement with each other. Here, the "exact" C_c refers to values calculated from Eq. (1).

Table 1. Comparison of the Expressions for the Single Fiber Efficiency by Diffusion (Previous Expression vs. Currently Derived Expression)

	Previous expression [2]	Current expression
Single fiber efficiency by diffusion (η_D)	$\bullet\ 2.6\left(\dfrac{1-\alpha}{K}\right)^{1/3}\left(\dfrac{kT}{3\pi\mu d_p u D_f}\right)^{2/3}$ for $\dfrac{\lambda}{d_p} \ll 1$ $\bullet\ 2.6\left(\dfrac{1-\alpha}{K}\right)^{1/3}\left(\dfrac{3.33kT\lambda}{3\pi\mu d_p^2 u D_f}\right)^{2/3}$ for $\dfrac{\lambda}{d_p} \gg 1$ $\bullet\ 2.6\left(\dfrac{1-\alpha}{K}\right)^{1/3}\left(\dfrac{kTC_c}{3\pi\mu d_p^2 u D_f}(\sqrt{3.69\lambda})\right)^{2/3}$ for intermediate size.	For the entire size range, $2.6(0.6)^{2/3}\left(\dfrac{1-\alpha}{K}\right)^{1/3}\left(\dfrac{1}{\lambda}\right)^{1/15}\left(\dfrac{kT}{3D_f u\pi\mu}\right)^{2/3}d_p^{-3/5}$ $+\dfrac{5.2(3.34)(0.6)^{2/3}}{3}\left(\dfrac{1-\alpha}{K}\right)^{1/3}(\lambda)^{14/15}\left(\dfrac{kT}{3D_f u\pi\mu}\right)^{2/3}d_p^{-8/5}$

Fig. 1. Comparison of the exact and approximated (a) Cunningham slip correction factors, (b) Diffusion coefficients, and (c) Peclet numbers.

Fig. 2. Comparison of the exact and approximated (a) Cunningham slip correction factors, (b) Diffusion coefficients, and (c) Peclet numbers, as functions of the particle diameter (D_f = 0.02 mm).

The correlation coefficients are 0.9801, 0.9956, and 0.9984 with slopes of 0.9971, 1.0771, and 1.0309 for C_c, D, and Pe, respectively.

Further, the total η_D, including that owing to interception and gravitational settling, is expressed as a function of the d_p:

$$\eta_T = \eta_D + \eta_R + \eta_G$$
$$= 2.6(0.6)^{2/3}\left(\frac{1-\alpha}{K}\right)^{1/3}\left(\frac{1}{\lambda}\right)^{1/15}\left(\frac{kT}{3D_f u\pi\mu}\right)^{2/3} d_p^{-3/5}$$
$$+ \frac{5.2(3.34)(0.6)^{2/3}}{3}\left(\frac{1-\alpha}{K}\right)^{1/3}(\lambda)^{14/15}$$
$$\left(\frac{kT}{3D_f u\pi\mu}\right)^{2/3} d_p^{-8/5} + \left(\frac{1-\alpha}{K}\right)\left(\frac{d_p}{D_f}\right)^2 + \frac{\rho_p d_p^2 g}{18\mu u} \quad (11)$$

where η_R and η_G are the single fiber efficiencies due to interception and gravitational settling, respectively.

$$\eta_R = \left(\frac{1-\alpha}{K}\right)\frac{R^2}{1+R} \cong \left(\frac{1-\alpha}{K}\right)R^2 = \left(\frac{1-\alpha}{K}\right)\left(\frac{d_p}{D_f}\right)^2,$$
$$\eta_G = \frac{\rho_p d_p^2 g}{18\mu u} \quad (12)$$

where R is the interception parameter $\left(= \frac{d_p}{D_f}\right)$ and g is the gravitational coefficient. It should be noted that the newly approximated diffusion single fiber efficiency (η_D) in Eq. (11) can now be expressed as a linear combination of the d_p.

Based on Fig. 1, we conclude that the approximated analytical expression for C_c estimates the exact C_c, and related parameters, such as D and Pe, with good confidence. Fig. 2 shows C_c, D, and Pe as functions of the d_p. The exact values of C_c which are calculated using Eq. (1) is compared with the approximated values calculated using Eq. (10). The filter diameter (D_f) is set at 0.02 mm, and the packing density and flow velocity are set at 0.08 and 1 cm/s, respectively. The results exhibit a good agreement (Fig. 2).

Fig. 3 shows the total single fiber efficiency as a function of the d_p for different assumptions of C_c. For a filter diameter of 0.02 mm (D_f = 0.02 mm), and flow velocities of 1 cm/s and 10 cm/s, the values of the single fiber efficiencies for the exact C_c, C_c = 1, and the approximated C_c from this study are compared. As seen
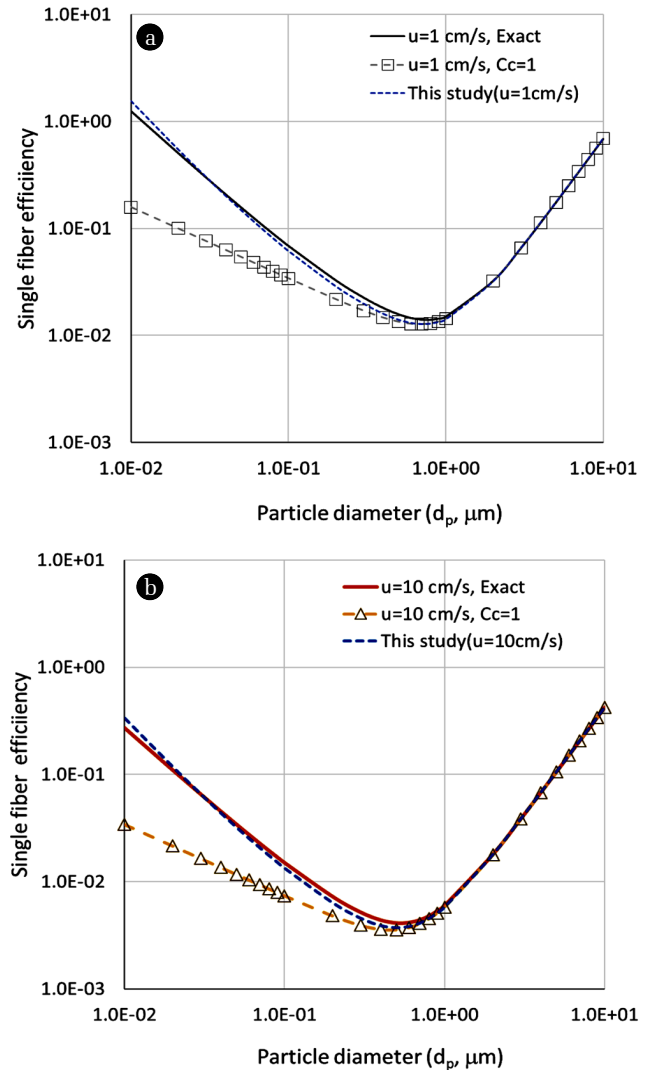
Fig. 3. Single fiber efficiency as a function of the particle diameter for different Cunningham slip correction factors (C_c = 1, exact solution with Eq. (1), and approximated value in this study; D_f =0.02mm).

in Fig. 3, the approximated solutions of the single fiber efficiencies using the approximated C_c agree with those using the exact C_c. The single fiber efficiency with unity C_c exhibits a discrepancy compared to the exact values, increasing with the decreasing particle size.

3.2. Analytic Expression for the MPPS of Fibrous Filters

As explained in the introduction, an increase in the particle size increases the single fiber efficiency due to interception and gravitational settling, whereas a reduction in the particle size causes an increase in the single fiber efficiency because of Brownian diffusion. It is notable that there is a regime with an intermediate particle size, where two or more mechanisms operate simultaneously, but no mechanism predominates over others. In this regime, the single fiber efficiency is minimum. The minimum single fiber efficiency diameter or the MPPS can be acquired by the differentiation of the single fiber efficiency with respect to the particle size, and the acquired value can be used to establish a value of zero. Previous studies determined the MPPS and the corresponding minimum single fiber efficiency with no consideration of C_c or with complex expressions [15-16]. Thus, the effects of C_c and the corresponding expression for the minimum single fiber efficiency, as well as the MPPS, must be improved in a way that considers C_c and simplifies those expressions. In this study, we approximated C_c to accurately estimate the single fiber efficiency and MPPS, as well as the minimum single fiber efficiency.

The consequent equation for the MPPS of fibrous filters can be obtained as follows, by differentiating Eq. (11) with respect to the d_p:

$$\frac{\partial \eta_T}{\partial d_p} = \frac{-3(2.6)(0.6)^{2/3}}{3\pi\mu d_p}\left(\frac{1-\alpha}{K}\right)^{1/3}\lambda^{-1/15}\left(\frac{kT}{3D_f u\pi\mu}\right)^{2/3}d_p^{-\frac{8}{5}}$$

$$-\frac{8(5.2)(3.34)(0.6)^{2/3}}{(3)(5)}\left(\frac{1-\alpha}{K}\right)^{1/3}\lambda^{14/15}\left(\frac{kT}{3D_f u\pi\mu}\right)^{2/3}d_p^{-13/5}$$

$$+\left\{2\left(\frac{1-\alpha}{K}\right)\left(\frac{1}{D_f}\right)^2+\frac{\rho_p g}{9\mu u}\right\}d_p=0 \tag{13}$$

Eq. (13) can be expressed in a simpler form as follows:

$$\frac{\partial \eta_T}{\partial d_p}=\varphi_1 d_p^{-8/5}+\varphi_2 d_p^{-13/5}+\varphi_3 d_p=0.$$

where

$$\varphi_1=-\frac{3(2.6)(0.6)^{2/3}}{3\pi\mu d_p}\left(\frac{1-\alpha}{K}\right)^{1/3}\lambda^{-1/15}\left(\frac{kT}{3D_f u\pi\mu}\right)^{2/3}$$

$$\varphi_2=-\frac{8(5.2)(3.34)(0.6)^{2/3}}{(3)(5)}\left(\frac{1-\alpha}{K}\right)^{1/3}\lambda^{14/15}\left(\frac{kT}{3D_f u\pi\mu}\right)^{2/3}$$

$$\varphi_3=\left\{2\left(\frac{1-\alpha}{K}\right)\left(\frac{1}{D_f}\right)^2+\frac{\rho_p g}{9\mu u}\right\} \tag{14}$$

The first and second terms (φ_1 and φ_2) refer to the diffusion, and the third term (φ_3) refers to the interception, and gravitational settling mechanisms in Eq. (14). The single fiber efficiency due to diffusion increases as the d_p decreases, while that due to interception increases as the d_p increases. Thus, the leading terms can

be estimated for each dominant particle or collector-size region. If the two dominant terms in Eq. (14) are determined, the resulting approximated equations can be generated as follows [6, 7, 16]:

$$\varphi_1 d_p^{-8/5}+\varphi_3 d_p=0$$
$$\varphi_2 d_p^{-13/5}+\varphi_3 d_p=0. \tag{15}$$

The subsequent solutions from Eq. (15) are expressed as follows:

$$d_{s1}=\left(\frac{-\varphi_1}{\varphi_3}\right)^{13/5}, \ d_{s2}=\left(\frac{-\varphi_2}{\varphi_3}\right)^{5/18} \tag{16}$$

Further, the minimum scavenging coefficient particle size is expressed as in Eq. (17) by applying the harmonic-mean type approximation [16]:

$$d_{p,min}=\overline{\omega}\sqrt{\frac{1}{\frac{1}{d_{s1}^2}+\frac{1}{d_{s2}^2}}} \tag{17}$$

Here, the correction parameter, $\overline{\omega}$, can be adapted to minimize the errors between the exact and approximated MPPS. In this study, an $\overline{\omega}$ of 0.6 is suggested by comparing the exact and harmonic mean results.

Fig. 4 shows the MPPS as a function of the velocity, considering D_f of 0.02 mm and 0.1 mm. The MPPS can be determined in accordance with a dominant mechanism, for any given condition. As seen in Fig. 4, the MPPS increases as the filtration velocity increases up to a point, and after the point, decreases with the increasing velocity. According to the previous study [5], the MPPS shifts to a larger size as the flow velocity increases for fiber diameter (d_f) of 0.02 mm, when Brownian diffusion and the gravitational force are dominant. The MPPS shifts to a smaller size as the flow velocity increases for d_f of 0.1 mm, when Brownian diffusion and interception are dominant. As shown in Fig. 4, the MPPS obtained using the analytical expression of this study agrees well with the exact MPPS. Fig. 5 depicts the MPPS as a function of the filter packing density; d_f of 0.02 and 0.2 mm are compared for filtration velocities of 1 and 10 cm/s, respectively. The actual MPPS and the one determined in this study show a good agreement. For a D_f of 0.2 mm, the MPPS increases with the increase in packing density up to a value
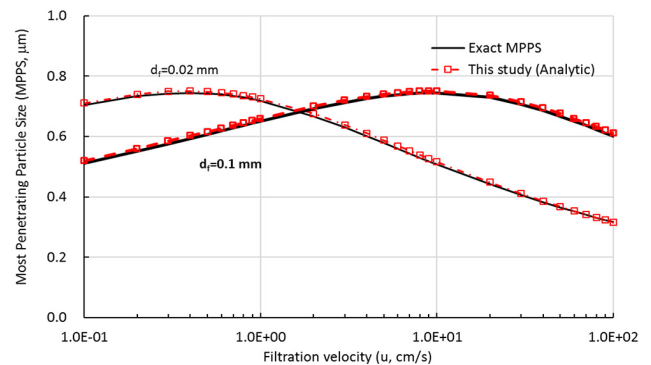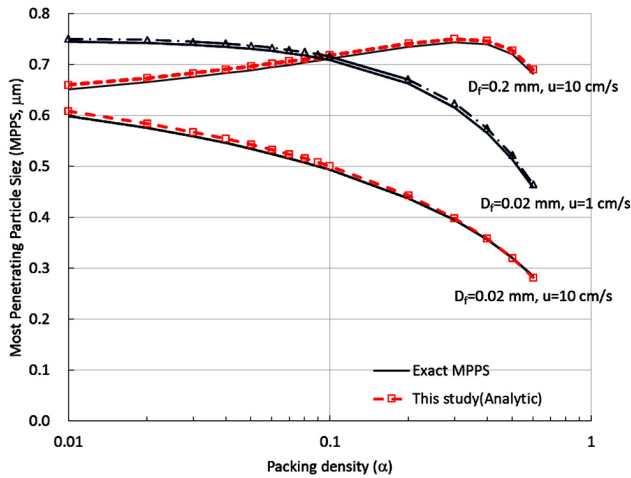

Fig. 4. Most penetrating particle size as a function of the filtration velocity for different filter diameters of 0.02 mm and 0.1 mm.
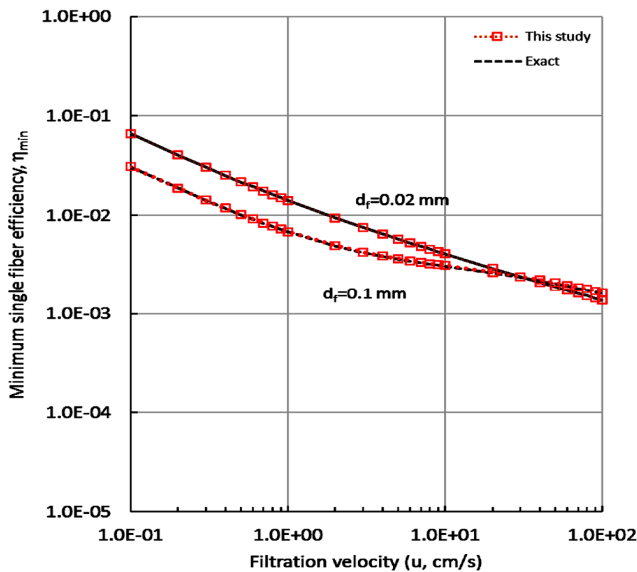
Fig. 5. Comparison of the exact solution of the most penetrating particle size with its analytical counterpart, as a function of the packing density.

of approximately 0.3, after which the MPPS starts to decrease. However, for a D_f of 0.02 mm, the MPPS decreases with the increase in packing density over the density range in Fig. 5.

3.3. Minimum Single Fiber Efficiency

Based on the MPPS, the corresponding minimum single fiber efficiency (η_{min}) is obtained by substituting d_p in Eq. (11) with $d_{p,min}$ in Eq. (17). Fig. 6 shows the minimum single fiber efficiency as a function of the filtration velocity; the exact solution and approximated one from this study are compared. The minimum single fiber efficiency decreases with the increase in filtration velocity. The minimum single fiber efficiency is greater at smaller d_f over

Fig. 6. Comparison of the exact solution of the minimum single fiber efficiency, as a function of the filtration velocity, with its analytical counterpart, for different filter diameters of 0.02 mm and 0.1 mm.

the filtration velocities up to ∼ 20 cm/s and for lower filtration velocities. Nevertheless, the difference in the minimum single fiber efficiency between different d_f decreases with the increase in filtration velocity.

4. Conclusions

The C_c is significant, for particle sizes below 2-3 μm, in air at ambient conditions. In filtration studies, the effects of C_c are critical and can be considered for the exact estimation of the collection efficiency and MPPS. For instance, in the regime where the diffusion mechanism is dominant, the aerosol slip effects cannot be neglected for small particle sizes, and the effects should be calibrated using C_c. However, the complex formula for C_c requires a simplified expression, for parameterization studies.

In this study, an expression for the MPPS and corresponding minimum single fiber efficiency were analytically derived based on the newly developed analytically simplified C_c expression in association with the slip effects of small particles. Slip correction was implicitly embedded in the D for aerosol particles with low Kn. The comparison of the obtained analytical solution for the MPPS, which considers the particle slip of small particles, with the exact solution demonstrated a good agreement. This result indicates that the simplified expression for C_c, demonstrated in this study, can be used for obtaining the analytical solution for the MPPS, including particles with low Knudsen numbers, with a high confidence level.

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