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An Exploratory Study on How Math Stories Engage Young Learners in Mathematical Sense-Making

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AN EXPLORATORY STUDY ON HOW MATH STORIES ENGAGE YOUNG LEARNERS IN MATHEMATICAL SENSE-MAKING

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ABSTRACT

AN EXPLORATORY STUDY ON HOW MATH STORIES ENGAGE YOUNG LEARNERS IN MATHEMATICAL SENSE-MAKING

by Maria Gigi Carunungan

This is an exploratory study of how pedagogy in the form of math stories, shapes young learners' perceptions, motivations, and sense-making of math concepts. The research is presented in an exploratory documentary, with audio-video data collected through the iPhone. The pilot test of story-driven math learning solutions was conducted by two teachers with eight first grade children from diverse backgrounds in an afterschool program. This study also includes interviews of the teachers, educational leaders and specialists in primary school curricula, children's literature, and math education.

The results of the pilot validated the efficacy of story-driven math learning solutions for mathematical sensemaking and reasoning. By helping the characters students were empowered as young mathematicians. They were motivated and engaged in mathematical modeling, for example, building equations deepened understanding from concrete problems to abstract concepts. The teachers observed the accelerated rate of students' learning through stories, games, and multimodal activities shaped by a creative, socially interactive, and culturally responsive pedagogy not typically used in their math classes.

ACKNOWLEDGEMENTS

I would like to thank the following people, without whom I would not have been able to complete this research and my EdD.

Angela G. opened the doors of her school and convinced her teachers of the value of the study. Teachers Rebecca and Marisa prepared and taught the pilot after-school hours. The students participated in the after-school program and relentlessly surprised us with their enthusiasm and math ideas.

My Committee: Dr. Bob Gliner taught me to see through the lens of a video camera, listen to the audio of headphones, and organize my narrative in a non-linear suite, so higher education studies can be made accessible to a wider audience. Dr. Reis inspired with her empathy, intellect, and thoughtfulness, making every student feel unique and special. Dr. Susan Charles took time from her busy schedule to read, watch, and critique the research. Peter and Jessica Liou, my partners and supporters in reinventing math learning.

My children Bikoy and Amyel and my stepchildren Misha and Iliana who encouraged me to go for the EdD and my husband David, who understood how much this program meant for my profession and supported me along the way.

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I. Introduction

> Knowledge emerges only through invention and re-invention, through the restless, impatient, continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other (Freire, 1972, p.72).

Background Setting

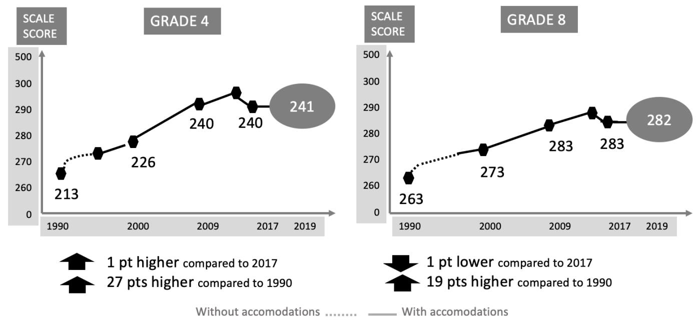
Majority of U.S. Students Have Low Math Proficiency Levels

The National Assessment of Educational Progress (NAEP) is a set of examinations taken annually by students in American public schools. The scores determine U.S. students' knowledge and skills levels (NAEP, 2019). In 2019, an estimated 296,900 students took the mathematics assessment. Compared with 2017, the average math scores were 1 point higher at fourth grade and 1 point lower at eighth grade. Among the fourth graders who took the test, 41% scored at or above mathematically proficient levels (NAEP Report Card: Mathematics, Grade 4).

The National Research Council defines the term *mathematical proficiency* as "what is necessary for anyone to learn mathematics successfully" (Kilpatrick et al., 2001). This means an estimated 60% of students in the fourth grade will move up to the next grade level without the basic proficiency expectations for that grade level, which will impact student success rates in math. It is of no surprise, then, that eighth graders' proficiency levels are at 34% (NAEP Report Card: Mathematics, Grade 8).

In 1990, NAEP scores were 27 points lower than 2017 among fourth graders and 19 points lower among eighth graders (Figure 1).

Figure 1. Trend in NAEP Mathematics Average Scores, by Grade



In 2010, the National Governors Association and the Council of Chief State School Officers published final versions of the Common Core Initiatives for math and language arts. The math standards responded to criticisms of American math curricula as "a mile wide and an inch deep" (Common Core State Standards Initiative, 2014) and focused on eight principles of mathematical practice. The standards included place value and the laws of arithmetic as foundational knowledge for conceptual understanding of key ideas. (Common Core Standards Initiative, 2014).

American Students Trailing Behind in Math

A report produced by an advisory panel contracted by the NMAP (2008) stated "International and domestic comparisons show that American students have not been succeeding in the mathematical part of their education at anything like a level expected of an international leader" (p. xii).

Math Test Results Reveal Flaws in U.S. Math Education

The Program for International Student Assessment (PISA), Third International Mathematics and Science Study (TIMSS), and NAEP scores signal the need for U.S. schools to take math reforms seriously by recalibrating and elevating their teachers' instruction and their students' math learning. Johnston (2020), interviewed in a webinar hosted by the American Institute of Research, explained through an example of a math question in a TIMSS test (Figure 2), that the answer necessitates a conceptual understanding of math ideas. The problem required eighth grade students to demonstrate understanding of linear equations to solve a problem in context and to show their solution. Johnston explained that, because the numbers used were asking for the combined costs of two pens and three pencils, the problem looked simple; however, it required a student's understanding of the algebraic substitution principle, where a variable is substituted in terms of one other variable. Only one out of four students who took the test responded correctly. Johnston explained, "Algebraic thinking needs to start in the elementary grades." These questions take root in a student's conceptual understanding of numbers, sets, and solving for unknown values. Johnston is critical of worksheets that waste students' time with repetitive equations without contexts. These types of math tasks undermine the importance of giving young learners opportunities to learn algebraic thinking. Johnston highlights algebraic thinking and conceptual understanding as key foundational abilities to mathematical thinking.

Figure 2. Algebra Item – TIMSS Grade 8 Sample Question

TIMSS SAMPLE QUESTION

Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils? Show your work.

Possible correct responses with work shown

x = cost of pen	2x + 3y = 17
y = cost of a pencil	2(y +1) + 3y = 17
X = y + 1	2y + 2 + 3y = 17
2x + 3y = 17	5y + 2 = 17
	-2 = -2
x = 3 + 1	5y = 15
x = 4	-5 -5
	y = 3
x + 2y = 4 + 2 · 3 = 10 zeds	

2 pens + 3 pencils = 17
2 · 5 + 3 · 4
10 +12 = 22 ✗
2 · 4 + 3 · 3
8 + 9 = 17
10 zeds ✓

Correct answer = 10 Zeds

1. Correct/work shown
2. Correct/no work shown
3. Incorrect
4. Omitted

Students who answered correctly: 25%

Across countries, only one in four students found the correct solution of 10 zeds and provided workl to support their answer.
These percentages reflect the average across fove countries: Italy, Norway, the russian federation, Slovenia, and the United states.

Statement of the Problem

There is a dearth of math learning resources with the appropriate pedagogy for meaningful math instruction that cultivates students' mathematical sensemaking and reasoning toward proficiency. The mechanical and shallow understanding resulting from rote learning and what Van de Walle (2007) calls the "I, We, You" approach of math learning. The *I* is the first activity in math class: the teacher writes an equation or a word problem and guides the class by showing the steps on the white- or blackboard to get the correct answer. The *we* part is the second activity: the teacher writes a second similar equation or problem on the board and asks students to raise hands and share what they remember about the next steps. Finally, the *you* part is when the teacher distributes worksheets filled with similar equations and problems for each student to answer correctly. This learning formula involves memorizing steps, repetitive drills, and practice tests that deter students from developing their capacity to analyze and express problems, apply mathematical ideas, build math models, and transfer and generalize math ideas while learning the fundamentals of math operations.

Rote Learning Persists in the Age of the Common Core

After-school companies promote rote learning as the best way to achieve fluency with math facts. These companies attract parents who want to make sure their children know how to count and compute according to prescribed algorithms. “Everyone can memorize the facts, but just not all at the same speed. Everyone can do rote learning, but it takes time and patient practice” (Crawford, 2020). These statements cause more confusion, which tends to escalate parents’ frustrations to anger against the math Common Core Standards. Parents fear their children are not learning the fundamentals in the same way they did when they were in elementary grades. Math is just that: memorization and repetitive practice.

Advanced Math Knowledge Defied by Rote Learning

The National Council of Teachers of Mathematics (NCTM) dissuades teachers from using rote learning because it focuses on memorization rather than on developing a deep and authentic understanding of mathematical ideas in contexts (NCTM, 2000). Supporting the new framework of the NCTM, de Walle warns “this follow-the-rules, computation-dominated, correct-answer-oriented view of mathematics is a gross distortion of what mathematics is all about” (Van de Walle, 2007, p.13).

An analysis of the PISA math results explains how students whose countries support memorization to learn math garnered the lowest scores. Among the lowest achievers, the United States ranked number three. Students who learned math through memorization and repetition were at least half a year behind high-achieving countries. Moreover, when compared to students in France and Japan who learned with metacognition and relational strategies, memorizing students were outperformed by more than a year of school (Boaler and Zoido, 2016).

A less visible effect of rote learning in math is making students believe that there is only one way to get the right answer. That way is the teacher's way. Following the teacher and instilling a learning culture of compliance is key to high grades, which students unconsciously correlate to educational achievement. Claims such as grit, hard work, and perseverance are used in rote learning as criteria for success. This framework of learning does not take into consideration, for example, students' inability to understand and follow instructions due to lack of context, teachers not building upon students' prior knowledge, or students feeling marginalized due to race, culture, and economic hardships. These factors effectively cause students to shut down.

Statement of the Purpose

This exploratory study tested the efficacy of a proposed pedagogy in the form of math stories and learning extension resources imbued with a sophisticated design framework, which included a creative, interdisciplinary, multicultural, socially interactive, and story-based approach. The goal of the study was to determine, based on student responses to the learning resources, the combined elements of a pedagogical design toward an effective, alternative math learning strategy. Based on the level of success demonstrated by the math stories and learning activities used in the research, these resources have the potential to raise students' proficiency levels and to address the challenges faced by K–2 students who are discouraged by rote and mechanical math learning pedagogies.

Research Questions

- How can children's physical, social, and cognitive responses to math stories inform us of the efficacy of this pedagogy?
- How do students' responses to the math stories and activity book reflect perceptions, motivations, and sense making of math concepts?
- How do the math stories and activities support teachers in guiding students toward mathematical sense making?

Assumptions

Elements of the Pedagogy

Meaning Making is Defined as Core Math-Learning Criteria.

The Mathematics Learning Study Committee described U.S. math education in comparison with other nations as "shallow, undemanding, and diffused in content coverage" (National Research Council, 2001, p.4). The committee created new and integrative criteria for math proficiency, which contains five strands: (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition (Kilpatrick, 2001, p.5).

The committee synthesized research on math and learning, taking a different stance from behaviorist thinking. Influenced by the new and evolving interdisciplinary field of learning sciences, which integrates constructivist philosophy, cognitive sciences, and neuroscience, internal and external factors that influence children's learning abilities were outlined. Additionally, with the invention of functional magnetic resonance imaging the field of developmental cognitive neuroscience was making big leaps in mapping the brain and developing understanding of children's emotional and cognitive functions.

Adding It Up (National Research Council ,2001), a landmark mathematics book sponsored by the National Research Council, reviewed and summarized K–8 math education in the United States, proposed a focus on numbers and numerical operations without being limited by computing for one correct answer. Following are key mathematical concepts outlined in the publication:

- Numbers and operations are abstractions. These ideas come from experiences but are also not attached to a particular context and experience. Math makes more sense when students learn multiple representations, such as representations and graphs that highlight the meanings of abstract concepts.
- Number systems guide students' understanding of numbers and operations. For example, whole numbers comprise a system.
- Algorithms are necessary for numerical computations. Students' understanding of mathematical algorithms guide their applications.
- K–8 math learning is not limited to arithmetic. Children will understand numbers and operations as these relate to algebra, measurement, space, data, and chance. (National Research Council, 2011, p.2)

Common Core Benchmarks

This exploratory documentary study used the Common Core Standards for math as the benchmark of students' expected knowledge and skills for age and grade levels. This benchmark measures the understanding and practices of mathematics expected in K–12. To be relevant, this study took into consideration the expected outcomes in mathematical learning of the majority of U.S. public elementary schools. The guideposts are based on the Common Core benchmarks outlined for each grade level. However, the study focused on the first-grade level benchmarks, with math stories as pedagogy for conceptual math learning. In its applications of the Common Core, the math stories and the extension activities provide open-ended questions and unstructured math problems, which were theoretically guided to build upon the background knowledge of diverse students and to raise the levels of students' mathematical thinking and practices. In this way, the study ensured students' diverse levels of mathematical abilities are able to connect and grow from their interactions with the stories, the activities, and their peers.

Common Core Unites National Math Benchmarks.

In 2009, the state school chiefs and governors that comprise Council of Chief State School Officers and the National Governors Association Center for Best Practices coordinated a state-led effort to develop the Common Core State Standards. These committees were concerned with how the United States, compared with its international peers, stagnated through the academic standards and the increasing remediation rates for undergraduates in colleges and universities. To address the nation's math education crisis, the committee wrote a set of globally competitive standards and benchmarks for each grade level that they called the Common Core.

Benchmarks do not Equate to Pedagogy.

It is vital to differentiate between standards and student-centered pedagogy. While the goal of the Common Core standards is to improve student outcomes for all children irrespective of their backgrounds and risk factors (Entz, 2007), *student-centered pedagogy*, on the other hand, refers to learning experiences that should take into consideration student backgrounds and risk factors to achieve efficacy.

To date, 41 states have adopted the Common Core standards (Common Core State Standards, 2014). Each state develops and adopts curricula, which are the pedagogical expressions of the Common Core. Kruger (2007), a writer on education policy, clarifies that the controversies surrounding the Common Core are influenced by historical reforms in math education. He explains that the Common Core comprises a list of standards called *benchmarks* or *skills goals* and not curricula.

Multiculturalism and America's Diverse Student Population

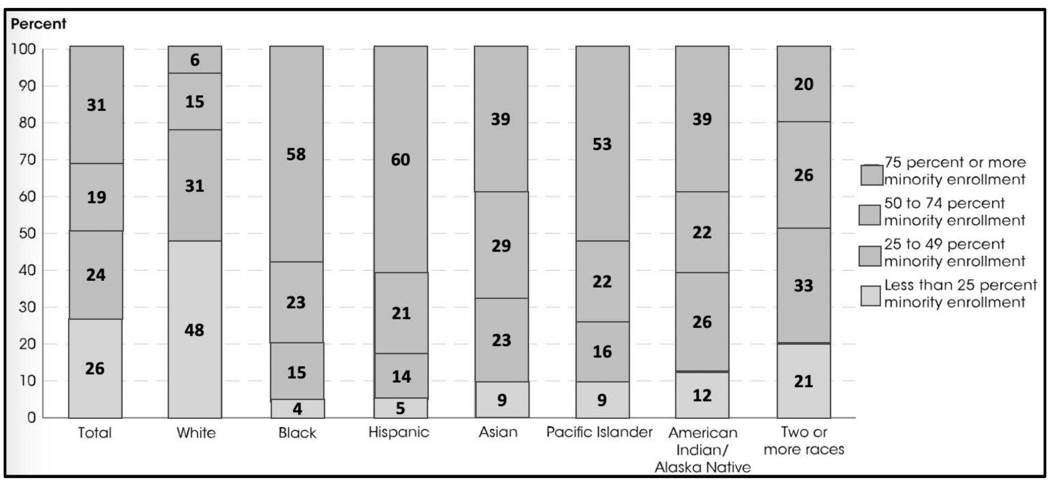
According to the National Center for Education Statistics (NCES), the percentage of English language learners among public school students in the United States grew to 10.1%, or five million students, in fall 2017. From fall 2000 to 2017, the percentage of White students in public schools decreased from 61% to 48%, Black students decreased from 17% to 15%, Hispanic students increased from 16% to 27%, and Asian/Pacific Islanders increased from 4% to 6%. The student population in the United States is growing more culturally diverse (NCES, 2019).

Less Experienced Teachers in Schools with Majority Students of Color.

In schools where African American and Latinx students comprise most of the population, most of the teachers have less than three years of teaching experience compared to schools where the majority of students are White (Wilkins & Education Trust Staff, 2006). Darling-Hammond (2001) explains that experienced teachers are not available where they are most needed. Barely able to keep up with the challenges of teaching, new teachers do not use (or do not have) mathematics materials that connect to the cultures of African American and Latinx students (Malloy, 1997). To address inequity in schools, it is critical to investigate the experiences of students in the way they learn in the classrooms (Cohen, 2000).

Figure 3 shows that 48% of White students were enrolled in public schools that were predominantly White and 25% of Black students were enrolled in schools that are predominantly Black.

Figure 3. Race and Ethnicity in the Public School Population



Culturally Responsive Teaching.

Teaching to students' strengths raises their confidence and demonstrates how the teacher values students' perspectives and abilities. Gay (2000) defined *culturally responsive teaching* (CRT) as "using cultural knowledge, prior experiences, frames of reference, and performance styles of ethnically diverse students, to make learning more appropriate." CRT is only possible when teachers shift their mindsets and make fundamental changes in their pedagogical practices (Gay, 2000). CRT requires teachers to be open to learn from their students and create a classroom culture of inclusion, authenticity, and respect. Mathematical meaning making necessitates creating connections with what is being learned through a participative and collaborative learning environment.

CRT In Mathematics Pedagogy.

This exploratory documentary study explored math fiction stories that reflect elements of multiple cultures. By mixing cultural elements to simulate the growing patterns of student populations in the United States and by positioning students as problem solvers, the study explored how integrating the CRT framework into the pedagogy can create experiences of learning math that empower all students.

Limitations of the Study

The exploratory study was implemented with a small group of students and was a single program of six sessions. The size of the group does not represent a conclusive study.

Significance of the Study

United States Failing Math Education

If a typical classroom rubric is used to measure math test scores, the Unites States would get a *needs improvement* mark. Historically low scores reflect the inefficacy of math pedagogies. There is a clear need for research-based, usable teacher materials that require minimal teacher training to provide better and immediate opportunities to achieve higher-order thinking in math for young learners.

Rote learning continues to be a widely used pedagogy in math learning. Merriam-Webster's online dictionary (2021) provides two definitions of *rote*: "the use of memory usually with little intelligence" and "mechanical or unthinking routine or repetition." Yet, schools and after-school programs continue to back up rote as the best way for young learners to learn basic math.

Exploring Pedagogical Innovation from Users' Reactions

Observations of end-users' responses provided real feedback on how a relatively new pedagogy, with a contrary approach to rote learning, affected student engagement and understanding of math ideas. Inferring perceptions, motivations, and sense making of math concepts from the students' physical, social, and cognitive responses during the activities is unlike the standard test-taking or verbal assessments typically employed in studies of mathematical proficiency.

Pedagogy, in this case, through its corresponding learning activities, shaped how students engage, question, thrive, and cultivate agency. Likewise, the way students were assessed for growth in relation to their responses to the modes of learning that the stories, games, and activities produced was based on observations documented by the video camera, which did not trigger fear or similar negative reactions.

The exploratory approach of observing students' learning, in terms of their perceptions, motivations, and sense making of math concepts from physical, social, and cognitive responses is a paradigm that seeks to see math learning not as a paper-and-pencil subject but as a necessary set of knowledge skills that people interact with in their everyday lives and work. The study sought to use evidence to demonstrate how the math stories and activities impacted the students' growth in mathematical understanding, other than a math test, and to focus more on the students' natural responses. This suggests a relationship between young learners and math ideas, processes, and learning that is not merely based on paper-and-pencil testing or interviews. Rather, the exploratory study sought to distill meaning from physical, social, and cognitive responses to the learning experiences, to the learning atmosphere that the materials create, and to the social interactions and relations among students and teachers and among students and peers. In this way, students and teachers were observed as their most human and intuitive selves, a perspective not typically used in assessing math learning resources.

I took the *us* perspective about academic fields and, as such, should reflect in the way we provide access and authenticity in engaging our students and teachers. Science, for example, is not the text in the textbook but rather how humans have learned about who we are in relation to our emotions and to our physical, intellectual, and social selves. History is not what is written in history books, but rather it is how humans, influenced by their biases, have interpreted how we have become who we are as individuals and societies.

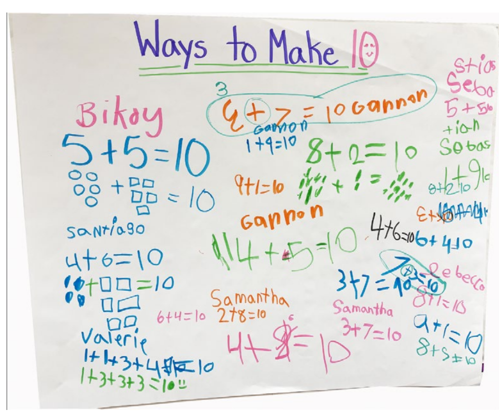
The experience of observing natural reactions to the learning materials values the intuitive and the natural as authentic reactions that can objectively define the efficacy of the proposed pedagogy. Moreover, this aligns with the constructivist framework of the pedagogy, specifically the humanistic perspective, to highlight the unique individual with perceptions, motivations, and sense making of math concepts that are reflected not only in pencil and paper or verbal tests, but also in physical, social, and cognitive responses.

User responses shape the research questions.

The exploratory study documented the young learners' perceptions, motivations, and sense making of math concepts with math stories and extension activities. Observations of the students' responses were based on the students' physical, social, and cognitive responses during the activities. The teacher shared how, in the beginning, the students tended to please the teacher and researcher by giving answers that were clearly reflective of what adults would like to hear from children, such as concepts and words that the teacher taught the class in past sessions. However, almost immediately after the children started reading the story and drawing themselves as characters in the storybook, the atmosphere in the class changed and became more relaxed. The students were responding with more authentic answers.

Changes in students' perceptions, motivations, and abilities in mathematical sensemaking and reasoning were demonstrated in their body language, for example, from squirmy and unfocused to collected and attentive to the story and activities. Socially, boys and girls were more at ease learning and working together. Figure 4 shows activity on a white board where writing multiple ways to get to 10 demonstrated the children's excitement for the many ways they could reconfigure smaller numbers to make the bigger number 10.

Figure 4. Multiple Ways to 10



A Pilot Test Provides Proof of Concept

This study explored a pedagogy based on math stories for developing mathematical sensemaking and reasoning with a group of first graders with teachers facilitating using the math stories and learning extension resources in the context of an after-school program. To effectively see the value a math story can bring to young learners required a design learning research methodology that generated evidence of success. The study provided proof of concept and underscored the combined variables that developed the students' mathematical proficiency that could be improved, from the topics in the math stories to the feedback on aspects of learning resources that underscored the pedagogy used.

Scale Optimal Opportunity Learning Contexts

The study accentuates the role of pedagogy in the context of the United States' historical challenge of the achievement gap in math learning. The decades-long challenge highlights the disparity between children experiencing discrimination by virtue of race and low-income families compared to middle-class families. A study by Tucker-Drob et al. (2013), which used the transactional model of cognitive development, suggests children of advantageous socioeconomic backgrounds can harness "high-opportunity contexts," that grow cognition built upon student potential. This exploratory study observed how math stories fostering conceptual mathematical thinking and peer-to-peer learning offered "high opportunity contexts" of learning for all students.

Overview of the Study

This is an exploratory study of how pedagogy in the form of math stories shapes young learners' perceptions, motivations, and sense making of math concepts. Structured in an exploratory video documentary, I documented a pilot test of the math stories and learning extension resources conducted by two teachers with eight first-grade children from diverse backgrounds. The pilot was held in the context of an after-school program. The exploratory study also includes interviews of the teachers regarding their experiences in testing the math stories and learning extension resources, which were provided by me. Additionally, I interviewed educational leaders and specialists in curricula, literature, and math pedagogy. Research data was gathered in the form of video footage of children participating in math activities taught by the teachers as well as participant interviews.

Math Learning Forms Student Ideas About Math

Children's perceptions shape their thinking and, consequently, influence their motivations in math learning. As an exploratory research project, the goal was to document how pedagogy affects children's interests and abilities in math learning. The study explored how storytelling with conceptual math pedagogies defined mathematical learning experiences in a class of first graders. Through video technology, the documentary captured participating children's physical and cognitive responses to two math storybooks, a math activity book, as well as math learning games and design projects. The children's responses to the stories, questions, and math problems were demonstrated in their facial expressions, body language, verbal responses, and social interactions; this demonstrated their growth (or not) in mathematical thinking, motivations, and interest in mathematical learning.

Exploratory Research Validates Creative Math Solutions

The framework of the study was shaped by my interest in developing usable math learning resources for teachers and parents of children 5 to 7 years old, specifically in mathematical sensemaking and reasoning. So much of educational research is left unused and unread, rendering it unable to provide alternative solutions that address concerns and challenges of everyday math teaching and learning practices in the classroom.

Some studies cite student economic background and lack of parental support as the main reasons for the lack of access to good educational math materials and support. These studies blame parents as the cause of the disparities among students' math education attitudes and test results (Ferguson, 2017). In an opinion article, Wexler (2018) blamed elementary school teachers for poor test results after the 2019 NAEP test scores were released. She expressed deep concern as these scores had not improved since 2009.

Other studies highlight the effects of racial discrimination, which create emotional challenges among students of color as well as an imbalance of resources resulting in inequality among students in U.S. schools. In a formidable podcast series titled *Nice White Parents*, Daniels and Gonchar (2020) exposed how White parents manipulated the trajectory of a middle school, taking away resources for mostly Black and Latinx children while fundraising millions of dollars for their children's programs at the school. The podcast correlated socioeconomic and racial issues to deterring low-income and students of color to succeed in math learning in comparison to high-income and White peer-equivalent students. Moreover, high-income and White students access resources such as after-school tutoring to either keep up or advance in math learning. Racial entitlements sustain the United States' historical learning opportunity gap. However, the fact that students must attend additional classes to keep up with math learning should be highlighted as a learning design problem in the way math is taught in schools.

Why Write, Design, and Publish Math Storybooks and an Activity Book?

The impetus to create math storybooks as pedagogical resources to address the achievement gap was to forge a creative solution in response to the following:

1. Larson and Kanold's (2016) call for educators and parents "to break the intractable cycle of resistance to change."
2. National Research Council's (NRC 2001) calls for closing the chasm between the educational researcher and practitioner with a new paradigm for research that interacts directly with practice and creates research materials that lead to "usable knowledge" (Lagemann, 2002).
3. The practices set forth by the NCTM (Huinker & Bill, 2017) to support positive mathematical identities and to provide equitable access for young learners to practice math and shape their own mathematical thinking.
4. The dearth of usable research-based curricula and resources available to primary school teachers to personalize and raise the levels of math learning in multicultural classrooms, especially in low-income and diverse communities.
5. Monroe and Young's (2018) definition of the current research on the use of children's literature in math as "wisdom of practice" without sufficient research to support that the proposed pedagogy works.

Video Documentary as a Data Gathering Tool and Research Format

Audio and visual images documented the impact of pedagogy in math learning. Pedagogical efficacy was determined by observing and analyzing student reactions and interactions with the story learning resources vis-à-vis the overarching goal of teaching and learning mathematical sense making and reasoning. The goal of the documentary is to make learning visible by showcasing the processes and experiences of math learning. The value of documenting children's learning is also inspired by the Reggio Emilia philosophy of early childhood education (Edwards et al., 2012).

Video technology captured how the pedagogy engaged students in math learning. In effect, video and photo images are evidence from which the efficacy of the pedagogy of math stories and the related learning activities are assessed in the study.

The exploratory study seeks to make the research accessible to teachers and parents. Video as medium of learning is increasingly preferred by today's teachers and parents of primary grade level students in comparison to reading written dissertations. Millennial parents and most new teachers of young children represent 92% of the online video viewership (Bevan, 2021).

Overview of the Study

I engaged in an exploratory design research of how math stories can generate better opportunities for students to succeed in foundational math learning. The study, which was an after-school learning program for first graders, is integral to the process of innovating alternative math learning solutions. The research is a user-test experience of two math fiction storybooks, a creative math activity book, and learning extension resources related to the stories that are taught by two teachers.

These learning resources are designed as creative and integrative applications of pedagogical theories (i.e., CRT, cognitive and learning sciences, and constructivist learning) that are integrated with foundational concepts of numeracy, an introduction to algebraic thinking and addition principles, and spatial thinking with shapes. The overarching goals of the books are to provide multiple ways for children to connect to mathematical ideas and actively co-build their mathematical knowledge toward developing their abilities in mathematical sensemaking and agency.

The dual use of video documentary was to document user responses as well as to share the results of the study with teachers and parents.

II. Review of the Literature

Introduction

The literature review is structured thematically. The first part triangulates the math scores of U.S. students among three organizations' standardized assessments. These scores lead to the same conclusion: the lack of depth of most U.S. students' mathematical understanding. The second part is a survey of the reasons most widely attributed to the opportunity learning gap. None of these reasons refer to pedagogy as the weak link to gaining mathematical proficiency.

The third and chief component of the literature review is a discussion of pedagogy. The first topic is the rationale for pedagogy, the *how* defines *what* students learn about math and math education and why one's sense of self connects to one's ability in math. Pedagogy is the bridge to mathematical proficiency; therefore, when done right, it can create an equal playing field for *all* students. The next part is a review of pedagogical variables that can make math learning empowering, build children's confidence, and develop agency. The learning sciences, CRT, and constructivist philosophy form the framework of the proposed math stories and learning activities that were used in the pilot study.

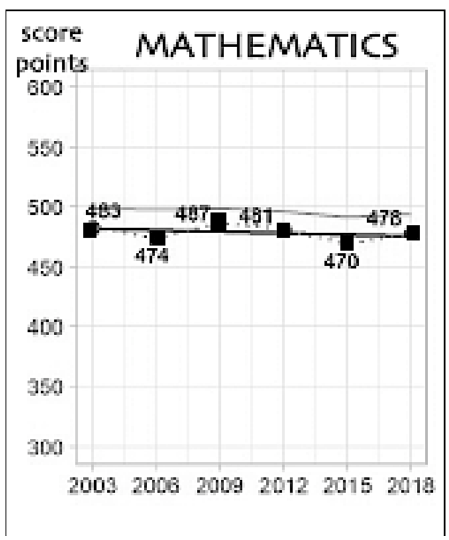
The fourth part clarifies why mathematical sensemaking and reasoning is the focus of the math stories. Finally, the fifth part has to do with the variables that make up the math stories and the math learning activities.

Triangulating with Three Mathematics Standardized Testing Companies

Program for International Student Assessment

America's K–12 math education is not improving. For more than 20 years, test scores in PISA, an examination developed by the Organization for Economic Cooperation and Development, have been stagnant (Figure 5). U.S. students scored lower than 30 countries. The United States' overall mean performance in mathematics was hardly unchanged (PISA, 2018). Eight percent of U.S. students had mathematics scores at Level 5 or higher, relative to the international average of 11%. The largest share of students with Level 5 or higher scores were China at 44% and Singapore at 37%. The largest percentage of American students scored Level 2 or higher; at this level, 15-year-old students were able to solve simple situations mathematically, such as converting price to a different currency.

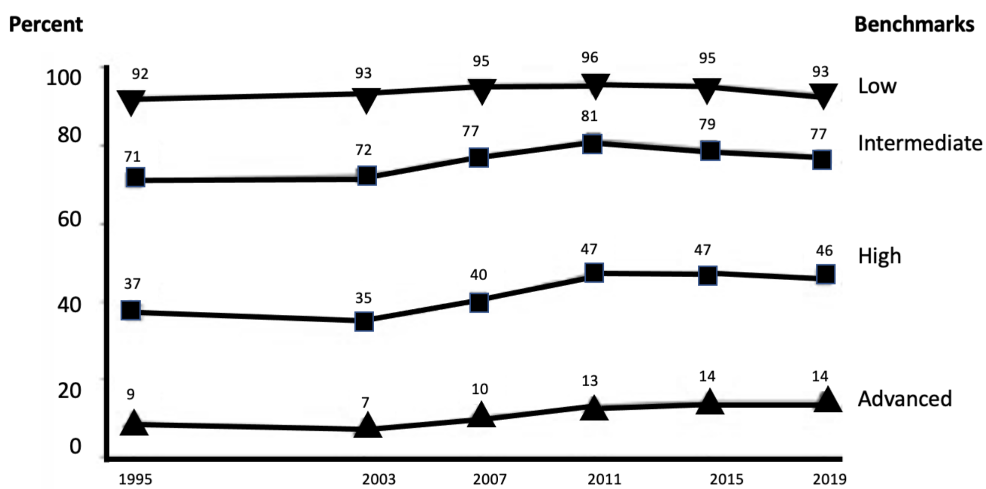
Figure 5. PISA Comparative Scores Over the Years 2003-2018



Trends in International Mathematics and Science Study

Since 1995, the United States has been actively taking the TIMSS examinations for Grades 4, 8, and 12 (Figure 6). The examinations in the United States are overseen by the NCES, with the international sponsorship of the International Association for the Evaluation of Educational Achievement. Like PISA, TIMSS is an international comparative study with representing 64 nations. In the most recent math examinations, fourth graders in the United States garnered an average of 535 points, compared to 297 points for the Philippines (the lowest performing education system) and 625 points for Singapore (the highest math performing education system). In the 2019 TIMSS results, the United States came out with highest average scores in math; however, the United States also had the highest increase in the gaps of its scores between the top and lowest performing students with scores decreasing within the lowest benchmark (Herz et al., 2021).

Figure 6. TIMSS Comparative Scores Over the Years 1995-2019. Trends in Percentage of U.S. 4th-Grade Students Reaching the TIMSS International Benchmarks in Mathematics.



National Assessment of Educational Progress

With the NAEP test, it is critical to note that only 41% of students in Grade 4 were performing at or above NAEP proficient, and 59% scored below proficiency (NAEP, 2019; Figure 7). The NAEP defines mathematical proficiency as “demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter” (NAEP, 2019). This implies, therefore, that almost 60% of U.S. fourth graders did not meet the grade-level benchmarks. Furthermore, average scores in mathematics tend to go down as students move to higher grade levels. Figure 8 visualizes the downward trend of students achieving grade level proficiency in math. This means an increasing number of students fail to meet the benchmarks set by the Common Core in mathematics, which suggests that the grades for a majority of low-income students and students of color will affect their range of options for careers and social mobility.

Figure 7. 2019 Trend in Fourth Grade NAEP Mathematics Achievement-Level Results

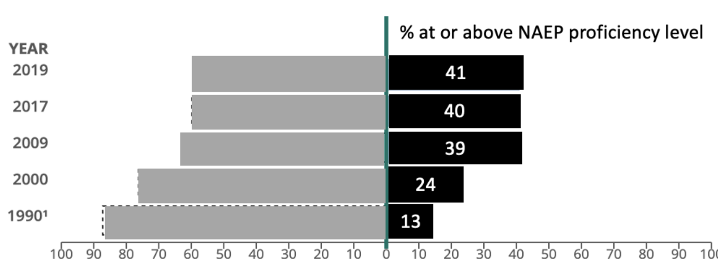


Figure 8. NAEP Trends in Assessment Score Averages for Grades 4, 8, and 12

GRADES	4th	8th	12th
Mathematics Assessment Score	41	34	24

Math Anxiety Derails Math Learning

Standardized assessments and examination scores do not necessarily reflect everything that a student knows in math. Test designers make it a point to choose items that discriminate among students' abilities. This means vital to the test design parameters is to choose applications of mathematical ideas that are expected to be answered correctly by half or less than half of the student population in a grade level (Poham, 1999). An unfortunate by-product resulting from the stress generated by these tests is what is now popularly called *math anxiety*. Tobias and Weissbrod (1980) define math anxiety as "the panic, helplessness, paralysis, and mental disorganization that arises among some people when they are required to solve a mathematical problem" (p. 1). Resulting from letdowns and humiliations in traditional math classes, negative math experiences belittle students, cultivate self-doubt, and result in disinterest in math learning (Young et al., 2012).

Different Perspectives on the Math Achievement Gap

The way a problem is framed determines its analysis and solutions (Flores, 2007). It is easiest to blame those at the lowest level of the totem pole of decisions and policymaking: families of poor and minority students and inept teachers.

Blame Low-Income and Parents of Color

A superintendent issues a statement (quoted by Heffter, 2006): "Students of color continue to lag behind white students and some Asian students, and the so-called achievement gap still exists." The National Governors' Association (2005) states, "Across the U.S. an achievement gap persists between minority and disadvantaged students and their white counterparts." Gardner (2017), an English teacher of 28 years, pinpoints the lack of participation from parents in "inculcating the importance of school and instilling reasonable discipline with their children so real learning can take place." He underscores that this can be difficult for low-income households, effectively implying that these parents are liable for their children's failure in school. In a letter to the editor, Eugene Oh, president and CEO of the I Have a Dream Foundation, criticizes Gardner's statements as "failing to weigh the complexity of systemic inequity" (2018). Today, more than 50% (or 25 million) students in U.S. public schools are classified as low-income. According to a report by the Pell Institute for the Study of Opportunity in Higher Education and the University of Pennsylvania Alliance for Higher Education and Democracy (2020), by the time students in the lowest quartile of income are 24 years old, less than 10% graduate from college, while 77% of their peer counterparts belonging to the highest quartile graduate from college. The report raises concerns that the education system can choose to either provide equity and social mobility or to sort students. Citing a relevant quote from the Commission on Higher Education to President Truman in 1947:

> If the ladder of educational opportunity rises high at the doors of some youth and scarcely rises at the doors of others, while at the same time formal education is made a prerequisite to occupational and social advance, then education may become the means, not of eliminating race and class distinctions, but of deepening and solidifying them. (Cahalan & Perna, 2015, p.5)

Blame Primary School Teachers

The reality is that teachers are themselves products of the American educational system from which they learned the concept and delivery of rote memorization as the only way math is taught and learned. The Los Angeles Times headline "Opinion: Math scares your child's elementary school teacher — and that should frighten you" (Willingham, 2006) works right into the already heightened fears of parents that their children's teachers are not adept at teaching math. Willingham further explains that American students are only taught with curricula based on textbooks that are "diffuse, shallow and repetitious," which is how teachers learned math themselves. This curriculum consists of memorizing a set of math facts and knowing to solve addition and subtraction algorithms without understanding why these yield the correct answer (Willingham, 2006).

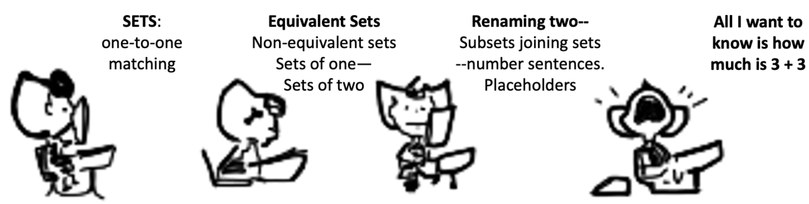
How Children Learn Influences Their Sense of Self and Math

Pedagogy, as defined by Li (2012), is the theory and practice of learning. Learning experiences are influenced by the social, political, and psychological background of the learners. As an academic discipline, pedagogy is the study of how knowledge and skills are taught and the interactions that occur in an educational setting. The Meriam-Webster dictionary (2021) defines pedagogy as the act of teaching. As such, pedagogy creates the learning culture, the types of interactions among students and their teacher, and the learning methods employed or how students learn. Rote learning is one type of pedagogy.

Rote Methods Persist in the Age of the Common Core

The year that standardized testing with the Common Core was launched, there were no new curricula, teachers had not undergone professional development, and learning materials were not ready. Green (2014) explains a pattern in the way math reform is planned and implemented in the United States: teachers are instructed to change the way they teach and to adopt new ideas without sufficient guidance. When teachers implement unclear ideas and pedagogies about math, the situation causes more confusion among students. Parents of young children become perplexed, as in the case of the Common Core, that math homework does not look anything like the math they learned when they were in the same grade levels. Green uses the metaphor of the 1965 Peanuts cartoon strip (Figure 9) to highlight the frustration caused by math education reform that fails to prepare teachers for big changes in the way they teach.

Figure 9. Peanuts cartoon (1965)



As such, without professional development and lacking in culturally responsive curricular materials and research-based teaching resources, most teachers continue to teach like they were taught—rote memorization and the I, We, You classroom strategy (Van de Walle, 2007).

> Some teachers call the pattern I, We, You. After checking homework, teachers announce the day's topic, demonstrating a new procedure: Today, I'm going to show you how to divide a three-digit number by a two-digit number (I). Then they lead the class in trying out a sample problem: Let's try out the steps for 242 ÷ 16 (We). Finally, they let students work through similar problems on their own, usually by silently making their way through a work sheet: Keep your eyes on your own paper! (Green, 2014)

Most students continue to learn by rote and mechanical math learning experiences, such as memorizing steps, repetitive drills, and practice tests. Van de Walle (2007) explains most adults continue to think about math as rules for solving equations and word problems, and when one is unable to follow each single step as presented by the teacher on the board or in the textbook, most adults would be proud to say, "I was never good at mathematics" (p. 12).

Traditional math curriculum teaches discrete algorithms, with a set of rules that elicit a correct answer, such as how to do long division or how to use the Pythagorean theorem. Students are then expected to learn the material by memorizing the algorithms and practicing with a large quantity of similar problems.

Among schools today, there is still confusion on what high-quality math means for primary school and how pedagogy can either deter or raise math learning levels for young learners. In a study on the efficacy of after-school tutoring for math, Tomic (2016) assessed the impact of after-school programs on student achievement. Most of the schoolteachers agreed, "The tutoring program should focus primarily on students' skill deficiencies and provide additional support for students who need skills remediation" (p. 70). Tutoring is overrated as a strategy for raising math achievement level considering the study concluded the program had minimal efficacy in improving students' math grades The discussion among teachers reflects how their understanding of mathematics and pedagogy reflects in their choice of strategies for math teaching. Many elementary school teachers continue to teach math using rote memorization and lack the knowledge and skills to teach high-level mathematics successfully (Ball et al., 2005). By continuing to teach in the same way they learned math, the cycle of breadth versus depth continues in math education.

Most Math Tutoring Programs Teach with Rote Pedagogy

Over a quarter of U.S. students attend after-school tutoring. These students struggle in math or have parents that pay for one-on-one tutoring to ensure their children advance in their math abilities (Scholarship Media, 2017). Students of low-income families are mostly unable to afford after-school tutoring.

Rote Pedagogy is Behaviorist

When devoid of free will, students forced to learn using rote methodology are like lab rats. Unknowingly or knowingly, rote learning is built upon behaviorism. “Behaviorist philosophy purports to explain human and animal behavior in terms of external physical stimuli, responses, learning histories, and reinforcements” (Graham, 2019). When children’s behavioral responses are based on external stimuli, like grades and other types of rewards, the teacher is, in effect, knowingly or unknowingly, applying elements of behavioral philosophy.

Prominent behaviorist B.F. Skinner promoted methods on operant conditioning and radical behaviorism. He even used his infant daughter in experiments similarly designed for lab rats, locking his daughter in a highly controlled box for months. When questioned, Skinner (1971) claimed how the desire for free will is misdirected. In a shattering attack of Skinner’s ideas, linguist and highly influential intellectual of the modern world Noam Chomsky accused Skinner of seemingly using the goal of goodness as the goal of operant conditioning when in fact he was laying the premise and methods for a police state (1971, pp. 66-74).

Watters (2015) warns of the implications of transferring behaviorist theories used with pigeons and rats to students in the classroom. Behaviorists experiments with animals prove that an organism is inclined to replicate a reinforced reaction and to restrain a response that has been punished. When students are rewarded to behave in a certain way, such as following the teacher’s instructions at any cost, and are punished by standing in the corner or failing grades, the teacher is using behaviorist methods.

The effects of rote learning among young children can cause a lifelong imprint on their sense of "math-selves." Children who are good memorizers and have had early exposure to math concepts do well in math and consider themselves "good in math." Children who are uninspired and bored with rote methods, mentally drop-off from math class. When math learning results in children shutting down, they lose a lifetime of opportunities to learn to think mathematically.

Despite criticism of behaviorist methods, debates abound on whether children should be made to memorize the times table and other math facts. For example, Resilient Educator is a coalition and one of numerous organizations with online resources for teachers and students that support rote learning. Supported by Northcentral University, Kaiser Permanente Thriving Schools, and many more colleges and universities, it promotes rote as the foundation for higher-level thinking.

The first mathematics textbook in the United States, *The New and Complete System of Arithmetic – Composed for the Use of the Citizens of the United States*, was written by Nicholas Pike (1809). Pike prescribed teaching through memorization of arithmetic terminologies and through direct teaching. This was followed up with rote practice to master the rules for basic arithmetic operations. According to the New England Historical Society, Pike's book was the standard for teaching arithmetic at a time when Congress was phasing out pounds and shillings and math teachers were adjusting to the new currency, the U.S. dollar. Henceforth, the influence of Pike's teaching math book has permeated school classrooms and math education debates for almost 200 years.

In his book *The Common School Arithmetic*, Botham (1837) explains how parents prefer their children learn math in a straightforward way. Parents want them to be educated in arithmetic ideas with clear, routine explanations of the concepts. Botham calls this strategy the "good old-fashioned way," with brief and basic descriptions of math directions. No chaos, ambiguity, and, of course, no questions about the math concepts and strategies. Just follow, memorize, practice, and learn the right way.

Larson and Kanold (2016) expresses concern on how this pedagogy has been forever embedded in American math education culture. At the core of the United States' so-called math wars is an extension of the conflict between the traditionalist preference for rote pedagogy and content-learning focused procedural or instructive approach and the reformist humanistic or student-centered and process orientation for math learning and teaching. Driscoll (2000) explains that a learner's mental models affect their perceptions and understanding, and they construct their own interpretation of what is being taught. While it seems at the onset that the differences between these two opposing blocks are not about conflicting views on the fundamentals of math concepts, these ideas get blurry as the discussions proceed. Heated arguments evolve into elaborate entanglements of math pedagogy and concepts as well as the reasons as to how and why, or how not to and why not to, prime students for mathematical reasoning and/or provide them a more open and analytical or structured and closed approach to gain the ability to do math (Shoenfeld, 2004).

***How* Children Learn is *What* They Learn**

This study hypothesizes that pedagogy, as a transformative process, affects the way in which children learn math. It can address the complex issues that contribute to the historical achievement gap in the United States. The math stories and activities are predicated on humanistic learning theories, specifically the constructivist philosophy and the integrative field of the learning sciences. The study sought to, through design, publication, and user-research, explore how a pedagogy of learning math through fiction stories creates an equal playing field for concept-driven math learning for first graders by cultivating intuitive connections with math ideas. More importantly, these stories and activities are guided by CRT theories, with the goal of opening high-level math learning opportunities for students of color and low-income backgrounds.

Adding It Up and Math Reform

Concerned with the low mathematical performance of America's K–12 students for 30 years, the National Research Council organized a Committee on Mathematics Learning in 1998. The committee was tasked to review and synthesize math education research and to publish their findings as a guide to transform math education for K–8 teachers, educators, researchers, policymakers, publishers, and parents (Kilpatrick et al., 2001).

In 2001, the committee published *Adding It Up*, a landmark publication that outlined and defined new criteria for improving math education in the United States. This publication influenced the development of the mathematics Common Core. Unfortunately, its recommendations were unable to influence a coordinated, strategic implementation program to transform math learning in U.S. public schools. This is reflected in the continuing use of rote for math teaching among the majority of primary school teachers. Davenport et al. (2019) were concerned that many teachers believe it is sufficient for students simply to memorize math facts through rote learning. As directors of the K–12 mathematics program

for Boston Public Schools, Davenport et al. have observed how most teachers gravitate toward efficiency in teaching and managing classrooms with worksheets and flashcards. For these teachers, this was how they learned math, and it worked well enough for them.
Most K–8 teachers use textbooks as their main instructional resource. To the detriment of students' educational math experiences, Tyson-Bernstein (1988) explained how textbook companies continue to sell millions of copies of their publications to schools: "Their books incorporate this mélange of test-oriented trivia, pedagogical faddism, and inconsistent social messages" (p. 7).

Pedagogy Impacts the Way Students Learn Math

Without the awareness of how pedagogy affects what children learn, math education will continue to be difficult and painful. For most students who are alienated by rote learning of discrete concepts and algorithms, their math learning experiences do not make it possible to connect with and make sense of mathematical ideas.

Pedagogy is the medium by which students learn. In the past 30 years, only 35%–40% of America's 4th and 8th graders who took the TIMSS and the NAEP achieved grade level proficiency. To achieve grade level proficiency, students enroll in after-school math tutoring to review and master what they learned in math class. The fact that students need additional tutoring after school, indicates the inadequate pedagogical tools used by schools for optimal math learning. As such, inequality begins in the pedagogy used in classrooms because the learning design is deficient and unable to connect with diverse students so as to elevate their abilities and achieve grade-level proficiency in mathematics.

Gojak (2014), in her final message as NCTM President, was concerned that teachers continue to fall back to traditional show-and-tell strategies in teaching math. Instead of moving into dynamic, hands-on, and participatory approaches, teachers repeat much of the same with bigger numbers or use tricks without deepening students' understanding. Falling back to ineffective pedagogy reflects a lack of training and understanding of the role of pedagogy in developing mathematical understanding.

The achievement gap commences with considerable disproportions in math achievement levels in younger students. Disparities in ability levels are already apparent with kindergarteners from low socioeconomic status backgrounds in comparison with peers coming from higher socioeconomic levels. Students of the former score 1.3 standard deviation points lower on math assessments than their higher socioeconomic status peers (Duncan & Magnuson, 2011). Furthermore, growing indicators of documented disparity in learning math in school are exacerbated with rote learning. It is typical that in elementary schools, children from a lower socioeconomic status receive less high-order and conceptual-oriented math activities. Lessons are more procedural instruction-oriented on basic math skills than their higher socioeconomic status peers (Desimone et al., 2010; Means & Knapp, 1991).

Crawford (2018), citing a conversation with Stanford University professor Jo Boaler, the executive director of YouCubed, Stanford University's K12 math resource center, asserted that the United States is "stuck in the Victorian Age." Boaler remarked on the irony of most teachers tending to slide into the use of archaic and static math learning strategies, considering higher education studies have proven how much more effective exploration and creativity build children's conceptual mathematical understanding. When students learn math in the way their teachers learned through rote pedagogy, the achievement gap is not resolved and more students fall through the cracks, develop math anxiety, and "hate math." Kurzenhauser (2020), who is a math major, said people tell her, "Oh man, I hate math. How can you do that?" "It makes me feel really stupid, I have a very hard time comprehending it." "It makes me feel bad. I feel less intelligent." and "I just get frustrated." Kurzenhauser follows up by proposing a solution: "We don't need to change what we're teaching kids but change how we're teaching it." Gowers (2016) explained that math is not the problem; it is the way it is taught.

Pedagogy Can Create an Equal Playing Field for Math Learning

Tomlinson & Erickson (2007) explains, "facts devoid of meaning are stillborn. When we deliver information to students without breathing life into it, we have done no more than throw sand in their faces" A math concept such as *number* is defined as an abstract symbol used for counting (p. viii). However, decisions of what pedagogy to use in teaching children how to count and the choice of who gets to learn what type of math concepts are mired in biases of race, class, politics, gender, and mobility (Schoenfeld, 2004). Paulo Freire criticizes how society uses pedagogy to deprive low-income and poor populations. In his 1970 landmark publication *Pedagogy of the Oppressed*, Freire described the banking system of education:

> The teacher talks about reality as if it were motionless, static, compartmentalized, and predictable. Or else he expounds on a topic completely alien to the existential experience of the students. His task is to "fill" the students with the contents of his narration—contents which are detached from reality, disconnected from the totality that engendered them and could give them significance. Words are

> emptied of their concreteness and become a hollow, alienated, and alienating verbosity. The outstanding characteristic of this narrative education, then, is the sonority of words, not their transforming power. "Four times four is sixteen.” (p.70)

Freire describes the metaphor of “turning students’ minds into receptacles to be filled by the teacher” (p.72) as oppressive and disempowering. This method disregards what students know and prevents them from recognizing their abilities to actively participate in shaping their knowledge.

Developing proficiency for all students in math is best achieved when the pedagogy engages students in a class as a community of learners, collectively spawning solutions and methods in problem solving. Learning activities empower students to figure out mistakes and correct solutions by departing from memorization and pivoting toward building understanding of the logic and structures of mathematics problems, concepts, and applications (Kilpatrick et al., 2001, p. 24). Creating an equal playing field in mathematics education necessitates “developing curricula that are catalysts for relevance, coherence, connectivity, and power” (Tomlinson & Erickson, 2007, p. x).

An emerging academic field questioning traditional concepts of math learning and learning in general has been growing with the interdisciplinary field of cognitive science. Born in the 1990s, *learning sciences* is an evolving, formidable field with scientifically driven learning theories based on interdisciplinary cognitive research. Learning sciences professionals study how children and adults learn in an integrative fashion to design learning environments that generate optimal results in formal and informal as well as real and virtual learning contexts. By taking into consideration learners’ backgrounds, abilities, and contexts, efficacy is achieved by using pedagogies to help make sense of different contexts, providing learners multiple ways to connect, and formulating new and/or mixed strategies (Schoenfeld, 2004, p.263). In 2004, the *Journal of the Learning Sciences* was listed as the highest-impact journal in education and educational research by the Institute for Scientific Information Journal Citation Reports (ISI, 2004). This suggests how the learning sciences field is developing interest among professionals in the cognitive and education theory domains.

Math Education at a Crossroad

The emotionally charged U.S. history of math reform is a cycle of memorized and rote-oriented content versus critical and creative pedagogy. Larson and Kanold (2016), poses questions on the great debates of opposing ideas on math education: “Should teachers offer students rules and facts to memorize? Or should they give students material to reason about to discover and develop understanding of underlying mathematical principles?” The nation’s achievement gap is telling on the relentless math education problem in the United States. Disagreements never end on how math is best taught, what math concepts and algorithms to teach, and the scope and sequence for each grade level. Innovation is stunted by resistance to what is not familiar and tested. Extraordinary.

Van de Walle (2007) reflects on how, historically, parents and educators know the importance of mathematics as a subject, yet very few really know the core essentials of the discipline. “For many, mathematics is a collection of rules to be mastered, arithmetic computations, mysterious algebraic equations, and geometric proofs” (p. 12). Traditionally driven math pedagogies are still the dominant way of teaching. Classroom curricula is anchored to textbook chapters and activities. Students are made to read and/or listen, observe how the teacher demonstrates assigned activities, whether this be on the black/white board, an overhead projector, or a video from the textbook company or YouTube, and mirror the example of the teacher while working on the assigned exercises. Then the teacher checks for the correct answers. This pedagogy results in shaping children's perception of math as following a set of rules taught by the teacher from the textbook. "This follow-the-rules, computation-dominated, answer-oriented view of mathematics is a gross distortion of what mathematics is all about" (p. 13). Students who are good at memorizing and following rules get high grades, yet there is no correlation to this result and high-level math thinking.

Publicly reported test scores place pressure on superintendents, then on principals, and, ultimately, on teachers. For a teacher who has little to no experience with the spirit of the math Common Core, it is very difficult to adopt a student-centered approach to mathematics with untested, science-based learning theories and strategies in mathematics. Van de Walle (2013) states how achievement gap test scores, considered a benchmark of quality education, multiplies the stress placed on educators—the superintendents, principals, and, mostly, the teachers.

Moreover, when teachers are unfamiliar with the Common Core, it makes it more challenging to translate learning theory, manage the classroom, and ensure every student works on the requisites as prescribed by the school district for their grade level while in a student-centered class culture, which they did not experience as learners. Sliding back to the familiar way is tempting and comforting even if rote memorization is not providing students nourishment for success. Unfortunately for children, math learning means excessive drill, review, and practice tests. This is not the learning experience they would be excited to be part of. If at all, it pushes students away from math classes.

Pedagogical Metamorphosis

Metamorphosizing from how today's primary school teachers, who teach multiple subjects in their classrooms, learned about math education, requires a reframing of the math opportunity-gap challenge from blaming teachers and low-income parents to being a national historic and systemic issue. Policymakers and educational leaders who have the power and direct resources can provide the necessary support for teachers to develop new skills and to shift mindsets on math education. Teachers should be guided and given access to resources with math curricula that neutralizes anxiety and nourishes all students to succeed.

Effective math pedagogy intersects the mind and body by engaging a mix of student intelligence, perspective, fears, and everything else that comes with being a multifaceted human in a complex, multicultural world. Math education research has consistently highlighted that students can reach enduring understanding when they engage in productive struggle with new ideas and challenges that they can relate to, where they merge their background knowledge to come up with solutions, and when they work with a community of learners (Van de Walle, 2013, p. xvii).

Learning Math Through Learning Sciences Guided Pedagogy

The learning sciences uses an integrative framework of the cognitive sciences and generates a holistic and scientific understanding of how people learn. Cognitive science is an interdisciplinary field studying the mind and intelligence, intersecting the domains of psychology, linguistics, neuroscience, anthropology, philosophy, and artificial intelligence (Thagard, 2018).

Children Learn Best When They Construct Their Math Knowledge

Schlechty (1977) prompts educators to take note of the fact that

> Students are not products. They are people with motives, wills, capacities, needs to be satisfied, desires, and longings. They are not clay, to be molded or widgets on an assembly line, though sometimes we make them feel as though they are. (p.58)

Good curriculum, according to Tomlinson & Erickson (2007), connects with the cultures of students and opens windows to the world. Activities should inspire and keep students curious so they can make sense and have a thorough understanding of what they are learning. The choice of math pedagogy is key to build upon students' prior knowledge as wellsprings from which they grow their abilities and understanding of new ideas. While doing so, students should learn to develop their metacognition. Learners grow in awareness of their thought processes, which in turn, raises their abilities to learn better.

Culture and the Social Context of Knowledge Construction

The constructivist pedagogy fosters learning as a social activity where students and teachers work on problems and projects together, rather than memorizing abstract concepts (Dewey, 1902). The framework conveys that knowledge is socially constructed. One of the most influential constructivist thinkers is Russian psychologist, Lev Vygotsky. His sociocultural theory (1978) emphasizes the role of community and culture as instrumental in the way learners make meaning. Students in formal and informal learning contexts, are constantly negotiating their beliefs, values, ideas, and ways of constructing ideas while learning. *Making meaning* is a process of developing cognitive abilities and involves interaction with the social world. Vygotsky's cognitive growth theories are contrary to Piaget

(1959), a cognitive constructivist and developmental psychologist who developed the theory of equilibrium, which fostered a single principle that accounts for growth in a child's ability. In the exploratory study, observations of children doing math, provided indicators of the aptness of the cognitive development concepts of Piaget and Vygotsky. With the two participating teachers aligned with the vision of developing students as mathematical thinkers, the social and individual oriented learning activities allowed students opportunities to self-navigate their participation in math activities in math stories, games, and design projects.

Building upon children's curiosity, of which both Piaget and Vygotsky have a unified view, the types of engagement in math learning activities in Piaget's ideas explains that the child learns through self-initiated discovery and Vygotsky's ideas, which emphasize learning through social interactions, were both present among the children that participated. Piaget's ideas, based on his observations of his own children, explained how children's cognitive development precedes their learning. Vygotsky argued differently, stating that "learning is a necessary and universal aspect of the process of developing culturally organized, specifically human psychological function" (1978, p. 90). In other words, social learning tends to precede development.

Key elements of the pedagogy of learning math through stories are influenced by two of Vygotsky's developmental concepts: (1) the More Knowledgeable Other (MKO) and (2) Zone of Proximal Development (ZPD). MKO suggests a person with more advanced knowledge or a higher skill level with respect to a particular task, process, or concept. Vygotsky (1978) defines ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (p. 86).

ZPD refers to how well a learner can solve problems without help and the necessary guidance or scaffolding required to grow a learner's ability to accomplish any learning task. For teachers and parents, this means being aware of children's understanding, providing activities they connect with, and elevating from what they know. This is where scaffolding, a metaphor likened to a physical support for a structure, which in pedagogical applications, can come in different forms (e.g., modeling, questions, problem-solving activities) comes in. The math stories and activities that I created have features that were applications of these theories and tested in the pilot. The overall narrative and visual designs of the stories, with a math-driven plot, were intended as embedded MKO and ZPD, which served as prompts for the teacher to elevate learning as well as identified and supported students within the ZPD framework. In these cases, the teachers' feedback on the effectiveness of the pedagogy during the field user-tests affirmed the efficacy of the creative applications of the above theories as well as served as feedback to the applicability of these theories in terms of the design of the extension activities in this study.

Culturally Responsive Teaching Strategies

Also titled "culturally relevant teaching," Childers-McKee (2016) explains CRT is a pedagogy that takes into consideration students' cultural orientations, actions, symbols, and more in designing learning experiences. CRT builds upon the constructivist framework by articulating the shift from a White-monopolized school culture to weaving threads of a diverse mix of cultures. By shifting power relationships of learners and teachers, CRT transforms learning experiences from a homogenous pedagogy to a rich mix of multicultural interactions, which serve as rich wellsprings that shape learning. Embedding CRT within lessons transforms the classroom culture into a more inclusive and egalitarian learning environment. CRT "(1) Strengthens all students' sense of identity, (2) Promotes equity and inclusivity in the classroom, (3) Engages/connects students to the course material, and (4) Cultivates critical thinking" (Burnham, 2020).

Essential to CRT is valuing students for their unique backgrounds and abilities. This means eradicating the monopoly of the dominant White culture, most especially in seemingly culture-neutral subjects such as math. To realize students' potential, Burnham (2020) describes key CRT criteria that should be embedded in curricula, including:

1. Activate students' prior knowledge and potential.
2. Make learning contextual.
3. Encourage students to leverage their cultural capital.
4. Include diverse races in the learning materials.
5. Build respectful relationships; make students feel valued and seen for who they are.

Building Upon Young Children's Math Abilities

There are six longitudinal studies on which subjects and developmental aptitudes could predict school success among early learners, including reading, math, attention, and socio-emotional skills. The meta-analysis of the study revealed that early math skills had the highest predictive power followed by reading and attention skills. The findings were consistent between boys and girls from high to low socioeconomic levels (Duncan et al., 2007). The study achieved its gal as a learning design review in the context of a pilot with end users and created a close connection between research and pedagogy in an integrated and interactive process (Clements, 2002).

The math stories and activities were guided by the following early childhood studies of math skills. Academic achievement is a cumulative process involving both mastering new skills and improving already existing skills (Entwisle & Alexander, 1990; Pungello et al., 1996). Tapping into the intrinsic potential of young children, the study validated how the math stories and extended activities connected with and accelerated children's math learning. The fiction narratives developed foundational aptitudes and knowledge in the math big ideas, which are embedded in the two storybooks. Knowing and applying the ideas in context helped the students become successful math learners.

Subitizing Before Counting

Studies on the development of children's mathematical abilities, from as early as 1912, reveal that children as young as 6 months old can recognize basic math concepts through the natural development of their brains. Kaufman et al. (1949) called this skill *subitizing*, a word derived from the Latin term *subitus*, which means "suddenly" (p.15). Unless hindered by a disability, children can place a numerical idea of 1:2 ratio starting at 6 months old, and by 9 months old, they can distinguish sets of 2:3 ratio of the items. Clements (1999) describes subitizing as "instantly seeing how many" (p.14); i.e., children can see quantities in a small group of objects, without counting.

Teachers and parents aware of the perceptual subitizing abilities of young children, can nurture and build upon this innate mental ability to learn addition through what is dubbed as conceptual subitizing, that is, learning to combine perceptually subitized numbers to a bigger number without having to count each item in groups or sets. Starkey and McCandliss (2014) explain that by the time they are in third grade, students who know conceptual subitizing can use more complex and comprehensive computational strategies compared to student peers that did not learn to conceptually subitize. "Subitizing is too often a neglected quantifier in educational practice, it has been extensively studied as a critical cognitive process." (Clements et al., 2014).

MacDonald (2015) expounds that there are six types of subitizing. Knowing the nuances within both perceptual and conceptual subitizing helps educators and parents of young learners to fully understand the mathematical levels of their students. Specifically, when parents and educators are able to identify the scaffolding, they can help prompt and navigate children's mathematical growth. In an instructional module, Clements (1999) demonstrate how children can develop understanding of mathematical concepts, such as unitizing or counting on, cardinality, by knowing the number of units in a set, and developing more sophisticated arithmetic faculties and making deeper sense of the relational nature of numbers, for example, 9 is one more than 8. These numeracy skills are vital foundational math abilities.

Recognizing and Making Comparisons

In a study to show children's capabilities to engage in analogical thinking, Ferry et al. (2015) shared their findings that 7-month-old infants comprehend sameness and differences between two objects. In the study, the infants were trained with six to nine examples, after which infants were able to tell if two objects were alike or different. Results of this study indicate that it is not a requisite for children to develop language skills to learn conceptual relations among objects.

Spatial Sensibility: Sense-Making and Reasoning

Days after infants are born, they begin to know the world by seeing different shapes, spaces, sizes, and more. By 18 months, except for those with visual impairments, babies' object perceptions are mostly developed. Babies can generally learn the names of and identify shapes as well as identify sameness attributes by the ages of 2-3 years (Ginsburg & Oppenzato, n.d.). Tapping into young children's spatial sensibility is an opportunity to develop abilities in mathematical sense making and reasoning. Punkoney (2021) lists key skills children can learn from shapes: construct, compare, describe spatial relationships, compose, and decompose. The big idea that "shapes make shapes" is a concept that cuts across math to every field for both people-made and natural objects and spaces.

Learning mathematical thinking through triangles and shapes provided access to spatial math understanding for the participants of the study. Some studies say the young children's spatial skills predict mathematics achievement (Young et al., 2018). Clements and Sarama (2009) provide learning trajectories for shapes based on developmental levels. Learning progresses as children can visualize locations, predict the results of movements, and interpret models and drawings, leading to the construction and understanding of maps and diagrams later.

Big Ideas in Math and Cognitively Guided Learning Trajectories

The process of creating the math stories, the accompanying activity book, the extended learning activities, and design projects in this study, is an integrative application of designing for learning trajectories.

Learning trajectories are descriptions of the tapestries of children's thinking and learning in a specific mathematical domain. These can also be multiple fields that are embedded and integrated in a conjectured direction of a lesson with a set of instructions designed to generate thinking, decisions, or actions. Curricular design with integrative learning trajectories is envisioned to engage children in a dynamic process of growing their thinking abilities within the context of mathematical domain goals. (Clements 2002; Gravemeijer, 1999; Simon, 1995)

The hypothetical learning trajectory in this case is based on the heuristic at engaging children in solving both structured and unstructured math problems. Structured problems generate opportunities for interactive explorations of the math concept(s) with different combinations of numbers or shapes. The latter problems are designed as open-ended and do not have clear right or wrong answers. Likened to a sandbox, open-ended questions are thinking prompts that surface students' authentic thoughts and logic in relation to the big mathematical ideas presented in the story. With an open-ended trajectory, students are expected to figure out how to frame the problem, formulate a mathematical statement, and possibly come up with an equation or whatever answer or solution that comes to mind with individual students and the larger group. The stories and questions are likened to the sandbox that, on the one hand, allows students freedom and choice to shape the results, yet, on the other hand, the elements and mathematical ideas in the math stories and activity book influence the degree and extent of their mathematical thinking and explorations. The hypothetical trajectories in this study also included reframing math learning from procedural learning activities where students are expected to answer equations and math problems correctly, to conceptual math, which focuses on mathematical meaning making.

Conceptual Math

Conceptual math is math learning that explains the reasons behind how operations work and mathematical ideas. In contrast, *procedural math* is teaching by practicing the steps to solve a problem. Using the computer as an analogy, conceptual math is likened to knowing the science behind the software and hardware engineering that makes a computer work. Procedural math is knowing how to use hardware and software components of a computer. During the pilot, students mainly engaged in conceptual math learning. Likewise, the process of computing solutions engaged students in exploring equations, shapes, and counting with small to bigger number combinations.

Van de Walle (2013) define conceptual understanding as an adaptable web of connections and relationships of ideas within domains and subdomains, applications, and multimodal representations of mathematical concepts. Conceptual understanding creates an interrelated conception of big ideas in math across domains and contexts.
Demonstrating competence requires the ability to translate a math idea into different representations. Studies have shown that students unable to interpret the same idea in different modes and contexts also struggle with problem solving and working on calculations (Lesh et al., 2003).

Conceptual understanding engages students in the process of generalization. *Generalization* is recognizing the big idea in specific problems and contexts and the specific problems and contexts in a big idea. Conceptual understanding is present as a key goal of learning and performing in different fields. Specific to the educator is to be able to oversee a lesson or learning process in relation to its learning goals (Dumitrascu, 2017).
Branford (1908) reminds teachers of the tendency to take away the practice of generalization from students, by defining the math ideas for the students.

> Beware, my fellow teacher, lest you unconsciously and incautiously supply the children with a generalization, which they have not as yet themselves perceived and reached . . . such a blunder I find myself repeatedly committing, so easy a trap is it to fall into. (p. 43)

To be able to see the general in the particular and vice versa is one of the key tenets of mathematical thinking. The purpose of mathematical proofs is to generalize, that is, to give a sense of order and relation to its domain and other domains. The structure of learning a concept necessitates allowing students to observe and to analyze patterns and cause and effect; whatever process they shape toward a solution, students need to be able to relate to other contexts and explore variables that may impact their results. Polya (1957) explains that, as teachers, “we need to adopt the inductive attitude which requires a ready ascent from observations to generalizations, and a ready descent from the highest generalizations to the most concrete observations” (p. 7).

To *generalize* means students should be able to reason and make sense of problems and situations. Herein is the crux of why rote memorization fails to teach real math. Ferlazzo and Sypnieski (2022) remark on how students typically do not understand when they simply memorize algorithms. Lack of understanding also reflects on the general pattern of American students' responses in PISA, TIMSS, and NAEP examinations. In an analysis of 20 years of TIMSS results, Johnston (2020) explains that a good majority of American students' responses reflect a lack of conceptual understanding starting from the early grades. Higher-level concepts, including algebraic thinking, must be introduced in the lower grades. Math curriculum and instruction in the United States are not as high level as high-achieving countries; this, considering U.S. students do more homework than their international counterparts. Furthermore, in high-achieving countries, math curricula are designed for students to engage in conceptual problem solving, which takes up an average of 50% of class time. In the United States, more than 95% of the lessons revert to teachers showing the solutions. The focus is on mimicking what the teacher does. Unfortunately, the NCTM standards and expectations are implemented in international high-achieving countries rather than in its home country.

Context Connects Math to Everyday Life and Work

In a 1993 article, Boaler explains that the use of contexts will develop the abilities of students to transfer level math applications. However, based on studies that she cites, findings show that contexts unfamiliar with students do not do much for transfer.

Boaler (1993; p.13) explored opposing views on the use of context for conceptual math learning. The conversation on math in context is a reaction to views of teaching devoid of context to highlight the abstract nature of the field. Walkerdine (1989), explains how he used context in a situation in class when he asked Palestinian students why there were higher levels of absentees on Saturdays. He used math to develop greater understanding of the people and culture of the community.

Contexts are considered as able to motivate students to engage in math learning and uplift their mathematical understanding to an abstract level. Kilpatrick (1988) explains that in a constructivist framework, not every context offers a common connection for students. He asserts that activities in context may commence rich mathematical discussions but should be open enough for students to follow their own ways. The value of context according to Boaler (1993), teaches math in ways "that there can be more than one answer, that mathematics can involve more than one discussion, negotiation, interpretation as well as encourage students to discover, explore, negotiate, discuss, and understand in a process view of mathematics education" (p. 15). Boaler (1993) concludes that contexts can inspire, motivate, engage interests, and combat underachievement and disconnection from math as cold and detached in its abstract form. "Context makes mathematics more meaningful to the individual" (Boaler, 1993, p.15).

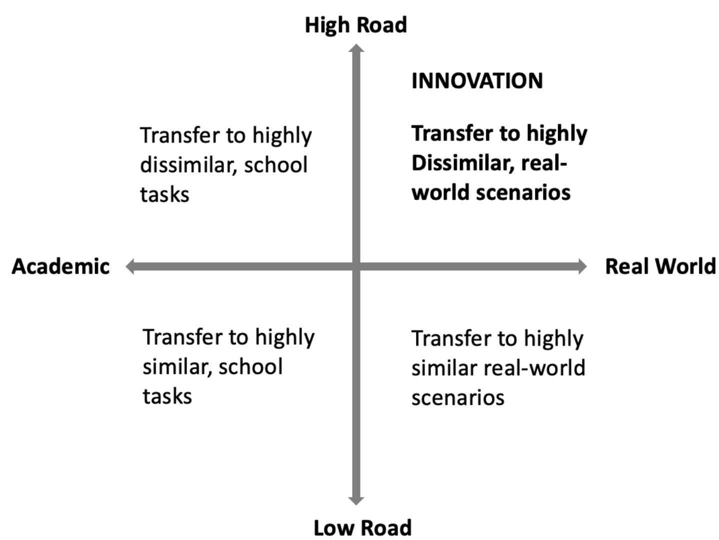
Transfer is the Ultimate Goal of Generalizing

"Mathematics is the science of pattern and order" (Van de Walle, 2007, p.13). To really do the math requires the following actions, which are expressed with scientific verbs that mean *figuring out* and *making sense*: "explore, conjecture, solve, justify, represent, formulate, discover, investigate, construct, verify, explain, predict, develop, describe, and use" (Van de Walle, 2007, p.13). Consistent with numerous studies, effective math learning requires students construct mathematical ideas and make sense of situations and problems. They need to tap into their prior knowledge and make personal sense of their observations, logic, and solutions. "In order to learn skills so that they are remembered, can be applied when they are needed, and can be adjusted to solve new problems, they must be learned with understanding (they need to make sense)" (Hiebert et al. 1997, p.6).

> Generalization can be seen as a statement that is true for a whole category of objects; it can be understood as the process through which we obtain a general statement; or it can be the way to transfer knowledge from one setting to a different one. (Dumitrascu, 2017, p.47)

Mathematics is a field with big ideas that cut through its multiple domains. According to Brown and Campione (1984) the term *transfer* is used in many ways but is considered to be a process where students leap across domains of knowledge (p.144). For teachers aware of the level of learning involved in transfer, there is a general understanding of its value, that is, for students to optimize conceptual understanding at higher levels of applications (Bransford & Schwartz, 1999). Stern et al. (2016) describes different types and levels of transfer in Figure 10 below. Learning design can address each of these levels with intention realized through active learning.

Figure 10. Real-World High-Road Transfer From Stern et al. (2016)



Efficacy of Math Stories as Math Learning Pedagogy

Real-World Applications in Fiction

It is common to hear parents share about a book they read to their child as part of their before-sleep ritual. Monroe and Young, (2018; pp. 2–3), explain that children's literature for math learning is a relatively new practice. They list three criteria for teachers when teaching mathematics with children's books:

1. Use trade books versus textbooks and reference books,
2. Reading level is less of a concern for as long as the mathematics is accurate and the content is developmentally appropriate, and
3. Books can have explicit, implicit, and invisible mathematics content.

Monroe and Young, (2018) affirm how much teachers of young children agree without doubt that constructing mathematical understanding through concrete experiences is when children learn math best. Teachers are cautioned that when reading picture books for math learning, they will have to factor for time. Otherwise, math learning will not go beyond a procedural-oriented pedagogy. Whitin (1992), worries that most children perceive mathematics to be a set of rules and facts to be memorized. Dubbed as math-related children's literature, children's books provide opportunities that make visible ways people use math in multiple situations.

"Books appearing to be mathematics practice pages sandwiched between two colorful covers are really not 'literature' at all" (Monroe & Young, 2018). Eula Ewing Monroe and Olivia Haworth Dial, with Stevie Carr and Bree De La Mare, conducted and published a very recent study on teachers using picture books to teach mathematics. Most of the studies focused on children in Grade 3 and younger (p.16). For their research, they chose studies that were mathematics content explicit and tallied the results accordingly:

- Increases mathematics achievement
- Builds interest in and a positive attitude toward mathematics
- Engages students in mathematical discourse
- Deepens students' mathematical understanding with the literary narrative
- Cultivates critical-thinking abilities

The authors claim they found no negative feedback on the use of children's literature for math teaching, though there were two concerns articulated by the teachers: reading takes up too much of their math time and the need for a list of children's books that have worthwhile mathematical tasks.

Mink and Fraser (2005) illuminate how the new standards created by the NCTM, migrating from rote learning to "apply, adopt, and extend" math knowledge, opens doors for including picture books as resources for math learning. Furner et al. (2018) state how children's literature addresses issues of inequity in the math classroom. Students have a better appreciation of math when it relates to everyday life. Children's literature "provides a context through which mathematical concepts, patterns, problem solving, and real-world contexts may be explored" (Moyer, 2000). Marilyn Burns, founder of Math Solutions, stipulates evidence that

> Picture books can spark children's imaginations [in ways] that textbooks or workbooks don't. Connecting math to literature can boost confidence for children who love books but are wary of math. And students who already love math can learn to appreciate stories in an entirely new way (2015).

While most studies on using children's literature to teach math have generated positive feedback, teachers continue to struggle with transforming math knowledge and pedagogical theories to constructivist learning experiences where activities are designed with thoughtful attention to engaging the complex nature of young people. Tomlinson & Erickson (2007) explains how shaping curricula, "is a complex intellectual task" (p. 2). Applying constructivist pedagogical principles into curricula requires mastery of key concepts as practiced and recognized in professional fields while making sure students' perspectives are employed in active and dynamic construction of knowledge.

Intersections of Literature and Mathematics Learning

From every class I taught, one thing that calms children and at the same time catches their attention is when being read stories. Children like being read to and particularly enjoy stories read repeatedly. Children learn from pictures, words, and their interactions with adults while reading (Duursma et al., 2018).

Children in Grades K–3 are learning how to read, but students in Grades 4 and above read to learn. Building reading skills typically commences in kindergarten and first grade. In these grade levels, children learn about word, phrase, and sentence formations in print media as they also practice how to shape text with handwriting. Concurrently, young learners learn how to deduce ideas, feelings, and meanings from stories and poems that are read to them. Recognizing the value of interdisciplinary teaching curricula promoted by the State of California, for example, encourages teachers to "support literacy across content areas and interdisciplinary instruction by focusing on the relationships and convergences of mathematics, science, and English language arts practices" (Krizo & Calinsky, 2020).

The NCTM Connections Standard (2000) also encourages teachers to intersect mathematics with other subjects, so students see how each subject weaves upon the other, effectively learning from a multidimensional frame of understanding and practice. However, Tomlinson & Erickson (2007), explains that designing "quality curricula is a complex intellectual task" (p. 2). Developing curricula is time-consuming, requiring understanding of conceptual learning and teaching concepts, curriculum design, and expertise in a field, or in multiple fields for multiple subject teaching. Curriculum designers must be knowledgeable in key concepts and principles and in major practices and skills in a discipline (p. 2). Based on research, students should learn actively and be engaged; it is not enough to develop a lesson activity with verbs such as "list, define, and identify."

Math Picture Books Intersect Mathematics and Language Arts

Here is an example of a typical first-grade math word problem found on a worksheet: "5 fish swim. 2 more fish swim. How many fish swim in all?" Another sample problem: "My uncle Bill has 10 kids. My uncles Hall and Tom have 5 kids each. How many cousins do I have in all?" (Keefer, 2021). Most worksheets found online have 10 of these types of problems. In these word problems, there are no stories or characters that keep students curious and engaged.

The nature of word problems brings to light the challenges faced by students in mathematics classes and in solving mathematics work sheets. Word problems include text, numbers, and mathematical symbols. The context and characters for every problem are different. To solve correctly, students must know how to read, identify clues, understand text, and do the math. Students then must translate these word problems into mathematic sentences and get the correct answers. These word problems reflect rote learning strategies and, based on international examinations, minimize enduring and high-level mathematics learning. Analyzing international tests results and test takers, Boaler et al. (2016), observed that those who learned through memorizing received the lowest scores. In fact, these students were a half year behind students that used relational strategies. She correlates America's low scores with the continuance of teaching math procedurally, i.e., having students memorize steps in calculating for solutions.

Despite the calls of state education departments and the NCTM to engage students in sense making and mathematical thinking, even teacher-designed lesson plans such as the example above, as I observed in multiple teacher-authored lesson websites, continue to be designed with rote learning.

The intersections of math and literature provide a new way of learning math while extending students' understanding of English terminologies. Literary elements can make math more interesting and dynamically presented. Keat and Wilburne (2009) conducted a study to analyze the influence of storybooks on achieving math-learning objectives and positive math-learning strategies. They found the children to be "enthusiastically interested and actively engaged" (p. 66) in figuring out the reasons and implications of the characters, plot, and contexts. The connections made with the elements of the story sustained their interest in the related mathematics activities for a significant period. Also, the story context provided children with motivation to persist amid learning difficult math concepts as they productively struggled for understanding.

Intersections of Fields Create new Ideas and new Fields

In his book titled *The Medici Effect*, Johansson (2004) reasons how the intersection of industries, cultures, and disciplines result in game changing innovations. Johansson defined the word *field* as a "discipline, requiring specialization" (p.40). There are differences between single field ideas, resulting in what is called *directional ideas*, and intersectional ideas, which combine concepts of multiple fields (p.40). Directional ideas use old ideas and refine and enhance so that the results are predictable. Contrary, intersectional innovations are "surprising and fascinating, take leaps in new directions, open up entirely new fields, and can affect the world in unprecedented ways" (p.43)

The intersections framework is not the way schools are traditionally structured, i.e., silos. Subjects are taught from subject-focused textbooks and discreetly scheduled, defined by blocks of time, from 30 to 90 minutes to a session. Students close their books and notebooks for the previous subject then open their books and notebooks for the next subject. This is the standard, hundreds-of-years-old structure of kindergarten to doctoral degree educational programs.

In the context of this exploratory study, mathematics and literary studies have typically been taught as separate subjects and in separate class periods. This exploratory study promotes a framework of intersectionality, predicated on a proposition that the intersection of two or more fields results in increased engagement in mathematical sense making. Adams et al. (2007) illuminate on how complex mathematical languages are embedded in the fiction book *Moira's Birthday*. Children developed understanding of the vocabulary in the book by engaging in mathematical sense making. Likewise, in a study on learning multiplication and division involving an experimental and control group of third graders, Morgan (2007) observed how much better the experimental group that learned through fiction stories performed in the posttests. Employing mixed methods, Monroe and Young (2018) observed an elevated level of students' "mathematical and literary justification and reasoning, understanding, connections, and vocabulary" (p.21). These studies argue how intersectionality builds upon each field toward enriching results for both fields.

Intersecting Creativity, Culture, Critical, and Developmental Theories

As creator of the picture books, I constantly returned to these questions. What are the considerations for reinventing math learning that fosters conceptual, creative, meaningful, and context-based pedagogy? How can math stories cultivate a more engaging and connected community?

Application of Educational Theories in the Books

How We Teach is What Students Learn About Mathematics Education

This study builds upon a growing number of research findings and examples on the best way children learn mathematics. The findings categorically explain how the pedagogies of rote learning have turned off a great majority of children, adults, and, more importantly, teachers of primarily school children from math learning. By shutting minds off math learning, rote learning continues to significantly contribute to the achievement gap.

Conclusion

The historical challenge of the achievement gap continues to be an unresolved issue plaguing American public schools. On one hand, teachers have been accused of not knowing how to teach math, and parents of low-income and children of color have been blamed for not fostering a culture of appreciation for math learning. On the other hand, this crisis puts into question the inequity of America's public educational system, which was established as a program to make possible social mobility. It reflects a deficient pedagogy promoted by textbooks, which are required by states and districts, as primary resources for math curricula. Additionally, without appropriate training on concept-based math teaching, teachers will only slide back to methods that are familiar to them—rote learning.

Students Do not Learn Math with Rote and Memorization

Children's traditional math learning experiences influence their negative sense of self as math learners as well as their perceptions of math. With rote learning, math is perceived to be difficult and detached from students' interests, culture, and daily lives. Studies on transfer find that children are unable to connect and apply the math they learn in school to other contexts. In their minds, from their math learning experiences in school, what matters in math is to solve for the one correct answer in worksheets and examinations. Rote learning diminishes students' exposure to the nature of mathematical thinking, which is meaning making and reasoning. Standardized tests generate stress, frustration, and fear of failing, thereby depleting students' self-confidence and positive mindsets about their abilities to learn math.

Children are Multifaceted Humans

The constructivist philosophy and the learning sciences employ a more accurate, holistic, and humanistic way of understanding of how children learn. Math education is not just about the math but rather about the learners and the experience of learning, which are determinant considerations for achieving grade-level proficiency, interest, and confidence in one's ability to do the math. Studies from this humanist field illuminate the complex make-up of children as learners and how pedagogy affects the way children learn and perceive math. Additionally, interdisciplinary studies have inspired landmark publications initiated by the National Research Council and supported by the NCTM such as *How People Learn I & II* and *It All Adds Up*, which influence the framework for the new Common Core in math and the campaign to explain to teachers how to migrate from rote learning to mathematical sense making and reasoning as one of the key elements of the NCTM's *Mathematical Practices*.

Student Ideas and Community Valued in Mathematical Meaning Making

Mathematical sense making and reasoning shifts the power of mathematical thinking from textbooks, teachers, and one correct answer to more than one way to solve problems and, more importantly, to acknowledging that children's ideas matter in the process of figuring out how to do the math. With this framework, students' ideas are wellsprings that make-up a community of active math learners. Math learning happens through peer-to-peer learning in the form of conversations, project-based activities, exchanging ideas while problem-solving, and formulating inventive math concepts.

Generalizing is Mathematical Thinking

Generalizing from patterns is a core process of thinking mathematically. To develop mathematical mindsets, children should be exposed to and engage in generalizing from patterns from their observations, so they practice conceptual understanding at higher levels of applications across math problems and domains. Moreover, transfer should extend to multiple subject domains, which can provide a plethora of contexts to make math visible when applied as a tool toward analytical understanding and problem solving.

CRT Highlights Embedded Whiteness in the Educational System

CRT provides another layer of analysis for highlighting authenticity in multicultural classrooms. The cultural reference lens builds upon students' ideas and emotions, which reflect the individual and the culture that shapes the perceptions of oneself and the world. CRT provides a nuanced lens in critical research theory that is relevant to understanding and transforming an educational system historically defined by the culture of the White, western world. It is only possible to realize CRT ideals with a pedagogy that includes students' prior knowledge and that leverages their cultural capital. In the case of math learning, CRT sharpens our understanding of how the *culture of Whiteness* pervades in the pedagogies, language use, and contexts used in math learning activities. CRT highlights the debilitating effects of embedded racism on students of color.

Research Questions

- How can children's emotional, physical, social, and cognitive responses to math stories inform us of the efficacy of this pedagogy?
- How do students' responses to the math stories and activity book reflect perceptions, motivations, and sense making of math concepts?
- How do the math stories and activities support teachers and parents in guiding students toward mathematical sense making?

III. Methods and Procedures

Introduction

In this exploratory study, intentional creative applications of theories on how children learn best are envisioned to inspire, engage, and enjoy math learning. The learning sciences, constructivism, critical race theories, and NCTM guidelines on mathematical practices intersect in two math picture books and extension learning resources, which will be the materials reviewed and explored. I developed the creative math learning resources.
The study will use the format of a video documentary in a mixed methods framework, which includes design and qualitative research. This chapter will discuss the research methodology for this study.

There are three general phases of the study: (1) Teachers conducting while I documented the pilot study with eight students through video technology, (2) Video interviews of teachers, educator-specialists, and leaders (3) Making sense of and communicating the findings by choosing and organizing the video clips and creating a narrative in a documentary format.

Statement of the Purpose

This exploratory study's purpose is to create and test an alternative study that has the potential to engage children in mathematics learning, inspired by pedagogical theories rooted in how a child learns best. This is predicated on the design of learning experiences that can provide optimal learning for children. Specifically, the study explores children's responses to a set of math-driven, fiction picture books designed to ignite children's curiosity and engage them in context-based mathematical sense making. The causality of how students actively participate in learning in general, and in math in particular, is key to the development of mathematical thinking abilities as well as developing children's perceptions on how math affects everyday life and work and what children get out of math learning.

Research Questions

- How can children's emotional, physical, social, and cognitive responses to math stories inform us of the efficacy of this pedagogy?
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- How do the math stories and activities support teachers and parents in guiding students toward mathematical sense making?

Research Design

The Math Stories

Two storybooks, one activity book, and various math learning games and activities were used in the pilot study. These learning resources were written by me and illustrated by Jessica Liou. The math story books offer young learners and teachers a pedagogy away from rote and mechanical memorization of definitions and algorithms.

These math stories invite children to become active creators of their knowledge. They are designed to develop young children's interests and abilities in concept-driven mathematics. Creative narratives and illustrations tap into young learners' natural aptitudes in numeracy, sense making, modeling, and inventing.

The stories provide prompts and explorations in conceptual and logical reasoning abilities through open-ended and context-based problem solving. Children learn by empathizing with the characters' challenges. Math is associated with helping the characters who are in contexts that are fictional applications of real-world problems. Through a process of figuring out, communicating, inventing, and testing solutions, children learn how to do math by modeling the processes of professional mathematicians. Open-ended math challenges engage children in discovery and exploration of math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear engagement of solving for the correct answer. The expected responses to the story books are learning experiences that are multidimensional in process, exciting because students' experience how much their views matter, and meaningful because problems involve helping characters in the story.

Findings in brain research and publications of studies on how people learn underscore the importance of children connecting to what they are learning. For enduring understanding, background knowledge must interact with new knowledge when children practice and explain in a complex presentation (Caine & Caine, 1997; Perkins 1992). Tomlinson & Erickson (2007) clarifies that interpreting standards into engaging curricula is not easy to achieve: "The arrangement of words on paper to effect instructional improvement in the classroom is a sophisticated task" (p. 4). Most teachers use resources that are prepared and published. These books are created to act as supplemental curricula for conceptual, context-based, and mathematical sense making that build upon children's background knowledge and abilities.

Math Extension Learning Games and Design Activities

A key purpose of the stories is for children to learn as they construct their own understanding of mathematical concepts. Math activities in the forms of math games and design projects related to the story support student learning, which is intended to connect what they know or their prior knowledge to new math ideas. The goal of these activities is for children to achieve enduring understanding and develop confidence in mathematical sensemaking and reasoning themes discussed in the stories.

Each of the books focuses on a set of big mathematical ideas. Math big ideas are crosscutting concepts that are guideposts to mathematical sensemaking and understanding. Four characters lead in the learning of math ideas in each of the stories (Figure 11). Multicultural specificities come into play in the overall aesthetics of the books and workbook. The four major characters are named Amyel, Misha, Bikoy, and Iliana; their names represent hybrids of different cultures. There are no clear ethnic boundaries and the characters' features also reflect mixed cultures, which include gradients of skin tones while maintaining an urban-like and quirky aesthetic, as reflected in their hairstyles and clothes.

Figure 11. Four Children who Lead Math Learning in the Stories

Every storybook has a page that invites the reader to draw themselves as a character and to add their favorite things to do (Figure 12). This page is intentional as it disrupts the concept of readers as mere spectators of picture storybooks. Instead, the reader gets to take part in the narrative as one of the characters tasked to help the people of the town of Whatever with solving their problems through the application (and invention) of mathematical ideas.

Figure 12. Join the Team of Characters

In the opening page of every book, the mathematical concepts embedded in the story are presented. An explicit statement on the first page for those who read to young children, including teachers, parents, and older children, explains that children's reading levels should not be considered a hindrance to learning math:

> Provide guidance by reading the text. Encourage children to figure out solutions and make mathematical sense of tasks and problems. Build their confidence by tapping into what they already know. Children ages 5 to 7 years are not expected to be able to read all the text to do the math. (Opening page)

There are two stories involved in the exploratory study, and each story weaves a different but interconnected set of mathematical ideas. These are outlined in the opening page of every book. In this way, an adult or older child guiding young learners is made aware of the key mathematical concepts that are built into the narrative.

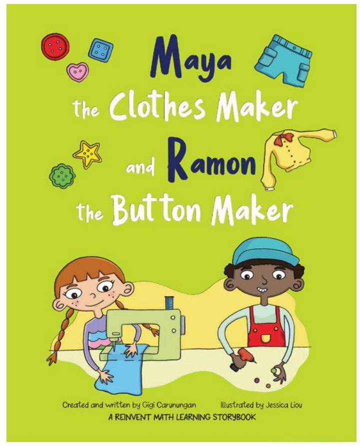
Maya the Clothes Maker and Ramon the Button Maker

- Recognize relationships between numbers by exploring different ways a large number can be formed from smaller numbers. Visualize math ideas with objects, =, +, >, and < symbols.
- Engage in deductive reasoning and logical thinking through real-world problem solving.
- Arrange objects in sets and assign the equivalent numerical symbols.
- Practice algebraic thinking by adding to, putting together, and using the equal sign to represent a numeric answer.

Summary of the Story

Maya and her friend Ramon find ways to make her uniquely designed clothing for a very tall customer. They and their MathXplorer friends come up with special ideas and invite young readers to share in their challenge while exploring patterns and possibilities. Figure 13 shows the characters Maya and Ramon.

Figure 13. Book Cover Maya the Clothes Maker and Ramon the Button Maker



Math presented in this book includes Common Core standards such as sets, groups, patterns, and comparisons. Children learn these concepts as they help Maya figure out how to design her clothes. MathPack activities include helping Maya sort her buttons, a card game of pattern recognition, and a design activity. Open-ended questions invite children to provide their own observations and solutions throughout the story.

Manuel, the Table Maker

- Analyze a variety of two-dimensional shapes that are bound by a finite set of line segments. Compare and/or contrast elements of simple polygons.
- Explore the concept of *shapes make shapes* through creative mosaic activities.
- Discover why triangles are the simplest polygon. Familiarize with three types of triangles and differences in the length of their sides.
- Identify different polygons by the length, direction, and the number of their sides.

Summary of the Story

Manuel and his nephew Amyel help a family design and build their new table. Working with triangles, the family learns how to recognize the different shapes that they will use for their mosaic and how they fit together to make something unique and wonderful. Figure 14 shows the character Manuel.

Figure 14. Book Cover:
Manuel the Table Maker

Math presented in this book includes Common Core standards such as shape recognition and comparison, identification of object features, how polygons are formed of other polygons, and how smaller numbers make bigger numbers.

Children join the MathXplorers team in Manuel's table studio in design activities such as making polygons, creating mosaic patterns, and learning how shapes combine to make other shapes. As the story progresses, they take their knowledge from the studio and learn about the importance of shapes and combinations in an encounter with bridge construction and design.

The Town of Whatever: Visualizing Culturally Responsive Teaching

Visual elements of the books and workbook expound on the narrative and the mathematical ideas of each story and activities. Tunnell et al. (2016) describes the role of illustration as independent on its own and, at the same time, woven along a narrative. As such, "illustrations take on dual positions as simultaneous and sequential" (p.38). The images of the books are designed to accentuate the text and the mathematical ideas (Figures 15–17).

Figure 15. Sample Page of Book 1

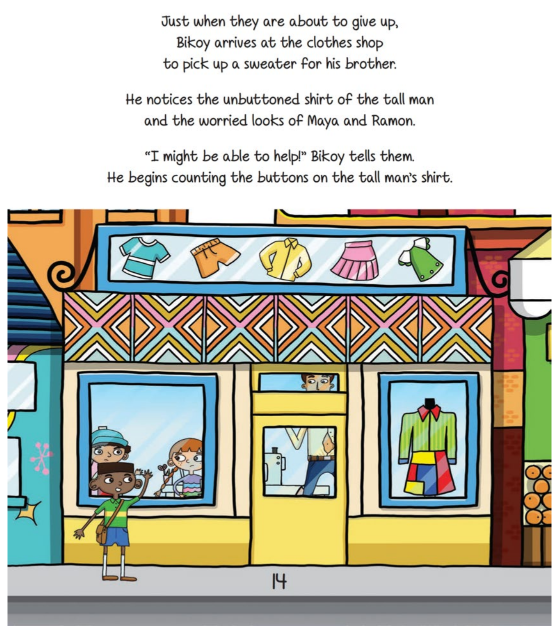


Figure 16. Sample Page of Book 1

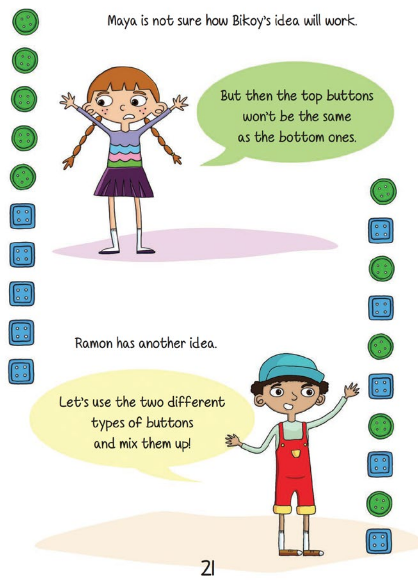


Figure 17. Sample Page of Book 2

Research Methods: Documentary

This exploratory study used the video documentary research method. A video camera recorded the pilot teaching and learning experiences in an after-school program setting in video and audio formats. The iPhone Camera was used to document and collect video footage of the reactions of the children and teachers while learning and teaching with the math stories. The dissertation's final form is presented in a video narrative. Photo and video footage are part of data collection, analysis, and distillation of educational theories. Students' emotional responses, attitudes, motivation to do the work, and feelings about the teacher and the tasks are all part of their learning experiences and are recorded in the study. The teachers' emotions, dilemmas, instructions, guidance, and the actions and words used in the classroom are documented. Through the lens of a video camera, the measure of the program's impact is visible through visual representations of students and teachers in class.

In the context of a design research review, Brown (1992) underscored the importance of using video (tapes). She sees the advantage that this media offers in innovating solutions for the classrooms both for archiving and for analyzing phenomena with different theoretical lenses (p.174). Schoenfeld (2004) contends that videos can be used as powerful sources of learning theories from which coexistent models can evolve.

Demonstrating Innovative Math Learning in a Research-Based Documentary

It is critical to note that to share video research results requires a genre that translates data and analysis that the public can connect with—a story (Walker & Boyer, 2018). Children like stories. Adults like stories too, even more so if it is presented in video documentary format. However, what differentiates a commercial from a research documentary are the ethics of truth in communicating and evaluating data, most especially because education deals with human subjects. The purpose and end-product determines how data is gathered and organized into a finished form.

Video in Mixed Methods Research

Video clips provide real-world images, voices, and sounds of human experiences. These are like powerful windows to see beyond or mirror our everyday experiences (Goldman & McDermott, 2007). Another critical consideration in the production of a research-based documentary is how the researcher converts video clips, as records of observations, to data sets, trends, and findings (Newbury, 2011; Knoblauch & Tuma, 2011). Walker & Boyer (2018). explains how video research provides a rich source of raw data:

> Reflections of the experience, in the direct words of participants, that contain insights provided by body language and tone, an immersive glimpse into the research world as it unfolds, and the potential to capture footage throughout the entire research process rather than just during prescribed times. (p. 10)

I used camera techniques employed in documentary films with a hybrid of qualitative and participatory research frameworks. Data gathering through video required taking more than enough footage to achieve what Kridel (2021) describes as "representativeness, or whether the material represents a collection of produced materials rather than an idiosyncratic portrayal." With the iPhone camera, I captured live reactions from students and teachers with the books and workbook activities, as they were reading, doing the math, and learning during the pilot, with one shot of a child with her mother at home. Recording included every type of reaction, both positive and negative, as the most truthful way to determine the efficacy of the materials. Different camera fields and angles highlighted multiple responses within learning contexts. Group work, for example, required a full and a medium shot to capture the dynamics of the team. Individual work was shown with close-up shots of a student's hands working on a math project. The camera was able to record multiple ways children engaged (or not) in solving problems, for example, emotional reactions such as excitement or boredom, confusion or clarity, motivated or apathetic, and more. The camera also recorded how the teachers read the books, interacted with students, and other things the teachers did in the classroom.

Interviews on camera provided articulated feedback and insights from students during the sessions, from the teachers at the culmination of the pilot, and from educators and specialists in the fields of curricula, math, and literature.

Preparations for the Pilot

The COVID pandemic affected the implementation of the study. I contacted several teachers and school principals and directors of after-school programs and received few replies to emails and phone calls. It was in late fall when a school leader agreed to host the pilot study. I delivered the math picture books to the school, and a week later, a meeting was held with two teachers via online video conference to discuss expectations and preparation for the pilot study.

Since the picture books were bundled with the activities, I recommended that the teachers interpret the stories and activities as they would other trade books. In this way, I did not tell the teachers how to teach with the resource materials, instead allowing the teachers to interpret the materials in their own ways.

The teachers agreed with the proposal. The pilot was scheduled. Each teacher prepared to lead one of the books. The teachers notified the parents who gave written consent. Written consent was also obtained from the school principal. (See Appendix A for the consent forms).

Population and Sample

Video footage was taken at a K–5 school in Silicon Valley with eight first-grade students at an after-school program. The demographics of this school provide perspective on the participating children. In terms of the opportunity gap, the children of this school fall into the lower end of the spectrum of math learning readiness among early learners. During the 2018–2019 school year, reading and math proficiency levels were in the bottom 50%. The percentage of students with grade-level math proficiency was 15%–19%. This is lower than the California state average of 40% for the 2018–2019 school year. Minority enrollment at this school is 94%, which is higher than the California average of 77%. Sixty-nine percent of the population is eligible for free lunch and 7% for reduced lunch.

Selection Criteria for the Sample

All parents were sent invitations for their child(ren) to participate in the after-school program. Included with the invitation from the teachers was the Institutional Review Board consent form. Parents self-selected their child(ren) to participate in the program.

Data Collection Procedures

The exploratory pilot: Where the rubber meets the road.

The exploratory pilot study was held in the context of an after-school learning program with two 3-day, 1-hour sessions with the same first-grade level children. Given the safety rules of COVID, the teachers and the children needed to wear masks during the sessions. The pilot tested the efficacy of two books, an activity book, and extension learning activities as part of an after-school learning program.

Students' responses in class and the teachers' assessment of students' responses in class activities were recorded to serve as evidence of the efficacy of the resource materials. Students' responses to the activities would also indicate their growth (or not) in sense making and mathematical reasoning as well as feedback from the teachers, based on how students responded to the activities on the efficacy of the materials in terms of deepening students' mathematical thinking skills.

Data gathering involved me being in the classroom primarily as an observer, while video recording the learning experiences from beginning to end and avoiding actions that would disrupt the class. Students were informed at the start about the study and the reason I was documenting the activities with an iPhone camera. It was only at the end of the program when I asked one or two questions regarding how students felt about the stories and learning activities.

All the video that is captured throughout a study is data and potentially part of the final research narrative (Walker & Boyer, 2018, p. 9). The researcher video recorded every class session from beginning to end, hoping to capture every reaction and action of the teachers and students. Video footage is composed of emotional responses, attitudes, motivation to do the work, and feelings about the teacher and the tasks. Video footage was then categorized according to emergent themes that demonstrated (or not) the learning process, levels of learning, social interactions, responses to the stories and the extension learning activities, comments, teachers giving instructions, students' behaviors while teachers were giving instructions and reading the stories, and throughout the entire session.

At the end of each day, the researcher took notes of key themes that were observed in class as a method of record keeping (Corbin & Strauss, 2008). The memos supported the researcher's aim to organize the clips thematically as the video documentary narrative structure.

Teacher Interviews After the Sessions

Both teachers requested that their interviews be conducted after each of their corresponding 3-day sessions. These were done through Zoom video technology.

Specialists and Educational Leaders Interviews

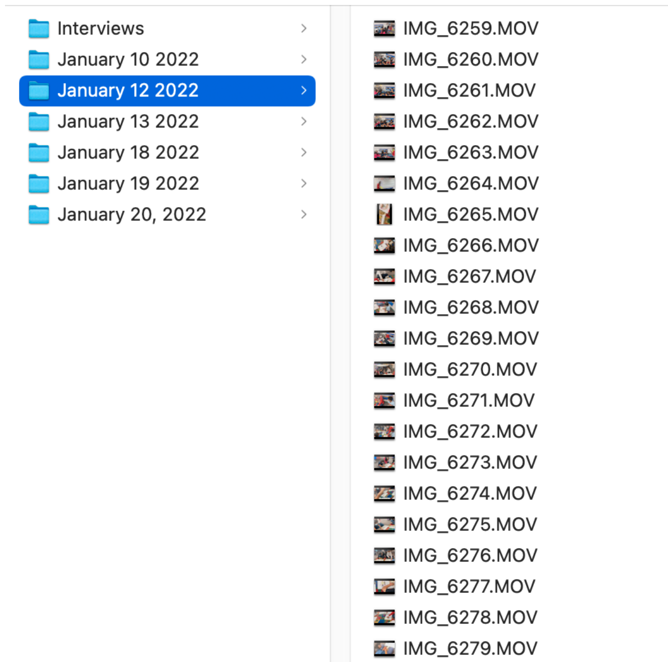
Math education is a contentious and delicate subject. Moreover, my positionality as the author of the story books helped create objectivity with individuals who were indirectly involved in the learning experiences, which provided additional informed perspectives to the books and the pilot. These individuals were composed of a professor and educational leader, a primary school leader, an instructional leader, and an education professor and author of a recent NCTM publication on children's literature and math learning. They were interviewed individually via Zoom video technology.

Organization of the Data Analysis: Qualitative Analysis

Choosing and Organizing the Video Clips

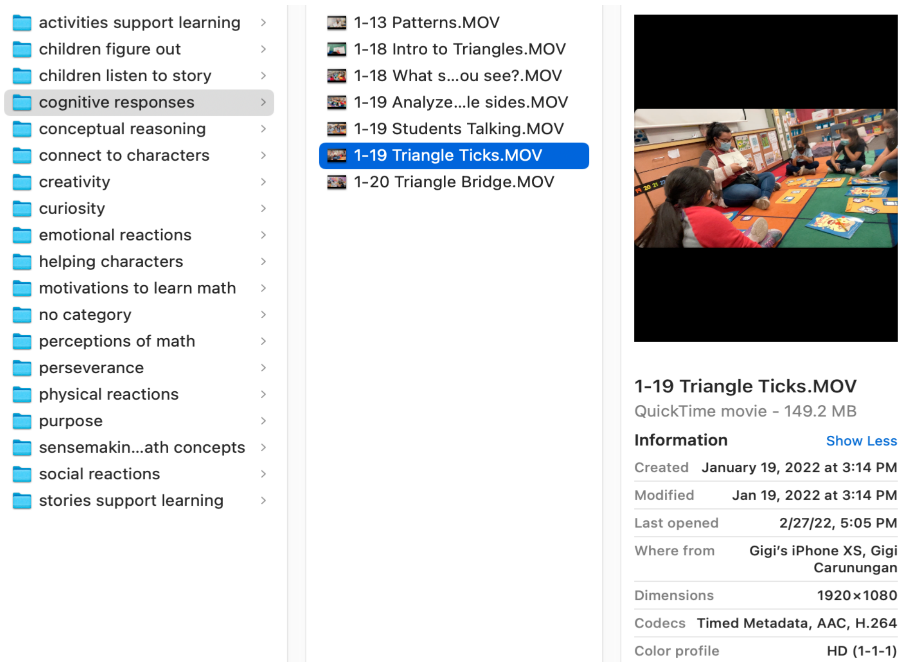
Most of the themes that emerged were convergent statements and actions, as well as unexpected responses from students (Knoblauch & Tuma, 2011). The video clips were organized in two ways. First, according to the date the video was recorded; this gave the study a database based on the linear flow of the events. Figure 18 presents a screenshot of the video clips on file.

Figure 18. Linear Organization of Clips



Then, the clips were organized according to common themes related to the process of mathematical sensemaking and reasoning as these related to the stories. Figure 19 is a screenshot of how the themes were organized as video files as the first step to categorizing the video clips.

Figure 19. Vertical Organization of Clips



To identify themes, the researcher first had to log and take notes of the recorded video footage after each day's pilot class. This was done with Final Cut Pro, a nonlinear editing suite software that makes it possible to access, cut, and/or extend video footage irrespective of the video and audio clip's location in the timeline. To organize video clips, these need to be uploaded to the computer and reviewed in the editing suite. Organization of data included the person, date, time, place, and learning context of the recorded clips. Checks of footage in the editing suite verified problems with sound quality in the first 2 days, which prompted the need to buy and use another microphone. Image quality was good; however, given the limitations of one camera and a whole class moving at the same time, it was impossible to capture everything going on.

Positionality

As the researcher in this exploratory study, I did not read to and teach the children directly. The books were read and taught by the teachers, who are the target users of the resource materials. I was present in the classroom, video recording.

I am both the researcher and the author of the books and the extension learning activities. Naturally, as the author, I bring with me a personal bias toward a preference for the narrative, the images, and the learning activities. Negative feedback, for example can cause me to be defensive and protective of my work. I could also accentuate the positive over the negative to elevate my ego. However, if I allow this to happen, then the very purpose of the study, which is to test the efficacy of the books and learning activities, would diminish to waste. More importantly, I would be detracting from a commitment to the teachers, the school leaders, and the students to provide an alternative to math education, that Jessica Liou, the illustrator of the books, and I envisioned to connect with children of multicultural backgrounds.

Aware of the possibility that my connection to the books as the author could influence the results of the research, I sought independent specialists and educational leaders to review the materials and the videos of the children. One of these individuals is an author of an NCTM-published book on literature and math, and the educational leaders from the school and district were not my acquaintances. I sought to interview and include these individuals because they provide a balanced perspective to maintain the level of objectivity to the documentary study. Moreover, to further ensure that the accuracy and interpretation of the words of the interviewees, the draft documentary video was shown to the participating teachers, school leaders, and specialists for review in the early and final versions of the documentary. This process gave them the opportunity to give their feedback on the accuracy of their views and the themes that emerged from the pilot. We agreed I would make changes as necessary upon request. In a sense, the interviewees were also reviewers with the right to request for changes of the exploratory documentary study. So far, feedback from the individual interviewees has not resulted in requests for any changes except for technical issues, such as fixing transitions between clips.

The insights of the individuals and experiences of the teachers with the classes form a triangulation of perspectives on the organization of the themes and formulation of the research findings (Merriam & Tisdell, 2016).

Editing the Footage

The narrative of the documentary is an intersection of the linear dates of the activities and the themes. With more than 15 hours of footage, there was a need to choose video clips that highlighted key findings and then organize these into a logical flow that made sense to the viewers of the documentary. Writing a narrative script guided the flow of the narrative and outlined the themes accordingly. This required reviewing the dates of the clips for each of the themes to avoid repetition while maintaining accuracy in what happened. As demonstrated in Figure 19, the times and dates when these were shot are automatically embedded in the data. I maintained three files of all the clips, requiring almost 500 gigabytes of digital storage. The process of editing is nonlinear and required a constant back-and-forth, deleting, accessing, cutting, inserting, and etc. of video files. Video footage was organized into themes and along the timeline of the study. The flow of the presentation was both thematic and linear. In this way, key points were highlighted and the flow from beginning to end of the students' learning processes showcased the learning progression and transformation from discomfort to enjoying and having fun learning math as a result of the story pedagogy.

Ethical Considerations

Every participant in the study was given an Institutional Review Board-approved permission form to sign. The children's parents signed on their behalf. Copies of each type of form are available in Appendices A, B, and C. During the pilot, only students whose parents had to pick them up early did not complete the program; otherwise, none of the participants chose to leave the study. Likewise, the teachers fulfilled their commitments to complete the study and to be interviewed thereafter.

The pilot study was conducted in a classroom setting in the school where these teachers taught their classes. As such, the teachers had control of the safety of the location and the students. The interviewees all signed an Institutional Review Board form that ensured that they could choose to refuse to answer any question during the interview. See Appendices A, B, and C

Limitations

The exploratory study was with a small group of students and was a single program of six sessions. The size of the group does not represent a conclusive study.

Summary

The objective of this chapter was to outline the research methodology that was used by the study to respond to the research questions. It also provided a brief overview of the learning materials used for the pilot study, the interview questions, the process of accessing, organizing, analyzing, and preparing the documentary, the positionality of the researcher, and ethical considerations of the research. The exploratory video documentary was produced to demonstrate a pilot study of an after-school program where the efficacy of two math stores and extension learning activities were tested.

IV. Findings of the Study

Introduction

The findings of the study comprise video clips from the pilot and interviews of the teachers and specialists in lower school instruction and educational leadership. Initial findings were based on the observed responses during and verbal feedback after the interaction of teachers and students with the math storybooks, games, and activities.
I also interviewed teachers, school leaders, and educational leaders, who were directly involved with the pilot testing; authors of literature relevant to the study; and educational leaders with extensive experiences working with a population of similar elementary teachers and students. The insights gathered from these interviews were also organized according to themes and have been integrated into relevant parts of the video documentary.

Statement of the Purpose

Through video documentary, this exploratory study demonstrates the significant difference that the researcher-created math story books made in how 5- to 7-year-old public school students understand and are motivated to learn math. The researcher-created books included in this study are *Maya the Clothes Maker and Ramon the Button Maker*, *Manuel the Table Maker*, and *Math Is So Much Fun* (Figure 20).

Figure 20. Book Covers of Researcher-Created Math Story Books

Research Questions

Following are the questions prepared for and given to the teachers ahead of the interview:

Teacher Interview Questions

1. Please describe from your observations, how the students felt about their learning experiences with the math stories.
2. How did the stories contribute to your students' math learning?
3. How did the stories, prompts, characters, and activities provide the students' access (or not) to learning math concepts/ideas? Can you give some examples?
4. How did you determine that the children learned (or not) math concepts? Were they able to articulate the concepts?

5. How were the stories able to build upon the children's prior knowledge to deepen and grow their understanding of math ideas?
6. What new math ideas did the students learn?
7. What was your favorite part of the math storybook?
8. How can we improve the storybook?

Following are the questions prepared for and given to the specialists ahead of the interview:

Specialists and Educational Leaders Interview Questions

1. The United States continues to face challenges in raising students' achievement levels in math. Sixty percent of students are not meeting the expected achievement levels in math. According to research, by the time students are in the third grade, they have decided whether they are good or not good in math. From your experiences as a curriculum coach, what do you think is causing the opportunity gap and math phobia?
2. The traditional and mainstream perception of math learning is that it is necessarily repetitive, and memorization is imperative. However, neuroscience presents a different perspective. If the approach is toward unlocking the inherent math abilities of children, children will feel joy and be empowered in learning math. How do schools make choices on math pedagogy given the many different views about math learning?
3. What are the general attitudes of children in the first grade in the schools you work with, toward math?

4. The NCTM is promoting innovation in math education, specifically in the curriculum. Your role as the **principal** influences the school’s pedagogical choices. Math stories combine two separate fields: English Language Arts and Math. Teachers learned these subjects separately. What does it take to prepare teachers to teach differently from how they learned?
5. How do you think stories build upon students' prior knowledge to deepen and grow their understanding of math ideas?
6. How can stories, prompts, characters, and activities provide students access (or not) to learning math concepts/ideas?
7. The books that I have developed are designed as wellsprings of math ideas. The pedagogy is guided by current research from the learning sciences. Children work with unstructured problems that provide the ingredients for mathematical sensemaking and reasoning. Additionally, the stories are about fictional characters representing the multicultural populations of our schools and communities. What would be your advice for author-educators like me who are producing math stories of these types?

Author Questions

1. What inspired you to edit and publish Deepening Students’ Mathematical Understanding with Children’s Literature?
2. Your book introduced multiple concepts of time, for example, beyond the European 24-hour/day cycle. The notion of time as mathematical with students computing seconds, minutes, hours, days, etc. becomes just one way of understanding time.

What is the mathematical implication of this expanded framework of a child's study of time?

Sample Profile

Through a series of pilot tests of the stories and accompanying games and design activities, the study got direct feedback from end-users, specifically, from children 5- to 7-years-old and their teachers, with real reactions in the forms of physical, social, and cognitive responses on how math stories with interactive hands-on activities affect student engagement and understanding of mathematical sensemaking and reasoning.

All the children and teachers involved in the study were given individual copies of the books and workbook. These were distributed to the students through their teachers.

Presentation of Themes From the Data

Throughout the pilot, the responses of the teachers, children, and the interviews were documented in video and photography. These were categorized into themes and formed the results of the study. The themes were categorized into (1) evidence collected to validate the hypotheses and (2) descriptive themes from the videos and photos:

1. Evidence collected to validate the hypothesis:

- Activities support learning
- Children figuring out problems and solutions
- Children listening to the stories
- Conceptual reasoning reflected
- Connecting with the characters
- Helping the characters

- Engagement with creative activities
- Growing levels of curiosity
- Verbal reactions reflecting emotions and engagement levels
- Physical reactions reflecting emotions and engagement levels
- Motivation to learn math
- Perceptions of math and learning
- Perseverance to learn and explore
- How purpose affects engagement
- Sensemaking and reasoning
- Social interactions
- How story elements support students' learning

2. Descriptive themes from the videos and photos:

- Open-ended and context-based problem solving are wellsprings for explorations in conceptual and logical reasoning.
- Children learn by empathizing with the characters' challenges. Doing math is associated with helping the characters.
- Students generate responses to challenges from story contexts with fictional applications of real-world problems that are seedbeds of math ideas.
- Children learn mathematical thinking by simulating the modeling processes of mathematicians through a nonlinear process of figuring out, modeling, generalizing, and communicating.
- Children discover and explore math concepts through open-ended math challenges.

- Math learning becomes more than just a linear engagement of solving for the correct answer through tasks and questions embedded in the stories that stimulate logical and analytical thinking.
- Math learning is exciting when students' experience how much their views matter, and it is meaningful because children are driven by purpose, i.e., to help the characters solve their problems in the story.
- The stories keep students focused because children are enthralled by listening to the text being read and through the images, are visualizing the narrative in their minds.
- The story shifts students' attitudes from fear of having to be correct to get teacher's approval, to excitement toward being able to contribute their knowledge, abilities, and insights in open-ended mathematical discovery and exploration.
- Students see themselves in the characters.
- Cognitive learning is achieved without the use of rote/memorization.
- The learning designs of the extension activities, such as the card games and creative projects, generate connections to the story, cultivate positive social interactions, and motivate children to practice and master learning the math concepts.

Limitations

It was not possible to get consistent attendance from the participants of the pilot. Only seven to eight participants attended consistently. After-school programs were dependent on parent pickup times and cases of COVID. Parents decided whether their children attended after-school activities. These were the givens of the programs, and I had no control of the said variables. As such, the study is exploratory as much as it is preliminary.

V. Summary, Conclusions, and Recommendations

Introduction

The exploratory documentary research presents a learning design study, through the test of the efficacy of a pedagogy, in the form of math-driven picture storybooks accompanied by a math activity book, games, and activities. Imbued with a creative, interdisciplinary, multicultural, and story-based approach, these materials were designed to reinvent math learning for young children. The goal of the study is to determine whether these learning resources have the potential to address challenges faced by students in the primary grades who are discouraged with learning math through rote methods.

The study accentuates the role of pedagogical design as influenced by two opposing worldviews of education and their roles in the opportunity gap in math learning. The first pedagogy, influenced by the behaviorist worldview of education (Skinner 1971; Watters, 2015), which mostly utilizes rote, discrete, and testing of math procedures, has been cited by multiple studies as a key variable in impacting students' failure to develop mathematical proficiency (Chomsky, 1971; Crawford, 2020; NCTM, 2000; Van de Walle, 2007; Green, 2014; Ball et al., 2005; Graham, 2019). What is common among K–12 students in terms of their math learning experiences is the challenges that are brought about by rote teaching strategies and the pedagogical design of math textbooks and math teaching and learning resources that tend to be a mile wide but an inch deep, delivered historically and currently through memory- and test-based learning resulting in a majority of students disconnecting with math learning. (Larson & Kanold, 2016; Pike, 1809; Johnston, 2020; Van de Walle, 2007). Moreover, the said pedagogy carries a culture of "Whiteness," influenced by an

Anglo-Europeancentric worldview, reflective of its images and examples of mathematical problems. The concept of math as a neutral subject devoid of race and culture is hardly true, as evidenced in numerous studies on racism, classism, and other forms of systematic oppression in mathematics education (Freire, 1970; Schoenfeld, 2004; Gay, 2000; Burnham, 2020).

As of the NAEP 2019 results, 59% of fourth grade students are not meeting grade level math proficiency in the United States. This decades-long problem highlights the disparity between children experiencing discrimination by virtue of race and low-income families compared to middle- and upper-class families. Though there is a stark disparity in math achievement scores among students from upper and lower economic classes, research data presents access to after-school math tutoring and early exposure to math concepts and practices as differentiating variables impacting math achievement levels (Scholarship Media, 2017).

Reinventing math learning through math stories and hands-on math learning activities.

The exploratory dissertation focuses on constructivist theory-based math stories and story-based activities such as games and design projects that foster conceptual mathematical thinking highlighted in the mathematical practices of the NCTM as mathematical sensemaking and reasoning. Through video documentary, the research determined the efficacy of these math resources as pedagogical tools which offered “high opportunity contexts” of learning, also commonly described as *rich learning experiences,* for all students.

The purpose of Chapter 5 is to synthesize the findings of the pilot study by answering the research questions, validate the findings vis-à-vis the literature review, and discuss implications for future studies. The following research questions guided this study:

- How do students' responses to the math stories and activity book reflect perceptions, motivations, and sense making of math concepts?
- How do children's physical, social, and cognitive responses to math stories inform research of the efficacy of this pedagogy?
- How do the math stories and activities support teachers and parents in guiding students toward mathematical sense making?

Summary of Findings

The findings from the exploratory study, which are responses to the research questions, are presented in the documentary video titled *Math Makes Sense with Math Stories*. Video footage was taken at a K–5 school in Silicon Valley with first grade students at an after-school program. The video clips document the children's responses to the stories, games, and design activities. These also include the audio-video documentation of the teachers leading the classes and interviews of educators and an author specializing in the field of math and children's literature. Screenshots of video clips are presented in Appendix D.

Research Question 1

How do students' responses to the math storybooks and activity book reflect perceptions, motivations, and sense making of math concepts?

Benefits.

The main themes distilled from observations of the children's responses include: a shift in perceptions of math from fear of the subject to connecting with the stories and authentic engagement with math learning; from discomfort because they felt unsure of knowing the right answer to enthusiasm in responding to questions based on the wellsprings of student-created ideas, inspired by the connections with the characters, the contexts, and the math ideas the children found in the math stories; and a feeling of relief and excitement as they grew in confidence with their ability to make sense of math ideas.

Children connected to the stories. Children's interest in the stories was evident in how they listened intently as the stories were told by the teacher. On day one, most of the students came to class unmotivated and anxious, visible in their body movements in the first session of the after-school program. The notion of math as not fun and difficult was observable in the hesitancy in the way some children dragged their feet as they walked to class. Some students were fidgety, hands playing with items on the classroom wall, and a few demonstrated inattentiveness and resistance to the teacher's instructions. However, when the storybooks were distributed to each child, and the teacher explained that the class would be reading a story, confusion was perceivable on the children's faces. This could have been since stories are not typically read in math classes. The teacher did not respond to the confused faces and continued along with the lesson. As soon as the teacher commenced reading the story, there was a clear shift among the students' sense of the kind of math they were learning, suggested in their verbal responses and discernible in their eyes and body language.

On the first day of the sessions, the students' responses and non-responses revealed how they perceived math as the subject requiring the need to know the correct answer or else face the social shame of being the kid to answer wrongly. Not wanting to make mistakes was a key premise influencing children's motivations to respond to the teacher's questions during the early phase of the pilot sessions. However, as the teacher read the story and as the open-ended questions made it progressively clear to the students that there was no one right answer, students demonstrated increasing interest in participating in the activities and excitement in their evolving sense of confidence as they engaged in mathematical discovery and exploration. This was apparent in most students going beyond providing one correct answer and in persevering through the math problems and challenges.

Motivation for children to participate was also encouraged by the setup of the classroom, with buttons as manipulable, writeable tables as opposed to worksheets, having a blank poster paper on the board where students showcased their ideas for equations to 10, and the story followed up by teacher prompts that encouraged students to think of more than one way of writing equations that total to 10, without being judged as right or wrong. This created an atmosphere where children realized they can learn from peers in a design-learning environment that promotes multiple ways of getting to answers. Multiple answers eased the students' fear of being alone and failing in math. Furthermore, in terms of mathematical sensemaking, this approach elevated students' understanding of big ideas in numeracy and equations.

Stories and illustrations are wellsprings of math ideas. The context and examples in the stories diminished students' fear of needing to be correct and the necessity to get their teacher's approval for every answer. Fear and the need for approval foster a sense of dependency and a learning culture of compliance, which have negative effects on the ability of a student to develop agency. When the teacher asked what the students could do to help the characters, specifically, to expand the character's ideas about equations, the students were motivated to find solutions. During the reflection session, some students expressed how doing math felt good to them because they were helping the characters.

By drawing characters of themselves and solving the problems, students saw themselves as part of the stories. Engagement and perseverance were evident in the students' persistence in finding solutions for the characters' problems, for example in creating button sets. The growth in confidence in their ability to make sense of math ideas was shown in students' working on multiple equations to 10, multiple patterns, playing the card game multiple times, and their increasing interest in attending and participating in the sessions. This was also evident in the number of students' hands raised, increasingly representing the majority of the class, ready with answers to challenges and questions raised by the teacher.

The narratives and illustrations provided fictional applications of real-world problems and the everyday situations encountered by the characters became wellsprings of math ideas where students generate responses to challenges. The questions embedded in the stories were formulated to nurture mindsets to think of math as "figuring out" rather than a "one correct answer" learning experience. This approach is key to guiding students' interests in mathematical sensemaking and reasoning.

There was no concern of the story as fiction or not real among the students. It was evident that navigating both fiction and nonfiction worlds in almost the same realm, was reflective of children's consciousness in their developmental age level.

Teachers' pedagogical alignment mattered. The teacher's alignment with the pedagogy of creating a culture of math learning wherein students are allowed to make and learn from mistakes and learning like young mathematicians supported the goal of developing mathematical sensemaking and reasoning. This is evident for example in the one-on-one support provided by the teachers when they observed individual students struggling with the activity. The teachers also allowed the students to find their way into the problems and solutions as unique individuals. An example is when one of the students took longer than the others and kept rocking and falling off his chair. Eventually, with the teacher's patience and understanding of the child's unique process, he came up with a solution as the number of buttonholes as the defining feature of his sets. It was evident in the responses that the children felt relieved when they discovered how to make sense of the problem and then excitement when they continued to develop multiple solutions.

Challenges.

Pilot testing a learning design in context means the conditions for intervention will be complex with many unknowns, and the variable for success will be different for each day and for each teacher and group of students.

The complexity of the variables impacting the pilot test in a school and in a classroom, affected the expected results of the study. These were extraordinarily difficult times for the school system. Schools were barely surviving the challenges posed by the COVID-19 pandemic, and then Omicron set foot on the world. As an outsider, it was difficult to get permission to conduct any research at almost every school; I was turned down by about 20 schools, principals, and teachers. As such, multiple delays and adjustments in the ways that the research could be conducted had to be considered and reconsidered.

In almost all the schools contacted, only half of the expected students came to school each day during the escalation of Omicron positive cases. There was no assurance that the children who participated on day one would be in the next session. Instead of having 15 students, there was an average of seven students in each session, and some students were inconsistent in their attendance. The teachers and school leaders explained that each day was a challenge for the whole school since safety was a prime concern.

Since each story necessitated at least three sessions to complete, continuity is key to the research results. The number of students participating generates unique variables to support conclusions. Continuity and quantity of participants are defining considerations that influenced the results of the study. Among the programs to include in the study, it was only this program where one or two students failed to return to the sessions. This was due to a positive COVID case in one family and a student's family traveled to Mexico after two sessions in the other. These cases were also unknowns and unexpected situations for the teachers. Additionally, in this school, the study found support from the administration and the teachers.

The findings from the study are consistent with the literature review. Most, if not all, students came to the first session affected by math anxiety, which creates a negative attitude toward math (Driscoll 2000; Van de Walle, 2007; Boaler, 1993; Boaler & Zoido, 2016;

Young et al., 2012). When children connect with the stories and characters and see themselves as problem-solvers, their interest grows in learning math (Schoenfeld, 2004; Schlechty, 1977; Tomlinson & Erickson, 2007; Moyer, 2000; Furner, 2005). With a context that children can culturally identify with and problems that connect with their everyday lives, meaning making facilitates mathematical thinking (Vygotsky, 1978; Gay, 2000; Boaler, 1993; Kilpatrick, 2001; Burnham, 2020; Krizo & Calinsky, 2020).

Research Question 2

How do children's physical, social, and cognitive responses to math stories inform the research of the efficacy of this pedagogy?

Benefits.

The main themes distilled from observations of the children's responses included shifting from how they felt in the first session, which was uncertainty to delight, to eagerness and then keenness on reading stories and working on the activities, and finally in the last session, students were adamant about not wanting the sessions to end. Physical responses were evident in the children's shift in body language expressing their unsureness in the earlier sessions and growing interest and understanding of the stories, math ideas, and activities. Their body language and expressions through their eyes changed as students realized how much math they learned while showcasing their abilities in getting mathematical sensemaking and reasoning. Socially, it was observable that girls and boys preferred not to partner-learn with opposite genders in the early sessions. This attitude loosened toward the latter sessions, signaling the diminishing interpersonal barriers among students and community-building and peer-to-peer learning as a social aspect of the pedagogy.

Motion indicated a shift in attitude from disinterest to thriving. On the first session, hesitation to being in the pilot class was visible. Students were making motions signaling disinterest with the teacher's opening conversation with the class. The teacher asked a question, and the underwhelming response was palpable. Math was clearly not a subject these students were enthusiastic or sure about. For example, when a student responded, "Yes," and the teacher asked why, the student replied, "It is easy to help and do the math for the characters." This could be interpreted in many ways; however, in the context of the class, the tone of the student's very soft-spoken and diminishing voice, one could hardly hear what was said. As such, the teacher asked the student to repeat her answer and her tone reflected a sense of hesitance, which, on the first day, is typically indicative of the fear of being right or wrong in one's responses in math classes. Her tone was very different on the last day of class when she shared what she learned from the class, evident of a growing sense of confidence after only a few days of the program. The student's voice was assertive, and she was communicating from a personalized perspective rather than regurgitating the value of what she learned about triangles.

Likewise, in the early sessions, the body language of most students visibly communicated that they were not sure about being in the after-school class. For example, a child was playing with his fingers and touching objects on the wall. He appeared to be in a different mental space; his physical behavior was expressing a lack of interest in participating in the class and, as his hands told us, he would rather escape and be immersed in his imagination.

Social responses reflected positive learning interactions among boys and girls. It was interesting to observe how girls and boys preferred to work with the same gender and were not interacting with each other in the early parts of the pilot, considering this was the second semester of the school year. These children had been classmates since the fall. Furthermore, a culture of respect for one another and learning from each other were constantly underscored by the teachers. However, the social dynamics changed after a few sessions and, thereafter, boys and girls were learning, working, and playing together. This was evident when they shared their solutions in the button activity and more so in the card game activity.

Efficacy of the pedagogy was validated by student's growth in mathematical abilities. As hypothesized in the proposal of the study, results confirmed that the stories provided the students with a different way to experience math, which influenced the way they saw themselves growing as young mathematicians. Children learned by empathizing with the characters' challenges. Students saw themselves in the characters. Doing math is associated with helping the characters. Prompted by the story, through a process of figuring out, communicating, inventing, and testing solutions, children learned how to do math by simulating the modeling techniques of mathematicians. Responses to the story books are learning experiences that are multidimensional in process. Students verbalized and physicalized their excitement. Students' experiences validated how much their views mattered and were meaningful because problems involved helping characters in the story.

The children responded to the stories, games, and activities in enthusiastic ways. This could be observed in the shift of their emotional, physical, and social responses and interactions. Some activities provided higher cognitive experiences. For example, learning how many ways to make the number 10 in the form of equations meant understanding math big ideas such as "equal means the same," "numbers tell how many," "smaller numbers make bigger numbers” and “there are many ways to take numbers apart.” However, it was also observable that in the book *Manuel the Table Maker*, the activities did not sufficiently provide children with a critical learning experience that builds understanding toward the big idea “triangles have 2 or 3 sides of the same length.” The question on the page did not result into a natural exploration into the sides of the triangle. When the children did not demonstrate their understanding of triangles in the way envisioned me and Jessica Lieu, the illustrator, a revision had to be considered for the book and its support learning resources to ensure that teachers were equipped with resources for children to understand this big idea. This was designed for the next day as an additional activity to ensure that the children understood this core concept about triangles.

Challenges

The masks limited visibility of facial reactions and the children were clearly uncomfortable wearing masks all day.

The findings from the study are consistent with the literature review. Effective math pedagogy intersects the mind and body by engaging a mix of students’ intelligences, perspectives, fears, and everything else that comes with being a multifaceted human. (Gay, 2000; Tomlinson & Erickson, 2007; Li, 2012; Burnham, 2020; Van de Walle, 2007). Students reach enduring understanding when they engage in productive struggle with new ideas and challenges that they can relate to; where they merge their background knowledge to come up with solutions; and when they work with a community of learners (Van der Walle, 2020; Entwisle & Alexander, 1990; Pungello et al., 1996; Clements, 1999; Clements & Sarama, 2009; Lesh et al., 2003; Kilpatrick et al., 2001). Generalization is recognizing the big idea in specific problems and contexts and the specific problems and contexts in a big idea (Polya, 1957; Boaler, 1993; Walkerdine, 1989; Hiebert et al. 1997; Dumitrascu, 2017; Bransford & Schwartz, 1999; Brown & Campione, 1984).

Research Question 3

How do the math stories and activities support teachers and parents in guiding students toward mathematical sense making?

Benefits.

The main themes distilled from observations of the pilot sessions and interviews of teachers after the pilot included their appreciation of the stories and pictures that gave context and visualized mathematical concepts that students struggle to understand otherwise. The connections to the students' lives, cultural backgrounds, and interests that the stories and illustrations provided made the idea of learning math like young mathematicians so much more real. The open-ended questions in different parts of the story provided teachers with concept-oriented math questions. The activities and games kept students engaged and interested in learning math concepts.

Children learning as mathematicians. The teachers expressed their observations on how quickly students learned math concepts through the stories and activities that they continued to struggle with in classes many months before introduction of the study. One of the teachers explained how concepts such as equation and equal and not equal were challenging for children to understand. While this teacher used stories in math class to explain concepts, she explained that the books were able to successfully make students understand the concepts in two sessions with the literary and visual elements in the picture books highlighting the math concepts.

Noticeable to the teachers were students who were willing to persevere with problem solving during the sessions. The teachers explained that students acted differently in their regular math classes. The experience of the after-school program on math stories gave teachers hope that the vision of students learning as young mathematicians is possible because they saw examples of students persevering and working on problems during the sessions. A teacher explained that while fiction stories do not necessarily mirror real lives, the children connected to the characters like they were real people. This was indicative of children at this age level able to navigate both fiction and nonfiction worlds like they were in one universe.

Open-ended prompts generate empathy and engagement in math sensemaking. Open-ended and context-based prompts in the form of questions and characters asking for help with problem solving were kindled explorations in sensemaking and conceptual and logical reasoning. The questions and tasks embedded in the stories motivate with empathy and provide the scaffold for analytical thinking, making math learning more than just a linear engagement of solving for the correct answer.

Most teachers did not learn and did not have mathematical sensemaking and reasoning as a way of learning math as part of their curricula in their teacher training programs. The teachers underscored their appreciation of the prompts and framing of the math problems, within the contexts of the stories. They explained how it would have taken them more time to figure out the proper analytical and conceptual math prompts and problems. For one of the teachers, teaching in this way was a new experience.

Stories and math big ideas were sources of conceptual and contextual math ideas. The stories are wellsprings of context-driven and unstructured math ideas that bode well for the goal of learning mathematical sensemaking and reasoning. A teacher shared how grateful she was that she did not have to think of the questions and analyze the story to teach math with the story. In a sense, the teachers shared, they also learned about mathematical sensemaking and reasoning as they taught with the stories, games, and activities.

Behavior and differentiation in math classes was managed by the math stories and learning resources. Managing behavior in math class is always a challenge while differentiating instruction and making sure students learn at optimal levels. The story-based learning design of the math games and activities made it possible for teachers to support individual students without needing to constantly remind the rest of the class to focus on their learning. Moreover, the games provided concept extension practice. The design activities engaged students in understanding math concepts in creative applications interconnecting big ideas with real-world solutions, such as building the bridge with triangles and creating different patterns for the tall man's shirt.

Challenges

Common Core Versus Big Ideas. The teachers were more familiar with the Common Core than the framework of math big ideas. Most of the schools I approached for consideration to host the study required documentation on how the storybooks, games, and activities connected to the Common Core and lesson guides for the book and the extension learning resources. The teachers in some of the schools were worried that using stories for math classes would take up more time than they had to cover their math topics for the quarter.

While both teachers who participated in the study preferred more training on math stories, the onslaught of Omicron affected the teachers' availability for training and access for collaboration. The overall goal of testing with 40 children could not be achieved within the context of the pandemic. School districts created new rules that disallowed any nonemployees from entering the campuses.

Math Stories Meet Math Common Core Standards. When a storybook, games, and accompanying activities are introduced for use in the classroom, stringent requirements are to submit a document that demonstrates how the book meets math Common Core standards. In conversations, teachers explained a trade book like a storybook can be introduced and read to the class in language arts without worry that this will disrupt the curriculum. Reading a storybook is considered a part of reading skills. Introducing a trade book to enrich math learning is not the same, most especially if it is in the form of a storybook, which does not typically register as a math book. For many, fiction stories are not recognized as the equivalent enrichment resource for math learning as they are for reading.

Innovation is a Complex Proposition for School Systems. Unless one is an established entity, like a textbook company, school districts are restrictive about accepting proposals for alternative math resources. Consideration could be granted upon review of the resource's referencing to the Common Core standards. Partnering with schools to innovate and address the opportunity gap depended on the levels of awareness and openness of the educators and education leaders to the pedagogy of math stories, games, and design activities. It was fortunate that a constructivist and innovation-oriented public school leader gave the study a chance as an after-school activity. It was only after the teachers saw the level of engagement and mathematical sensemaking and reasoning achieved by the students that they wished the rest of the class could have been part of the experience. Appreciative of the level of learning achieved during the program, the teachers extended the experience to their classes. This was one of the positive results of the pilot.

Math and reading are two separate subjects. Most primary school teachers I spoke to tend to think that reading a storybook during math class takes up more time. Whereas primary schools are designed in a multiple subject framework, actual integration of subjects in thematic or topical ways, is not common practice. Specifically for math stories, this has to do with the traditional structure of math and language arts as separate subjects and math and language arts as unconnected textbooks. With guidelines from district leadership regarding class scheduling, the integrative approach of math stories is considered as a disruption to the normal school day structure.

The Mask Mandate. This made it a challenge to observe facial expressions and to hear children's responses in class. Upon checking the video after the first two sessions, there was a hum that was not evident in the classroom but was particularly strong in the video clips. This necessitated close captioning during editing to ensure that the teacher's and children's words are communicated to the viewer. In addition, the time limitations and the rules on external people coming into the classroom to help with camera work also diminished opportunities to spend time interviewing and videorecording all students at every given moment.

COVID presented new challenges for external researchers in the school system. The public school system instituted a new rule during COVID, which disallowed any visitors, substitutes, and teacher aides on school grounds. This made it almost impossible to conduct the study. The criteria the study outlined as basis for observing participants included facial expressions and verbal reactions, which were very difficult to observe when children and teachers were required to wear masks in class. Fortunately, the young learners had other ways to communicate their responses and emotions in class. This does not discount the fact that not having access to complete facial expressions now stands as a limitation of this research. COVID also impacted the time and focus the teachers could give the pilot project. Asking teachers to support this study was tantamount to one more thing they had to deal with in the chaos created by COVID.

The teachers want lesson plans to accompany the materials. The teachers are overworked, and with the pandemic, their load increased every day in the school year. Requesting a lesson plan is justifiable with new materials.

The findings from the study are consistent with the literature review. Learning math through stories and concrete experiences connect children to math ideas that have meaning and purpose and connect to their interests. (Monroe & Young, 2018; Moyer, 2000; Duursma et al., 2018, Burns, 2015, Furner, 2018, Whitin, 1992; Keat & Wilburne, 2009; Adams et al., 2007; Morgan, 2007). Contexts are considered as able to motivate students to engage in math learning and uplift their mathematical understanding to an abstract level (Kilpatrick, 1988;

Boaler, 1993; Morgan, 2007; Monroe & Young, 2018). Teachers need support in shaping curricula along constructivist principles and key concepts such as mathematical sensemaking and reasoning (Davenport, et al., 2019, Tomlinson & Erickson, 2007, Keat & Wilburne, 2009, Gojak, 2014, Larson & Kanold, 2016, Crawford, 2018, Kurzenhauser, 2020, Gowers, 2016).

Implications for Practice

The NCTM Beckons.

Matt Larson, the president of the NCTM from 2016 to 2018, calls on educators and parents, "to break the intractable cycle of resistance to change" (Larson & Kanold, 2016). Current president, Trena Wilkerson, underscores the importance of preparing children's mathematical mindsets toward deep mathematical understanding by emphasizing sense making and reasoning, from the early and primary years. This necessitates teachers learning math in ways different from the way they were taught.

Support Teacher Training.

With all the work teachers are accountable for, schools and districts should incentivize and support this training. If teachers are to trust the constructivist philosophy of learning, they should first experience this themselves. The teachers who taught with the stories in this study came to their realization of the power of the pedagogy while they were teaching and upon reflecting on the experiences during the interviews.

Lesson Plan With the User in Mind.

In the field of math education, when introducing a new pedagogical solution to schools, one needs to make sure that it comes with a lesson plan. Teachers are overworked and prepare for teaching multiple subjects every day. The lesson plan should take into consideration what matters to schools, districts, and learning organizations when making decisions for adopting a pedagogical solution, including picture books.

Recommendations for Future Research

While it was clear, even within a brief period, that having stories as context for math learning transformed students' perceptions and attitudes, this was a very short study. The research was limited to two books and a workbook, within a limited number of hours for one group of children. The teachers expressed that they wished the study had more time so they could expand on the concepts with their students during regular class time. At the end of the program, students did not want to go home; they wanted more math stories and socially interactive and design-oriented math learning activities.

The NCTM launched the Catalyzing Change campaign, encouraging schools to innovate the way math is taught to defeat the opportunity gap. For early childhood and elementary mathematics levels, there are four key recommendations: (1) broaden the purposes of learning mathematics, (2) create equitable structures, (3) implement equitable instruction, and (4) develop deep mathematical understanding (NCTM, 2000). All of these recommendations were integrated into the design of math stories, which also included games and design-oriented, hands-on activities.

Currently, there is dearth of usable, research-based pedagogy and resources available to primary school teachers to personalize and raise the levels of math learning in diverse communities. Based upon the results of this study, there is a need for more learning design studies to innovate the way math is taught. This study integrated three theoretical frameworks and recommended research questions for consideration for future design studies:

1. Learning Sciences: How can math curricula recognize the value and tap into the innate aptitudes of children, such as curiosity, creativity, and math ability, for example, subitizing? What are the implications of building upon the natural mathematical abilities to raising students' mathematical sensemaking, reasoning, and agency for early learners?
2. Constructivist Learning: How can the dynamics of power and control over math learning, as reflected in the way math is taught and learned in the classroom, influence students' motivations and interests to persevere in solving math problems?
3. CRT: How do children's responses compare when learning math trade books with animals, White people, and people of color as lead characters, in terms of engagement and mathematical understanding?

Research-Based Innovative Solutions.

This study is also a response to the National Research Council's (NRC 2002), call for closing the chasm between the educational researcher and practitioner with a new paradigm for research that interacts directly with practice and creates research materials that lead to "usable knowledge" (Lagemann,2002). There is a need for dissertations about design learning that can use analytical tools and merge these with creative approaches to produce research-based educational solutions.

The findings, as showcased in a documentary video, demonstrated the power of stories in creating a context for understanding math ideas and wellsprings for math problem solving.

However, this was a brief and small study. The challenge is to find more schools and districts to partner with and generate studies of integrating stories within the regular curricula with extension activities, such as games and design projects that build on math ideas presented by the stories. This could be a 1-year design-learning study comparing two cohorts of children, one learning with math stories and the other with the traditional math curriculum, with parents choosing their children's curricula, so they take accountability for their children's math education choices. The study can showcase the math skills, attitudes, and applications children learn with two opposing pedagogies. This study would provide vital, multilevel insights and recommendations for educators seeking to reinvent math learning.

Reflection and Conclusion

There is a lot at stake for math education in primary school. Math is a gateway to social mobility and, for most of the U.S. population, one that is a challenging pathway to get through. When fourth graders fail to meet proficiency expectations, as per the results of assessment outcomes, students' confidence in their ability to learn math depletes through Grades K–12. It matters especially when career choices with a promise of a better future are limited for most of the population who see themselves as not capable of math learning. Many school systems continue to fail to nurture children's interest in learning math, and teachers are still teaching math in mostly discrete ways. This is concerning as the upward infusion of technology in the world's industries is resulting in greater need for mathematical sensemaking and reasoning skills.

Research informs us that the failure to connect children with math has more to do with how we have been teaching and learning the subject. This hypothesis was strongly validated in this study, with the teachers sharing how the experiences of learning with the stories connected children with the math concepts so naturally.

Authoring and publishing my own storybooks were a calculated risk. Putting the books into a test through a dissertation was even more of a risk. The journey posed potential consequences of a math solution failing to meet the goals that were predicated on children deserving better math learning experiences. Some conversations were tough, and some were enlightening. I am grateful for the opportunities I had in conversing with all the people I encountered on this journey. I am indebted to the time and effort the teachers provided to make the pilot test possible during the COVID pandemic.

In a relatively short period of time, as per the words of the teachers, the children learned, practiced, and intersected the following big ideas: (1) Numbers tell how many, (2) Smaller numbers make bigger numbers, (3) Equal means the same, (4) Sets are groups of the same types of objects, (5) There are many ways to take numbers apart, (6) A pattern is a repeating order of objects, (7) Smaller shapes make bigger shapes, (8) You can draw animals with shapes, (9) All polygons have three or more sides, (10) Triangles have three sides and three corners, (11) Triangles can have two or three sides of the same length, (12) Polygons are closed, flat shapes with straight sides. If students can do this with two storybooks and 6 days, there is so much they can learn beyond the grade level benchmarks with additional storybooks throughout the schoolyear, while enjoying their math learning.

During the process of this research, I discovered how math tends to be a sensitive topic among K–12 educators. It is a fragile theme that touches on many unhappy and traumatic childhood experiences with math education, just like my own. As a child, not connecting with math learning experiences led me to think that having a creative and inquiring mind was a curse. I was so bored in math class. The mechanical approach to math learning made no meaning. Whatever the teacher was explaining and the miles of worksheets that we were forced to solve had no connection to what I took interest in, until I grew up to be a creative entrepreneur. Mathematical sensemaking and reasoning were key to understanding and developing spreadsheet algorithms as well as logic and design applications in science, technology, engineering, crafts, and business.

As educators, we owe it to today's children to not be afraid to recognize the shortcomings of our generation, specifically in the way we were taught and learned math. The consequences of our choices as educational leaders and teachers can take away opportunities for millions to achieve social mobility. Pedagogical decisions should always take into consideration how we want children to think of math. Let us nurture their potential by simulating math learning, likened to going into the rainforest with rich learning experiences so their senses can be stimulated, and their curiosity nurtured to develop their abilities to learn as active thinkers, as young mathematicians, and as creative doers that intersect math with every other field that attracts their interest. Working on this dissertation has exposed me to the things that math can do for individuals who can harness its tools and use them to make sense, reason, problem-solve, predict, and understand every facet of life and work with mathematical sense making and reasoning.

There is so much to fill with the huge gap between higher education research and K–12 education practice. So much of what I read and learned in my doctoral program are unknowns to the teachers. I hope that with a video documentary and with the support of math learning organizations I will be able to collaborate with many more primary school teachers and educational leaders to reinvent math learning.

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APPENDIX A:

IRB Consent Form A: Educators

REQUEST FOR YOUR PARTICIPATION IN RESEARCH
Title of the Study
An Exploratory Study on How Math Stories Engage Young Learners in Mathematical Sense-Making and Reasoning
Author Faculty Adviser
Gigi Carunungan, San Jose State University graduate student **Dr. Robert Gliner**
Purpose
This pilot program is an integral part of an exploratory documentary study on how math stories engage young learners in mathematical sensemaking and reasoning.
Explanation of the Research
The exploratory documentary study's purpose is to pilot and test an alternative study that has the potential to engage children in mathematics learning, inspired by pedagogical theories rooted in how a child learns best. This is predicated on the design of learning experiences that can provide optimal learning for children.
The stories provide prompts and explorations in conceptual and logical reasoning through open-ended and context-based problem-solving. Children learn by empathizing with the characters' challenges. Math is associated with helping the characters who are in contexts that are fictional applications of real-world problems. Through a process of figuring out, communicating, inventing, and testing solutions, children learn how to do math by modeling the processes of professional mathematicians. Open-ended math challenges engage children in discovery and exploration of math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear engagement of solving for the correct answer. The expected responses to the story books are learning experiences that are multidimensional in process, exciting because students' experience how much their views matter, and meaningful because problems involve helping characters in the story.
The exploratory research will be held with the purpose of enriching math learning with children ages 5-7 years.
The pilot will test the efficacy of the three books and workbook in teaching mathematical sensemaking and reasoning. Research questions:

- How do students' responses to the math stories and activity book reflect perceptions, motivations, and sense-making of math concepts?
- Do children's emotional, physical, social, and cognitive responses to math stories inform research of the efficacy of this pedagogy?
- How do the math stories and activities support teachers and parents in guiding students towards mathematical sense making?

As educator, you consent to be interviewed and participate in the research.
The interview is estimated to be about 30 Minutes. Following are the guide questions that the researcher will be asking you in the interview. You can reframe any of the questions below

as you deem appropriate.
You may also choose to answer in general or specific to the schools your work in.
1) The US continues to face challenges in raising students' achievement levels in math. 60% of students are not meeting the expected achievement levels in math. According to research, by the time students are in the 3rd grade, they have decided whether they are good or not good in math. From your experiences as a curriculum coach, what do you think is causing the opportunity gap and math phobia?
2) The traditional and mainstream perception of math learning is that it is necessarily repetitive, and memorization is imperative. However, neuroscience presents a different perspective. If the approach is towards unlocking the inherent math abilities of children, children will feel joy and be empowered in learning math. How do schools make choices on math pedagogy given the many different views about math learning?
3) What are the general attitudes of children in the first grade in the schools you work with, towards math?
4) The NCTM is promoting innovation in math education, specifically in the curriculum. Your role as the **principal** influences the school's pedagogical choices. Math stories combine two separate fields English Language Arts and Math. Teachers learned these subjects separately. What does it take to prepare teachers to teach differently from how they learned?
5) How do you think stories build upon students' prior knowledge to deepen and grow their understanding of math ideas?
6) How can stories, prompts, characters, and activities provide students access (or not) to learning math concepts/ideas?
7) The books that I have developed are designed as wellsprings of math ideas. The pedagogy is guided by current research from the learning sciences. Children work with unstructured problems that provide the ingredients for mathematical sensemaking and reasoning. Additionally, the stories are about fictional characters representing the multicultural populations of our schools and communities. What would be your advice for author-educators like me who are producing math stories of these types?
About the Study and the Documentary Video
The video documentary involves a mixed methods research framework and format. This includes (1) experimental—testing the book and activity books in real contexts of children's learning and (2) descriptive- using video technology to capture with audio-video tools the learning experiences. These will include responses, attitudes, motivation to do the work, and feelings about the tasks. Additionally, these also include the teachers' and parents' emotions, dilemmas, instructions, guidance, and the actions and words used in the classroom and at home.
Potential Risks
The interview will be done with an iPhone camera. The study poses no more than minimal risk to participants. Participants have a choice about the extent they wish to be identified in the research.
Potential Benefits
Your participation in the interviews will contribute to this exploratory study which aims to explore the potential of math stories, inspired by pedagogical theories rooted in how children learn best. The stories, which are in the picture books given to your child, provide

opportunities to explore conceptual and logical reasoning through open-ended and context-based problem-solving. The goal is for children to learn by empathizing with the characters' challenges and helping them solve math-related real-world problems. Through a process of inventing and communicating their unique solutions, the hope is that your child will learn to do math by modeling the processes of mathematicians. Open-ended, multidimensional math challenges will engage your child in discovery and exploration of new math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear process of solving for a single correct answer.

Confidentiality

As an adult participant you can choose to agree or not for the author to use your real name and position in the final edited version. Please write your consent in this form.

Participant Rights

As educator, your participation in this study is completely voluntary. You can refuse to participate in the entire study or any part of the study without any negative effect on your relations with San Jose State University or [name any other participating institutions]. You also have the right to skip any question you do not wish to answer. This consent form is not a contract. It is a written explanation of what will happen during the study if you decide to participate. You will not waive any rights if you choose not to participate, and there is no penalty for stopping your participation in the study.

Questions Or Problems

You are encouraged to ask questions at any time during this study.

- For further information about the study, please contact Gigi Carunungan at 650.714.9620 or email at gigi.carunungan@sjsu.edu.
- Complaints about the research may be presented to Dr. Bradley Portfillo, Bradley.portfillo@sjsu.edu, Ed.D. Program Director
- For questions about participants' rights or if you feel your child has been harmed by participating in this study, please contact Dr. Mohamed Abousalem, Vice President for Research & Innovation, San Jose State University, at 408-924-2479 or irb@sjsu.edu

___Yes___No I am allowing the use of my full name
___Yes___No I am allowing the use of my first name
___Yes___No I am allowing the use of my last name
___Yes___No I am allowing the researcher to choose a pseudonym for myself.

Visual/Audio Image Release Form

I grant permission for the researcher Gigi Carunungan to take and use visual/audio images of me. Visual/audio images are any type of recording, including but not limited to photographs, digital images, drawings, renderings, voices, sounds, video recordings, audio clips or accompanying written descriptions. The researcher will not materially alter the original images. I agree that the researcher owns the images and all rights related to them. The images may be used in any manner or media without notifying me, such as university-sponsored web sites, publications, promotions, broadcasts, advertisements, posters, and theater slides, as well as for non-university uses. I waive any right to inspect or approve the finished images or any printed or electronic matter that may be used with them, or to be compensated for them. I release Gigi Carunungan, including any firm authorized by the researcher to publish and/or distribute a finished product containing the images, from any claims, damages, or liability which I may ever have in connection with the taking of use of the images or printed material used with the images. I am at least 18 years of age and competent to sign this release. I have read this release before signing, I understand its contents, meaning and impact, and I freely accept the terms.

Signatures

Your signature indicates that you voluntarily agree to be a part of the study, that the details of the study have been explained to you, that you have been given time to read this document, and that your questions have been answered. You will receive a copy of this consent form for your records.

Participant Signature

__________________ ______________________ ____________________

Participant's Name (printed) Participant's Signature Date

Researcher Statement

I certify that the participant has been given adequate time to learn about the study and ask questions. It is my opinion that the participant understands his/her rights and the purpose, risks, benefits, and procedures of the research and has voluntarily agreed to participate.

____

Signature of Person Obtaining Informed Consent

APPENDIX B:

IRB Consent Form B: Parents

REQUEST FOR YOUR CHILD'S OR WARD'S PARTICIPATION IN RESEARCH
An Exploratory Study on how Math Stories Engage Young Learners
in Mathematical Sense-Making and Reasoning
Gigi Carunungan, San Jose State University graduate student. Dr. Robert Gliner, Faculty Adviser
Your child will participate in an exploratory documentary research which will be held with the purpose of enriching math learning for children ages 5-7 years.
The pilot will test the efficacy of the three books and workbook in teaching mathematical sensemaking and reasoning. Research questions:

- How do students' responses to the math stories and activity book reflect perceptions, motivations, and sense-making of math concepts?
- Do children's emotional, physical, social, and cognitive responses to math stories inform research of the efficacy of this pedagogy?
- How do the math stories and activities support teachers and parents in guiding students towards mathematical sense making?

The number of hours and days will be determined in consultation with the teachers and the principal. Initial estimates require about 3-5 hours/book with the corresponding activities in the activity book.
The exploratory documentary study's purpose is to pilot and test an alternative study that has the potential to engage children in mathematics learning, inspired by pedagogical theories rooted in how a child learns best. This is predicated on the design of learning experiences that can provide optimal learning for children.
The stories provide prompts and explorations in conceptual and logical reasoning through open-ended and context-based problem-solving**. Your child** will learn by empathizing with the characters' challenges. Math is associated with helping the characters who are in contexts that are fictional applications of real-world problems. Through a process of figuring out, communicating, inventing, and testing solutions, **your child** will learn how to do math by modeling the processes of professional mathematicians. Open-ended math challenges will engage **your child** in discovery and exploration of math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear engagement of solving for the correct answer. The expected responses to the story books are learning experiences that are multidimensional in process, exciting because students' experience how much their views matter, and meaningful because problems involve helping characters in the story. Your child will be interviewed after class in the classroom for no more than 15 minutes in the presence of the teacher.
POTENTIAL RISKS
The study will be a class observation. The class is led by a teacher. Potential risks are the same as the everyday classes participated in by your child and classmates. There are minimal risks involved in the participation of this study. As a participant your child discomfort when completing the survey, since it is another task to complete in their school

day. If necessary, students may choose not to be interviewed.

POTENTIAL BENEFITS

The stories provide opportunities for students like your child to explore conceptual and logical reasoning through open-ended and context-based problem-solving. **Your child** will learn by empathizing with the characters' challenges and helping them solve math-related real-world problems. Through a process of inventing and communicating their unique solutions, **your child** will learn how to do math by modeling the processes of advanced mathematicians. Open-ended, multidimensional math challenges will engage **your child** in discovery and exploration of new math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear process of solving for a single correct answer.

COMPENSATION

Your child will get a free set of picture books and the activity book used for the study.

CONFIDENTIALITY

Children participants will be given pseudonyms in the final edited version of the documentary.

PARTICIPANT RIGHTS

Your child's participation in this study is completely voluntary. You may refuse to allow his or her participation in the entire study or any part of the study without any negative effect on your relations with San Jose State University. Your child also has the right to skip any question that he or she does not wish to answer. This consent form is not a contract. It is a written explanation of what will happen during the study if you decide to allow your child to participate. You will not waive any rights if you choose not to allow your child to participate and there is no penalty for stopping your child's participation in the study. Your child may also decide to stop at any time.

Visual/Audio Image Release Form

I grant permission for the researcher Gigi Carunungan to take and use visual/audio images of me. Visual/audio images are any type of recording, including but not limited to photographs, digital images, drawings, renderings, voices, sounds, video recordings, audio clips or accompanying written descriptions. The researcher will not materially alter the original images. I agree that the researcher owns the images and all rights related to them. The images may be used in any manner or media without notifying me, such as university-sponsored web sites, publications, promotions, broadcasts, advertisements, posters, and theater slides, as well as for non-university uses. I waive any right to inspect or approve the finished images or any printed or electronic matter that may be used with them, or to be compensated for them. I release Gigi Carunungan, including any firm authorized by the researcher to publish and/or distribute a finished product containing the images, from any claims, damages, or liability which I may ever have in connection with the taking of use of the images or printed material used with the images. I am at least 18 years of age and competent to sign this release. I have read this release before signing, I understand its contents, meaning and impact, and I freely accept the terms.

QUESTIONS OR PROBLEMS

You are encouraged to ask questions and to have your child ask questions at any time during this study.

For further information about the study, please contact Gigi Carunungan at 650.714.9620 or email at gigi.carunungan@sjsu.edu.
Complaints about the research may be presented to Dr. Bradley Portfillo, Bradley.portfillo@sjsu.edu, Ed.D. Program Director
For questions about participants' rights or if you feel your child has been harmed by participating in this study, please contact Dr. Mohamed Abousalem, Vice President for Research & Innovation, San Jose State University, at 408-924-2479 or irb@sjsu.edu
__ Yes __ No I am allowing the use of a pseudonym for my child.
__ Yes __ No I am allowing the research author to choose a pseudonym for my child.
SIGNATURES
Parent/Guardian Signature
Your signature indicates that you voluntarily agree to allow your child to be part of the study, that the details of the study have been explained to you and your child, that you have been given time to read this document, and that your questions have been answered. You will be given a copy of this consent form, signed and dated by the researcher, to keep for your records.

____
Name of Child or Minor Parent or Guardian Name (Printed)

____
Relationship to Child or Minor Parent or Guardian Signature Date
Researcher Statement
I certify that the minor's parent/guardian has been given adequate time to learn about the study and ask questions. It is my opinion that the parent/guardian understands his/her child's rights and the purpose, risks, benefits, and procedures of the research and has voluntarily agreed to allow his/her child to participate. I have also explained the study to the minor in language appropriate to his/her age and have received assent from the minor.

____
Signature of Person Obtaining Informed Consent and Assent Date

APPENDIX C:

Parent as Participant

REQUEST FOR YOUR PARTICIPATION IN RESEARCH
TITLE OF THE STUDY
An Exploratory Study on How Math Stories Engage Young Learners in Mathematical Sense-Making and Reasoning
AUTHOR Faculty Adviser
Gigi Carunungan, San Jose State University graduate student **Dr. Robert Gliner**
PURPOSE
This pilot program is an integral part of an exploratory documentary study on how math stories engage young learners in mathematical sensemaking and reasoning.
EXPLANATION OF THE RESEARCH
The exploratory documentary study's purpose is to pilot and test an alternative study that has the potential to engage children in mathematics learning, inspired by pedagogical theories rooted in how a child learns best. This is predicated on the design of learning experiences that can provide optimal learning for children.
The stories provide prompts and explorations in conceptual and logical reasoning through open-ended and context-based problem-solving. Children learn by empathizing with the characters' challenges. Math is associated with helping the characters who are in contexts that are fictional applications of real-world problems. Through a process of figuring out, communicating, inventing, and testing solutions, children learn how to do math by modeling the processes of professional mathematicians. Open-ended math challenges engage children in discovery and exploration of math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear engagement of solving for the correct answer. The expected responses to the story books are learning experiences that are multidimensional in process, exciting because students' experience how much their views matter, and meaningful because problems involve helping characters in the story.
The exploratory research will be held with the purpose of enriching math learning with children ages 5-7 years.
The pilot will test the efficacy of the three books and workbook in teaching mathematical sensemaking and reasoning. Research questions:

- How do students' responses to the math stories and activity book reflect perceptions, motivations, and sense-making of math concepts?
- Do children's emotional, physical, social, and cognitive responses to math stories inform research of the efficacy of this pedagogy?
- How do the math stories and activities support teachers and parents in guiding students towards mathematical sense making?

As parent/guardian, you consent to be interviewed and participate in the research.
The interview is estimated to be about 15-20 Minutes. Following are the guide questions that the researcher will be asking you in the interview:

- How did your child feel about their learning experiences with the math stories?
- How did the stories contribute to your child's math learning? Did your child share examples?
- How did the stories, prompts, characters, and activities provide children access (or not) to learning math concepts/ideas? What examples did your child share with you?
- How did the stories, prompts, characters, and activities make math learning more meaningful (or not)? What examples did your child share with you?
- How did the children describe the math concepts they learned?
- How were the stories able to build upon the children's prior knowledge to deepen and grow their understanding of math ideas?
- What new math ideas did your child learn from the math stories?
- What was your child's favorite/least favorite part of the math stories/activity book?
- What can we do to improve these stories?

About the Study and the Documentary Video

The video documentary involves a mixed methods research framework and format. This includes (1) experimental—testing the book and activity books in real contexts of children's learning and (2) descriptive- using video technology to capture with audio-video tools the learning experiences. These will include responses, attitudes, motivation to do the work, and feelings about the tasks. Additionally, these also include the teachers' and parents' emotions, dilemmas, instructions, guidance, and the actions and words used in the classroom and at home.

Potential Risks

The interview will be done with an iPhone camera. The study poses no more than minimal risk to participants. Participants have a choice about the extent they wish to be identified in the research.

Potential Benefits

Your participation in the interviews will contribute to this exploratory study which aims to explore the potential of math stories, inspired by pedagogical theories rooted in how children learn best. The stories, which are in the picture books given to your child, provide opportunities to explore conceptual and logical reasoning through open-ended and context-based problem-solving. The goal is for children to learn by empathizing with the characters' challenges and helping them solve math-related real-world problems. Through a process of inventing and communicating their unique solutions, the hope is that your child will learn to do math by modeling the processes of mathematicians. Open-ended, multidimensional math challenges will engage your child in discovery and exploration of new math concepts. Tasks and questions embedded in the stories are envisioned to stimulate logical and analytical thinking, making math learning more than just a linear process of solving for a single correct answer.

Confidentiality

As an adult participant you can choose to agree or not for the author to use your real name and position in the final edited version. Please write your consent in this form.

Participant Rights

As parent/guardian, your participation in this study is completely voluntary. You can refuse to participate in the entire study or any part of the study without any negative effect on your relations with San Jose State University or [name any other participating institutions]. You also have the right to skip any question you do not wish to answer. This consent form is not a contract. It is a written explanation of what will happen during the study if you decide to participate. You will not waive any rights if you choose not to participate, and there is no penalty for stopping your participation in the study.

Questions Or Problems

You are encouraged to ask questions at any time during this study.

- For further information about the study, please contact Gigi Carunungan at 650.714.9620 or email at gigi.carunungan@sjsu.edu.
- Complaints about the research may be presented to Dr. Bradley Portfillo, Bradley.portfillo@sjsu.edu, Ed.D. Program Director
- For questions about participants' rights or if you feel your child has been harmed by participating in this study, please contact Dr. Mohamed Abousalem, Vice President for Research & Innovation, San Jose State University, at 408-924-2479 or irb@sjsu.edu

___Yes___No As parent/guardian, I am allowing the use of my full name
___Yes___No As parent/guardian, I am allowing the use of my first name
___Yes___No As parent/guardian, I am allowing the use of my last name
___Yes___No As parent/guardian, I am allowing the researcher to choose a pseudonym for myself.

Visual/Audio Image Release Form

I grant permission for the researcher Gigi Carunungan to take and use visual/audio images of me. Visual/audio images are any type of recording, including but not limited to photographs, digital images, drawings, renderings, voices, sounds, video recordings, audio clips or accompanying written descriptions. The researcher will not materially alter the original images. I agree that the researcher owns the images and all rights related to them. The images may be used in any manner or media without notifying me, such as university-sponsored web sites, publications, promotions, broadcasts, advertisements, posters, and theater slides, as well as for non-university uses. I waive any right to inspect or approve the finished images or any printed or electronic matter that may be used with them, or to be compensated for them. I release Gigi Carunungan, including any firm authorized by the researcher to publish and/or distribute a finished product containing the images, from any claims, damages, or liability which I may ever have in connection with the taking of use of the images or printed material used with the images. I am at least 18 years of age and competent to sign this release. I have read this release before signing, I understand its contents, meaning and impact, and I freely accept the terms.

Signatures

Your signature indicates that you voluntarily agree to be a part of the study, that the details of the study have been explained to you, that you have been given time to read this document, and that your questions have been answered. You will receive a copy of this consent form for your records.

Participant Signature

Participant's Name (printed) Participant's Signature Date

Researcher Statement

I certify that the participant has been given adequate time to learn about the study and ask questions. It is my opinion that the participant understands his/her rights and the purpose, risks, benefits, and procedures of the research and has voluntarily agreed to participate.

Signature of Person Obtaining Informed Consent Date

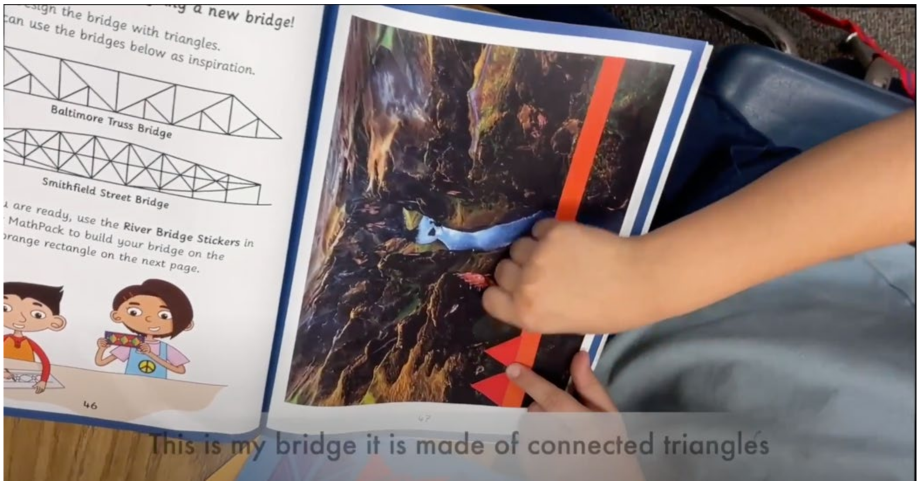
APPENDIX D

Clips from the Documentary

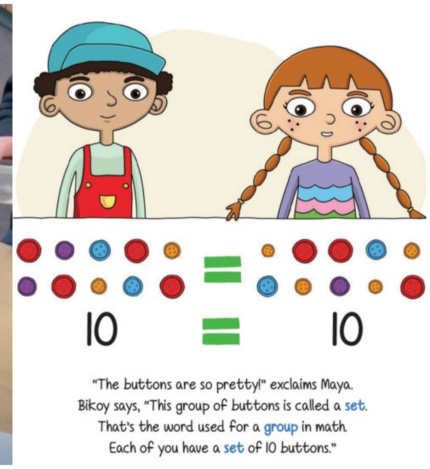
Open-ended and context-based problem-solving were ideas for explorations in conceptual and logical reasoning.

Children learned by empathizing with the characters' challenges. Doing math was associated with helping the characters.

Students generated responses to challenges from story contexts with fictional applications of real-world problems that became seedbeds of math ideas.

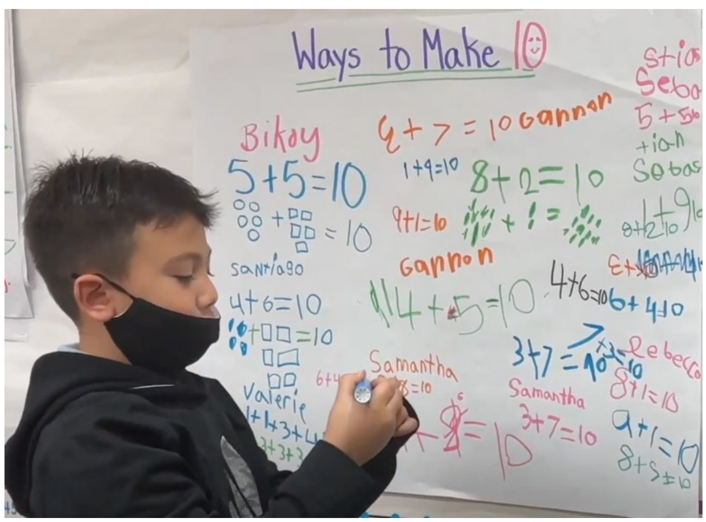


Children learned mathematical thinking by simulating the modeling processes of mathematicians through a non-linear process of figuring out, modeling, generalizing, and communicating.

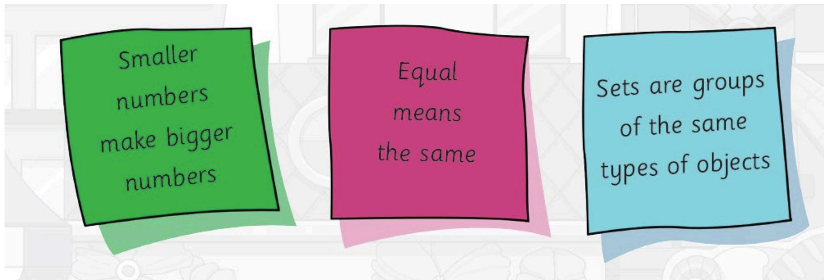


Children discovered and explored math concepts through open-ended math challenges.

Math learning became more than just a linear engagement of solving for the correct answer.



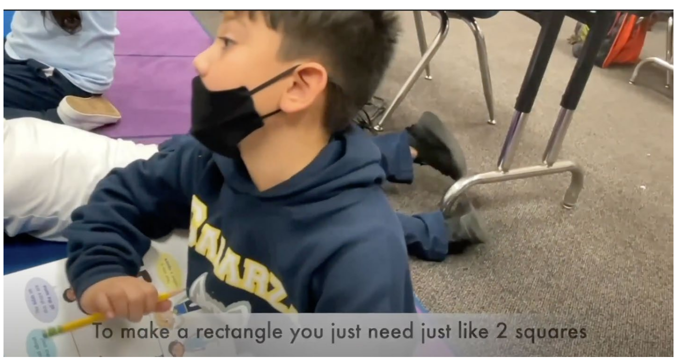
The questions embedded in the stories, stimulated logical and analytical thinking exemplifying how math ideas guide problem solving.



Math learning was exciting because students experienced how much their views mattered.

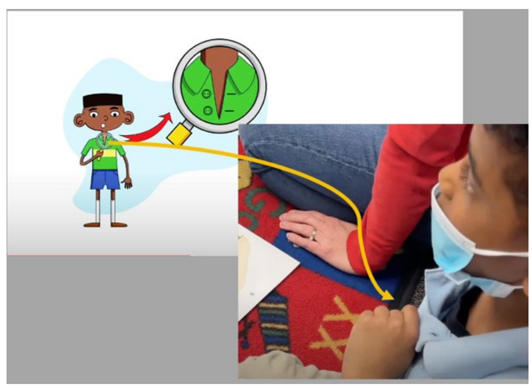
The experiences were meaningful because children were driven by purpose, i.e., to help the characters solve their problems in the story.

The stories kept students focused. The children were enthralled as they listened to the text being read. With the support of the images, they were able to visualize the narrative and the math ideas.



The story shifted students' attitudes from fear of having to be correct to get the teacher's approval, to excitement towards being able to contribute their knowledge, abilities, and insights in open-ended mathematical discovery and exploration.

The students saw themselves in the characters.



Mathematical sense making and reasoning was achieved without the use of rote learning.

The learning designs of the extension activities such as the card games and creative projects, generated connections to the story, cultivated positive social interactions, and motivated children to practice and master learning the math concepts.