AN EVALUATION OF ACTIVITY-BASED COSTING AND FUNCTIONAL BASED COSTING: A GAME THEORETIC APPROACH

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CHAPTER 1: INTRODUCTION

1.1 Statement of Problem

Activity-based costing (ABC) has been implemented by numerous entities because they believe it is superior to functional based costing (FBC). ABC's *assumed* superiority stems from the belief that it provides a relatively more accurate cost assignment than FBC. The accuracy belief, which is based on a notion of relative accuracy, is founded in the causal criterion. Under the causal criterion, relative accuracy is assumed to increase as the number of cause-and-effect relationships used in a cost assignment system increases. Thus, since ABC uses *more* cause-and-effect relationships in assigning costs than FBC, it is considered relatively more accurate and consequently a superior cost assignment system.

The evaluation of competing cost systems is often based on materiality. Since ABC provides numbers that are materially different from the numbers generated by FBC, it is believed that the new numbers lead to desirable changes in product pricing, product mix, make or buy and other cost based decisions. However, Dopuch (1993, pg. 618) states that "no materiality study can demonstrate that one set of cost estimates is more accurate than another unless the researcher knows the true cost function." So far there have been few, if any, efforts to identify the true cost to enable firms to evaluate competing cost systems in terms of relative accuracy. This is potentially costly to a firm that bases cost system implementation on the *assumption* that new cost numbers are more accurate. Thus, a demand is created for criteria that can be used to assess the relative accuracy of competing cost assignments. In fact, Dopuch argues that economically relevant benchmarks are needed to evaluate alternative accounting numbers. These statements suggest that a demand exists for identifying an economically relevant benchmark that can be used to evaluate cost assignment accuracy.

1.2 Purpose

The primary purpose of this study is to develop a theoretical benchmark that can be used to evaluate competing cost assignment systems. A theoretical benchmark represents a measure of product cost that can be used as a reference to compare alternative cost assignments. Microeconomic theory and cooperative game theory are two possible analytical frameworks by which a theoretical measure of product cost can be developed. Both frameworks contribute in the development of a theoretical measure of product cost that is independent of cost assignment concepts. However, cooperative game theory provides a richer framework for identifying conditions under which statements can be made regarding the superiority of a specific cost assignment system.

This study identifies conditions under which ABC can be argued to be a rationally superior cost assignment system using a game-theoretic paradigm. This study also develops a measure for the degree of product diversity that allows conditions to be identified that cause FBC to violate the requirements of cooperative game theory solution concepts. Identification of these conditions also allows the identification of limitations and possible theoretical improvements to ABC.

A secondary purpose of the study is an empirical investigation of conditions affecting cost assignments identified in the theoretical analysis. Specifically, the effect of increased product diversity on cost assignments and the choice of competing ABC cost drivers will be investigated.

1.3 Literature Review: Evaluation of ABC

In evaluating the accuracy of ABC, prior research has embraced the assumption that a complete disaggregation of a company's activities leads to the identification of all cause-and-effect relationships, which then leads to a benchmark of the underlying true cost. Previous ABC studies have been motivated by the need to provide better accounting numbers to managers than what they had been receiving in the past. A subjective criterion of better is often assumed to follow from the premise that increasing the number of cause-and-effect relationships, as measured by the number of cost drivers, will bring researchers closer to the true product cost (Dopuch, 1993).

Studies that evaluate ABC assume the best benchmark to assess accuracy is one that identifies all activities of the company. Babad and Balachandran (1993) assume that improved accuracy in cost assignments is achieved "by using multiple cost drivers to trace the cost of activities to the products associated with the resource consumed by those activities…"(p. 563). They suggest that to derive true cost, all activities of a company must be identified. Datar and Gupta (1994) agree that true cost is achieved when all activities are identified and proceed to use this benchmark to assess loss of accuracy when cost drivers are misspecified. These two studies attempt to address a trade-off between information processing costs and cost assignment accuracy.

Other studies that evaluate ABC use the same benchmark to suggest that developing methods for improving the identification of cause-and-effect relationships increases accuracy. Datar et al. (1993) suggest that cost drivers should be determined by considering the interdependencies of the activities of the company. Accuracy is then evaluated based on a statistical criterion for measuring when cost drivers are better identified. In a related study, Datar and Gupta (1996) suggest that accuracy in a cost system increases if refinements are made to the system which consider the interdependence of processes and product costs in identifying cost drivers. These two studies assume that the benchmark defined is correct and proceed to use the benchmark to assess the accuracy of ABC.

Of the studies identified above, none have attempted to evaluate ABC and FBC as competing cost assignment systems. Each study evaluated ABC and assumed that the identification of all activities leads to a true cost benchmark and increasing the detail in a cost system will increase the accuracy of the cost assignment. The benchmark used in these studies was developed from the causal criterion, which is the foundation of ABC, and therefore is not independent of ABC. The conclusion that can be drawn from the above studies is that no effort has been made to develop an *independent* theoretical measure of product cost.

Dopuch argues that accuracy cannot be assessed unless the true cost function is known, which implies true product cost must be known. In a single product setting, true product cost is known and accuracy is not an issue as ABC and FBC provide the same cost assignment. It is only in a multiple product setting, where products use shared resources, that the accuracy of competing cost systems becomes an issue. The cost of shared resources represents the cost of producing multiple products in a single plant. A theoretical measure of product cost can be developed by separating, or decomposing, the cost of shared resources to create a separable cost function that allows costs to be traced to individual products. Therefore, in addition to a multiple product setting and the identification of multiple product cost functions, developing a theoretical measure of product cost also requires a method of decomposing the cost of shared resources. One possible method of decomposing a shared cost is cooperative game theory, which provides a concept of accuracy based on rationality axioms. This concept of accuracy becomes an independent theoretical product cost benchmark by which competing cost systems can be evaluated.

The remaining sections of the paper are organized as follows. Chapter 2 identifies two possible subadditive cost functions that exist in multiple product plants. Chapter 3 presents the development of a theoretical product cost benchmark that relies on cooperative game theory as a method of decomposition. Chapter 4 defines four possible cooperative game theory solution concepts. Chapter 5 presents a development of a measure for the degree of product diversity. Chapter 6 and Chapter 7 provide an analytical evaluation of ABC and FBC and the identification of superiority conditions based on cooperative game theory solution concepts. An empirical investigation of factors affecting cost assignments identified in the theoretical analysis is presented in Chapter 8 and the conclusions are summarized in Chapter 9

CHAPTER 2: SUBADDITIVE COST FUNCTIONS

Developing a theoretical measure of product cost, based on a decomposition of the cost of shared resources in a multiple product setting, requires the specification of cost functions associated with shared resources. In a single product setting, microeconomic theory specifies cost functions that represent measures of true product cost. However, to assess accuracy, the single product measure must be extended to a multiple product setting. Economies of scope provide justification for multiple product settings and also the opportunity to identify cost functions of shared resources.

2.1 Single Product Settings: Microeconomic Theory

Microeconomic theory has been used to examine the consistency of neoclassical economic theory with the concepts of product costing. Using the microeconomic framework, Christenson and Demski (1992) develop a true product cost benchmark based on cost function separability and derived demand elasticities (derived demand elasticities indicate how an input factor demand changes in response to price changes of another input factor). They derive input factor demands and group similar derived input demand elasticities to form cost pools. The true product cost benchmark is then determined by assigning the cost pools to products using cost drivers based on a measure of output specified by the production function, where the production function is assumed to be known. Although they attempt to apply these concepts beyond a single product setting, their analysis relies on a single production function, which fails to capture the properties of a multiple product setting.

Most microeconomic analysis of product costing relies exclusively on a single-product production function. However, in a single-product setting, ABC and FBC provide the same cost assignment and, therefore, both ABC and FBC advocates admit that there is no accuracy problem for a single product firm. Only multiple product firms that offer an environment where products use shared resources create a product-costing problem. Thus, microeconomic theory can establish a true product cost for a single product but alone it appears inadequate for evaluating the relative accuracy of competing cost systems because of an inability to discriminate between ABC and FBC. Therefore, the single product concept of product cost must be extended to a multiple product setting to develop an independent theoretical measure of product cost to evaluate relative accuracy.

For multiple product plants to exist, a multiple product setting must provide a benefit over single product settings. Justification for a multiple product plant exists if it is less costly to produce multiple products in a single plant than to produce them individually in multiple, single product plants. The cost savings that arise by combining multiple single product plants represent a shared benefit related to production. As described in the next section, the existence of economies of scope, which implies the possibility of cost savings, drives the demand for multiple product plants.

2.2 Multiple Product Settings: Economies of Scope

Economies of scope is a concept provided by industrial economics and organizational structure theory that addresses the issue of why multiple product plants exist. Panzar and Willig (1981) coined the term economies of scope to describe conditions where it is less costly to combine two or more product lines in one plant than to produce them separately. Baumol, et al., (1982), describe economies of scope as a necessary condition for the existence of multiple product plants. Bailey and Friedlaender (1982) state that economies of scope measure the cost advantages to firms of providing a large number of diversified products as opposed to specializing in the production of a single product. With the existence of economies of scope, cost savings result from the scope of the firm and economies of sharing, rather than the scale of the firm. Therefore, when economies of scope is invoked, a multiple product plant is expected.

A necessary condition for invoking economies of scope is cost function subadditivity, which requires the total cost of providing a shared input to be less than the total cost of providing the input to each plant separately. When the cost of a sharable input is subadditive, cost savings are possible in a multiple product plant, which is necessary for economies of scope to exist. Cost function subadditivity can be illustrated in a multiple product setting by assuming n independent firms exist, $A_1,\ldots, A_n$, each producing a different product with characteristics represented by $a_1,\ldots,a_n$, respectively. The cost functions of each independent firm can be expressed as $C_1(a_1),\ldots, C_n(a_n)$ [1]. Economies of scope imply an n-product plant will exist when the cost function of a shared resource is strictly subadditive such that, $C_N(a_1,\ldots,a_n) < \Sigma\ C_i(a_i)$, where $i = 1,\ldots, n$ and $C_N(a_1,\ldots,a_n)$ represents the total cost of the shared resource in the n-product plant. A cost function exhibits weak subadditivity where $C_N(a_1,\ldots,a_n) \leq \Sigma\ C_i(a_i)$.

[1] For a single product firm, microeconomic theory expresses the cost function as $C(q_i)$, where q_i is the quantity of a_i produced. Thus, $C(a_i) = C(q_i)$ and the true product cost is a function of output. Expressing cost as a function of the characteristics of a product, a_i, is desirable in a multiple product environment since it allows for a less restrictive expression which does not imply a microeconomic framework where cost is assumed only to be a function of output.

When economies of scope exist, a shared benefit (i.e., cost savings) arises from forming the n-product plant and is defined as $\Sigma\ C_i(a_i) - C_N(a_1,\ldots,a_n)$. For example, in a two firm setting the necessary condition is satisfied if $C_{12}(a_1, a_2) < C_1(a_1) + C_2(a_2)$. The cost savings for the two-product plant is defined as $C_1(a_1) + C_2(a_2) - C_{12}(a_1, a_2)$, which drives the demand for the two-product setting.

By invoking economies of scope, the formation of a multiple product plant is expected and a justification for moving from a single product setting to a multiple product setting exists. A theoretical measure of product cost will be identified when subadditive shared resources can be decomposed. For example, in the two-product case $C_1(a_1) + C_2(a_2) > C_{12}$ implies the existence of economies of scope and a subadditive cost function, C_{12}. If the total cost, C_{12}, can be separated into $C_1^*(a_i)+ C_2^*(a_2) = C_{12}$, then determining $C_1^*(a_1)$ and $C_2^*(a_2)$ will provide a theoretical measure of product cost for the two product plant. However, before the cost of the shared resource can be decomposed and traced to individual products to specify $C_i^*(a_i)$, it is necessary to specify C_N.

2.3 Subadditive Cost Functions

Economies of scope imply the existence of a multiple product plant and a necessary condition for economies of scope is the presence of a sharable input exhibiting a subadditive cost function. This means that when economies of scope is invoked, shared inputs with subadditive cost functions exist in the multiple product plant. The identification and decomposition of subadditive cost functions is justified because of their required existence for invoking economies of scope.

Economies of scope literature is limited regarding the identification of specific sharable inputs that exhibit cost function subadditivity. According to Panzar and Willig (1981, p. 268), "when there are economies of scope, there exists some input … that is shared by two or more product lines without complete congestion," which implies "…the presence of a sharable, 'quasi-public' input." Panzar and Willig present a model of input sharing to address the identification of sharable, quasi-public inputs. Their model assumes a multiple product plant producing quantities $\boldsymbol{y} = (\boldsymbol{y_1},\ldots,\boldsymbol{y_n})$ with n independent processes which are able to share one input, $\boldsymbol{K} = (k_1,\ldots,k_n)$ with factor price β. The model is specified as follows:

$$C(y_S) \equiv \min_{k} \sum_{i \in S} V^i(y_i, k_i) + \psi(k,\beta) \tag{1}$$

$C(y_S)$ = the minimum cost of producing S products, $S \subseteq N$,

$V^i\,(y_i, k_i)$ = the minimum variable cost of producing output y_i using k_i units of capital services,

$\psi(k, \beta)$ = the cost of acquiring $\boldsymbol{K}$.

The model is used to illustrate two extreme examples of inputs where the resulting cost functions do not exhibit strict subadditivity. At one extreme $\boldsymbol{K}$ is a pure public input which is not subadditive, while at the other extreme $\boldsymbol{K}$ is a pure private input which is only weakly subadditive. When $\boldsymbol{K}$ is a pure public input, the cost of acquiring $\boldsymbol{K}$ is $\psi(k, \beta) = \beta(max\ k_i)$. A pure public input is openly available for consumption and the formation of multiple product plants does not create cost savings where

$C_N(a_1,\ldots,a_n) = \Sigma\, C_i(a_i)$ is not subadditive. When $\boldsymbol{K}$ is a pure private input, the cost of acquiring the input is $\psi(k, \beta) = \beta \sum_i k_i$. A pure private input is divisible in such a way that a plant can purchase only what is needed and the possibility of cost savings may not always exist for a multiple product plant, where $C_N(a_1,\ldots,a_n) \leq \Sigma\, C_i(a_i)$ is only weakly subadditive.

Panzar and Willig provide two general examples of quasi-public inputs falling in between the two extremes that exhibit strict subadditivity and allow economies of scope to be invoked. The first example is an input that is imperfectly divisible in such a way that the production of one or a small set of products would leave unused capacity of that input. The second example is an input that has some properties of a public input such that when it is purchased for one production process it would be freely available for the production of another product. These two examples provide general characteristics of quasi-public inputs but are too general to prevent the identification of specific subadditive cost functions.

Bailey and Friedlaender (1982) recognize the need to identify sources of economies of scope and provide specific examples of shared resources with subadditive cost functions. Two examples identified in their study are described in the next sections; fixed factors of production and inputs exhibiting economies of scale effects[2].

2.3.1 Fixed Factors of Production

One possible source of economies of scope results from the sharing of imperfectly divisible inputs that once acquired for the production of one output would also be available, in part or in whole, for the production of other outputs. This type of input, identified by Bailey and Friedlaender as a fixed factor of production, provides the opportunity to exploit some type of unused capacity. Examples include indivisible equipment usable for more than one manufacturing process and human capital applicable to the production of more than one output. Fixed factors of production are typically committed resources that are purchased in lumps, or steps, prior to consumption such as production facilities or purchasing agents. It is not known at the time of acquisition if the fixed factor will be fully utilized. If the fixed factor is underutilized, unused capacity exists that may be reduced or eliminated in a multiple product plant. For example, cost savings will arise due to fixed factor inputs when two independent firms, each employing one underutilized purchasing agent, combine operations. After combining, the multiple product plant only requires one purchasing agent and cost savings arise due to the reduction or elimination of unused resource capacity.

Fixed factors of production are represented by step cost functions. Assume input j is a fixed factor of production. A step of input j is defined as S_j, which represents a fixed amount of input that must be purchased, regardless of actual consumption. Let the number of steps of input j needed to be acquired for product i be defined as K_{ij}, where the total amount available is S_jK_{ij}. Let z_{ij} represent the demand of product i for input j, where the amount of unused resource capacity can be expressed as $S_jK_{ij} - z_{ij}$. The cost of product i is represented by the following step cost function:

[2] Other subadditive cost functions exists. The two chosen for this study occur more frequently and also reflect basic characteristics desired for evaluating competing cost assignments.

$$C_i = \sum_j P_j K_{ij} \tag{2}$$

P_j = price of one step of fixed input j,

K_{ij} = number of steps required for product i, fixed input j,

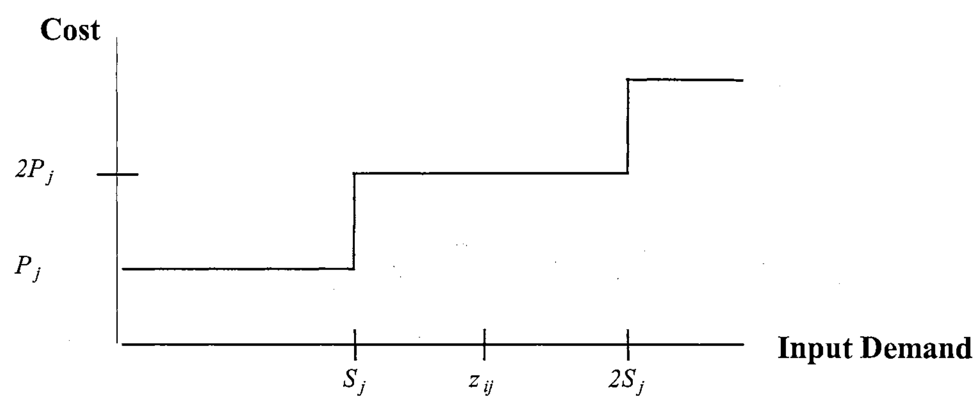
S_j = step size (i.e., capacity purchased in one step) for fixed input j,

z_{ij} = amount of fixed input j demanded by product i, where

$S_j (K_{ij} - 1) < z_{ij} \leq S_j K_{ij}$

where i is the number of products, $i = 1, ..., n$ and j is the number of fixed inputs in the n product plant, $j = 1, \ldots, m$. Figure 2-1 graphically illustrates the cost function of a fixed factor of production, where $K_{ij} = 2$.

Figure 2-1. Fixed Factor of Production



By definition, when economies of scope arise from fixed factors of production, unused capacity exists in the single product plant such that, $z_{ij} < S_j K_{ij}$, or $\frac{z_{ij}}{S_j} < K_{ij}$. When a multiple product plant is formed, cost savings result when this unused capacity is reduced or eliminated. Therefore, cost savings imply that the number of steps required to be purchased in the single product settings, $\sum_i K_{ij}$, is reduced in the multiple product plant, where $\sum_i \frac{z_{ij}}{S_j} < \sum_i K_{ij}$. For example, if at least one step is saved in the multiple product plant then $\sum_i \frac{z_{ij}}{S_j} \leq \sum_i K_{ij} - 1$, or $\sum_i z_{ij} \leq S_j(\sum_i K_{ij} - 1)$. The maximum number of steps that can be saved in an n-product plant is defined when the total input demand, $\sum_i z_{ij}$, is determined. By definition, $S_j\,(K_{ij} - 1) < z_{ij} \leq S_jK_{ij}$. Total input demand is found by summing this condition over i:

$$S_j\,(\sum_i K_{ij} - n) < \sum_i z_{ij} \leq S_j \sum_i K_{ij}. \tag{3}$$

The left-hand side of equation (3) indicates that the number of steps required for the n-product plant is greater than $\sum_i K_{ij} - n$, which implies that the minimum number of steps required is $\sum_i K_{ij} - (n - 1)$. Therefore, the maximum number of steps that can be saved in an n-product plant is the difference between the number of steps required for independent production and the minimum number required in the n-product plant, which is $\sum_i K_{ij} - (\sum_i K_{ij} - (n - 1))$, or $(n - 1)$ steps.

2.3.2 Fixed Factor of Production - Example

The properties of a fixed factor of production can be illustrated for a three-product setting, where each product requires purchasing labor input. The cost of purchasing labor input is a function of the number of labor hours consumed by purchasing. However, purchasing labor is acquired based on the volume of purchase orders required for production. Assuming that the same amount of time is needed to process each purchase order, a product's consumption of the purchasing labor input can also be represented by the number of purchase orders needed to be processed. The following example assumes one purchasing agent is capable of processing 500 purchase orders at a cost of $5,000 per period. Information regarding the purchasing labor demands of each single product plant and the three-product plant are as follows:

Purchasing Labor Input:		Single Product Plants			Three-Product Plant
S_1=500, P_1=\$5,000		1	2	3	123
Purchase orders needed	z_{ij}	945	1,050	1,505	3,500
Number of steps required	K_{ij}	2	3	4	7
Practical capacity	S_jK_{ij}	1,000	1,500	2,000	3,500
Unused capacity	S_jK_{ij}-z_{ij}	55	450	495	0

For this example, the formation of the three-product plant eliminates all unused capacity existing in the single product plants. The three-product plant requires seven purchasing agents, which is a savings of two agents over the number required if each plant operated independently.

The cost structure of a fixed factor of production can also be analytically illustrated in a three-product setting, $i = 1,\ldots,3$, with three inputs, $j = 1, \ldots,3$. If t_j represents the number of steps saved in a three-product plant for input j, where $1 \leq t_j \leq 2$, the production costs can be represented by the following:

Cost of Single Product Plants:

$$C_1 = P_1K_{11} + P_2K_{12} + P_3K_{13},$$

$$C_2 = P_1K_{21} + P_2K_{22} + P_3K_{23},$$

$$C_3 = P_1K_{31} + P_2K_{32} + P_3K_{33}.$$

Cost of Three-Product Plant:

$$C_{123} = P_1(K_{11}+ K_{21}+K_{31} - t_1) + P_2(K_{12} + K_{22} + K_{32} - t_2) + P_3(K_{13} + K_{23} + K_{33} - t_3).$$

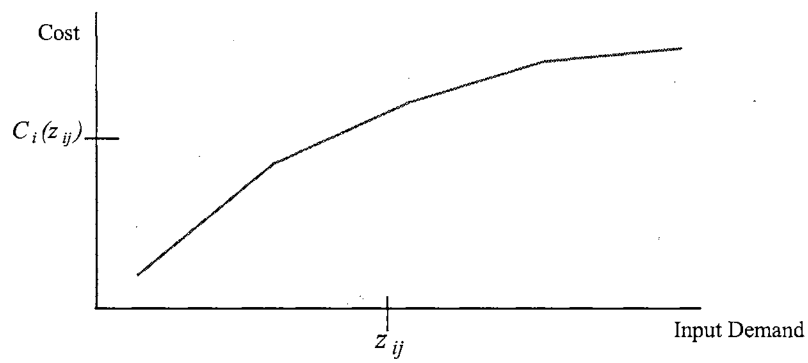
2.3.3 Economies of Scale Effects

Bailey and Friedlaender suggest sharable inputs exhibiting economies of scale effects as another source of economies of scope with subadditive cost functions. Inputs exhibiting economies of scale effects have some properties of a public input. When economies of scale effects exist, the underlying cost function of the sharable input exhibits a decreasing average cost per unit, which indicates subadditivity. If firms combine and share inputs with this type of cost function, the average cost per unit decreases and cost savings result for the multiple product plant. For example, assume the production process of two single product plants requires the use of similar production machines. If the single product plants combine, it is possible that one machine capable of handling the production volume of both products can replace the two smaller machines used in the single product plants. Cost savings will arise due to a lower average cost per unit associated with using one larger machine in the multiple product plant. For inputs exhibiting economies of scale effects, cost is a function of input usage and is specified as follows:

$$C_i = \sum_j C(z_{ij}), \tag{4}$$

where $C'(\cdot) > 0$ and $C''(\cdot) < 0$. Figure 2-2 graphically illustrates the cost function of an input exhibiting economies of scale effects.

Figure 2-2. Economies of Scale



Cost savings in a multiple product setting are due to a decreasing average cost per unit. The possible cost savings are a function of total input demand and are defined as $\sum_i C(z_{ij}) - C(\sum_i z_{ij})$, for input j.

2.3.4 Economies of Scale Effects - Example

The properties of an input exhibiting economies of scale effects can be illustrated with a three-product setting where the production of each product can be achieved with the same type of machine in the multiple product plant. The input demands and the average cost per unit for the single product plants and the three-product plant are as follows:

Machining Input:

		Single Product Plants 1	Single Product Plants 2	Single Product Plants 3	Three-Product Plant 123
Machine hours needed	z_{ij}	3,960	6,600	22,440	33,000
Input cost	$C(z_{ij})$	\$31,680	\$39,600	\$89,760	\$115,500
Average cost/machine hour	$C(z_{ij})/z_{ij}$	\$8.00	\$6.00	\$4.00	\$3.50

The cost structure of a sharable input exhibiting economies of scale effects can also be illustrated analytically in a three-product setting, $i = 1,\ldots,3$, with three inputs, $j = 1,\ldots,3$, as follows:

Cost of Single Product Plants:

$$C_1 = C(z_{11}) + C(z_{12}) + C(z_{13}),$$

$$C_2 = C(z_{21}) + C(z_{22}) + C(z_{23}),$$

$$C_3 = C(z_{31}) + C(z_{32}) + C(z_{33}).$$

Cost of Three-Product Plant:

$$C_{123} = C(z_{11} + z_{21} + z_{31}) + C(z_{12} + z_{22} + z_{32}) + C(z_{13} + z_{23} + z_{33}).$$

Two subadditive cost functions have been identified and a theoretical measure of product cost can be developed by decomposing C_N, where C_N is represented by either a fixed factor of production, an input exhibiting economies of scale effects, or a combination of both. A method to decompose C_N must now be identified. One possible method of decomposition is cooperative game theory, which relies on rationality axioms. Chapter 3 develops a theoretical measure of product cost based on a rational decomposition of C_N using cooperative game theory concepts.

CHAPTER 3: A THEORETICAL MEASURE OF PRODUCT COST

At least two frameworks provide potential guidance for developing an independent theoretical measure of product cost that can be used in evaluating cost assignment accuracy: microeconomic theory and cooperative game theory. Both frameworks offer the possibility of identifying a theoretical measure of product cost. Microeconomic theory, which was discussed in Chapter 2, provides a theoretical measure of product cost in a single product setting. However, cooperative game theory is needed to extend the microeconomic concepts of product cost to multiple product settings where product-costing accuracy becomes an issue.

3.1 Cooperative Game Theory

Cooperative game theory applies in settings where two or more parties participate in dividing a joint benefit. This framework has been used to solve the joint costing problem of cost accounting (Hamlen et al., 1980, Gangolly, 1981, and Loehman and Whinston, 1976). For example, a multiple division firm may have a single purchasing department because it is less costly than each division conducting purchasing activities independently. The firm faces the problem of dividing the cost of the single purchasing department among the divisions. Cooperative game theory provides the framework by which divisional managers, acting as rational economic agents, decide how the shared cost will be divided. Thus, the amount assigned to each division is a *rational cost assignment*. This suggests the possibility of using rationality as the basis for decomposing the cost of shared resources in a multiple product setting into separable cost functions – one for each product. Thus, in a multiple product setting, the game-theoretic paradigm defines theoretical product cost as a rational cost assignment.

3.1.1 Cooperative Game Theory and Product Costing

To apply cooperative game theory concepts to the product-costing problem, a justification must exist for producing multiple products in a single plant. The justification for multiple product settings is provided by economies of scope, as described in Chapter 2. The cost savings that arise by combining multiple single product plants into one multiple product plant represent a shared benefit related to production, which must be divided among the products. Under a cooperative game theory framework, a rational division of the benefit will take place. A rational division of the benefit, combined with the single product measure of true product cost, will provide a theoretical measure of product cost in a multiple product setting.[3]

The characteristics of a multiple product setting can be expressed in terms of game-theoretic concepts. Generally, a game can be expressed in extensive, normal or characteristic function form, with the characteristic function form being the most abstract. The characteristic function form of a game can be used to express the costs and cost savings from forming a multiple product plant. Thus, it can also be used to capture the economics of a rational decomposition of a shared cost.

[3] Microeconomic theory specifies the production cost function and, thus, true product cost for a single product setting.

3.1.2 Characteristic Function Form of a Game

The characteristic function assigns a value to a combination, i.e., a coalition of players acting together. The value represents the maximum the coalition can guarantee itself by coordinating the strategies of its members, no matter what the other players do.

Formally, the characteristic function is defined over a set of N players in which $S \subset N$ represents a possible coalition. The characteristic function is represented by $v(S)$, the value of forming coalition S. In a non-zero sum game, the characteristic function must satisfy the following conditions (Luce and Raiffa, 1957):

1. $v(\phi) = 0$, where ϕ represents the empty set
2. $v(R \cup S) \geq v(R) + v(S)$, where R and S represent disjoint subsets of N.

Condition 1 indicates that the subset involving no players has no value, i.e. no players win, but no players lose. Condition 2 implies that a coalition of R and S can achieve anything R and S can do acting alone, and possibly more. After forming, the members of coalition S will share in the benefit of cooperating. The benefit in forming S is defined as $\sum v(i) - v(S)$, where $v(i)$ represents the value to player i acting alone. The benefit must be divided among the players in S. Player i's allocation of the shared benefit is represented by x_i, where $i \in S \subset N$, and the set of all rational allocations is the payoff vector defined by $\boldsymbol{x} = (x_1,\ldots, x_n)$ for an n-player game.

The characteristic function can be applied to the product-costing problem to represent the value of forming a multiple product plant. A necessary condition for cooperative behavior, and for invoking economies of scope, is $C_N(a_1,.., a_n) < \Sigma\ C_i(a_i)$. The ability to invoke economies of scope insures that a multiple product plant will form. Let the total cost of an n product plant be defined as $v(N) = C_N(a_1,.., a_n)$ and let the cost of each plant operating independently be defined as $v(i) = C_i(a_i)$. The cost savings from forming an n-product plant, $\Sigma\ C_i(a_i) - C_N(a_1,.., a_n)$, is the shared benefit that must be divided among the n products, which is specified by $\boldsymbol{x} = (x_1,\ldots, x_n)$. For example, in a setting where two independent plants combine, a characteristic function represents the total cost to the multiple product plant, $v(A_1, A_2) = C_{12}(a_1, a_2)$, and the cost to each firm operating independently, $v(A_1) = C_1(a_1)$ and $v(A_2) = C_2(a_2)$. The cost savings achieved by forming a multiple product plant is $C_1(a_1) + C_2(a_2) - C_{12}(a_1, a_2)$. The two owners, acting as rational economic agents, will divide the cost savings. A payoff vector, $\boldsymbol{x} = (x_1, x_2)$ where $x_1, x_2 > 0$, represents the set of rational divisions of the cost savings to each owner.

3.2 A Theoretical Measure of Product Cost

By combining both microeconomic and cooperative game theory concepts, a theoretical measure of product cost is derived. Microeconomic theory provides a measure of the theoretical product cost in a single product setting, $C_i(a_i)$ (i.e., all costs are assigned to the single product in a single product setting). This concept is extended to a multiple product setting to address the accuracy of competing cost assignments. Under cooperative game theory, the shared benefit in forming a multiple product plant is defined as $\Sigma\ C_i(a_i) - C_N(a_1,.., a_n)$ where $\boldsymbol{x} =(x_1,\ldots,x_n)$ is the payoff vector specifying a rational division of the shared benefit. Combining these two concepts gives a theoretical measure of cost for product i in a multiple product setting, defined as $C_i^*(a_i) = C_i(a_i) - x_i$. That is,

$C_i^*(a_i)$ is the microeconomic measure of true product cost in a single product setting less the rational division of the shared benefit assigned to each product. Thus, $C_i^*(a_i)$ represents a rational decomposition of C_N.

This theoretical measure of cost represents an independent economically relevant benchmark that provides one possible concept of accuracy that can be used to evaluate competing cost systems. Accuracy, under the game-theoretic paradigm, is based on how well competing cost systems comply with rational decompositions. Benchmarks are identified when the payoff vector, $\boldsymbol{x}$, is specified. Cooperative game theory offers a number of potential methods for specifying $\boldsymbol{x}$, referred to as solution concepts. Four possible solution concepts include; imputations, the core, the simple Shapley value, and the generalized Shapley value. These four solution concepts are defined and discussed in Chapter 4.

CHAPTER 4: COOPERATIVE GAME THEORY SOLUTION CONCEPTS

The key to identifying the theoretical product cost benchmark is specifying a solution for the payoff vector $\boldsymbol{x}$. For most games many solutions are possible, in fact for some the number of possible solutions is infinite. Cooperative game theory offers suggestions for identifying solutions that conform to rational behavior referred to as solution concepts. Each solution concept provides an opportunity for identifying a benchmark, or set of benchmarks, to evaluate competing cost systems. Four possible solution concepts are used in this study to provide guidelines for evaluating ABC and FBC. Imputations and the core are set solution concepts, while the simple Shapley and the generalized Shapley provide theoretical point predictions for $\boldsymbol{x}$. Each solution concept is described in the following sections.

4.1 Set Solution Concepts

4.1.1 The Set of Imputations

Every player of a cooperative game expects to receive a share of the total benefit. Intuitively, a rational player will not accept a payment that provides him or her with less than they could achieve acting alone. Any payment that divides the total benefit so that this rationality requirement is satisfied is called an *imputation*. For a cost assignment to belong to the set of imputations, two requirements must be met. First, the cost assignment must be *pareto* optimal; thus, for any cost assignment to be an imputation, it must not be possible to reduce the cost of one of the products without increasing the cost of another. Second, cost assignments must satisfy individual rationality. This condition requires that the cost assigned to a product in an n-product plant must be less than or equal to the cost if produced independently in a single product plant. Formally, these conditions can be stated as follows:

1. $\sum_{i} C_B(a_i) = C_N(a_1, \ldots, a_n)$,

2. $C_B(a_i) \leq C_i(a_i)$,

where $C_B(a_i)$ represents the cost assigned to product i in the multiple product plant.

The imputation requirements can be illustrated with the following three-product example, where the independent cost for each product, the cost of each possible two product combination and the total cost of a three product plant are as follows:

$v(A_1) = C_1 = 20$, $\quad v(A_2) = C_2 = 30$, $\quad v(A_2) = C_3 = 40$

$v(A_1,A_2) = C_{12} = 45$, $\quad v(A_1,A_3) = C_{13} = 55$, $\quad v(A_2,A_3) = C_{23} = 65$

$v(A_1,A_2,A_3) = C_{123} = 70$

For this example, a cost assignment will be an imputation when the following requirements are met simultaneously:

1. $\sum_{i} C_B(a_i) = C_N(a_1, \ldots, a_n)$,

   $(20 - x_1) + (30 - x_2) + (40 - x_3) = 70$ or,

   $x_1 + x_2 + x_3 = 20$.

2. $C_B(a_i) \leq C_i(a_i)$,

$20 - x_1 \leq 20,\ \ 30 - x_2 \leq 30,\ \ 40 - x_3 \leq 40$ or,

$x_1, x_2, x_3 \geq 0.$

Thus, any set of cost assignments satisfying $x_1 + x_2 + x_3 = 20$ and $x_1, x_2, x_3 \geq 0$ will be an imputation. For example, if $\boldsymbol{x} = \{5, 5, 10\}$, the cost assignment is an imputation where $C_B(a_1) = 15$, $C_B(a_2) = 25$, and $C_B(a_3) = 30$. Any cost assignment not satisfying these conditions is not rational and fails to be an imputation. For example, if $\boldsymbol{x} = \{-5, 10, 5\}$, the cost assignment is $C_B(a_1) = 25$, $C_B(a_2) = 20$, and $C_B(a_3) = 35$, which violates both *pareto* optimality and individual rationality and is not an imputation.

4.1.2 The Core

Generally, a set of imputations can be defined for an n-product plant that identifies all possible rational cost assignments. However, within this set it is possible for some imputations to be "better" than other imputations. For example, with two imputations, $\boldsymbol{x}$ and $\boldsymbol{y}$, it is possible that for a particular n-product plant, $\boldsymbol{x}$ is better than $\boldsymbol{y}$ for all products. In this sense, $\boldsymbol{x}$ dominates $\boldsymbol{y}$. The set of non-dominated imputations defines the core of a game. A cost assignment will be considered a member of the core when three requirements are met: *pareto* optimality, individual rationality, and group rationality. Group rationality requires that the benefit of any subcoalition, $S \subset N$, be less than the benefit from N. For an n-product game these three conditions define the core and are expressed as follows:

1. $\sum_i C_B(a_i) = C_N(a_1, \ldots, a_n)$,

2. $C_B(a_i) \leq C_i(a_i)$,

3. $\sum_i C_B(a_i) \leq C_S(S)$, where $i \in S$ and $i = 1, \ldots, n$.

It is not guaranteed that the core will always exist. In fact, for many games the core is empty. Fortunately, for a multiple product setting, the core always exists. Proposition 1 states this important outcome.

Proposition 1:

The core will always exist given the formation of a multiple product plant.

Proof:

The core will always exist when the three conditions of core membership are satisfied simultaneously. By definition, $C_B(a_i) = C_i - x_i$, and the core conditions can be expressed in terms of a set of x_i's as follows for a three product setting:

$$1.\ \sum_i C_B(a_i) = C_N(a_1, \ldots, a_n) \Rightarrow x_1 + x_2 + x_3 = \sum_i C_i - C_N, \quad (5)$$

$$2.\ C_B(a_i) \leq C_i(a_i) \Rightarrow x_i > 0, \quad (6)$$

$$3.\ \sum_i C_B(a_i) \leq C_S(S) \Rightarrow x_1 + x_2 \geq C_1 + C_2 - C_{12},$$

$$\Rightarrow x_1 + x_3 \geq C_1 + C_3 - C_{13},$$

$$\Rightarrow x_2 + x_3 \geq C_2 + C_3 - C_{23} \quad (7)$$

The core will exist when a set of x_i's can be chosen to satisfy (5) – (7) simultaneously. It is possible to choose a set of $x_i > 0$ such that $\sum_i x_i = \sum_i C_i - C_N$, which satisfies (5) and (6). By assumption, a three-product plant has formed, which implies economies of scope and the existence of cost savings. This implies that a set of x_i's must also exist that satisfies (7), otherwise two product plants would form and a three product plant would not exist.

If an n-product plant forms there must be a benefit to forming for all n firms, whereby all core requirements are met, otherwise some other combination less than n would define a multiple product plant. Therefore, a set of non-dominated assignments, i.e., the core, will always exist for an n-product plant. Q.E.D.

The boundaries that define the core, given that the core exists under Proposition 1 assumptions, can be illustrated using the three-product example defined previously. The core is a set of non-dominated rational assignments, $\boldsymbol{x} = (x_1, x_2, x_3)$, that satisfy the following requirements simultaneously:

1. $\sum_i C_B(a_i) = C_N(N)$;

   $(20 - x_1) + (30 - x_2) + (40 - x_3) = 70,$

   $x_1 + x_2 + x_3 = 20.$

2. $C_B(a_i) \leq C_i(a_i)$;

   $20 - x_1 < 20, \;\; 30 - x_2 < 30, \;\; 40 - x_3 < 40$

   $x_1, x_2, x_3 \geq 0.$

3. $\sum_{i} C_B(a_i) \leq C_S(S)$, where $i \in S$;

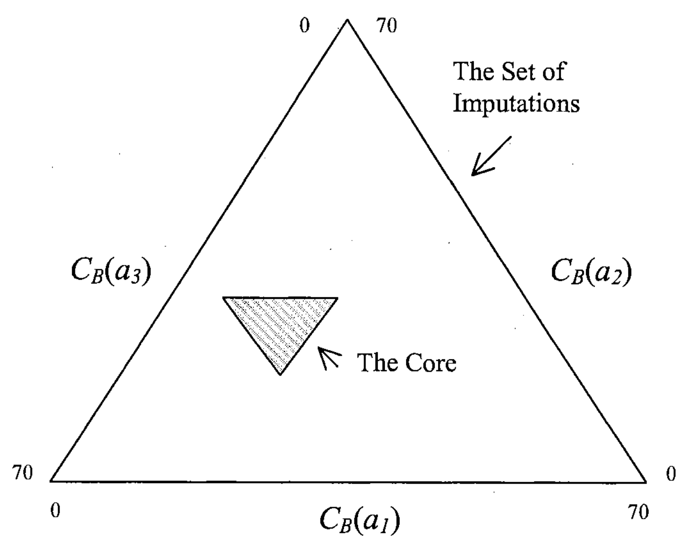
$(20 - x_1) + (30 - x_2) \leq 45,$

$(20 - x_1) + (40 - x_3) \leq 55,$

$(30 - x_2) + (40 - x_3) \leq 65.$

Solving the above requirements simultaneously results in three points that define the boundaries for the set that includes all core assignments of the total benefit: (15, 2.5, 2.5), (2.5, 15, 2.5) and (2.5, 2.5, 15). Figure 4-1 illustrates the boundaries for the set that includes all core assignments of the total cost defined by (5, 27.5, 37.5), (17.5, 15, 37.5) and (17.5, 27.5, 25).

Figure 4-1. Illustration of the Core



4.2 Point Solution Concepts

The simple Shapley value and the generalized Shapley value represent two possible solution concepts that provide theoretical point predictions for the payoff vector, $\boldsymbol{x}$. The simple Shapley value bases allocations of a shared benefit on a set of axioms. The generalized Shapley is derived by relaxing one of the axioms of the simple Shapley. Each of the point predictions is described in the following sections.

4.2.1 Simple Shapley Value

The simple Shapley value prescribes a specific allocation of a shared benefit between participants in a game. Two alternative methods exist for computing the simple Shapley value, with each ultimately specifying the same assignment of the shared benefit. The first method specifies an allocation of the total value of the game, C_N, to each player. The second method specifies an allocation of the shared benefit of cooperation to each player. Each method relies on the following three axioms, stated in a cost assignment context:

1. A product's assignment of cost savings is independent of the product's contribution to the total cost savings;
2. The total cost savings is completely assigned to the n products;
3. For any two residual contributions, an acceptable assignment of the cost savings, x_i, must be additive, or $x_i(d_s + d_s') = x_i(d_s) + x_i(d_s')$, where d_s defines the residual contribution of a subcoalition with s members.

The first method assigns $C_N(a_1,\ldots, a_n)$ to each product and specifies the total cost assigned to each product based on the following formula:

$$C_B(a_i) \;=\; \sum_{S\subset N}\frac{(n-s)!(s-1)!}{n!}[v(S)-v(S-\{i\})], \tag{8}$$

where $C_B(a_i)$ = the cost assigned to product i, $i \in N$; s = the number of products in S, $S \subseteq N$; and n = the total number of products in the n-product firm.

The second method assigns the total benefit, or the cost savings, to each product, which allows for a specification of x_i. This method relies on axiom 3 to decompose the total cost savings and recognizes subcoalitions' contribution to the total cost savings. "Shapley showed that the general function $v(S)$, $S\subset N$ can always be decomposed into the sum of the residual contributions made by all subcoalitions within the coalition S, including S itself" (Hamlen, et al., 1980, p. 273).

The decomposition of the total cost savings into residual contributions can be illustrated with a three-product formula as follows, where the residual contribution of the independent firm is assumed to be zero (i.e. no cost savings are available to the independent firm):

$$\begin{aligned}
d_i \; &= 0,\; i = 1, 2, 3 \\
d_{12} &= (C_1 + C_2 - C_{12}) - d_1 - d_2 = C_1 + C_2 - C_{12}, \\
d_{13} &= (C_1 + C_3 - C_{13}) - d_1 - d_3 = C_1 + C_3 - C_{13}, \\
d_{23} &= (C_2 + C_3 - C_{23}) - d_2 - d_3 = C_2 + C_3 - C_{23}, \\
d_{123} &= (C_1 + C_2 + C_3 - C_{123}) - d_{12} - d_{13} - d_{23} - d_1 - d_2 - d_3 \\
&= (C_1 + C_2 + C_3 - C_{123}) - d_{12} - d_{13} - d_{23}.
\end{aligned}$$

The sum of the residual contributions is equal to the total cost savings of the n-product firm, $\sum d_s = (C_1 + C_2 + C_3 - C_{123})$. This allows the specification of x_i to be expressed as follows:

$$x_i = \sum_i w_i(S)\, d_s\,, \tag{9}$$

where $S \subseteq N$; $i \in S$; w_i = the share of the residual contribution of S assigned to product i; and $w_i = 1/s$, $\sum_i w_i = 1$ and $w_i\,(R) = 0$, $i \notin R$.

Both methods will provide the same value for $C_B(a_i)$ and can be illustrated using the three–product example described previously in Section 3.1. Equation (8) specifies $C_B(a_i)$ as follows:

$$C_B(a_1) = \frac{0!2!}{3!}(C_{123} - C_{23}) + \frac{1!1!}{3!}(C_{12} - C_2) + \frac{1!1!}{3!}(C_{13} - C_3) + \frac{2!0!}{3!}(C_1 - \phi),$$

$$= \frac{1}{3}(70 - 65) + \frac{1}{6}(45 - 30) + \frac{1}{6}(55 - 40) + \frac{1}{3}(20 - 0),$$

$$= 13.33$$

$$C_B(a_2) = \frac{1}{3}(70 - 55) + \frac{1}{6}(45 - 20) + \frac{1}{6}(65 - 40) + \frac{1}{3}(30 - 0),$$

$$= 23.33$$

$$C_B(a_3) = \frac{1}{3}(70 - 45) + \frac{1}{6}(55 - 20) + \frac{1}{6}(65 - 30) + \frac{1}{3}(40 - 0),$$

$$= 33.34$$

The specification of x_i under equation (9), requires the computation of the residual contributions as follows, where the cost savings to the independent plant are assumed to be zero:

$d_1 = d_2 = d_3 = 0,$

$d_{12} = (C_1 + C_2 - C_{12}) = 20 + 30 - 45 = 5,$

$d_{13} = (C_1 + C_3 - C_{13}) = 20 + 40 - 55 = 5,$

$d_{23} = (C_2 + C_3 - C_{23}) = 30 + 40 - 65 = 5,$

$d_{123} = (C_1 + C_2 + C_3 - C_{123}) - d_{12} - d_{13} - d_{23} = (20 + 30 + 40 - 70) - (5) - (5) - (5) = 5.$

Equation (9) specifies x_i as follows:

$$x_1 = \frac{1}{2}(d_{12}) + \frac{1}{2}(d_{13}) + \frac{1}{3}(d_{123})$$

$$x_1 = \frac{1}{2}(5) + \frac{1}{2}(5) + \frac{1}{3}(5) = 6.67,$$

$$x_2 = \frac{1}{2}(5) + \frac{1}{2}(5) + \frac{1}{3}(5) = 6.67,$$

$$x_3 = \frac{1}{2}(5) + \frac{1}{2}(5) + \frac{1}{3}(5) = 6.67,$$

where $C_B(a_1) = 20 - 6.67 = 13.33$, $C_B(a_2) = 30 - 6.67 = 23.33$, and $C_B(a_3) = 40 - 6.67 = 33.34$.

Note that the assignment specified by the simple Shapley value is an imputation and a core member for the example provided. In fact, if the core exists the simple Shapley will provide assignments in the core (Hamlen, et al. 1980).

4.2.2 Generalized Shapley Value

Axiom 1 of the simple Shapley value has been criticized since it does not allow consideration of the relative contribution each product makes in generating the shared benefit from forming a multiple product plant. The relaxation of axiom 1 has led to a class of generalized Shapley assignments that allow the total cost savings to be fully assigned, but the assignment to each product does not need to be independent of contribution. Replacing axiom 1 of the simple Shapley with a set of weights that reflect relative contribution derives an appealing candidate for a rational point solution. For example, in a joint costing setting where costs are shared between divisions, Hamlen et al. (1980) suggest that the relative resource demands of a division be used as the weight for assignment. For a product-costing setting, measures that define relative contribution include the use of the relative resource demands of the products or the relative amount of unused resource capacity. Therefore, by replacing axiom 1 of the simple Shapley solution concept with relative contribution measures, the generalized Shapley solution is derived as another theoretical point prediction of a rational product cost.

The generalized Shapley relies on the decomposition properties of the simple Shapley value and specifies x_i as follows:

$$x_i = \sum_i w_i(S)\, d_s, \tag{10}$$

where $S \subseteq N$; $i \in S$; $w_i(S)$ = the assignment weight for product i; $\sum_i w_i(S) = 1$, and

$w_i(R) = 0,\ i \notin R$.

The generalized Shapley can be illustrated using the three-product example described previously. The example assumes that a measure of relative contribution is represented by relative cost and axiom 1 of the simple Shapley is replaced with a set of weights based on the relative cost to each independent firm. Equation (10) specifies x_i as follows:

$$x_1 = \frac{C_1}{(C_1 + C_2)}(d_{12}) + \frac{C_1}{(C_1 + C_3)}(d_{13}) + \frac{C_1}{(C_1 + C_2 + C_3)}(d_{123})$$

$$x_1 = \frac{2}{5}(5) + \frac{1}{3}(5) + \frac{2}{9}(5) = \ 4.78,$$

$$x_2 = \frac{3}{5}(5) + \frac{3}{7}(5) + \frac{3}{9}(5) = \ 6.81,$$

$$x_3 = \frac{4}{6}(5) + \frac{4}{7}(5) + \frac{4}{9}(5) = \ 8.41,$$

where $C_B(a_1) = 20 - 4.78 = 15.22$, $C_B(a_2) = 30 - 6.81 = 23.19$, and $C_B(a_3) = 40 - 8.41 = 31.59$.

Note that the point solution specified by the generalized Shapley is an imputation and a core member. In general, if the core exists and the relative weights sum to one, the generalized Shapley will provide assignments in the core (Sharkey, 1990).

The generalized Shapley represents a class of point solutions with substantial flexibility in how the assignment weights, $w_i(S)$, are specified. If weights are chosen according to cost allocation concepts, any cost assignment can be represented by the generalized Shapley value. Of course many of the allocations in this case are arbitrary and may not be measures of how rational assignments are made. This study is specifically interested only in those representations of axiom 1 that correspond to candidates for rational assignments.

Proposition 2

Any cost assignment system can be represented by a generalized Shapley value that follows a nonspecific relaxation of axiom 1 of the simple Shapley.

Proof: See Appendix A.

Proposition 2 suggests that assignment weights can be defined that allow any cost assignment to result from a generalized Shapley value. This result depends on the ability to define the weights according to cost allocation concepts. However, this does not imply that every cost assignment system complies with the rationality axioms of the generalized Shapley value. Axiom 1 of the simple Shapley value should be replaced with a reasonable set of weights, for example, a set of weights based on relative contribution. Not all cost assignment systems fulfill this conceptual requirement.

The solution concepts described provide possible theoretical benchmarks that represent rational decompositions of a shared cost. The benchmarks allow an evaluation of ABC and FBC, which will indicate how well each cost assignment system complies with a rational decomposition.

An evaluation of ABC and FBC according to the imputation and core solution concepts also requires an understanding and identification of what causes cost assignment systems to differ. The existence of product diversity in a multiple product plant causes ABC and FBC assignments to differ. Product diversity exists when the activity consumption ratios of non-unit level and unit level activities differ.[4] As product diversity increases, the difference between ABC and FBC assignments also increases. Therefore, a measure of the degree of product diversity will help to identify conditions under which ABC and FBC are sufficiently different that one cost assignment fails to comply with the requirements of a rational decomposition suggested by cooperative game theory solution concepts. An operational measure for the degree of product diversity is developed in Chapter 5.

In addition to the set solution concepts, the point solution concepts provide possible theoretical point predictions of a theoretical product cost. The simple Shapley provides a specific point prediction that is independent of a product's contribution to the total cost savings in a multiple product plant. The generalized Shapley provides a class of solutions by relaxing the independence axiom of the simple Shapley. Proposition 2 implies that any cost assignment system can be represented by a generalized Shapley solution. Thus, for the generalized Shapley to be used as a benchmark, axiom 1 of the simple Shapley must be replaced with a set of assignment weights that potentially reflect rational behavior. A discussion of the use of point solutions as theoretical product cost benchmarks is presented in Chapter 7.

[4] Activity consumption ratios measure the relative consumption of an activity by a product. Unit level activities are performed each time a unit is produced. Unit level activity drivers are correlated with production. Non-unit level activities are not correlated with production. Non-unit level activity drivers measure the consumption of non-unit level activities. The use of consumption ratios is explained more fully in Chapter 5.

CHAPTER 5: AN OPERATIONAL MEASURE OF PRODUCT DIVERSITY

ABC and FBC represent two possible alternatives for assigning shared costs in a multiple product setting. Obviously, an evaluation would not be required if ABC and FBC provided the same cost assignment. When ABC and FBC assignments differ, an evaluation of the cost assignments is necessary to determine which cost system provides the most accurate product cost. Therefore, it is necessary to identify when and how ABC and FBC assignments differ.

An ABC assignment system recognizes that both non-unit and unit level factors drive production costs, while FBC assumes all costs are driven by unit level factors. It is likely that a product will consume non-unit level and unit level activities in different proportions. This concept has been identified as product diversity, and conceptually, the existence of product diversity indicates that there is a difference in ABC and FBC assignments. However, this conceptual definition of product diversity lacks the structure necessary to measure how much product diversity exists in a multiple product plant. Identifying a measure of the degree of product diversity will allow for a quantification of the difference between ABC and FBC assignments, which is important for evaluating of ABC and FBC cost assignments. Thus, the development of an operational measure of product diversity is needed.

The development of an operational measure of product diversity relies on ABC and FBC assignment concepts. The next section presents basic cost assignment concepts and general definitions of cost assignments provided by ABC and FBC that will be used in the subsequent development.

5.1 Definition of ABC and FBC Assignments

An ABC assignment system recognizes that the consumption of both non-unit and unit level activities drive production costs and assigns the cost of each activity based on the proportion of an activity consumed by a product. The proportion of an activity consumed by products is measured by consumption ratios, where non-unit level consumption ratios measure the consumption of non-unit level activities, and unit level consumption ratios measure the consumption of unit level activities. The calculation of consumption ratios rely on a measure of an activity defined by an activity cost driver, which represent the cause and effect relationship between production and activity cost. For example, inspection cost is a function of the number of inspection hours worked. ABC would identify inspection hours as an appropriate cost driver and assign inspection cost based on consumption ratios that measure the proportion of inspection hours consumed by each product.

A general definition of the cost assignment provided by ABC, $C_\rho(a_i)$, in a three-product, three-activity setting can be expressed as follows[5]:

$$C_\rho(a_i) = \rho_{i1}\alpha_1 + \rho_{i2}\alpha_2 + \mu_{i3}\alpha_3\,, \tag{11}$$

where $\rho_{ij}, j=1, 2$, represent the non-unit level consumption ratios, $\mu_{ij}, j=3$, represents the unit level consumption ratio and α_j represents the cost of activity j in the multiple product plant.

[5] For simplicity and ease of illustration, a three-product setting, with two non-unit level activities and one unit level activity, is assumed in all subsequent analysis.

An FBC assignment system assigns all shared costs, both non-unit level and unit level activities, to products based only on unit level consumption ratios. The analysis assumes the use of a plant-wide rate. FBC ignores the possibility that different cause and effect relationships exist for each activity. A general definition of the cost assignment provided by FBC, $C_{\mu}(a_i)$, can be expressed as follows:

$$C_{\mu}(a_i) = \mu_{i3} \sum_{j} \alpha_j \, , \tag{12}$$

where μ_{i3} represents the unit level assignment ratio.

ABC and FBC assignments can be illustrated in a three-product plant with three activities; purchasing, material handling and machining. The following information is used to illustrate calculations of ABC and FBC assignments:

		Single Product Plants			Three Product Plant
		1	2	3	123
Non-unit level activities:					
Purchasing :					
Purchase orders needed	z_{ij}	945	1,050	1,505	3,500
Activity cost	P_jK_{ij}	\$10,000	\$15,000	\$20,000	\$35,000
ρ_{ij}	$z_{ij}/\Sigma z_{ij}$	0.27	0.30	0.43	

		Single Product Plants 1	2	3	Three Product Plant 123
Material Handling :					
	z_{ij}	3,375	1,125	3,000	7,500
Number of moves needed	P_jK_{ij}	\$20,000	\$10,000	\$20,000	\$40,000
Activity cost	$z_{ij}/\Sigma z_{ij}$	0.45	0.15	0.40	
ρ_{ij}					
<u>**Unit level activity:**</u>					
Machining :	z_{ij}	3,960	6,600	22,440	33,000
Machine hours needed	$C(z_{ij})$	\$31,680	\$39,600	\$89,760	\$115,500
Activity cost	$z_{ij}/\Sigma z_{ij}$	0.12	0.20	0.68	
μ_{ij}					

Using equation (11), an ABC assignment can be calculated as follows:

$$C_\rho(a_1) = .27(35{,}000) + .45(40{,}000) + .12(115{,}500) = \$41{,}310,$$

$$C_\rho(a_2) = .30(35{,}000) + .15(40{,}000) + .20(115{,}500) = \$39{,}600,$$

$$C_\rho(a_3) = .43(35{,}000) + .40(40{,}000) + .68(115{,}500) = \$109{,}590.$$

The FBC assignment, defined by equation (12), is calculated as follows:

$$C_\mu(a_1) = .12(190{,}500) = \$22{,}860,$$

$$C_\mu(a_2) = .20(190{,}500) = \$38{,}100,$$

$$C_\mu(a_3) = .68(190{,}500) = \$129{,}540.$$

5.1.1 An Aggregate ABC Assignment Ratio

Assigning the cost of shared resources under an ABC assignment system requires the identification of assignment ratios for each activity in a multiple product plant. Since ABC uses so many ratios to assign costs to products, it appears more complex than FBC, which uses a single ratio under the assumption of a plant-wide rate. ABC has been criticized for its apparent complexity and often, because of its simplicity, FBC appears more desirable even though it represents a less detailed effort in assigning costs than ABC. However, if the multiple ABC ratios can be replaced by a single ratio and achieve the same ABC assignment, this criticism can be challenged.

Traditionally, an ABC assignment is defined as follows for a multiple product plant with m activities; m-1 non-unit level activities and one unit level activity:

$$C_\rho(a_i) = \sum_{j=1}^{m-1} \rho_{ij}\alpha_j + \mu_i\alpha_m . \tag{13}$$

Let $\overline{\varepsilon_i}$ represent a possible aggregate measure of the ABC assignment ratios, where the existence of $\overline{\varepsilon_i}$ implies a single assignment ratio exists that, when applied to the total activity cost, $\sum_j \alpha_j$, would provide the same cost assignment as specified by equation (13). This implies the following:

$$\overline{\varepsilon_i} \sum_j \alpha_j = \sum_{j=1}^{m-1} \rho_{ij}\alpha_j + \mu_i\alpha_m , \tag{14}$$

where $\sum_i \overline{\varepsilon_i} = 1$. Dividing both sides of equation (14) by $\sum_j \alpha_j$ provides:

$$\overline{\varepsilon_i} = \sum_{j=1}^{m-1} \rho_{ij} \frac{\alpha_j}{\sum_j \alpha_j} + \mu_i \frac{\alpha_m}{\sum_j \alpha_j}. \tag{15}$$

Equation (15) implies that the weighted average of the *m* ABC assignment ratios can be represented by a single ABC assignment ratio, $\overline{\varepsilon_i}$. The identification of a single ABC assignment ratio reduces the apparent complexity of the traditional ABC assignment method, where an ABC assignment can be expressed as follows:

$$C_\rho(a_i) = \overline{\varepsilon_i} \sum_j \alpha_j \tag{16}$$

The ABC assignment using equation (16) can be illustrated with the previous example, where $\overline{\varepsilon_i}$ is calculated as follows:

$$\overline{\varepsilon_1} = .27 \left(\frac{35{,}000}{(190{,}500)}\right) + .45\left(\frac{40{,}000}{(190{,}500)}\right) + .12 \left(\frac{115{,}500}{(190{,}500)}\right) = .21685,$$

$$\overline{\varepsilon_2} = .30\left(\frac{35{,}000}{(190{,}500)}\right) + .15\left(\frac{40{,}000}{(190{,}500)}\right) + .20 \left(\frac{115{,}500}{(190{,}500)}\right) = .20787,$$

$$\overline{\varepsilon_3} = .43 \left(\frac{35{,}000}{(190{,}500)}\right) + .40\left(\frac{40{,}000}{(190{,}500)}\right) + .68 \left(\frac{115{,}500}{(190{,}500)}\right) = .57528.$$

The ABC assignment, defined by equation (16), is calculated as follows:

$C_\rho(a_1) = .21685(190{,}500) = \$41{,}310,$

$C_\rho(a_2) = .20787(190{,}500) = \$39{,}600,$

$C_\rho(a_3) = .57528(190{,}500) = \$109{,}590.$

One implication of the identification of a single ABC assignment ratio is that it may allow for the development of a standard cost system based on activity consumption ratios. A multiple product plant could identify activity consumption ratios that represent "standards" for efficiency for each product. The standard consumption ratios could be used to determine $\overline{\varepsilon_i}$, which would allow for an ABC costing system based on equation (16).

5.2 Development of an Operational Measure of the Degree of Product Diversity

The development of an operational measure of product diversity is needed to determine when and how cost assignments provided by ABC and FBC will differ. For simplicity, a measure for the degree of product diversity (PD) is developed for a multiple product setting with two non-unit level activities and one unit level activity.[6] Let ρ_{i1}, ρ_{i2} be the non-unit level consumption ratios and μ_{i3} the unit level consumption ratio for product i. Let α_j be the cost of activity j in the n product plant, j=1, …,3, where j = 1, 2 for the non-unit level activities and j = 3 for the unit level activity.

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[6] The results are easily extended to n non-unit level activities and m unit level activities.

PD is defined to be zero when the cost assignments under ABC and FBC are equal. The cost assigned to product i under ABC is defined as

$C_\rho(a_i) = \sum_{j=1}^{2} \rho_{ij}\, \alpha_j + \mu_{i3}(\alpha_3)$. The cost assigned to product i under FBC is defined as

$C_\mu(a_i) = \mu_{i3}(\sum_{j=1}^{3} \alpha_j)$. PD=0 implies $C_\rho(a_i) = C_\mu(a_i)$:

$$\sum_{j=1}^{2} \rho_{ij}\, \alpha_j + \mu_{i3}(\alpha_3) = \mu_{i3}(\sum_{j=1}^{3} \alpha_j),$$

$$\sum_{j=1}^{2} \rho_{ij}\, \alpha_j = \mu_{i3}(\sum_{j=1}^{2} \alpha_j),$$

$$\sum_{j=1}^{2} \rho_{ij} \left(\frac{\alpha_j}{\sum_{j=1}^{2} \alpha_j} \right) = \mu_{i3}. \tag{17}$$

Equation (17) implies that PD=0 when the weighted average of the non-unit level consumption ratios is equal to the unit level consumption ratio. Define

$\overline{\rho}_i = \sum_{j=1}^{2} \rho_{ij} \left(\frac{\alpha_j}{\sum_{j=1}^{2} \alpha_j} \right)$, which is an aggregate measure that captures the characteristics of product i's consumption of the non-unit level activities, where $\sum_i \overline{\rho}_i = 1$. The identification of $\overline{\rho}_i$ is significant as it allows for a direct comparison of the non-unit level and the unit level consumption ratios. This comparison becomes the basis for measuring product diversity. Let $\underline{\rho} = (\bar{\rho}_1, \bar{\rho}_2, \ldots, \bar{\rho}_n)$ be the vector of non-unit level consumption ratios and $\underline{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ be the vector of unit level consumption ratios in an n-product plant. Also, assume that the unit level consumption ratios, which are correlated with volume, can be ordered such that $\mu_1 < \mu_2 < \ldots < \mu_n$, where μ_1 represents the low volume product. The degree of PD is determined by a measure of the distance between the two vectors, $\underline{\rho}$ and $\underline{\mu}$:

$$PD = [(\bar{\rho}_1 - \mu_1)^2 + (\bar{\rho}_2 - \mu_2)^2 + \ldots + (\bar{\rho}_n - \mu_n)^2] \text{ or,}$$

$$PD = \sum_{i=1}^{n} [(\bar{\rho}_i - \mu_i)^2]. \tag{18}$$

Equation (18) allows for a measurement of the degree of difference between ABC and FBC assignments. This measure helps quantify one phenomenon that has traditionally indicated the existence of product diversity: the over-costing of high volume products under an FBC assignment. For example, assume PD moves away from zero such that $\bar{\rho}_n < \mu_n$. Since μ_n represents consumption of the high volume product, the high volume product becomes over-costed under an FBC assignment.

The minimum degree of PD is defined to be zero. Intuitively, the maximum degree of product diversity will occur when the product with the maximum distance for $(\bar{\rho}_i - \mu_i)$ can be identified. This maximum distance occurs when $\bar{\rho}_1 = 1$. If $\bar{\rho}_1 = 1$, then $\bar{\rho}_2 = \ldots = \bar{\rho}_n = 0$ and the maximum degree of product diversity is defined to be:

$$\mathrm{PD}_{\max} = [\,(1 - \mu_1)^2 + \sum_{i=2}^{n} (\mu_i)^2\,], \tag{19}$$

where $\mu_1 < \mu_2 < \ldots < \mu_n$, with μ_1 representing the low volume product.

An n-product setting is used to illustrate the validity of equation (19). The maximum degree of PD occurs when $\bar{\rho}_1 = 1$. To show this, assume that $\bar{\rho}_1 < 1$ and $\bar{\rho}_i > 0$, $i = 2, \ldots, n$. This assumption implies:

$$(1 - \mu_1)^2 > (\bar{\rho}_1 - \mu_1)^2 \text{ and} \tag{20}$$

$$\sum_{i=2}^{n} \mu_i^2 > \sum_{i=2}^{n} (\bar{\rho}_i - \mu_i)^2. \tag{21}$$

Summing equations (20) and (21) provides

$$(1 - \mu_1)^2 + \sum_{i=2}^{n} \mu_i^2 > (\bar{\rho}_1 - \mu_1)^2 + \sum_{i=2}^{n} (\bar{\rho}_i - \mu_i)^2. \tag{22}$$

Equation (22) confirms the claim that PD is greatest when $\bar{\rho}_1 = 1$ and begins to decrease as $\bar{\rho}_1$ becomes less than one.

5.3 Product Diversity - Example

The measure of the degree of product diversity can be illustrated using a three-product setting with two non-unit level activities and one unit level activity. Purchasing and material handling represent non-unit level activities, where $j = 1, 2$. Machining represents a unit level activity, where $j = 3$. The information associated with these activities was presented in a previous section and will be used to illustrate how $\bar{\rho}_i$ is defined for each product and how the degree of product diversity is calculated.

The weighted average consumption ratio of the non-unit level activities for each product is defined by $\bar{\rho}_i = \sum_{j=1}^{2} \rho_{ij} \left(\frac{\alpha_j}{\sum_{j=1}^{2} \alpha_j} \right)$ and calculated as follows:

$$\bar{\rho}_1 = .27 \left(\frac{35{,}000}{(35{,}000 + 40{,}000)} \right) + .45 \left(\frac{40{,}000}{(35{,}000 + 40{,}000)} \right) = .366,$$

$$\bar{\rho}_2 = .30 \left(\frac{35{,}000}{(35{,}000 + 40{,}000)} \right) + .15 \left(\frac{40{,}000}{(35{,}000 + 40{,}000)} \right) = .220,$$

$$\bar{\rho}_3 = .43 \left(\frac{35{,}000}{(35{,}000 + 40{,}000)} \right) + .40 \left(\frac{40{,}000}{(35{,}000 + 40{,}000)} \right) = .414.$$

The measure of the degree of product diversity is defined as PD = $\sum_{i=1}^{3}[(\bar{\rho}_i - \mu_i)^2]$ and is calculated as follows:

$$\text{PD} = (.366 - .12)^2 + (.22 - .20)^2 + (.414 - .68)^2 = .132.$$

The maximum degree of PD is defined as $\text{PD}_{\max} = (1 - \mu_1)^2 + \sum_{i=2}^{3}\mu_i^2$ and is calculated as follows:

$$\text{PD}_{\max} = (1 - .12)^2 + (.20)^2 + (.68)^2 = 1.277.$$

By definition, when PD=0 in a multiple product setting, the use of ABC or FBC will provide the same cost assignment. In terms of simplicity and ease of application, FBC represents the preferred cost assignment method under this condition. However, as PD increases the cost assignments begin to differ. As the difference in cost assignments provided by ABC and FBC increases, a materiality threshold will be reached whereby the difference in cost assignments becomes so significant that it causes a switch from the simpler FBC method to a more detailed ABC method. The question becomes by how much can the degree of product diversity increase before this threshold is exceeded and ABC and FBC produce significantly different cost assignments? Does a low degree of product diversity necessarily indicate an immaterial difference in the cost assignments provided by ABC and FBC? In the example above, a low degree of product diversity exists, .132, relative to the maximum degree possible, 1.277. However, the following comparison of the cost assignments provided by ABC and FBC when PD = .132 reveals what appears to be a significant difference in the cost assignments.

	Product 1	*Product 2*	*Product 3*
ABC assignment	\$41,310	\$39,600	\$109,590
FBC assignment	\$22,860	\$38,100	\$129,540
Difference	**\$18,450**	**\$ 1,500**	**(\$ 19,950)**

The comparison of ABC and FBC assignments above implies that a relatively low degree of product diversity does not always indicate that an insignificant difference between the cost assignments will be observed. The example can be extended to illustrate how the difference in the cost assignments provided by ABC and FBC is affected by various degrees of product diversity. Assume the degree of product diversity changes as a result of changes in the unit level consumption ratios, where $\overline{\rho}_i$ remains constant. This assumption represents only one way in which the degree of product diversity can change. However, it is adequate for illustrating how cost assignments differ under different degrees of product diversity. The non-unit level consumption ratio, $\overline{\rho}_i$, and the unit level consumption ratio μ_i, under increasing degrees of product diversity are presented in Table 5-1:

Table 5-1. Consumption Ratios for Different Degrees of Product Diversity

Degree of Product Diversity	***Consumption Ratios***					
	$\overline{\rho}_1$	μ_1	$\overline{\rho}_2$	μ_2	$\overline{\rho}_3$	μ_3
PD = 0.00000	0.366	0.366	0.220	0.220	0.414	0.414
PD = 0.00103	0.366	0.350	0.220	0.210	0.414	0.440
PD = 0.01215	0.366	0.300	0.220	0.200	0.414	0.500
PD = 0.06255	0.366	0.200	0.220	0.200	0.414	0.600
PD = 0.13167	0.366	0.120	0.220	0.200	0.414	0.680
PD = 0.18855	0.366	0.100	0.220	0.150	0.414	0.750
PD = 0.36495	0.366	0.050	0.220	0.050	0.414	0.900
PD = 0.50261	0.366	0.010	0.220	0.010	0.414	0.990

A comparison of cost assignments provided by ABC and FBC for each product are illustrated in Table 5-2.

Table 5-2. Product Diversity: ABC vs. FBC

Degree of Product Diversity	*ABC Assignment*	*FBC Assignment*	*Cost Assignment Difference*
		Product 1	
PD = 0.00000	$ 69,723	$ 69,723	$ 0
PD = 0.00103	$ 67,875	$ 66,675	$ 1,200
PD = 0.01215	$ 62,100	$ 57,150	$ 4,950
PD = 0.06255	$ 50,550	$ 38,100	$ 12,450
PD = 0.13167	$ 41,310	$ 22,860	$ 18,450
PD = 0.18855	$ 39,000	$ 19,050	$ 19,950
PD = 0.36495	$ 33,225	$ 9,525	$ 23,700
PD = 0.50261	$ 28,605	$ 1,905	$ 26,700
		Product 2	
PD = 0.00000	$ 41,910	$ 41,910	$ 0
PD = 0.00103	$ 40,755	$ 40,005	$ 750
PD = 0.01215	$ 39,600	$ 38,100	$ 1,500
PD = 0.06255	$ 39,600	$ 38,100	$ 1,500
PD = 0.13167	$ 39,600	$ 38,100	$ 1,500
PD = 0.18855	$ 33,825	$ 28,575	$ 5,250
PD = 0.36495	$ 22,275	$ 9,525	$ 12,750
PD = 0.50261	$ 17,655	$ 1,905	$ 15,750
		Product 3	
PD = 0.00000	$ 78,867	$ 78,867	$ 0
PD = 0.00103	$ 81,870	$ 83,820	($ 1,950)
PD = 0.01215	$ 88,800	$ 95,250	($ 6,450)
PD = 0.06255	$100,350	$114,300	($ 13,950)
PD = 0.13167	$109,590	$129,540	($ 19,950)
PD = 0.18855	$117,675	$142,875	($ 25,200)
PD = 0.36495	$135,000	$171,450	($ 36,450)
PD = 0.50261	$145,395	$188,595	($ 43,200)

Clearly, as PD increases, the difference in the cost assignment for each product increases. When PD = .00103, the difference between cost assignments for each product does not appear to be significant. However, as PD increases to .06255, the difference becomes more material. Although the question of how much PD has to increase before the difference in cost assignments becomes significant is not specifically answered by this analysis, the comparison suggests that a materiality threshold does exist. Further investigation of this question, either analytically or empirically, may reveal a materiality threshold to define the degree of PD that causes multiple product plants to switch from FBC to a more complex ABC assignment method. This investigation is left as a possible extension to this study.

The measure of PD captures all underlying causes of the difference between cost assignments provided by ABC and FBC in a multiple product plant. By definition, product diversity exists when systematic differences are present between the consumption ratios of non-unit level and unit level activities. Unsystematic differences produce a "wash" effect whereby an FBC assignment will over-cost a product related to some non-unit level activities and under-cost the product related to other non-unit level activities. Essentially, the wash effect causes ABC and FBC assignments to be the same even though the consumption ratios of non-unit and unit level activities differ. The operational measure of PD captures the effects of unsystematic differences in consumption ratios, where PD $\cong 0$. This concept can be illustrated with minor modifications to the information presented previously related to the purchasing, material handling and machining activities.

Assume the following consumption ratios for each product:

	Product 1	***Product 2***	***Product 3***
Purchasing	$\rho_{11} = .18$	$\rho_{21} = .32$	$\rho_{31} = .88$
Material Handling	$\rho_{12} = .06$	$\rho_{22} = .12$	$\rho_{32} = .48$
Machining	$\mu_{13} = .12$	$\mu_{23} = .22$	$\mu_{33} = .68$

The weighted average non-unit level consumption ratios are calculated as follows:

$$\bar{\rho}_1 = .18\left(\frac{35{,}000}{(35{,}000+40{,}000)}\right) + .06\left(\frac{40{,}000}{(35{,}000+40{,}000)}\right) = .116,$$

$$\bar{\rho}_2 = .32\left(\frac{35{,}000}{(35{,}000+40{,}000)}\right) + .12\left(\frac{40{,}000}{(35{,}000+40{,}000)}\right) = .213,$$

$$\bar{\rho}_3 = .88\left(\frac{35{,}000}{(35{,}000+40{,}000)}\right) + .48\left(\frac{40{,}000}{(35{,}000+40{,}000)}\right) = .670.$$

The degree of product diversity is calculated as follows:

$$\text{PD} = (.116 - .120)^2 + (.213 - .200)^2 + (.670 - .680)^2 = .000165.$$

This example illustrates that unsystematic differences in the consumption ratios are captured by the measure of product diversity, where PD $\cong 0$. A wash effect has occurred and there is essentially no difference in the cost assignments provided by ABC and FBC, as illustrated below.

	Product 1	***Product 2***	***Product 3***
ABC assignment	\$22,560	\$41,385	\$128,790
FBC assignment	\$22,860	\$41,910	\$129,540
Difference	**(\$ 300)**	**(\$ 525)**	**(\$ 750)**

An important implication of the operational measure of PD is that it can be used in the evaluation of ABC and FBC assignments. Theoretically, ABC is assumed to provide a more accurate product cost than FBC. If this assumption is valid, then as PD increases, FBC becomes a less accurate product cost compared to ABC. Subsequent analysis in Chapters 6 and 7 will reveal that ABC assignments conform to rational decompositions under a game-theoretic paradigm. The operational measure of the degree of product diversity will allow conditions to be identified that indicate when FBC assignments will violate rationality axioms.

The operational measure of PD developed in this chapter is derived under the assumption that an FBC assignment is calculated using a single plant-wide unit level assignment ratio. However, the use of multiple unit level departmental rates is often observed. Measuring the degree of product diversity under the assumption of departmental rates would require some modifications to the measure developed in this study. This development is left as a possible extension to this study.

CHAPTER 6: AN EVALUATION OF ABC AND FBC – IMPUTATION BENCHMARK

The game-theoretic paradigm provides one possible independent measure of theoretical product cost that can be used to evaluate the relative accuracy of ABC and FBC assignments. This measure of product cost is a concept of accuracy based on the ability to decompose costs of shared resources in multiple product settings. Chapter 3 defined one possible method for decomposing a shared cost based on game-theoretic concepts. Four possible cooperative game theory solution concepts were identified and described in Chapter 4 that specify rational decompositions that can be used as measures of theoretical product cost. An evaluation of ABC and FBC assignments can be performed based on how well each cost assignment conforms to the rational decompositions suggested by the solution concepts.

Two of these decompositions correspond to solution concepts known as imputations and core solutions. Imputations and the core offer a set of solutions: solutions that, as a group, meet certain key rationality concepts. ABC and FBC assignments can be evaluated to determine under what conditions they qualify as members of these two solution sets. The identification of these conditions provides a basis for evaluation, where a cost assignment conforming to the rationality concepts of a solution set will be superior to a cost assignment that does not. Therefore, if one of the cost assignments always qualifies as a member of a solution set, the degree of difference between the cost assignments, as defined by the degree of product diversity, will indicate when the other cost assignment will or will not be a member of the same set.

The analysis presented in this chapter evaluates ABC and FBC assignments based on their conformance to the imputation conditions. General conditions are derived that define when ABC and FBC assignments will qualify as members of the set of imputations. These conditions provide the basis for a comparison of ABC and FBC assignments under two specific cost functions; fixed factors of production and inputs exhibiting economies of scale effects. An evaluation based on core membership is presented in Chapter 7.

6.1 Imputation Benchmark

The set of imputations represents all rational decompositions of a shared cost. To be a member of this set, cost assignments must satisfy two rationality conditions that were described in Chapter 4; *pareto* optimality and individual rationality. The rationality conditions defining the set of imputations define an imputation benchmark that can be used to evaluate ABC and FBC assignments. When ABC and FBC assignments satisfy the imputation conditions, the cost assignments qualify as members of the set of imputations and are said to conform to the imputation benchmark.

General conditions that indicate when cost assignments provided by ABC and FBC conform to the imputation benchmark are derived in Proposition 3.

Proposition 3:

Cost assignments provided by ABC and FBC are imputations provided the following conditions are satisfied:

$$\textbf{ABC: } \bar{\varepsilon}_i \leq \frac{C_i}{C_N}, \ \sum_i \bar{\varepsilon}_i = 1, \tag{23}$$

$$\textbf{FBC: } \mu_i \leq \frac{C_i}{C_N}, \ \sum_i \mu_i = 1. \tag{24}$$

Proof: See Appendix A.

Proposition 3 derives the general conditions under which ABC and FBC assignments will conform to rational decompositions defined by the set of imputations. The ratio, $\frac{C_i}{C_N}$, defines a critical assignment ratio that indicates when cost assignments qualify as members of the set of imputations. An analysis of ABC and FBC assignments is needed to determine if conditions exist that cause either the ABC or FBC assignment ratio to exceed this critical assignment ratio, which allow superiority conditions to be identified. For example, if the ABC assignment ratio never exceeds $\frac{C_i}{C_N}$, ABC assignments will always conform to the imputation benchmark. Then, if it is possible for the FBC assignment ratio to exceed $\frac{C_i}{C_N}$, and thus fail to conform to the imputation benchmark, conditions can be identified when ABC will be argued to be a rationally superior cost assignment system. The superiority claim will depend on the difference between the ABC and FBC assignments, which is quantified by the operational measure of product diversity (PD) developed in Chapter 5. The superiority claim will be supported when PD exists such that $\mu_i > \frac{C_i}{C_N}$. Therefore, PD proves to be the key in identifying when FBC assignments are not members of the set of imputations, which will imply when ABC assignments are rationally superior.

Equations (23) and (24) define the general conditions that indicate when ABC and FBC assignments will conform to the imputation benchmark. However, an evaluation requires additional structure be imposed on the cost functions in a multiple product setting. Cost assignments provided by ABC and FBC depend on the specification of cost functions of shared resources. Two possible subadditive cost functions were identified and described in Chapter 2; fixed factors of production and inputs exhibiting economies of scale effects. The following sections provide an evaluation of ABC and FBC assignments, based on the imputation benchmark, assuming shared inputs in a multiple product plant are fixed factors of production or inputs exhibiting economies of scale, or a combination of both.

6.1.1 Fixed Factors of Production

Subadditive cost functions generate the cost savings necessary to invoke economies of scope, which drives the existence of multiple product plants. Fixed factors of production are identified in economies of scope literature as one type of sharable input exhibiting a subadditive cost function. Thus, the existence of fixed inputs in a multiple product plant is identified independent of cost assignment concepts, which is important for an evaluation of ABC and FBC assignments based on independent criteria. Recall that the cost of fixed inputs in a three-product plant is expressed as follows:

$$C_N = P_1(K_{11} + K_{21} + K_{31} - t_1) + P_2(K_{12} + K_{22} + K_{32} - t_2) + P_3(K_{13} + K_{23} + K_{33} - t_3).$$

The cost of fixed inputs can be assigned to products under an ABC or FBC assignment system. The cost assignment specified by both systems is described and defined below.

An ABC assignment system is a causal system that identifies causal criteria to assign costs to products. Causal criteria are defined by cause and effect relationships between products and input consumption. These relationships logically would be specified by the variables of the input cost functions. Therefore, the causal factor, or cost driver, used in ABC assignments should be defined by the input cost function variable, or some factor highly correlated with the cost function variable.

The cost function of a fixed input was defined in Chapter 2 as $C_i = \sum_j P_j K_{ij}$, where K_{ij} is a function of z_{ij}. The cost function variable and therefore, the causal factor of a fixed input, is specified by the input demand, z_{ij}. For example, purchasing labor input is a fixed input, where z_{ij} is measured by purchasing labor hours. An ABC assignment system assigns the cost of activities to products, where the cost of an activity is a function of the inputs consumed by the activity. Using purchasing hours to assign the cost of a purchasing activity to products is completely consistent with an ABC system. ABC assignment systems do recognize and use duration drivers associated with input cost functions. ABC also uses other drivers that are highly correlated with z_{ij} (factors called transaction drivers) to assign activity costs to products. For example, if it takes the same amount of time to process each purchase order, purchase orders can replace purchasing labor hours as the causal measure used by ABC. Therefore, ABC either uses z_{ij}, or a measure highly correlated with z_{ij}, to assign the cost of fixed inputs.

Thus, an ABC assignment system would use the relative input usage, $\frac{z_{ij}}{\sum_i z_{ij}}$, to define consumption ratios used in assigning the cost of fixed inputs to products. An ABC assignment of fixed inputs can be expressed as follows for a three-product setting with two non-unit level fixed inputs, j=1, 2, and one unit level fixed input, j=3:

$$C_\rho(a_i) = \rho_{i1}(P_1(\sum_i K_{i1} - t_1)) + \rho_{i2}(P_2(\sum_i K_{i2} - t_2)) + \mu_{i3}(P_3(\sum_i K_{i3} - t_3)). \qquad (25)$$

where $\rho_{ij} = \frac{z_{ij}}{\sum_i z_{ij}}$ is a non-unit level input consumption ratio and $\mu_{i3} = \frac{z_{i3}}{\sum_i z_{ij}}$ is the unit level input consumption ratio.

To determine if ABC assignments of fixed factors conform to the imputation benchmark, the cost assignment must be evaluated against the imputation conditions. An analysis of the ABC assignment specified by equation (25) against the imputation conditions reveals that the ABC system will always provide cost assignments that qualify as members of the set of imputations. This result is derived in Proposition 4.

Proposition 4:

ABC assignments always conform to the imputation benchmark if shared resources in a multiple product setting are fixed factors of production.

Proof: See Appendix A.

Proposition 4 states a significant result: ABC assignments of fixed factors of production will always qualify as members of the set of imputations, which implies

$\bar{\varepsilon}_i \leq \frac{C_i}{C_N}$. Since ABC assignments will always conform to the imputation benchmark, an evaluation must determine if FBC assignments have the same properties. If FBC assignments do not always conform, conditions can be defined that indicate when ABC assignments are rationally superior.

An FBC assignment system assigns the cost of all inputs assuming the relationship between products and the inputs consumed is represented by a unit level factor correlated with production, such as machine hours or direct labor hours. The unit level factor may or may not be correlated with the cost drivers specified by the input cost functions. Therefore, FBC assignments of fixed inputs often do not reflect the underlying cause and effect relationship between products and inputs consumed.

An FBC assignment of fixed factors of production assigns the total cost of all fixed inputs using unit level consumption ratios, μ_{ij}, based on a measure correlated with production. A cost assignment of fixed inputs provided by FBC can be expressed as follows for a three-product setting, with two non-unit level fixed inputs, j=1, 2, and one unit level fixed input, j=3:

$$C_\mu(a_i) = \mu_{i3}\,(P_1(\sum_i K_{i1} - t_1) + P_2(\sum_i K_{i2} - t_2) + P_3(\sum_i K_{i3} - t_3)). \qquad (26)$$

The FBC assignment specified in equation (26) must be evaluated against the imputation conditions to determine when FBC assignments qualify as members of the set of imputations. When μ_i is not correlated with the cost driver specified by the input cost function, it is possible for $\mu_i > \dfrac{C_i}{C_N}$, which indicates FBC assignments violate the rationality conditions of the set of imputations.[7] Therefore, FBC does not always provide cost assignments that qualify as members of the set of imputations. This result is stated in Proposition 5.

Proposition 5:

FBC assignments do not always conform to the imputation benchmark if shared resources in a multiple product setting are fixed factors of production.

Proof:

Assume FBC assignments of fixed inputs always qualify as members of the set of imputations. Then, if at least one example can be constructed that shows an FBC assignment of a fixed input that does not belong to the set of imputations, Proposition 5 is established.

For simplicity, a three-product setting with one non-unit level fixed input, purchasing labor input, and one unit level fixed input, machining input is used to construct the example. The results are easily extended to *n* fixed inputs. For the purchasing labor input, assume purchase orders processed is highly correlated with purchasing labor hours, and the causal factor, z_{ij}, is represented by purchase orders demanded. The causal factor, (i.e. the unit level factor), for machining is machine hours

[7] The activity subscript for unit level consumption ratios is suppressed in subsequent presentation under the assumption of a plant wide FBC rate.

demanded. Each fixed input is represented by the cost function, $C_i = \sum_j P_j K_{ij}$.

Information related to each input is as follows:

		Single Product Plants			Three Product Plant
Purchasing Labor – **S_1 = 500 purchase orders** **P_1 = \$5,000**		1	2	3	123
Purchase orders demanded	z_{ij}	945	1,050	1,505	3,500
Number of steps required	K_{ij}	2	3	4	7
Input cost	P_jK_{ij}	\$10,000	\$15,000	\$20,000	\$35,000
ρ_{ij}	$z_{ij}/\sum z_{ij}$	.27	.30	.43	
Machining- **S_2= 5,000 machine hours** **P_2= \$10,000**					
Machine hours demanded	z_{ij}	9,000	2,500	7,500	19,000
Number of steps required	K_{ij}	2	1	2	4
Input cost	P_jK_{ij}	\$20,000	\$10,000	\$20,000	\$40,000
μ_i	$z_{ij}/\sum z_{ij}$	.47	.13	.40	
C_i		\$30,000	\$25,000	\$40,000	

The FBC assignment of the fixed inputs is calculated as follows:

$C_\mu(a_1)$ = .47 (\$35,000 + \$40,000) = \$35,250,

$C_\mu(a_2)$ = .13 (\$35,000 + \$40,000) = \$ 9,750,

$C_\mu(a_3)$ = .40 (\$35,000 + \$40,000) = \$12,000.

The imputation conditions require $C_\mu(a_i) < C_i$, which is clearly violated by the cost assignment for product 1. Therefore, FBC does not always provide cost assignments of fixed inputs that qualify as members of the set of imputations. Q.E.D.

Taken together, the results of Proposition 4 and Proposition 5 provide the opportunity to identify conditions under which ABC assignments are rationally superior. Proposition 4 states that ABC assignments will always qualify as members of the set of imputations. Proposition 5 reveals that FBC assignments do not have this property and under certain situations do not belong to the set of imputations. Therefore, when the difference between ABC and FBC assignments increases, conditions can be identified that indicate when FBC assignments fail to conform to the imputation benchmark. These conditions are driven by product diversity. For example, PD = 0 implies $\bar{\varepsilon}_i = \bar{\rho}_i = \mu_i$. When PD =0, ABC and FBC assignments are equal, indicating FBC assignments will also qualify as members of the set of imputations. As PD increases, Proposition 4 implies an ABC assignment will remain an imputation, while Proposition 5 implies that an FBC assignment ratio will approach, and eventually exceed, the critical assignment ratio, $\frac{C_i}{C_N}$. Thus, a critical degree of product diversity exists that defines when the FBC assignment ratio will satisfy $\mu_i \leq \frac{C_i}{C_N}$. Proposition 4 implies $\bar{\rho}_i \leq \frac{C_i}{C_N}$, therefore, the maximum increment by which ABC and FBC assignments can differ before FBC fails to conform to the imputation benchmark can be defined as $\frac{C_i}{C_N}$ - $\bar{\rho}_i$. If μ_i^0 is defined as the FBC assignment ratio at PD = 0, then the threshold FBC assignment ratio that allows FBC assignments to qualify as members of the set of imputations is

$\mu_i^I = \mu_i^0 + \left(\frac{C_i}{C_N} - \overline{\rho}_i \right)$, where $\sum_i \mu_i^I = 1$. When product diversity causes FBC assignment ratios to exceed μ_i^I, FBC no longer provides a cost assignment conforming to the imputation benchmark and ABC can be identified as the rationally superior cost assignment system. Let PD_I be the critical degree of product diversity that prevents FBC assignments from belonging to the set of imputations. Using the operational measure of product diversity developed in Chapter 5, PD_I is defined as the degree of difference between ABC and FBC assignments when $\mu_i = \mu_i^I$. PD_I is defined for an *n*-product plant as follows:

$$PD_I = \sum_i (\overline{\rho}_i - \mu_i^I)^2. \tag{27}$$

Thus, when PD > PD_I, $\mu_i > \frac{C_i}{C_N}$ and FBC assignments do not qualify as members of the set of imputations, which implies ABC assignments are rationally superior.

The critical degree of product diversity for the imputation conditions can be defined for a three-product, three-input setting, where each μ_i^I is specified as follows:

$$\mu_1^I = \mu_1^0 + \left(\frac{C_1}{C_N} - \overline{\rho}_1 \right),$$

$$\mu_2^I = \mu_2^0 + \left(\frac{C_2}{C_N} - \overline{\rho}_2 \right),$$

$$\mu_3^I = \mu_3^0 + \left(\frac{C_3}{C_N} - \bar{\rho}_3\right).$$

The critical degree of product diversity, PD_I, is expressed as:

$$PD_I = (\bar{\rho}_1 - \mu_1^I)^2 + (\bar{\rho}_2 - \mu_2^I)^2 + (\bar{\rho}_3 - \mu_3^I)^2$$

Substituting μ_2^I, μ_3^I and $\mu_1^I = 1 - \mu_2^I - \mu_3^I$ provides the following specification of PD_I for a three-product plant:

$$PD_I = \left[\left(\frac{C_3}{C_N} - \bar{\rho}_3\right) + \left(\frac{C_2}{C_N} - \bar{\rho}_2\right)\right]^2 + \left(\frac{C_3}{C_N} - \bar{\rho}_3\right)^2 + \left(\frac{C_2}{C_N} - \bar{\rho}_2\right)^2. \quad (28)$$

It is important to note that the development of PD_I is independent of specific cost functions. If, for any cost structure, ABC assignments always conform to the imputation benchmark and FBC assignments do not, ABC assignments will be rationally superior to FBC assignments when $PD > PD_I$.

6.1.2 Inputs Exhibiting the Effects of Economies of Scale

The cost function of inputs exhibiting the effects of economies of scale has subadditive properties, which allows economies of scope to be invoked and drives the existence of multiple product plants. The possible existence of inputs exhibiting economies of scale in multiple product plants is identified by economies of scope literature and is therefore, independent of cost assignment concepts. Recall that the cost of inputs exhibiting the effects of economies of scale in a three-product plant is expressed as follows:

$$C_N = C(z_{11} + z_{21} + z_{31}) + C(z_{12} + z_{22} + z_{32}) + C(z_{13} + z_{23} + z_{33}).$$

The cost function of an input exhibiting the effects of economies of scale was defined in Chapter 2 as $C_i = \sum_j C(z_{ij})$, where the cost function variable, z_{ij}, identifies a causal relationship between input cost and input usage. However, the nature of inputs exhibiting the effects of economies of scale implies a decreasing average cost function, where the input cost per unit decreases as input consumption increases. This implies an alternative causal relationship where products with higher input consumption contribute more to the lower cost per unit achieved by the multiple product plant.

An ABC assignment is based on causal criteria and should reflect a cost assignment that is based on relative contribution. The causal factor typically used by ABC in assigning the cost of inputs exhibiting the effects of economies of scale is identified by the cost function variable, z_{ij}, which typically implies a transaction or a duration driver. However, the cost assigned to the high consumption products using a relative measure of z_{ij} does not reflect the effects of a decreasing average cost. This implies that an alternative causal factor reflecting a product's contribution to the lower cost per unit achieved in the multiple product plant should be identified. For example, a financial driver such as relative independent input cost, would capture the incremental effects of different levels of input consumption. A financial driver may provide a better measure of the contribution of high consumption products to the lower average cost per unit achieved by the multiple product plant. Thus, two alternatives exist for ABC cost drivers used in assigning the cost of inputs exhibiting the effects of economies of scale: drivers based on z_{ij} and financial drivers.

ABC assignments of inputs exhibiting the effects of economies of scale, using transaction or duration drivers based on z_{ij}, and financial drivers, $C(z_{ij})$, can be expressed as follows for a three-product setting with three inputs that exhibit economies of scale, with two non-unit level inputs, j=1,2, and one unit level input, j=3:

ABC – Transaction or Duration Driver:

$$C_{\rho}(a_i) = \frac{z_{i1}}{\sum_i z_{ij}} C(\sum_i z_{i1}) + \frac{z_{i2}}{\sum_i z_{ij}} C(\sum_i z_{i2}) + \frac{z_{i3}}{\sum_i z_{ij}} C(\sum_i z_{i3}). \tag{29}$$

ABC - Financial Driver:

$$C_{\rho}(a_i) = \frac{C(z_{i1})}{\sum_j C(z_{ij})} (C(\sum_i z_{i1})) + \frac{C(z_{i2})}{\sum_j C(z_{ij})} (C(\sum_i z_{i2})) + \frac{C(z_{i3})}{\sum_j C(z_{ij})} (C(\sum_i z_{i3})). \tag{30}$$

To determine if ABC assignments of inputs exhibiting the effects of economies of scale qualify as members of the set of imputations, the cost assignments specified by equations (29) and (30) must be evaluated against the imputation conditions. A comparison of the assignments reveal that the ABC assignment system, using either drivers based on z_{ij} or financial drivers, will always provide cost assignments that conform to the imputation benchmark. This result is derived in Proposition 6.

Proposition 6:

ABC assignments always conform to the imputation benchmark if shared resources in a multiple product setting are inputs exhibiting economies of scale effects.

Proof: See Appendix A.

Proposition 6 implies that ABC assignments of inputs exhibiting the effects of economies of scale will always be members of the set of imputations. This result implies $\bar{\varepsilon}_i \leq \frac{C_i}{C_N}$ and allows for the identification of superiority conditions when FBC assignments fail to have the same property.

An FBC assignment assigns the cost of all inputs exhibiting the effects of economies of scale using a unit level factor correlated with production. The unit level factor may or may not be correlated with the cost drivers specified by the input cost functions. Therefore, FBC assignments of inputs exhibiting the effects of economies of scale often do not reflect the underlying cause and effect relationship between products and inputs consumed. A cost assignment of inputs exhibiting the effects of economies of scale provided by FBC can be expressed as follows for a three-product setting with two non-unit level inputs and one unit level input:

$$C_{\mu}(a_i) = \mu_i(\sum_j C(\sum_i z_{ij})). \tag{31}$$

The FBC assignment specified in equation (31) must be evaluated against the imputation conditions to determine when FBC assignments qualify as members of the set of imputations. When μ_i is not correlated with the cost driver specified by the input cost function, it is possible for $\mu_i > \frac{C_i}{C_N}$. Proposition 7 states this possibility.

Proposition 7:

FBC assignments do not always conform to the imputation benchmark if shared resources in a multiple product setting are inputs exhibiting the effects of economies of scale.

Proof:

Assume FBC assignments of inputs exhibiting the effects of economies of scale always qualify as members of the set of imputations. Then, if at least one example can be constructed that shows an FBC assignment that does not belong to the set of imputations, Proposition 7 is established by contradiction.

For simplicity, a three-product setting with one non-unit level input, purchasing material input, and one unit level input, machining material input is used to construct the example. The results are easily extended to *n* inputs. Assume the causal factor for purchasing material input is represented by purchase orders demanded, and the unit level factor for the machining material input is machine hours. Each input is represented by the cost function, $C_i = \sum_j C(z_{ij})$. Information related to each input is as follows:

		Single Product Plants			Three Product Plant
		1	2	3	123
Purchasing Material –					
Purchase orders demanded	z_{ij}	945	1,050	1,505	3,500
Average cost per unit		\$20	\$19	\$18	\$15
Input cost	$C(z_{ij})$	\$18,900	\$19,950	\$27,090	\$52,500
ρ_{ij}	$z_{ij}/\Sigma z_{ij}$	.27	.30	.43	
Machining Material-					
Machine hours demanded	z_{ij}	10,450	2,850	5,700	19,000
Average cost per unit		\$4.50	\$8.00	\$6.00	\$4.00
Input cost	$C(z_{ij})$	\$47,025	\$22,800	\$34,200	\$76,000
μ_i	$z_{ij}/\Sigma z_{ij}$	.55	.15	.30	
C_i		\$65,925	\$42,750	\$61,290	

The FBC assignment is calculated as follows:

$$C_{\mu}(a_1) = .55\ (\$52{,}500 + \$76{,}000) = \$70{,}675,$$

$$C_{\mu}(a_2) = .15\ (\$52{,}500 + \$76{,}000) = \$19{,}275,$$

$$C_{\mu}(a_3) = .30\ (\$52{,}500 + \$76{,}000) = \$38{,}550.$$

The imputation conditions require $C_{\mu}(a_i) \leq C_i$, which is clearly violated by the cost assignment for product 1. Therefore, FBC does not always provide cost assignments of inputs exhibiting the effects of economies of scale that qualify as members of the set of imputations. Q.E.D.

The results of Propositions 6 and 7 imply that ABC assignments of inputs exhibiting the effects of economies of scale always conform to the imputation benchmark, while FBC assignments will not always conform. Therefore, the critical degree of product diversity for the imputation benchmark, PD_I, will define when FBC assignments do not belong to the set of imputations. Thus, when $PD > PD_I$, ABC is identified as a rationally superior cost assignment system.

6.2 A Combination of Shared Inputs

An ABC assignment system assigns the cost of activities to products, where activity cost is a function of the inputs consumed by the activity, such as labor, capital, materials, and energy. The analysis in the previous sections assumed that activities only consume one type of sharable input, either fixed inputs, such as labor and capital, or inputs exhibiting the effects of economies of scale, such as material and energy. However, generally activities consume a combination of inputs. For example, an activity could consume labor input and a material input, where each input could be represented by a different cost function. This implies that there may be more than one input cost function variable and therefore, potentially different causal factors. In principle, the most accurate cost assignment would assign the cost of each input to products using the causal factor identified by the input cost function variable. However, an ABC assignment system identifies and uses only one causal factor to assign activity costs to products.

Although assigning the cost of each input based on the input cost function variable is a much more detailed approach to product costing, it represents a potential refinement to the ABC assignment system.

The consumption of a combination of inputs does not always imply that different causal factors exist, or that the causal factors of the inputs are not highly correlated. It is possible that each input consumed by an activity has the same input cost function variable, which would support ABC's use of a single cost driver for assigning activity cost. For example, purchasing is an activity that consumes purchasing labor (labor), purchase orders (materials), computers (capital), and electricity (energy). When purchase orders consumed is highly correlated with purchasing labor hours, the cost of purchasing labor can be assumed to be a function of purchase orders. The cost of materials is also a function of purchase orders. Computers are acquired based a certain capacity for processing purchase orders and this cost is a function of purchase orders. Electricity is consumed based on the number of purchase orders processed. Therefore, the cost function of all four inputs identifies the same causal factor. An ABC assignment would then assign the cost of the purchasing activity based on the number of purchase orders.

However, as stated previously, the consumption of inputs with different input cost function variables is possible. Does the possible existence of multiple input cost functions, with different causal factors, affect an ABC assignment's status as a member of the set of imputations? The analysis of fixed inputs identified z_{ij}, or a measure highly correlated with z_{ij}, as the causal factor used in ABC assignments of fixed inputs. Proposition 4 states that ABC will always provide cost assignments of fixed inputs that belong to the set of imputations. The analysis of inputs exhibiting the effects of economies of scale identified z_{ij} or $C(z_{ij})$ as appropriate cost drivers for ABC assignments of this type of input. Proposition 6 states the ABC assignments of input exhibiting the effects of economies of scale will always be members of the set of imputations. Therefore, using either z_{ij} or $C(z_{ij})$ as the cost driver in an ABC assignment results in an imputation. Then, if ABC assigns the cost of an activity consuming a combination of inputs using either z_{ij} or $C(z_{ij})$, or both, the resulting cost assignment will always be a member of the set of imputations.

6.3 Economies of Scale Effects: Direct Tracing vs. Driver Tracing

Direct tracing is used to assign the cost of inputs that are exclusively associated with a product, which implies products consume inputs that can be traced directly by a physically observed relationship. The cost assigned to a product is determined by applying the average cost per unit of input to actual input consumption. Typically, direct tracing is used in ABC assignment systems to assign the cost of direct material and direct labor. The basic concept of direct tracing can be illustrated by the following three-product setting where purchasing materials are consumed based on a physically observable relationship.

		Single Product Plants			Three Product Plant
		1	2	3	123
Purchasing Materials					
Material demanded	z_{ij}	500	800	1,200	2,500
Average cost per unit of material		\$3.00	\$3.00	\$3.00	\$3.00
Material cost	$C(z_{ij})$	\$1,500	\$2,400	\$3,600	\$7,500

Direct tracing would assign the material cost to each product based on the observable usage and the average cost per unit in the three-product plant as follows:

$$C_B(a_1) = \$3\ (500) \quad = \$1{,}500,$$

$$C_B(a_2) = \$3\ (800) \quad = \$2{,}400,$$

$$C_B(a_3) = \$3\ (1{,}200) = \$3{,}600.$$

The example above is an illustration of pure direct tracing. Pure direct tracing implies a direct relationship between material cost and material usage in which the average cost per unit is the same for all levels of consumption and no cost savings are possible. However, material inputs often exhibit the effects of economies of scale, where cost savings are possible due to a decreasing average cost function. Does direct tracing still achieve the most accurate cost assignment of direct inputs exhibiting the effects of economies of scale? Or, is driver tracing more appropriate? Driver tracing assigns the cost of inputs based on causal factors that measure a product's consumption and is used when an observable relationship does not exist between input cost and input usage.

An example can be used to investigate the use of direct tracing vs. driver tracing when a direct input exhibits the effects of economies of scale. Information related to a three-product setting in which purchasing materials exhibits a decreasing average cost function, $C(z_{ij})$, is presented as follows:

		Single Product Plants			Three Product Plant
		1	2	3	123
Purchasing Materials					
Material demanded	z_{ij}	500	800	1,200	2,500
Average cost per unit		\$6.00	\$5.00	\$4.00	\$3.00
Material cost	$C(z_{ij})$	\$3,600	\$4,000	\$4,800	\$7,500

Direct tracing would assign the material input cost by applying the average cost per unit of the three-product plant to actual consumption as follows:

$$C_B(a_1) = \$3\ (500) = \$1{,}500,$$

$$C_B(a_2) = \$3\ (800) = \$2{,}400,$$

$$C_B(a_3) = \$3\ (1{,}200) = \$3{,}600.$$

Driver tracing assigns the cost based on the material demanded, which is the causal factor identified by the input cost function. Material demanded is a transaction driver that provides the following cost assignment:

$$C_B(a_1) = \frac{500}{2{,}500}\ (\$7{,}500) = \$1{,}500,$$

$$C_B(a_2) = \frac{800}{2{,}500}\ (\$7{,}500) = \$2{,}400,$$

$$C_B(a_3) = \frac{1{,}200}{2{,}500}\ (\$7{,}500) = \$3{,}600.$$

The example illustrates that direct tracing is equivalent to driver tracing when a transaction driver is used to assign the cost of a direct input that exhibits the effects of economies of scale. This result indicates a possible improvement to the ABC assignment system. Recall that when inputs exhibit the effects of economies of scale, the use of a financial driver in an ABC assignment system may provide a cost assignment that better reflects relative contribution. This implies that, in an ABC assignment system, direct tracing can be improved by using a financial driver.

Evaluating ABC and FBC against the imputation benchmark reveals a condition under which ABC can be identified as the superior cost assignment system. When economies of scope arises from fixed factors of production, inputs exhibiting economies of scope or a combination of both, $PD > PD_I$ indicates a violation of the imputation conditions by FBC assignments. Thus, using a benchmark defined by the set of imputations, the superiority claim of ABC is supported when $PD > PD_I$. The analysis in Chapter 7 provides similar results when an evaluation of ABC and FBC assignments is conducted against a benchmark specified by the core conditions.

CHAPTER 7: AN EVALUATION OF ABC AND FBC – THE CORE BENCHMARK AND POINT SOLUTIONS

A core solution offers a set of solutions that meet certain key rationality concepts. ABC and FBC assignments can be analyzed to determine under what conditions each qualifies as a member of the core. The conditions for core membership provide the basis for an evaluation of competing cost assignments, where an assignment qualifying for membership in the core is superior to an assignment that does not. An evaluation of ABC and FBC assignments against the core conditions is similar to the analysis presented in Chapter 6, where the degree of product diversity indicates when the difference between cost assignments is sufficiently different to cause the FBC assignment to violate the conditions of core membership.

Point solutions are specific predictions of shared cost decompositions and potentially provide additional methods of evaluating ABC and FBC assignments. Two possible point solutions were identified in Chapter 4; the simple Shapley value and the generalized Shapley value. These two point predictions can be considered theoretical product cost benchmarks if they correspond to actual rational outcomes.

The analysis presented in this chapter evaluates ABC and FBC assignments of fixed inputs and inputs exhibiting the effects of economies of scale based on their conformance to the core conditions and their correspondence to point solution concepts.

7.1 The Core Benchmark

The core represents a more narrowly defined, more restrictive, rational theoretical product cost benchmark. The core benchmark refines the imputation benchmark by including only non-dominated imputations. To be a member of the core, a cost assignment must satisfy three rationality conditions that were described in Chapter 4; *pareto* optimality, individual rationality and group rationality. Comparing cost assignments provided by ABC and FBC to the rationality conditions that define the core facilitates an evaluation of ABC and FBC assignments. When ABC and FBC assignments satisfy these conditions, the cost assignments qualify as members of the core and are said to conform to the core benchmark.

General conditions that indicate when cost assignments provided by ABC and FBC conform to the core benchmark are derived in Proposition 8.

Proposition 8:

Cost assignments provided by ABC and FBC are members of the core provided the following conditions are satisfied:

$$\textbf{ABC:}\ 1 - \frac{C_{\{N-i\}}}{C_N} \leq \bar{\varepsilon}_i \leq \frac{C_i}{C_N}. \tag{32}$$

$$\textbf{FBC:}\ 1 - \frac{C_{\{N-i\}}}{C_N} \leq \mu_i \leq \frac{C_i}{C_N}. \tag{33}$$

Proof: See Appendix A.

Proposition 8 states the general conditions under which ABC and FBC assignments will conform to a rational decomposition defined by the core. The ratio $\frac{C_i}{C_N}$ was identified in Chapter 6 as the critical assignment ratio that defines membership in the set of imputations. The existence of this condition in equation (32) and (33) implies that members of the core must also qualify as members of the set of imputations. Further restrictions are placed on the ABC and FBC assignment ratios for core membership, as indicated by $\bar{\varepsilon}_i \geq 1 - \frac{C_{\{N-i\}}}{C_N}$ and $\mu_i \geq 1 - \frac{C_{\{N-i\}}}{C_N}$. ABC and FBC assignments must satisfy both conditions to be members of the core.

In an evaluation of competing cost systems, a cost assignment belonging to the core is rationally superior to a cost assignment that does not qualify as a member of the core. If ABC assignments always satisfy the core conditions, and FBC assignments do not always belong to the core, then conditions can be identified that indicate when ABC provides a rationally superior cost assignment. These superiority conditions are defined by identifying a critical degree of product diversity that indicates when FBC assignments violate the core conditions.

The following sections provide an evaluation of ABC and FBC assignments of fixed inputs and inputs exhibiting the effects of economies of scale based on the core benchmark.

7.1.1 Fixed Factors of Production

As stated in Chapter 6, the existence of fixed inputs in a multiple product plant is independent of cost assignment concepts. Recall that the cost of fixed inputs in a three-product plant is expressed as follows:

$$C_N = P_1(K_{11} + K_{21} + K_{31} - t_1) + P_2(K_{12} + K_{22} + K_{32} - t_2) + P_3(K_{13} + K_{23} + K_{33} - t_3).$$

Cost assignments of fixed inputs must satisfy all core conditions to qualify as members of the core. For an analysis under the core conditions, it is necessary to express the cost of all two-product combinations to evaluate the group rationality condition. Given a three-product plant has formed, then $t_j \leq 2$ and the number of steps of input j saved by any two-product combination must be strictly less than t_j. If v_j represents the number of steps of input j saved in a two-product combination, then the cost of all possible two-product combinations in a three-product setting can be expressed as follows:

$$C_{12} = P_1(K_{11} + K_{21} - v_j) + P_2(K_{12} + K_{22} - v_j),$$

$$C_{13} = P_1(K_{11} + K_{31} - v_j) + P_3(K_{13} + K_{33} - v_j),$$

$$C_{23} = P_1(K_{22} + K_{32} - v_j) + P_3(K_{23} + K_{33} - v_j),$$

where $0 \leq v_j \leq 1$ and $v_j < t_j$.

An ABC assignment of fixed inputs is based on the causal factor identified by the fixed input cost function variable, z_{ij}. Using z_{ij} as the cost driver, ABC will always provide cost assignments of fixed inputs that qualify as members of the core. This result is derived in Proposition 9.

Proposition 9:

ABC assignments always conform to the core benchmark if shared resources in a multiple product setting are fixed factors of production.

Proof: See Appendix A.

Proposition 9 states an important result that is used to determine when ABC assignments will be superior to FBC assignments under the core benchmark. Proposition 9 states that ABC assignments of fixed inputs will always conform to the core benchmark, which implies group rationality is met and $\sum_{i \in S} \bar{\varepsilon}_i C_N \leq C_S$. To evaluate ABC and FBC assignments, it is necessary to determine if FBC assignments have the same property. If FBC assignments do not always conform to the core benchmark, a critical degree of product diversity can be identified to indicate when ABC assignments are rationally superior to FBC assignments.

An FBC assignment is based on a unit level factor correlated with production, μ_i, and equation (26) specifies an FBC assignment of fixed inputs in a three-product setting. By Proposition 5, FBC does not always belong to the set of imputations, which implies FBC assignments will also fail to satisfy the conditions for core membership. When FBC assignments of fixed inputs do conform to the imputation benchmark it is possible, however, for the assignments to violate the conditions of core membership. This implies that FBC assignments of fixed factors qualifying as members of the set of imputations can be dominated by other imputations. This result is stated in Proposition 10.

Proposition 10:

If FBC assignments qualify as members of the set of imputations, the assignments do not always conform to the core benchmark if shared resources in a multiple product plant are fixed inputs.

Proof:

Assume FBC assignments of fixed inputs are imputations and always qualify as members of the core. Then, if at least one example can be constructed that shows an FBC assignment belonging to the set of imputations that does not belong to the core, Proposition 10 is established by contradiction.

For simplicity, a three-product setting with one non-unit level input, purchasing labor input, and one unit level input, machining input is used to construct the example. The results are easily extended to *n* inputs. Assume the causal factor for purchasing labor input is represented by purchase orders demanded, and the unit level factor of the machining input is machine hours. Each fixed input is represented by the cost function, $C_i = \sum_j P_j K_{ij}$, and information related to each input is as follows:

		Single Product Plants			Three Product Plant
		1	2	3	123
Purchasing Labor – $\boldsymbol{S_1}$ **= 500 purchase orders** $\boldsymbol{P_1}$ **= \$5,000**					
Purchase orders demanded	z_{ij}	945	1,050	1,505	3,500
Number of steps required	$\boldsymbol{K_{ij}}$	2	3	4	7
Input cost	$\boldsymbol{P_j K_{ij}}$	\$10,000	\$15,000	\$20,000	\$35,000
ρ_{ij}	$z_{ij}/\sum z_{ij}$	.27	.30	.43	

		Single Product Plants			Three Product Plant
		1	2	3	123

Machining-
S_2= 5,000 machine hours
P_2= $10,000

Machine hours demanded	z_{ij}	11,250	12,500	1,250	25,000
Number of steps required	K_{ij}	3	3	1	5
Input cost	P_jK_{ij}	$30,000	$30,000	$10,000	$50,000
μ_i	$z_{ij}/\Sigma z_{ij}$	.45	.50	.05	

The single product cost, C_i, and the cost of each possible two-product combination are as follows:

C_1	C_2	C_3
$40,000	$45,000	$30,000

C_{12}	C_{13}	C_{23}
$70,000	$55,000	$60,000

The FBC assignment of the fixed inputs is calculated as follows:

$C_\mu(a_1)$ = .45 ($35,000 + $50,000) = $38,250,

$C_\mu(a_2)$ = .50 ($35,000 + $50,000) = $42,500,

$C_\mu(a_3)$ = .05 ($35,000 + $50,000) = $ 4,250.

The above FBC assignment satisfies the conditions for membership in the set of imputations such that $\sum_i C_\mu(a_i) = C_N$ and $C_\mu(a_i) \leq C_i$. However, the core condition of group rationality requires $\sum_{i \in S} C_\mu(a_i) \leq C_S$ and this condition is violated by the combination of product 1 and product 2, where $C_\mu(a_1) + C_\mu(a_2) > C_{12}$. Therefore, FBC assignments of fixed inputs that are imputations do not always qualify as members of the core. Q.E.D.

The results of Proposition 9 and Proposition 10 provide the opportunity to identify when ABC assignments are rationally superior under the core benchmark. Proposition 9 states that ABC assignments will always be members of the core, while Propositions 5 and 10 reveal that FBC assignments do not always qualify as members of the core. When PD=0, $\sum_{i\in S} \bar{\varepsilon}_i = \sum_{i\in S} \bar{\rho}_i = \sum_{i\in S} \mu_i$, which implies FBC assignments will also be members of the set of imputations and the core under this condition. However, as PD increases, FBC assignments that qualify as members of the set of imputations will be driven away from satisfying the conditions of the core such that $\mu_i < 1 - \frac{C_{\{N-i\}}}{C_N}$. Proposition 9 implies $\sum_{i\in S} \bar{\rho}_i\, C_N < C_S$, and the maximum increment by which ABC and FBC assignments can differ before FBC fails to conform to the core benchmark is, $\frac{C_S}{C_N} - \sum_{i\in S} \bar{\rho}_i$. If $\sum_{i\in S} \mu_i^0 = \sum_{i\in S} \bar{\rho}_i$ when PD=0, then the threshold FBC assignment ratios which allow FBC to conform to the core benchmark are

$\sum_{i\in S} \mu_i^C = \sum_{i\in S} \mu_i^0 + \left(\frac{C_S}{C_N} - \sum_{i\in S} \bar{\rho}_i\right)$, where $\sum_i \mu_i^C = 1$. When PD causes FBC assignment ratios to exceed $\sum_{i\in S} \mu_i^C$, FBC no longer provides assignments conforming to the core benchmark. Under this condition $\mu_i < 1 - \frac{C_{\{N-i\}}}{C_N}$ and the ABC assignment dominates the FBC assignment which allows ABC to be identified as the superior cost assignment system. Let PD_C be the critical degree of product diversity that defines when FBC assignments belong to the core, where PD_C is defined as the degree of difference between ABC and FBC when $\mu_i = \mu_i^C$. PD_C is defined as follows:

$$PD_C = \sum_i (\overline{\rho}_i - \mu_i^C)^2 . \tag{34}$$

Thus, when PD > PD_C, $\mu_i < 1 - \frac{C_{\{N-i\}}}{C_N}$ and FBC assignments that qualify as members of the set of imputations do not qualify as members of the core, which implies ABC assignments are rationally superior under the core benchmark. PD_C can be defined for a three-product, three input setting where each $\sum_{i \in S} \mu_i^C$ is specified as follows:

$$S=\{1,2\}: \qquad (\mu_1^C + \mu_2^C) = (\mu_1^0 + \mu_2^0) + \left(\frac{C_{12}}{C_N} - (\overline{\rho}_1 + \overline{\rho}_2)\right), \tag{35}$$

$$S=\{1,3\}: \qquad (\mu_1^C + \mu_3^C) = (\mu_1^0 + \mu_3^0) + \left(\frac{C_{13}}{C_N} - (\overline{\rho}_1 + \overline{\rho}_3)\right), \tag{36}$$

$$S=\{2,3\}: \qquad (\mu_2^C + \mu_3^C) = (\mu_2^0 + \mu_3^0) + \left(\frac{C_{23}}{C_N} - (\overline{\rho}_2 + \overline{\rho}_3)\right). \tag{37}$$

Substituting equations (35) – (37) into ($\sum_{i \in S} \overline{\rho}_i$ - $\sum_{i \in S} \mu_i^C$) and simplifying provides the following:

$$(\overline{\rho}_1 - \mu_1^C) + (\overline{\rho}_2 - \mu_2^C) = - \left(\frac{C_{12}}{C_N} - (\overline{\rho}_1 + \overline{\rho}_2)\right). \tag{38}$$

$$(\bar{\rho}_1 - \mu_1^C) + (\bar{\rho}_3 - \mu_3^C) = -\left(\frac{C_{13}}{C_N} - (\bar{\rho}_1 + \bar{\rho}_3)\right), \tag{39}$$

$$(\bar{\rho}_2 - \mu_2^C) + (\bar{\rho}_3 - \mu_3^C) = -\left(\frac{C_{23}}{C_N} - (\bar{\rho}_2 + \bar{\rho}_3)\right). \tag{40}$$

Solving equations (38) – (40) simultaneously provides:

$$(\bar{\rho}_1 - \mu_1^C) = \left(\frac{C_{23} - C_{13} - C_{12}}{2C_N} + \bar{\rho}_1\right),$$

$$(\bar{\rho}_2 - \mu_2^C) = \left(\frac{C_{13} - C_{12} - C_{23}}{2C_N} + \bar{\rho}_2\right),$$

$$(\bar{\rho}_3 - \mu_3^C) = \left(\frac{C_{12} - C_{13} - C_{23}}{2C_N} + \bar{\rho}_3\right).$$

PD_C in a three-product plant is defined as:

$$PD_C = \left(\frac{C_{23} - C_{13} - C_{12}}{2C_N} + \bar{\rho}_1\right)^2 + \left(\frac{C_{13} - C_{12} - C_{23}}{2C_N} + \bar{\rho}_2\right)^2 + \left(\frac{C_{12} - C_{13} - C_{23}}{2C_N} + \bar{\rho}_3\right)^2. \tag{41}$$

ABC assignments of fixed inputs will be rationally superior to FBC assignments when the degree of product diversity causes the FBC assignment to violate the rationality conditions of the set of imputations and/or the core. If $PD > PD_I > PD_C$, FBC fails to satisfy both the imputation and core conditions and ABC provides a rationally superior cost assignment. When $PD_I > PD > PD_C$, an ABC assignment dominates the FBC assignment and ABC is identified as a rationally superior cost system.

7.1.2 Inputs Exhibiting the Effects of Economies of Scale

Inputs exhibiting the effects of economies of scale are identified by economies of scope literature as inputs exhibiting subadditivity and, therefore, may exist in a multiple product plant. The cost of inputs exhibiting the effects of economies of scale in a three-product plant is expressed as follows:

$$C_N = C(z_{11} + z_{21} + z_{31}) + C(z_{12} + z_{22} + z_{32}) + C(z_{13} + z_{23} + z_{33}).$$

As presented in Chapter 6, an ABC assignment of inputs exhibiting the effects of economies of scale can be defined using a driver based on z_{ij}, specified by equation (29), or financial drivers, specified by equation (30). To determine if the ABC assignments specified by equations (29) and (30) qualify as members of the core, the assignments must be evaluated against the core requirements. The analysis reveals that the ABC assignment system will always provide cost assignments that conform to the core benchmark, regardless of the cost driver chosen. This result is derived in Proposition 11.

Proposition 11:

ABC assignments always conform to the core benchmark in a multiple product setting if shared resources exhibit economies of scale effects.

Proof: See Appendix A.

The result of Proposition 11 indicates ABC assignments of inputs exhibiting the effects of economies of scale will always qualify as members of the core. This result allows for the identification of superiority conditions if FBC assignments do not always conform to the core benchmark.

An FBC assignment of inputs exhibiting the effects of economies of scale was specified in equation (25). This cost assignment must be evaluated against the core conditions to determine when FBC assignments qualify as members of the core. By Proposition 7, FBC assignments of inputs exhibiting the effects of economies of scale do not always conform to the imputation benchmark, which implies the assignments also fail to qualify as members of the core. The analysis of FBC assignments reveals that even if FBC assignments are imputations, it is possible that they may not be members of the core. This implies the FBC assignments of inputs exhibiting the effects of economies of scale qualifying as members of the set of imputations can be dominated by other imputations. This result is stated in Proposition 12.

Proposition 12:

If FBC assignments qualify as members of the set of imputations, the assignments do not always conform to the core benchmark if shared resources in a multiple product setting are inputs exhibiting the effects of economies of scale.

Proof:

Assume FBC assignments of inputs exhibiting the effects of economies of scale are imputations and always qualify as members of the core. Then, if at least one example can be constructed that shows an FBC assignment belonging to the set of imputations that does not belong to the core, Proposition 12 is established by contradiction.

For simplicity, a three-product setting with one non-unit level input, purchasing material input, and one unit level input, machining material input is used to construct the example. The results are easily extended to n inputs. Assume the causal factor for purchasing material input is represented by purchase orders demanded, and the unit level factor for machining material input is machine hours. Each input is represented by the cost function, $C_i = \sum_j C(z_{ij})$, and information related to each input is as follows:

		Single Product Plants			Three Product Plant
		1	2	3	123
Purchasing Material –					
Purchase orders demanded	z_{ij}	10,450	2,850	5,700	19,000
Average cost per unit		\$5.00	\$8.00	\$6.00	\$4.50
Input cost	$C(z_{ij})$	\$52,250	\$22,800	\$34,200	\$85,500
ρ_{ij}	$z_{ij}/\sum z_{ij}$	.55	.15	.30	
Machining Material-					
Machine hours demanded	z_{ij}	9,000	3,750	2,250	15,000
Average cost per unit		\$4.80	\$5.00	\$5.00	\$4.50
Input cost	$C(z_{ij})$	\$43,200	\$18,750	\$11,250	\$67,500
μ_i	$z_{ij}/\sum z_{ij}$	.60	.25	.15	

The single product cost and the cost of each two-product combination are as follows:

C_1	C_2	C_3
\$95,450	\$41,550	\$45,450

C_{12}	C_{13}	C_{23}
\$123,875	\$123,300	\$80,100

The FBC assignment of the fixed inputs is calculated as follows:

$C_{\mu}(a_1) = .60\ (\ \$85{,}500 + \$67{,}500) = \$91{,}800,$

$C_{\mu}(a_2) = .25\ (\ \$85{,}500 + \$67{,}500) = \$38{,}250,$

$C_{\mu}(a_3) = .15\ (\ \$85{,}500 + \$67{,}500) = \$22{,}950.$

The above FBC assignment qualifies as a member of the set of imputations since $\sum_i C_{\mu}(a_i) = C_N$ and $C_{\mu}(a_i) \leq C_i$. However, the core condition of group rationality requires $\sum_{i \in S} C_{\mu}(a_i) < C_S$, which is violated by the combination of product 1 and product 2, where $C_{\mu}(a_1) + C_{\mu}(a_2) > C_{12}$. Therefore, FBC assignments of inputs exhibiting the effects of economies of scale belonging to the set of imputations do not always qualify as members of the core. Q.E.D.

The results of Proposition 11 imply that ABC assignments of inputs exhibiting the effects of economies of scale always conform to the core benchmark, while Propositions 7 and 12 imply that FBC assignments do not always have the same property. Therefore, when $PD > PD_I > PD_C$, FBC does not provide a rational cost assignment and ABC is identified as the superior cost system. When $PD_I > PD > PD_C$, FBC does provide a rational cost assignment, but this assignment is dominated by the assignment provided by ABC and thus, ABC is identified as a rationally superior cost assignment system.

7.2 Critical Degrees of Product Diversity – An Example

Two critical degrees of product diversity have been identified by the analytical investigation; PD_I indicating when FBC assignments fail to conform to the imputation benchmark and PD_C indicating when FBC assignments fail to conform to the core benchmark. PD, PD_I and PD_C can be illustrated using a three-product, three-input example, where purchasing labor and material handling labor represent non-unit level inputs and machining represents a unit level input.

		Single Product Plants			Three Product Plant
		1	2	3	123
Purchasing Labor :					
Purchase orders needed	z_{ij}	945	1,050	1,505	3,500
Input cost	P_jK_{ij}	\$10,000	\$15,000	\$20,000	\$35,000
ρ_{ij}	$z_{ij}/\Sigma z_{ij}$	0.27	0.30	0.43	
Material Handling Labor :					
Number of moves needed	z_{ij}	3,375	1,125	3,000	7,500
Input cost	P_jK_{ij}	\$20,000	\$10,000	\$20,000	\$40,000
ρ_{ij}	$z_{ij}/\Sigma z_{ij}$	0.45	0.15	0.40	
Machining :					
Machine hours needed	z_{ij}	3,960	6,600	22,440	33,000
Activity cost	$C(z_{ij})$	\$31,680	\$39,600	\$89,760	\$115,500
μ_i	$z_{ij}/\Sigma z_{ij}$	**0.12**	**0.20**	**0.68**	
$\overline{\rho}_i$		**.366**	**.220**	**.414**	
C_i		\$61,680	\$64,600	\$129,760	

PD, defined by equation (18), is calculated as follows:

$$PD = (.366 - .12)^2 + (.22 - .20)^2 + (.414 - .68)^2 = .132.$$

PD_I, defined by equation (23), is calculated as follows:

$$PD_I = \left[\left(\frac{129{,}760}{190{,}500} - .414\right) + \left(\frac{64{,}600}{190{,}500} - .220\right)\right]^2 + \left(\frac{129{,}760}{190{,}500} - .414\right)^2 + \left(\frac{64{,}600}{190{,}500} - .220\right)^2,$$

$$PD_I = .2348.$$

PD_C is calculated using equation (41) as follows, where C_{12} = \$92,800, C_{13} = \$162,680, C_{23} = \$167,448, and C_N = \$190,500:

$$PD_C = \left(\frac{167{,}448 - 162{,}680 - 92{,}800}{2(190{,}500)} + .366\right)^2$$

$$+ \left(\frac{162{,}680 - 92{,}800 - 167{,}448}{2(190{,}500)} + .220\right)^2$$

$$+ \left(\frac{92{,}800 - 162{,}680 - 167{,}448}{2(190{,}500)} + .414\right)^2,$$

$$PD_C = .06315.$$

The example illustrates that both ABC and FBC assignments conform to the imputation benchmark as indicated by PD < .2348. However, the FBC assignment fails to satisfy the core conditions since PD > .06315. This implies that the FBC assignment qualifies as an imputation but is dominated by other imputations. Specifically, it is dominated by the ABC assignment, which is a member of the core. Therefore, ABC will be identified as the rationally superior cost assignment system.

7.3 Point Solutions

Point solutions, such as the simple Shapley and the generalized Shapley, are point predictions of rational decompositions of shared costs that provide an additional opportunity to identify theoretical product cost benchmarks. A point prediction is a specific solution, or a class of solutions, for allocating a shared benefit. However, only point predictions corresponding to *actual* rational outcomes should be used as theoretical product cost benchmarks in an evaluation of ABC and FBC assignments. The following sections present discussions related to the use of the simple and generalized Shapley values as theoretical product cost benchmarks.

7.3.1 Simple Shapley Value

The simple Shapley value, which provides a specific allocation of a shared benefit based on a set of rationality axioms, was described in Chapter 4. The simple Shapley value can be interpreted as an expected payoff where the main axiom of the simple Shapley requires an allocation of the shared benefit independent of contribution. In a multiple product cost assignment setting, the main axiom implies that each product in the multiple product plant, regardless of contribution, shares equally in the cost savings that arise from economies of scope.

The main axiom of the simple Shapley has been criticized in game theory and cost allocation literature. Kalai and Samet (1987) state that the main axiom of the simple Shapley requires acceptance of an assumption of symmetry between the players in a game. Kalai and Samet argue that the assumption of symmetry is not realistic since, in most situations, players in a game are not symmetric. Greater bargaining power and/or greater effort by one of the players in a game threatens the validity of this symmetry assumption, which implies the simple Shapley may not correspond to an actual rational outcome when symmetry does not exist. Roth and Verrecchia (1979) state that the simple Shapley represents a fair and equitable allocation of cost when it can be assumed that managers are indifferent between bargaining for an uncertain outcome and receiving an equal share of the benefit for certain. The authors state that without this assumption the simple Shapley might not yield an appropriate cost assignment scheme. This assumption of indifference may not be realistic for cost assignment settings in which the contribution of products is not equal, since managers would be less likely to accept an equal allocation of cost savings.

It is unlikely that an assumption of symmetry or indifference could be invoked in a multiple product cost assignment setting. For example, inputs exhibiting the effects of economies of scale generate cost savings due to a decreasing average cost function. This implies that higher input demand is associated with a lower cost per unit of input. Individual products with higher input demand would contribute more to the cost savings of a multiple product plant than products with lower input demand, which implies symmetry does not exist. The cost savings in a multiple product plant would be shared equally among the products using the simple Shapley value. However, a rational outcome would be an assignment based on relative contribution since the product with higher input demand would create greater bargaining power for a rational manager. Therefore, the simple Shapley is unable to provide a point prediction corresponding to an actual rational outcome when symmetry does not exist, which prevents its use as a theoretical product cost benchmark under these conditions.

Point solution benchmarks based on contribution that correspond to actual rational outcomes should be used in an evaluation of ABC and FBC assignments. One possible point solution that specifies a class of solutions based on contribution is the generalized Shapley value.

7.3.2 Generalized Shapley Value

The generalized Shapley value, which provides a class of point solutions, was described in Chapter 4. The generalized Shapley replaces the main axiom of the simple Shapley with a reasonable set of weights based on relative contribution. When a set of weights can be chosen that reflect rational behavior, the generalized Shapley provides a point prediction that corresponds to an actual rational outcome. Rational behavior, and the definition of actual rational outcomes, depends on specific input cost structures. The discussion that follows assumes an input cost structure represented by fixed factors of production. A similar analysis would be necessary for other cost structures, such as inputs exhibiting economies of scale.

For fixed inputs, a possible choice for a set of weights that reflects rational behavior, and a correspondence to an actual rational outcome, is a set of weights defined by relative unused input capacity. This set of weights reflects the contribution of each single product plant in reducing or eliminating unused input capacity in the multiple product plant and, therefore, implies a possible correspondence to actual rational outcomes.

The use of the simple Shapley value and the generalized Shapley value as possible theoretical product cost benchmarks can be illustrated with two cases. Case 1 provides an illustration where each product contributes equally to the cost savings and the symmetry assumption of the simple Shapley is valid. This implies that both the simple Shapley and the generalized Shapley may correspond to an actual rational outcome, which suggests that both are candidates for theoretical product cost benchmarks. In Case 2 symmetry between the products does not exist and the simple Shapley may no longer be considered an actual rational outcome, which prevents its use as a theoretical benchmark. Assuming no unused capacity exists in the three-product plant, each case can be illustrated using information related to purchasing labor, a non-unit level input, and machining, a unit level input. A comparison of the proposed generalized Shapley benchmark, $C_{GS}(a_i)$, and the proposed simple Shapley benchmark, $C_{SS}(a_i)$, is presented for each case.

Case 1:

Purchasing Labor		Single Product Plants			Three-Product Plant
S_1=500, P_1=\$5,000		1	2	3	123
Expected capacity	z_{ij}	**667**	**1,167**	**1,666**	**3,500**
Practical capacity	S_jK_{ij}	1,000	1,500	2,000	3,500
Unused capacity	$S_jK_{ij}-z_{ij}$	**333**	**333**	**334**	**0**
Input cost	P_jK_{ij}	\$10,000	\$15,000	\$20,000	\$35,000
ρ_{ij}	$z_{ij}/\sum z_{ij}$	.19	.33	.48	

Machining Input S_1=5,000, P_1=\$10,000		Single Product Plants 1	2	3	Three-Product Plant 123
Expected capacity	z_{ij}	**3,333**	**8,333**	**13,334**	**25,000**
Practical capacity	S_jK_{ij}	5,000	10,000	15,000	25,000
Unused capacity	$S_jK_{ij}-z_{ij}$	**1,667**	**1,667**	**1,666**	**0**
Input cost	P_jK_{ij}	\$10,000	\$20,000	\$30,000	\$50,000
μ_i	$z_{ij}/\sum z_{ij}$	.13	.33	.54	

The single product cost and the cost of each two-product combination are as follows:

	C_1	C_2	C_3	C_{12}	C_{13}	C_{23}
Purchasing	\$10,000	\$15,000	\$20,000	\$20,000	\$25,000	\$30,000
Machining	\$10,000	\$20,000	\$30,000	\$30,000	\$40,000	\$50,000
Total	**\$20,000**	**\$35,000**	**\$50,000**	**\$50,000**	**\$65,000**	**\$80,000**

Both the simple Shapley and the generalized Shapley specify an assignment based on the allocation of the residual contribution of each subcoalition. Each method was described in Chapter 4. The simple Shapley provides an equal allocation of the residual contributions, while the generalized Shapley uses a sets of weights, $w_i(S)$, to assign the residual contribution of each possible combination. The residual contributions, d_S, are as follows for each input:

	d_{12}	d_{13}	d_{23}	d_{123}
Purchasing	\$5,000	\$5,000	\$5,000	(\$5,000)
Machining	\$0	\$0	\$0	\$10,000

The generalized Shapley weights, $w_i(S)$, based on relative unused capacity for each product and input are summarized in Table 7-1.

Table 7-1. Generalized Shapley Weights: Case 1 - Symmetry

	Generalized Shapley Weights		
Product 1	$w_1(12)$	$w_1(13)$	$w_1(123)$
Purchasing	.50	.50	.33
Machining	.50	.50	.33
Product 2	$w_2(12)$	$w_2(23)$	$w_2(123)$
Purchasing	.50	.50	.33
Machining	.50	.50	.33
Product 3	$w_3(13)$	$w_3(23)$	$w_3(123)$
Purchasing	.50	.50	.33
Machining	.50	.50	.33

The proposed benchmarks are compared in Table 7-2, where equation (6) is used to calculate the simple Shapley value, $C_{SS}(a_i)$, and equation (7) is used to calculate the generalized Shapley value, $C_{GS}(a_i)$. The ABC and FBC assignments are also presented to illustrate a comparison of each to the proposed benchmarks.

Table 7-2. Simple Shapley and Generalized Shapley Values: Case 1

	Product 1	**Product 2**	**Product 3**
$C_{SS}(a_i)$	\$13,350	\$28,350	\$43,300
$C_{GS}(a_i)$	\$13,335	\$28,335	\$43,330
$C_{\rho}(a_i)$	\$13,336	\$28,336	\$43,328
$C_{\mu}(a_i)$	\$11,332	\$28,332	\$45,336

Symmetry, i.e. equal contribution, exists between the products in Case 1 and the benchmarks are approximately equal. Under the assumptions of Case 1, both point predictions potentially correspond to actual rational outcomes that can be used as theoretical product cost benchmarks.

An evaluation of the ABC and FBC assignments against the proposed point solution benchmarks is a comparison of "closeness". Closeness can be measured using a distance measure, $\mathbf{d} = [\sum(x_i - y_i)^2]^{1/2}$, where x_i represents a theoretical product cost benchmark and y_i represents a cost assignment using ABC or FBC. For Case 1, the distance between each assignment and the proposed benchmarks, defined by **d**, is illustrated as follows:

	$\underline{C_{SS}(a_i)}$	$\underline{C_{GS}(a_i)}$
ABC assignment	29.58	3.06
FBC assignment	2,830.98	2,833.83

A comparison using the distance measure illustrates that an ABC assignment corresponds more closely to each of the proposed benchmarks in this example, which implies ABC may be rationally superior to FBC.

Case 1 illustrates equal contribution among the products. However, as contribution begins to differ, the simple Shapley will no longer provide an outcome that is rational. In Case 2 contribution is no longer equal, and the generalized Shapley value based on relative unused capacity remains a valid candidate for an actual rational outcome since it considers the relative contribution of each product. The following information is used to illustrate the assumptions of Case 2, where initially PD=0:

Case 2:

Purchasing Labor $S_1=500, P_1=\$5,000$		Single Product Plants 1	2	3	Three-Product Plant 123
Expected capacity	z_{ij}	**945**	**1,050**	**1,505**	**3,500**
Practical capacity	S_jK_{ij}	1,000	1,500	2,000	3,500
Unused capacity	$S_jK_{ij}-z_{ij}$	**55**	**450**	**495**	**0**
Input cost	P_jK_{ij}	\$10,000	\$15,000	\$20,000	\$35,000
ρ_{ij}	$z_{ij}/\sum z_{ij}$	.27	.30	.43	

Machining Input $S_1=5,000, P_1=\$10,000$					
Expected capacity	z_{ij}	**6,750**	**7,500**	**10,750**	**25,000**
Practical capacity	S_jK_{ij}	10,000	10,000	15,000	25,000
Unused capacity	$S_jK_{ij}-z_{ij}$	**3,250**	**2,500**	**4,250**	**0**
Input cost	P_jK_{ij}	\$20,000	\$20,000	\$30,000	\$50,000
μ_i	$z_{ij}/\sum z_{ij}$	.27	.30	.43	

The generalized Shapley weights based on relative unused capacity for each product and input under Case 2 are presented in Table 7-3.

Table 7-3. Generalized Shapley Weights: Case 2 - No Symmetry

	Generalized Shapley Weights		
Product 1	$w_1(12)$	$w_1(13)$	$w_1(123)$
Purchasing	.110	.100	.055
Machining	.570	.430	.325
Product 2	$w_2(12)$	$w_2(23)$	$w_2(123)$
Purchasing	.890	.480	.450
Machining	.430	.370	.250
Product 3	$w_3(13)$	$w_3(23)$	$w_3(123)$
Purchasing	.900	.520	.495
Machining	.570	.630	.425

The proposed benchmarks and the ABC and FBC assignments are summarized in Table 7-4:

Table 7-4. Simple Shapley and Generalized Shapley Values: Case 2

	Product 1	**Product 2**	**Product 3**
$C_{SS}(a_i)$	$19,950	$24,950	$39,950
$C_{GS}(a_i)$	$22,494	$24,862	$37,642
$C_{\rho}(a_i)$	$22,950	$25,500	$36,550
$C_{\mu}(a_i)$	$22,950	$25,500	$36,550

To evaluate ABC and FBC assignments using point solutions, the correspondence of each assignment to proposed benchmarks can be measured. The results of previous evaluations indicate that product diversity plays a key role in determining when ABC and FBC assignments will differ. Therefore, the question becomes, how does product diversity affect the correspondence between ABC and FBC assignments and the proposed point predictions? To address this question, a comparison of the proposed benchmarks to each cost assignment when the degree of product diversity changes is presented in Table 7-5.

Table 7-5. Product Diversity: Simple Shapley Value, Generalized Shapley Value, ABC and FBC Assignments

Degree of Product Diversity	$C_{SS}(a_i)$	$C_{GS}(a_i)$	*ABC Assignment*	*FBC Assignment*
		Product 1		
PD = 0.0000	\$19,950	\$22,494	\$22,950	\$22,950
PD = 0.0518	\$29,950	\$31,980	\$31,950	\$38,250
PD = 0.0878	\$29,950	\$33,730	\$34,450	\$42,500
PD = 0.1718	\$36,650	\$39,230	\$39,450	\$51,000
PD = 0.2558	\$39,950	\$41,980	\$41,950	\$55,250
PD = 0.3318	\$39,950	\$43,730	\$44,450	\$59,500
		Product 2		
PD = 0.0000	\$24,950	\$24,862	\$25,500	\$25,500
PD = 0.0518	\$24,950	\$23,164	\$23,000	\$21,250
PD = 0.0878	\$24,950	\$23,164	\$23,000	\$21,250
PD = 0.1718	\$21,650	\$20,414	\$20,500	\$17,000
PD = 0.2558	\$24,950	\$23,164	\$23,000	\$21,250
PD = 0.3318	\$24,950	\$23,164	\$23,000	\$21,250
		Product 3		
PD = 0.0000	\$39,950	\$37,643	\$36,550	\$36,550
PD = 0.0518	\$29,950	\$29,856	\$30,050	\$25,500
PD = 0.0878	\$29,950	\$28,106	\$27,750	\$21,250
PD = 0.1718	\$26,650	\$25,356	\$25,050	\$17,000
PD = 0.2558	\$19,950	\$19,856	\$20,050	\$ 8,500
PD = 0.3318	\$19,950	\$18,106	\$17,550	\$ 4,250

The distance between each cost assignment and the proposed benchmarks, measured by $\mathbf{d} = [(x_i - y_i)^2]^{1/2}$, is presented in Table 7-6.

Table 7-6. Distance Measure: ABC and FBC Assignments vs. Proposed Benchmarks

Degree of Product Diversity	***Distance: Cost Assignment/Benchmark***			
	$ABC/C_{SS}(a_i)$	$FBC/C_{SS}(a_i)$	$ABC/C_{GS}(a_i)$	$FBC/C_{GS}(a_i)$
PD = 0.0000	4,567	4,567	1,345	1,345
PD = 0.0518	2,795	10,118	256	7,870
PD = 0.0878	5,460	15,713	924	11,295
PD = 0.1718	3,424	17,907	387	14,832
PD = 0.2558	2,795	19,465	256	17,570
PD = 0.3318	5,460	25,345	924	21,079

The example shows that as PD increases, the distance between FBC assignments and the proposed point solution benchmarks becomes greater, indicating a possible violation of rationality conditions. The example also indicates that ABC assignments of fixed inputs correspond more closely to either the simple Shapley or the generalized Shapley benchmark than FBC assignments. Clearly, ABC corresponds most closely with the generalized Shapley benchmark and, if the generalized Shapley corresponds to an actual rational outcome, this implies that ABC is rationally superior to FBC.

The evaluation of ABC and FBC assignments against point solution benchmarks presented in this section is limited, which suggests possible extensions to this study. For example, only point solutions that correspond to actual rational outcomes should be used in evaluating competing cost assignments. A more formal investigation is needed to determine the extent to which the generalized Shapley, and other point solutions such as the modified Shapley[8], correspond to actual rational outcomes. Once a correspondence is established, an evaluation of competing cost assignments would require a measure to indicate the closeness of each cost assignment to point solution benchmarks. However, it may be possible that the closest cost assignment to the benchmark is not a rational assignment. Therefore, a closeness measure should be developed that incorporates rationality conditions to identify the closest *rational* assignment. In addition, the analysis presented in this section was limited to one cost structure, fixed inputs. Similar analysis is needed for other cost structures such as inputs exhibiting the effects of economies of scale.

7.4 Fixed Factors of Production: Practical vs. Expected Input Capacity

The analytical evaluations of ABC and FBC assignments of fixed inputs have been performed under the assumption that expected and practical input capacity are equal in the three-product plant. Under the traditional framework, ABC assigns cost by applying an assignment rate to the actual input consumption of a product. An ABC assignment rate is calculated by dividing total input cost by practical capacity,

$\frac{\alpha_j}{S_j(\sum_i K_{ij} - t_j)}$. When expected and practical input capacity are equal, the total input cost in the multiple product plant will be assigned under an ABC assignment. However, when this assumption is relaxed, and expected capacity is less than practical capacity for any input, the cost of any unused input capacity will not be assigned under an ABC

[8] The modified Shapley value, introduced by Harsanyi (1977), redefines the Shapley value in terms of a *modified* characteristic function that incorporates the utility functions of the players.

assignment system, which implies $\sum_i C_p(a_i) < C_N$. A *rational* cost assignment requires the total cost of all inputs be assigned to products such that $\sum_i C_p(a_i) = C_N$. This implies that the cost of unused input capacity is assigned under a rational cost assignment, which suggests a possible limitation to the ABC assignment system. This limitation can be illustrated with a purchasing labor input, a fixed input represented by the step cost function, $C_i = P_jK_{ij}$, where it is assumed that purchase orders is highly correlated with purchasing labor hours:

Purchasing Labor S_1=2,000, P_1=\$10,000		Single Product Plants 1	2	3	Three-Product Plant 123
Expected capacity	z_{ij}	3,375	1,125	3,000	7,500
Practical capacity	S_jK_{ij}	4,000	2,000	4,000	8,000
Unused capacity	$S_jK_{ij}-z_{ij}$	625	875	1,000	500
Input cost	P_jK_{ij}	\$20,000	\$10,000	\$20,000	\$40,000

In this example, expected input capacity is 7,500 orders, practical input capacity is 8,000 orders and unused input capacity is 500 orders in the three-product plant. The ABC assignment of the purchasing labor input is calculated as follows:

ABC Assignment Rate:

$$\frac{\alpha_j}{S_j(\sum_i K_{ij} - t_j)} = \frac{\$40{,}000}{8{,}000 \text{ orders}} = \$5 \text{ per order.}$$

ABC Assignment:

$C_\rho(a_1)$:	\$5/order x 3,375 orders	=	\$16,875
$C_\rho(a_2)$:	\$5/order x 1,125 orders	=	\$ 5,625
$C_\rho(a_3)$:	\$5/order x 3,000 orders	=	\$15,000
Total cost assigned under ABC			\$37,500
Total input cost			\$40,000
Input cost not assigned = cost of unused capacity, \$5/order x 500 orders			**\$ 2,500**

The example clearly illustrates that the existence of unused capacity in a multiple product plant causes an ABC assignment to violate the rationality condition of the imputation and core benchmark that requires $\sum_i C_\rho(a_i) = C_N$. The total cost assigned under ABC is \$37,500, but the total cost of the input is \$40,000, which implies $\sum_i C_\rho(a_i) < C_N$. A rational cost assignment will assign the total cost of \$40,000 to the products, which includes the cost of the unused input capacity. Identifying this limitation suggests a modification may be necessary to the existing ABC assignment system when unused capacity exists in the multiple product plant.

7.5 Inputs Exhibiting the Effects of Economies of Scale: Competing ABC Cost Drivers

Two possible cost drivers have been identified for use in an ABC assignment of inputs exhibiting the effects of economies of scale; transaction/duration drivers based on z_{ij} and financial drivers based on $C(z_{ij})$. The cost function of inputs exhibiting the effects of economies of scale is characterized by a decreasing average cost per unit. This implies that a higher input consumption results in a lower cost per unit. Therefore, products that consume higher levels of input contribute more to the total cost savings in a multiple product plant than products with lower input consumption. Conceptually, a cost assignment based on relative contribution would assign the cost savings of the multiple product plant according to each product's contribution. A cost assignment should reflect this contribution and the choice of cost driver should be based on which provides a better measure of relative contribution.

ABC typically uses a driver based on z_{ij} to assign input cost to products, which implies that a product's contribution to total input cost is measured by relative consumption of z_{ij}. However, for inputs exhibiting the effects of economies of scale, higher levels of consumption result in a lower cost per unit, indicating a greater contribution to the total cost savings in a multiple product plant by high consumption products. If a z_{ij} driver is used to assign the cost of inputs exhibiting the effects of economies of scale, the cost assigned to a high consumption product reflects relative consumption but not the incremental effects of high consumption on average cost per unit of input. Thus, a z_{ij} driver is unable to capture the effects of a decreasing average cost and an ABC assignment does not reflect relative contribution. Therefore, input demand may not be an appropriate causal measure for an ABC assignment of inputs exhibiting the effects of economies of scale.

One alternative to the z_{ij} driver is a financial driver based on relative independent input cost, $C(z_{ij})$. This financial driver would capture the incremental effects on the average cost per unit achieved by different levels of input consumption. This suggests that the financial driver may provide a better measure of the contribution each product makes to the total input cost in the multiple product plant.

An evaluation of z_{ij} and financial cost drivers is based on which driver provides an assignment closest to a theoretical product cost benchmark reflecting relative contribution. The generalized Shapley value, where $w_i(S)$ is based on input cost, represents one possible benchmark that can be used in evaluating these competing ABC cost drivers and can be illustrated by the following three-product setting with a single input, purchasing material:

		Single Product Plants			Three-Product Plant
		1	2	3	123
Purchasing Material					
Input demand	z_{ij}	10,450	2,850	5,700	19,000
Average cost per unit		\$5.00	\$8.00	\$6.00	\$4.50
Input cost	$C(z_{ij})$	\$52,250	\$22,800	\$34,200	\$85,500

The generalized Shapley value and ABC assignments using z_{ij} and financial drivers is presented below:

	Product 1	***Product 2***	***Product 3***
$\boldsymbol{C_{GS}(a_i)}$	\$40,016	\$18,814	\$26,669
$\boldsymbol{C_\rho(a_i)}$ **–** z_{ij} **Driver**	\$47,025	\$12,825	\$25,650
$\boldsymbol{C_\rho(a_i)}$ **– Financial Driver**	\$40,891	\$17,843	\$26,765

The distance between the generalized Shapley value and each ABC assignment, measured by $\mathbf{d} = [(x_i - y_i)^2]^{1/2}$,is as follows:

	$\underline{C_{GS}(a_i)}$
ABC – z_{ij} Driver	9,275.44
ABC – Financial Driver	1,310.68

The example illustrate that an ABC assignment of inputs exhibiting the effects of economies of scale based on a financial driver is closer to the benchmark than the ABC assignment using a z_{ij} driver. Based on this numeric analysis it may be argued that a financial driver provides a better measure of relative contribution for inputs exhibiting the effects of economies of scale. A possible extension to this study is an analytical investigation of this claim.

CHAPTER 8: AN EMPIRICAL INVESTIGATION

8.1 Empirical Research Questions and Hypotheses

The primary purpose of this study is an analytical investigation of ABC and FBC assignments. However, the analytical evaluation suggests a secondary opportunity for an empirical analysis whereby two factors that theoretically affect cost assignments can be explored in an experimental setting.

First, the degree of product diversity was identified in the analytical evaluation as a key element in determining the conditions under which a cost assignment provided by ABC is rationally superior to one provided by FBC. The theoretical analysis revealed two critical values of product diversity that indicate when FBC assignments violate the rationality conditions of cooperative game theory; PD_I, which indicates when FBC assignments violate the imputation conditions and PD_C, which indicates when FBC assignments violate the core conditions. When product diversity exceeds the critical values such that $PD > PD_I$ or $PD > PD_C$, rational individuals would not be expected to choose FBC as a cost assignment method. An empirical investigation will help determine if increased degrees of product diversity influence the choice of FBC assignments by rational individuals in a multiple product cost assignment setting. This provides for the following hypothesis:

H_1: As the degree of product diversity increases to exceed PD_I or PD_C in a multiple product plant, an ABC assignment method will be chosen a higher percentage of the time to assign the cost of shared resources than FBC.

Second, the theoretical analysis of inputs exhibiting the effects of economies of scale revealed that two cost drivers exist for ABC assignments; a driver based on the input cost function variable z_{ij} and a financial driver. These cost drivers compete on the basis of providing an assignment that best reflects relative contribution. For inputs exhibiting the effects of economies of scale, a financial driver capturing the effects of a decreasing average cost was identified in the analytical investigation as possibly providing a better reflection of a product's contribution to total input cost in a multiple product plant. An empirical investigation will help determine if individuals recognize the properties of a financial driver when inputs exhibiting the effects of economies of scale exist in a multiple product setting. This provides for the following hypothesis:

> H_2: If inputs exhibiting the effects of economies of scale exist in a multiple product plant, an ABC assignment using a financial driver will be chosen a higher percentage of the time than either FBC or an ABC assignment using a transaction driver.

The degree of product diversity and the use of a financial driver for inputs exhibiting the effects of economies of scale were identified in the analytical evaluation of ABC and FBC assignments using a game-theoretic framework. An empirical analysis examining the effects of game-theoretic concepts on actual cost assignment behavior is considered exploratory in nature. Although empirical studies in game theory have examined the accuracy of predicted solution concepts (Bonacich, 1979; Rapoport, 1987), no such study exists in cost assignment literature.

8.2 Methodology

8.2.1 Experimental Setting

The experimental setting is based on the existence of three single product plants, where the ability to share common resources with subadditive cost functions provides the opportunity for cost savings when multiple product plants are formed, i.e. economies of scope exist. The three single product plants Alpha, Beta and Gamma independently produce baseballs, footballs and softballs, respectively, where four multiple product plants are possible; Alpha/Beta, Alpha/Gamma, Beta/Gamma and Alpha/Beta/Gamma. After forming, common resources are shared and the cost of the shared resources must be assigned to each product in the multiple product plant.

The cost savings in this setting are generated by two shared activities with subadditive cost functions; inspection and machining. Inspection, a non-unit level activity, represents a fixed factor of production and machining, a unit level activity, exhibits the effects of economies of scale. Although cost savings are possible for four all combinations, the greatest amount of cost savings, and thus, the most beneficial combination, is achieved by forming Alpha/Beta/Gamma.

8.2.2 Experimental Design

A one way design is used to test the hypotheses in this study, where the dependent variable is the method chosen to assign the cost of shared resources in multiple product plants. The methods available for assigning shared costs are described as follows:

Method A: FBC assignment
Total cost is divided based on the number of machine hours.

Method B: ABC assignment: Transaction drivers
Inspection cost is divided based on the number of inspections.
Machining cost is divided based on the number of machine hours.

Method C: ABC assignment : Financial and Duration drivers
Inspection cost is divided based on the number of inspection hours (duration driver).
Machining cost is divided based on the relative machining cost (financial driver).

Method B represents an ABC assignment using the transaction drivers number of inspections and number of machine hours. Method C represents an ABC assignment using the number of inspection hours, a duration driver, to assign inspection cost and relative machining cost, a financial driver, to assign the cost of machining. Since financial drivers theoretically provide a better measure of relative contribution for inputs with a decreasing average cost, it is expected that Method C will be chosen more often than Method A or Method B.

One independent variable is the degree of product diversity. Product diversity is defined to exist when a product's relative consumption of non-unit level and unit level activities differs. An operational measure of the degree of product diversity (PD) was developed in Chapter 5 and is defined as $PD = \sum_{i=1}^{n} (\bar{\rho}_i - \mu_i)^2$. Two critical degrees of product diversity were identified in the analytical evaluation, PD_I and PD_C. When either is exceeded, ABC assignments are analytically identified as being rationally superior to FBC assignments. The empirical analysis is based on three different levels of product diversity, where PD = 0, $PD > PD_I > PD_C$, and $PD_I > PD > PD_C$. Table 8-1 summarizes the manipulation of the degree of product diversity for each case.

For this experiment, the degree of product diversity is defined as the difference between the assignment ratios of Method A (FBC) and Method C (ABC using financial and duration drivers). Method C is used for the ABC assignment ratio because, theoretically, it represents an ABC assignment that best reflects relative contribution and thus, the underlying cause and effect relationships between input cost and consumption.

Table 8-1. Independent Variable: Product Diversity

	Product Diversity*		
	PD	**PD_C**	**PD_I**
Case 1: $PD<PD_C<PD_I$ (No PD)	0.00000	0.01265	0.09490
Case 2: $PD_C<PD<PD_I$ (Medium PD)	0.02337	0.00620	0.06371
Case 3: $PD_C<PD_I<PD$ (High PD)	0.05438	0.00490	0.04499

*Calculations are presented in Appendix B.

In Case 1, PD = 0, and there is no difference in ABC and FBC assignments. This implies that, since it was shown that ABC assignments are always members of the set of imputations and the core, FBC also conforms to the imputation and core benchmarks in Case 1. The degree of product diversity is increased in Case 2, where PD > PD_C, and FBC theoretically fails to conform to the core benchmark. For Case 3, PD > PD_I, which indicates FBC theoretically fails to conform to the imputation benchmark. To support H_1, it is expected that as PD increases from Case 1 to Case 3, the number of subjects choosing Method B and Method C will be significantly greater than those choosing Method A.

The existence of inputs exhibiting the effects of economies of scale in a multiple product setting is required for a test of H_2. Each multiple product plant includes the shared resource machining, which exhibits a decreasing average cost function. Since financial drivers were argued to be a better measure of relative contribution for inputs exhibiting a decreasing average cost function, H_2 is supported if more subjects choose Method C over Method A and Method B.

8.2.3 Experimental Procedure

The experiment requested subjects to participate in a role playing exercise. The subjects were provided with background information regarding the three single product plants and were informed of the opportunity to form multiple product plants to reduce product cost. Each single product owner maintained responsibility for their respective profits and subjects were instructed to act in the best interest of all owners involved in a multiple product plant.

In addition to the background information, each subject also received a methods sheet and four fact sheets. The methods sheet described the shared resources and the methods available for assigning the cost of the shared resource in a multiple product setting. The four fact sheets, one for each possible multiple product plant, provided cost and activity information for the multiple product plant as well as for each single product plant involved. The fact sheets also presented the cost that would be assigned to products in the multiple product plant for each cost assignment method available. Supporting calculations of the cost assignments were provided. Appendix B includes examples of all materials given to the subjects.

After receiving the packets of information, but before beginning the experimental task, the subjects were given a brief explanation about the experimental setting and the materials they had received. The subjects were also taken through a short example to gain familiarity with the information presented on each of the four fact sheets. The subjects were then asked to analyze the information provided and make two decisions to be recorded on a question sheet included in their packet of materials. First, a decision had to be made regarding which multiple product plant to form. Second, the subjects had to decide which of the three methods to use in assigning the cost of the shared resources in the multiple product plant formed.

The subjects were also asked to respond to a four item involvement scale, which was intended to measure how well subjects related to the experimental task. The scale is presented in Exhibit 8-1. Note that the first two items on the scale are reverse coded, thus, Exhibit 8-1 differs from the scale actually presented to the subjects as shown in Appendix B. Adding the score given on each of the items indicated a subject's score.

For example, if a subject indicated scores of 5, 4, 4, 5 for the respective items on the scale in Appendix B, a score of 14 would be observed. The possible range of scores on the involvement scale is 4-24. The minimum score indicates a high knowledge of and use of cost assignment methods, which implies that a subject with a low score identified well with the experimental task. On the other hand, a high score on the scale indicated that a subject knew little about cost assignment methods and possibly did not perceive them to be relevant to their career. Thus a high score implied that a subject did not relate well to the experimental task.

8.2.4 Subjects

A total of 113 business students enrolled in a senior level business strategy course participated in the study. Demographic information and responses to the involvement scale were gathered from the subjects. The descriptive statistics are summarized in Table 8-2. The experiment was conducted during a regularly scheduled class time and required approximately 30 minutes to complete. Each subject was randomly assigned to one of the three cases.

Exhibit 8-1. Involvement Scale

Please rate the following statements as they relate to you.

I do not know much about cost assignment systems. Score

6	5	4	3	2	1	
Strongly Agree					Strongly Disagree	______

I am not familiar with many cost assignment methods.

6	5	4	3	2	1	
Strongly Agree					Strongly Disagree	______

My choice of cost assignment is relevant to my future career.

1	2	3	4	5	6	
Strongly Agree					Strongly Disagree	______

Cost assignment methods are relevant to my future career goals.

1	2	3	4	5	6	
Strongly Agree					Strongly Disagree	______

Involvement Score ______

Table 8-2. Descriptive Statistics

	Frequency	%
Class:		
Junior	18	15.9
Senior	94	83.2
Graduate Student	1	.9
Major:		
Marketing	38	33.6
Accounting/Finance	21	18.6
MIS	28	24.8
Other	26	23.0
Gender:		
Female	50	44.3
Male	63	55.7
Mean Age	23.70 years	
Mean Self Reported GPA	3.13 (4 point scale)	
Mean Involvement Score	16.38 (24=low involvement, 4=high involvement)	
Median Involvement Score	16.00	

8.3 Results

Table 8-3 presents a summary of observations by case and assignment method chosen. This information is used in the following sections to test H_1 and H_2.

Table 8-3. Summary of Assignment Choice

	Cost Assignment Method			Total
	FBC	ABC: Transaction Drivers	ABC: Financial and Duration Drivers	
Case 1:PD=0.00000 (No PD)	7 (18.9%)	11 (29.7%)	19 (51.4%)	37 (100%)
Case 2:PD=0.02337 (Medium PD)	4 (10.5%)	10 (26.3%)	24 (63.2%)	38 (100%)
Case 3:PD=0.05438 (High PD)	5 (13.2%)	6 (15.8%)	27 (71.1%)	38 (100%)
Total	**16** (14.2%)	**27** (23.9%)	**70** (61.9%)	**113** (100%)

8.3.1 Hypothesis 1: Product Diversity

Hypothesis 1 predicted that as the degree of product diversity increased, the number of subjects choosing an ABC assignment method would be greater than those choosing the FBC assignment method. Table 8-4 summarizes the cost assignment method chosen for each case, where Method A represents an FBC assignment and Methods B and C represent ABC assignments. By observation, ABC assignments were consistently chosen a higher percentage of the time for each case. To test the effect of the degree of product diversity on the cost assignment choice, a chi-square test of independence was performed. This is a test of the null hypothesis that assignment method is independent of the degree of product diversity. The results of the test imply that the null hypothesis cannot be rejected ($\chi^2 = 1.134$, df = 2, p=.430). The test was also performed on the data after eliminating subjects who had indicated a low involvement in the task. An involvement score in the upper quartile defined low involvement, which was indicated by an involvement score ≥ 19. This procedure reduced the number of subjects to 73, see Table 8-5. A chi-square test of independence on this data provided similar results to the full sample ($\chi^2 = .468$, df = 2, p = .791). The results of the independence test on both sets of data imply that the choice of ABC or FBC assignment methods is independent of the degree of product diversity, and therefore, Hypothesis 1 is not supported.

Table 8-4. Hypothesis 1 - Product Diversity

	Cost Assignment Method		
	FBC	**ABC Approaches**	**Total**
Case 1:PD=0.00000 (No PD)	7 (18.9%)	30 (81.1%)	37 (100%)
Case 2:PD=0.02337 (Medium PD)	4 (10.5%)	34 (89.5%)	38 (100%)
Case 3:PD=0.05438 (High PD)	5 (13.2%)	33 (86.8%)	38 (100%)
Total	**16** (14.2%)	**97** (85.8%)	**113** (100%)

Table 8-5. Hypothesis 1 – High Involvement

	Cost Assignment Method		
	FBC	**ABC Approaches**	**Total**
Case 1:PD=0.00000 (No PD)	3 (13.0%)	20 (87.0%)	23 (100%)
Case 2:PD=0.02337 (Medium PD)	2 (8.3%)	22 (91.7%)	24 (100%)
Case 3:PD=0.05438 (High PD)	2 (7.7%)	24 (92.3%)	26 (100%)
Total	**7** (9.6%)	**66** (90.4%)	**73** (100%)

8.3.2 Hypothesis 2: Competing Cost Drivers

Hypothesis 2 predicted that an ABC assignment method using a financial driver, Method C, would be chosen a higher percentage of the time than either Method A or Method B when inputs exhibiting the effects of economies of scale exist in a multiple product plant. In total, 43 subjects chose either Method A or Method B, while 70 subjects chose Method C. A chi-square goodness of fit test was performed to test the null hypothesis that Method A and B were chosen in the same proportion as Method C. The results of the test imply that the null hypothesis is rejected ($\chi^2 = 6.451$, df = 1, p=0.011), indicating significantly different proportions. Similar results were noted when the test was performed on the data after eliminating subjects with involvement scores in the upper quartile, with 23 subjects choosing Method A or B and 50 subjects choosing Method C ($\chi^2 = 9.986$, df = 1, p=0.002). The results of the chi-square tests on both sets of data indicate a significant difference in the choice of cost assignment method. Subjects chose an ABC assignment using financial and duration drivers significantly more often than the other two methods available. This implies that a financial driver is perceived to be a better choice (i.e. a better reflection of relative contribution), for assigning the cost of inputs with decreasing average cost functions. These results provide support for Hypothesis 2 and also indicate support for a possible improvement to the ABC assignment system through the use of a financial driver for input exhibiting the effects of economies of scale.

An additional test was performed to further examine the effects of subject involvement. The involvement scale was intended to measure how subjects related to the experimental task, where the involvement of the subjects may have influenced their approach to the task and their responses. To investigate whether involvement had an effect on cost assignment choice, a median split was performed on the data, where subjects were classified as either high involvement (involvement ≤ 16) or low involvement (involvement > 16). Table 8-6 presents a summary of assignment method choice by this involvement classification. A chi-square test of independence was performed and the results imply a significant interaction between the involvement score and assignment method choice, ($\chi^2 = 9.29$, df = 2, p=0.010). This implies that those subjects having a higher knowledge and use of cost assignment methods chose assignment methods in significantly different proportions than those who did not relate well to the experimental task. By observation, Table 8-6 reveals that an ABC approach was chosen by 96.6% of the subjects who identified well with the cost assignment task, and chosen by 76.4% by those who did not. One implication of these results is that the use of strategy students, which included students with little knowledge of cost assignment methods, may have biased the results of the study against ABC approaches. This provides some explanation for the lack of support for Hypothesis 1, but on the other hand strengthens the support found for Hypothesis 2.

Table 8-6. Assignment Choice by Involvement Score

Involvement	Cost Assignment Method: FBC	Cost Assignment Method: ABC: Transaction Drivers	Cost Assignment Method: ABC: Financial and Duration Drivers	Total
Low Involvement (score > 16)	13 (23.6%)	14 (25.5)	28 (50.9%)	55 (100%)
High Involvement (score ≤ 16)	2 (3.4%)	14 (24.2%)	42 (72.4%)	58 (100%)
Total	**15** (13.2%)	**28** (24.9%)	**70** (61.9%)	**113** (100%)

8.4 Discussion and Limitations

The results of the statistical analysis provide several interesting insights and directions for future research. The analytical evaluation suggested that competing cost drivers exist for inputs exhibiting the effects of economies of scale. The empirical analysis found that subjects chose an ABC assignment using a financial driver more often than ABC assignments using transaction drivers or FBC assignments. These results suggest that if rational individuals assign costs based on equity, or contribution, a financial driver is recognized as a better reflection of relative contribution for inputs with decreasing average cost functions.

No support was found in this study to indicate that product diversity plays a role in cost assignment choice. One possible factor contributing to this result is the use of a measure that has not been previously tested in a laboratory environment. The measure of product diversity was developed in the analytical component of the study and the variation in product diversity may not have been great enough for individuals to notice the difference in cost assignments. Additional research is needed to determine noticeable levels of product diversity. A manipulation check would have measured the subjects' awareness of different degrees of product diversity. This check was not performed in this study, which implies a limitation in the manipulation of the independent variable.

The empirical component of the study has several additional limitations. First, the study was intended to test two conditions based on game-theoretic concepts. However, cooperative game theory is based on the cooperative behavior of rational individuals, which implies individuals working together to resolve a conflict. This empirical study was conducted with individuals making decisions in isolation with no interaction or negotiation. A possible extension to the study would require a subject to assume the role of an owner of a single product plant and negotiate a cost assignment with two other players assuming the roles of the other two owners.

Low subject involvement in the experimental task is another limitation. An involvement score was tabulated based on the subjects' response to a four-item scale (see Exhibit 8-2 and Appendix B). The mean involvement score for this study was 16.38, indicating a relatively low involvement. Statistically, it was found that subjects' responses on the involvement scale were significantly associated with their choice of assignment method, which may have biased the results against ABC approaches. This implies that future administration of this experiment may require subjects that are more knowledgeable regarding cost assignment concepts.

The use of students as subjects provides several limitations. First, the use of non-accounting students was intended to reduce a potential bias towards the ABC assignment system. However, another interesting bias entered the study with the use of business strategy students. Many of the students analyzed the choice of the multiple product plant independent of costs or possible cost savings. The subjects were advised that each single product owner would remain responsible for the profits of their respective product. However, many chose multiple product plants that were sub-optimal for the owners of the single product plants, stating strategy and marketing reasons. The subjects may have been led to concentrate on irrelevant factors, which implies the possibility of an extraneous factor related to demand artifacts.

Lack of subject motivation is another limitation. Although each student participating in the study received extra credit points from their instructor, the points awarded did not depend on the results of their decisions. Therefore, nothing was at stake for the students and no incentive existed for thoughtful completion of the experimental task. A possible modification to the study would include an incentive scheme related to the cost assignment outcome, such as profits.

Another limitation is the amount of information that the individuals were expected to analyze. A large amount of information was needed for the subjects to make a decision regarding the cost assignment method and all of the information was provided at once. Providing this much information at the same time may have introduced an additional extraneous variable associated with information overload. One possible improvement to this experiment is to have individuals receive the information in smaller steps or stages.

This empirical study provides an initial attempt in exploring game-theoretic concepts in a cost assignment setting and the results are promising for future research. Although additional research is needed, the support found for Hypothesis 2 and the use of financial drivers is encouraging since the result may be a more accurate ABC assignment.

The use of the measure of the degree of product diversity as a tool to quantify and manipulate the difference in ABC and FBC assignments will be valuable for future research related to cost system evaluation and choice. Although an association between the degree of product diversity and assignment method choice was not found in this study, improvements to the design may provide different results. For example, one possibility is the exploration of the recognition of noticeable differences in the degree of product diversity. If noticeable differences can be identified, the manipulation of product diversity may be strengthened.

CHAPTER 9: SUMMARY AND CONCLUSIONS

This study presents an analytical evaluation of ABC and FBC assignments under a game-theoretic paradigm. A theoretical measure of product cost is derived that allows for an evaluation using cooperative game theory solution concepts as benchmarks for comparing competing cost assignments. The following sections summarize the conclusions of this study, describe possible implications for ABC systems, and provide suggestions for future research.

9.1 Conclusions: Analytical Evaluation of ABC and FBC Assignments

This study developed an operational measure of the degree of product diversity (PD) that allows for a quantification of the difference between ABC and FBC assignments. The ability to measure the difference between cost assignments is the key element in an evaluation of ABC and FBC assignments. As PD increases, the difference between the cost assignments becomes so great that the FBC assignment no longer satisfies certain rationality conditions suggested by cooperative game theory, and thus, the identification of ABC as a rationally superior cost system is possible.

This study first evaluated ABC and FBC assignments against two possible set solution concepts; the set of imputations and the core. The analysis of ABC assignments of fixed inputs and inputs exhibiting the effects of economies of scale revealed that ABC assignments will always be members of the set of imputations and the core. It was also shown that FBC assignments do not always have these same properties and can violate the rationality conditions of either one or both of these sets. These results allow for the definition of two critical degrees of product diversity that identify when FBC is no longer a rational assignment. PD_I defines when FBC assignments violate the imputation conditions, while PD_C defines a violation of the core conditions. When PD exceeds either of these critical values, the superiority claim of ABC is supported.

An evaluation against possible point solutions was also presented in the study. The use of point solution concepts as theoretical product cost benchmarks requires that a point solution correspond to an actual rational outcome. This study identified one such possible point solution – the generalized Shapley value with assignment weights defined by relative unused resource capacity. Although additional research is needed in this area, this study found support for a close correspondence between ABC assignments and the proposed generalized Shapley benchmark, which implies that the ABC system is a rationally superior cost system to FBC related to this point solution benchmark.

9.2 Summary of Implications for ABC Systems

Several insights and implications relating to the ABC system were identified in this study. One insight is the identification of a single aggregate ABC assignment ratio, $\overline{\varepsilon_i}$, which allows a seemingly complex ABC system to be simplified. With one assignment ratio capturing the characteristics of multiple cost drivers, the information processing cost of an ABC system may be reduced. In addition, the use of an aggregate assignment ratio could facilitate the development of a standard $\overline{\varepsilon_i}$ based on efficiency for use in product costing.

This study also revealed that competing cost drivers exist for inputs exhibiting the effects of economies of scale. Two possible alternatives include the traditional driver based on the input cost function variable, z_{ij}, and a financial driver. In limited numerical analysis, the financial driver was argued to provide a better reflection of relative contribution than the z_{ij} driver. This was revealed through a comparison of ABC assignments for each driver to the generalized Shapley value, which is a point solution concept based on relative contribution. Additional analytical work is needed to extend the support for this claim.

A potential improvement to the ABC system was identified by an analysis of direct tracing. It was revealed that direct tracing of direct inputs with decreasing average cost functions is essentially equivalent to driver tracing using a transaction driver. Since it was argued that a financial cost driver may be a better reflection of relative contribution for inputs exhibiting the effects of economies of scale, using driver tracing with a financial driver may provide a possible improvement to the ABC system.

A possible limitation to the ABC system identified in this study relates to the assignment of unused resource capacity. In an ABC system, cost is assigned by applying a rate based on practical capacity to a product's expected capacity. This leaves a portion of the resource cost unassigned in an ABC system. A rational cost assignment, however, assigns the entire resource cost, including the amount attributable to unused resource capacity.

9.3 Future Research

This study provides several opportunities for extensions and future research. The ability to measure the degree of product diversity provides opportunities for future research in the evaluation of competing cost assignments. For example, PD could be modified to incorporate the existence of multiple FBC rates at departmental levels. The operational measure of the degree of product diversity may also provide a tool to assist management in deciding which cost assignment system to implement, or if a change to a more detailed system will provide a more accurate product cost. The measure of PD could also be used in empirical studies to investigate the degree of product diversity existing in companies when a switch is made from FBC to ABC.

Second, the analytical evaluation was based on two possible set solution concepts, imputations and the core, and two possible point solution concepts, the simple and generalized Shapley values. Additional solution concepts do exist, such as the nucleolus, the kernel, or the modified Shapley value, which could provide additional insight in an evaluation of ABC and FBC assignments.

Additional research is also needed related to the use of point solutions as theoretical product cost benchmarks. This study argued that point solution must correspond to actual rational outcomes. This correspondence must be established, either analytically or empirically, before a point solution concept can be used in an evaluation of competing cost assignments. For example, stronger support is needed to establish the correspondence between the generalized Shapley value used in this study and actual rational outcomes. The correspondence of other point solutions should also be investigated for future evaluations. Once a correspondence is established, additional analytical work is needed to develop a distance measure that is able to identify the conditions under which ABC is rationally superior to FBC using point solution benchmarks.

The empirical component of this study tested the effects of PD on cost assignment choice. Future empirical research using the operational measure of the degree of product diversity is possible. For example, a study examining noticeable differences of product diversity could aid in determining materiality thresholds of PD that cause companies to switch from FBC to ABC.

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APPENDIX A: PROOFS OF PROPOSITIONS

Proof of Proposition 2

A cost assignment for any cost system is defined as:

$$C_B(a_i) = r_i C_N, \tag{A1}$$

where r_i is the general allocation ratio for product i, and C_N is the total cost of the multiple product plant, for $i = 1,\ldots, n$.

A cost assignment provided by the generalized Shapley solution concept is defined as:

$$C_B(a_i) = C_i - \sum_{S \subseteq N} w_i(S) d_s, \tag{A2}$$

where $i \in S$, and $S \subseteq N$.

Setting equation (A1) and (A2) equal provides:

$$C_i - \sum_{S \subseteq N} w_i(S) d_s = r_i C_N \tag{A3}$$

Solving the set of equations provided by (A3) for $w_i(S)$ results in a set of n equations with $(n + 1)$ unknowns which implies that an infinite number of solutions are possible for $w_i(S)$ when C_i and $r_i C_N$ are known. Therefore, assignment weights can be defined based on cost allocation concepts that will force the generalized Shapley value to correspond to any cost assignment system. Q.E.D.

Proof of Proposition 3

A cost assignment, $C_B(a_i)$, is an imputation if

$$\sum_i C_B(a_i) = C_N, \text{ and} \tag{A4}$$

$$C_B(a_i) \leq C_i, \forall i. \tag{A5}$$

ABC Assignments:

$$\sum_i \bar{\varepsilon}_i = 1 \qquad \Rightarrow \qquad C_\rho(a_i) = C_N.$$

$$\bar{\varepsilon}_i C_N \leq C_i \qquad \Rightarrow \qquad \bar{\varepsilon}_i \leq \frac{C_i}{C_N}, \forall i. \tag{A6}$$

FBC Assignments:

$$\sum_i \mu_i = 1 \qquad \Rightarrow \qquad C_\mu(a_i) = C_N.$$

$$\mu_i C_N \leq C_i \qquad \Rightarrow \qquad \mu_i \leq \frac{C_i}{C_N}, \forall i. \tag{A7}$$

Q.E.D.

Proof of Proposition 4

An ABC assignment of a fixed input is a member of the set of imputations provided the following are satisfied:

$$\sum_i \sum_j \left(\frac{z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - t_j)) \right) = \sum_j (P_j \sum_i K_{ij} - t_j). \tag{A8}$$

$$\sum_j \left(\frac{z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - t_j)) \right) \leq \sum_j (P_j K_{ij}). \tag{A9}$$

By definition, $\sum_i \frac{z_{ij}}{\sum_i z_{ij}} = 1$, which implies (A8).

Let v_j represent the number of steps of activity j saved in a two-product plant and let ω_{ij} represent the portion of one step that is unused in a single product plant, such that $\sum_{i \in S} \omega_{ij} \geq v_j$, where $0 \leq \omega_{ij} \leq 1$.

By definition, $S_j(K_{ij} - 1) < z_{ij} \leq S_j K_{ij}$, which implies

$$z_{ij} = S_j(K_{ij} - \omega_{ij}) \text{ and} \tag{A10}$$

$$\sum_i z_{ij} = S_j(\sum_i K_{ij} - \sum_i \omega_{ij}). \tag{A11}$$

The existence of a three-product plant implies $v_j < t_j$ and $\sum_{i \in S} \omega_{ij} < t_j$, which provides

$$\sum_{i \in S} \omega_{ij} - t_j < 0 \tag{A12}$$

Adding ω_{ij}, $i \notin S$, to both sides of equation (A12) and dividing by $\sum_i \omega_{ij} - t_j$ provides:

$$1 < \frac{\omega_{ij}}{\sum_i \omega_{ij} - t_j} \tag{A13}$$

By definition, $K_{ij} \geq 1$ and

$$\frac{K_{ij}}{\sum_i K_{ij} - t_j} \leq 1. \tag{A14}$$

Equation (A13) and (A14) imply the following:

$$\frac{K_{ij}}{\sum_i K_{ij} - t_j} \leq \frac{\omega_{ij}}{\sum_i \omega_{ij} - t_j},$$

$$K_{ij} \sum_i \omega_{ij} - K_{ij} t_j \leq \omega_{ij} \sum_i K_{ij} - \omega_{ij} t_j. \tag{A15}$$

Adding $K_{ij} \sum_i K_{ij}$ to both sides of equation (A15) and simplifying provides:

$$\frac{K_{ij} - \omega_{ij}}{\sum_i K_{ij} - \sum_i \omega_{ij}} (\sum_i K_{ij} - t_j) \leq K_{ij}. \tag{A16}$$

Multiplying the left hand term by $\frac{S_j}{S_j}$ and multiplying both sides by P_j provides:

$$\frac{S_j (K_{ij} - \omega_{ij})}{S_j (\sum_i K_{ij} - \sum_i \omega_{ij})} P_j (\sum_i K_{ij} - t_j) \leq P_j K_{ij}. \tag{A17}$$

Substituting equations (A10) and (A11) into equation (A17) and summing over j provides:

$$\sum_j \left(\frac{z_{ij}}{\sum_i z_{ij}} (P_j (\sum_i K_{ij} - t_j)) \right) \leq \sum_j (P_j K_{ij}) . \qquad \text{(A18)}$$

Equation (A18) implies an ABC assignment of fixed inputs will always be a member of the set of imputations. Q.E.D.

Proof of Proposition 6

The proof follows for an ABC assignment using drivers based on z_{ij} (transaction or duration) and an ABC assignment using financial drivers.

Transaction/Duration Driver:

An ABC assignment specified by equation (29) is a member of the set of imputations provided the following are satisfied:

$$\sum_i \sum_j \frac{z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) = \sum_j C(\sum_i z_{ij}). \tag{A19}$$

$$\sum_j \frac{z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) \leq \sum_j C(z_{ij}). \tag{A20}$$

By definition, $\sum_i \frac{z_{ij}}{\sum_i z_{ij}}=1$, which implies (A19).

By definition, a decreasing average cost function of an input exhibiting the effects of economies of scale implies:

$$\frac{C(\sum_i z_{ij})}{\sum_i z_{ij}} < \frac{C(z_{ij})}{z_{ij}}. \tag{A21}$$

Multiplying both sides of equation (A21) by z_{ij} and summing over j provides:

$$\sum_j \frac{z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) < \sum_j C(z_{ij}). \tag{A22}$$

Equation (22) implies an ABC assignment of inputs exhibiting the effects of economies of scale assuming the use of a transaction driver will always qualify as a member of the set of imputations.

Financial Driver:

An ABC assignment specified by (30) is a member of the set of imputations provided the following are satisfied:

$$\sum_i \sum_j \frac{C(z_{ij})}{\sum_i C(z_{ij})} (C(\sum_i z_{ij})) = C(\sum_i z_{ij}). \tag{A23}$$

$$\sum_j \frac{C(z_{ij})}{\sum_i C(z_{ij})} (C(\sum_i z_{ij})) \leq \sum_j C(z_{ij}). \tag{A24}$$

By definition, $\sum_i \frac{C(z_{ij})}{\sum_i C(z_{ij})} = 1$, which implies (A23).

By definition, economies of scope implies the following for each activity:

$$C(\sum_i z_{ij}) < \sum_i C(z_{ij}). \tag{A25}$$

Multiplying both sides of (A25) by $C(z_{ij})$, dividing both sides by $\sum_i C(z_{ij})$ and summing over j provides:

$$\sum_j \frac{C(z_{ij})}{\sum_i C(z_{ij})} C(\sum_i z_{ij}) < \sum_j C(z_{ij}). \tag{A26}$$

Equation (A26) implies an ABC assignment of inputs exhibiting economies of scale effects using a financial driver will always qualify as a member of the set of imputations. Q.E.D.

Proof of Proposition 8

ABC Assignments:

An ABC assignment is a member of the core provided the following conditions are satisfied:

$$\sum_{i} C_\rho(a_i) = C_N, \tag{A27}$$

$$C_\rho(a_i) \leq C_i, \tag{A28}$$

$$\sum_{i \in S} C_\rho(a_i) \leq C_S,\ S \subset N, \tag{A29}$$

where $C_\rho(a_i) = \bar{\varepsilon}_i\ C_N$ and $\sum_{i} \bar{\varepsilon}_i = 1$.

Condition (A27) and (A28) are satisfied when $\bar{\varepsilon}_i < \frac{C_i}{C_N}$ (see Proposition 3, (A6)).

For ease of illustration, condition (A29) is satisfied when the following conditions are met in a three-product plant:

$$\bar{\varepsilon}_1 C_N + \bar{\varepsilon}_2 C_N \leq C_{12}, \tag{A30}$$

$$\bar{\varepsilon}_2 C_N + \bar{\varepsilon}_3 C_N \leq C_{13}, \tag{A31}$$

$$\bar{\varepsilon}_2 C_N + \bar{\varepsilon}_3 C_N \leq C_{23}, \tag{A32}$$

Condition (A30) is satisfied by the following:

$$C_N(\bar{\varepsilon}_1 + \bar{\varepsilon}_2) \leq C_{12},$$

$$C_N(1 - \bar{\varepsilon}_3) \leq C_{12},$$

$$1 - \frac{C_{12}}{C_N} \leq \bar{\varepsilon}_3.$$

Similarly for (A31) and (A32); $1 - \frac{C_{13}}{C_N} \leq \bar{\varepsilon}_2$ and $1 - \frac{C_{23}}{C_N} \leq \bar{\varepsilon}_1$, respectively.

Thus, an ABC assignment conforms to the core benchmark when the following is satisfied:

$$1 - \frac{C_{\{N-i\}}}{C_N} \leq \bar{\varepsilon}_i \leq \frac{C_i}{C_N}. \tag{A33}$$

<u>FBC Assignments</u>:

An FBC assignment is a member of the core provided the following conditions are satisfied:

$$\sum_i C_\mu(a_i) = C_N, \tag{A34}$$

$$C_\mu(a_i) \leq C_i, \tag{A35}$$

$$\sum_{i \in S} C_\mu(a_i) \leq C_S,\ S \subset N. \tag{A36}$$

where $C_\mu(a_i) = \mu_i\, C_N$ and $\sum_i \mu_i = 1$.

Conditions (A34) and (A35) are satisfied when $\mu_i < \frac{C_i}{C_N}$ (see Proposition 3, (A7)).

For ease of illustration, condition (A36) is satisfied when the following conditions are met in a three-product plant:

$$\mu_1 C_N + \mu_2 C_N \leq C_{12}, \tag{A37}$$

$$\mu_1 C_N + \mu_3 C_N \leq C_{13}, \tag{A38}$$

$$\mu_2 C_N + \mu_3 C_N \leq C_{23}, \tag{A39}$$

Condition (A39) is satisfied by the following:

$$C_N(\mu_1+\mu_2) \le C_{12},$$

$$C_N(1-\mu_3) \le C_{12},$$

$$1-\frac{C_{12}}{C_N} \le \mu_3.$$

Similarly for (A38) and (A39): $1-\frac{C_{13}}{C_N} \le \mu_2$ and $1-\frac{C_{23}}{C_N} \le \mu_1$, respectively.

Thus, an FBC assignment conforms to the core benchmark when the following is satisfied:

$$1-\frac{C_{\{N-i\}}}{C_N} \le \mu_i \le \frac{C_i}{C_N}. \tag{A40}$$

Q.E.D.

Proof of Proposition 9

An ABC assignment of a fixed input is a member of the core provided the following are satisfied:

$$\sum_i \sum_j \left(\frac{z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - t_j)) \right) = \sum_j (P_j \sum_i K_{ij} - t_j), \tag{A41}$$

$$\sum_j \left(\frac{z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - t_j)) \right) \leq \sum_j (P_j K_{ij}), \tag{A42}$$

$$\sum_j \left(\frac{\sum_{i \in S} z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - t_j)) \right) \leq \sum_j (P_j(\sum_{i \in S} K_{ij} - v_j). \tag{A43}$$

ABC assignments always satisfy (A41) and (A42), see Proposition 4.

Let v_j represent the number of steps of activity j saved in a two-product plant and let ω_{ij} represent the portion of one step that is unused in a single product plant, such that $\sum_{i \in S} \omega_{ij} \geq v_j$ and $\sum_i \omega_{ij} \geq t_j$, where $0 \leq \omega_{ij} \leq 1$.

By definition, $S_j(K_{ij} - 1) < z_{ij} \leq S_j K_{ij}$, which implies

$$z_{ij} = S_j(K_{ij} - \omega_{ij}), \tag{A44}$$

$$\sum_{i \in S} z_{ij} = S_j(\sum_{i \in S} K_{ij} - \sum_{i \in S} \omega_{ij}), \tag{A45}$$

$$\sum_i z_{ij} = S_j(\sum_i K_{ij} - \sum_i \omega_{ij}). \tag{A46}$$

Two cases must be considered for satisfying (A43) in a three-product plant.

Case 1: Assume $t_1 = \ldots = t_j = 1$, $v_1 = \ldots = v_j = 0$, $\sum_{i \in S} \omega_{ij} = 1$ and $\sum_{i} \omega_{ij} = 2$.

The following are implied under the assumptions of Case 1:

$$\sum_{i \in S} z_{ij} = S_j\Big(\sum_{i \in S} K_{ij} - 1\Big), \tag{A47}$$

$$\sum_{i} z_{ij} = S_j\Big(\sum_{i} K_{ij} - 2\Big). \tag{A48}$$

Condition (A43), which must be shown for Case 1, becomes:

$$\sum_j \left(\frac{\sum_{i \in S} z_{ij}}{\sum_i z_{ij}} \big(P_j(\sum_i K_{ij} - 1)\big) \right) \le \sum_j P_j\Big(\sum_{i \in S} K_{ij}\Big). \tag{A49}$$

By definition, $1 \le K_{ij}$ and adding $\sum_{i \in S} K_{ij}$ to both sides of this definition provides:

$$1 + \sum_{i \in S} K_{ij} \le \sum_i K_{ij} \tag{A50}$$

Subtracting $(2\sum_{i \in S} K_{ij} - \sum_i K_{ij} \sum_{i \in S} K_{ij})$ from both sides of (A50) provides:

$$\sum_i K_{ij} \sum_{i \in S} K_{ij} - \sum_{i \in S} K_{ij} - \sum_i K_{ij} + 1 \le \sum_i K_{ij} \sum_{i \in S} K_{ij} - 2\sum_{i \in S} K_{ij}. \tag{A50}$$

Equation (A50) can be expressed as:

$$\Big(\sum_i K_{ij} - 1\Big)\Big(\sum_{i \in S} K_{ij} - 1\Big) \le \Big(\sum_i K_{ij} - 2\Big) \sum_{i \in S} K_{ij}. \tag{A51}$$

Dividing both sides of equation (A51) by $(\sum_i K_{ij} - 2)$ provides:

$$\frac{(\sum_{i\in S} K_{ij} - 1)}{(\sum_i K_{ij} - 2)}(\sum_i K_{ij} - 1) \le \sum_{i\in S} K_{ij}. \tag{A52}$$

Multiplying the left hand term by $\frac{S_j}{S_j}$ and multiplying both sides of (A52) by P_j,

provides:

$$\frac{S_j}{S_j}\frac{(\sum_{i\in S} K_{ij} - 1)}{(\sum_i K_{ij} - 2)} P_j(\sum_i K_{ij} - 1) \le P_j \sum_{i\in S} K_{ij}. \tag{A53}$$

Substituting equations (A48) and (A49) into (A53) and summing over j, provides:

$$\sum_j \left(\frac{\sum_{i\in S} z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - 1)) \right) \le \sum_j P_j(\sum_{i\in S} K_{ij}). \tag{A54}$$

Equation (A54) implies (A49) and ABC assignments are members of the core under the assumptions of Case 1.

Case 2: Assume $t_1 = \ldots = t_j = 2$, $v_1 = \ldots = v_j = 1$, $\sum_{i\in S} \omega_{ij} = 2$ and $\sum_i \omega_{ij} = 3$.

The following are implied under the assumptions of Case 2:

$$\sum_{i\in S} z_{ij} = S_j(\sum_{i\in S} K_{ij} - 2), \tag{A55}$$

$$\sum_i z_{ij} = S_j(\sum_i K_{ij} - 3). \tag{A56}$$

Condition (A43), which must be shown for Case 2, becomes:

$$\sum_j \left(\frac{\sum_{i \in S} z_{ij}}{\sum_i z_{ij}} (P_j (\sum_i K_{ij} - 2)) \right) \leq \sum_j P_j (\sum_{i \in S} K_{ij} - 1). \tag{A57}$$

By definition, $1 \leq K_{ij}$ and adding $\sum_{i \notin S} K_{ij}$ to both sides of this definition provides:

$$1 + \sum_{i \notin S} K_{ij} \leq \sum_i K_{ij} \tag{A58}$$

Subtracting $(3 \sum_{i \notin S} K_{ij} - \sum_i K_{ij})$ and adding $(\sum_i K_{ij} \sum_{i \notin S} K_{ij} + 3)$ to both sides of (A58) provides:

$$\sum_i K_{ij} \sum_{i \notin S} K_{ij} - 2 \sum_{i \notin S} K_{ij} - 2 \sum_i K_{ij} + 4 \leq \sum_i K_{ij} \sum_{i \notin S} K_{ij} - 3 \sum_{i \notin S} K_{ij} - \sum_i K_{ij} + 3. \tag{A59}$$

Equation (A59) can be expressed as:

$$(\sum_i K_{ij} - 2)(\sum_{i \notin S} K_{ij} - 2) \leq (\sum_i K_{ij} - 3)(\sum_{i \notin S} K_{ij} - 1). \tag{A60}$$

Dividing both sides of equation (A60) by $(\sum_i K_{ij} - 3)$ provides:

$$\frac{(\sum_{i \in S} K_{ij} - 2)}{(\sum_i K_{ij} - 3)} (\sum_i K_{ij} - 2) \leq (\sum_{i \notin S} K_{ij} - 1) \tag{A61}$$

Multiplying the left hand term by $\frac{S_j}{S_j}$ and multiplying both sides of (A61) by P_j,

provides:

$$\frac{S_j}{S_j} \frac{(\sum_{i \in S} K_{ij} - 2)}{(\sum_i K_{ij} - 3)} P_j(\sum_i K_{ij} - 2) \leq P_j (\sum_{i \notin S} K_{ij} - 1). \tag{A62}$$

Substituting equations (A55) and (A56) into (A62) and summing over j, provides:

$$\sum_j \left(\frac{\sum_{i \in S} z_{ij}}{\sum_i z_{ij}} (P_j(\sum_i K_{ij} - 2)) \right) \leq \sum_j P_j(\sum_{i \in S} K_{ij} - 1). \tag{A63}$$

Equation (A63) implies (A57) and ABC assignments are members of the core under the assumptions of Case 2.

Q.E.D.

Proof of Proposition 11

The proof follows for an ABC assignment using drivers based on z_{ij} (transaction or duration) and an ABC assignment using financial drivers.

Transaction/Duration Driver:

An ABC assignment specified by equation (29) is a member of the core provided the following are satisfied:

$$\sum_i \sum_j \frac{z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) = \sum_j C(\sum_i z_{ij}), \tag{A64}$$

$$\sum_j \frac{z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) \le \sum_j C(z_{ij}), \tag{A65}$$

$$\sum_j \frac{\sum_{i \in S} z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) \le \sum_j C(\sum_{i \in S} z_{ij}). \tag{A66}$$

ABC assignments always satisfy (A64) and (A65), see Proposition 6.

By definition, a decreasing average cost function of an input exhibiting the effects of economies of scale implies:

$$\frac{C(\sum_i z_{ij})}{\sum_i z_{ij}} < \frac{C(\sum_{i \in S} z_{ij})}{\sum_{i \in S} z_{ij}}. \tag{A67}$$

Multiplying both sides of equation (A67) by $\sum_{i \in S} z_{ij}$ and summing over j provides:

$$\sum_j \frac{\sum_{i \in S} z_{ij}}{\sum_i z_{ij}} (C(\sum_i z_{ij})) < \sum_j C(\sum_{i \in S} z_{ij}). \tag{A68}$$

Equation (A68) implies (A66) and ABC assignments of inputs exhibiting the effects of economies of scale using transaction drivers will always qualify as a member of the core.

<u>Financial Driver:</u>

An ABC assignment specified by equation (30) is a member of the core provided the following are satisfied:

$$\sum_i \sum_j \frac{C(z_{ij})}{\sum_i C(z_{ij})} (C(\sum_i z_{ij})) = \sum_j C(\sum_i z_{ij}), \tag{A69}$$

$$\sum_j \frac{C(z_{ij})}{\sum_i C(z_{ij})} (C(\sum_i z_{ij})) \leq \sum_j C(z_{ij}), \tag{A70}$$

$$\sum_j \frac{\sum_{i \in S} C(z_{ij})}{\sum_i C(z_{ij})} (C(\sum_i z_{ij})) \leq \sum_j C(\sum_{i \in S} z_{ij}). \tag{A71}$$

ABC assignments always satisfy (A69) and (A70), see Proposition 6.

A decreasing average cost function implies:

$$C(\sum_i z_{ij}) < \sum_i C(z_{ij}). \tag{A72}$$

Subtracting $C(\sum_{i\in S} z_{ij})$ from both sides of equation (A72) provides:

$$C(\sum_{i} z_{ij}) - C(\sum_{i\in S} z_{ij}) < \sum_{i} C(z_{ij}) - C(\sum_{i\in S} z_{ij}). \tag{A73}$$

Equation (A73) implies $C(\sum_{i} z_{ij}) - C(\sum_{i\in S} z_{ij})$ increases at a smaller rate than

$\sum_{i} C(z_{ij}) - C(\sum_{i\in S} z_{ij})$, which implies:

$$\frac{C(\sum_{i} z_{ij})}{\sum_{i} C(z_{ij})} < \frac{C(\sum_{i\in S} z_{ij})}{\sum_{i\in S} C(z_{ij})}. \tag{A74}$$

Multiply both sides of equation (A74) by $\sum_{i\in S} C(z_{ij})$ and summing over j provides:

$$\sum_{j} \frac{\sum_{i\in S} C(z_{ij})}{\sum_{i} C(z_{ij})} C(\sum_{i} z_{ij}) < \sum_{j} C(\sum_{i\in S} z_{ij}). \tag{A75}$$

Equation (A75) implies (A71) and ABC assignments of inputs exhibiting the effects of economies of scale using financial drivers will always qualify as members of the core. Q.E.D.

APPENDIX B: EXPERIMENTAL MATERIALS

Cover Sheet

The following material is part of a research project intended to investigate how individuals divide a shared production cost between two or three products.

Your participation is fully voluntary. If you choose to participate you will receive 5 extra credit points for this course. You are not obligated to participate. If you choose not to participate, an additional assignment (i.e., a case write-up) is available from your instructor that is worth 5 extra points.

This packet of material contains:

1. **A <u>Background Sheet</u> that describes the companies involved in your decision.**
2. **A <u>Question Sheet</u> where you will record the choices you make in the experiment.**
3. **A <u>Methods Sheet</u> that describes the methods available for dividing the shared production cost.**
4. **Four <u>Fact Sheets</u> that provide you with the information to make your decisions.**
5. **A Demographic Sheet**

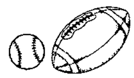
Background Sheet:

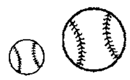
 Alpha, Inc. produces baseballs in a plant located east of town.

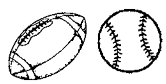
 Beta Corp. produces footballs in a plant located west of town.

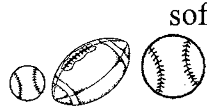
 Gamma Co. produces softballs in a plant located south of town.

The owners of Alpha, Beta and Gamma have the opportunity to combine their operations. If operations are combined, four possible outcomes can occur:

Merger 1:  **Alpha** and **Beta** combine to produce baseballs and footballs

Merger 2:  **Alpha** and **Gamma** combine to produce baseballs and softballs

Merger 3:  **Beta** and **Gamma** combine to produce footballs and softballs

Merger 4:  **Alpha**, **Beta** and **Gamma** combine to produce baseballs, footballs and softballs.

If any of the mergers occur, each owner continues to be responsible for the profits from the sale of their respective products. For example, the owner of Alpha will continue to be responsible for the profits from the sales of baseballs, the owner of Beta for the profits from the sales of footballs, and the owner of Gamma for the profits from the sale of softballs.

Sales of each product will not be affected if the companies combine. However, costs will be lower because some of the production activities can be shared if more than one product can be produced in a single plant. The cost of these shared activities must be divided between the products produced in the plant. In the case of Mergers 1, 2, and 3, the cost of the shared activities is divided between two products. For Merger 4, the cost of the shared activities must be divided between all three products.

You will do the following on the Question Sheet:

1. Decide which merger should take place and provide a justification for your choice.
2. Decide which method to use to divide the cost of shared production activities that is best for all owners involved in the merger and provide a justification for your choice.

Question Sheet

Step #1: Based on the information given on the Fact Sheets, which merger is best for all owners? (Circle one)

Merger 1 : Alpha/Beta

Merger 2 : Alpha/Gamma

Merger 3 : Beta/Gamma

Merger 4 : Alpha/Beta/Gamma

Why did you choose this merger?

Step #2: Choose a method to divide the production cost of the merged company you chose in Step #1. (Check one and complete the sentence by giving reasons for your choice.)

_____ Method A : **Method A is better than Method B or Method C because....**

_____ Method B : **Method B is better than Method A or Method C because...**

_____ Method C : **Method C is better than Method A or Method B because...**

Methods Sheet

METHODS FOR DIVIDING A SHARED COST

Method A — **Total cost** is divided based on the **number of machine hours**

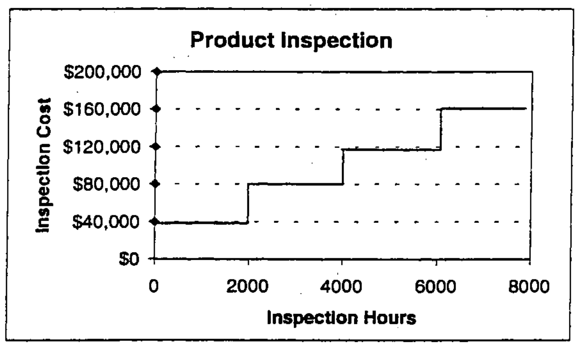
Method B — **Inspection cost** is divided based on the **number of inspections**
Machining cost is divided based on the **number of machine hours**

Method C — **Inspection cost** is divided based on the **number of inspection hours**
Machining cost is divided based on the **relative machining cost**

SHARED PRODUCTION ACTIVITIES

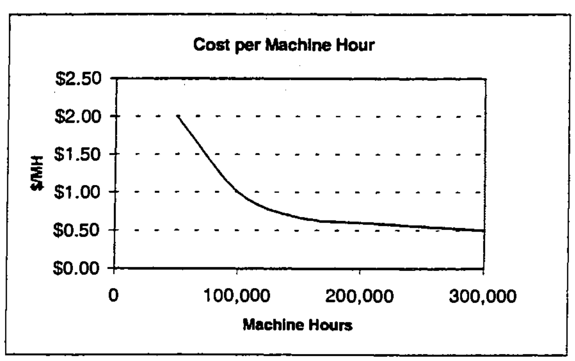
Product Inspection

Inspectors are needed to inspect baseballs, footballs and softballs. One inspector can work up to 2,000 hours in a year. Each inspector is paid a salary of $40,000. Inspecting is a step cost, which means that a company cannot hire a fraction of an inspector. Whether the demand is 1,000 or 2,000 inspection hours, the company will pay $40,000. This is shown in the graph below.



Machining

Specialized machines are used for cutting and stitching baseballs, footballs and softballs. As the number of machine hours demanded increases, the cost per machine hour decreases. This is shown in the graph below.



Fact Sheet **Merger 1:** **Alpha/Beta**

Alpha and Beta operate with the following activity demands and production costs:

	Alpha		Beta		Total		Merger 1: Alpha/Beta
	Amount	%	Amount	%	Amount	%	Amount
Activity cost							
Inspection	$ 40,000	0.25	$ 120,000	0.75	$ 160,000	1.00	$ 120,000
Machining	$ 100,000	0.50	$ 100,000	0.50	$ 200,000	1.00	$ 100,000
Total cost	$ 140,000	0.39	$ 220,000	0.61	$ 360,000	1.00	$ 220,000
Production activity demanded							
Number of inspections needed	1,600	0.40	2,400	0.60	4,000	1.00	4,000
Total inspection hours	1,083	0.19	4,667	0.81	5,750	1.00	5,750
Machine hours needed	50,000	0.33	100,000	0.67	150,000	1.00	150,000

The division of the production cost under each method and the savings for each company from forming Alpha/Beta are as follows:
Note: If the savings are positive, the company has lower costs under the merger; if the savings are negative, the company has higher costs under the merger.

	Merger 1: Alpha/Beta			
	Alpha		Beta	
	Cost	Savings	Cost	Savings
Method A	$ 73,333	**$ 66,667**	$ 146,667	**$ 73,333**
Method B	$ 81,333	**$ 58,667**	$ 138,667	**$ 81,333**
Method C	$ 72,602	**$ 67,398**	$ 147,398	**$ 72,602**

Supporting calculations for dividing the production cost:

Method A: Alpha : $220,000 x (50,000/150,000) = $73,333
Beta : $220,000 x (100,000/150,000) = $146,667
Method B: Alpha : [$120,000 x (1,600/4,000)] + [$100,000 x (50,000/150,000)] = $81,333
Beta : [$120,000 x (2,400/4000)] + [$100,000 x (100,000/150,000)] = $138,667
Method C: Alpha : [$120,000 x (1083/5750)] + [$100,000 x ($100,000/$200,000)] = $72,602
Beta : [$120,000 x (4,667/5,750)] + [$100,000 x ($100,000/$200,000)] = $147,398

Supporting calculations for the savings:

Method A : Alpha : $140,000 - $73,333 = $66,667
Beta : $220,000 - $146,667 = $73,333
Method B : Alpha : $140,000 - $81,333 = $58,667
Beta : $220,000 - $138,667 = $81,333
Method C : Alpha : $140,000 - $72,602 = $67,398
Beta : $220,000 - $147,398 = $72,602

Fact Sheet **Merger 2:** **Alpha/Gamma**

Separately, Alpha and Gamma operate with the following activity demands and production costs:

	Alpha		Gamma		Total		Merger 2: Alpha/Gamma
	Amount	%	Amount	%	Amount	%	Amount
Activity cost							
Inspection	$ 40,000	0.17	$ 200,000	0.83	$ 240,000	1.00	$ 200,000
Machining	$ 100,000	0.50	$ 100,000	0.50	$ 200,000	1.00	$ 120,000
Total cost	$ 140,000	0.32	$ 300,000	0.68	$ 440,000	1.00	$ 320,000
Production activity demanded							
Number of inspections needed	1,600	0.44	2,000	0.56	3,600	1.00	3,600
Total inspection hours	1,083	0.12	8,250	0.88	9,333	1.00	9,333
Machine hours needed	50,000	0.25	150,000	0.75	200,000	1.00	200,000

The division of the production cost under each method and the savings for each company from forming Alpha/Gamma are as follows:
Note: If the savings are positive, the company has lower costs under the merger; if the savings are negative, the company has higher costs under the merger.

	Merger 2: Alpha/Gamma			
	Alpha		Gamma	
	Cost	Savings	Cost	Savings
Method A	$ 80,000	**$ 60,000**	$ 240,000	**$ 60,000**
Method B	$ 118,889	**$ 21,111**	$ 201,111	**$ 98,889**
Method C	$ 83,208	**$ 56,792**	$ 236,792	**$ 63,208**

Supporting calculations for dividing the production cost:

Method A: Alpha : \$320,000 x (50,000/200,000) = \$80,000
Gamma : \$320,000 x (150,000/200,000) = \$240,000
Method B: Alpha : [\$200,000 x (1,600/3,600)] + [\$120,000 x (50,000/200,000)] = \$118,889
Gamma : [\$200,000 x (2,000/3,600)] + [\$120,000 x (150,000/200,000)] = \$201,111
Method C: Alpha : [\$200,000 x (1,083/9,333)] + [\$120,000 x (\$100,000/\$200,000)] = \$83,208
Gamma : [\$200,000 x (8,250/9,333)] + [\$120,000 x (\$100,000/\$200,000)] = \$236,792

Supporting calculations for the savings:

Method A : Alpha : \$140,000 - \$80,000 = \$60,000
Gamma : \$300,000 - \$240,000 = \$60,000
Method B : Alpha : \$140,000 - \$118,889 = \$21,111
Gamma : \$300,000 - \$201,111 = \$98,889
Method C : Alpha : \$140,000 - \$83,208 = \$56,792
Gamma : \$300,000 - \$236,792 = \$63,208

case 1

Fact Sheet **Merger 3:**  **Beta/Gamma**

Beta and Gamma operate with the following activity demands and production costs:

	Beta		Gamma		Total		Merger 3: Beta/Gamma
	Amount	%	Amount	%	Amount	%	Amount
Activity cost							
Inspection	$ 120,000	0.38	$ 200,000	0.63	$ 320,000	1.00	$ 280,000
Machining	$ 100,000	0.50	$ 100,000	0.50	$ 200,000	1.00	$ 137,500
Total cost	$ 220,000	0.42	$ 300,000	0.58	$ 520,000	1.00	$ 417,500
Production activity demanded							
Number of inspections needed	2,400	0.55	2,000	0.45	4,400	1.00	4,400
Total inspection hours	4,667	0.36	8,250	0.64	12,917	1.00	12,917
Machine hours needed	100,000	0.40	150,000	0.60	250,000	1.00	250,000

The division of the production cost under each method and the savings for each company from forming Beta/Gamma are as follows:
Note: If the savings are positive, the company has lower costs under the merger; if the savings are negative, the company has higher costs under the merger.

	Merger 3: Beta/Gamma			
	Beta		**Gamma**	
	Cost	**Savings**	**Cost**	**Savings**
Method A	$ 167,000	**$ 53,000**	$ 250,500	**$ 49,500**
Method B	$ 207,727	**$ 12,273**	$ 209,773	**$ 90,227**
Method C	$ 169,916	**$ 50,084**	$ 247,584	**$ 52,416**

Supporting calculations for dividing the production cost:

Method A: Beta : $417,500 x (100,000/250,000) = $167,000
Gamma : $417,500 x (150,000/250,000) = $250,500
Method B: Beta : [$280,000 x (2,400/4,400)] + [$137,500 x (100,000/250,000)] = $207,727
Gamma : [$280,000 x (2,000/4,400)] + [$137,500 x (150,000/250,000)] = $209,773
Method C: Beta : [$280,000 x (4,667/12,917)] + [$137,500 x ($100,000/$200,000)] = $169,916
Gamma : [$280,000 x (8,250/12,917)] + [$137,500 x ($100,000/$200,000)] = $247,584

Supporting calculations for the savings:

Method A : Beta : $220,000 - $167,000 = $53,000
Gamma : $300,000 - $250,500 = $49,500
Method B : Beta : $220,000 - $207,727 = $12,273
Gamma : $300,000 - $209,773 = $90,227
Method C : Beta : $220,000 - $169,916 = $50,084
Gamma : $300,000 - $247,584 = $52,416

case 1

Fact Sheet **Merger 4:** **Alpha/Beta/Gamma**

Alpha, Beta and Gamma operate with the following activity demands and production costs:

	Alpha		Beta		Gamma		Total		Merger 4: Alpha/Beta/Gamma
	Amount	%	Amount	%	Amount	%	Amount	%	Amount
Activity cost									
Inspection	$ 40,000	0.11	$ 120,000	0.33	$ 200,000	0.56	$ 360,000	1.00	$ 280,000
Machining	$ 100,000	0.33	$ 100,000	0.33	$ 100,000	0.33	$ 300,000	1.00	$ 150,000
Total cost	$ 140,000	0.21	$ 220,000	0.33	$ 300,000	0.45	$ 660,000	1.00	$ 430,000
Production activity demanded									
Number of inspections needed	1,600	0.27	2,400	0.40	2,000	0.33	6,000	1.00	6,000
Total inspection hours	1,083	0.08	4,667	0.33	8,250	0.59	14,000	1.00	14,000
Machine hours needed	50,000	0.17	100,000	0.33	150,000	0.50	300,000	1.00	300,000

The division of the production cost under each method and the savings for each company from forming Alpha/Beta/Gamma are as follows:
Note: If the savings are positive, the company has lower costs under the merger; if the savings are negative, the company has higher costs under the merger.

	Merger 4: Alpha/Beta/Gamma					
	Alpha		Beta		Gamma	
	Cost	Savings	Cost	Savings	Cost	Savings
Method A	$ 71,667	**$ 68,333**	$ 143,333	**$ 76,667**	$ 215,000	**$ 85,000**
Method B	$ 99,667	**$ 40,333**	$ 162,000	**$ 58,000**	$ 168,333	**$ 131,667**
Method C	$ 71,667	**$ 68,333**	$ 143,333	**$ 76,667**	$ 215,000	**$ 85,000**

Supporting calculations for dividing the production cost:

Method A : Alpha : $430,000 x (50,000/300,000) = $71,667
Beta : $430,000 x (100,000/300,000) = $143,333
Gamma : $430,000 x (150,000/300,000) = $215,000
Method B : Alpha : [$280,000 x (1,600/6,000)] + [$150,000 x (50,000/300,000)] = $99,667
Beta : [$280,000 x (2,400/6,000)] + [$150,000 x (100,000/300,000)] = $162,000
Gamma : [$280,000 x (2,000/6,000)] + [$150,000 x (150,000/300,000)] = $168,333
Method C : Alpha : [$280,000 x (1,083/14,000)] + [$150,000 x ($100,000/$300,000)] = $71,667
Beta : [$280,000 x (4,667/14,000)] + [$150,000 x ($100,000/$300,000)] = $143,333
Gamma : [$280,000 x (8,250/14,000)] + [$150,000 x ($100,000/$300,000)] = $215,000

Supporting calculations for the savings:

Method A : Alpha : $140,000 - $71,667 = $68,333
Beta : $220,000 - $143,333= $76,667
Gamma : $300,000 - $215,000 = $85,000
Method B : Alpha : $140,000 - $99,667 = $40,333
Beta : $220,000 - $162,000 = $58,000
Gamma : $300,000 - $168,333 = $131,667
Method C : Alpha : $140,000 - $71,667 = $68,333
Beta : $220,000 - $143,333= $76,667
Gamma : $300,000 - $215,000 = $85,000

case 1

Demographic Sheet

Please provide the following background information:

1. Gender Female _____

   Male _____

2. Age ______

3. Class Freshman _____ Junior _____ Graduate student____

   Sophomore _____ Senior _____

4. Major ___________________________________

5. GPA _____________

6. List all college level accounting courses you have taken, including the those you are currently taking

7. List any work experience you may have.

8. List any ***accounting*** related work experience you may have.

9. Please rate the following statements as they relate to you.

I do not know much about cost assignment systems.

1	2	3	4	5	6
Strongly Agree					Strongly Disagree

I am not familiar with many cost assignment methods.

1	2	3	4	5	6
Strongly Agree					Strongly Disagree

My choice of cost assignment is relevant to my future career.

1	2	3	4	5	6
Strongly Agree					Strongly Disagree

Cost assignment methods are relevant to my future career goals.

1	2	3	4	5	6
Strongly Agree					Strongly Disagree

Product Diversity Manipulation:

The experiment was designed to test three different levels of product diversity, resulting in the following three cases:

	Product Diversity*		
Case	**PD**	**PD_C**	**PD_I**
Case 1: $PD<PD_C<PD_I$	0.00000	0.01265	0.09490
Case 2: $PD_C<PD<PD_I$	0.02337	0.00620	0.06371
Case 3: $PD_C<PD_I<PD$	0.05438	0.00490	0.04499

$$PD = \sum_{i=1}^{n} [(\bar{\rho}_i - \mu_i)^2],$$

$$PD_I = \left[\left(\frac{C_3}{C_N} - \bar{\rho}_3\right) + \left(\frac{C_2}{C_N} - \bar{\rho}_2\right)\right]^2 + \left(\frac{C_3}{C_N} - \bar{\rho}_3\right)^2 + \left(\frac{C_2}{C_N} - \bar{\rho}_2\right)^2 \text{ and}$$

$$PD_C =$$

$$\left(\frac{C_{23} - C_{13} - C_{12}}{2C_N} + \bar{\rho}_1\right)^2 + \left(\frac{C_{13} - C_{12} - C_{23}}{2C_N} + \bar{\rho}_2\right)^2 + \left(\frac{C_{12} - C_{13} - C_{23}}{2C_N} + \bar{\rho}_3\right)^2.$$

The information used in the above is summarized by the following:

	$\bar{\rho}_1$	μ_1	$\bar{\rho}_2$	μ_2	$\bar{\rho}_3$	μ_3
Case 1	.170	.170	.330	.330	.500	.500
Case 2	.174	.300	.316	.250	.506	.450
Case 3	.160	.350	.330	.250	.509	.400

	C_1	C_2	C_3	C_{12}	C_{13}	C_{23}	C_N
Case 1	140,000	220,000	300,000	220,000	320,000	417,500	430,000
Case 2	130,000	195,000	290,450	219,000	323,750	395,500	430,000
Case 3	110,350	195,000	280,400	228,000	323,750	397,000	430,000

Oklahoma State University
Institutional Review Board

Protocol Expires: 4/18/01

Date : Tuesday, April 18, 2000

IRB Application No: BU00007

Proposal Title: AN EVALUATION OF ACTIVITY-BASED COSTING AND FUNCTIONAL BASED COSTING: A GAME THEORETIC APPROACH

Principal Investigator(s) :

Shannon L. Leikam	Maryanne Mowen	Don R. Hansen
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Reviewed and Processed as: Exempt

Approval Status Recommended by Reviewer(s) : Approved

Signature : Carol Olson

Carol Olson, Director of University Research Compliance

Date

Approvals are valid for one calendar year, after which time a request for continuation must be submitted. Any modifications to the research project approved by the IRB must be submitted for approval with the advisor's signature. The IRB office MUST be notified in writing when a project is complete. Approved projects are subject to monitoring by the IRB. Expedited and exempt projects may be reviewed by the full Institutional Review Board.

VITA

Shannon L. Leikam

Candidate for the Degree of

Doctor of Philosophy

Thesis: AN EVALUATION OF ACTIVITY-BASED COSTING AND FUNCTIONAL BASED COSTING: A GAME THEORETIC APPROACH

Major Field: Business Administration

Biographical:

Education: Graduated from Rocky Ford High School, Rocky Ford, Colorado in June 1983; received Bachelor of Science in Business with a focus in Accounting from Western Oregon State College, Monmouth, Oregon in June 1989; received Master of Business Administration from Oregon State University, Corvallis, Oregon in August 1995; Completed the requirements for the Doctor of Philosophy at Oklahoma State University, Stillwater, Oklahoma, December 2000.

Experience: Senior Accountant, Maginnis and Carey, Portland, Oregon from 1989-1992; Audit Senior, Aldrich, Kilbride and Tatone, Salem, Oregon from 1992-1993; Controller, Buffalo Products, Salem, Oregon from 1993-1994; Accounting Instructor, Western Oregon University, Monmouth Oregon from 1995-1997; Graduate teaching Assistant, Oklahoma State University, Stillwater, Oklahoma from 1997-2000.

Professional Memberships: Certified public Accountant, Oregon; American Accounting Association.