INVESTIGATION, EXTENSION, AND GENERALIZATION OF A METHODOLOGY FOR TWO STAGE SHORT RUN VARIABLES CONTROL CHARTING

By

MATTHEW EDWARD ELAM

Bachelor of Science in Mathematics
University of Texas at Tyler
Tyler, Texas
1991

Master of Science in Mathematics
University of Texas at Tyler
Tyler, Texas
1994

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
DOCTOR OF PHILOSOPHY
May, 2001

INVESTIGATION, EXTENSION, AND GENERALIZATION

OF A METHODOLOGY FOR TWO STAGE SHORT

RUN VARIABLES CONTROL CHARTING

Thesis Approved:

Kenneth E Case
Thesis Adviser

Allen C Schuermann

David B Pratt

Camille F. DeYong

P. Larry Claypool

Alfred Carlozzi
Dean of the Graduate College

ACKNOWLEDGMENTS

I would like to mention several people who have made important contributions to the process I have undertaken to complete this dissertation and to receive my Ph.D. in Industrial Engineering at Oklahoma State University.

Dr. Case, my advisor and committee chair, undoubtedly deserves much credit for my knowledge in the area of Quality and for my growth as a researcher. I consider it a privilege to have witnessed and learned from his professionalism, knowledge, and expertise that he displays in his capacities as a teacher, a researcher, and as an advisor. Thank you, Dr. Case, for your time and effort spent on my behalf.

I would also like to thank Dr. Schuermann, Dr. Pratt, Dr. DeYong, and Dr. Claypool for their time and effort in serving on my committee. Special thanks go to Dr. Pratt for allowing me to be his teaching assistant and to Dr. Schuermann for his patience and kindness while I wrestled with the decision to switch to Industrial Engineering.

I consider myself fortunate to have met Brad Beaird when I first arrived at Oklahoma State. In addition to being a good friend, he introduced me to Industrial Engineering and helped me to see that Quality Control was exactly what I had been looking for in an area to study.

I cannot say enough about the positive impact that Dr. McClaran has had on my life. He provided me with my first college teaching experience at East Texas Baptist University. He also introduced me to Oklahoma State. It is because of him that I came to know both how much I enjoy college teaching and how much I would enjoy attending Oklahoma State.

Completing this dissertation and my Ph.D. program has required significant mathematical skills. These were taught to me by Dr. Cranford, Dr. Kraut, Dr. Mitchell, Dr. Morris, and Dr. Pace at the University of Texas at Tyler. A special thanks goes to Dr. Cranford for informing me of the summer teaching opportunity at East Texas Baptist University that would start me out on the road to Oklahoma State.

Standing beside me throughout this entire process has been my parents. Without all that they have freely and abundantly given, none of this would have been possible. I dedicate this dissertation to them, to the memories of my grandparents, and to the memories of Mitzie and Mopsy.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure Page

CHAPTER I

THE RESEARCH PROBLEM

Introduction

Control charts have been used since their introduction by Shewhart (1925, 1926, 1927, 1931) to monitor both products and processes to determine if and when action should be taken to adjust a process because of changes in centering and/or spread of the quality characteristic being measured. Shewhart control charts are constructed using estimates of the process mean and standard deviation obtained from subgrouped data, as well as conventional control chart constants that are widely available in table form. These conventional control chart constants assume that an infinite number of subgroups are available to estimate the process mean and standard deviation.

Hillier (1969) presents three situations in which this assumption is invalid. The first is in the initiation of a new process. The second is during the startup of a process just brought into statistical control again. The third is for a process whose total output is not large enough to use conventional control chart constants. Each of these is an example of a short run situation. A short run situation is one in which little or no historical information is available about a process in order to estimate process parameters to begin control charting. Consequently, the initial data obtained from the early run of the process must be used for this purpose.

In recent years, manufacturing companies have increasingly faced each of these short run situations. One reason is the widespread application of the just-in-time (JIT) philosophy, which has caused much shorter continuous runs of products. Other reasons are frequently changing product lines and product characteristics caused by shorter-lived products, fast-paced product innovation, and changing consumer demand. Fortunately, flexible manufacturing technology has provided companies with the ability to alter their processes in order to face these challenges. Unfortunately, existing statistical process control (SPC) methodologies in general have not provided companies with the ability to reliably monitor quality in each of the previously mentioned short run situations.

One of these methodologies for short run control charting is from Hillier (1969). It is implemented in exactly the same way as Shewhart control charting, but with control chart factors that are based on a finite number of subgroups. As the number of subgroups grows to infinity, Hillier's (1969) control chart factors converge to the respective conventional control chart constants used to construct Shewhart control charts. Two problems exist with this methodology that limit its application. This research effort solves these problems by investigating, extending, and generalizing Hillier's (1969) theory, resulting in a comprehensive, theoretically sound, easy-to-implement, and effective methodology that is immediately applicable in industry due to the creation of computer programs that implement the research.

Problem

In Shewhart control charting, m subgroups of size n consisting of measurements of a quality characteristic of a part or process are collected. The mean $(\overline{X})$ in combination with the range (R), variance (v), or standard deviation ($\sqrt{v}$ or s) is calculated for each subgroup. When the subgroup size is one, individual values (denoted by X) are used in combination with moving ranges (denoted by MR) of size two. The mean of the subgroup means $(\overline{\overline{X}})$ and subgroup ranges $(\overline{R})$, variances $(\overline{v})$, or standard deviations $(\overline{s})$ are calculated and used to determine estimates of the process mean and standard deviation, respectively. When the subgroup size is one, the mean of the individual values $(\overline{X})$ and moving ranges $(\overline{MR})$ are calculated and used to determine estimates of the process mean and standard deviation, respectively. These parameter estimates are then used to construct control limits using conventional control chart constants for monitoring the performance of the process.

A common rule of thumb, which has been widely accepted despite evidence that it may be incorrect, states that twenty to thirty subgroups of size four or five are necessary before parameter estimates may be obtained to construct control limits using conventional control chart constants. This is a difficult if not impossible rule to satisfy in a short run situation. As a result, papers appear in the literature starting several decades ago detailing methodologies that allow for control charting when it is not possible to collect enough data to satisfy the rule.

The prevalent methodologies focus on pooling data from different parts onto a single control chart combination (i.e., onto $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts) in order to have enough data to satisfy the rule. It should be noted that the difference between $(\overline{X}, \sqrt{v})$ and $(\overline{X}, s)$ control charts is that the former are constructed using the statistic $\sqrt{\overline{v}}$ and the latter are constructed using the statistic $\overline{s}$. Pooling data is advantageous because it reduces the number of control charts in use, which greatly simplifies control chart management programs. Also, in most cases, control charting can begin almost immediately after the startup of a process because control limits are known and constant. However, pooling data has several disadvantages. One is that few situations in industry allow for its application. Another is that the values used as estimates of the process parameters (i.e., estimates of the process mean and standard deviation) are either not representative of the process or violate the original motivations for pooling data. A final disadvantage is that some of the methodologies are difficult to implement.

A second approach to control charting in a short run situation is using control charts with greater sensitivity (i.e., more statistical power) than Shewhart control charts. An advantage of this approach is that it allows for the quick detection of special cause signals, which takes on added importance in a short run situation where the total output of the process is not large. A disadvantage is that initial estimates of the process parameters must be close to their true values in order for the control charts to perform well. Also, the methodologies that comprise this approach are difficult to implement.

A third approach to control charting in a short run situation is to monitor and control process inputs rather than process outputs. The assumption upon which this approach is based is that, by correctly selecting and monitoring critical input variables, one can control the output of the process. An advantage of this approach is that, since large amounts of process input data may be available even in a short run situation, Shewhart control charting may be used. A disadvantage of this approach is that few situations in industry allow for its application.

A fourth approach to control charting in a short run situation is using control charts with modified limits. Control limits are modified in order to achieve a specified Type I error probability (i.e., the probability of a false alarm). Quesenberry's (1991) Q chart methodology falls under this approach. Q charts are advantageous in that, not only do they allow for the pooling of data from different parts, but different statistics may be plotted on the same Q chart. Also, control charting can begin almost immediately after the start-up of a process because control limits are known and constant. Disadvantages of Q charts are their inability to detect a process that starts out-of-control and their general lack of sensitivity in detecting process changes. Also, process standard deviation estimates used to calculate Q statistics to be plotted on Q charts are unreliable.

Hillier's (1969) methodology also falls under the fourth approach. It has significant advantages over Quesenberry's (1991) methodology as well as the methodologies from the other approaches. It overcomes their endemic problems of relying on the common rule of thumb, using parameter estimates that are not representative of the process, assuming the process starts in-control, and complex implementation.

An integral part of Hillier's (1969) methodology is its two stage procedure, which is used to determine both the initial state of the process and the control limits for testing future performance of the process. In the first stage, the initial subgroups drawn from the process are used to determine the control limits. The initial subgroups are plotted against the control limits to retrospectively test if the process was in-control while the initial subgroups were being drawn. Any out-of-control initial subgroups are deleted using a delete and revise (D&R) procedure. Once control is established, the procedure moves to the second stage, where the initial subgroups that were not deleted in the first stage are used to determine the control limits for testing if the process remains in-control while future subgroups are drawn. Each stage uses a different set of control chart factors called first stage short run control chart factors and second stage short run control chart factors.

Two problems exist with Hillier's (1969) methodology that present research opportunities. The first one is that it has been applied to only $(\overline{X}, R)$ control charts (see Hillier (1969)) and to $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts (see Yang and Hillier (1970)). Additionally, limited and in some cases incorrect results are presented in the literature for these charts. A particularly important deficiency of Hillier's (1969) methodology is that it has not been applied to (X, MR) control charts (see Del Castillo and Montgomery (1994) and Quesenberry (1995b)).

The second problem is that the process of establishing control in the first stage of the two stage procedure is not clear (see Faltin, Mastrangelo, Runger, and Ryan (1997)). Several D&R procedures exist in the literature with no evidence to suggest which one establishes the most reliable control limits for monitoring the future performance of a process. In a short run situation, the D&R process takes on added importance. The reason is that deleting subgroups is equivalent to throwing away information about a process, which, in a short run situation, is limited even before the D&R process begins. Since the reliability of control limits for monitoring the future performance of a process is directly related to the amount of information from the process that is used to construct them, the choice of the D&R procedure used in a short run situation would seem to have serious implications.

From these problems it is clear that opportunities exist not only to correct and generalize results currently available in the literature, but also to extend and generalize Hillier's (1969) methodology to other control chart combinations, namely $(\overline{X}, s)$ and (X, MR) charts. Also, an opportunity exists to develop a methodology to determine the appropriate execution (i.e., the appropriate D&R procedure to use to establish control in the first stage) of the two stage procedure.

Research Objective

The objective of this research is to investigate, extend, and generalize a methodology for two stage short run variables control charting.

Research Sub-Objectives and Tasks

The research objective is achieved by accomplishing the following five research sub-objectives (in order of appearance) and their respective tasks:

1. Generalize Hillier's (1969) theory so that it can be used for $(\overline{X}, R)$ control charts regardless of the subgroup size, number of subgroups, alpha for the $\overline{X}$ control chart, alpha for the R control chart above the upper control limit, and alpha for the R control chart below the lower control limit (alpha is the probability of a Type I error). As a part of this generalization, correct previous results in the literature for two stage short run control chart factors for $(\overline{X}, R)$ charts.

   The first research sub-objective is achieved by accomplishing the following tasks:

   a. Develop a computer program using the software *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997) that accurately calculates first and second stage short run control chart factors for $(\overline{X}, R)$ charts.

b. Use exact equations for the probability integral of the range, the expected values of the first and second powers of the distribution of the range, the probability integral of the studentized range, degrees of freedom calculations, short run calculations, and conventional control chart calculations in the program.

c. Use numerical routines provided by the software in the program.

d. Have the program accept values for subgroup size, number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the R chart both above the upper control limit and below the lower control limit.

e. Use the program to generate tables for specific values of these inputs.

f. Compare the tabulated results to legitimate results in the literature to validate the program.

g. Use the tables to correct and extend previous results in the literature.

2. Generalize Yang and Hillier's (1970) theory so that it can be used for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts regardless of the subgroup size, number of subgroups, alpha for the $\overline{X}$ control chart, alpha for the v and $\sqrt{v}$ control charts above the upper control limit, and alpha for the v and $\sqrt{v}$ control charts below the lower control limit. As a part of this generalization, correct Yang and Hillier's (1970) results for two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts.

The second research sub-objective is achieved by accomplishing the following tasks:

a. Develop a computer program using the software *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997) that accurately calculates first and second stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts.

b. Use exact equations for the distributions of the variance and the studentized variance, degrees of freedom calculations, short run calculations, and conventional control chart calculations in the program.

c. Use numerical routines provided by the software in the program.

d. Have the program accept values for subgroup size, number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the v or $\sqrt{v}$ chart both above the upper control limit and below the lower control limit.

e. Use the program to generate tables for specific values of these inputs.

f. Compare the tabulated results to legitimate results in the literature to validate the program.

g. Use the tables to correct and extend previous results in the literature.

3. Extend and generalize Hillier's (1969) theory so that it can be used for $(\overline{X}, s)$ control charts regardless of the subgroup size, number of subgroups, alpha for the $\overline{X}$ control chart, alpha for the s control chart above the upper control limit, and alpha for the s control chart below the lower control limit.

The third research sub-objective is achieved by accomplishing the following tasks:

a. Extend Hillier's (1969) theory to allow for the derivation of equations to calculate first and second stage short run control chart factors for $(\overline{X}, s)$ charts.

b. Derive equations to calculate first and second stage short run control chart factors, as well as conventional control chart constants, for $(\overline{X}, s)$ charts.

c. Develop a computer program using the software *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997) that accurately calculates the factors using the derived equations.

d. Use exact equations for the distribution of the standard deviation, the mean and standard deviation of the distribution of the standard deviation, the distribution of the studentized standard deviation, and degrees of freedom calculations in the program.

e. Use numerical routines provided by the software in the program.

f. Have the program accept values for subgroup size, number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the s chart both above the upper control limit and below the lower control limit.

g. Use the program to generate tables for specific values of these inputs.

h. Compare the tabulated results to legitimate results in the literature to validate the program.

4. Extend and generalize Hillier's (1969) theory so that it can be used for (X, MR) control charts regardless of the number of subgroups, alpha for the X control chart, alpha for the MR control chart above the upper control limit, and alpha for the MR control chart below the lower control limit. As a part of this extension and generalization, correct previous results in the literature for two stage short run control chart factors for (X, MR) charts.

The fourth research sub-objective is achieved by accomplishing the following tasks:

a. Extend Hillier's (1969) theory to allow for the derivation of equations to calculate first and second stage short run control chart factors for (X, MR) charts.

b. Derive equations to calculate first and second stage short run control chart factors, as well as conventional control chart constants, for (X, MR) charts.

c. Develop a computer program using the software *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997) that accurately calculates the factors using these derived equations.

d. Use exact equations for the probability integral of the range, the mean of the distribution of the range, the probability integral of the studentized range (all three for subgroup size two), and degrees of freedom calculations in the program.

e. Use numerical routines provided by the software in the program.

f. Have the program accept values for number of subgroups, alpha for the X chart, and alpha for the MR chart both above the upper control limit and below the lower control limit.

g. Use the program to generate tables for specific values of these inputs.

h. Compare the tabulated results to legitimate results in the literature to validate the program.

i. Use the tables to correct and extend previous results in the literature.

5. Develop a methodology to determine the appropriate execution of the two stage procedure.

The fifth research sub-objective is achieved by accomplishing the following tasks:

a. Develop a computer program using FORTRAN (1999) and the Marse-Roberts Uniform (0, 1) random variate generator (see Marse and Roberts (1983)) to simulate two stage short run control charting for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts for in-control and various out-of-control conditions in both stages.

b. Determine the delete and revise (D&R) procedures to include in the program by reviewing the relevant literature. Also, develop reasonable hybrids of existing procedures.

c. Determine the measurements (i.e., the information) that the program needs to provide so that one can choose the appropriate D&R procedure to use. This is accomplished by reviewing the literature concerning measurements to use when control charting in a short run situation.

d. Determine any additional information that the program needs to provide. This is accomplished by studying sample runs of the program to detect occurrences of events that need to be recorded.

e. Use sample runs from the program to show how to interpret its output.

Research Contributions

This research makes important contributions to the statistical process control body of knowledge. The application of Hillier's (1969) theory to $(\overline{X}, s)$ and (X, MR) control charts is a new contribution. It is important because two stage short run $(\overline{X}, s)$ control charts provide another alternative to two stage short run $(\overline{X}, R)$ control charts that use a more efficient estimate of the process standard deviation and that may be easier to use in industry than two stage short run $(\overline{X}, \sqrt{v})$ control charts. It is also important because two stage short run (X, MR) control charts provide a means by which two stage short run control charting can occur in situations where subgrouping is infeasible. It should be noted that two stage short run $(\overline{X}, s)$ and (X, MR) control charts previously did not exist.

The computer programs are important contributions because they calculate theoretically precise control chart factors to determine control limits for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts regardless of the subgroup size, number of subgroups, and alpha values. Previously these capabilities did not exist. This flexibility is valuable in that process monitoring in industry will no longer have to be adjusted to use the limited, and in some cases incorrect, results previously available in the literature for two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, and $(\overline{X}, \sqrt{v})$ control charts.

The development of a methodology for determining the appropriate execution of the two stage procedure is another new contribution. This methodology is important because, in a short run situation, the implications of choosing different D&R procedures for establishing control in the first stage can now be investigated. The information provided by the methodology allows one to choose the D&R procedure that most closely balances two competing issues. The first is avoiding losing too much important information about a process by deleting an already limited number of subgroups in stage one. The second is having control limits to start stage two control charting that have both the desired probability of a false alarm (i.e., the desired probability of signaling a change in the process when there is none) and a high probability of detecting a special cause signal (i.e., a high probability of detecting a signal indicating a change in the process).

Another contribution is two new equations to calculate unbiased estimates of a population variance. The first equation uses the average standard deviation calculated from m standard deviations, each of which is based on a subgroup of size n. The second equation uses the average moving range calculated from (m-1) moving ranges, each of which is based on a subgroup of size two.

It is evident that the contributions of this research result in the development of a comprehensive, theoretically sound, easy-to-implement, and effective methodology for two stage short run control charting using $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts. Additionally, the programs allow for the immediate use of this methodology in industry.

CHAPTER II

LITERATURE REVIEW

Introduction

For several decades and with much higher frequency in recent years, different methods of monitoring processes in a short run situation with $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts have appeared in the literature. These methods belong to at least one of four general approaches to control charting in a short run situation (see Woodall, Crowder, and Wade (1995) and Crowder and Halbleib (2000)).

The first approach is pooling data from different parts onto a single control chart combination (i.e., onto $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts). The second is using control charts that have greater sensitivity (i.e., more statistical power) than Shewhart control charts. The third is emphasizing the monitoring and controlling of process inputs rather than product characteristics (i.e., process outputs). The fourth is modifying control chart limits to achieve the desired Type I error probability (i.e., the desired probability of a false alarm).

This chapter first reviews the literature comprising each of these approaches as they concern $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts. Next, this chapter reviews the different ways of executing the two stage procedure. The last topic this chapter reviews is the different metrics used to determine control chart performance in a short run situation.

Pooling Data

In a short run situation it is not likely that enough data will be available to estimate process parameters to construct control charts for single parts. The widely accepted guideline for how much data is enough is the common rule of thumb. This rule states that twenty to thirty subgroups of size four or five are necessary before process parameters may be estimated and conventional control chart constants used to construct control limits. By pooling data from different parts, it is hoped that enough data is available to satisfy this rule.

Pooling data is the procedure of taking measurements of quality characteristics from different parts, performing a transformation on the measurements, and plotting the transformed measurements from the different parts on the same control chart. Typically, all of the part numbers on the same control chart are produced by one machine or process. Hence, control charting using pooled data is often termed a process-focused approach rather than a product-focused approach to control charting.

Transformations for Pooling Data

Early attempts at pooling data on a single control chart focused on using the deviation-from-nominal transformation (see Grubbs (1946) and Occasione (1956)). Bothe (1988) calls this a Nom-i-nal (i.e., Nominal) transformation and applies it to a short run situation. Each measurement X of a quality characteristic on a given part number is adjusted using the transformation given as equation (2.1) (see Bothe (1988)):

$$X' = X - \text{Nominal} \tag{2.1}$$

where

Nominal: the blueprint specification for the measurement taken from that given part number

Shewhart control chart techniques are then applied to these adjusted values to construct control charts using conventional control chart constants. Both Occasione (1956) and Bothe (1988) give examples of applying the deviation-from-nominal transformation to construct pooled $(\overline{X}, R)$ control charts. Koons and Luner (1988, 1991) give an example of applying it to construct pooled $(\overline{X}, v)$ control charts with varying subgroup sizes.

When expressed in terms of averages, equation (2.1) becomes equation (2.2) (see Bothe (1989)):

$$\overline{X}\ \text{PLOT POINT} = \overline{X} - \text{TARGET}\ \overline{\overline{X}} \tag{2.2}$$

where

$\overline{\overline{X}}$: the average of m values of $\overline{X}$ for a specific part number

The transformation given as equation (2.2) is not suitable in the situation where the standard deviation estimates for different part numbers are not close to each other (this can be determined using the range test (see Griffith (1996)) or Hartley's F-max test (see Nelson (1987))). Consequently, Bothe (1989) suggests the use of his Short Run $\overline{X}$ and R chart.

The control chart statistic for the Short Run R (Range) chart is given as equation

(2.3a):

$$\text{R PLOT POINT} = \frac{\text{R}}{\text{TARGET } \overline{\overline{\text{R}}}} \qquad (2.3a)$$

where

$\overline{\text{R}}$: the average of m values of R for a specific part number

Equation (2.3a) standardizes the range from any part number so that it fits on the same Short Run Range chart as long as the subgroup sizes remain constant. The upper control limit (UCL) for the Short Run Range chart is the conventional control chart constant D_4. The lower control limit (LCL) is the conventional control chart constant D_3.

The control chart statistic for the Short Run $\overline{\text{X}}$ chart is given as equation (2.3b):

$$\overline{\text{X}}\text{ PLOT POINT} = \frac{\overline{\text{X}} - \text{TARGET } \overline{\overline{\text{X}}}}{\text{TARGET } \overline{\overline{\text{R}}}} \qquad (2.3b)$$

Equation (2.3b) standardizes the average from any part number so it fits on the same Short Run $\overline{\text{X}}$ chart as long as the subgroup sizes remain constant. The UCL for the Short Run $\overline{\text{X}}$ chart is the conventional control chart constant A_2. The LCL is equal to $-A_2$.

The $\text{TARGET } \overline{\text{R}}$ value in equations (2.3a) and (2.3b) and the $\text{TARGET } \overline{\overline{\text{X}}}$ value in equation (2.3b) are determined in one of four different ways (see Bothe (1989)). The first is by using prior control charts for the specific part number. The second is by using historical data for the specific part number. The third is by using prior experience on similar part numbers. The fourth is by using specification limits.

Bothe (1989) states several advantages to his Short Run $\overline{X}$ and R charts. The first is that the Short Run $\overline{X}$ chart is independent of both $\overline{\overline{X}}$ and $\overline{R}$ and the Short Run Range chart is independent of $\overline{R}$. This means that part numbers with significantly different $\overline{\overline{X}}$ and $\overline{R}$ values may be plotted on the same Short Run $\overline{X}$ and R charts. The second is that the control limits for the Short Run $\overline{X}$ and R chart can be used when beginning the first control chart with the first plot point. The third is that the control limits do not need to be calculated or recalculated unless process changes are detected.

Quesenberry (1998) and Crowder and Halbleib (2000) point out two problems with Bothe's (1989) transformations, which are similar to many of the transformations used for pooling data. Quesenberry (1998) states that Bothe's (1989) Short Run $\overline{X}$ and R chart is not valid since point patterns on them are not predictable, even for a stable process. Crowder and Halbleib (2000) state that the distribution of Bothe's (1989) transformation given as equation (2.3b) depends on m (the number of subgroups) as well as the subgroup size n. Consequently, plotting it against the conventional control chart constants $-A_2$ and A_2 (which do not depend on m) for the $\overline{X}$ chart is problematic.

Burr (1989) applies his deviation-from-tolerance transformation (similar to equation (2.1) except Tolerance is used instead of Nominal) to construct pooled (X, MR) control charts when the tolerance widths for the measured quality characteristics of different products to be pooled are close. When they are not (i.e., when they differ by a factor of two), Burr (1989) recommends the Q-statistic control chart. The Q-statistic is given as equation (2.4):

$$Q = \frac{X - \text{Nominal}}{0.5 \cdot (\text{Tolerance})} \quad (2.4)$$

The motivation for the Q-statistic is similar to that used by Bothe (1989) to derive his plot points given earlier as equations (2.3a) and (2.3b).

Similar to Burr (1989), Wheeler (1991) shows how to construct pooled (X, MR) control charts, except with the deviation-from-nominal transformation given as equation (2.1). Shewhart control chart techniques are applied to the adjusted values to construct control charts using conventional control chart constants. The resulting control charts are called Difference Charts.

As a test to determine if the Difference Charts are adequate to display the process data, Wheeler (1991) suggests plotting average moving ranges for each product on a chart for Mean Ranges. The control limits for this chart are given as equations (2.5a)-(2.5c):

$$UCL_{\overline{R}} = \overline{\overline{R}} + \frac{H \cdot d_3 \cdot \overline{\overline{R}}}{d_2 \cdot \sqrt{k}} \quad (2.5a)$$

$$CL_{\overline{R}} = \overline{\overline{R}} \quad (2.5b)$$

$$LCL_{\overline{R}} = \overline{\overline{R}} - \frac{H \cdot d_3 \cdot \overline{\overline{R}}}{d_2 \cdot \sqrt{k}} \quad (2.5c)$$

where

$\overline{\overline{R}}$: the average of m average moving ranges (m is also the number of different products)

H: a tabled constant that depends on m

d_2, d_3: the mean and standard deviation, respectively, of the distribution of the range (these are tabled constants that depend on n (see Table M in the appendix of Duncan (1974)))

k: the number of moving ranges (i.e., the number of subgroups) for each product

CL: the center line for the chart for Mean Ranges

If an average moving range for a product is not within the control limits, then there is evidence to suggest that variation between products is too inconsistent to use Difference Charts. In this case, Wheeler (1991) recommends the use of Zed Charts (also called Z-Charts) or Z^* charts. The transformations for the Z-Chart are given as equations (2.6a) and (2.6b):

$$Z = \frac{X - \text{Nominal}}{\overline{R}/d_2} \qquad (2.6a)$$

$$W = \frac{MR}{\overline{\overline{R}}/d_2} \qquad (2.6b)$$

where

Nominal: the target value for the product specific quality characteristic being measured

$\overline{R}$: the mean of k moving ranges determined from the initial subgroups for a specific product drawn from the process.

The control chart for the Z statistic has UCL=3.0, CL=0.0, and LCL=-3.0. The control chart for the W statistic has $UCL = d_2 + 3 \cdot d_3 = 3.686$ and $CL = d_2 = 1.128$.

The transformations for the Z^* chart are exactly like those for the Z-Chart, except the denominators are $\overline{R}$ instead of $\overline{R}/d_2$. The control chart for the Z^* statistic has UCL, CL, and LCL equal to 2.660, 0.0, and -2.660, respectively. The control chart for the W^* statistic has $UCL = D_4 = 3.268$ and CL=1.0.

Equations (2.6a) and (2.6b) differ from equations (2.3a), (2.3b), and (2.4) in that the standard deviation used is an estimate from initial subgroups drawn from the process; it is not target or tolerance values. It should be noted that Wheeler (1991) also gives equations to calculate Difference Charts, Zed charts (called Zed-Bar charts), and Z^* charts (called $\overline{Z^*}$ charts) for subgrouped data (i.e., pooled $(\overline{X}, R)$ control charts).

Farnum (1992), like Bothe (1989), Burr (1989), and Wheeler (1991), proposes a modification of the deviation-from-nominal (which he calls DNOM) procedure in the case where variances are not constant among different parts. For processes with an approximately constant coefficient of variation, together with measurement systems whose errors are reported as percentages of the instrument's reading, Farnum (1992) recommends a DNOM chart that monitors how much $\overline{X}_i/T_i$ deviates from one. The value $\overline{X}_i$ is the average of a subgroup for part i. The value T_i is the nominal dimension for the quality characteristic being measured for part i. The ratio $\overline{X}_i/T_i$ is interpreted as percent of nominal.

The control chart for the ratio $\overline{X}_i/T_i$ has $UCL = 1 + \left((3 \cdot s)/\sqrt{n}\right)$, CL=1.0, and $LCL = 1 - \left((3 \cdot s)/\sqrt{n}\right)$. The value s is the square root of the average of m values of $(s_i/T_i)^2$, where s_i is the standard deviation of a subgroup for part i.

Pyzdek (1993) presents a variation on Bothe's (1988) Nom-i-nal transformation (see equation (2.1)). It is given as equation (2.7):

$$\hat{x} = \frac{X - \text{target}}{\text{Unit of measure}} \tag{2.7}$$

Dividing by the unit of measure allows for integer values of $\hat{x}$ to be plotted on pooled $(\overline{X}, R)$ control charts using Hillier's (1969) methodology, which is reviewed later in the Two Stage Short Run Control Charts subsection of the Control Charts with Modified Limits section of this chapter.

Pyzdek (1993) also presents a methodology called Stabilized control charts that is similar to Bothe's (1989) Short Run $\overline{X}$ and R chart. The difference is that, instead of using target values to estimate the process average and standard deviation for a specific part, a grand average and an average range, respectively, are used from initial subgroups drawn from the process for that specific part. Conventional control chart constants are then used to construct control limits as in Bothe's (1989) approach.

Advanced Methodologies for Pooling Data

Al-Salti and Statham (1994) present a more comprehensive approach to determine which parts should be pooled. It is called the group technology (GT) concept. The main idea is to group parts together into component families based on design and manufacturing similarities. When a new part is scheduled for production, the component family in which it belongs is determined. Historical information obtained from this component family is used to estimate process parameters for the new part.

The most important part of applying the GT concept is the use of a suitable Classification and Coding (C&C) system. This system determines the similarity structure in component machining as a basis for family formation. A C&C system for statistical process control consists of two main codes. The first is a primary code that is based on an existing design-oriented system. A secondary code incorporates the manufacturing similarities of machined components.

The formation of the component families involves identifying the most important variables affecting the quality characteristic of the process output. As part of the primary code, examples of such variables are the basic shape, size, material, and the initial form of the component. As part of the secondary code, examples of such variables are the machine tool used, the machining process monitored, the quality characteristic, the measuring device used, the dimensional class and accuracy of the machined surface, the cutting tool, and the component and tool holding methods.

The procedure for estimating the process parameters for a new component using the GT concept is as follows. First, determine the code number for the component to be machined. Second, identify the important variables affecting the quality characteristic of the process output. Third, use the results of step two to establish the family in which the component belongs. Fourth, retrieve from the family any data that is related to the measurements taken from the component. Fifth, calculate the transformed values of the retrieved data using appropriate upper and lower specification limits. Sixth, estimate the process mean and standard deviation using the transformed retrieved data. Seventh, establish target values to use as estimates of the process parameters for the component to be machined using the estimated process parameters from step six and appropriate upper and lower specification limits.

The process parameter estimates from step seven in the previous paragraph are used to transform component measurements, which are then plotted on pooled $(\overline{X}, R)$ control charts for the machining process being monitored.

Lin, Lai, and Chang (1997) propose a multicriteria part family formation to improve upon the group technology concept for placing parts into families. In this methodology, deviations-from-nominal for each part type are calculated using equation (2.1). The standard deviation of the deviations-from-nominal for each part type are calculated and ranked in ascending order. Ratios of these standard deviations are formed and different part types are placed in the same family if the ratios satisfy certain criteria.

Once families are formed, control chart statistics for each family are calculated using equation (2.2). The family-specific control charts have $UCL = 3 \cdot \left(S_{p(r)} / \sqrt{n}\right)$, CL=0.0, and $LCL = -3 \cdot \left(S_{p(r)} / \sqrt{n}\right)$, where $S_{p(r)}$ is the family-specific pooled standard deviation for a family with r parts. The resulting control charts are pooled $\overline{X}$ charts.

Lin, Lai, and Chang (1997) state two advantages of their methodology over the group technology concept. First, it is simpler to implement for small manufacturers with inadequate statistical staffs. Second, a multicriteria part family formation methodology improves process variation estimates based on pooled observations from different quality characteristics. Statistics calculated from poorly pooled observations tend to be underestimated for some quality characteristics and overestimated for others. This can create pooled control charts that for some parts will have a higher false alarm rate and for others will have less sensitivity to detect special cause signals.

Conclusions for Pooling Data

Several problems exist with each of these methodologies for pooling data. In a true short run situation, one will often find it difficult to even proceed to pool data (Crowder and Halbleib (2000)). The reason is that, in order to construct control limits from pooled data, many part types or operations with similar characteristics must be produced or performed, respectively, by the same process.

Another problem is process parameters for each part number are estimated using target or nominal values, tolerances, specification limits, initial subgroups drawn from the process, or historical data. Quesenberry (1991) states that using target or nominal values is equivalent to using specification limits instead of statistical control limits on control charts, which Deming (1986) asserts is a serious mistake. The same can be said for tolerances and specification limits. The reason is that the process target (what you want), the process aim (what you set), and the process average (what you get) are never the same. The magnitude of the differences depends on how well the process is performing. The result is a control chart that in general will be useless in delineating special cause variation from common cause variation (i.e., variation that is the result of an in-control process).

Using initial subgroups drawn from the process to obtain parameter estimates for part numbers begs the original short run problem that motivates the use of pooled data. If one has enough data (as defined by the common rule of thumb) from a process for a single part to estimate its process parameters, then pooling data is not necessary in the first place.

When one has historical data to estimate process parameters for part numbers, then by definition one is not in a short run situation. Consequently, pooling data is not even necessary, other than to reduce the number of control charts in use.

Finally, an original motivation for pooling data was to satisfy the common rule of thumb. However, Ng and Case (1992) and Quesenberry (1993) show in detail that satisfying the rule does not guarantee control limits that result in a low false alarm rate and have a high probability of detecting a special cause signal.

Control Charts with Greater Sensitivity

In a short run situation where the total output of the process is not large, the quick detection of special cause signals takes on added importance. It is well known that cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control schemes are more sensitive to detecting small process shifts than Shewhart control charts (e.g., see Lucas and Saccucci (1990) and Ch.22, p.464 in Duncan (1974), respectively). Also, economically designed control charts have greater sensitivity (see Woodall, Crowder, and Wade (1995) and Crowder and Halbleib (2000)). Consequently, these have been adapted for use in short run situations.

CUSUM and EWMA Control Schemes

Hawkins (1987) introduces a short run CUSUM control scheme called self-starting CUSUM charts in which process parameters are estimated using the running mean and standard deviation of all of the data obtained since the startup of the process. This scheme has increased sensitivity in detecting shifts at the startup of a process over using parameter estimates obtained from initial subgroups drawn from the process. This sensitivity improves as more data are used in the calculation of the running mean and standard deviation.

Del Castillo and Montgomery (1994) show results originally given in a 1992 Arizona State University technical report that adapts the EWMA control scheme to short run situations. The methodology is called the adaptive Kalman filtering method. Other names given to this methodology are the dynamic EWMA, the adaptive EWMA, and a first-order, constant variance, dynamic linear model (Wasserman (1994)).

Wasserman's (1994) dynamic EWMA control chart is a generalization of the EWMA control chart. It allows for prior information about the process to be incorporated into the model in the form of a prior distribution. Prior information may consist of engineering judgment, expert knowledge, engineering specifications, or information obtained from similar processes. This prior information is updated as individual observations are obtained from the process. Initial estimates of the process mean and standard deviation are obtained using the prior information along with a Bayesian estimation scheme. Updated estimates of these two process parameters are obtained using the updated prior information.

The dynamic EWMA control chart statistic is calculated using equation (2.8):

$$m_t = \lambda_t \cdot Y_t + (1 - \lambda_t) \cdot m_{t-1} \tag{2.8}$$

where

m_t : mean level of the process at time t

λ_t : adaptive weighting factor at time t (adaptive means that the variance terms are estimated)

Y_t : individual observation at time t

Since individual observations are used to determine the control chart statistic, this is a short run application of the X chart.

Wasserman and Sudjianto (1993) present a second order, constant variance, dynamic linear model version of the dynamic EWMA. This model also performs well in detecting small process shifts in a short run situation.

The methodologies of Del Castillo and Montgomery (1994), Wasserman (1994), and Wasserman and Sudjianto (1993) have a common problem. Initial estimates of the process mean and standard deviation must be close to their true values. If not, the ability of the control mechanisms to detect shifts is significantly hampered.

Chan (1994) uses simulation techniques to determine control chart parameter values for the usual EWMA control chart (where $\lambda_t = \lambda$ is constant in equation (2.8)) that allow for the application of this chart to short run situations. Chan's (1994) two assumptions that the process starts in-control and the process mean and standard deviation are known undermine his results. If process parameters are known, then by definition one is not in a short run situation. Also, it is possible for a process to start out-of-control. Consequently, Chan's results (1994) may not be applicable in a short run situation.

Combined Methodologies

Quesenberry (1995a) applies EWMA and CUSUM monitoring schemes to his Q chart (this Q is different from Burr's (1989) Q-statistic given earlier as equation (2.4)) methodology, which is reviewed later in the Q Charts subsection of the Control Charts with Modified Limits section of this chapter, to improve the detection of small process shifts. The Q statistic is used to calculate the EWMA control chart statistic as shown in equation (2.9):

$$Z_t = \lambda \cdot Q_t + (1-\lambda) \cdot Z_{t-1} \tag{2.9}$$

where

Z_t : the EWMA control chart statistic at time t, t: 1, 2, ... ($Z_0 = 0.0$)

λ : constant weighting factor

Q_t : the Q statistic at time t

The Q statistic is used to calculate the CUSUM statistics as shown in equations (2.10a) and (2.10b):

$$S_t^+ = \max\left\{0, S_{t-1}^+ + Q_t - k_s\right\} \tag{2.10a}$$

$$S_t^- = \min\left\{0, S_{t-1}^- + Q_t + k_s\right\} \tag{2.10b}$$

where

S_t^+, S_t^- : the CUSUM control chart statistics at time t, t: 1, 2, ... ($S_0^+ = 0.0, S_0^- = 0.0$)

k_s : reference value (Quesenberry (1995a) uses $k_s = 0.75$)

Problems with Quesenberry's (1995a) methodology are given later in the Issues with Q Charts subsection of the Control Charts with Modified Limits section of this chapter.

Doganaksoy and Vandeven (1997) apply an EWMA monitoring scheme to control charts for pooled data. Charting pooled data in this manner results in earlier notification of process changes. The transformation used to allow for pooling is given as equation (2.11):

$$z_{gcl;t} = \frac{y_{gcl;t} - \bar{y}_{gc}}{s_{gc}} \tag{2.11}$$

where

g, c, l: product grade, color, and line, respectively

$z_{gcl;t}$: pooled control chart statistic for product gcl at time t

$y_{gcl;t}$: measured quality characteristic for product gcl at time t

$\bar{y}_{gc}$: historical mean of the measured quality characteristic for product gc

s_{gc} : historical standard deviation of the measured quality characteristic for product gc

The historical mean and standard deviation for each product can be estimated using data collected from a previous production period.

The EWMA control chart statistic is calculated using equation (2.12):

$$\text{EWMA}_t = \lambda \cdot z_{gcl;t} + (1-\lambda) \cdot \text{EWMA}_{t-1} \tag{2.12}$$

where

EWMA_t: the EWMA control chart statistic at time t, t: 1, 2, ... ($\text{EWMA}_0 = 0.0$)

λ: constant weighting factor

Problems with Doganaksoy and Vandeven's (1997) methodology are the same as those given earlier in the Pooling Data section of this chapter.

Economic Design

Del Castillo and Montgomery (1996) develop a model for the optimal economic design of $\overline{X}$ charts for short run situations. It assumes a finite production run whose length is determined separately from the model. Incorporated in the model is the consideration of the effect the setup operation has on the chart design. An imperfect setup corresponds to a process that has a nonzero probability of starting out-of-control. As the production run lengthens to infinity and as the probability of a perfect setup converges to one, the model converges to Duncan's (1956) model.

Del Castillo and Montgomery (1996) use designed experiments to conclude that the length of the production run, the probability of having a correct setup, and the power of the chart design are related. Another conclusion is that the model is sensitive to the value of the parameter that represents the probability of a perfect setup.

Del Castillo and Montgomery (1996) give several examples illustrating these conclusions. As the setup improves or as the production run increases, charts with higher power are needed. If there is a high probability of an incorrect setup, then a high power chart is not recommended because there is no point in stopping the process for a setup that will not bring a process to an in-control state. If the setup is perfect and the production run length is short, a low power chart can be used because an out-of-control state will reset to an in-control state through the perfect setup operation.

Del Castillo (1996b) presents an algorithm for the constrained optimization of Del Castillo and Montgomery's (1996) model. For the situation in which cost and parameter estimation is impractical, Del Castillo (1996b) presents a graphical method for finding a feasible chart design. The constraints, which are statistical and production-related in nature, link the chart design variables with the production process to make the model more realistic and to obtain chart designs with better statistical properties.

Process Inputs

The third approach to applying $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts to short run situations is the monitoring and controlling of process inputs (e.g., temperature, pressure, rpms) rather than product characteristics (e.g., diameter, thickness, number of defects). By controlling the process inputs, one can control the quality of the process output. This approach is applicable when large amounts of process input data are available.

Foster (1988) gives a three-phase model for monitoring process inputs. The first step of phase one is the creation of a Master Process Requirements List. This is a compilation of all the individual specification requirements for a particular process. When separate specification requirements overlap, the most stringent requirement is used. The second step is to flowchart the process. The third step is to select and rank critical inputs. The last step of phase one is to perform a capability analysis on each critical input parameter. If any are not capable, the process should be adjusted and the last step repeated.

Phase two is the evaluation of the process output. If the output is unacceptable, then the selection and/or capability of the critical input parameters should be re-evaluated.

This phase should be repeated until the output is acceptable.

In phase three, the focus is on maintaining control, establishing and refining relationships between critical input parameters, and improving process requirements. Monitoring of the process inputs in this phase is done with $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, or (X, MR) charts constructed using conventional control chart constants.

A problem with this approach is that critical input parameters for a new part to be produced in a short run may not match all of the critical input parameters for which large amounts of data are available. Also, Foster (1988) assumes that the process input nominal values are the same for all product fabricated on that process. If nominal values are different, a transformation of the process input data may be required (Crowder and Halbleib (2000)).

Control Charts with Modified Limits

In a true short run situation, the process mean and standard deviation are unknown and must be estimated from a small number of subgroups with only a few samples each drawn from the startup of a process. When these estimates are used with conventional control chart constants to construct control limits for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts, the Type I error probability (i.e., the probability of a false alarm) becomes distorted. Consequently, modified control chart factors need to be used to achieve the desired Type I error probability.

Two methodologies exist that use control charts with modified limits for short run control charting. This section first reviews Quesenberry's (1991) Q chart methodology.

This section then reviews Hillier's (1969) two stage short run control chart methodology.

Q Charts

Quesenberry (1991) introduces Q charts (this Q is different from Burr's (1989) Q-statistic given earlier as equation (2.4)) for short run situations that allow for the specification of the desired Type I error probability as well as the plotting of measurements of quality characteristics from multiple part types on a single chart. This second characteristic establishes a relationship between Q charts and the pooled control charts presented earlier in the Pooling Data section of this chapter. The Q chart methodology is for measurements of quality characteristics that are independent and identically distributed Normal random variables.

Quesenberry (1991) derives equations to calculate Q-chart complements of (X, MR) and $(\overline{X}, v)$ control charts. These equations convert a measurement of a quality characteristic into a standard Normal variable called a Q statistic. These equations also update the estimated process mean and standard deviation as measurements are made and subgroups are formed. The Q statistic is plotted on a control chart that has control limits in a standardized Normal scale. These known, constant control limits on Q charts allow for meaningful control charting to begin almost at the start-up of a process, even if the process mean and standard deviation are unknown.

The Q statistic for the X control chart when the process mean and standard deviation are unknown is calculated using equation (2.13):

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-2}\left[\left(\frac{r-1}{r}\right)^{0.5} \cdot \left(\frac{X_r - \overline{X}_{r-1}}{S_{r-1}}\right)\right]\right\} \tag{2.13}$$

where

$r = 3, 4, \ldots$: the number of the individual measurement

Q_r: the rth Q statistic

X_r: the rth individual measurement

Φ^{-1}: the inverse of the standard normal distribution function

G_{r-2}: the Student t distribution with $\nu = (r-2)$ degrees of freedom

$\overline{X}_{r-1} = \frac{\sum_{j=2}^{r-1} X_j}{r-1}$: the average of the first (r-1) measurements

$S_{r-1} = \sqrt{\frac{\sum_{j=2}^{r-1}\left(X_j - \overline{X}_{r-1}\right)^2}{r-2}}$: the standard deviation of the first (r-1) measurements

The Q statistic for the MR control chart when the process mean and standard deviation are unknown is calculated using equation (2.14):

$$Q(R_r) = \Phi^{-1}\left\{F_{1,\nu}\left(\frac{\nu \cdot R_r^2}{R_2^2 + R_4^2 + \cdots + R_{r-2}^2}\right)\right\} \tag{2.14}$$

where

$r = 4, 6, \ldots$: the number of the moving range

$R_r = X_r - X_{r-1}$: the rth moving range

$F_{1,v}$: the F distribution with $v_1 = 1$ numerator degrees of freedom and $v_2 = v = ((r/2)-1)$ denominator degrees of freedom

Equation (2.14) avoids overlapping moving ranges to maintain independence among the Q statistics.

The Q statistic for the $\overline{X}$ control chart when the process mean and standard deviation are unknown is calculated using equation (2.15):

$$Q_i(\overline{X}_i) = \Phi^{-1}\left[G_{n_1+n_2+\cdots+n_{i-1}}\left(\sqrt{\frac{n_i \cdot (n_1+n_2+\cdots+n_{i-1})}{n_1+n_2+\cdots+n_{i-1}}} \cdot \left(\frac{\overline{X}_i - \overline{\overline{X}}_{i-1}}{S_{p,i}}\right)\right)\right] \qquad (2.15)$$

where

$i = 2, 3, \ldots$: the number of the subgroup

$\overline{X}_i$: the average of the ith subgroup

$\overline{\overline{X}}_{i-1} = \dfrac{n_1 \cdot \overline{X}_1 + n_2 \cdot \overline{X}_2 + \cdots n_{i-1} \cdot \overline{X}_{i-1}}{n_1 + n_2 + \cdots n_i}$: the average of the first (i-1) subgroup averages

$S_{p,i} = \sqrt{\dfrac{(n_1-1)\cdot v_1 + (n_2-1)\cdot v_2 + \cdots (n_i-1)\cdot v_i}{n_1+n_2+\cdots n_i - i}}$: the square root of the pooled variance of the first i subgroup variances

The Q statistic for the v control chart when the process mean and standard deviation are unknown is calculated using equation (2.16):

$$Q_i(v_i) = \Phi^{-1}\left[F_{n_i-1,\, n_1+n_2+\cdots n_{i-1}-i+1}\left(\frac{(n_1+n_2+\cdots n_{i-1}-i+1)\cdot v_i}{(n_1-1)\cdot v_1+(n_2-1)\cdot v_2+\cdots(n_{i-1}-1)\cdot v_{i-1}}\right)\right] \tag{2.16}$$

where

v_i : the variance of the ith subgroup

It should be noted that equations (2.15) and (2.16) allow for unequal subgroup sizes.

The upper and lower control limits for the Q chart are q_{α_1} and $q_{1-\alpha_2}$, where q_α is the (1-α)th fractile of the standard Normal distribution. The center line is zero. Since each of the Q statistics given in equations (2.13), (2.14), (2.15), and (2.16) are standard Normal variables, each may be plotted on the same Q chart, even though each is for a different statistic.

Issues with Q Charts

Quesenberry (1991) gives two precautions when using Q charts. Both affect the sensitivity of Q charts to detect changes in a process. Consider the situation when the process mean μ shifts to a larger value. Because the Q statistic calculated using equation (2.13) utilizes all of the information prior to the rth observation to calculate estimates for μ, the Q statistics following the shift will eventually settle into an in-control pattern. The reason is that, as more data are collected following the shift, the parameter estimates will reflect the shifted value for μ. A similar problem occurs when the process standard deviation σ shifts to a larger value. Wade (1992) investigates this issue further and concludes that Q charts for individual measurements can be insensitive to large shifts in the estimated process parameters when the shifts occur early in the production run.

The second precaution given by Quesenberry (1991) is that data from processes that start out-of-control and need time to settle into an in-control state should not be used in the calculations of the process parameter estimates for Q statistics. Wasserman and Sudjianto (1993) state that if Q charts are used at the start-up of an out-of-control process, then they would be useless because the Q statistics would be formed from a running process average of the process parameter which has existed solely in an out-of-control state. The resulting Q chart would not detect the out-of-control state. They conclude that Q charts cannot be used prior to the establishment of an in-control state. Woodall, Crowder, and Wade (1995) suggest the use of a two stage procedure to overcome Quesenberry's (1991) implicit assumption that the process being monitored starts in-control. Otherwise, when a process starts out-of-control, the Q chart results would be difficult to interpret. Crowder and Halbleib (2000) also state that Q charts will not detect the situation where a process commences with an off target mean.

In general, when control charts with modified limits are used in short run situations, sensitivity issues are inherent because of the tradeoff between having a low false alarm rate and a high probability of detecting a special cause signal (Del Castillo (1995)). To deal with these sensitivity issues, Quesenberry (1991) suggests using the tests for special causes given by Nelson (1984) with Q charts. Also, as mentioned earlier in the Combined Methodologies subsection of the Control Charts with Greater Sensitivity section of this chapter, Quesenberry (1995a) applies his Q statistics to EWMA and CUSUM control schemes to improve detection capabilities.

Problems exist with the Q statistics in equations (2.13) and (2.15). According to Del

Castillo and Montgomery (1994), the standard deviation estimate S_{r-1} in equation (2.13) is biased and should be divided by the factor c_4 for n equal to $(r-1)$. The factor c_4 is the mean of the distribution of the standard deviation and is tabled for several values of n (e.g., see Table M in the appendix of Duncan (1974)). Del Castillo and Montgomery (1994) investigate the performance of Q charts using equation (2.13) and conclude that using S_{r-1}/c_4 instead of S_{r-1} improves the sensitivity of Q charts. In equation (2.15), the standard deviation estimate $S_{p,i}$ is biased and should be divided by the factor c_4 for a subgroup size of $((n_1 + n_2 + ... + n_i) - i + 1)$ (see Nelson (1990)).

Del Castillo (1995) states an additional problem with the standard deviation estimate S_{r-1} in equation (2.13). When the process shifts to an out-of-control state, S_{r-1} will overestimate the process standard deviation σ. The reason is that S_{r-1} combines within subgroup variability and between subgroup variability. The result is that, when a small amount of data from a process is used to obtain parameter estimates, the probability of detecting a shift in the observations immediately following the shift may decrease as the shift size increases. Del Castillo and Montgomery (1994) and Quesenberry (1995a) also investigate this problem and arrive at identical conclusions.

It should be noted that, instead of using the Q statistics in equations (2.13) and (2.14), Wade (1992) suggests the use of a sequential X-chart in a short run situation. This is similar to (X, MR) control charts, except the process parameters are re-estimated as each measurement is obtained from the process (as with Quesenberry's (1991) Q statistics given as equations (2.13) and (2.15)). Also, as explained earlier in the CUSUM and EWMA Control Schemes subsection of the Control Charts with Greater Sensitivity section of this chapter, Hawkins (1987) uses running estimates of process parameters in his short run CUSUM control scheme. Wade (1992) states that the sequential X-chart is more sensitive than the Q chart for individual values and moving ranges for a broad range of process shifts, especially those occurring after only a few in-control observations.

Two Stage Short Run Control Charts

Hillier (1969) presents a methodology for two stage short run control charting for $(\overline{X}, R)$ charts that allows for the specification of the desired Type I error probability. It includes the methodology for second stage short run control charting for $\overline{X}$ charts and R charts presented by Hillier in his 1964 and 1967 papers, respectively. Earlier papers by King (1954) and Proschan and Savage (1960) also consider only one of the two stages.

King (1954) investigates the probability of a Type I error during retrospective testing (stage one) when only a small number of subgroups are available to construct $\overline{X}$ control charts. Proschan and Savage (1960) do the same when testing for future subgroups (stage two). The results of both papers indicate that control chart factors different from conventional control chart constants need to be used in both stages to prevent distortion of the Type I error probability.

Hillier (1964) shows that the probability of a Type I error is exceedingly high when estimates of the process mean and standard deviation based on a small number of subgroups are used together with conventional control chart constants to construct $\overline{X}$ charts for future testing (stage two). To resolve this issue, Hillier (1964) derives an equation for A_2^*, the second stage short run control chart factor for the $\overline{X}$ chart. Using this factor, which depends on m (the number of subgroups) as well as the subgroup size n, instead of the conventional control chart constant A_2 results in control limits that give the desired Type I error probability. The value A_2^* is related to A_2 in that, as m→∞, $A_2^* \to A_2$. Second stage short run $\overline{X}$ control charts are constructed by following the same procedure for constructing Shewhart control charts, except A_2^* is used instead of A_2.

The derivation for A_2^* proceeds as follows (see Hillier (1964) and (1969)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup averages as $\overline{\overline{X}}$ and the average of the subgroup ranges as $\overline{R}$. Suppose again that an additional subgroup of size n is sampled from the same population. Denote the average and range of this subgroup as $\overline{X}$ and R, respectively. In order to achieve the desired Type I error probability for future testing, we need to determine the value A_2^* such that equation (2.17a) holds:

$$P\left(\overline{\overline{X}} - A_2^* \cdot \overline{R} \le \overline{X} \le \overline{\overline{X}} + A_2^* \cdot \overline{R}\right) = 1 - \text{alphaMean} \qquad (2.17a)$$

where

alphaMean: probability of a Type I error on the $\overline{X}$ control chart

Rearranging equation (2.17a) results in equation (2.17b):

$$P\left(-A_2^* \le \frac{\overline{X} - \overline{\overline{X}}}{\overline{R}} \le A_2^*\right) = 1 - \text{alphaMean} \qquad (2.17b)$$

It is necessary to determine the distribution of $\left(\overline{X}-\overline{\overline{X}}\right)/\overline{R}$. First consider $\left(\overline{X}-\overline{\overline{X}}\right)$. Both $\overline{X}$ and $\overline{\overline{X}}$ are normally distributed, hence their difference is normally distributed. The expected value of $\left(\overline{X}-\overline{\overline{X}}\right)$ is equal to zero and is derived in Appendix A of this dissertation. The standard deviation of $\left(\overline{X}-\overline{\overline{X}}\right)$ is equal to $\left(\left(\sqrt{(m+1)/(n\cdot m)}\right)\cdot\sigma\right)$ and is also derived in Appendix A.

Now consider the distribution of $\overline{R}$. Patnaik (1950) shows that $\left(\nu\cdot\left(\overline{R}\right)^2\right)/\left(\left(d_2^*\right)^2\cdot\sigma^2\right)$ has approximately a χ^2 distribution with ν degrees of freedom, where ν and d_2^* are both functions of m and n. This means that, since $\left(\overline{X}-\overline{\overline{X}}\right)$ and $\overline{R}$ are independent for a Normal distribution, the ratio given as (2.18a) has approximately a Student's t distribution with ν degrees of freedom:

$$\frac{\left(\left(\overline{X}-\overline{\overline{X}}\right)/\left(\sqrt{\frac{m+1}{n\cdot m}}\cdot\sigma\right)\right)}{\sqrt{\left(\left(\frac{\nu\cdot\left(\overline{R}\right)^2}{\left(d_2^*\right)^2\cdot\sigma^2}\right)/\nu\right)}} \tag{2.18a}$$

Simplifying the ratio in (2.18a) results in (2.18b):

$$d_2^*\cdot\sqrt{\frac{n\cdot m}{m+1}}\cdot\left(\frac{\overline{X}-\overline{\overline{X}}}{\overline{R}}\right) \tag{2.18b}$$

Since equation (2.18b) has approximately a Student's t distribution with ν degrees of freedom, we have the probability relationship given as equation (2.19a):

$$P\left(-t_{(\text{alphaMean}/2),\nu} \le \left(d_2^* \cdot \sqrt{\frac{n \cdot m}{m+1}} \cdot \left(\frac{\overline{X} - \overline{\overline{X}}}{\overline{R}}\right)\right) \le t_{(\text{alphaMean}/2),\nu}\right) = 1 - \text{alphaMean} \qquad (2.19a)$$

where

$t_{(\text{alphaMean}/2),\nu}$: the critical value for an area of (alphaMean/2) in each tail of the Student's t-distribution with ν degrees of freedom

Rearranging equation (2.19a) results in equation (2.19b):

$$P\left(\left(\frac{-t_{(\text{alphaMean}/2),\nu}}{d_2^*} \cdot \sqrt{\frac{m+1}{n \cdot m}}\right) \le \frac{\overline{X} - \overline{\overline{X}}}{\overline{R}} \le \left(\frac{t_{(\text{alphaMean}/2),\nu}}{d_2^*} \cdot \sqrt{\frac{m+1}{n \cdot m}}\right)\right) = 1 - \text{alphaMean} \quad (2.19b)$$

Comparing equation (2.19b) with equation (2.17b) reveals the equation for A_2^*, which is given as equation (2.20):

$$A_2^* = \frac{t_{(\text{alphaMean}/2),\nu}}{d_2^*} \cdot \sqrt{\frac{m+1}{n \cdot m}} \qquad (2.20)$$

Hillier (1967) shows that the probability of a Type I error is exceedingly high when estimates of the process standard deviation based on a small number of subgroups are used together with conventional control chart constants to construct R charts for future testing (stage two). To resolve this issue, Hillier (1967) derives equations for D_4^* and D_3^*, the second stage short run upper and lower control chart factors, respectively, for the R chart. Using these factors, which depend on m as well as n, instead of the corresponding alpha-based (i.e., probability based) conventional upper and lower control chart constants D_4 and D_3, respectively, results in control limits that give the desired Type I error probability. The value D_4^* is related to D_4 in that, as $m \to \infty$, $D_4^* \to D_4$. Similarly, the value D_3^* is related to D_3 in that, as $m \to \infty$, $D_3^* \to D_3$.

The derivation for D_4^* proceeds as follows (see Hillier (1967) and (1969)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup ranges as $\overline{R}$. Suppose again that an additional subgroup of size n is sampled from the same population. Denote the range of this subgroup as R. In order to achieve the desired Type I error probability for future testing, we need to determine the value D_4^* such that equation (2.21a) holds:

$$P\left(R \le D_4^* \cdot \overline{R}\right) = 1\text{-alphaRangeUCL} \qquad (2.21a)$$

where

alphaRangeUCL: probability of a Type I error on the R control chart above the upper control limit (UCL)

Rearranging equation (2.21a) results in equation (2.21b):

$$P\left(\frac{R}{\overline{\overline{R}}} \le D_4^*\right) = 1\text{-alphaRangeUCL} \tag{2.21b}$$

It is necessary to determine the distribution of $R/\overline{\overline{R}}$. Consider first the distribution of the range R/σ. Through the application of Patnaik's (1950) theory, σ may be replaced with the independent estimate of the population standard deviation denoted by $\overline{\overline{R}}/d_2^*$, which is based on ν degrees of freedom (ν and d_2^* are both functions of m and n). The resulting ratio $\left(d_2^* \cdot R\right)/\overline{\overline{R}}$ is by definition the distribution of the studentized range with ν degrees of freedom.

Consequently, we have the probability relationship given as equation (2.22a):

$$P\left(\frac{d_2^* \cdot R}{\overline{\overline{R}}} \le q_{1\text{-alphaRangeUCL},\,\nu}\right) \le 1\text{-alphaRangeUCL} \tag{2.22a}$$

where

$q_{1\text{-alphaRangeUCL},\,\nu}$: the critical value for a cumulative area of (1-alphaRangeUCL) under the curve of the distribution of the studentized range with ν degrees of freedom

Rearranging equation (2.22a) results in equation (2.22b):

$$P\left(\frac{R}{\overline{\overline{R}}} \le \frac{q_{1\text{-alphaRangeUCL},\,\nu}}{d_2^*}\right) \le 1\text{-alphaRangeUCL} \tag{2.22b}$$

Comparing equation (2.22b) with equation (2.21b) reveals the equation for D_4^*, which is given as equation (2.23):

$$D_4^* = \frac{q_{1\text{-alphaRangeUCL},\nu}}{d_2^*} \tag{2.23}$$

The equation for D_3^* is derived in exactly the same way as the equation for D_4^*, except alphaRangeLCL replaces (1-alphaRangeUCL) (alphaRangeLCL is the probability of a Type I error on the R control chart below the lower control limit (LCL)). It is given as equation (2.24):

$$D_3^* = \frac{q_{\text{alphaRangeLCL},\nu}}{d_2^*} \tag{2.24}$$

where

$q_{\text{alphaRangeLCL},\nu}$: the critical value for a cumulative area of alphaRangeLCL under the curve of the distribution of the studentized range with ν degrees of freedom

Hillier (1969) incorporates the two stage procedure with his (1964) and (1967) results and derives equations to calculate first and second stage short run control chart factors for $(\overline{X}, R)$ charts. Using these factors when process parameter estimates come from a small number of subgroups results in control chart limits that reliably indicate when a process has gone out of control. The first stage short run control chart factor for the $\overline{X}$ chart is denoted by A_2^{**}. It depends on m as well as n. The value A_2^{**} is related to A_2 in that, as $m \to \infty$, $A_2^{**} \to A_2$. First stage short run $\overline{X}$ control charts are constructed by following the same procedure for constructing Shewhart control charts, except A_2^{**} is used instead of A_2.

The derivation for A_2^{**} proceeds as follows (see Hillier (1969)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup averages as $\overline{\overline{X}}$ and the average of the subgroup ranges as $\overline{R}$. Denote one of the initial subgroup averages used to calculate $\overline{\overline{X}}$ as $\overline{X}_k$ (k: 1, 2, ..., m). In order to achieve the desired Type I error probability for retrospective testing, we need to determine the value A_2^{**} such that equation (2.25) holds:

$$P\left(-A_2^{**} \le \frac{\overline{X}_k - \overline{\overline{X}}}{\overline{R}} \le A_2^{**}\right) = 1 - \text{alphaMean} \qquad (2.25)$$

where

alphaMean: probability of a Type I error on the $\overline{X}$ control chart

The expected value and standard deviation of $\left(\overline{X}_k - \overline{\overline{X}}\right)$ are derived in Appendix A. Using these in place of the expected value and standard deviation, respectively, of $\left(\overline{X} - \overline{\overline{X}}\right)$ in equations (2.18a), (2.18b), (2.19a), and (2.19b) results in equation (2.26):

$$A_2^{**} = \frac{t_{(\text{alphaMean}/2),\,v}}{d_2^*} \cdot \sqrt{\frac{m-1}{n \cdot m}} \qquad (2.26)$$

The first stage short run upper and lower control chart factors for the R chart are denoted by D_4^{**} and D_3^{**}, respectively. Each of these factors depends on m as well as n. As $m \to \infty$, $D_4^{**} \to D_4$ and $D_3^{**} \to D_3$.

The derivation for D_4^{**} proceeds as follows (see Hillier (1969)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup ranges as $\overline{R}$. Denote one of the initial subgroup ranges used to calculate $\overline{R}$ as R_k (k: 1, 2, ..., m). In order to achieve the desired Type I error probability for retrospective testing, we need to determine the value D_4^{**} such that equation (2.27) holds:

$$P\left(R_k \le D_4^{**} \cdot \overline{R}\right) = 1\text{-alphaRangeUCL} \tag{2.27}$$

where

alphaRangeUCL: probability of a Type I error on the R control chart above the UCL

When equation (2.27) is expressed in terms of D_4^*, equation (2.28a) is the result:

$$P\left(R_k \le D_{4,m-1}^* \cdot \left(\frac{m \cdot \overline{R} - R_k}{m-1}\right)\right) = 1\text{-alphaRangeUCL} \tag{2.28a}$$

where

$D_{4,m-1}^*$: the second stage short run upper control chart factor for the R chart based on (m-1) subgroups

$\frac{m \cdot \overline{R} - R_k}{m-1}$: the average (based on $\overline{R}$) of (m-1) subgroup ranges

Collecting R_k on the left side of the inequality in equation (2.28a) results in equation (2.28b):

$$P\left(R_k \le \left(\frac{m \cdot D^*_{4,m\text{-}1}}{m-1+D^*_{4,m\text{-}1}} \right) \cdot \overline{R} \right) = 1\text{-alphaRangeUCL} \qquad (2.28b)$$

Comparing equation (2.28b) to equation (2.27) reveals the equation for D_4^{**}, which is given as equation (2.29):

$$D_4^{**} = \frac{m \cdot D^*_{4,m\text{-}1}}{m-1+D^*_{4,m\text{-}1}} \qquad (2.29)$$

The equation for D_3^{**} is derived in exactly the same way as the equation for D_4^{**}, except alphaRangeLCL replaces (1-alphaRangeUCL) (alphaRangeLCL is the probability of a Type I error on the R control chart below the LCL). It is given as equation (2.30):

$$D_3^{**} = \frac{m \cdot D^*_{3,m\text{-}1}}{m-1+D^*_{3,m\text{-}1}} \qquad (2.30)$$

where

$D^*_{3,m\text{-}1}$: the second stage short run lower control chart factor for the R chart based on (m-1) subgroups

Hillier (1969) gives tables of two stage short run control chart factors for $(\overline{X}, R)$ charts for the following values:

- n: 5
- m: 1 (for second stage only), 2-10, 15, 20, 25, 50, 100, ∞
- alphaMean: 0.001, 0.0027, 0.01, 0.025, 0.05
- alphaRangeUCL, alphaRangeLCL: 0.001, 0.005, 0.01, 0.025, 0.05

These values give limited results that have two consequences. First, further study of two stage short run $(\overline{X}, R)$ control charts is hindered. Second, in order to use the limited results, those involved with quality control in industry would most likely have to adjust their process monitoring to the above values. Otherwise, they would have to incorrectly use conventional control chart constants.

To allow for the use of more efficient estimates of the process variance and standard deviation, Yang and Hillier (1970) use exact distributional results to derive equations to calculate two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ using Hillier's (1969) methodology. Using these factors when process parameter estimates come from a small number of subgroups results in control chart limits that reliably indicate when a process has gone out of control.

The first and second stage short run control chart factors for the $\overline{X}$ chart are denoted by A_4^{**} and A_4^{*}, respectively. These factors depend on m as well as n. As m→∞, both

A_4^{**} and A_4^{*} converge to A_4, the conventional control chart constant for the $\overline{X}$ chart. First and second stage short run $\overline{X}$ control charts are constructed by following the same procedure for constructing Shewhart control charts, except A_4^{**} and A_4^{*}, respectively, are used instead of A_4.

The derivation for A_4^{*} proceeds as follows (see Yang and Hillier (1970)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup averages as $\overline{\overline{X}}$ and the average of the subgroup variances as $\overline{v}$. Suppose again that an additional subgroup of size n is sampled from the same population. Denote the average and variance of this subgroup as $\overline{X}$ and v, respectively. In order to achieve the desired Type I error probability for future testing, we need to determine the value A_4^{*} such that equation (2.31a) holds:

$$P\left(\overline{\overline{X}} - A_4^{*} \cdot \sqrt{\overline{v}} \le \overline{X} \le \overline{\overline{X}} + A_4^{*} \cdot \sqrt{\overline{v}}\right) = 1 - \text{alphaMean} \qquad (2.31a)$$

Rearranging equation (2.31a) results in equation (2.31b):

$$P\left(-A_4^{*} \le \frac{\overline{X} - \overline{\overline{X}}}{\sqrt{\overline{v}}} \le A_4^{*}\right) = 1 - \text{alphaMean} \qquad (2.31b)$$

It is necessary to determine the distribution of $\left(\overline{X} - \overline{\overline{X}}\right) / \sqrt{\overline{v}}$. First consider $\left(\overline{X} - \overline{\overline{X}}\right)$.

It was determined earlier that $\left(\overline{X}-\overline{\overline{X}}\right)$ is normally distributed with mean zero and standard deviation $\left(\left(\sqrt{(m+1)/(n\cdot m)}\right)\cdot\sigma\right)$ (see Appendix A).

Now consider the distribution of $\overline{v}$. The ratio $\left((m\cdot(n-1))\cdot\overline{v}\right)/\left(\sigma^2\right)$ has a χ^2 distribution with $(m\cdot(n-1))$ degrees of freedom. This means that, since $\left(\overline{X}-\overline{\overline{X}}\right)$ and $\overline{v}$ are independent for a Normal distribution, the ratio given as (2.32a) has approximately a Student's t distribution with $(m\cdot(n-1))$ degrees of freedom:

$$\frac{\left(\left(\overline{X}-\overline{\overline{X}}\right)\Big/\left(\sqrt{\frac{m+1}{n\cdot m}}\cdot\sigma\right)\right)}{\sqrt{\left(\frac{(m\cdot(n-1))\cdot\overline{v}}{\sigma^2}\right)\Big/(m\cdot(n-1))}} \tag{2.32a}$$

Simplifying the ratio in (2.32a) results in (2.32b):

$$\sqrt{\frac{n\cdot m}{m+1}}\cdot\left(\frac{\overline{X}-\overline{\overline{X}}}{\sqrt{\overline{v}}}\right) \tag{2.32b}$$

Since equation (2.32b) has a Student's t distribution with $(m\cdot(n-1))$ degrees of freedom, we have the probability relationship given as equation (2.33a):

$$P\left(-t_{(alphaMean/2),\,m\cdot(n-1)}\le\left(\sqrt{\frac{n\cdot m}{m+1}}\cdot\left(\frac{\overline{X}-\overline{\overline{X}}}{\sqrt{\overline{v}}}\right)\right)\le t_{(alphaMean/2),\,m\cdot(n-1)}\right)=1-alphaMean \tag{2.33a}$$

where

$t_{(alphaMean/2), m\cdot(n-1)}$: the critical value for an area of (alphaMean/2) in each tail of the Student's t distribution with $(m\cdot(n-1))$ degrees of freedom

Rearranging equation (2.33a) results in equation (2.33b):

$$P\left(\left(-t_{(alphaMean/2), m\cdot(n-1)}\cdot\sqrt{\frac{m+1}{n\cdot m}}\right)\leq\frac{\overline{X}-\overline{\overline{X}}}{\sqrt{\overline{v}}}\leq\left(t_{(alphaMean/2), m\cdot(n-1)}\cdot\sqrt{\frac{m+1}{n\cdot m}}\right)\right)=1-alphaMean \tag{2.33b}$$

Comparing equation (2.33b) with equation (2.31b) reveals the equation for A_4^*, which is given as equation (2.34):

$$A_4^* = t_{(alphaMean/2), m\cdot(n-1)}\cdot\sqrt{\frac{m+1}{n\cdot m}} \tag{2.34}$$

The derivation for A_4^{**} proceeds as follows (see Yang and Hillier (1970)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup averages as $\overline{\overline{X}}$ and the average of the subgroup variances as $\overline{v}$. Denote one of the initial subgroup averages used to calculate $\overline{\overline{X}}$ as $\overline{X}_k$ (k: 1, 2, ..., m). In order to achieve the desired Type I error probability for retrospective testing, we need to determine the value A_4^{**} such that equation (2.35) holds:

$$P\left(-A_4^{**} \le \frac{\overline{X}_k - \overline{\overline{X}}}{\sqrt{v}} \le A_4^{**}\right) = 1 - \text{alphaMean} \tag{2.35}$$

The expected value and standard deviation of $\left(\overline{X}_k - \overline{\overline{X}}\right)$ are derived in Appendix A. Using these in place of the expected value and standard deviation, respectively, of $\left(\overline{X} - \overline{\overline{X}}\right)$ in equations (2.32a), (2.32b), (2.33a), and (2.33b) results in equation (2.36):

$$A_4^{**} = t_{(\text{alphaMean}/2),\, m\cdot(n-1)} \cdot \sqrt{\frac{m-1}{n \cdot m}} \tag{2.36}$$

The first stage short run upper and lower control chart factors for the v chart are denoted by B_8^{**} and B_7^{**}, respectively. The second stage short run upper and lower control chart factors for the v chart are denoted by B_8^{*} and B_7^{*}, respectively. These factors depend on m as well as n. As m→∞, both B_8^{**} and B_8^{*} converge to B_8, the alpha-based conventional upper control chart constant for the v chart. Similarly, as m→∞, both B_7^{**} and B_7^{*} converge to B_7, the alpha-based conventional lower control chart constant for the v chart.

The derivation for B_8^{*} proceeds as follows (see Yang and Hillier (1970)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup variances as

$\bar{v}$. Suppose again that an additional subgroup of size n is sampled from the same population. Denote the variance of this subgroup as v. In order to achieve the desired Type I error probability for future testing, we need to determine the value B_8^* such that equation (2.37a) holds:

$$P\left(v \leq B_8^* \cdot \bar{v}\right) = 1\text{-alphaVarUCL} \tag{2.37a}$$

where

alphaVarUCL: probability of a Type I error on the v and $\sqrt{v}$ control charts above the UCL

Rearranging equation (2.37a) results in equation (2.37b):

$$P\left(\frac{v}{\bar{v}} \leq B_8^*\right) = 1\text{-alphaVarUCL} \tag{2.37b}$$

The ratio $v/\bar{v}$ is the F distribution with (n-1) degrees of freedom for v and $(m \cdot (n-1))$ degrees of freedom for $\bar{v}$. Consequently, B_8^* is calculated using equation (2.38):

$$B_8^* = F_{1-\text{alphaVarUCL},\, n-1,\, m \cdot (n-1)} \tag{2.38}$$

where

$F_{1\text{-alphaVarUCL},\, n\text{-}1,\, m \cdot (n-1)}$: the critical value for a cumulative area of (1-alphaVarUCL) under the curve of the F distribution with (n-1) numerator degrees of freedom and $(m \cdot (n-1))$ denominator degrees of freedom

The equation for B_7^* is derived in exactly the same way as the equation for B_8^*, except alphaVarLCL replaces (1-alphaVarUCL) (alphaVarLCL is the probability of a Type I error on the v and $\sqrt{v}$ control charts below the LCL). It is given as equation (2.39):

$$B_7^* = F_{\text{alphaVarLCL}, n-1, m\cdot(n-1)} \tag{2.39}$$

where

$F_{\text{alphaVarLCL}, n-1, m\cdot(n-1)}$: the critical value for a cumulative area of alphaVarLCL under the curve of the F distribution with (n-1) numerator degrees of freedom and $(m \cdot (n-1))$ denominator degrees of freedom

The derivation for B_8^{**} proceeds as follows (see Yang and Hillier (1970)). Consider a Normal population with mean μ and standard deviation σ. Suppose that m subgroups of size n are sampled from this population. Denote the average of the subgroup variances as $\bar{v}$. Denote one of the initial subgroup variances used to calculate $\bar{v}$ as v_k (k: 1, 2, ..., m). In order to achieve the desired Type I error probability for retrospective testing, we need to determine the value B_8^{**} such that equation (2.40) holds:

$$P\left(v_k \leq B_8^{**} \cdot \bar{v}\right) = \text{1-alphaVarUCL} \tag{2.40}$$

When equation (2.40) is expressed in terms of B_8^*, equation (2.41a) is the result:

$$P\left(v_k \le B^*_{8,\,m\text{-}1} \cdot \left(\frac{m \cdot \bar{v} - v_k}{m-1}\right)\right) = 1\text{-alphaVarUCL} \qquad (2.41a)$$

where

$$B^*_{8,\,m\text{-}1} = F_{1-\text{alphaVarUCL},\, n\text{-}1,\, (m\text{-}1)\cdot(n-1)}$$

$\frac{m \cdot \bar{v} - v_k}{m-1}$: the average (based on $\bar{v}$) of (m-1) subgroup variances

Collecting v_k on the left side of the inequality in equation (2.41a) results in equation (2.41b):

$$P\left(v_k \le \left(\frac{m \cdot B^*_{8,\,m\text{-}1}}{m-1+B^*_{8,\,m\text{-}1}}\right) \cdot \bar{v}\right) = 1\text{-alphaVarUCL} \qquad (2.41b)$$

Comparing equation (2.41b) to equation (2.40) reveals the equation for B_8^{**}, which is given as equation (2.42):

$$B_8^{**} = \frac{m \cdot B^*_{8,\,m\text{-}1}}{m-1+B^*_{8,\,m\text{-}1}} \qquad (2.42)$$

The equation for B_7^{**} is derived in exactly the same way as the equation for B_8^{**}, except alphaVarLCL replaces (1-alphaVarUCL). It is given as equation (2.43):

$$B_7^{**} = \frac{m \cdot B_{7,m-1}^*}{m - 1 + B_{7,m-1}^*} \tag{2.43}$$

where

$$B_{7,m-1}^* = F_{\text{alphaVarLCL}, n-1, (m-1)\cdot(n-1)}$$

The first stage short run upper and lower control chart factors for the $\sqrt{v}$ chart are the square roots of B_8^{**} and B_7^{**}, respectively. The second stage short run upper and lower control chart factors for the $\sqrt{v}$ chart are the square roots of B_8^* and B_7^*, respectively. These factors, which depend on m as well as n, result in control limits that give the desired Type I error probability. As $m \to \infty$, both $\sqrt{B_8^{**}}$ and $\sqrt{B_8^*}$ converge to $\sqrt{B_8}$, the alpha-based conventional upper control chart constant for the $\sqrt{v}$ chart. Similarly, as $m \to \infty$, both $\sqrt{B_7^{**}}$ and $\sqrt{B_7^*}$ converge to $\sqrt{B_7}$, the alpha-based conventional lower control chart constant for the $\sqrt{v}$ chart.

Yang and Hillier (1970) give tables of two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts for the following values:

- n: 5
- m: 1 (for second stage only), 2-10, 15, 20, 25, 50, 100, ∞
- alphaMean: 0.001, 0.002, 0.01, 0.05
- alphaVarUCL, alphaVarLCL: 0.001, 0.005, 0.025

These values give limited results that have two consequences. First, further study of two stage short run $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts is hindered. Second, in order to use the limited results, those involved with quality control in industry would most likely have to adjust their process monitoring to the above values. Otherwise, they would have to incorrectly use conventional control chart constants.

Additionally, Yang and Hillier (1970) neglect to include appropriate bias correction factors in their two stage short run control chart factor equations that involve $\sqrt{v}$, which is a biased estimate of the population standard deviation. This omission renders much of their tables as incorrect. Also, some of their results calculated using the correct equations are incorrect in the last decimal place shown by one and in some cases two digits. These issues are explained in complete detail in Chapter V of this dissertation.

Two attempts appear in the literature to expand Hillier's (1969) results for two stage short run $(\overline{X}, R)$ charts. Pyzdek (1993) gives two stage short run control chart factors for $(\overline{X}, R)$ charts using Hillier's (1969) theory for the following values:

- n: 2-5
- m: 1 (for second stage only), 2-10, 15, 20, 25
- alphaMean: 0.0027
- alphaRangeUCL: 0.005

In addition to these values offering even more limited results for m, alphaMean, and alphaRangeUCL (with no alphaRangeLCL) than those presented by Hillier (1969), several of Pyzdek's values are incorrect (see Chapter IV of this dissertation).

Yang (1995) gives two stage short run control chart factors for $(\overline{X}, R)$ charts using Hillier's (1969) theory for the following values:

- n: 2-25 for the $\overline{X}$ chart and 2-20 for the R chart
- m: 1 (for second stage only), 2-25
- alphaMean: 0.0027, 0.01, 0.05
- alphaRangeUCL: 0.00135 and 0.0027

Similar to Pyzdek (1993), Yang (1995) does not give two stage short run control chart factors for the R chart below the lower control limit. Many of the values given by Yang (1995) are incorrect because inaccurate equations and numerical techniques are used to calculate the results (see Chapter IV). It should be noted that Yang (1999 and 2000) contain some of the results from Yang (1995).

Elam and Case (2001) describe the development and execution of a computer program that overcomes the problems associated with Hillier's (1969), Pyzdek's (1993), and Yang's (1995, 1999, 2000) efforts to present two stage short run control chart factors for $(\overline{X}, R)$ charts. Chapter IV and Appendix B of this dissertation include the entire contents of Elam and Case (2001).

Other than Yang and Hillier (1970), one attempt appears in the literature to extend Hillier's (1969) methodology to other control chart combinations. Pyzdek (1993) attempts to present two stage short run control chart factors for (X, MR) charts for the following values (alphaInd is the probability of a Type I error on the X chart and alphaMRUCL is the probability of a Type I error on the MR chart above the UCL):

- m: 1 (for second stage only), 2-10, 15, 20, 25
- alphaInd: 0.0027
- alphaMRUCL: 0.005

However, all of Pyzdek's (1993) Table 1 results for subgroup size one are incorrect because he uses invalid theory (this is explained in complete detail in Chapter VII of this dissertation).

Sensitivity Issues with Two Stage Short Run Control Charts

As with Quesenberry's (1991) Q charts, two stage short run control charts based on Hillier's (1969) theory, in general, are not very sensitive in detecting process changes (see Del Castillo (1996a) and Crowder and Halbleib (2000)). Using the average run length (ARL), which is the average number of subgroups that must be plotted on a control chart before an out-of-control condition is indicated, Del Castillo (1996a) evaluates Yang and Hillier's (1970) second stage short run $\overline{X}$ control chart. For an in-control situation, Del Castillo (1996a) concludes that fewer short runs and more very long runs occur between false alarms. This is a desirable situation. However, for an out-of-control situation, fewer short runs and more very long runs occur until detection. This is clearly an undesirable situation.

In order to deal with these sensitivity issues, one may use the tests for special causes given by Nelson (1984), which Quesenberry (1991) suggests for his Q charts, or runs rules (i.e., the four tests for instability in Western Electric Co., Inc. (1956)). However, using techniques to increase the sensitivity of two stage short run control charts based on Hillier's (1969) methodology increases the probability of a false alarm. This is because of the inherent tradeoff between these two issues when control charts with modified limits are used in short run situations (Del Castillo (1995)).

The Two Stage Procedure

A two stage (i.e., two phase, delete and revise) procedure for initiating control charting serves two distinct purposes. The first is retrospective testing. The second is future testing. In the first stage of the two stage procedure, the initial subgroups drawn from the process are used to determine the control limits. The initial subgroups are plotted against the control limits to retrospectively test if the process was in control while the initial subgroups were being drawn. Once control is established, the procedure moves to the second stage, where the subgroups that were not deleted in the first stage are used to determine the control limits for testing if the process remains in control while future subgroups are drawn.

Stage One Control Limits

Two approaches are given in the literature for setting up control limits in stage one. Hillier (1969) uses each of the initial subgroups to estimate parameters to determine stage one control limits, which only have to be calculated once. All of the initial subgroups are tested simultaneously against these control limits (Yang and Hillier (1970)). Roes, Does, and Schurink (1993) suggest an approach by which the initial subgroup that is going to be tested is not used to estimate parameters to determine stage one control limits. This requires that stage one control limits be recalculated for each initial subgroup. It should be noted that Yang and Hillier (1970) also mention the procedure suggested by Roes, Does, and Schurink (1993), but do not use it. Also, King (1954) seems to have suggested this approach.

Establishment of Control

A point of contention with the two stage procedure in the literature has been how to establish control in the first stage; i.e., how to make the transition from stage one to stage two. Faltin, Mastrangelo, Runger, and Ryan (1997) state that there is a failure to distinguish between these two stages in much of the relevant literature. The tendency is to focus on stage one without considering the ramifications for stage two.

Several approaches (i.e., delete and revise (D&R) procedures) have been suggested for establishing control in stage one. The first approach, and the one that seems to appear most often in the literature, is to repeat the following procedure until no subgroups show out-of-control on either the control chart for centering or the control chart for spread:

1. Delete the out-of-control initial subgroups on either control chart entirely (i.e., if a subgroup shows out-of-control on either the control chart for centering or spread, it should be deleted from both charts).
2. Recalculate the control limits.

Hillier (1969), Ryan (1989), and Montgomery (1997) all advocate this approach. Ryan (1989) states that a subgroup should be deleted only if an assignable (special) cause is detected and removed. Since an assignable cause that affects the standard deviation estimate does not necessarily affect the average estimate, it may not be necessary to delete a subgroup from the chart for centering that shows out-of-control only on the chart for spread. However, for the sake of simplicity, Ryan (1989) recommends deleting the out-of-control subgroup entirely, stating that the exclusion of such points will not make a difference in the end result unless they are near one of the control limits.

Montgomery (1997) states that it may not be possible to find an assignable cause for a subgroup that plots out-of-control on either chart. In this case, one option is to eliminate the subgroup anyway. The other option is to keep the subgroup, which is a risk because if the subgroup is really out-of-control because of an assignable cause, then the control limits will be distorted.

When many subgroups plot out-of-control and each is subsequently deleted, an undesirable situation arises because few subgroups will remain to estimate process parameters to construct control limits. The fewer the initial subgroups, the less information one has about the process. Less information results in less reliable control limits. In this situation, Montgomery (1997) suggests that one should not search for an assignable cause for each out-of-control subgroup, but should instead determine the pattern of the out-of-control subgroups and determine the assignable cause associated with the pattern.

Pyzdek (1993) suggests an approach for establishing control in the first stage that uses the following procedure:

1. Delete the out-of-control initial subgroups on the control chart for spread.
2. Recalculate the control limits.
3. Repeat steps 1 and 2 until no initial subgroups show out-of-control on the control chart for spread.
4. Using the parameter estimate for spread obtained after completing steps 1-3 and the overall average obtained from all of the initial subgroups, determine the control limits for the control chart for centering.
5. Perform steps 1-3 for the control chart for centering.

Except for the fact that the deletion of subgroups is performed on the charts for centering and spread separately, Pyzdek's (1993) approach is exactly like the one advocated by Hillier (1969), Ryan (1989), and Montgomery (1997).

A third approach is to delete out-of-control subgroups only on the chart for spread just once (Case (1998)). The resulting parameter estimate for spread is used with the overall average from all of the initial subgroups to determine control limits for the control chart for centering. This approach has the advantage of requiring recalculation of control limits just once on only one chart.

A fourth approach is to not perform any revision of the control chart limits regardless of whether or not initial subgroups plot out-of-control. Doty (1997) bases his justification for supporting this approach on two assumptions. The first is that trial control charts constructed from all of the initial subgroups are perfectly adequate for controlling the process. The second is that, since control chart limits are periodically revised anyway, it is not necessary to establish control using the initial subgroups. For additional justification, Doty (1997) also states that much of the statistical process control computer programs do not recognize revised charts.

Control Chart Factors for the Two Stage Procedure

As was shown in the Two Stage Short Run Control Charts subsection of the Control Charts with Modified Limits section earlier in this chapter, Hillier (1969) expresses analytically the two distinct purposes of two stage control charting in a short run situation. Even if no subgroups are deleted in stage one when establishing control, stage one control limits are still different from stage two control limits. This means that the values for the control chart factors depend upon the two distinct purposes of two stage control charting when in a short run situation (i.e., when only a finite number of subgroups is available).

The approaches by Ryan (1989), Montgomery (1997), and Case (1998) use conventional control chart constants for each stage. This means that, if no subgroups are deleted in stage one when establishing control, then stage one control limits are equal to stage two control limits. This implies that values for the control chart factors do not depend upon the two distinct purposes of two stage control charting when operating under the assumption that an infinite number of subgroups is available. This statement is theoretically validated when one considers that, for a specific control chart, Hillier's (1969) and Yang and Hillier's (1970) first stage and second stage short run control chart factors converge to the same conventional control chart constant as the number of subgroups approaches infinity.

Performance Evaluation of Short Run Control Charts

The one performance metric that is used extensively to evaluate the performance of short run control charts is the average run length (ARL). The ARL is the average number of subgroups that must be plotted on a control chart before an out-of-control condition is indicated. It is desirable to have a large value for the ARL when a process is in-control. When a process is out-of-control, a small ARL is preferred.

By its very definition, the ARL would seem difficult to apply in a short run situation. The reason is that, in a short run situation, a process may not run long enough in order to draw enough subgroups to even come close to equaling the ARL. Nevertheless, the ARL seems to be the metric of choice for those evaluating the performance of short run control charts in the literature (see Quesenberry (1993), Wasserman and Sudjianto (1993), Del Castillo and Montgomery (1994), Del Castillo (1996a), Doganaksoy and Vandeven (1997), and Lin, Lai, and Chang (1997)).

A more meaningful performance metric for short run control charts is the probability of detection (POD). This is the probability that a control chart will signal, within a given number of subgroups following a shift, that a process is out-of-control (see Woodall, Crowder, and Wade (1995) and Crowder and Halbleib (2000)). Wade (1992) uses the POD within ten subgroups following a shift. Quesenberry (1995a) and Del Castillo (1995) use the POD within thirty subgroups following a shift. It should be noted that determining the POD is the same thing as characterizing the run length distribution.

Summary

It is clear from this literature review that Hillier's (1969) methodology overcomes the endemic problems associated with the other methodologies that apply $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts to short run situations. These problems include relying on the common rule of thumb, using target or nominal values, tolerances, or specification limits to estimate process parameters, assuming the process starts in-control, and complex implementation. However, Hillier's (1969) methodology has its own problems that present research opportunities.

The first problem is that Hillier's (1969) methodology is limited to $(\overline{X}, R)$ control charts (see Hillier (1969)) and to $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts (see Yang and Hillier (1970)). Additionally, limited and in some cases incorrect results are presented in the literature for these charts. A particularly important deficiency of Hillier's (1969) methodology is that it has not been applied to (X, MR) control charts (see Del Castillo and Montgomery (1994) and Quesenberry (1995b)).

The second problem is that the execution of the two stage procedure is not clear (see Faltin, Mastrangelo, Runger, and Ryan (1997)). Using the approach advocated by Hillier (1969), Ryan (1989), and Montgomery (1997) is problematic because, in a short run situation, one does not have a lot of initial data to estimate process parameters. By continually deleting subgroups from both control charts in the first stage, one is creating a situation in which an even more limited amount of data will be available to initially estimate process parameters for stage two. This is a problem because the reliability of the control limits decreases as the amount of data used to obtain initial estimates of the process parameters decreases. However, control limits are also less reliable if subgroups reflecting process changes are used in their calculation. A methodology is required that can provide information to investigate this tradeoff.

CHAPTER III

TWO STAGE SHORT RUN VARIABLES CONTROL CHARTING

Introduction

The purpose of this chapter is to describe the process required to perform two stage short run variables control charting, with reference to the research in Chapters IV-VIII of this dissertation. Tables are presented that indicate, based on the choice of the two stage short run control chart ($(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, or (X, MR)), the appropriate program to use from Chapters IV-VII, the output to use from these programs, and the equations to use to construct Stage 1 and Stage 2 control limits. Additionally, a table is presented that indicates, based on the choice of the statistic ($\overline{R}$, $\overline{v}$, $\overline{\sqrt{v}}$, $\overline{s}$, or $\overline{MR}$), the appropriate program to use from Chapters IV-VII, the output to use from these programs, and the equations to use to calculate unbiased estimates of the process variance and standard deviation.

Stage One Control Charting

In the first stage of the two stage procedure, initial subgroups are collected from the process. Tables 3.1 and 3.2 have, based on the choice of the two stage short run control chart ($(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, or (X, MR)), the appropriate program to use from Chapters IV-VII, the output to use from these programs (the last three columns of each table), and the equations to use to construct upper (Table 3.1) and lower (Table 3.2) Stage 1 control limits. It should be noted that the notation in these tables is explained in

Table 3.1. Upper Control Limit (UCL) Calculations for Two Stage Short Run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) Control Charts

Control Chart	Mathcad Program (extension .mcd)	Center Line (CL)	General Form for the UCL	Stage 1 ccf	Stage 2 ccf	Conventional ccf
$\overline{X}$	ccfsR	$\overline{\overline{X}}$	$\overline{\overline{X}} + ccf \cdot \overline{R}$	A21	A22	A2 (i.e., A_2)
R		$\overline{R}$	$ccf \cdot \overline{R}$	D41	D42	D4 (i.e., D_4)
$\overline{X}$	ccfsv	$\overline{\overline{X}}$	$\overline{\overline{X}} + ccf \cdot \sqrt{\overline{v}}$	A41	A42	A4 (i.e., A_4)
v		$\overline{v}$	$ccf \cdot \overline{v}$	B81	B82	B8 (i.e., B_8)
$\overline{X}$	ccfsv	$\overline{\overline{X}}$	$\overline{\overline{X}} + ccf \cdot \sqrt{\overline{v}}$	A41	A42	A4 (i.e., A_4)
$\sqrt{v}$		$\sqrt{\overline{v}}$	$ccf \cdot \sqrt{\overline{v}}$	B81sqrt	B82sqrt	B8sqrt (i.e., $\sqrt{B_8}$)
$\overline{X}$	ccfss	$\overline{\overline{X}}$	$\overline{\overline{X}} + ccf \cdot \overline{s}$	A31	A32	A3 (i.e., A_3)
s		$\overline{s}$	$ccf \cdot \overline{s}$	B41	B42	B4 (i.e., B_4)
X	ccfsMR	$\overline{X}$	$\overline{X} + ccf \cdot \overline{MR}$	E21	E22	E2 (i.e., E_2)
MR		$\overline{MR}$	$ccf \cdot \overline{MR}$	D41	D42	D4 (i.e., D_4)

Chapters IV-VII.

For example, suppose one wants to construct first stage control limits for $(\overline{X}, R)$ charts. Referring to the first two rows of the fourth columns of Tables 3.1 and 3.2, three pieces of information are required: $\overline{\overline{X}}$, $\overline{R}$, and ccf (ccf stands for control chart factor). $\overline{\overline{X}}$ and $\overline{R}$ are, respectively, the average of the initial subgroup averages (which are denoted by $\overline{X}$) and the average of the initial subgroup ranges (which are denoted by R).

The value for ccf is from the output of the *Mathcad* (1998) program ccfsR.mcd, which is in Chapter IV and Appendix B.2 of this dissertation. For the $\overline{X}$ control chart, ccf is equal to A21 for both the upper and lower Stage 1 control limits. For the R control chart, ccf is equal to D41 for the upper Stage 1 control limit and it is equal to D31 for the lower

Table 3.2. Lower Control Limit (LCL) Calculations for Two Stage Short Run $(\bar{X}, R)$, $(\bar{X}, v)$, $(\bar{X}, \sqrt{v})$, $(\bar{X}, s)$, and (X, MR) Control Charts

Control Chart	**Mathcad Program (extension .mcd)**	**Center Line (CL)**	**General Form for the LCL**	**Stage 1 ccf**	**Stage 2 ccf**	**Conventional ccf**
$\bar{X}$	ccfsR	$\bar{\bar{X}}$	$\bar{\bar{X}} - ccf \cdot \bar{R}$	A21	A22	A2 (i.e., A_2)
R		$\bar{R}$	$ccf \cdot \bar{R}$	D31	D32	D3 (i.e., D_3)
$\bar{X}$	ccfsv	$\bar{\bar{X}}$	$\bar{\bar{X}} - ccf \cdot \sqrt{\bar{v}}$	A41	A42	A4 (i.e., A_4)
v		$\bar{v}$	$ccf \cdot \bar{v}$	B71	B72	B7 (i.e., B_7)
$\bar{X}$	ccfsv	$\bar{\bar{X}}$	$\bar{\bar{X}} - ccf \cdot \sqrt{\bar{v}}$	A41	A42	A4 (i.e., A_4)
$\sqrt{v}$		$\sqrt{\bar{v}}$	$ccf \cdot \sqrt{\bar{v}}$	B71sqrt	B72sqrt	B7sqrt (i.e., $\sqrt{B_7}$)
$\bar{X}$	ccfss	$\bar{\bar{X}}$	$\bar{\bar{X}} - ccf \cdot \bar{s}$	A31	A32	A3 (i.e., A_3)
s		$\bar{s}$	$ccf \cdot \bar{s}$	B31	B32	B3 (i.e., B_3)
X	ccfsMR	$\bar{X}$	$\bar{X} - ccf \cdot \overline{MR}$	E21	E22	E2 (i.e., E_2)
MR		$\overline{MR}$	$ccf \cdot \overline{MR}$	D31	D32	D3 (i.e., D_3)

Stage 1 control limit.

After constructing Stage 1 control limits, the initial subgroups are plotted against them to retrospectively test if the process was in-control while the initial subgroups were being drawn. If all of the subgroups are in-control, then one is ready to construct Stage 2 control limits using all of the initial subgroups. The construction of Stage 2 control limits is explained later in the Stage Two Control Charting section of this chapter. If any subgroups are out-of-control, then one needs to determine which delete and revise (D&R) procedure to use to establish control of the process. This is explained in the next section.

The Delete and Revise (D&R) Process

Six D&R procedures are described in detail in the Delete and Revise (D&R) Procedures section of Chapter VIII of this dissertation. Chapter VIII also presents a methodology that provides information to assist one in determining which D&R procedure to use. The methodology consists of three elements, each of which is described in complete detail in Chapter VIII. The main element is the computer program that simulates two stage short run variables control charting. The next element, which is included in the operation of the program, is the measurements that one may use to determine which D&R procedure establishes the most reliable second stage control limits. The third element is the interpretation of the results from the program.

Once a D&R procedure has been chosen and completed, then one is ready to construct Stage 2 control limits.

Stage Two Control Charting

In the second stage of the two stage procedure, the initial subgroups that remain after completing Stage 1 control charting are used to construct Stage 2 control limits. Tables 3.1 and 3.2 have, based on the choice of the two stage short run control chart ($(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, or (X, MR)), the appropriate program to use from Chapters IV-VII, the output to use from these programs (the last three columns of each table), and the equations to use to construct upper (Table 3.1) and lower (Table 3.2) Stage 2 control limits.

For example, suppose one wants to construct second stage control limits for $(\overline{X}, R)$

charts. Referring to the first two rows of the fourth columns of Tables 3.1 and 3.2, three pieces of information are required: $\overline{\overline{X}}$, $\overline{R}$, and ccf. $\overline{\overline{X}}$ and $\overline{R}$ are, respectively, the average of the remaining initial subgroup averages (which are denoted by $\overline{X}$) and the average of the remaining initial subgroup ranges (which are denoted by R).

The value for ccf is from the output of the *Mathcad* (1998) program ccfsR.mcd. For the $\overline{X}$ control chart, ccf is equal to A22 for both the upper and lower Stage 2 control limits. For the R control chart, ccf is equal to D42 for the upper Stage 2 control limit and it is equal to D32 for the lower Stage 2 control limit.

After constructing Stage 2 control limits, one is ready to monitor the future performance of the process. If one is interested in updating Stage 2 control limits as more subgroups are accumulated, then an approach to do this may be found in Hillier's (1969) example. However, no methodology is presented in this dissertation that determines the approach for updating that results in Stage 2 control limits that perform the best.

Unbiased Estimates of the Process Variance and Standard Deviation

Table 3.3 presents equations to calculate unbiased estimates of the process variance (σ^2) and standard deviation (σ) based on $\overline{R}$, $\overline{v}$, $\overline{\sqrt{v}}$, $\overline{s}$, and $\overline{MR}$. For any one of these statistics calculated from m subgroups of size n, the table gives the appropriate *Mathcad* (1998) program from Chapter IV, V, VI, or VII that must be used to determine the value for the bias correction factor. Using the notation from the programs, the tables then give the equations to calculate unbiased estimates of σ and σ^2 using the bias correction

Table 3.3. Unbiased Estimates of the Process Variance (σ^2) and Standard Deviation (σ)

Statistic	**Mathcad Program (extension .mcd)**	**Unbiasing Factor**		**Unbiased Estimate**	
		σ	σ^2	σ	σ^2
$\overline{\mathbf{R}}$	ccfsR	d2 (i.e., d_2)	d2star (i.e., d_2^*)	$\overline{R}/d2$	$(\overline{R}/d2star)^2$
$\overline{v}$	ccfsv	c4(v2+1) (i.e., c_4 with subgroup size (v2+1))	-----	$\sqrt{\overline{v}}/c4(v2+1)$	$\overline{v}$
$\sqrt{\overline{v}}$	ccfsv	c4(v2+1)	-----	$\sqrt{\overline{v}}/c4(v2+1)$	$(\sqrt{\overline{v}})^2$
$\overline{s}$	ccfss	c4 (i.e., c_4 with subgroup size n)	c4star (i.e., c_4^*)	$\overline{s}/c4$	$(\overline{s}/c4star)^2$
$\overline{\mathbf{MR}}$	ccfsMR	d2 (i.e., d_2)	d2starMR (i.e., $d_2^*(MR)$)	$\overline{MR}/d2$	$(\overline{MR}/d2starMR)^2$

factors. It should be noted that columns three and four of Table 3.3 represent output from the respective programs. Also, the notation in this table is explained in Chapters IV-VII.

For example, suppose one wants to determine unbiased estimates of σ and σ^2 based on $\overline{R}$. Referring to the first row of Table 3.3, three pieces of information are required: $\overline{R}$, d2 (i.e., d_2), and d2star (i.e., d_2^*). $\overline{R}$ is the average of m subgroup ranges (which are denoted by R), each of which is based on a subgroup of size n. Values for the unbiasing factors d2 and d2star are from the output of the *Mathcad* (1998) program ccfsR.mcd. The equations to calculate the unbiased estimates of σ and σ^2 based on $\overline{R}$ are in the first rows of the last two columns, respectively, of Table 3.3.

As will be explained in Chapters VI and VII, the unbiasing factors c4star (i.e., c_4^*) and d2starMR (i.e., $d_2^*(\mathrm{MR})$), respectively, in Table 3.3 are new developments from the research presented in this dissertation. This means that, for the first time, one may obtain an unbiased estimate of σ^2 based on $\bar{s}$ and $\overline{\mathrm{MR}}$ using the equations in the last two rows, respectively, of the last column of Table 3.3.

Conclusions

The description of the process required to perform two stage short run variables control charting together with the notation and equations presented in this chapter is meant to indicate where and how to use the research presented in Chapters IV-VIII of this dissertation in this process. By addressing the tasks associated with research sub-objectives 1, 2, 3, 4, and 5 from Chapter I of this dissertation, the research presented in Chapters IV, V, VI, VII, and VIII, respectively, results in a comprehensive, theoretically sound, easy-to-implement, and effective methodology for two stage short run control charting using $(\overline{\mathrm{X}}, \mathrm{R})$, $(\overline{\mathrm{X}}, \mathrm{v})$, $(\overline{\mathrm{X}}, \sqrt{\mathrm{v}})$, $(\overline{\mathrm{X}}, \mathrm{s})$, and (X, MR) charts.

CHAPTER IV

TWO STAGE SHORT RUN $(\overline{X}, R)$ CONTROL CHARTS AND A COMPUTER PROGRAM TO CALCULATE THE FACTORS

Introduction

Hillier (1969) presents equations to calculate two stage short run control chart factors for $(\overline{X}, R)$ charts and gives extensive tabulated results, but for subgroup size five only. Using Hillier's (1969) theory, Pyzdek (1993) gives two stage short run control chart factors for $(\overline{X}, R)$ charts for subgroup sizes 2-5, but with less numbers of subgroups than Hillier (1969) and only one set of values for alpha for the $\overline{X}$ chart and alpha for the R chart above the upper control limit (alpha is the probability of a Type I error). Unlike Hillier's (1969) results, Pyzdek (1993) does not give two stage short run control chart factors for the R chart below the lower control limit.

Also using Hillier's (1969) theory, Yang (1995) presents two stage short run control chart factors for $(\overline{X}, R)$ charts for subgroup sizes 2-25 for the $\overline{X}$ chart and 2-20 for the R chart, number of subgroups 1 (for second stage only) and 2-25, alpha values of 0.05, 0.01, and 0.0027 for the $\overline{X}$ chart, and alpha values of 0.00135 and 0.0027 for the R chart above the upper control limit. Similar to Pyzdek (1993), Yang (1995) does not give two stage short run control chart factors for the R chart below the lower control limit. It should be noted that Yang (1999 and 2000) contain some of the results from Yang (1995).

Problem

Hillier (1969), Pyzdek (1993), and Yang (1995, 1999, 2000) represent the only attempts in the literature to present two stage short run control chart factors for $(\overline{X}, R)$ charts based on Hillier's (1969) theory. In addition to the limitations already presented, Pyzdek's Table 1: Exact Method Control Chart Factors contains some incorrect values. Also, many of the values in Yang's (1995) Tables 2.1-2.7 and 3.1-3.4 are incorrect because inaccurate equations and numerical techniques are used to calculate the results. It should be noted that Tables 1 and 2 in Yang (1999) are exact replications of Tables 3.4 and 3.2, respectively, in Yang (1995). Also, Tables 1 and 2 in Yang (2000) are exact replications of Tables 2.4 and 2.7, respectively, in Yang (1995).

Solution

This chapter describes the development and execution of a computer program that overcomes these limitations. It will accurately calculate first and second stage short run control chart factors for $(\overline{X}, R)$ charts. The program uses exact equations for the probability integral of the range, the expected values of the first and second powers of the distribution of the range, the probability integral of the studentized range, degrees of freedom calculations, short run calculations, and conventional control chart calculations. The program accepts values for subgroup size, number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the R chart both above the upper control limit and below the lower control limit. Tables are generated for specific values of these inputs. Comparison of the tabulated results to legitimate results in the literature validates the program. The tables correct and extend previous results in the literature.

The software used for the program is *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997). The program uses numerical routines provided by the software.

Outline

This chapter first presents the probability integrals of the range and the studentized range. These are essential in the application of Hillier's (1969) theory to $(\overline{X}, R)$ control charts and are required for the program to perform accurate calculations. Next, the computer program is described. Tables generated by the program are then presented and compared with legitimate results in the literature. Also, implications of the tabulated results are discussed. Following a numerical example that illustrates the use of the program, final conclusions describing the impact of the program on industry and research are given.

Note

Results from the program are for processes generating parts with independent measurements that follow a Normal distribution.

The Probability Integral of the Range

The probability integral (or cumulative distribution function (cdf)) of the range for subgroups of size n sampled from a standard Normal population is given by Pachares

(1959) as equation (4.1) (with some modifications in notation):

$$P(W) = n \cdot \int_{-\infty}^{\infty} f(x) \cdot (F(x+W) - F(x))^{n-1} dx \tag{4.1}$$

W represents the (standardized) range w/σ, where w is the range of a subgroup and σ is the population standard deviation. Throughout this chapter, F(x) is the cdf of the standard Normal probability density function (pdf) f(x).

The expected values of the first and second powers (or moments) of the distribution of the range $W = (w/\sigma)$ for subgroups of size n sampled from a Normal population with mean μ and variance equal to one given by Harter (1960) are equations (4.2) and (4.3), respectively (with some modifications in notation):

$$W1 = n \cdot (n-1) \cdot \int_{-\infty}^{\infty} \left[\int_{0}^{\infty} W \cdot (F(x+W) - F(x))^{n-2} \cdot f(x+W)\, dW \right] \cdot f(x)\, dx \tag{4.2}$$

$$W2 = n \cdot (n-1) \cdot \int_{-\infty}^{\infty} \left[\int_{0}^{\infty} W^2 \cdot (F(x+W) - F(x))^{n-2} \cdot f(x+W)\, dW \right] \cdot f(x)\, dx \tag{4.3}$$

The mean of the distribution of the range (E(W)) is W1 and is the control chart constant denoted by d_2 (see Table M in the appendix of Duncan (1974)). The variance of the distribution of the range (Var(W)) is calculated using equation (4.4):

$$Var = W2 - W1^2 \tag{4.4}$$

The control chart constant d_3 (see Table M in the appendix of Duncan (1974)) is the square root of the variance.

The values d_2, d_3, and m (the number of subgroups) are used to generate the degrees of freedom (ν) and d_2^* (d2star) values for Table D3 in the appendix of Duncan (1974). The value d2star is calculated using the exact equation (equation (4.5)) from David (1951) (note: $d2 \equiv d_2$ and $d3 \equiv d_3$):

$$d2star = \left(d2^2 + \frac{d3^2}{m} \right)^{0.5} \tag{4.5}$$

The value ν has two possible calculations. The first calculation is an estimate. It is given by David (1951) as equation (4.6):

$$\nu = A^{-1} + \left(\frac{1}{4}\right) - \left(\frac{3}{16}\right) \cdot A + \left(\frac{3}{64}\right) \cdot A^2 \tag{4.6}$$

where A is determined using equation (4.7) (with some modifications in notation):

$$A = \left(\frac{2}{m}\right) \cdot \left(\frac{d3}{d2}\right)^2 \tag{4.7}$$

This estimate is also given by Pearson (1952) and Prescott (1971). However, this estimate for ν is highly inaccurate for small m (e.g., for m=1 and n less than 11, the inaccuracy is in the third place or less to the right of the decimal). As $m \rightarrow \infty$ for any n, the accuracy of the estimate for ν improves.

Consequently, the program presented by this chapter uses the second calculation for ν, which is exact. Two equations are involved. The first equation (equation (4.8)) is derived in Appendix B.1 of this dissertation from results given by David (1951) and Prescott (1971):

$$r = \frac{d3^2}{m \cdot d2^2} \tag{4.8}$$

The second equation (equation (4.9)) is derived in Appendix B.1 from results given by Prescott (1971):

$$h(x) = \frac{x \cdot e^{2 \cdot (\mathrm{gammln}(0.5 \cdot x) - \mathrm{gammln}(0.5 \cdot x + 0.5))} - 2}{2} \tag{4.9}$$

where gammln is a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that calculates the natural logarithm of the gamma function. Using gammln in equation (4.9) allows for large values of ν (hence large values for m and n) in the program. The exact value for ν is the value of x such that equation (4.10) holds:

$$h(x) = r \tag{4.10}$$

The Probability Integral of the Studentized Range

The probability integral of the studentized range for subgroups of size n sampled from a Normal population is given by Harter, Clemm, and Guthrie (1959) as equation (4.11a):

$$P3(z) = \left(\frac{5}{z}\right) \cdot e^{cv} \cdot (P1(z) + P2(z)) \qquad (4.11a)$$

where

$$cv = \ln(2) + \left(\frac{v}{2}\right) \cdot \ln\left(\frac{v}{2}\right) - \left(\frac{v}{2}\right) - \text{gammln}\left(\frac{v}{2}\right) \qquad (4.11b)$$

$$P1(z) = \int_0^{11} \left[\left(5 \cdot \frac{W}{z}\right) \cdot e^{\frac{z^2 - 25 \cdot W^2}{2 \cdot z^2}}\right]^{v-1} \cdot e^{\frac{z^2 - 25 \cdot W^2}{2 \cdot z^2}} \cdot P(W)\, dW \qquad (4.11c)$$

$$P2(z) = \left(\frac{z}{5}\right) \cdot \int_{\frac{55}{z}}^{\infty} \left(x \cdot e^{\frac{1-x^2}{2}}\right)^{v-1} \cdot e^{\frac{1-x^2}{2}}\, dx \qquad (4.11d)$$

The variable z is equal to $5 \cdot Q$. Q represents the studentized range w/s, where w is the range of a subgroup and s is an independent estimate (based on v degrees of freedom) of the population standard deviation. The equation for cv (equation (4.11b)) is the natural logarithm of the equation for $C(v)$ given by Harter, Clemm, and Guthrie (1959). It is derived in Appendix B.1. Using gammln in equation (4.11b) allows for large values of v (hence large values for m and n) in the program. The equation to calculate v is given earlier as equation (4.10). In equation (4.11c), P(W) is the probability integral of the range $W = (w/\sigma)$ (see equation (4.1)).

As $v \rightarrow \infty$ (i.e., as $m \rightarrow \infty$) for any n, the distribution of the studentized range $Q = (w/s)$

converges to the distribution of the range $W = (w/\sigma)$ (see Pearson and Hartley (1943)). This fact is used to calculate alpha-based conventional control chart constants for the R chart.

The Computer Program

This section of the chapter presents the computer program, which is in Appendix B.2 of this dissertation. The program has seven pages, each of which is further divided into sections.

Mathcad (1998) Note

It is possible for a section of code in the program to turn red and have the error message "Unknown Error". To correct this, delete one character in the red code and type it back in. Click the mouse arrow outside of the code. The code should turn black, indicating that the error has been eliminated. If not, repeat the procedure (it will eventually correct the problem).

Page 1

The first page of the program begins with the data entry section. The program requires the user to enter the following values: alphaMean (alpha for the $\overline{X}$ chart), alphaRangeUCL (alpha for the R chart above the UCL), alphaRangeLCL (alpha for the R chart below the LCL), m (number of subgroups), and n (subgroup size for the $(\overline{X}, R)$ charts). If no lower control limit on the R chart is desired, the entry for alphaRangeLCL should be left blank (do not enter zero). Before a value can be entered, the cursor must be moved to the right side of the appropriate equal sign. This may be done using the arrow keys on the keyboard or by moving the mouse arrow to the right side of the equal sign and clicking once with the left mouse button. The program is activated by paging down once the last entry is made. When using *Mathcad 8.03 Professional* (1998), paging down is not allowed while a calculation is taking place. However, *Mathcad 2000 Professional* (1999) allows the user to page down to the output section of the program (explained later) after the last entry is made.

The next part of page 1 is section 1.1 of the program. The value TOL is the tolerance. The calculations that use this value will be accurate to ten places to the right of the decimal. The functions dnorm(x, 0, 1) and pnorm(x, 0, 1) in *Mathcad* (1998) are the pdf and cdf, respectively, of the standard Normal distribution. The equations for the pdf and cdf are also given in case the dnorm or pnorm function fails to calculate a result. In *Mathcad* (1998), an equation turns red if it does not calculate a result due to an error. If the dnorm function gives an error, type f(x) on the left side of the equal sign in equation (4.12):

$$= \left[(2 \cdot \pi)^{-0.5}\right] \cdot e^{\frac{-x^2}{2}} \tag{4.12}$$

If the pnorm function gives an error, type F(x) on the left side of the equal sign in equation (4.13):

$$= \int_0^x f(t)\,dt \qquad (4.13)$$

W1, W2, and Var, which depend only on n, are given earlier as equations (4.2), (4.3), and (4.4), respectively. The value d2 is used to calculate the conventional control chart constant for the $\overline{X}$ chart. It is also used to calculate alpha-based conventional control chart constants for the R chart. Both d2 and d3 are used to calculate two stage short run control chart factors for the $\overline{X}$ chart as well as the R chart.

Page 2

Page 2 of the program begins with section 2.1. P(W) is given earlier as equation (4.1). The remainder of the code in this section determines wD4 and wD3, the (1-alphaRangeUCL) and alphaRangeLCL percentage points, respectively, of the distribution of the range $W = (w/\sigma)$ for a given n and infinite ν (i.e., infinite m) (recall the earlier statement that as $\nu \to \infty$ (i.e., as $m \to \infty$) for any n, the distribution of the studentized range $Q = (w/s)$ converges to the distribution of the range $W = (w/\sigma)$). The values wD4 and wD3 are used to calculate alpha-based conventional upper and lower control chart constants, respectively, for the R chart. The roots of the equations DUCL(W) and DLCL(W) are wD4 and wD3, respectively, and are determined using zbrent (a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that uses Brent's method to find the roots of an equation). The subprograms Wseed1 and Wseed2 generate seed values seedD4 and seedD3, respectively, for Brent's method.

The subprogram Wseed1 works as follows. Initially, W_0 and W_1 are set equal to 0.01

and 0.02, respectively. A_0 and A_1 result from evaluating DUCL(W) at W_0 and W_1, respectively. The while loop begins by checking if the product of A_0 and A_1 is negative. If so, the root for DUCL(W) lies between 0.01 and 0.02. If not, W_0 and W_1 are incremented by 0.01. A_0 and A_1 are recalculated and if their product is negative, the root for DUCL(W) lies between 0.02 and 0.03. Otherwise, the while loop repeats. Once a root for DUCL(W) is bracketed, the bracketing values are passed out of the subprogram into the 2×1 vector seedD4 to be used by Brent's method to determine wD4. The subprogram Wseed2 works similarly to construct the 2×1 vector seedD3 to be used by Brent's method to determine wD3, except the starting value is 0.001.

The next part of page 2 is section 2.2 of the program. The two stage short run control chart factor calculations require ν and νprevm (i.e., ν for (m-1) subgroups). The value rprevm has the same meaning as r (given earlier as equation (4.8)), except it is for (m-1) subgroups. The equation for h(x) is described earlier (see equation (4.9)). Brent's method is used to find the root ν of d(x) using the seed value x. Similarly, Brent's method is used to find the root νprevm of dprevm(x) using the seed value xprevm. The equations for x and xprevm are from the footnote to Table D3 in the appendix of Duncan (1974). Patnaik (1950) also gives a form for these equations.

Page 3

Page 3 of the program begins with section 3.1. P3(z), cν, P1(z), and P2(z) are all given earlier as equations (4.11a), (4.11b), (4.11c), and (4.11d), respectively. Section 3.2 contains the calculations required to determine qD4, the (1-alphaRangeUCL) percentage point of the distribution of the studentized range $Q = (w/s)$ with ν degrees of freedom (which is calculated earlier in the program). The value qD4 is used to calculate the second stage short run upper control chart factor for the R chart. The subprogram Zseed1 generates the seed value seed1 for Brent's method or for root (root is a numerical routine in *Mathcad* (1998) that uses the Secant method for determining the roots of an equation). Either root-finding method determines the root of D(x). The result of dividing this root by five is qD4. Both Brent's method and the Secant method are given because one may not work when the other one does. If Brent's method fails, type qD4 on the left side of the equal sign in equation (4.14):

$$= \frac{\mathrm{root}\left[\left|\mathrm{P3(seed1)} - (1 - \mathrm{alphaRangeUCL})\right|, \mathrm{seed1}\right]}{5} \tag{4.14}$$

The subprogram Zseed1 begins by generating values for Z_0 and Z_1. A_0 and A_1 result from evaluating P3(z) at Z_0 and Z_1, respectively. The while loop continually increments Z_0 and Z_1 by 5.0 and evaluates P3(z) at these two values until A_1 becomes greater than (1-alphaRangeUCL) for the first time, at which point A_0 will be less than (1-alphaRangeUCL). When this occurs, P3(z) is equal to (1-alphaRangeUCL) for some value z between Z_0 and Z_1. An initial guess for this value is determined using linterp (a numerical routine in *Mathcad* (1998) that performs linear interpolation) and stored in Zguess. The initial guess is passed out of the subprogram as seed1.

Page 4

Page 4 of the program is section 4.1. The code in this section is used to determine qD3, the alphaRangeLCL percentage point of the distribution of the studentized range $Q = (w/s)$ with ν degrees of freedom (which is calculated earlier in the program). The value qD3 is used to calculate the second stage short run lower control chart factor for the R chart. The subprogram Zseed2 generates the value seed2 that is used to determine an initial value for qD3. An improved value for qD3 is then calculated by determining the root of the equation (P3(z)-alphaRangeLCL) using the Secant method with the seed value seed2 and dividing this root by five.

For some values of n in combination with mostly large m, the Secant method fails to work (Brent's method should not be used). This is not a problem because the initial value for qD3 and the improved value match to several places to the right of the decimal. This phenomenon is discussed in more detail when the tabulated results of the program are presented later in this chapter. The Monitor Results area in the bottom right hand corner of section 4.1 indicates how closely the two values for qD3 match until the root routine fails. This will dictate the number of decimal places that can be used to display qD3 and the second stage short run lower control chart factor for the R chart.

The code in the subprogram Zseed2 that begins with the first line of code and includes the while loop and the two for loops constructs 21×1 vectors Zv for z and Av for P3(z). The first row of each vector is zero. The while loop determines the first value Z where P3(Z) is greater than alphaRangeLCL. This Z and the corresponding value P3(Z) are stored in the second rows of Zv and Av, respectively. The two for loops generate values for the remaining rows of Zv and Av. Two different for loops are used because P3(z) may encounter an error for some i (i: 1, 2, ..., 20). The value for i where the error occurs can be skipped using the dual for loop construction. When the execution of this section of code is complete, P3(z) is equal to alphaRangeLCL for some value z between Zv_0 and Zv_1.

The code in the subprogram Zseed2 that starts in the line where the variable Zguess first appears to the last line of the subprogram is derived from Harter, Clemm, and Guthrie (1959). This code searches for and estimates the value z where P3(z) is equal to alphaRangeLCL. Zguess is the initial guess for this value z. It is determined using linterp, the 21×1 vectors for P3(z) and z previously determined, and alphaRangeLCL. The 2×1 vector A is determined using ratint (a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that performs rational interpolation), the 21×1 vectors for z and P3(z), and Zguess. Aguess is the entry in the first row of A and is the estimated value for P3(Zguess). The while loop first checks if Aguess is an accurate estimate (within 10^{-15}) of alphaRangeLCL. If so, Zguess is passed out of the subprogram as the value seed2. If not, Aguess and Zguess are entered into the second rows of the previously determined vectors Av and Zv, respectively, if Aguess is more than 10^{-15} larger than alphaRangeLCL. If Aguess is more than 10^{-15} smaller than alphaRangeLCL, Aguess and Zguess are entered into the first rows of the vectors Av and Zv, respectively. New values for Zguess and Aguess are determined using the same procedure as before and execution is returned to the beginning of the while loop.

Page 5

Page 5 of the program contains sections 5.1 and 5.2. These sections correspond to sections 3.1 and 3.2, respectively, described earlier. The only difference is that the calculations in sections 5.1 and 5.2 use νprevm instead of ν. The calculations are for qD4prevm, which is used to determine the first stage short run upper control chart factor for the R chart.

Page 6

Page 6 of the program is section 6.1. This section corresponds to section 4.1 described earlier. The only difference is that the calculations in section 6.1 use νprevm instead of ν. The calculations are for qD3prevm, which is used to determine the first stage short run lower control chart factor for the R chart.

Page 7

Page 7 of the program begins with section 7.1. It has the equations for d2star (given earlier as equation (4.5)) and d2starprevm (d2star for (m-1) subgroups). The value d2star is used to calculate first and second stage short run control chart factors for the $\overline{X}$ chart. It is also used to calculate second stage short run control chart factors for the upper and lower control limits for the R chart. The value d2starprevm is used to calculate first stage short run control chart factors for the upper and lower control limits for the R chart. The function qt(adj_alpha, ν) in *Mathcad* (1998) determines the critical value crit_t for a cumulative area of adj_alpha under the Student's t curve with ν degrees of freedom. The value crit_t is used to calculate first and second stage short run control chart factors for the $\overline{X}$ chart. The function qnorm(adj_alpha, 0, 1) in *Mathcad* (1998) determines the critical value crit_z for a cumulative area of adj_alpha under the standard Normal curve. The value crit_z is used to calculate the conventional control chart constant for the $\overline{X}$ chart.

Section 7.2 of the program has the two stage short run control chart factor equations from Hillier (1969). A21 and A22 are, respectively, the first and second stage short run control chart factors for the $\overline{X}$ chart. D41 and D42 are, respectively, the first and second stage short run upper control chart factors for the R chart. D31 and D32 are, respectively, the first and second stage short run lower control chart factors for the R chart. Table 4.1 compares the notation for these factors from Hillier (1969), Pyzdek (1993), and this chapter (Yang (1995, 1999, 2000) uses the same notation as Pyzdek (1993)).

Section 7.2 also has the conventional control chart equations for A2 and alpha-based D4 and D3. A2 is the conventional control chart constant for the $\overline{X}$ chart. The equation for A2 is a generalization of the equation for A_2 from Table M in the appendix of Duncan (1974) to allow for different values of alphaMean. It is obtained by taking the limit of either A21 or A22 as m→∞ (i.e., as ν→∞) for any n. D4 is the conventional upper control chart constant for the R chart. It is obtained by taking the limit of either D41 as m→∞ (i.e., as vprevm→∞) or D42 as m→∞ (i.e., as ν→∞) for any n. D3 is the

Table 4.1. Comparison of Two Stage Short Run Control Chart Factor Notation

	A21	**D41**	**D31**	**A22**	**D42**	**D32**
Hillier (1969)	A_2^{**}	D_4^{**}	D_3^{**}	A_2^{*}	D_4^{*}	D_3^{*}
Pyzdek (1993)	A_{2F}	D_{4F}	-----	A_{2S}	D_{4S}	-----

conventional lower control chart constant for the R chart. It is obtained by taking the limit of either D31 as $m \to \infty$ (i.e., as $\nu prevm \to \infty$) or D32 as $m \to \infty$ (i.e., as $\nu \to \infty$) for any n.

The last part of page 7 is the output section of the program. The five values entered at the beginning of the program are given. The control chart factors are broken down into first stage, second stage, and conventional. The mean, standard deviation, and variance of the distribution of the range $W = (w/\sigma)$, Duncan's (1974) Table D3 results, and Harter, Clemm, and Guthrie's (1959) Table II.2 results complete the output of the program. To copy results into another software package (like Excel), follow the directions from *Mathcad's* (1998) help menu or highlight a value and copy and paste it into the other software package. When highlighting a value with the mouse arrow, place the arrow in the middle of the value, depress the left mouse button, and drag the arrow to the right. This will ensure just the numerical value of the result is copied and pasted.

Tabulated Results of the Program

The four tables (Tables B.3.1-B.3.4) in Appendix B.3 of this dissertation were generated using the program with the following input values:

- alphaMean=0.0027, alphaRangeUCL=0.005, alphaRangeLCL=0.001
- m: 1-20, 25, 30, 50, 75, 100, 150, 200, 250, 300
- n: 2-8, 10, 25, 50

The values ν, d2star, νprevm, d2starprevm, d2, d3, and $d3^2$ (Var.) are in Table B.3.1.

The results in this table compare favorably to Duncan's (1974) Table D3. If the values in Table B.3.1 are rounded as in Duncan's Table D3, some values differ from those in Duncan's Table D3 by one digit in the last decimal place. A possible explanation is that the Table B.3.1 calculations were performed with more places to the right of the decimal and with ν determined exactly. Nelson (1975) uses the exact calculation for ν (referenced from Pearson (1952)) for some combinations of subgroup size and number of subgroups in his re-creation of Duncan's (1958) Table 548 (a separate publication equivalent to Duncan's (1974) Table D3). Nelson also encountered differences between his results and Duncan's (1958) similar to the differences found here. It should be noted that the program eliminates the need for the estimations for ν and d_2^* given by Duncan (1974) in the footnote to his Table D3.

The values qD4, qD4prevm, and wD4 are in Table B.3.2. The values qD3, qD3prevm, and wD3 are in Table B.3.3. The results in these tables compare favorably to Harter, Clemm, and Guthrie's (1959) Table II.2. The blanks in Table B.3.3 indicate where Zseed2 was not able to generate an initial value for qD3. This problem may be attributable to the low value used for alphaRangeLCL (0.001).

As explained earlier in this chapter, in the calculations for qD3 and qD3prevm, the Secant method fails to work for some values of n in combination with mostly large m. For Table B.3.3, this is true for n=2 (m≥2), n=3 (m≥50), n=5 (m≥150), n=6 (m=250), n=7 (m=200), n=10 (m=200), and n=25 (m≥150). This problem may also be attributable to the low value used for alphaRangeLCL. As mentioned previously, this is not a serious issue, especially for n less than seven. For these values of n, the initial value for qD3 matches the improved value for qD3 (before the Secant method fails) to at least six places to the right of the decimal. For n=7 and n=10, the match is five places to the right of the decimal. This is why the values for m=200 when n=7 and n=10 are displayed with four places to the right of the decimal in Table B.3.3. For n=25, the match is four places to the right of the decimal. Consequently, the values for m≥150 when n=25 are displayed with three places to the right of the decimal in Table B.3.3.

The entry for n=50 and m=300 in Table B.3.3 is blank because the initial value for qD3 was incorrect. The Secant method also failed to work. Again, this is probably attributable to the low value for alphaRangeLCL. This brings up the important point that the results from the program should converge smoothly to their respective infinite values. If not, the program may have performed an incorrect calculation.

Values for A21, D41, D31, A22, D42, D32, A2, D4, and D3 are in Table B.3.4. Results from Table B.3.4 for n=5 compare favorably to Hillier's (1969) results. Any differences may be attributable to Hillier using ν and d_2^* from Duncan's (1974) Table D3, which shows fewer places to the right of the decimal than the results used in the program. The blanks in Table B.3.4 are where Zseed2 and Zseed4 were not able to generate initial values for qD3 and qD3prevm, respectively. D31 and D32 for m=200 when both n=7 and n=10 are displayed to four places to the right of the decimal for reasons previously explained. Similarly, D31 and D32 for n=25 and m≥150 are displayed to three places to the right of the decimal. It should be noted that the values wD4, wD3, and D4 and D3 in Tables B.3.2, B.3.3, and B.3.4, respectively, may differ in the ninth or tenth decimal place for different root routines used to calculate wD4 and wD3.

These favorable comparisons validate the program. Consequently, Table B.3.4 results for n: 2-5 and m: 1-10, 15, 20, 25 may be considered corrections to Pyzdek's (1993)

Table 1. Table 4.2 illustrates a smaller magnitude correction and a larger magnitude correction to Pyzdek's Table 1.

Also, results in Tables B.3.1 and B.3.4 for n: 1-8, 10, 25 and m: 1-20, 25 may be considered corrections to Yang's (1995) Tables 2.1, 2.4, and 2.7. Results in Yang's (1995) Table 2.1 for V (i.e., ν) are inaccurate regardless of the values for m and n. However, for many values of n, the inaccuracies of the results in Yang's (1995) Tables 2.1, 2.4, and 2.7 for C (i.e., d_2^*), A_{2F} (i.e., A21), and A_{2S} (i.e., A22), respectively, decrease as $m \to \infty$.

Yang's (1995) results are inaccurate for several reasons. Yang (1995) uses equations that give estimates for ν and d_2^*. Additionally, Yang's (1995) equation for the cdf of the standard Normal distribution gives estimated results. Also, the numerical techniques used by Yang (1995) do not give accurate results.

It should be noted that Tables 2.2-2.4 in Yang (1995) incorrectly show zeroes as the value of A_{2F} (i.e., A21) when m=1. A21 does not exist when m=1. This does not mean the same thing as having a value of zero. Also, Yang (1999 and 2000) incorrectly states that Pyzdek (1993) uses an alpha value of 0.0027 for both the $\overline{X}$ control chart and the R control chart above the upper control limit. Pyzdek (1993) uses an alpha value of 0.005 for the R control chart above the upper control limit.

Table 4.2. Examples of Corrections to Pyzdek's (1993) Table 1

	n	m	Factor	Table B.3.4	Pyzdek
Smaller Magnitude Correction	2	2	A21	8.27583	8.49
Larger Magnitude Correction	4	1	D42	7.13456	13

Implications of the Tabulated Results

Values in Table B.3.4 show some interesting properties. Consider Table 4.3, which contains selected A22 and corresponding A2 values from Table B.3.4. As n increases for a particular m, the A22 values decrease. For larger values of m, the difference between A22 for n=2 and n=50 decreases. Of more interest is that as m increases for a particular n, the A22 values converge in a decreasing manner to their respective A2 values. For larger values of n, the difference between A22 for m=1 and the respective A2 value decreases. This means that as m increases the convergence of A22 to A2 is faster for larger values of n. These results make sense because more information about the process is at hand for larger n and m.

Further investigation of Table B.3.4 reveals that, as m increases for a particular n, the D31 and D42 values also converge to their respective D3 and D4 values in a decreasing manner. The convergence pattern for D41 and D32 differs in that as m increases for a particular n, the D41 and D32 values converge in an increasing manner to their respective D4 and D3 values. The convergence pattern for A21 is unique. For n equal to 2, 3, and 4, A21 converges in a decreasing manner to A2 as m increases. For n=5, A21 also

Table 4.3. Selected A22 and Corresponding A2 Values from Table B.3.4

	A22						**A2**
n	**m=1**	**m=2**	**m=20**	**m=30**	**m=100**	**m=300**	**m=∞**
2	166.72424	14.33417	2.20516	2.08810	1.93901	1.89934	1.87996
3	8.35221	2.70257	1.11739	1.08487	1.04132	1.02927	1.02332
4	3.01070	1.43980	0.77844	0.76144	0.73829	0.73181	0.72859
5	1.76214	1.00199	0.60994	0.59872	0.58331	0.57897	0.57681
10	0.61168	0.44314	0.32071	0.31654	0.31074	0.30909	0.30826
25	0.25204	0.20157	0.15757	0.15593	0.15363	0.15297	0.15265
50	0.14716	0.12122	0.09711	0.09618	0.09488	0.09451	0.09432

converges in a decreasing manner to A2, but starting at m=3. For n equal to 6, 7, 8, 10, 25, and 50, A21 converges in an increasing manner to A2 as m increases.

These results have major implications. A common rule of thumb is that 20 to 30 subgroups of size 4 or 5 are necessary to use conventional control chart constants for constructing control limits. The results in Table B.3.4 indicate that this may be an incorrect rule. Consider again the A22 and corresponding A2 values in Table 4.3. When n=4, A2 is 6.404% smaller than A22 for m=20. When n=5, A2 is 3.659% smaller than A22 for m=30. These results indicate that if one were to construct $\overline{X}$ charts using conventional control chart constants when only 20 to 30 subgroups of size 4 or 5 are available to estimate the process mean and standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

D42 and corresponding D4 values, as well as D32 and corresponding D3 values, in Table B.3.4 also indicate that the common rule of thumb may be an incorrect rule. When n=4, D4 is 4.748% smaller than D42 for m=20 and D3 is 0.896% larger than D32 for m=20. When n=5, D4 is 2.581% smaller than D42 for m=30 and D3 is 0.663% larger than D32 for m=30. Consequently, if one were to construct R charts using conventional control chart constants when only 20 to 30 subgroups of size 4 or 5 are available to estimate the process standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

Quesenberry (1993) also investigated the validity of the common rule of thumb and concluded that $400/(n-1)$ subgroups are needed for the $\overline{X}$ chart before conventional control chart constants may be used. However, for all practical purposes, the program presented by this chapter eliminates the need for these rules.

A Numerical Example

Consider the data in Table 4.4 obtained from a process requiring short run control charting techniques (assume alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001). For m=5 and n=4, the following first stage short run control chart factors are obtained from Table B.3.4: A21=0.77660, D41=2.11840, and D31=0.11338. UCL(R), LCL(R), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(R) = D41 \cdot \overline{R} = 2.11840 \cdot 0.21600 = 0.45757$$

$$LCL(R) = D31 \cdot \overline{R} = 0.11338 \cdot 0.21600 = 0.02449$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A21 \cdot \overline{R} = 1.28600 + 0.77660 \cdot 0.21600 = 1.45375$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A21 \cdot \overline{R} = 1.28600 - 0.77660 \cdot 0.21600 = 1.11825$$

R for subgroup five (R=0.49000) is above UCL(R). Find, investigate, and remove from the process the special cause (or causes) that created this out of control point, delete subgroup five, recalculate averages (shown as the Revised Averages in Table 4.4), and

Table 4.4. A Numerical Example

Subgroup	$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{X_3}$	$\mathbf{X_4}$	$\overline{\mathbf{X}}$	**R**
1	1.17	1.14	1.20	1.18	1.17250	0.06000
2	1.38	1.29	1.36	1.44	1.36750	0.15000
3	1.20	1.21	1.30	1.14	1.21250	0.16000
4	1.40	1.40	1.21	1.43	1.36000	0.22000
5	1.12	1.20	1.61	1.34	1.31750	0.49000
	Averages				1.28600	0.21600
	Revised Averages				1.27813	0.14750

reconstruct first stage control limits (this approach is from Hillier's (1969) example). For m=4 and n=4, the following first stage short run control chart factors are obtained from Table B.3.4: A21=0.78832, D41=2.07041, and D31=0.11848. Revised UCL(R), LCL(R), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(R) = D41 \cdot \overline{R} = 2.07041 \cdot 0.14750 = 0.30539$$

$$LCL(R) = D31 \cdot \overline{R} = 0.11848 \cdot 0.14750 = 0.01748$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A21 \cdot \overline{R} = 1.27813 + 0.78832 \cdot 0.14750 = 1.39441$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A21 \cdot \overline{R} = 1.27813 - 0.78832 \cdot 0.14750 = 1.16185$$

Since none of the remaining values plot out of control (i.e., control has been established), the next step is to construct second stage control limits using the following second stage short run control chart factors from Table B.3.4 (for m=4 and n=4): A22=1.01772, D42=2.94060, and D32=0.09281. UCL(R), LCL(R), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(R) = D42 \cdot \overline{R} = 2.94060 \cdot 0.14750 = 0.43374$$

$$LCL(R) = D32 \cdot \overline{R} = 0.09281 \cdot 0.14750 = 0.01369$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A22 \cdot \overline{R} = 1.27813 + 1.01772 \cdot 0.14750 = 1.42824$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A22 \cdot \overline{R} = 1.27813 - 1.01772 \cdot 0.14750 = 1.12802$$

These control limits may be used to monitor the future performance of the process.

Conclusions

This chapter and the program it presents make important contributions to both industry and research. Those involved with quality control in industry will, for the first time, be able to use theoretically precise control chart factors to determine control limits for $(\overline{X}, R)$ charts regardless of the subgroup size, number of subgroups, and alpha values. This flexibility is valuable in that process monitoring will no longer have to be adjusted to use the limited, and in some cases incorrect, results previously available in the literature. Concerning research, this chapter provides a valuable reference for anyone interested in anything having to do with $(\overline{X}, R)$ control charts. Also, as already mentioned, the program eliminates the need for the research question of how many subgroups are enough before conventional control chart constants may be used.

CHAPTER V

TWO STAGE SHORT RUN $(\overline{X}, v)$ AND $(\overline{X}, \sqrt{v})$ CONTROL CHARTS AND A COMPUTER PROGRAM TO CALCULATE THE FACTORS

Introduction

Yang and Hillier (1970) follow Hillier's (1969) theory to derive equations to calculate two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts. The tables presented by Yang and Hillier (1970) are for several values for number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the v and $\sqrt{v}$ charts both above the upper control limit and below the lower control limit (alpha is the probability of a Type I error). However, as in Hillier's 1969 paper, the results are for subgroup size five only.

Problem

Yang and Hillier (1970) represent the only attempt in the literature to present two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts based on Hillier's (1969) theory. In addition to the limitations already presented, Yang and Hillier (1970) neglect to include appropriate bias correction factor calculations in some of their two stage short run control chart factor equations, rendering much of their tables as incorrect. Also, some of the results that were calculated using the correct equations are inaccurate in the last decimal place shown by one and in some cases two digits.

Solution

This chapter describes the development and execution of a computer program that overcomes these limitations. It will accurately calculate first and second stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts using the appropriate bias correction factor calculations. The program uses exact equations for the distributions of the variance and the studentized variance, degrees of freedom calculations, short run calculations (which are corrected for bias), and conventional control chart calculations. The program accepts values for subgroup size, number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the v or $\sqrt{v}$ chart both above the upper control limit and below the lower control limit. Tables are generated for specific values of these inputs. Comparison of the tabulated results to legitimate results in the literature validates the program. The tables correct and extend previous results in the literature.

The software used for the program is *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997). The program uses numerical routines provided by the software.

Outline

This chapter first presents the distributions of the variance and the studentized variance. These are essential in the application of Hillier's (1969) theory to $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts and are required for the program to perform accurate calculations. Next, the equation to calculate the bias correction factors is presented, as well as justification for its use. From this, corrected equations to calculate two stage short run control chart factors for $(\bar{X}, v)$ and $(\bar{X}, \sqrt{v})$ charts are given. Next, the computer program is described. Tables generated by the program are then presented and compared with legitimate results in the literature. Also, implications of the tabulated results are discussed. Following a numerical example that illustrates the use of the program, final conclusions describing the impact of the program on industry and research are given.

Note

Results from the program are for processes generating parts with independent measurements that follow a Normal distribution.

The Distribution of the Variance

The distribution of the variance for subgroups of size n sampled from a Normal population with mean μ and variance σ^2 is given by Pearson and Hartley (1962) as equation (5.1a) (with some modifications in notation):

$$p(v) = \left(\frac{v1}{2}\right)^{\frac{v1}{2}} \cdot \left(\Gamma\left(\frac{v1}{2}\right)\right)^{-1} \cdot \sigma^{-v1} \cdot v^{\frac{v1}{2}-1} \cdot e^{\frac{-v1 \cdot v}{2 \cdot \sigma^2}} \qquad (5.1a)$$

The value v (the variance) is an independent estimate of σ^2 based on $v1 = (n-1)$ degrees of freedom. Equation (5.1a) may also be represented as equation (5.1b) (see Appendix C.1 of this dissertation):

$$p(v) = \left(\frac{1}{\sigma^{v1}}\right) \cdot \left[e^{\left(\frac{v1}{2}\right) \cdot \ln\left(\frac{v1}{2}\right) - \text{gammln}\left(\frac{v1}{2}\right) + \left(\frac{v1}{2} - 1\right) \cdot \ln(v) - \frac{v1 \cdot v}{2 \cdot \sigma^2}} \right] \tag{5.1b}$$

Equation (5.1b) is the form used in the program. The function gammln is a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that calculates the natural logarithm of the gamma function. Using gammln in equation (5.1b) allows for large values of ν1 (hence large values for n) in the program. The cumulative distribution function (cdf) of the variance v with ν1 degrees of freedom is equation (5.2):

$$P(V) = \int_0^V p(v)\, dv \tag{5.2}$$

The program uses equation (5.2) (with σ=1.0) to determine alpha-based conventional control chart constants for the v and $\sqrt{v}$ charts.

The Distribution of the Studentized Variance

The distribution of the studentized variance (i.e., the F distribution) for subgroups of size n sampled from a Normal population with mean μ and variance σ^2 is given by Bain and Engelhardt (1992) as equation (5.3a) (with some modifications in notation):

$$p3(f) = \frac{\Gamma\left(\frac{v1+v2}{2}\right)}{\Gamma\left(\frac{v1}{2}\right) \cdot \Gamma\left(\frac{v2}{2}\right)} \cdot \left(\frac{v1}{v2}\right)^{\frac{v1}{2}} \cdot f^{\frac{v1}{2}-1} \cdot \left(1 + \frac{v1}{v2} \cdot f\right)^{-\frac{v1+v2}{2}} \tag{5.3a}$$

The value f (the studentized variance) is equal to v/v', where v' is a second independent estimate of σ^2 based on $\nu 2 = m \cdot (n-1)$ degrees of freedom (m is the number of subgroups). Equation (5.3a) may also be represented as equation (5.3b) (see Appendix C.1):

$$p3(f) = e^{p1+p2(f)} \tag{5.3b}$$

where

$$p1 = \text{gammln}\left(\frac{\nu 1+\nu 2}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right) - \text{gammln}\left(\frac{\nu 2}{2}\right) \tag{5.3c}$$

$$p2(f) = \left(\frac{\nu 1}{2}\right) \cdot (\ln(\nu 1) - \ln(\nu 2)) + \left(\frac{\nu 1}{2} - 1\right) \cdot \ln(f) - \left(\frac{\nu 1+\nu 2}{2}\right) \cdot \ln\left(1 + \frac{\nu 1}{\nu 2} \cdot f\right) \tag{5.3d}$$

Equations (5.3b)-(5.3d) are used in the program. Using gammln in equation (5.3c) allows for large values of ν1 (hence large values for n) and large values of ν2 (hence large values for m and n) in the program. The cdf of the studentized variance $f = (v/v')$ with ν1 degrees of freedom for v and ν2 degrees of freedom for v' is equation (5.4):

$$P3(F) = \int_0^F p3(f)\,df \tag{5.4}$$

The program uses equation (5.4) to determine two stage short run control chart factors for the v and $\sqrt{v}$ charts.

As $\nu 2 \to \infty$ (i.e., as $m \to \infty$) for any n, the distribution of the studentized variance $f = (v/v')$ converges to the distribution of the variance v (when σ=1.0). This fact is used to calculate alpha-based conventional control chart constants for the v and $\sqrt{v}$ charts.

The Equation to Calculate the Bias Correction Factors

As mentioned earlier in the Problem subsection, Yang and Hillier (1970) neglect to include appropriate bias correction factor calculations in some of their two stage short run control chart factor equations. The equations that involve $\bar{v}$ are correct ($\bar{v}$ is the average of m values of v, each of which is based on a subgroup of size n), since $\bar{v}$ is an unbiased estimate of σ^2 (see Appendix C.1). The problem occurs in those equations that involve $\sqrt{\bar{v}}$, which is a biased estimate of σ. This bias is revealed when one considers the fact that $\sqrt{\bar{v}} = s_p$, where s_p is the pooled standard deviation (this equivalency is shown in Appendix C.1). King (1953), Burr (1969), Nelson (1990), and Wheeler (1995) all state that s_p is a biased estimate of σ, and that this bias is corrected by dividing s_p by c_4, where c_4 is calculated using equation (5.5a) from Mead (1966) (with σ=1.0):

$$c_4 = \sigma \cdot \left(\frac{2}{\nu 2}\right)^{0.5} \cdot \frac{\Gamma\left(\frac{\nu 2+1}{2}\right)}{\Gamma\left(\frac{\nu 2}{2}\right)} \tag{5.5a}$$

Wheeler (1995) also gives this equation as his c_4' (with σ=1.0). The control chart constant c_4 is the mean of the distribution of the standard deviation. The equation for ν2 is given earlier in relation to equation (5.3a). Equation (5.5a) may also be represented as equation (5.5b) (see Appendix C.1) (note: $c4 \equiv c_4$):

$$c4(x) = \sigma \cdot \left(\frac{2}{x-1}\right)^{0.5} \cdot \left(e^{\operatorname{gammln}\left(\frac{x}{2}\right) - \operatorname{gammln}\left(\frac{x-1}{2}\right)}\right) \tag{5.5b}$$

where x is the appropriate value for subgroup size (in the case of $\sqrt{v}$, $x = (\nu 2 + 1)$). Equation (5.5b) is the form used in the program. Using gammln in equation (5.5b) allows for large values of ν2 (hence large values for m and n) in the program.

Corrected Two Stage Short Run Control Chart Factor Equations

Since $\sqrt{v}/c4(\nu 2 + 1)$ is an unbiased estimate of σ, six of Yang and Hillier's (1970) equations to calculate two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts require correcting. The first one is the equation for A_4^*, the second stage short run control chart factor for the $\overline{X}$ chart. Yang and Hillier (1970) calculate second stage short run upper and lower control limits for the $\overline{X}$ chart using equations (5.6) and (5.7), respectively:

$$UCL = \overline{\overline{X}} + A_4^* \cdot \sqrt{v} \tag{5.6}$$

$$LCL = \overline{\overline{X}} - A_4^* \cdot \sqrt{v} \tag{5.7}$$

Consequently, the bias correction factor calculated using equation (5.5b) with $x = (\nu 2 + 1)$ should be incorporated into the equation for A_4^*. The result is given as equation (5.8) (note: $A42 \equiv A_4^*$):

$$A42 = \left(\frac{crit\_t}{c4(\nu 2+1)}\right) \cdot \left(\frac{m+1}{n \cdot m}\right)^{0.5} \tag{5.8}$$

where crit_t is the critical value for a cumulative area of $(1-(alphaMean/2))$ under the Student's t curve with ν2 degrees of freedom (alphaMean is the probability of a Type I error on the $\overline{X}$ control chart). Similarly, the correct equation for A_4^{**}, the first stage short run control chart factor for the $\overline{X}$ chart, is given as equation (5.9) (note: $A41 \equiv A_4^{**}$):

$$A41 = \left(\frac{crit\_t}{c4(\nu 2+1)}\right) \cdot \left(\frac{m-1}{n \cdot m}\right)^{0.5} \tag{5.9}$$

The value crit_t has the same meaning here as in equation (5.8).

The next two equations that require correcting are for $\sqrt{B_8^*}$ and $\sqrt{B_7^*}$, the second stage short run upper and lower control chart factors, respectively, for the $\sqrt{v}$ chart. Yang and Hillier (1970) calculate second stage short run upper and lower control limits for the $\sqrt{v}$ chart using equations (5.10) and (5.11), respectively:

$$UCL = \sqrt{B_8^*} \cdot \sqrt{v} \tag{5.10}$$

$$LCL = \sqrt{B_7^*} \cdot \sqrt{v} \tag{5.11}$$

Consequently, the bias correction factor calculated using equation (5.5b) with $x = (\nu 2 + 1)$ should be incorporated into the equations for the control chart factors used in equations (5.10) and (5.11). The results are given as equations (5.12) and (5.13), respectively (note: $B82^{0.5} \equiv \sqrt{B_8^*}$ and $B72^{0.5} \equiv \sqrt{B_7^*}$):

$$B82sqrt = \frac{B82^{0.5}}{c4(\nu 2 + 1)} \tag{5.12}$$

$$B72sqrt = \frac{B72^{0.5}}{c4(\nu 2 + 1)} \tag{5.13}$$

B82sqrt replaces $\sqrt{B_8^*}$ in equation (5.10) and B72sqrt replaces $\sqrt{B_7^*}$ in equation (5.11). B82 is the second stage short run upper control chart factor for the v chart. It is equal to fB8, the (1-alphaVarUCL) percentage point of the distribution of the studentized variance $f = (v/v')$ with $\nu 1$ degrees of freedom for v and $\nu 2$ degrees of freedom for v' (alphaVarUCL is the probability of a Type I error on the v and $\sqrt{v}$ charts above the upper control limit). B72 is the second stage short run lower control chart factor for the v chart. It is equal to fB7, the alphaVarLCL percentage point of the distribution of the studentized variance $f = (v/v')$ with $\nu 1$ degrees of freedom for v and $\nu 2$ degrees of freedom for v' (alphaVarLCL is the probability of a Type I error on the v and $\sqrt{v}$ charts below the lower control limit).

Similarly, the correct equations for the first stage short run upper and lower control chart factors for the $\sqrt{v}$ chart are given as equations (5.14) and (5.15), respectively (note: $B81^{0.5} \equiv \sqrt{B_8^{**}}$ and $B71^{0.5} \equiv \sqrt{B_7^{**}}$):

$$B81sqrt = \frac{B81^{0.5}}{c4(v2prevm+1)} \tag{5.14}$$

$$B71sqrt = \frac{B71^{0.5}}{c4(v2prevm+1)} \tag{5.15}$$

B81sqrt and B71sqrt replace $\sqrt{B_8^{**}}$ and $\sqrt{B_7^{**}}$, respectively. The value v2prevm has the same meaning as v2, except it is for (m-1) subgroups (i.e., $v2prevm = (m-1)\cdot(n-1)$). B81, the first stage short run upper control chart factor for the v chart, is calculated using equation (5.16):

$$B81 = \frac{m \cdot fB8prevm}{m-1+fB8prevm} \tag{5.16}$$

The value fB8prevm is the (1-alphaVarUCL) percentage point of the distribution of the studentized variance $f = (v/v')$ with v1 degrees of freedom for v and v2prevm degrees of freedom for v'. B71, the first stage short run lower control chart factor for the v chart, is calculated using equation (5.17):

$$B71 = \frac{m \cdot fB7prevm}{m - 1 + fB7prevm} \tag{5.17}$$

The value fB7prevm is the alphaVarLCL percentage point of the distribution of the studentized variance $f = (v/v')$ with ν1 degrees of freedom for v and ν2prevm degrees of freedom for v'.

Since c4(x)→1.0 as x→∞ (i.e., as m→∞) for any n, Yang and Hillier's (1970) results for infinite m are calculated using the correct equations. The equation for A4, the conventional control chart constant for the $\overline{X}$ chart, may be obtained by taking the limit of either A41 or A42 as m→∞ (i.e., as ν2→∞) for any n. The resulting equation for A4 is given as equation (5.18):

$$A4 = \frac{crit\_z}{n^{0.5}} \tag{5.18}$$

The value crit_z is the critical value for a cumulative area of $(1 - (alphaMean/2))$ under the standard Normal curve.

The equation for B8, the alpha-based conventional upper control chart constant for the v chart, may be obtained by taking the limit of either B81 as m→∞ (i.e., as ν2prevm→∞) or B82 as m→∞ (i.e., as ν2→∞) for any n. The resulting equation for B8 is given as equation (5.19):

$$B8 = vB8 \tag{5.19}$$

The value vB8 is the (1-alphaVarUCL) percentage point of the distribution of the variance v with ν1 degrees of freedom.

The equation for B7, the alpha-based conventional lower control chart constant for the v chart, may be obtained by taking the limit of either B71 as $m \to \infty$ (i.e., as ν2prevm$\to\infty$) or B72 as $m \to \infty$ (i.e., as ν2$\to\infty$) for any n. The resulting equation for B7 is given as equation (5.20):

$$B7 = vB7 \tag{5.20}$$

The value vB7 is the alphaVarLCL percentage point of the distribution of the variance v with ν1 degrees of freedom.

The equation for B8sqrt, the alpha-based conventional upper control chart constant for the $\sqrt{v}$ chart, may be obtained by taking the limit of either B81sqrt as $m \to \infty$ (i.e., as ν2prevm$\to\infty$) or B82sqrt as $m \to \infty$ (i.e., as ν2$\to\infty$) for any n. The resulting equation for B8sqrt is given as equation (5.21):

$$B8sqrt = B8^{0.5} \tag{5.21}$$

The equation for B7sqrt, the alpha-based conventional lower control chart constant for the $\sqrt{v}$ chart, may be obtained by taking the limit of either B71sqrt as $m \to \infty$ (i.e., as ν2prevm$\to\infty$) or B72sqrt as $m \to \infty$ (i.e., as ν2$\to\infty$) for any n. The resulting equation for B7sqrt is given as equation (5.22):

$$B7sqrt = B7^{0.5} \tag{5.22}$$

The Computer Program

This section of the chapter presents the computer program, which is in Appendix C.2 of this dissertation. The program has seven pages, each of which is further divided into sections.

Mathcad (1998) Note

It is possible for a section of code in the program to turn red and have the error message "Unknown Error". To correct this, delete one character in the red code and type it back in. Click the mouse arrow outside of the code. The code should turn black, indicating that the error has been eliminated. If not, repeat the procedure (it will eventually correct the problem).

Page 1

The first page of the program begins with the data entry section. The program requires the user to enter the following values: alphaMean (alpha for the $\overline{X}$ chart), alphaVarUCL (alpha for the v or $\sqrt{v}$ chart above the UCL), alphaVarLCL (alpha for the v or $\sqrt{v}$ chart below the LCL), m (number of subgroups), and n (subgroup size for the $(\overline{X}, v)$ or $(\overline{X}, \sqrt{v})$ charts). If no lower control limit on the v or $\sqrt{v}$ chart is desired, the entry for alphaVarLCL should be left blank (do not enter zero). Before a value can be entered, the cursor must be moved to the right side of the appropriate equal sign. This may be done using the arrow keys on the keyboard or by moving the mouse arrow to the right side of the equal sign and clicking once with the left mouse button. The program is activated by paging down once the last entry is made. When using *Mathcad 8.03 Professional* (1998), paging down is not allowed while a calculation is taking place. However, *Mathcad 2000 Professional* (1999) allows the user to page down to the output section of the program (explained later) after the last entry is made.

The next part of page 1 is section 1.1 of the program. The value TOL is the tolerance. The calculations that use this value will be accurate to twelve places to the right of the decimal. The population standard deviation σ is set equal to one for two reasons. The first is to achieve the convergence of the distribution of the studentized variance $f = (v/v')$ with ν1 degrees of freedom for v and ν2 degrees of freedom for v' to the distribution of the variance v with ν1 degrees of freedom as $\nu 2 \rightarrow \infty$ (i.e., as $m \rightarrow \infty$) for any n. The second is to have the appropriate calculation for the bias correction factors. As mentioned earlier in relation to equation (5.1a), the degrees of freedom ν1 for the variance v is equal to $(n-1)$. The equation for c4(x) is given earlier as equation (5.5b).

Page 2

Page 2 of the program begins with section 2.1. The equations for p(v) and P(V) are given earlier as equations (5.1b) and (5.2), respectively. The next part of page 2 is section 2.2 of the program. The code in this section determines vB8 and vB7, the (1-alphaVarUCL) and alphaVarLCL percentage points, respectively, of the distribution of the variance v with $\nu 1$ degrees of freedom and infinite $\nu 2$ (i.e., infinite m) (recall the earlier statement that as $\nu 2 \rightarrow \infty$ (i.e., as $m \rightarrow \infty$) for any n, the distribution of the studentized variance $f = (v/v')$ converges to the distribution of the variance v (when σ=1.0)). As shown earlier in equations (5.19) and (5.20), vB8 is equal to B8 and vB7 is equal to B7, respectively. The roots of the equations DUCL(V) and DLCL(V) are vB8 and vB7, respectively, and are determined using zbrent (a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that uses Brent's method to find the roots of an equation). The subprograms Vseed1 and Vseed2 generate seed values seedB8 and seedB7, respectively, for Brent's method.

The subprogram Vseed1 works as follows. Initially, V_0 and V_1 are set equal to 0.01 and 0.02, respectively. A_0 and A_1 result from evaluating DUCL(V) at V_0 and V_1, respectively. The while loop begins by checking if the product of A_0 and A_1 is negative. If so, the root for DUCL(V) lies between 0.01 and 0.02. If not, V_0 and V_1 are incremented by 0.01. A_0 and A_1 are recalculated and if their product is negative, the root for DUCL(V) lies between 0.02 and 0.03. Otherwise, the while loop repeats. Once a root for DUCL(V) is bracketed, the bracketing values are passed out of the subprogram into the 2×1 vector seedB8 to be used by Brent's method to determine vB8. The subprogram Vseed2 works similarly to construct the 2×1 vector seedB7 to be used by Brent's method to determine vB7, except the starting value is 0.000001.

The last part of page 2 is section 2.3 of the program. As shown earlier, the two stage short run control chart factor calculations require $\nu 2$ and ν2prevm. The equation for $\nu 2$ is given earlier in relation to equation (5.3a). The equation for ν2prevm is given earlier in relation to equations (5.14) and (5.15).

Page 3

Page 3 of the program begins with section 3.1. The equations for p3(f), p1, p2(f), and P3(F) are given earlier as equations (5.3b), (5.3c), (5.3d), and (5.4), respectively. Section 3.2 contains the calculations required to determine fB8, the (1-alphaVarUCL) percentage point of the distribution of the studentized variance $f = (v/v')$ with ν1 degrees of freedom for v and ν2 degrees of freedom for v' (both ν1 and ν2 are calculated earlier in the program). As explained earlier in relation to equation (5.12), fB8 is equal to B82. The subprogram Fseed1 generates the seed value seed1 for Brent's method or for root (root is a numerical routine in *Mathcad* (1998) that uses the Secant method to determine the roots of an equation). Either root-finding method determines the root fB8 of D1(x). Both Brent's method and the Secant method are given because one may not work when the other one does. If Brent's method fails (which is signified in *Mathcad* (1998) by the code turning red), type fB8 on the left side of the equal sign in equation (5.23):

$$= \mathrm{root}\left[\left|\mathrm{P3(seed1)} - (1 - \mathrm{alphaVarUCL})\right|, \mathrm{seed1}\right] \tag{5.23}$$

The subprogram Fseed1 begins by generating values for F_0 and F_1. A_0 and A_1 result from evaluating P3(F) at F_0 and F_1, respectively. The while loop continually increments F_0 and F_1 by delta1 and evaluates P3(F) at these two values until A_1 becomes greater than (1-alphaVarUCL) for the first time, at which point A_0 will be less than

(1-alphaVarUCL). When this occurs, P3(F) is equal to (1-alphaVarUCL) for some value F between F_0 and F_1. An initial guess for this value is determined using linterp (a numerical routine in *Mathcad* (1998) that performs linear interpolation) and stored in Fguess. The initial guess is passed out of the subprogram as seed1.

Page 4

Page 4 of the program is section 4.1. The code in this section is used to determine fB7, the alphaVarLCL percentage point of the distribution of the studentized variance $f = (v/v')$ with $\nu 1$ degrees of freedom for v and $\nu 2$ degrees of freedom for v' (both $\nu 1$ and $\nu 2$ are calculated earlier in the program). As explained earlier in relation to equation (5.13), fB7 is equal to B72. The subprogram Fseed2 generates the seed value seed2 for Brent's method or for root. Either root-finding method determines the root fB7 of D2(x). Both Brent's method and the Secant method are given because one may not work when the other one does. If Brent's method fails, type fB7 on the left side of the equal sign in equation (5.24):

$$= \text{root}\left(\left|\text{P3(seed2)} - \text{alphaVarLCL}\right|, \text{seed2}\right) \tag{5.24}$$

The subprogram Fseed2 begins by generating values for F_0 and F_1. A_0 and A_1 result from evaluating P3(F) at F_0 and F_1, respectively. The while loop continually increments F_0 and F_1 by delta2 and evaluates P3(F) at these two values until A_1 becomes greater than alphaVarLCL for the first time, at which point A_0 will be less than alphaVarLCL.

When this occurs, P3(F) is equal to alphaVarLCL for some value F between F_0 and F_1. An initial guess for this value is determined using linterp and stored in Fguess. The initial guess is passed out of the subprogram as seed2.

Page 5

Page 5 of the program contains sections 5.1 and 5.2. These sections correspond to sections 3.1 and 3.2, respectively, described earlier. The only difference is that the calculations in sections 5.1 and 5.2 use ν2prevm instead of ν2. The calculations are for fB8prevm, which is used in the equation for B81 (given earlier as equation (5.16)).

Page 6

Page 6 of the program is section 6.1. This section corresponds to section 4.1 described earlier. The only difference is that the calculations in section 6.1 use ν2prevm instead of ν2. The calculations are for fB7prevm, which is used in the equation for B71 (given earlier as equation (5.17)).

Page 7

Page 7 of the program begins with section 7.1. The function qt(adj_alpha, ν2) in *Mathcad* (1998) determines the critical value crit_t for a cumulative area of adj_alpha under the Student's t curve with ν2 degrees of freedom. The value crit_t is used in the equations for A42 and A41, both of which are given earlier as equations (5.8) and (5.9), respectively. The function qnorm(adj_alpha, 0, 1) in *Mathcad* (1998) determines the critical value crit_z for a cumulative area of adj_alpha under the standard Normal curve. The value crit_z is used in the equation for A4 (given earlier as equation (5.18)).

Section 7.2 of the program has the equations to calculate two stage short run control chart factors and conventional control chart constants given earlier in the Corrected Two Stage Short Run Control Chart Factor Equations section of this chapter. A41, B81, B71, A42, B82, B72, A4, B8, and B7 are for the $(\overline{X}, v)$ control charts. A41, B81sqrt, B71sqrt, A42, B82sqrt, B72sqrt, A4, B8sqrt, and B7sqrt are for the $(\overline{X}, \sqrt{v})$ control charts.

The last part of page 7 is the output section of the program. The five values entered at the beginning of the program are given. The control chart factors are broken down into first stage, second stage, and conventional. Values for ν1, ν2, $c4(\nu2+1)$, ν2prevm, and $c4(\nu2prevm+1)$, and the (1-alphaVarUCL) and alphaVarLCL percentage points of the distributions of the studentized variance $f = (v/v')$ with ν1 degrees of freedom for v and ν2 degrees of freedom for v' and the variance v with ν1 degrees of freedom complete the output of the program. To copy results into another software package (like Excel), follow the directions from *Mathcad's* (1998) help menu or highlight a value and copy and paste it into the other software package. When highlighting a value with the mouse arrow, place the arrow in the middle of the value, depress the left mouse button, and drag the arrow to the right. This will ensure just the numerical value of the result is copied and pasted.

Tabulated Results of the Program

The four tables (Tables C.3.1-C.3.4) in Appendix C.3 of this dissertation were generated using the program with the following input values:

- alphaMean=0.0027, alphaVarUCL=0.005, alphaVarLCL=0.001
- m: 1-20, 25, 30, 50, 75, 100, 150, 200, 250, 300
- n: 2-8, 10, 25, 50

The values $\nu 2$, $c4(\nu 2+1)$, ν2prevm, and $c4(\nu 2prevm+1)$ are in Table C.3.1. The $c4(\nu 2+1)$ values compare favorably to the c_4 values in Table M in the appendix of Duncan (1974) and Tables 1 and 20 in the appendix of Wheeler (1995).

The values fB8, fB8prevm, and vB8 are in Table C.3.2. The values fB7, fB7prevm, and vB7 are in Table C.3.3. The distribution of the studentized variance $f = (v/v')$ with $\nu 1$ degrees of freedom for v and $\nu 2$ degrees of freedom for v' is equivalent to the F distribution with $\nu 1$ numerator degrees of freedom and $\nu 2$ denominator degrees of freedom. Results in Table C.3.2 compare favorably to the upper 0.005 percentage points of the F distribution in Table 18 from Appendix II of Pearson and Hartley (1962).

The distribution of the variance v with $\nu 1$ degrees of freedom is equivalent to a second distribution as shown in equation (5.25):

$$p(v) = c\left(\frac{\nu 1 \cdot v}{\sigma^2}\right) \cdot \frac{\nu 1}{\sigma^2} \qquad (5.25)$$

where c is the χ^2 distribution with $\nu 1$ degrees of freedom (this equivalency is shown in Appendix C.1). Also, percentage points of the distribution of the variance v with $\nu 1$ degrees of freedom are equivalent to percentage points of the χ^2 distribution with $\nu 1$ degrees of freedom divided by $\nu 1$.

Values for A41, B81, B71, A42, B82, B72, B81sqrt, B71sqrt, B82sqrt, B72sqrt, A4, B8, B7, B8sqrt, and B7sqrt are in Table C.3.4. Results from Table C.3.4 for B81, B71, B82, B72, B8, B7, B8sqrt, and B7sqrt when n=5 compare favorably to Yang and Hillier's (1970) results. Any differences are attributable to the accuracy issues concerning Yang and Hillier's (1970) results mentioned earlier in the Problem subsection. It should be noted that the values vB8, vB7, and B8 and B7 in Tables C.3.2, C.3.3, and C.3.4, respectively, may differ in the ninth or tenth decimal place for different root routines used to calculate vB8 and vB7.

These favorable comparisons validate the program. Consequently, Table C.3.4 results for n=5, m: 1-10, 15, 20, 25, 50, 100, ∞, alphaVarUCL=0.005, and alphaVarLCL=0.001 may be considered corrections to Yang and Hillier's (1970) Tables 3-6.

Implications of the Tabulated Results

Values in Table C.3.4 show some interesting properties. Consider Table 5.1, which contains selected A42 and corresponding A4 values from Table C.3.4. As n increases for a particular m, the A42 values decrease. For larger values of m, the difference between A42 for n=2 and n=50 decreases. Of more interest is that as m increases for a particular n, the A42 values converge in a decreasing manner to their respective A4 values. For larger values of n, the difference between A42 for m=1 and the respective A4 value

Table 5.1: Selected A42 and Corresponding A4 Values from Table C.3.4

	A42						A4
n	**m=1**	**m=2**	**m=20**	**m=30**	**m=100**	**m=300**	**m=∞**
2	295.51103	18.76822	2.51074	2.37035	2.19190	2.14447	2.12130
3	17.69484	4.97997	1.90426	1.84459	1.76489	1.74290	1.73204
4	7.07531	3.13025	1.61030	1.57260	1.52140	1.50709	1.49999
5	4.45422	2.41654	1.42343	1.39568	1.35765	1.34695	1.34163
10	1.88245	1.36485	0.98715	0.97427	0.95633	0.95122	0.94868
25	0.95593	0.77906	0.61835	0.61225	0.60368	0.60122	0.60000
50	0.63533	0.53455	0.43596	0.43208	0.42662	0.42505	0.42426

decreases. This means that as m increases the convergence of A42 to A4 is faster for larger values of n. These results make sense because more information about the process is at hand for larger n and m.

Further investigation of Table C.3.4 reveals that, as m increases for a particular n, the B71, B82, B71sqrt, and B82sqrt values converge to B7, B8, B7sqrt, and B8sqrt, respectively, in a decreasing manner. The convergence pattern for B81 and B81sqrt differs in that as m increases for a particular n, the B81 and B81sqrt values converge in an increasing manner to B8 and B8sqrt, respectively.

The convergence patterns for A41, B72, and B72sqrt are unique. For n equal to 2, 3, and 4, A41 converges in a decreasing manner to A4 as m increases. For n=5, A41 converges in a decreasing manner to A4, but starting at m=3. For n=6, A41 also converges in a decreasing manner to A4, but starting at m=7. For n equal to 7, 8, 10, 25, and 50, A41 converges in an increasing manner to A4 as m increases. For n equal to 2 and 3, B72 converges in a decreasing manner to B7 as m increases. However, for n equal to 4-8, 10, 25, and 50, B72 converges in an increasing manner to B7 as m increases. For n equal to 2-4, B72sqrt converges in a decreasing manner to B7sqrt as m increases. For n equal to 5-8, 10, 25, and 50, B72sqrt converges in an increasing manner to B7sqrt as m increases.

These results have major implications. A common rule of thumb is that 20 to 30 subgroups of size 4 or 5 are necessary to use conventional control chart constants for constructing control limits. The results in Table C.3.4 indicate that this may be an incorrect rule. Consider again the A42 and corresponding A4 values in Table 5.1. When n=4, A4 is 6.850% smaller than A42 for m=20. When n=5, A4 is 3.873% smaller than A42 for m=30. These results indicate that if one were to construct $\overline{X}$ charts using conventional control chart constants when only 20 to 30 subgroups of size 4 or 5 are available to estimate the process mean and standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

B82 and corresponding B8 values, as well as B72 and corresponding B7 values, in Table C.3.4 also indicate that the common rule of thumb may be an incorrect rule. When n=4, B8 is 9.507% smaller than B82 for m=20 and B7 is 0.872% larger than B72 for m=20. When n=5, B8 is 5.244% smaller than B82 for m=30 and B7 is 0.799% larger than B72 for m=30. Consequently, if one were to construct v charts using conventional control chart constants when only 20 to 30 subgroups of size 4 or 5 are available to estimate the process variance, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

Lastly, B82sqrt and corresponding B8sqrt values, as well as B72sqrt and corresponding B7sqrt values, in Table C.3.4 indicate that the common rule of thumb may be an incorrect rule. When n=4, B8sqrt is 5.268% smaller than B82sqrt for m=20 and B7sqrt is 0.0111% smaller than B72sqrt for m=20. Consequently, if one were to construct $\sqrt{v}$ charts using conventional control chart constants when only 20 subgroups of size 4 are available to estimate the process standard deviation, the upper control limit would not be wide enough, resulting in a higher false alarm rate. Also, the lower control limit would be too tight, resulting in a decrease in the sensitivity of the chart. When n=5, B8sqrt is 2.860% smaller than B82sqrt for m=30 and B7sqrt is 0.186% larger than B72sqrt for m=30. Consequently, if one were to construct $\sqrt{v}$ charts using conventional control chart constants when only 30 subgroups of size 5 are available to estimate the process standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

Quesenberry (1993) also investigated the validity of the common rule of thumb and concluded that $400/(n-1)$ subgroups are needed for the $\overline{X}$ chart before conventional control chart constants may be used. However, for all practical purposes, the program presented by this chapter eliminates the need for these rules.

A Numerical Example

Consider the data in Table 5.2 obtained from a process requiring short run control charting techniques (assume alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001). This example will be worked two ways, the first with $(\overline{X}, v)$ control charts and the second with $(\overline{X}, \sqrt{v})$ control charts.

For m=5 and n=4, the following first stage short run control chart factors for $(\overline{X}, v)$ charts are obtained from Table C.3.4: A41=1.63082, B81=3.21838, and B71=0.00972. UCL(v), LCL(v), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

Table 5.2. A Numerical Example

Subgroup	X_1	X_2	X_3	X_4	$\overline{\overline{X}}$	v	$\sqrt{v}$
1	1.17	1.14	1.20	1.18	1.17250	0.00063	0.02500
2	1.38	1.29	1.36	1.44	1.36750	0.00382	0.06185
3	1.20	1.21	1.30	1.14	1.21250	0.00436	0.06602
4	1.40	1.40	1.21	1.43	1.36000	0.01020	0.10100
5	1.12	1.20	1.61	1.34	1.31750	0.04629	0.21515
	Averages				1.28600	0.01306	-----
	Revised Averages				1.27813	0.00475	-----

$$UCL(v) = B81 \cdot \overline{v} = 3.21838 \cdot 0.01306 = 0.04203$$

$$LCL(v) = B71 \cdot \overline{v} = 0.00972 \cdot 0.01306 = 0.00013$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A41 \cdot \sqrt{\overline{v}} = 1.28600 + 1.63082 \cdot \sqrt{0.01306} = 1.47237$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A41 \cdot \sqrt{\overline{v}} = 1.28600 - 1.63082 \cdot \sqrt{0.01306} = 1.09963$$

The variance for subgroup five (v=0.04629) is above UCL(v). Find, investigate, and remove from the process the special cause (or causes) that created this out of control point, delete subgroup five, recalculate averages (shown as the Revised Averages in Table 5.2), and reconstruct first stage control limits (this approach is from Hillier's (1969) example). For m=4 and n=4, the following first stage short run control chart factors are obtained from Table C.3.4: A41=1.66424, B81=2.97585, and B71=0.01024. Revised UCL(v), LCL(v), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(v) = B81 \cdot \overline{v} = 2.97585 \cdot 0.00475 = 0.01414$$

$$LCL(v) = B71 \cdot \overline{v} = 0.01024 \cdot 0.00475 = 0.000049$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A41 \cdot \sqrt{\overline{v}} = 1.27813 + 1.66424 \cdot \sqrt{0.00475} = 1.39283$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A41 \cdot \sqrt{\overline{v}} = 1.27813 - 1.66424 \cdot \sqrt{0.00475} = 1.16343$$

Since none of the remaining values plot out of control (i.e., control has been established), the next step is to construct second stage control limits using the following second stage short run control chart factors from Table C.3.4 (for m=4 and n=4): A42=2.14852, B82=7.22576, and B72=0.00779. UCL(v), LCL(v), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(v) = B82 \cdot \overline{v} = 7.22576 \cdot 0.00475 = 0.03432$$

$$LCL(v) = B72 \cdot \overline{v} = 0.00779 \cdot 0.00475 = 0.000037$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A42 \cdot \sqrt{\overline{v}} = 1.27813 + 2.14852 \cdot \sqrt{0.00475} = 1.42621$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A42 \cdot \sqrt{\overline{v}} = 1.27813 - 2.14852 \cdot \sqrt{0.00475} = 1.13005$$

These control limits may be used to monitor the future performance of the process.

For m=5 and n=4, the following first stage short run control chart factors for $(\overline{X}, \sqrt{v})$ charts are obtained from Table C.3.4: A41=1.63082, B81sqrt=1.83171, and B71sqrt=0.10068. UCL($\sqrt{v}$), LCL($\sqrt{v}$), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(\sqrt{v}) = B81sqrt \cdot \sqrt{\overline{v}} = 1.83171 \cdot \sqrt{0.01306} = 0.20933$$

$$LCL(\sqrt{v}) = B71sqrt \cdot \sqrt{\bar{v}} = 0.10068 \cdot \sqrt{0.01306} = 0.01151$$

$$UCL(\bar{X}) = \bar{\bar{X}} + A41 \cdot \sqrt{\bar{v}} = 1.28600 + 1.63082 \cdot \sqrt{0.01306} = 1.47237$$

$$LCL(\bar{X}) = \bar{\bar{X}} - A41 \cdot \sqrt{\bar{v}} = 1.28600 - 1.63082 \cdot \sqrt{0.01306} = 1.09963$$

The standard deviation for subgroup five ($\sqrt{v} = 0.21515$) is above UCL($\sqrt{v}$). Find, investigate, and remove from the process the special cause (or causes) that created this out of control point, delete subgroup five, recalculate averages (shown as the Revised Averages in Table 5.2), and reconstruct first stage control limits (this approach is from Hillier's (1969) example). For m=4 and n=4, the following first stage short run control chart factors are obtained from Table C.3.4: A41=1.66424, B81sqrt=1.77356, and B71sqrt=0.10404. Revised UCL($\sqrt{v}$), LCL($\sqrt{v}$), UCL($\bar{X}$), and LCL($\bar{X}$) are calculated as follows:

$$UCL(\sqrt{v}) = B81sqrt \cdot \sqrt{\bar{v}} = 1.77356 \cdot \sqrt{0.00475} = 0.12223$$

$$LCL(\sqrt{v}) = B71sqrt \cdot \sqrt{\bar{v}} = 0.10404 \cdot \sqrt{0.00475} = 0.00717$$

$$UCL(\bar{X}) = \bar{\bar{X}} + A41 \cdot \sqrt{\bar{v}} = 1.27813 + 1.66424 \cdot \sqrt{0.00475} = 1.39283$$

$$LCL(\bar{X}) = \bar{\bar{X}} - A41 \cdot \sqrt{\bar{v}} = 1.27813 - 1.66424 \cdot \sqrt{0.00475} = 1.16343$$

Since none of the remaining values plot out of control (i.e., control has been established), the next step is to construct second stage control limits using the following second stage short run control chart factors from Table C.3.4 (for m=4 and n=4): A42=2.14852, B82sqrt=2.74460, and B72sqrt=0.09014. UCL($\sqrt{v}$), LCL($\sqrt{v}$), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(\sqrt{v}) = B82sqrt \cdot \sqrt{\overline{v}} = 2.74460 \cdot \sqrt{0.00475} = 0.18916$$

$$LCL(\sqrt{v}) = B72sqrt \cdot \sqrt{\overline{v}} = 0.09014 \cdot \sqrt{0.00475} = 0.00621$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A42 \cdot \sqrt{\overline{v}} = 1.27813 + 2.14852 \cdot \sqrt{0.00475} = 1.42621$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A42 \cdot \sqrt{\overline{v}} = 1.27813 - 2.14852 \cdot \sqrt{0.00475} = 1.13005$$

These control limits may be used to monitor the future performance of the process.

Conclusions

This chapter and the program it presents make important contributions to both industry and research. Those involved with quality control in industry will, for the first time, be able to use theoretically precise control chart factors to determine control limits for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts regardless of the subgroup size, number of subgroups, and alpha values. This flexibility is valuable in that process monitoring will no longer have to be adjusted to use the limited, and in some cases incorrect, results previously available in the literature. Concerning research, this chapter provides a valuable reference for anyone interested in anything having to do with $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts. Also, as already mentioned, the program eliminates the need for the research question of how many subgroups are enough before conventional control chart constants may be used.

CHAPTER VI

TWO STAGE SHORT RUN $(\overline{X}, s)$ CONTROL CHARTS AND A COMPUTER PROGRAM TO CALCULATE THE FACTORS

Introduction

Hillier (1969) and Yang and Hillier (1970) represent the only attempts in the literature to develop two stage short run control charts based on Hillier's (1969) theory. Hillier (1969) derives equations to calculate two stage short run control chart factors for $(\overline{X}, R)$ charts. Yang and Hillier (1970) derive equations to calculate two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts.

Problem

Yang and Hillier (1970) mention that, for theoretical reasons, it does not appear to be possible to derive equations to calculate two stage short run control chart factors for $(\overline{X}, s)$ charts, where s is the standard deviation of a subgroup. It seems that no subsequent work appears in the literature that attempts to overcome this problem.

Solution

This chapter presents a solution to this problem, consequently allowing for the derivation of equations to calculate first and second stage short run control chart factors for $(\overline{X}, s)$ charts. It also describes the development and execution of a computer program that will accurately calculate the factors using these derived equations. Other exact equations that the program uses are the distribution of the standard deviation, the mean and standard deviation of the distribution of the standard deviation, the distribution of the studentized standard deviation, equations to calculate degrees of freedom, and derived conventional control chart equations. The program accepts values for subgroup size, number of subgroups, alpha for the $\overline{X}$ chart, and alpha for the s chart both above the upper control limit and below the lower control limit (alpha is the probability of a Type I error). Tables are generated for specific values of these inputs. Comparison of the tabulated results to legitimate results in the literature validates the program.

The software used for the program is *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997). The program uses numerical routines provided by the software.

Outline

This chapter first presents the distributions of the standard deviation and the studentized standard deviation. These are essential in the application of Hillier's (1969) theory to $(\overline{X}, s)$ control charts and are required for the program to perform accurate calculations. Next, Patnaik's (1950) theory is used to develop an approximation to the distribution of the mean standard deviation. From this result, equations to calculate two stage short run control chart factors for $(\overline{X}, s)$ charts are derived by following the work in the appendix of Hillier (1969). Also, equations to calculate conventional control chart constants for $(\overline{X}, s)$ charts are derived. Next, the computer program is described. Tables generated by the program are then presented and compared with legitimate results in the literature. Also, implications of the tabulated results are discussed. A numerical example illustrates the use of the program. Following a discussion of the advantages of two stage short run $(\overline{\overline{X}}, s)$ control charts, unbiased estimates of σ and σ^2 using $\bar{s}$ are given, as well as final conclusions describing the impact of the program on industry and research.

Note

Results from the program are for processes generating parts with independent measurements that follow a Normal distribution.

The Distribution of the Standard Deviation

The distribution of the standard deviation for subgroups of size n sampled from a Normal population with mean μ and standard deviation σ is given by Lord (1950) as equation (6.1a) (with some modifications in notation):

$$p(s) = \frac{\nu1^{\frac{\nu1}{2}}}{2^{\frac{\nu1}{2}-1} \cdot \Gamma\left(\frac{\nu1}{2}\right) \cdot \sigma^{\nu1}} \cdot s^{\nu1-1} \cdot e^{\frac{-\nu1 \cdot s^2}{2 \cdot \sigma^2}} \qquad (6.1a)$$

This equation may also be found in Irwin (1931). The value s (the standard deviation) is an independent estimate of σ based on $\nu1 = (n-1)$ degrees of freedom. Equation (6.1a) may also be represented as equation (6.1b) (see Appendix D.1 of this dissertation):

$$p(s) = \left(\frac{1}{\sigma^{\nu 1}}\right) \cdot \left[e^{\left(\frac{\nu 1}{2}\right)\cdot \ln(\nu 1) - \left(\frac{\nu 1}{2} - 1\right)\cdot \ln(2) - \text{gammln}\left(\frac{\nu 1}{2}\right) + (\nu 1 - 1)\cdot \ln(s) - \frac{\nu 1 \cdot s^2}{2\cdot \sigma^2}} \right] \qquad (6.1b)$$

Equation (6.1b) is the form used in the program. The function gammln is a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that calculates the natural logarithm of the gamma function. Using gammln in equation (6.1b) allows for large values of $\nu 1$ (hence large values for n) in the program. The cumulative distribution function (cdf) of the standard deviation s with $\nu 1$ degrees of freedom is equation (6.2):

$$P(S) = \int_0^S p(s)\, ds \qquad (6.2)$$

The program uses equation (6.2) (with σ=1.0) to determine alpha-based conventional control chart constants for the s chart.

The mean of the distribution of the standard deviation s with $\nu 1$ degrees of freedom is given by Mead (1966) as equation (6.3a) (with some modifications in notation):

$$E(s) = \sigma \cdot \left(\frac{2}{\nu 1}\right)^{0.5} \cdot \frac{\Gamma\left(\frac{\nu 1 + 1}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)} \qquad (6.3a)$$

E(s) is the control chart constant denoted by c_4 (when σ=1.0) (see Table M in the appendix of Duncan (1974) and Tables 1 and 20 in the appendix of Wheeler (1995)).

Equation (6.3a) may also be represented as equation (6.3b) (see Appendix D.1) (note: $c4 \equiv c_4$):

$$c4 = \sigma \cdot \left(\frac{2}{v1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{v1+1}{2}\right) - \mathrm{gammln}\left(\frac{v1}{2}\right)}\right) \tag{6.3b}$$

Equation (6.3b) is the form used in the program. Using gammln in equation (6.3b) allows for large values of v1 (hence large values for n) in the program.

The variance of the distribution of the standard deviation s with v1 degrees of freedom is also given by Mead (1966) as equation (6.4a) (with some modifications in notation):

$$\mathrm{var(s)} = \left(\frac{2 \cdot \sigma^2}{v1}\right) \cdot \left[\frac{\Gamma\left(\frac{v1+2}{2}\right)}{\Gamma\left(\frac{v1}{2}\right)} - \left(\frac{\Gamma\left(\frac{v1+1}{2}\right)}{\Gamma\left(\frac{v1}{2}\right)}\right)^2\right] \tag{6.4a}$$

The value $\sqrt{\mathrm{var(s)}}$ is the control chart constant denoted by c_5 (when σ=1.0) (see Wheeler's (1995) Table 20). It is also equal to $\sqrt{1 - c_4^2}$ (when σ=1.0). The square root of equation (6.4a) may be represented as equation (6.4b) (see Appendix D.1) (note: $c5 \equiv c_5$):

$$c5 = \sigma \cdot \left[\left(\frac{2}{v1}\right) \cdot \left[e^{\mathrm{gammln}\left(\frac{v1+2}{2}\right) - \mathrm{gammln}\left(\frac{v1}{2}\right)} - e^{2 \cdot \left(\mathrm{gammln}\left(\frac{v1+1}{2}\right) - \mathrm{gammln}\left(\frac{v1}{2}\right)\right)}\right]\right]^{0.5} \tag{6.4b}$$

Equation (6.4b) is the form used in the program. Using gammln in equation (6.4b) allows for large values of ν1 (hence large values for n) in the program.

The Distribution of the Studentized Standard Deviation

The distribution of the studentized standard deviation for subgroups of size n sampled from a Normal population with mean μ and standard deviation σ is given by Irwin (1931) as equation (6.5a) (with some modifications in notation):

$$p3(t) = \frac{2 \cdot \nu 1^{\frac{\nu 1}{2}} \cdot \nu 2^{\frac{\nu 2}{2}} \cdot \Gamma\left(\frac{\nu 1 + \nu 2}{2}\right) \cdot t^{\nu 1 - 1}}{\Gamma\left(\frac{\nu 1}{2}\right) \cdot \Gamma\left(\frac{\nu 2}{2}\right) \cdot \left(\nu 1 \cdot t^2 + \nu 2\right)^{\frac{\nu 1 + \nu 2}{2}}} \tag{6.5a}$$

The value t (the studentized standard deviation) is equal to s/s', where s' is a second independent estimate of σ based on ν2 degrees of freedom. Equation (6.5a) may also be represented as equation (6.5b) (see Appendix D.1):

$$p3(t) = e^{p1(t) - p2(t)} \tag{6.5b}$$

where

$$p1(t) = \ln(2) + \left(\frac{\nu 1}{2}\right) \cdot \ln(\nu 1) + \left(\frac{\nu 2}{2}\right) \cdot \ln(\nu 2) + \text{gammln}\left(\frac{\nu 1 + \nu 2}{2}\right) + (\nu 1 - 1) \cdot \ln(t) \tag{6.5c}$$

$$p2(t) = \text{gammln}\left(\frac{\nu 1}{2}\right) + \text{gammln}\left(\frac{\nu 2}{2}\right) + \left(\frac{\nu 1 + \nu 2}{2}\right) \cdot \ln\left(\nu 1 \cdot t^2 + \nu 2\right) \tag{6.5d}$$

Equations (6.5b)-(6.5d) are used in the program. Using gammln in equations (6.5c) and (6.5d) allows for large values of ν1 (hence large values for n) and large values of ν2 (hence large values for n and m (the number of subgroups)) in the program. The cdf of the studentized standard deviation $t = (s/s')$ with ν1 degrees of freedom for s and ν2 degrees of freedom for s' is equation (6.6):

$$P3(T) = \int_0^T p3(t)\,dt \tag{6.6}$$

The program uses equation (6.6) to determine two stage short run control chart factors for the s chart.

As ν2→∞ (i.e., as m→∞) for any n, the distribution of the studentized standard deviation $t = (s/s')$ converges to the distribution of the standard deviation s (when σ=1.0). This fact is used to calculate alpha-based conventional control chart constants for the s chart.

The Distribution of the Mean Standard Deviation

Consider the situation in which the mean of a statistic is calculated by averaging m values of the statistic, each of which is based on a subgroup of size n. Patnaik (1950) investigates this situation when the statistic is the range and develops an approximation to the distribution of the mean range $\overline{R}/\sigma$. The resulting distribution is the $(\chi \cdot d_2^*)/\sqrt{\nu}$ distribution, which is a function of the χ distribution with ν degrees of freedom (the χ distribution with ν degrees of freedom and its moments about zero may be found in Johnson and Welch (1939)). Equations for ν and d_2^* are derived from results obtained by equating the squared means as well as the variances of the distribution of the mean range $\overline{R}/\sigma$ and the $(\chi \cdot d_2^*)/\sqrt{\nu}$ distribution with ν degrees of freedom. Hillier (1964 and 1967) uses Patnaik's (1950) theory to derive equations to calculate short run control chart factors for $\overline{X}$ and R charts, respectively. Hillier (1969) then incorporates the two stage procedure into his short run control chart factor calculations for $(\overline{X}, R)$ charts.

Consider the situation in which the statistic is the standard deviation and the distribution of interest is the distribution of the mean standard deviation $\bar{s}/\sigma$. In order to be able to use Hillier's (1969) theory to derive equations to calculate two stage short run control chart factors for $(\overline{X}, s)$ charts, we apply Patnaik's (1950) theory to approximate $\bar{s}/\sigma$ by the $(\chi \cdot c_4^*)/\sqrt{\nu 2}$ distribution with $\nu 2$ degrees of freedom (this $\nu 2$ is the same as the one given earlier in equation (6.5a)). The equation for c_4^* is derived in Appendix D.1 and is given as equation (6.7) (note: $\text{c4star} \equiv c_4^*$):

$$\text{c4star} = \left(\text{c4}^2 + \frac{\text{c5}^2}{m} \right)^{0.5} \qquad (6.7)$$

The equations for the control chart constants c4 and c5 are given earlier as equations (6.3b) and (6.4b), respectively.

Using results from Prescott (1971), the equation for $\nu 2$ is determined by equating the ratio of the variance to the squared mean, both of the χ distribution with $\nu 2$ degrees of freedom, to the ratio of the variance to the squared mean, both of the distribution of the mean standard deviation $\bar{s}/\sigma$. The resulting equation for ν2 is equation (6.8):

$$d(x) = h(x) - r \qquad (6.8)$$

The exact value for ν2 is the value of x such that d(x) is equal to zero. The function h(x) is the ratio of the variance to the squared mean, both of the χ distribution with x degrees of freedom (x replaces ν2). The mean and variance of the χ distribution with ν2 degrees of freedom are given in Appendix D.1. The equation for h(x), which is derived in Appendix B.1 of this dissertation, is given as equation (6.9):

$$h(x) = \frac{x \cdot e^{2\cdot(\mathrm{gammln}(0.5\cdot x) - \mathrm{gammln}(0.5\cdot x + 0.5))} - 2}{2} \qquad (6.9)$$

The value r is the ratio of the variance to the squared mean, both of the distribution of the mean standard deviation $\bar{s}/\sigma$. The mean and the variance of the distribution of the mean standard deviation $\bar{s}/\sigma$ are derived in Appendix D.1. The equation for r is given as equation (6.10):

$$r = \frac{c5^2}{m \cdot c4^2} \qquad (6.10)$$

An equivalent form (also based on Patnaik's (1950) theory) of equation (6.8) may be found in Palm and Wheeler (1990), who use their result to calculate equivalent degrees of freedom for population standard deviation estimates based on subgroup standard deviations.

Table D.3.1 (the creation of which is explained in the Tabulated Results of the Program section later in this chapter) in Appendix D.3 of this dissertation has ν2 and c_4^* values for m: 1-20, 25, 30, 50, 75, 100, 150, 200, 250, 300 and n: 2-8, 10, 25, 50, as well as c_4 values. When m=1 for any n, c_4^* is equal to one. As m→∞ (i.e., as ν2→∞) for any n, c_4^* converges to c_4.

Approximating the distribution of the mean standard deviation $\bar{s}/\sigma$ by the $(\chi \cdot c_4^*)/\sqrt{\nu 2}$ distribution with ν2 degrees of freedom works well. In fact, based on how c_4^* is derived in Appendix D.1, the means and variances of these two distributions are equal.

Derivation of the Control Chart Factor Equations

Since the $(\chi \cdot c_4^*)/\sqrt{\nu 2}$ distribution with ν2 degrees of freedom approximates the distribution of the mean standard deviation $\bar{s}/\sigma$, the derivation of equations to calculate first and second stage short run control chart factors for $(\bar{X}, s)$ charts follows the work in the appendix of Hillier (1969). A32, the second stage short run control chart factor for the $\bar{X}$ chart, is derived in almost the same manner as Hillier's (1969) A_2^*. Differences are that A32, $\bar{s}$, ν2, and c_4^* in this chapter replace A_2^*, $\bar{R}$, ν, and c, respectively, in Hillier (1969). The resulting equation for A32 is given as equation (6.11) (note:

c4star ≡ c_4^*):

$$A32 = \left(\frac{crit\_t}{c4star}\right) \cdot \left(\frac{m+1}{n \cdot m}\right)^{0.5} \tag{6.11}$$

The value crit_t is the critical value for a cumulative area of $(1-(alphaMean/2))$ under the Student's t curve with ν2 degrees of freedom (alphaMean is the probability of a Type I error on the $\overline{X}$ control chart).

A31, the first stage short run control chart factor for the $\overline{X}$ chart, is derived in almost the same manner as Hillier's (1969) A_2^{**}. Differences are that A31, $\bar{s}$, ν2, and c_4^* in this chapter replace A_2^{**}, $\overline{R}$, ν, and c, respectively, in Hillier (1969). The resulting equation for A31 is given as equation (6.12):

$$A31 = \left(\frac{crit\_t}{c4star}\right) \cdot \left(\frac{m-1}{n \cdot m}\right)^{0.5} \tag{6.12}$$

The value crit_t has the same meaning here as in equation (6.11).

B42, the second stage short run upper control chart factor for the s chart, is derived in Appendix D.1. Other than differences in notation and distributions, this derivation follows that for Hillier's (1969) D_4^*. The resulting equation for B42 is given as equation (6.13):

$$B42 = \frac{tB4}{c4star} \tag{6.13}$$

The value tB4 is the (1-alphaStandUCL) percentage point of the distribution of the studentized standard deviation $t = (s/s')$ with $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s' (alphaStandUCL is the probability of a Type I error on the s chart above the upper control limit).

B32, the second stage short run lower control chart factor for the s chart, is derived in a manner similar to B42. Differences are that B32, tB3, and alphaStandLCL replace B42, tB4, and (1-alphaStandUCL), respectively (alphaStandLCL is the probability of a Type I error on the s chart below the lower control limit). The resulting equation for B32 is given as equation (6.14):

$$B32 = \frac{tB3}{c4star} \tag{6.14}$$

The value tB3 is the alphaStandLCL percentage point of the distribution of the studentized standard deviation $t = (s/s')$ with $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s'.

B41, the first stage short run upper control chart factor for the s chart, is derived in almost the same manner as Hillier's (1969) D_4^{**}. Differences are that B41, s_i, B42, and $\bar{s}$ in this chapter replace D_4^{**}, R_i, D_4^{*}, and $\bar{R}$, respectively, in Hillier (1969). The resulting equation for B41 is given as equation (6.15):

$$B41 = \frac{m \cdot tB4prevm}{c4starprevm \cdot (m-1) + tB4prevm} \tag{6.15}$$

The value tB4prevm has the same meaning as tB4 (given earlier in equation (6.13)), except it is for v2prevm (i.e., v2 for (m-1) subgroups). The value c4starprevm has the same equation as c4star (given earlier as equation (6.7)), except m is replaced with (m-1).

The equation for B31, the first stage short run lower control chart factor for the s chart, is derived in almost the same manner as Hillier's (1969) D_3^{**}. Differences are that B31, s_i, B32, and $\bar{s}$ in this chapter replace D_3^{**}, R_i, D_3^{*}, and $\bar{R}$, respectively, in Hillier (1969). The resulting equation for B31 is given as equation (6.16):

$$B31 = \frac{m \cdot tB3prevm}{c4starprevm \cdot (m-1) + tB3prevm} \tag{6.16}$$

The value tB3prevm has the same meaning as tB3 (given earlier in equation (6.14)), except it is for v2prevm instead of v2.

The equation for A3, the conventional control chart constant for the $\bar{X}$ chart, may be obtained by taking the limit of either A31 or A32 as m→∞ (i.e., as v2→∞) for any n. The resulting equation for A3 is given as equation (6.17):

$$A3 = \frac{crit\_z}{c4 \cdot n^{0.5}} \tag{6.17}$$

The value crit_z is the critical value for a cumulative area of $(1-(\text{alphaMean}/2))$ under the standard Normal curve. The equation for the control chart constant c4 is given earlier as equation (6.3b).

The equation for B4, the alpha-based conventional upper control chart constant for the s chart, may be obtained by taking the limit of either B41 as $m \to \infty$ (i.e., as $\nu 2\text{prevm} \to \infty$) or B42 as $m \to \infty$ (i.e., as $\nu 2 \to \infty$) for any n. The resulting equation for B4 is given as equation (6.18):

$$B4 = \frac{sB4}{c4} \tag{6.18}$$

The value sB4 is the (1-alphaStandUCL) percentage point of the distribution of the standard deviation s with $\nu 1$ degrees of freedom.

The equation for B3, the alpha-based conventional lower control chart constant for the s chart, may be obtained by taking the limit of either B31 as $m \to \infty$ (i.e., as $\nu 2\text{prevm} \to \infty$) or B32 as $m \to \infty$ (i.e., as $\nu 2 \to \infty$) for any n. The resulting equation for B3 is given as equation (6.19):

$$B3 = \frac{sB3}{c4} \tag{6.19}$$

The value sB3 is the alphaStandLCL percentage point of the distribution of the standard deviation s with $\nu 1$ degrees of freedom.

The Computer Program

This section of the chapter presents the computer program, which is in Appendix D.2 of this dissertation. The program has seven pages, each of which is further divided into sections.

Mathcad (1998) Note

It is possible for a section of code in the program to turn red and have the error message "Unknown Error". To correct this, delete one character in the red code and type it back in. Click the mouse arrow outside of the code. The code should turn black, indicating that the error has been eliminated. If not, repeat the procedure (it will eventually correct the problem).

Page 1

The first page of the program begins with the data entry section. The program requires the user to enter the following values: alphaMean (alpha for the $\overline{X}$ chart), alphaStandUCL (alpha for the s chart above the UCL), alphaStandLCL (alpha for the s chart below the LCL), m (number of subgroups), and n (subgroup size for the $(\overline{X}, s)$ charts). If no lower control limit on the s chart is desired, the entry for alphaStandLCL should be left blank (do not enter zero). Before a value can be entered, the cursor must be moved to the right side of the appropriate equal sign. This may be done using the arrow keys on the keyboard or by moving the mouse arrow to the right side of the equal sign and clicking once with the left mouse button. The program is activated by paging down once the last entry is made. When using *Mathcad 8.03 Professional* (1998), paging down is not allowed while a calculation is taking place. However, *Mathcad 2000 Professional* (1999) allows the user to page down to the output section of the program (explained later) after the last entry is made.

The next part of page 1 is section 1.1 of the program. The value TOL is the tolerance. The calculations that use this value will be accurate to twelve places to the right of the decimal. The population standard deviation σ is set equal to one for two reasons. The first is to achieve the convergence of the distribution of the studentized standard deviation $t = (s/s')$ with $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s' to the distribution of the standard deviation s with $\nu 1$ degrees of freedom as $\nu 2 \rightarrow \infty$ (i.e., as $m \rightarrow \infty$) for any n. The second is to have the correct calculations for c4 and c5. As mentioned earlier in relation to equation (6.1a), the degrees of freedom $\nu 1$ for the standard deviation s is equal to (n-1). The equations for p(s), c4, and c5 are given earlier as equations (6.1b), (6.3b), and (6.4b), respectively.

Page 2

Page 2 of the program begins with section 2.1. P(S) is given earlier as equation (6.2). The remainder of the code in this section determines sB4 and sB3, the (1-alphaStandUCL) and alphaStandLCL percentage points, respectively, of the distribution of the standard deviation s with $\nu 1$ degrees of freedom and infinite $\nu 2$ (i.e., infinite m) (recall the earlier statement that as $\nu 2 \rightarrow \infty$ (i.e., as $m \rightarrow \infty$) for any n, the distribution of the studentized standard deviation $t = (s/s')$ converges to the distribution of the standard deviation s (when σ=1.0)). The value sB4 is used in the equation for B4, which is given earlier as equation (6.18). The value sB3 is used in the equation for B3, which is given earlier as equation (6.19). The roots of the equations DUCL(S) and DLCL(S) are sB4 and sB3, respectively, and are determined using zbrent (a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that uses Brent's method to find the roots of an equation). The subprograms Sseed1 and Sseed2 generate seed values seedB4 and seedB3, respectively, for Brent's method.

The subprogram Sseed1 works as follows. Initially, S_0 and S_1 are set equal to 0.01 and 0.02, respectively. A_0 and A_1 result from evaluating DUCL(S) at S_0 and S_1, respectively. The while loop begins by checking if the product of A_0 and A_1 is negative. If so, the root for DUCL(S) lies between 0.01 and 0.02. If not, S_0 and S_1 are incremented by 0.01. A_0 and A_1 are recalculated and if their product is negative, the root for DUCL(S) lies between 0.02 and 0.03. Otherwise, the while loop repeats. Once a root for DUCL(S) is bracketed, the bracketing values are passed out of the subprogram into the 2×1 vector seedB4 to be used by Brent's method to determine sB4. The subprogram Sseed2 works similarly to construct the 2×1 vector seedB3 to be used by Brent's method to determine sB3, except the starting value is 0.001.

The next part of page 2 is section 2.2 of the program. As shown earlier, the two stage short run control chart factor calculations require ν2 and ν2prevm. The equation for h(x) is described earlier (see equation (6.9)). The value rprevm has the same meaning as r described earlier (see equation (6.10)), except it is for (m-1) subgroups. The equation for dprevm(x) is the same as that for d(x) (given earlier as equation (6.8)), except rprevm replaces r. The equation for ν(A) is from Prescott (1971). Brent's method is used to find the root $\nu 2$ of d(x) using the seed value $\nu(A)$, where A is given as equation (6.20):

$$A = \left(\frac{2}{m}\right)\cdot\left(\frac{c5}{c4}\right)^2 \tag{6.20}$$

This equation for A is the distribution of the mean standard deviation counterpart of the equation for A from Prescott (1971). Similarly, Brent's method is used to find the root ν2prevm of dprevm(x) using the seed value $\nu(A)$, where A is given as equation (6.21):

$$A = \left(\frac{2}{m-1}\right)\cdot\left(\frac{c5}{c4}\right)^2 \tag{6.21}$$

<u>Page 3</u>

Page 3 of the program begins with section 3.1. The equations for p3(t), p1(t), p2(t), and P3(T) are given earlier as equations (6.5b), (6.5c), (6.5d), and (6.6), respectively. Section 3.2 contains the calculations required to determine tB4, the (1-alphaStandUCL) percentage point of the distribution of the studentized standard deviation $t = (s/s')$ with $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s' (both $\nu 1$ and $\nu 2$ are calculated earlier in the program). The value tB4 is used in the equation for B42, which is given earlier as equation (6.13). The subprogram Tseed1 generates the seed value seed1 for Brent's method or for root (root is a numerical routine in *Mathcad* (1998) that uses the Secant method to determine the roots of an equation). Either root-finding method determines the root tB4 of D1(x). Both Brent's method and the Secant method are given because one may not work when the other one does. If Brent's method fails (which is signified in *Mathcad* (1998) by the code turning red), type tB4 on the left side of the equal sign in equation (6.22):

$$= \text{root}\left[\left|\text{P3(seed1)} - (1 - \text{alphaStandUCL})\right|, \text{seed1}\right] \tag{6.22}$$

The subprogram Tseed1 begins by generating values for T_0 and T_1. A_0 and A_1 result from evaluating P3(T) at T_0 and T_1, respectively. The while loop continually increments T_0 and T_1 by 0.1 and evaluates P3(T) at these two values until A_1 becomes greater than (1-alphaStandUCL) for the first time, at which point A_0 will be less than (1-alphaStandUCL). When this occurs, P3(T) is equal to (1-alphaStandUCL) for some value T between T_0 and T_1. An initial guess for this value is determined using linterp (a numerical routine in *Mathcad* (1998) that performs linear interpolation) and stored in Tguess. The initial guess is passed out of the subprogram as seed1.

Page 4

Page 4 of the program is section 4.1. The code in this section is used to determine tB3, the alphaStandLCL percentage point of the distribution of the studentized standard deviation $t = (s/s')$ with ν1 degrees of freedom for s and ν2 degrees of freedom for s' (both ν1 and ν2 are calculated earlier in the program). The value tB3 is used in the equation for B32, which is given earlier as equation (6.14). The subprogram Tseed2 generates the seed value seed2 for Brent's method or for root. Either root-finding method determines the root tB3 of D2(x). Both Brent's method and the Secant method are given because one may not work when the other one does. If Brent's method fails, type tB3 on the left side of the equal sign in equation (6.23):

$$= \text{root}\left(\left|\text{P3(seed2)} - \text{alphaStandLCL}\right|, \text{seed2}\right) \tag{6.23}$$

The subprogram Tseed2 begins by generating values for T_0 and T_1. A_0 and A_1 result from evaluating P3(T) at T_0 and T_1, respectively. The while loop continually increments T_0 and T_1 by 0.001 and evaluates P3(T) at these two values until A_1 becomes greater than alphaStandLCL for the first time, at which point A_0 will be less than alphaStandLCL. When this occurs, P3(T) is equal to alphaStandLCL for some value T between T_0 and T_1. An initial guess for this value is determined using linterp and stored in Tguess. The initial guess is passed out of the subprogram as seed2.

Page 5

Page 5 of the program contains sections 5.1 and 5.2. These sections correspond to sections 3.1 and 3.2, respectively, described earlier. The only difference is that the calculations in sections 5.1 and 5.2 use ν2prevm instead of ν2. The calculations are for tB4prevm, which is used in the equation for B41 (given earlier as equation (6.15)).

Page 6

Page 6 of the program is section 6.1. This section corresponds to section 4.1 described earlier. The only difference is that the calculations in section 6.1 use ν2prevm instead of ν2. The calculations are for tB3prevm, which is used in the equation for B31 (given earlier as equation (6.16)).

Page 7

Page 7 of the program begins with section 7.1. It has the equations for c4star (given earlier as equation (6.7)) and c4starprevm (c4star for (m-1) subgroups). The value c4star is used in the equations for A32, A31, B42, and B32, all of which are given earlier as equations (6.11), (6.12), (6.13), and (6.14), respectively. The value c4starprevm is used in the equations for B41 and B31, which are given earlier as equations (6.15) and (6.16), respectively. The function qt(adj_alpha, ν2) in *Mathcad* (1998) determines the critical value crit_t for a cumulative area of adj_alpha under the Student's t curve with ν2 degrees of freedom. The value crit_t is used in the equations for A31 and A32. The function qnorm(adj_alpha, 0, 1) in *Mathcad* (1998) determines the critical value crit_z for a cumulative area of adj_alpha under the standard Normal curve. The value crit_z is used in the equation for A3 (given earlier as equation (6.17)).

Section 7.2 of the program has the equations to calculate two stage short run control chart factors and conventional control chart constants given earlier in the Derivation of the Control Chart Factor Equations section of this chapter. The equation for A3 is a generalization of the equation for A_3 from Duncan's (1974) Table M to allow for different values of alphaMean.

The last part of page 7 is the output section of the program. The five values entered at the beginning of the program are given. The control chart factors are broken down into first stage, second stage, and conventional. The mean, standard deviation, and variance of the distribution of the standard deviation s with ν1 degrees of freedom, the values for ν1, ν2, c4star, ν2prevm, and c4starprevm, and the (1-alphaStandUCL) and alphaStandLCL percentage points of the distributions of the studentized standard deviation $t = (s/s')$ with ν1 degrees of freedom for s and ν2 degrees of freedom for s' and the standard deviation s with ν1 degrees of freedom complete the output of the program. To copy results into another software package (like Excel), follow the directions from *Mathcad's* (1998) help menu or highlight a value and copy and paste it into the other software package. When highlighting a value with the mouse arrow, place the arrow in the middle of the value, depress the left mouse button, and drag the arrow to the right. This will ensure just the numerical value of the result is copied and pasted.

Tabulated Results of the Program

The four tables (Tables D.3.1-D.3.4) in Appendix D.3 were generated using the program with the following input values:

- alphaMean=0.0027, alphaStandUCL=0.005, alphaStandLCL=0.001
- m: 1-20, 25, 30, 50, 75, 100, 150, 200, 250, 300
- n: 2-8, 10, 25, 50

The values v2, c4star, v2prevm, c4starprevm, c4, c5, and $c5^2$ (the variance of the distribution of the standard deviation s with v1 degrees of freedom) are in Table D.3.1. The v2 and v2prevm values compare favorably to the equivalent degrees of freedom in Table 2 of Palm and Wheeler (1990) and Table 25 in the appendix of Wheeler (1995). The c4 values compare favorably to the c_4 values in Duncan's (1974) Table M and Wheeler's (1995) Tables 1 and 20. The c5 values compare favorably to the c_5 values in Wheeler's (1995) Table 20.

The values tB4, tB4prevm, and sB4 are in Table D.3.2. The values tB3, tB3prevm, and sB3 are in Table D.3.3. The distribution of the studentized standard deviation $t = (s/s')$ with v1 degrees of freedom for s and v2 degrees of freedom for s' is equivalent to a second distribution as shown in equation (6.24):

$$p3(t) = f(t^2) \cdot 2 \cdot t \qquad (6.24)$$

where f is the F distribution with v1 numerator degrees of freedom and v2 denominator degrees of freedom (this equivalency is shown in Appendix D.1). Also, percentage points of the distribution of the studentized standard deviation $t = (s/s')$ with v1 degrees of freedom for s and v2 degrees of freedom for s' are equivalent to the square root of percentage points of the F distribution with v1 numerator degrees of freedom and v2 denominator degrees of freedom. Hartley (1944) also gives distributions that are transformations of the distribution of the studentized standard deviation $t = (s/s')$.

The distribution of the standard deviation s with v1 degrees of freedom is equivalent to a second distribution as shown in equation (6.25):

$$p(s) = c\left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right) \cdot \frac{2 \cdot \nu 1 \cdot s}{\sigma^2} \tag{6.25}$$

where c is the χ^2 distribution with $\nu 1$ degrees of freedom (this equivalency is shown in Appendix D.1). Also, percentage points of the distribution of the standard deviation s with $\nu 1$ degrees of freedom are equivalent to the square root of the percentage points of the χ^2 distribution with $\nu 1$ degrees of freedom divided by $\nu 1$.

Values for A31, B41, B31, A32, B42, B32, A3, B4, and B3 are in Table D.3.4. The A3 values compare favorably to the A_3 values in Duncan's (1974) Table M. It should be noted that the values sB4, sB3, and B4 and B3 in Tables D.3.2, D.3.3, and D.3.4, respectively, may differ in the ninth or tenth decimal place for different root routines used to calculate sB4 and sB3.

Implications of the Tabulated Results

Values in Table D.3.4 show some interesting properties. Consider Table 6.1, which contains selected A32 and corresponding A3 values from Table D.3.4. As n increases for a particular m, the A32 values decrease. For larger values of m, the difference between A32 for n=2 and n=50 decreases. Of more interest is that as m increases for a particular n, the A32 values converge in a decreasing manner to their respective A3 values. For larger values of n, the difference between A32 for m=1 and the respective A3 value decreases. This means that as m increases the convergence of A32 to A3 is faster for

Table 6.1. Selected A32 and Corresponding A3 Values from Table D.3.4

	A32						A3
n	**m=1**	**m=2**	**m=20**	**m=30**	**m=100**	**m=300**	**m=∞**
2	235.78369	20.27157	3.11857	2.95302	2.74218	2.68607	2.65866
3	15.68165	5.12390	2.13293	2.07124	1.98856	1.96570	1.95440
4	6.51861	3.17444	1.73764	1.70031	1.64942	1.63517	1.62809
5	4.18690	2.43647	1.50709	1.48008	1.44296	1.43250	1.42729
10	1.83098	1.36718	1.01240	1.00001	0.98273	0.97780	0.97534
25	0.94603	0.77925	0.62420	0.61825	0.60988	0.60748	0.60628
50	0.63210	0.53458	0.43797	0.43415	0.42876	0.42721	0.42643

larger values of n. These results make sense because more information about the process is at hand for larger n and m.

Further investigation of Table D.3.4 reveals that, as m increases for a particular n, the B31 and B42 values also converge to their respective B3 and B4 values in a decreasing manner. The convergence pattern for B41 and B32 differs in that as m increases for a particular n, the B41 and B32 values converge in an increasing manner to their respective B4 and B3 values. The convergence pattern for A31 is unique. For n equal to 2, 3, and 4, A31 converges in a decreasing manner to A3 as m increases. For n=5, A31 also converges in a decreasing manner to A3, but starting at m=4. For n equal to 6, 7, 8, 10, 25, and 50, A31 converges in an increasing manner to A3 as m increases.

These results have major implications. A common rule of thumb is that 20 to 30 subgroups of size 4 or 5 are necessary to use conventional control chart constants for constructing control limits. The results in Table D.3.4 indicate that this may be an incorrect rule. Consider again the A32 and corresponding A3 values in Table 6.1. When n=4, A3 is 6.305% smaller than A32 for m=20. When n=5, A3 is 3.567% smaller than A32 for m=30. These results indicate that if one were to construct $\overline{X}$ charts using conventional control chart constants when only 20 to 30 subgroups of size 4 or 5 are available to estimate the process mean and standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

B42 and corresponding B4 values, as well as B32 and corresponding B3 values, in Table D.3.4 also indicate that the common rule of thumb may be an incorrect rule. When n=4, B4 is 4.758% smaller than B42 for m=20 and B3 is 0.878% larger than B32 for m=20. When n=5, B4 is 2.580% smaller than B42 for m=30 and B3 is 0.634% larger than B32 for m=30. Consequently, if one were to construct s charts using conventional control chart constants when only 20 to 30 subgroups of size 4 or 5 are available to estimate the process standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

Quesenberry (1993) also investigated the validity of the common rule of thumb and concluded that $400/(n-1)$ subgroups are needed for the $\overline{X}$ chart before conventional control chart constants may be used. However, for all practical purposes, the program presented by this chapter eliminates the need for these rules.

A Numerical Example

Consider the data in Table 6.2 obtained from a process requiring short run control charting techniques (assume alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001). For m=5 and n=4, the following first stage short run control chart factors are obtained from Table D.3.4: A31=1.72737, B41=2.09812, and B31=0.11441. UCL(s), LCL(s), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

Table 6.2. A Numerical Example

Subgroup	X_1	X_2	X_3	X_4	$\overline{X}$	s
1	1.17	1.14	1.20	1.18	1.17250	0.02500
2	1.38	1.29	1.36	1.44	1.36750	0.06185
3	1.20	1.21	1.30	1.14	1.21250	0.06602
4	1.40	1.40	1.21	1.43	1.36000	0.10100
5	1.12	1.20	1.61	1.34	1.31750	0.21515
	Averages				1.28600	0.09380
	Revised Averages				1.27813	0.06346

$$UCL(s) = B41 \cdot \bar{s} = 2.09812 \cdot 0.09380 = 0.19680$$

$$LCL(s) = B31 \cdot \bar{s} = 0.11441 \cdot 0.09380 = 0.01073$$

$$UCL(\overline{X}) = \overline{\overline{X}} + A31 \cdot \bar{s} = 1.28600 + 1.72737 \cdot 0.09380 = 1.44803$$

$$LCL(\overline{X}) = \overline{\overline{X}} - A31 \cdot \bar{s} = 1.28600 - 1.72737 \cdot 0.09380 = 1.12397$$

The standard deviation for subgroup five (s=0.21515) is above UCL(s). Find, investigate, and remove from the process the special cause (or causes) that created this out of control point, delete subgroup five, recalculate averages (shown as the Revised Averages in Table 6.2), and reconstruct first stage control limits (this approach is from Hillier's (1969) example). For m=4 and n=4, the following first stage short run control chart factors are obtained from Table D.3.4: A31=1.75114, B41=2.05256, and B31=0.11958. Revised UCL(s), LCL(s), UCL($\overline{X}$), and LCL($\overline{X}$) are calculated as follows:

$$UCL(s) = B41 \cdot \bar{s} = 2.05256 \cdot 0.06346 = 0.13026$$

$$LCL(s) = B31 \cdot \bar{s} = 0.11958 \cdot 0.06346 = 0.00759$$

$$UCL(\bar{X}) = \bar{\bar{X}} + A31 \cdot \bar{s} = 1.27813 + 1.75114 \cdot 0.06346 = 1.38926$$

$$LCL(\bar{X}) = \bar{\bar{X}} - A31 \cdot \bar{s} = 1.27813 - 1.75114 \cdot 0.06346 = 1.16700$$

Since none of the remaining values plot out of control (i.e., control has been established), the next step is to construct second stage control limits using the following second stage short run control chart factors from Table D.3.4 (for m=4 and n=4): A32=2.26072, B42=2.89208, and B32=0.09367. UCL(s), LCL(s), UCL($\bar{X}$), and LCL($\bar{X}$) are calculated as follows:

$$UCL(s) = B42 \cdot \bar{s} = 2.89208 \cdot 0.06346 = 0.18353$$

$$LCL(s) = B32 \cdot \bar{s} = 0.09367 \cdot 0.06346 = 0.00594$$

$$UCL(\bar{X}) = \bar{\bar{X}} + A32 \cdot \bar{s} = 1.27813 + 2.26072 \cdot 0.06346 = 1.42160$$

$$LCL(\bar{X}) = \bar{\bar{X}} - A32 \cdot \bar{s} = 1.27813 - 2.26072 \cdot 0.06346 = 1.13466$$

These control limits may be used to monitor the future performance of the process.

Advantages of Two Stage Short Run $(\bar{X}, s)$ Control Charts

Several advantages exist to using two stage short run $(\bar{X}, s)$ control charts. A significant advantage is that there is a smaller loss in degrees of freedom from using the Patnaik (1950) approximation than with two stage short run $(\bar{X}, R)$ control charts. This is illustrated in Table 6.3, which has selected values for degrees of freedom for both c_4^* (from Table D.3.1 in Appendix D.3) and d_2^* (from Table B.3.1 in Appendix B.3 of this dissertation).

As expected, when n=2, the degrees of freedom for both c_4^* and d_2^* are equal. When m=1 for each value of n given, c_4^* suffers no loss in degrees of freedom, at least to the accuracy shown (the exact degrees of freedom is equal to $(m \cdot (n-1))$ (see Yang and Hillier (1970))). However, as n increases when m=1, d_2^* loses degrees of freedom at an increasing rate to the point that, when n=50, the degrees of freedom for d_2^* is less than half of that for c_4^*. Even when m=300 and n=2, the degrees of freedom for c_4^* is still approximately 88% of the exact value of 300 degrees of freedom. As expected, this percentage increases as n increases.

Many authors suggest that when n gets large (i.e., in the case of Duncan (1974), when n>12), the loss in efficiency (which is related to a loss in degrees of freedom) becomes too great to use the range to estimate process variability. The results in Table 6.3 seem to

Table 6.3. Comparison of Degrees of Freedom for c_4^* and d_2^*

n	**2**		**5**	
m	c_4^*	d_2^*	c_4^*	d_2^*
1	1.00000	1.00000	4.00000	3.82651
2	1.91952	1.91952	7.81543	7.47105
5	4.59060	4.59060	19.21294	18.35417
10	8.98907	8.98907	38.19043	36.47359
25	22.14078	22.14078	95.11138	90.81974
50	44.04420	44.04420	189.9757	181.3926
100	87.84479	87.84479	379.7029	362.5367
200	175.4428	175.4428	759.1566	724.8242
300	263.0400	263.0400	1138.610	1087.112

Table 6.3 continued. Comparison of Degrees of Freedom for c_4^* and d_2^*

n	10		25		50	
m	c_4^*	d_2^*	c_4^*	d_2^*	c_4^*	d_2^*
1	9.00000	7.68007	24.00000	15.62977	49.00000	24.02990
2	17.78069	15.14589	47.76168	31.02740	97.75573	47.82145
5	44.09875	37.51556	119.0374	77.20616	244.0184	119.1869
10	87.95388	74.78859	237.8272	154.1660	487.7879	238.1261
25	219.5142	186.6017	594.1947	385.0424	1219.095	594.9419
50	438.7796	372.9550	1188.140	769.8356	2437.941	1189.634
100	877.3099	745.6608	2376.030	1539.422	4875.632	2379.019
200	1754.370	1491.072	4751.810	3078.593	9751.014	4757.787
300	2631.430	2236.483	7127.590	4617.765	14626.39	7136.556

agree with this statement, even when compared to the degrees of freedom for c_4^* (when n=10, the degrees of freedom for d_2^* is approximately 85% of that for c_4^*).

These results are significant when one considers the fact that degrees of freedom is equivalent to information about the process. The more (less) degrees of freedom retained in relation to the exact value when estimating the process variability, the more (less) information is obtained from the process. The more (less) information obtained from the process, the more (less) reliable are the control limits calculated using this information.

Another possible advantage to using two stage short run $(\overline{X}, s)$ control charts relates to Yang and Hillier's (1970) $(\overline{X}, \sqrt{v})$ control charts (which is mentioned earlier in the Introduction). Both sets of charts may be used for plotting means and standard deviations of subgroups. However, two stage short run $(\overline{X}, s)$ control charts may be easier to implement and maintain in a production environment. Control limits for two stage short run $(\overline{X}, \sqrt{v})$ charts must be constructed using subgroup variances. This means that both the variance and the standard deviation of each subgroup must be recorded. If just subgroup standard deviations are recorded after control limits are set, then one must perform additional calculations to get the variances from past subgroups when it is time to update the control limits. Considering the small loss in degrees of freedom for c_4^* compared to the exact degrees of freedom (which is used in two stage short run $(\overline{X}, \sqrt{v})$ control charts), two stage short run $(\overline{X}, s)$ control charts may have an advantage since one only has to calculate and record the subgroup standard deviation.

A final advantage to using two stage short run $(\overline{X}, s)$ control charts relates to Yang and Hillier's (1970) $(\overline{X}, v)$ control charts (which is mentioned earlier in the Introduction). Yang and Hillier (1970) state that $\bar{s}$ is less affected proportionally than $\bar{v}$ if the process has gone out-of-control with increased dispersion when any of the initial subgroups are drawn. Burr (1976) states two objections to using v control charts instead of s control charts. The first objection is that $\bar{v}$, the center line on a v control chart, will be more affected by a single large v than will $\bar{s}$ by the square root of this single large v. The end result would be a more highly inflated center line on the v control chart, creating a situation in which a special cause signal may not be detected. The second objection is that the distribution of v is far more unsymmetrical than that for s. The notes under Table 17 in the appendix of Wheeler (1995) state that this extreme skewness of the distribution of v makes the v control chart somewhat unsatisfactory.

Unbiased Estimates of σ and σ^2 Using $\bar{s}$

It is well known that $\bar{s}/c_4$ is an unbiased estimate of σ (see Wheeler's (1995) Tables

3.6, 3.7, and 4.2). A proof of this is given in Appendix D.1. It is also shown in Appendix D.1 that $\left(\bar{s}/c_4^*\right)^2$ is an unbiased estimate of σ^2. Since the value c_4^* is a new result from this chapter, this means that, for the first time, an unbiased estimate of the population variance may be obtained from the average of m standard deviations, each based on a subgroup of size n. Also, since c_4^* retains increasingly more degrees of freedom as n gets larger when compared to the degrees of freedom for d_2^*, the variability in $\left(\bar{s}/c_4^*\right)^2$ will be increasingly smaller than that for $\left(\bar{R}/d_2^*\right)^2$ as n gets larger ($\left(\bar{R}/d_2^*\right)^2$ is also an unbiased estimate of σ^2 (see Duncan (1955a, 1955b, 1955c) and Ott (1990))).

Conclusions

This chapter and the program it presents make important contributions to both industry and research. Those involved with quality control in industry will, for the first time, be able to use theoretically precise control chart factors to determine control limits for $(\bar{X}, s)$ charts regardless of the subgroup size, number of subgroups, and alpha values. Concerning research, this chapter provides a valuable reference for anyone interested in anything having to do with $(\bar{X}, s)$ control charts. Also, as already mentioned, the program eliminates the need for the research question of how many subgroups are enough before conventional control chart constants may be used.

CHAPTER VII

TWO STAGE SHORT RUN (X, MR) CONTROL CHARTS AND A COMPUTER PROGRAM TO CALCULATE THE FACTORS

Introduction

Hillier (1969) and Yang and Hillier (1970) represent the only attempts in the literature to develop two stage short run control charts based on Hillier's (1969) theory. Hillier (1969) derives equations to calculate two stage short run control chart factors for $(\overline{X}, R)$ charts. Yang and Hillier (1970) derive equations to calculate two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts.

Problem

It seems that no attempt appears in the literature to derive equations to calculate two stage short run control chart factors for (X, MR) charts. Del Castillo and Montgomery (1994) and Quesenberry (1995) both point out this deficiency. The application of (X, MR) control charts is desirable because in a short run situation, it may be difficult to form subgroups (Del Castillo and Montgomery (1994)).

Pyzdek (1993) attempts to present two stage short run control chart factors for (X, MR) charts for several values for numbers of subgroups and one value each for alpha for the X chart and alpha for the MR chart above the upper control limit (alpha is the probability of a Type I error). However, all of Pyzdek's (1993) Table 1 results for subgroup size one are incorrect because he uses invalid theory (this is explained in detail in the Tabulated Results of the Program section later in this chapter).

Solution

This chapter presents a solution to this problem, consequently allowing for the derivation of equations to calculate first and second stage short run control chart factors for (X, MR) charts. It also describes the development and execution of a computer program that will accurately calculate the factors using these derived equations. Other exact equations that the program uses are the probability integral of the range, the mean of the distribution of the range, the probability integral of the studentized range (all three for subgroup size two), equations to calculate degrees of freedom, and derived conventional control chart equations. The program accepts values for number of subgroups, alpha for the X chart, and alpha for the MR chart both above the upper control limit and below the lower control limit. Tables are generated for specific values of these inputs. Comparison of the tabulated results to legitimate results in the literature validates the program. The tables correct and extend previous results in the literature.

The software used for the program is *Mathcad 8.03 Professional* (1998) with the *Numerical Recipes Extension Pack* (1997). The program uses numerical routines provided by the software.

Outline

This chapter first presents the probability integrals of the range and the studentized range, both for subgroup size two. These are essential in the application of Hillier's (1969) theory to (X, MR) control charts and are required for the program to perform accurate calculations. Next, Patnaik's (1950) theory is used to develop an approximation to the distribution of the mean moving range. From this result, equations to calculate two stage short run control chart factors for (X, MR) charts are derived by following the work in the appendix of Hillier (1969). Also, equations to calculate conventional control chart constants for (X, MR) charts are derived. Next, the computer program is described. Tables generated by the program are then presented and compared with legitimate results in the literature. Also, implications of the tabulated results are discussed. Following a numerical example that illustrates the use of the program, unbiased estimates of σ and σ^2 using $\overline{MR}$ are given, as well as final conclusions describing the impact of the program on industry and research.

Note

Results from the program are for processes generating parts with independent measurements that follow a Normal distribution.

The Probability Integral of the Range for Subgroup Size Two

The probability integral (or cumulative distribution function (cdf)) of the range for subgroups of size two sampled from a standard Normal population is given by Pachares (1959) as equation (7.1) (with some modifications in notation):

$$P(W) = 2 \cdot \int_{-\infty}^{\infty} f(x) \cdot (F(x+W) - F(x))dx \tag{7.1}$$

W represents the (standardized) range w/σ, where w is the range of a subgroup and σ is the population standard deviation. Throughout this chapter, F(x) is the cdf of the standard Normal probability density function (pdf) f(x).

The mean of the distribution of the range $W = (w/\sigma)$ for subgroups of size two sampled from a Normal population with mean μ and variance equal to one given by Harter (1960) is equation (7.2) (with some modifications in notation):

$$d2 = \frac{2}{\pi^{0.5}} \tag{7.2}$$

The value d2 is the control chart constant denoted by d_2 (see Table M in the appendix of Duncan (1974)). The equation for d2 for subgroup size two for any value of σ is given by Johnson, Kotz, and Balakrishnan (1994).

The Probability Integral of the Studentized Range for Subgroup Size Two

The probability integral of the studentized range for subgroups of size two sampled from a Normal population is given by Harter, Clemm, and Guthrie (1959) as equation (7.3a):

$$P3(z) = \left(\frac{5}{z}\right) \cdot e^{cv} \cdot (P1(z) + P2(z)) \tag{7.3a}$$

where

$$cv = \ln(2) + \left(\frac{v}{2}\right)\cdot\ln\left(\frac{v}{2}\right) - \left(\frac{v}{2}\right) - \text{gammln}\left(\frac{v}{2}\right) \tag{7.3b}$$

$$P1(z) = \int_0^{11} \left[\left(5\cdot\frac{W}{z}\right)\cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}}\right]^{v-1} \cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}} \cdot P(W)\,dW \tag{7.3c}$$

$$P2(z) = \left(\frac{z}{5}\right)\cdot\int_{\frac{55}{z}}^{\infty}\left(x\cdot e^{\frac{1-x^2}{2}}\right)^{v-1}\cdot e^{\frac{1-x^2}{2}}\,dx \tag{7.3d}$$

The variable z is equal to $5\cdot Q$. Q represents the studentized range w/s, where w is the range of a subgroup and s is an independent estimate (based on ν degrees of freedom) of the population standard deviation. The equation for cν (equation (7.3b)) is the natural logarithm of the equation for C(ν) given by Harter, Clemm, and Guthrie (1959). It is derived in Appendix B.1 of this dissertation. The function gammln is a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that calculates the natural logarithm of the gamma function. Using gammln in equation (7.3b) allows for large values of ν (hence large values for m (the number of subgroups)) in the program. In equation (7.3c), P(W) is the probability integral of the range $W = (w/\sigma)$ for subgroup size two (see equation (7.1)).

As ν→∞ (i.e., as m→∞), the distribution of the studentized range $Q = (w/s)$ for subgroup size two converges to the distribution of the range $W = (w/\sigma)$ for subgroup size two (see Pearson and Hartley (1943)). This fact is used to calculate alpha-based conventional control chart constants for the MR chart.

The Distribution of the Mean Moving Range

Consider the situation in which the mean of a statistic is calculated by averaging m values of the statistic, each of which is based on a subgroup of size n. Patnaik (1950) investigates this situation when the statistic is the range and develops an approximation to the distribution of the mean range $\overline{R}/\sigma$. The resulting distribution is the $(\chi \cdot d_2^*)/\sqrt{\nu}$ distribution, which is a function of the χ distribution with ν degrees of freedom (the χ distribution with ν degrees of freedom and its moments about zero may be found in Johnson and Welch (1939)). Equations for ν and d_2^* are derived from results obtained by equating the squared means as well as the variances of the distribution of the mean range $\overline{R}/\sigma$ and the $(\chi \cdot d_2^*)/\sqrt{\nu}$ distribution with ν degrees of freedom. Hillier (1964 and 1967) uses Patnaik's (1950) theory to derive equations to calculate short run control chart factors for $\overline{X}$ and R charts, respectively. Hillier (1969) then incorporates the two stage procedure into his short run control chart factor calculations for $(\overline{X}, R)$ charts.

Consider the situation in which the statistic is the moving range of size two and the distribution of interest is the distribution of the mean moving range $\overline{MR}/\sigma$. Evidence exists in the literature that $\overline{MR}/\sigma$ may be approximated by a distribution that is a function of either the χ^2 or the χ distribution. Sathe and Kamat (1957) use results given by Cadwell (1953, 1954) to approximate the distribution of the mean successive difference (i.e., the distribution of the mean moving range $\overline{MR}/\sigma$) by a distribution that is a function of a power of the χ^2 distribution. Roes, Does, and Schurink (1993) use theory that is similar to Patnaik's (1950) theory to approximate the distribution of the mean moving range $\overline{MR}/\sigma$ (with σ=1.0) by a distribution that is a function of the χ distribution.

In order to be able to use Hillier's (1969) theory to derive equations to calculate two stage short run control chart factors for (X, MR) charts, we apply Patnaik's (1950) theory to approximate the distribution of the mean moving range $\overline{MR}/\sigma$ by the $\left(\chi \cdot d_2^*(MR)\right)/\sqrt{v}$ distribution with ν degrees of freedom (this ν is the same as the one given earlier in equation (7.3a)). The equation for $d_2^*(MR)$ is derived in Appendix E.1 of this dissertation and is given as equation (7.4) (note: $d2starMR \equiv d_2^*(MR)$):

$$d2starMR = \left(d2^2 + d2^2 \cdot r\right)^{0.5} \tag{7.4}$$

The equation for the control chart constant d2 for subgroup size two is given earlier as equation (7.2). The value r represents the variance of $\overline{MR}/d2$. Its equation is given later as equation (7.7a).

Using results from Prescott (1971), the equation for ν is determined by equating the ratio of the variance to the squared mean, both of the χ distribution with ν degrees of freedom, to the ratio of the variance to the squared mean, both of the distribution of the mean moving range $\overline{MR}/\sigma$. The resulting equation for ν is equation (7.5):

$$d(x) = h(x) - r \tag{7.5}$$

The exact value for ν is the value of x such that d(x) is equal to zero. The function h(x) is the ratio of the variance to the squared mean, both of the χ distribution with x degrees of freedom (x replaces ν). The mean and variance of the χ distribution with ν degrees of freedom are given in Appendix E.1. The equation for h(x), which is derived in Appendix B.1, is given as equation (7.6):

$$h(x) = \frac{x \cdot e^{2\cdot(\text{gammln}(0.5\cdot x) - \text{gammln}(0.5\cdot x + 0.5))} - 2}{2} \tag{7.6}$$

The value r is the ratio of the variance to the squared mean, both of the distribution of the mean moving range $\overline{MR}/\sigma$. The mean and the variance of the distribution of the mean moving range $\overline{MR}/\sigma$ are derived in Appendix E.1. The equation for r is given by Palm and Wheeler (1990) as equation (7.7a):

$$r = \frac{b \cdot (m-1) - c}{(m-1)^2} \tag{7.7a}$$

where

$$b = \frac{2 \cdot \pi}{3} - 3 + 3^{0.5} \tag{7.7b}$$

$$c = \frac{\pi}{6} - 2 + 3^{0.5} \tag{7.7c}$$

Cryer and Ryan (1990) give an equivalent form for equation (7.7a). Hoel (1946) gives an equation for the variance of $\overline{MR}$ which, when multiplied by $1/d2^2$, gives the same results as those obtained by using equation (7.7a). It should be noted that an equivalent form (also based on Patnaik's (1950) theory) of equation (7.5) may be found in Palm and Wheeler (1990), who use their result to calculate equivalent degrees of freedom for population standard deviation estimates based on consecutive overlapping moving ranges of size two.

Table E.3.1 (the creation of which is explained in the Tabulated Results of the Program section later in this chapter) in Appendix E.3 of this dissertation has ν and $d_2^*(\text{MR})$ values for m: 2-20, 25, 30, 50, 75, 100, 150, 200, 250, 300, as well as d_2 for subgroup size two. As $m \to \infty$ (i.e., as $\nu \to \infty$), $d_2^*(\text{MR})$ converges to d_2 for subgroup size two.

Approximating the distribution of the mean moving range $\overline{\text{MR}}/\sigma$ by the $\left(\chi \cdot d_2^*(\text{MR})\right)/\sqrt{\nu}$ distribution with ν degrees of freedom works well. In fact, based on how $d_2^*(\text{MR})$ is derived in Appendix E.1, the means and variances of these two distributions are equal.

Derivation of the Control Chart Factor Equations

Since the $\left(\chi \cdot d_2^*(\text{MR})\right)/\sqrt{\nu}$ distribution with ν degrees of freedom approximates the distribution of the mean moving range $\overline{\text{MR}}/\sigma$, the derivation of equations to calculate first and second stage short run control chart factors for (X, MR) charts follows the work in the appendix of Hillier (1969). E22, the second stage short run control chart factor for the X chart, is derived in almost the same manner as Hillier's (1969) A_2^*. Differences are that n=1 and X, $\overline{X}$, E22, $\overline{\text{MR}}$, and $d_2^*(\text{MR})$ in this chapter replace $\overline{X}$, $\overline{\overline{X}}$, A_2^*, $\overline{R}$, and c, respectively, in Hillier (1969). The resulting equation for E22 is given as equation (7.8) (note: $\text{d2starMR} \equiv d_2^*(\text{MR})$):

$$E22 = \left(\frac{\text{crit\_t}}{\text{d2starMR}} \right) \cdot \left(\frac{m+1}{m} \right)^{0.5} \tag{7.8}$$

The value crit_t is the critical value for a cumulative area of $(1 - (\text{alphaInd}/2))$ under the Student's t curve with ν degrees of freedom (alphaInd is the probability of a Type I error on the X control chart).

E21, the first stage short run control chart factor for the X chart, is derived in almost the same manner as Hillier's (1969) A_2^{**}. Differences are that E21, X_i, $\overline{X}$, $\overline{MR}$, and $d_2^*(\text{MR})$ in this chapter replace A_2^{**}, $\overline{X}_i$, $\overline{\overline{X}}$, $\overline{R}$, and c, respectively, in Hillier (1969). The resulting equation for E21 is given as equation (7.9):

$$E21 = \left(\frac{\text{crit\_t}}{\text{d2starMR}} \right) \cdot \left(\frac{m-1}{m} \right)^{0.5} \tag{7.9}$$

The value crit_t has the same meaning here as in equation (7.8).

D42, the second stage short run upper control chart factor for the MR chart, is derived in Appendix E.1. Other than differences in notation, this derivation follows that for Hillier's (1969) D_4^*. The resulting equation for D42 is given as equation (7.10):

$$D42 = \frac{qD4}{d2starMR} \tag{7.10}$$

The value qD4 is the (1-alphaMRUCL) percentage point of the distribution of the studentized range $Q = (w/s)$ for subgroup size two with ν degrees of freedom (alphaMRUCL is the probability of a Type I error on the MR chart above the upper control limit).

D32, the second stage short run lower control chart factor for the MR chart, is derived in a manner similar to D42. Differences are that D32, qD3, and alphaMRLCL replace D42, qD4, and (1-alphaMRUCL), respectively (alphaMRLCL is the probability of a Type I error on the MR chart below the lower control limit). The resulting equation for D32 is given as equation (7.11):

$$D32 = \frac{qD3}{d2starMR} \tag{7.11}$$

The value qD3 is the alphaMRLCL percentage point of the distribution of the studentized range $Q = (w/s)$ for subgroup size two with ν degrees of freedom.

D41, the first stage short run upper control chart factor for the MR chart, is derived in almost the same manner as Hillier's (1969) D_4^{**}. Differences are that D41, MR_i, D42, and $\overline{MR}$ in this chapter replace D_4^{**}, R_i, D_4^{*}, and $\overline{R}$, respectively, in Hillier (1969). The resulting equation for D41 is given as equation (7.12):

$$D41 = \frac{m \cdot qD4prevm}{d2starMRprevm \cdot (m-1) + qD4prevm} \tag{7.12}$$

The value qD4prevm has the same meaning as qD4 (given earlier in equation (7.10)), except it is for νprevm (i.e., ν for (m-1) subgroups). The value d2starMRprevm has the same equation as d2starMR (given earlier as equation (7.4)), except m is replaced with (m-1).

The equation for D31, the first stage short run lower control chart factor for the MR chart, is derived in almost the same manner as Hillier's (1969) D_3^{**}. Differences are that D31, MR_i, D32, and $\overline{MR}$ in this chapter replace D_3^{**}, R_i, D_3^{*}, and $\overline{R}$, respectively, in Hillier (1969). The resulting equation for D31 is given as equation (7.13):

$$D31 = \frac{m \cdot qD3prevm}{d2starMRprevm \cdot (m-1) + qD3prevm} \tag{7.13}$$

The value qD3prevm has the same meaning as qD3 (given earlier in equation (7.11)), except it is for νprevm instead of ν.

The equation for E2, the conventional control chart constant for the X chart, may be obtained by taking the limit of either E21 or E22 as m→∞ (i.e., as ν→∞). The resulting equation for E2 is given as equation (7.14):

$$E2 = \frac{crit\_z}{d2} \tag{7.14}$$

The value crit_z is the critical value for a cumulative area of $(1-(\text{alphaInd}/2))$ under the standard Normal curve. The equation for the control chart constant d2 for subgroup size two is given earlier as equation (7.2).

The equation for D4, the alpha-based conventional upper control chart constant for the MR chart, may be obtained by taking the limit of either D41 as $m \rightarrow \infty$ (i.e., as $\text{vprevm} \rightarrow \infty$) or D42 as $m \rightarrow \infty$ (i.e., as $\nu \rightarrow \infty$). The resulting equation for D4 is given as equation (7.15):

$$D4 = \frac{wD4}{d2} \qquad (7.15)$$

The value wD4 is the (1-alphaMRUCL) percentage point of the distribution of the range $W = (w/\sigma)$ for subgroup size two.

The equation for D3, the alpha-based conventional lower control chart constant for the MR chart, may be obtained by taking the limit of either D31 as $m \rightarrow \infty$ (i.e., as $\text{vprevm} \rightarrow \infty$) or D32 as $m \rightarrow \infty$ (i.e., as $\nu \rightarrow \infty$). The resulting equation for D3 is given as equation (7.16):

$$D3 = \frac{wD3}{d2} \qquad (7.16)$$

The value wD3 is the alphaMRLCL percentage point of the distribution of the range $W = (w/\sigma)$ for subgroup size two.

The Computer Program

This section of the chapter presents the computer program, which is in Appendix E.2 of this dissertation. The program has seven pages, each of which is further divided into sections.

Mathcad (1998) Note

It is possible for a section of code in the program to turn red and have the error message "Unknown Error". To correct this, delete one character in the red code and type it back in. Click the mouse arrow outside of the code. The code should turn black, indicating that the error has been eliminated. If not, repeat the procedure (it will eventually correct the problem).

Page 1

The first page of the program begins with the data entry section. The program requires the user to enter the following values: alphaInd (alpha for the X chart), alphaMRUCL (alpha for the MR chart above the UCL), alphaMRLCL (alpha for the MR chart below the LCL), and m (number of subgroups (i.e., the number of MRs plus one)). If no lower control limit on the MR chart is desired, the entry for alphaMRLCL should be left blank (do not enter zero). Before a value can be entered, the cursor must be moved to the right side of the appropriate equal sign. This may be done using the arrow keys on the keyboard or by moving the mouse arrow to the right side of the equal sign and clicking once with the left mouse button. The program is activated by paging down once the last entry is made. When using *Mathcad 8.03 Professional* (1998), paging down is not allowed while a calculation is taking place. However, *Mathcad 2000 Professional* (1999) allows the user to page down to the output section of the program (explained later) after the last entry is made.

The next part of page 1 is section 1.1 of the program. The value TOL is the tolerance. The calculations that use this value will be accurate to ten places to the right of the decimal. The functions dnorm(x, 0, 1) and pnorm(x, 0, 1) in *Mathcad* (1998) are the pdf and cdf, respectively, of the standard Normal distribution. The equations for the pdf and cdf are also given in case the dnorm or pnorm function fails to calculate a result. In *Mathcad* (1998), an equation turns red if it does not calculate a result due to an error. If the dnorm function gives an error, type f(x) on the left side of the equal sign in equation (7.17):

$$= \left[(2 \cdot \pi)^{-0.5}\right] \cdot e^{\frac{-x^2}{2}} \tag{7.17}$$

If the pnorm function gives an error, type F(x) on the left side of the equal sign in equation (7.18):

$$= \int_0^x f(t)\,dt \tag{7.18}$$

The equations for P(W) and d2 are given earlier as equations (7.1) and (7.2), respectively.

<u>Page 2</u>

Page 2 of the program begins with section 2.1. The code in this section determines wD4 and wD3, the (1-alphaMRUCL) and alphaMRLCL percentage points, respectively, of the distribution of the range $W = (w/\sigma)$ for subgroup size two and infinite ν (i.e., infinite m) (recall the earlier statement that as $\nu \to \infty$ (i.e., as $m \to \infty$), the distribution of the studentized range $Q = (w/s)$ for subgroup size two converges to the distribution of the range $W = (w/\sigma)$ for subgroup size two). The value wD4 is used in the equation for D4, which is given earlier as equation (7.15). The value wD3 is used in the equation for D3, which is given earlier as equation (7.16). The roots of the equations DUCL(W) and DLCL(W) are wD4 and wD3, respectively, and are determined using zbrent (a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that uses Brent's method to find the roots of an equation). The subprograms Wseed1 and Wseed2 generate seed values seedD4 and seedD3, respectively, for Brent's method.

The subprogram Wseed1 works as follows. Initially, W_0 and W_1 are set equal to 0.01 and 0.02, respectively. A_0 and A_1 result from evaluating DUCL(W) at W_0 and W_1, respectively. The while loop begins by checking if the product of A_0 and A_1 is negative. If so, the root for DUCL(W) lies between 0.01 and 0.02. If not, W_0 and W_1 are incremented by 0.01. A_0 and A_1 are recalculated and if their product is negative, the root for DUCL(W) lies between 0.02 and 0.03. Otherwise, the while loop repeats. Once a root for DUCL(W) is bracketed, the bracketing values are passed out of the subprogram into the 2×1 vector seedD4 to be used by Brent's method to determine wD4. The subprogram Wseed2 works similarly to construct the 2×1 vector seedD3 to be used by

Brent's method to determine wD3, except the starting value is 0.001.

The next part of page 2 is section 2.2 of the program. As shown earlier, the two stage short run control chart factor calculations require ν and νprevm. The equation for h(x) is described earlier (see equation (7.6)). The value rprevm has the same meaning as r described earlier (see equation (7.7a)), except it is for (m-1) subgroups. The equations for b and c are given earlier as equations (7.7b) and (7.7c), respectively. The equation for dprevm(x) is the same as that for d(x) (given earlier as equation (7.5)), except rprevm replaces r. The value ν is the root of the equation d(x) and is determined using zbrent with seed value seedν. The value νprevm is the root of the equation dprevm(x) and is determined using zbrent with seed value seedνprevm. The subprogram dfseed generates the seed values seedν and seedνprevm for Brent's method.

The subprogram dfseed works as follows. Initially, df_0 and df_1 are set equal to 0.9 and 1.1, respectively. A_0 and A_1 result from evaluating y(x) (which is equal to either d(x) or dprevm(x)) at df_0 and df_1, respectively. The while loop begins by checking if the product of A_0 and A_1 is negative. If so, the root for y(x) lies between 0.9 and 1.1. If not, df_0 and df_1 are incremented by 0.5. A_0 and A_1 are recalculated and if their product is negative, the root for y(x) lies between 1.1 and 1.6. Otherwise, the while loop repeats. Once a root for y(x) is bracketed, the bracketing values are passed out of the subprogram into the 2×1 vector seedν (if y(x) is equal to d(x)) or seedνprevm (if y(x) is equal to dprevm(x)) to be used by Brent's method to determine ν or νprevm, respectively.

Page 3

Page 3 of the program begins with section 3.1. The equations for P3(z), cν, P1(z), and P2(z) are given earlier as equations (7.3a), (7.3b), (7.3c), and (7.3d), respectively. Section 3.2 contains the calculations required to determine qD4, the (1-alphaMRUCL) percentage point of the distribution of the studentized range $Q = (w/s)$ for subgroup size two with ν degrees of freedom (which is calculated earlier in the program). The value qD4 is used in the equation for D42, which is given earlier as equation (7.10). The subprogram Zseed1 generates the seed value seed1 for Brent's method or for root (root is a numerical routine in *Mathcad* (1998) that uses the Secant method for determining the roots of an equation). Either root-finding method determines the root of D(x). The result of dividing this root by five is qD4. Both Brent's method and the Secant method are given because one may not work when the other one does. If Brent's method fails, type qD4 on the left side of the equal sign in equation (7.19):

$$= \frac{\text{root}\left[\left|\text{P3(seed1)} - (1 - \text{alphaMRUCL})\right|, \text{seed1}\right]}{5} \tag{7.19}$$

The subprogram Zseed1 begins by generating values for Z_0 and Z_1. A_0 and A_1 result from evaluating P3(z) at Z_0 and Z_1, respectively. The while loop continually increments Z_0 and Z_1 by 5.0 and evaluates P3(z) at these two values until A_1 becomes greater than (1-alphaMRUCL) for the first time, at which point A_0 will be less than (1-alphaMRUCL). When this occurs, P3(z) is equal to (1-alphaMRUCL) for some value z between Z_0 and Z_1. An initial guess for this value is determined using linterp (a numerical routine in *Mathcad* (1998) that performs linear interpolation) and stored in Zguess. The initial guess is passed out of the subprogram as seed1.

Page 4

Page 4 of the program is section 4.1. The code in this section is used to determine qD3, the alphaMRLCL percentage point of the distribution of the studentized range $Q = (w/s)$ for subgroup size two with ν degrees of freedom (which is calculated earlier in the program). The value qD3 is used in the equation for D32, which is given earlier as equation (7.11). The subprogram Zseed2 generates the value seed2 that is used to determine an initial value for qD3. An improved value for qD3 is then calculated by determining the root of the equation (P3(z)-alphaMRLCL) using the Secant method with the seed value seed2 and dividing this root by five.

The ability of the Secant method to calculate a result depends upon the values for alphaMRLCL and m (Brent's method should not be used). It is not a problem if it does not calculate a result because the initial value for qD3 and the improved value match to several places to the right of the decimal. This phenomenon is discussed in more detail when the tabulated results of the program are presented later in this chapter. The Monitor Results area in the bottom right hand corner of section 4.1 indicates how closely the two values for qD3 match until the root routine fails. This will dictate the number of decimal places that can be used to display qD3 and the second stage short run lower control chart factor for the MR chart.

The code in the subprogram Zseed2 that begins with the first line of code and includes the while loop and the two for loops constructs 21×1 vectors Zv for z and Av for P3(z). The first row of each vector is zero. The while loop determines the first value Z where P3(Z) is greater than alphaMRLCL. This Z and the corresponding value P3(Z) are stored in the second rows of Zv and Av, respectively. The two for loops generate values for the remaining rows of Zv and Av. Two different for loops are used because P3(z) may encounter an error for some i (i: 1, 2, ..., 20). The value for i where the error occurs can be skipped using the dual for loop construction. When the execution of this section of code is complete, P3(z) is equal to alphaMRLCL for some value z between Zv_0 and Zv_1.

The code in the subprogram Zseed2 that starts in the line where the variable Zguess first appears to the last line of the subprogram is derived from Harter, Clemm, and Guthrie (1959). This code searches for and estimates the value z where P3(z) is equal to alphaMRLCL. Zguess is the initial guess for this value z. It is determined using linterp, the 21×1 vectors for P3(z) and z previously determined, and alphaMRLCL. The 2×1 vector A is determined using ratint (a numerical recipe in the *Numerical Recipes Extension Pack* (1997) that performs rational interpolation), the 21×1 vectors for z and P3(z), and Zguess. Aguess is the entry in the first row of A and is the estimated value for P3(Zguess). The while loop first checks if Aguess is an accurate estimate (within 10^{-15}) of alphaMRLCL. If so, Zguess is passed out of the subprogram as the value seed2. If not, Aguess and Zguess are entered into the second rows of the previously determined vectors Av and Zv, respectively, if Aguess is more than 10^{-15} larger than alphaMRLCL. If Aguess is more than 10^{-15} smaller than alphaMRLCL, Aguess and Zguess are entered into the first rows of the vectors Av and Zv, respectively. New values for Zguess and

Aguess are determined using the same procedure as before and execution is returned to the beginning of the while loop.

Page 5

Page 5 of the program contains sections 5.1 and 5.2. These sections correspond to sections 3.1 and 3.2, respectively, described earlier. The only difference is that the calculations in sections 5.1 and 5.2 use νprevm instead of ν. The calculations are for qD4prevm, which is used in the equation for D41 (given earlier as equation (7.12)).

Page 6

Page 6 of the program is section 6.1. This section corresponds to section 4.1 described earlier. The only difference is that the calculations in section 6.1 use νprevm instead of ν. The calculations are for qD3prevm, which is used in the equation for D31 (given earlier as equation (7.13)).

Page 7

Page 7 of the program begins with section 7.1. It has the equations for d2starMR (given earlier as equation (7.4)) and d2starMRprevm (d2starMR for (m-1) subgroups). The value d2starMR is used in the equations for E22, E21, D42, and D32, all of which are given earlier as equations (7.8), (7.9), (7.10), and (7.11), respectively. The value d2starMRprevm is used in the equations for D41 and D31, which are given earlier as equations (7.12) and (7.13), respectively. The function qt(adj_alpha, ν) in *Mathcad*

(1998) determines the critical value crit_t for a cumulative area of adj_alpha under the Student's t curve with ν degrees of freedom. The value crit_t is used in the equations for E21 and E22. The function qnorm(adj_alpha, 0, 1) in *Mathcad* (1998) determines the critical value crit_z for a cumulative area of adj_alpha under the standard Normal curve. The value crit_z is used in the equation for E2 (given earlier as equation (7.14)).

Section 7.2 of the program has the equations to calculate two stage short run control chart factors and conventional control chart constants given earlier in the Derivation of the Control Chart Factor Equations section of this chapter. The equation for E2 is a generalization of the equation for E_2 from Wheeler's (1995) Tables 3 and 4 to allow for different values of alphaInd.

The last part of page 7 is the output section of the program. The four values entered at the beginning of the program are given. The control chart factors are broken down into first stage, second stage, and conventional. The values for ν, d2starMR, νprevm, and d2starMRprevm, the mean of the distribution of the range $W = (w/\sigma)$ for subgroup size two and the variance of the distribution of the mean moving range $\overline{MR}/\sigma$, and Harter, Clemm, and Guthrie's (1959) Table II.2 results for n=2 (i.e., for subgroup size two) complete the output of the program. To copy results into another software package (like Excel), follow the directions from *Mathcad's* (1998) help menu or highlight a value and copy and paste it into the other software package. When highlighting a value with the mouse arrow, place the arrow in the middle of the value, depress the left mouse button, and drag the arrow to the right. This will ensure just the numerical value of the result is copied and pasted.

Tabulated Results of the Program

The three tables (Tables E.3.1-E.3.3) in Appendix E.3 were generated using the program with the following input values:

- alphaInd=0.0027, alphaMRUCL=0.005, alphaMRLCL=0.001
- m: 2-20, 25, 30, 50, 75, 100, 150, 200, 250, 300

The values ν, d2starMR, νprevm, d2starMRprevm, and d2 are in Table E.3.1. The ν and νprevm values compare favorably to the equivalent degrees of freedom in Table 3 of Palm and Wheeler (1990) and Table 23 in the appendix of Wheeler (1995). The d2 value compares favorably to the d_2 value for subgroup size two in Duncan's (1974) Table M and Wheeler's (1995) Tables 1 and 18.

The values qD4, qD4prevm, and wD4, as well as qD3, qD3prevm, and wD3, are in Table E.3.2. The results in these tables compare favorably to Harter, Clemm, and Guthrie's (1959) Table II.2 results for n=2 (i.e., for subgroup size two).

As explained earlier in the Page 4 subsection of The Computer Program section of this chapter, in the calculations for qD3 and qD3prevm, the ability of the Secant method to calculate a result depends upon the values for alphaMRLCL and m. For Table E.3.2, the Secant method fails to work for m≥3. As mentioned previously, this is not a serious issue. The reason is that the initial value for qD3 matches the improved value for qD3 (before the Secant method fails) to eight places to the right of the decimal.

Values for E21, D41, D31, E22, D42, D32, E2, D4, and D3 are in Table E.3.3. The

E2 value compares favorably to the E_2 value for n=2 in Wheeler's (1995) Table 4. It should be noted that the values wD4 and wD3 in Table E.3.2 and D4 and D3 in Table E.3.3 may differ in the ninth or tenth decimal place for different root routines used to calculate wD4 and wD3.

These favorable comparisons validate the program. Consequently, Table E.3.3 results for m: 2-10, 15, 20, 25 may be considered corrections to Pyzdek's (1993) Table 1 for subgroup size one. All of Pyzdek's (1993) Table 1 results for subgroup size one are incorrect for two reasons. The first is that he uses degrees of freedom based on Patnaik's (1950) approximation applied to the distribution of the mean range $\overline{R}/\sigma$, where $\overline{R}$ is the average of m values of R (the range), each based on a subgroup of size two, not the distribution of the mean moving range $\overline{MR}/\sigma$. In the latter case, the degrees of freedom reflect the fact that serial correlation exists among consecutive overlapping moving ranges of size two, which means that the average of these overlapping MRs reflects that serial correlation. The result is that degrees of freedom based on Patnaik's (1950) approximation applied to the distribution of the mean moving range $\overline{MR}/\sigma$ is less than that from applying Patnaik's (1950) approximation to the distribution of the mean range $\overline{R}/\sigma$, where R is the range of a subgroup of size two.

The second is that Pyzdek (1993) uses the equation for d_2^* (i.e., d2star) instead of that for d2starMR (given earlier as equation (7.4)). The equation for d_2^* is given as equation (7.20):

$$d_2^* = \left(d_2^2 + \frac{d_3^2}{m} \right)^{0.5} \tag{7.20}$$

where d_2 and d_3 are the mean and standard deviation, respectively, of the distribution of the range $W = (w/\sigma)$. Equations to calculate d_2 and d_3 for any subgroup size as well as the equation for d_2^* may be found in Chapter IV of this dissertation.

The difference between equations (7.4) and (7.20) is that equation (7.4) has $d2^2 \cdot r$, which is the variance of the distribution of the mean moving range $\overline{MR}/\sigma$, instead of d_3^2/m, which is the variance of the distribution of the mean range $\overline{R}/\sigma$. The equation for r in $d2^2 \cdot r$ reflects the fact that serial correlation exists among consecutive overlapping moving ranges of size two, which means that the average of these overlapping MRs reflects that serial correlation. The result is that values for d2starMR are less than those for d2star for subgroup size two; but, as m→∞, both converge to d2. It should be noted that d2starMR for m=2 is equal to d2star for n=2 and m=1 (see Table B.3.1 in Appendix B.3 of this dissertation).

One last issue regarding Pyzdek's (1993) Table 1 results is that he gives second stage short run control chart factors for number of subgroups equal to one. This is clearly an impossibility because one must have two subgroups in order to calculate one moving range. The results in Table E.3.3 show that for stage one short run control chart factors for the individuals and moving range charts, m must be at least two and three, respectively. For stage two short run control chart factors for the individuals and moving range charts, m must be at least two.

Implications of the Tabulated Results

Values in Table E.3.3 show some interesting properties. As m increases, the E22 and D42 values converge in a decreasing manner to E2 and D4, respectively. The D32 values also converge in a decreasing manner to D3, though it is not evident from the accuracy shown. This convergence makes sense because more information about the process is at hand for larger m.

These properties have major implications. A common rule of thumb is that 20 to 30 subgroups of size 4 or 5 are necessary to use conventional control chart constants for constructing control limits. The results in Table E.3.3 indicate that this may be an incorrect rule when applied to constructing (X, MR) control charts. Consider again the E22 values and E2 in Table E.3.3. E2 is 20.709% smaller than E22 for m=20 and 13.915% smaller than E22 for m=30. These results indicate that if one were to construct X charts using the conventional control chart constant E2 when only 20 to 30 subgroups of size one are available to estimate the process mean and standard deviation, the upper and lower control limits would not be wide enough, resulting in a higher false alarm rate.

D42 values and D4 in Table E.3.3 also indicate that the common rule of thumb, when applied to constructing (X, MR) control charts, may be an incorrect rule. D4 is 16.513% smaller than D42 for m=20 and 10.975% smaller than D42 for m=30. Consequently, if one were to construct the upper control limit of MR charts using the conventional control chart constant D4 when only 20 to 30 subgroups of size one are available to estimate the process standard deviation, the upper control limit would not be wide enough, resulting in a higher false alarm rate.

To the accuracy shown in Table E.3.3, there is little difference between D32 for any m and D3. If increased accuracy is used, then D3 is slightly less than D32 for any m. Consequently, if one were to construct the lower control limit of MR charts using the conventional control chart constant D3 when only 20 to 30 subgroups of size one are available to estimate the process standard deviation, the lower control limit would be slightly too wide, possibly creating a situation in which the probability of detecting a special cause signal is slightly diminished.

Quesenberry (1993) also investigated the validity of the common rule of thumb when applied to constructing (X, MR) control charts and concluded that 300 individual values are needed for the X chart before conventional control chart constants may be used. However, for all practical purposes, the program presented by this chapter eliminates the need for these rules.

A Numerical Example

Consider the data in Table 7.1 obtained from a process requiring short run control charting techniques (assume alphaInd=0.0027, alphaMRUCL=0.005, and alphaMRLCL=0.001). For m=5, the following first stage short run control chart factors for the MR chart are obtained from Table E.3.3: D41=3.83736 and D31=0.00196. UCL(MR) and LCL(MR) are calculated as follows:

$$UCL(MR) = D41 \cdot \overline{MR} = 3.83736 \cdot 0.03875 = 0.14870$$

$$LCL(MR) = D31 \cdot \overline{MR} = 0.00196 \cdot 0.03875 = 0.000076$$

Table 7.1. A Numerical Example

Subgroup	X	MR
1	1.280	-----
2	1.129	0.151
3	1.130	0.001
4	1.131	0.001
5	1.133	0.002
Averages	1.16060	0.03875
Revised Average		0.00133

The first moving range (MR=0.151) is above UCL(MR). Find, investigate, and remove from the process the special cause (or causes) that created this out of control point, delete the first moving range, recalculate the average moving range (shown as the Revised Average in Table 7.1), and construct second stage control limits for the (X, MR) charts (this approach is from Case (1998)). For m=4, the following second stage short run control chart factors for the MR chart are obtained from Table E.3.3: D42=13.20218 and D32=0.00157. For m=5, the following second stage short run control chart factor for the X chart is obtained from Table E.3.3: E22=9.00182. UCL(MR), LCL(MR), UCL(X), and LCL(X) are calculated as follows:

$$UCL(MR) = D42 \cdot \overline{MR} = 13.20218 \cdot 0.00133 = 0.017559$$

$$LCL(MR) = D32 \cdot \overline{MR} = 0.00157 \cdot 0.00133 = 0.0000021$$

$$UCL(X) = \overline{X} + E22 \cdot \overline{MR} = 1.16060 + 9.00182 \cdot 0.00133 = 1.17257$$

$$LCL(X) = \overline{X} - E22 \cdot \overline{MR} = 1.16060 - 9.00182 \cdot 0.00133 = 1.14863$$

These control limits may be used to monitor the future performance of the process.

Unbiased Estimates of σ and σ^2 Using $\overline{MR}$

It is well known that $\overline{MR}/d_2$ is an unbiased estimate of σ (e.g., see Wheeler's (1995) Table 3.7). A proof of this is given in Appendix E.1. It is also shown in Appendix E.1 that $\left(\overline{MR}/d_2^*(MR)\right)^2$ is an unbiased estimate of σ^2. Since the value $d_2^*(MR)$ is a new result from this chapter, this means that, for the first time, an unbiased estimate of the population variance may be obtained from the average of m moving ranges, each based on a subgroup of size two.

Conclusions

This chapter and the program it presents make important contributions to both industry and research. Those involved with quality control in industry will, for the first time, be able to use theoretically precise control chart factors to determine control limits for (X, MR) charts regardless of the number of subgroups and alpha values. This is valuable in that process monitoring will no longer have to be adjusted to use the incorrect and limited results previously available in the literature. Concerning research, this chapter provides a valuable reference for anyone interested in anything having to do with (X, MR) control charts. Also, as already mentioned, the program eliminates the need for the research question of how many subgroups are enough before conventional control chart constants may be used.

CHAPTER VIII

A METHODOLOGY FOR THE DETERMINATION OF THE APPROPRIATE EXECUTION OF THE TWO STAGE PROCEDURE

Introduction

Several approaches appear in the literature for establishing control of a process during the retrospective stage of control charting. No research has been put forth that provides a means by which one may determine the delete and revise procedure that will establish control limits for future testing that have both the desired Type I error probability and a high probability of detecting a special cause signal. This chapter presents a methodology that determines, when one is using two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts as presented in Chapters IV, V, VI, and VII, respectively, of this dissertation, the appropriate execution of the two stage procedure.

Delete and Revise (D&R) Procedures

This chapter considers six different D&R procedures for establishing control of a process in the first stage of the two stage procedure. Four of them are given in the Establishment of Control subsection of The Two Stage Procedure section of Chapter II of this dissertation. Detailed descriptions of all six follow.

D&R 1

The first D&R procedure is from Hillier (1969), Ryan (1989), and Montgomery

(1997). It is executed as follows:

i. Deletes out-of-control (OOC) initial subgroups on either the control chart for centering or spread entirely (i.e., if a subgroup shows OOC on either control chart, it is deleted from both charts).

ii. Recalculates the control limits for both charts using the subgroups remaining after step i.

iii. Determines OOC subgroups.

iv. Repeats steps i-iii until no initial subgroups show OOC on either chart.

D&R 2

The second D&R procedure is from Pyzdek (1993). It is executed as follows:

i. Deletes out-of-control (OOC) initial subgroups on the control chart for spread.

ii. Recalculates the control limits for the control chart for spread using the subgroups remaining after step i.

iii. Determines OOC subgroups.

iv. Repeats steps i-iii until no initial subgroups show OOC on the control chart for spread.

v. Determines the control limits for the chart for centering using the parameter estimate for spread obtained after completing steps i-iv and the overall average obtained from all of the initial subgroups.

vi. Repeats steps i-ii for the control chart for centering until no initial subgroups show OOC.

D&R 3

The third D&R procedure is from Case (1998). It deletes out-of-control (OOC) initial subgroups on the control chart for spread just once. No D&R is performed on the control chart for centering.

D&R 4

The fourth D&R procedure is from Doty (1997). It does not perform D&R. This means all of the initial subgroups will be used to determine second stage control limits for both the control charts for centering and spread.

D&R 5

The fifth D&R procedure is a hybrid of D&R 1 in that it iterates only once. It deletes out-of-control (OOC) initial subgroups on either the control chart for centering or spread entirely (i.e., if a subgroup shows OOC on either control chart, it is deleted from both charts). D&R is performed just once.

D&R 6

The sixth D&R procedure is a hybrid of D&R 2 in that it iterates only once. It is executed as follows:

i. Deletes out-of-control (OOC) initial subgroups on the control chart for spread just once.

ii. Determines the control limits for the chart for centering using the parameter estimate for spread obtained after completing step i and the overall average obtained from all of the initial subgroups.

iii. Performs step i for the control chart for centering.

Any of the above six D&R procedures may be used on two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, and $(\overline{X}, s)$ control charts. However, only D&Rs 2, 3, 4, and 6 may be used on two stage short run (X, MR) control charts. The reason is that, since the MR values are calculated from two consecutive X values, no single MR value can be associated with a single X value. Consequently, D&Rs 1 and 5, which delete out-of-control (OOC) initial subgroups on either the control chart for centering or spread entirely (i.e., if a subgroup shows OOC on either control chart, it is deleted from both charts), cannot be used on two stage short run (X, MR) control charts.

The Methodology

The methodology for the determination of the appropriate execution of the two stage procedure as presented in this chapter consists of three elements. The main element is the computer program that simulates two stage short run variables control charting. The next element, which is included in the operation of the program, is the measurements that one may use to determine which delete and revise (D&R) procedure establishes the most reliable second stage control limits. The third element, which is explained using sample runs from the program, is the interpretation of the results from the program.

Measurements

The computer program in this chapter uses two sets of measurements to provide information that one may use to determine the reliability of second stage control limits. The first set of measurements is the probability of detection (POD), the average run length (ARL), and the standard deviation of the run length (SDRL). The second set of measurements is the probability of a false alarm (P(false alarm)), the average probability of a false alarm (APFL), and the standard deviation of the probability of a false alarm (SDPFL).

POD, ARL, and SDRL

As mentioned in the Performance Evaluation of Short Run Control Charts section of Chapter II, the POD is the probability that a control chart will signal, within a given number of subgroups following a shift, that a process is out-of-control (OOC). Additionally, if a process is in-control (IC), the POD may be interpreted as the probability of a Type I error (i.e., the probability of a false alarm) within a given number of subgroups starting with the first subgroup drawn from the process.

Using the POD allows for the characterization of the run length (RL) distribution. This is particularly useful in a short run situation because it is desirable to know, for small numbers of subgroups, the probability of detecting a special cause signal or the probability of a false alarm. Using the ARL, which is the average number of subgroups that must be plotted on a control chart before an OOC condition is indicated, in a short run situation is not appropriate because a short run may not last long enough to even achieve the ARL. Additionally, as will be shown in the Interpretation of Results from the Computer Program section later in this chapter, the ARL can mislead one in choosing the appropriate D&R procedure.

The POD may be expressed mathematically as equation (8.1):

$$POD = P(RL \leq t) \tag{8.1}$$

where

RL: run length (in number of subgroups)

t: the subgroup number

$P(RL \leq t)$: the probability that the run length (RL) is less than or equal to subgroup number t

As calculated by the computer program in this chapter, for an OOC situation in the second stage of the two stage procedure, the subgroup count starts at one at the first OOC subgroup. For an IC situation, the subgroup count starts at one with the first subgroup drawn from the process in the second stage.

Each time the program simulates two stage short run variables control charting, an RL value is determined. As the simulation is repeated, RL and RL^2 values are summed, and counts for the number of RLs less than or equal to each integer value in the interval [1, 50000] are kept. Once the repeating of the simulation is complete, the two sums are used to calculate the ARL and the SDRL, which is the standard deviation of the number of subgroups that must be plotted on a control chart before an OOC condition is indicated. The counts are used to determine the POD values.

For an OOC situation in the second stage of the two stage procedure, it is desirable to have the highest possible POD values and the lowest possible ARL. For an IC situation in the second stage, it is desirable to have the lowest possible POD values and the highest possible ARL.

P(false alarm), APFL, and SDPFL

The probability of a false alarm (i.e., P(false alarm)) is the probability of a control chart indicating an OOC condition when none exists. As mentioned in the Two Stage Short Run Control Charts subsection of the Control Charts with Modified Limits section of Chapter II, Hillier's (1969) methodology, upon which the two stage short run variables control charts presented in Chapters IV-VII are based, allows for the specification of the desired probability of a false alarm (i.e., the desired Type I error probability).

The computer program in this chapter calculates the probability of a false alarm when an OOC situation occurs beyond the first subgroup drawn from the process in the second stage of the two stage procedure. Each time the program simulates two stage short run variables control charting under these conditions, a value for P(false alarm) is determined. As the simulation is repeated, P(false alarm) and $(\text{P(false alarm)})^2$ values are summed. Once the repeating of the simulation is complete, these two sums are used to calculate the APFL and the SDPFL. It is desirable for the P(false alarm) values, and consequently the APFL, to be as low as possible.

The Computer Program

The computer program that simulates two stage short run variables control charting is in Appendix F.1 of this dissertation. It is coded in FORTRAN (1999). The program is meant to simulate two stage short run variables control charting of a process before initiating it so that one can decide which D&R procedure to use when performing two stage short run variables control charting during the early run of the process. The D&R procedures that the program provides are described earlier in the Delete and Revise (D&R) Procedures section of this chapter.

The layout of the segments of the simulation program is illustrated in Figure 8.1. Each segment of the program and its operation is described in this section in reverse order of appearance in Figure 8.1 (i.e., in the order in which the program operates).

Main Program cc

The main program cc (cc stands for control charting) includes the data entry, file setup, subroutine calls, summations of various values determined by the subroutines, final ARL, SDRL, P(false alarm), APFL, and SDPFL calculations, and the output of information to a file. It is the only segment of the program requiring user interaction.

The following inputs (in order of appearance in the program) are requested from the user in main program cc:

- The process mean and standard deviation.

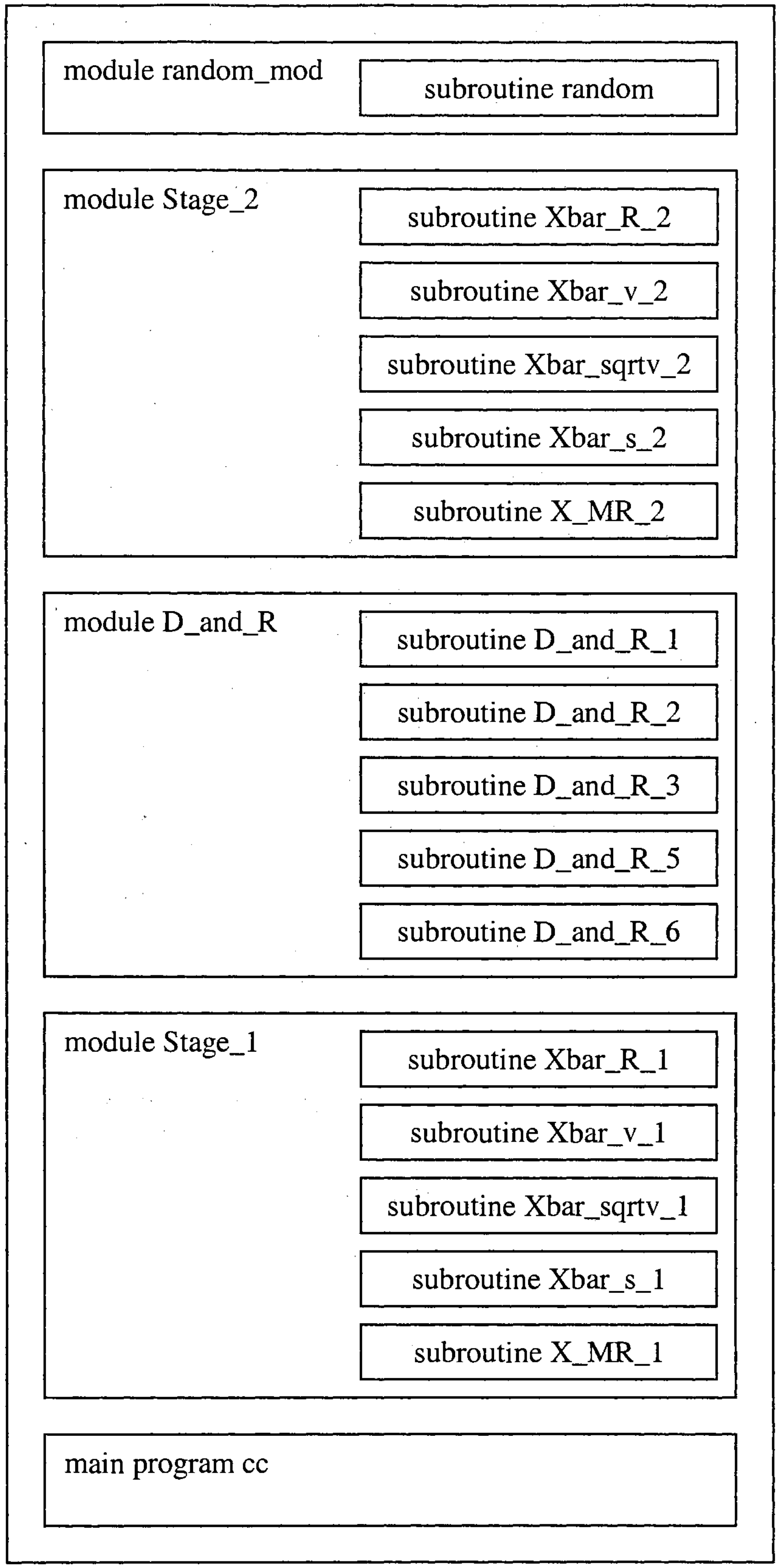

Figure 8.1. Layout of the Segments of the Computer Program

- The number of times to replicate the two stage short run control charting procedure.
- The control chart combination ($(\bar{X}, R)$, $(\bar{X}, v)$, $(\bar{X}, \sqrt{v})$, $(\bar{X}, s)$, or (X, MR)).
- The subgroup size (not applicable to (X, MR) control charts).
- The number of subgroups for Stage 1.
- The choice of simulating the process in Stage 1 as IC or OOC. If OOC is chosen, then the user is requested to enter the choice of a sustained shift in the mean, the standard deviation, or both. Once the user chooses a shift type, the program prompts for the shift size (in the same units as the parameter that has shifted) and the number of the first subgroup after the shift in Stage 1.
- The choice of simulating the process in Stage 2 as IC or OOC. If OOC is chosen, then the user is requested to enter the choice of a sustained shift in the mean, the standard deviation, or both. Once the user chooses a shift type, the program prompts for the shift size (in the same units as the parameter that has shifted) and the number of the first subgroup after the shift in Stage 2.
- The choice of using a different starting value for seed for the Marse-Roberts Uniform (0, 1) random variate generator (see Marse and Roberts (1983)) coded as subroutine random in module random_mod.
- The D&R procedure (entered as 1, 2, 3, 4, 5, or 6). The program describes the execution of each D&R procedure in detail for the user.
- The name (including the location) of the text file (extension .txt) that has the two stage short run control chart factors for the control chart combination entered earlier.
- The name (including the location) of the text file (extension .txt) that will store the results from the program.

The second to last bullet point above requires further explanation. Appendix F.2 of this dissertation has the five input files that were used to generate the results in the Interpretation of Results from the Computer Program section later in this chapter. The first input file contains the first and second stage short run control chart factors for $(\overline{X}, R)$ charts from Table B.3.4 in Appendix B.3 of this dissertation for n=3 and m: 1-5. The second input file contains the first and second stage short run control chart factors for $(\overline{X}, v)$ charts from Table C.3.4 in Appendix C.3 of this dissertation for n=3 and m: 1-5. The third input file contains the first and second stage short run control chart factors for $(\overline{X}, \sqrt{v})$ charts, also from Table C.3.4 in Appendix C.3 for n=3 and m: 1-5. The fourth input file contains the first and second stage short run control chart factors for $(\overline{X}, s)$ charts from Table D.3.4 in Appendix D.3 of this dissertation for n=3 and m: 1-5. The fifth input file contains the first and second stage short run control chart factors for (X, MR) charts from Table E.3.3 in Appendix E.3 of this dissertation for m: 2-15.

The only difference between the appearance of the input files and their corresponding tables in the appendices is that the first stage short run control chart factors in the first row of each input file are set to zero. This is required in order for the program to correctly read the second stage short run control chart factors from these input files when m=1 (in the case of $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, and $(\overline{X}, s)$ control charts) or m=2 (in the case of (X, MR) control charts).

Module Stage_1

When the data entry is complete, the first replication of the two stage short run control charting procedure begins as program execution proceeds from main program cc to module Stage_1 and the subroutine for the control chart combination entered by the user. Each of the five subroutines for Stage 1 control charting performs the following tasks:

- Reads first stage short run control chart factors from the input file.
- Generates first stage subgroups.
- Constructs first stage control limits.
- Determines OOC subgroups.

The tasks in the last two bullet points use Hillier's (1969) approach. When Stage 1 control charting is complete, program execution returns to main program cc.

Module D&R

Once program execution returns to main program cc, it immediately proceeds to module D_and_R and the subroutine for the D&R procedure entered by the user. All six D&R procedures are described earlier in the Delete and Revise (D&R) Procedures section of this chapter. When the D&R procedure is complete, program execution returns to main program cc. At this point, the program assumes that control of the process has been established.

Module Stage_2

Once program execution returns to main program cc, required summations are calculated and required variable assignments are made. Program execution then proceeds to module Stage_2 and the subroutine for the control chart combination entered by the user. Each of the five subroutines for Stage 2 control charting performs the following tasks:

- Reads second stage short run control chart factors from the input file.
- Constructs second stage control limits.
- Generates second stage subgroups.
- Determines the run length (RL) and, if applicable, if a false alarm occurs.

The calculations in the last bullet point are based on the signaling capabilities of combined control charts for centering and spread; i.e., a signal occurs if a subgroup plots OOC on either the control chart for centering or the control chart for spread. The number of the first subgroup that signals is the RL value. The second stage control limits are not updated as subgroups are accumulated. When an RL value is determined, Stage 2 control charting is complete and program execution returns to main program cc.

Replications

In main program cc after Stage 2 control charting, required summations are calculated. When this is complete, execution returns to the location in main program cc immediately before the five subroutine calls for Stage 1 control charting to begin the second replication. The entire procedure for two stage short run control charting just described repeats for the amount of times entered by the user.

Output

After the last replication, program execution in main program cc proceeds to writing the following information to the output file:

- The process mean and standard deviation.
- The number of replications of the two stage short run control charting procedure that were carried out.
- The control chart combination ($(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, or (X, MR)).
- The subgroup size (not applicable to (X, MR) control charts).
- The number of subgroups for Stage 1.
- The D&R procedure.
- The state of the process in Stage 1: IC or OOC. If it is OOC, then the type of sustained shift, the shift size (in the same units as the parameter that has shifted), and the number of the first subgroup after the shift in Stage 1 are given.
- The state of the process in Stage 2: IC or OOC. If it is OOC, then the type of sustained shift, the shift size (in the same units as the parameter that has shifted), and the number of the first subgroup after the shift in Stage 2 are given.
- The ARL and SDRL.
- The APFL and SDPFL (if applicable).

- A table of POD values.

The information in the first eight bullet points was entered by the user. The values in the last three bullet points are calculated by the program.

In addition to these calculated values, which are explained in the Measurements section of this chapter, the computer program determines counts of the number of occurrences of certain events (when applicable). These events are as follows:

- The number of times out of the total number of replications that D&R 1 iterated more than once.
- The number of times out of the total number of replications that D&R 2 iterated more than once for the control chart for spread as well as for the control chart for centering.
- The number of times out of the total number of replications the program skipped a replication because the number of subgroups dropped to zero (for two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts) or one (for two stage short run (X, MR) control charts) after OOC subgroups were deleted in a D&R procedure.
- The number of times out of the total number of replications a D&R procedure was stopped because the number of subgroups dropped to one (for two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, and $(\overline{X}, s)$ control charts) or two (for two stage short run (X, MR) control charts) after OOC subgroups were deleted.

These counts, if applicable, are also written to the output file.

Once the above information, applicable calculations, and applicable counts have been written to the output file, execution of the computer program is complete.

Interpretation of Results from the Computer Program

The fourteen pairs of tables (Tables 8.1a-8.14b) that appear in this section were constructed from output files generated from sample runs of the computer program. For example, Tables 8.12a and 8.12b were constructed from the six output files in Appendix F.3 of this dissertation. In addition to the notation already introduced in this chapter, Tables 8.1a-8.14b use the following notation:

- MN - a sustained shift in the mean
- SD - a sustained shift in the standard deviation
- MS - a sustained shift in both the mean and the standard deviation
- Replications (skipped) - the number of replications carried out and, in parentheses, the number of replications skipped because the number of subgroups dropped to zero (for two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts) or one (for two stage short run (X, MR) control charts) after OOC subgroups were deleted in a D&R procedure.
- Stops - the number of times out of the total number of replications carried out that a D&R procedure was stopped because the number of subgroups dropped to one (for two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, and $(\overline{X}, s)$ control charts) or two (for two stage short run (X, MR) control charts) after OOC subgroups were deleted.

The sample runs of the program that generated the information in Tables 8.1a-8.14b assumed the following:

- The process mean and standard deviation are always 0.0 and 1.0, respectively.
- The planned number of replications is always 5000.
- The subgroup size n is always 3 (not applicable to (X, MR) control charts).
- The number of Stage 1 subgroups (denoted by m) is always 5 for two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, and $(\overline{X}, s)$ control charts and it is always 15 for two stage short run (X, MR) control charts. This is why the first four sample input files in Appendix F.2 have two stage short run control chart factors for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, and $(\overline{X}, s)$ charts for m up to and including m=5 and the fifth sample input file in Appendix F.2 has two stage short run control chart factors for (X, MR) charts for m up to and including m=15.
- A shift in the mean is always of size 1.5 (same units as the mean).
- A shift in the standard deviation is always of size 1.0 (same units as the standard deviation).
- A shift in Stage 1 always occurs between subgroups 2 and 3.
- A shift in Stage 2 always occurs between subgroups 10 and 11.
- The process is IC immediately before Stage 2 control charting begins.

Sample Runs for an IC Process in Stages 1 and 2

The first 28 sample runs of the program are for the process being IC during both Stage 1 and Stage 2 control charting. Two stage short run control charting for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts was simulated using all six D&R procedures for each control chart combination. The results of these simulations appear in Tables 8.1a-8.5b.

Since the process is being simulated as IC in Stage 2, it is desirable for the ARL values in Tables 8.1a-8.5a to be as high as possible. Also, it is desirable for the $P(RL \leq t)$ values in Tables 8.1b-8.5b to be as low as possible (since they correspond to probabilities of false alarms within t or less subgroups after starting Stage 2 control charting), especially for small numbers of subgroups (since a short run situation is in effect).

Based on both of these criteria, the information in Tables 8.1a-8.5b indicates that D&R 4 is, for the most part, the delete and revise procedure of choice. The only exception is in Table 8.3a, where D&R 1 is the delete and revise procedure of choice based on the ARL. This implies that, under the assumptions of this simulation, it is preferable to use subgroups that signal false alarms in the construction of second stage control limits. The cost, in terms of the loss in reliability of second stage control limits, is higher by throwing out subgroups that signal false alarms than it is by including them in the construction of second stage control limits.

Comparing results in Tables 8.1a-8.5a reveals that two stage short run $(\overline{X}, s)$ control charts have the highest ARL for D&R 4. Comparing results in Tables 8.1b-8.5b reveals that two stage short run $(\overline{X}, \sqrt{v})$ control charts have, for most of the shown values of t,

Table 8.1a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: IC and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	552.89	701.12	5000 (0)	0
2	550.10	702.51	4999 (1)	1
3	552.87	701.72	5000 (0)	0
4	560.49	702.22	5000 (-----)	-----
5	552.08	700.49	5000 (0)	0
6	552.03	700.61	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 22 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 8 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 70				

Table 8.1b. P(RL ≤ t) for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: IC and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00940	0.01000	0.00900	0.00740	0.00820	0.00860
2	0.01640	0.01760	0.01600	0.01260	0.01520	0.01560
3	0.02540	0.02741	0.02520	0.02040	0.02440	0.02500
4	0.03360	0.03561	0.03300	0.02700	0.03260	0.03300
5	0.03820	0.04061	0.03760	0.03140	0.03700	0.03760
6	0.04400	0.04721	0.04400	0.03580	0.04320	0.04420
8	0.05380	0.05761	0.05460	0.04520	0.05320	0.05480
10	0.06400	0.06721	0.06480	0.05420	0.06380	0.06500
15	0.08880	0.09182	0.08880	0.07820	0.08840	0.08920
20	0.11040	0.11462	0.11100	0.09960	0.11000	0.11180
30	0.14040	0.14423	0.14100	0.12980	0.13960	0.14180
40	0.16480	0.16863	0.16520	0.15360	0.16420	0.16620
50	0.19180	0.19584	0.19160	0.17980	0.19120	0.19320
100	0.27440	0.27806	0.27460	0.26480	0.27440	0.27520
200	0.40740	0.41148	0.40800	0.40060	0.40820	0.40820
300	0.50200	0.50630	0.50340	0.49600	0.50360	0.50380
400	0.57760	0.58192	0.57900	0.57320	0.57900	0.57940
500	0.63500	0.63773	0.63640	0.63120	0.63600	0.63680
750	0.74900	0.75075	0.74840	0.74560	0.74920	0.74860
1000	0.82100	0.82156	0.82060	0.81840	0.82120	0.82080
2000	0.95460	0.95479	0.95460	0.95280	0.95460	0.95480
3000	0.98480	0.98480	0.98480	0.98440	0.98500	0.98500
5000	0.99840	0.99840	0.99840	0.99860	0.99840	0.99840
7500	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.2a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, v)$ Control Charts with Stage 1: IC and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	543.47	699.56	5000 (0)	1
2	540.76	698.13	5000 (0)	0
3	543.47	699.98	5000 (0)	0
4	557.40	705.40	5000 (-----)	-----
5	542.93	699.56	5000 (0)	0
6	543.01	699.50	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 14				
# of Times D&R 2 Iterated More Than Once for the v Control Chart: 5				
# of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 71				

Table 8.2b. P(RL ≤ t) for Two Stage Short Run $(\overline{X}, v)$ Control Charts with Stage 1: IC and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00900	0.01000	0.00860	0.00640	0.00880	0.00880
2	0.01580	0.01740	0.01660	0.01080	0.01620	0.01660
3	0.02460	0.02680	0.02560	0.01780	0.02480	0.02580
4	0.03200	0.03520	0.03340	0.02380	0.03240	0.03400
5	0.03740	0.04060	0.03860	0.02800	0.03760	0.03940
6	0.04440	0.04660	0.04460	0.03360	0.04400	0.04560
8	0.05320	0.05640	0.05400	0.04180	0.05300	0.05520
10	0.06380	0.06680	0.06520	0.05080	0.06420	0.06600
15	0.09140	0.09420	0.09220	0.07640	0.09180	0.09300
20	0.11180	0.11520	0.11340	0.09840	0.11220	0.11340
30	0.14180	0.14520	0.14340	0.12740	0.14220	0.14360
40	0.16640	0.17020	0.16760	0.15060	0.16680	0.16840
50	0.19260	0.19640	0.19360	0.17700	0.19300	0.19440
100	0.28300	0.28740	0.28400	0.26980	0.28380	0.28420
200	0.40940	0.41140	0.40900	0.39440	0.41020	0.40940
300	0.50240	0.50420	0.50280	0.49080	0.50320	0.50380
400	0.58040	0.58260	0.58100	0.57040	0.58140	0.58120
500	0.64260	0.64360	0.64220	0.63180	0.64320	0.64300
750	0.75760	0.75800	0.75720	0.75060	0.75800	0.75700
1000	0.82920	0.83040	0.82880	0.82460	0.82960	0.82920
2000	0.95560	0.95620	0.95580	0.95420	0.95560	0.95580
3000	0.98440	0.98460	0.98420	0.98340	0.98440	0.98440
5000	0.99860	0.99860	0.99860	0.99860	0.99860	0.99860
7500	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.3a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, \sqrt{v})$ Control Charts with Stage 1: IC and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	566.68	758.05	5000 (0)	8
2	550.63	675.26	5000 (0)	3
3	555.38	683.76	5000 (0)	0
4	561.88	682.24	5000 (-----)	-----
5	555.38	682.22	5000 (0)	0
6	555.38	683.51	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 93				
# of Times D&R 2 Iterated More Than Once for the $\sqrt{v}$ Control Chart: 28				
# of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 60				

Table 8.3b. P(RL ≤ t) for Two Stage Short Run $(\overline{X}, \sqrt{v})$ Control Charts with Stage 1: IC and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00680	0.00800	0.00760	0.00620	0.00740	0.00720
2	0.01060	0.01260	0.01240	0.00980	0.01160	0.01200
3	0.01740	0.02020	0.01900	0.01600	0.01780	0.01880
4	0.02260	0.02580	0.02460	0.02080	0.02380	0.02520
5	0.02660	0.03020	0.02860	0.02480	0.02760	0.02920
6	0.03240	0.03580	0.03420	0.03020	0.03320	0.03480
8	0.04100	0.04400	0.04240	0.03800	0.04140	0.04280
10	0.05040	0.05340	0.05180	0.04660	0.05080	0.05220
15	0.07520	0.07780	0.07560	0.06960	0.07440	0.07600
20	0.09680	0.09920	0.09720	0.09020	0.09580	0.09720
30	0.12340	0.12520	0.12360	0.11660	0.12220	0.12320
40	0.14660	0.14800	0.14620	0.13900	0.14520	0.14640
50	0.17040	0.17180	0.17040	0.16280	0.16900	0.17040
100	0.25760	0.26100	0.25900	0.25220	0.25760	0.25880
200	0.38660	0.39080	0.38820	0.38140	0.38720	0.38760
300	0.48040	0.48540	0.48380	0.47780	0.48320	0.48380
400	0.56220	0.56560	0.56560	0.55920	0.56540	0.56560
500	0.62380	0.62800	0.62760	0.62140	0.62760	0.62800
750	0.74480	0.74920	0.74820	0.74380	0.74860	0.74820
1000	0.82080	0.82440	0.82400	0.82020	0.82400	0.82400
2000	0.95520	0.95800	0.95680	0.95620	0.95680	0.95680
5000	0.99800	0.99900	0.99900	0.99900	0.99900	0.99900
10000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
30000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.4a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, s)$ Control Charts with Stage 1: IC and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	562.52	709.58	5000 (0)	0
2	561.89	709.13	5000 (0)	1
3	561.99	706.56	5000 (0)	0
4	566.35	702.87	5000 (-----)	-----
5	562.51	709.61	5000 (0)	0
6	561.99	707.42	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 17 # of Times D&R 2 Iterated More Than Once for the s Control Chart: 8 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 65				

Table 8.4b. P(RL ≤ t) for Two Stage Short Run $(\overline{X}, s)$ Control Charts with Stage 1: IC and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00940	0.01000	0.00860	0.00800	0.00860	0.00840
2	0.01700	0.01780	0.01580	0.01280	0.01600	0.01580
3	0.02520	0.02640	0.02420	0.02120	0.02420	0.02420
4	0.03120	0.03260	0.03020	0.02600	0.03020	0.03040
5	0.03640	0.03820	0.03560	0.03040	0.03540	0.03560
6	0.04320	0.04560	0.04320	0.03680	0.04260	0.04320
8	0.05260	0.05560	0.05320	0.04540	0.05200	0.05320
10	0.06220	0.06500	0.06260	0.05420	0.06140	0.06240
15	0.08540	0.08800	0.08600	0.07680	0.08480	0.08560
20	0.10620	0.11000	0.10780	0.09800	0.10560	0.10760
30	0.13460	0.13780	0.13600	0.12660	0.13380	0.13580
40	0.15960	0.16340	0.16100	0.15080	0.15900	0.16120
50	0.18800	0.19180	0.18900	0.17840	0.18740	0.18960
100	0.27540	0.27800	0.27640	0.26680	0.27520	0.27600
200	0.40340	0.40480	0.40380	0.39780	0.40300	0.40320
300	0.49200	0.49400	0.49280	0.48880	0.49240	0.49300
400	0.57040	0.57160	0.57100	0.56640	0.57100	0.57140
500	0.62740	0.62860	0.62800	0.62420	0.62780	0.62840
750	0.74240	0.74200	0.74160	0.74020	0.74240	0.74200
1000	0.81700	0.81660	0.81640	0.81620	0.81700	0.81660
2000	0.95400	0.95420	0.95400	0.95380	0.95400	0.95400
3000	0.98380	0.98380	0.98420	0.98420	0.98380	0.98400
5000	0.99840	0.99860	0.99860	0.99880	0.99840	0.99860
7500	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.5a. ARL, SDRL, Replications, and Stops for Two Stage Short Run (X, MR) Control Charts with Stage 1: IC and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
2	539.78	705.27	5000 (0)	0
3	540.46	705.74	5000 (0)	0
4	544.85	709.22	5000 (-----)	-----
6	540.76	705.61	5000 (0)	0
# of Times D&R 2 Iterated More Than Once for the MR Control Chart: 13				
# of Times D&R 2 Iterated More Than Once for the X Control Chart: 51				

Table 8.5b. $P(RL \leq t)$ for Two Stage Short Run (X, MR) Control Charts with Stage 1: IC and Stage 2: IC

t	Delete and Revise (D&R) Procedure			
	2	**3**	**4**	**6**
1	0.00340	0.00300	0.00220	0.00260
2	0.01200	0.01120	0.01000	0.01080
3	0.01840	0.01740	0.01620	0.01700
4	0.02500	0.02380	0.02260	0.02360
5	0.02940	0.02820	0.02680	0.02800
6	0.03540	0.03360	0.03180	0.03340
8	0.04440	0.04260	0.03960	0.04220
10	0.05480	0.05320	0.04940	0.05260
15	0.07660	0.07560	0.07080	0.07520
20	0.09580	0.09480	0.08940	0.09440
30	0.12960	0.12800	0.12160	0.12780
40	0.16020	0.15860	0.15320	0.15820
50	0.18460	0.18320	0.17760	0.18280
100	0.28000	0.27940	0.27380	0.27880
200	0.42000	0.41960	0.41580	0.41920
300	0.51940	0.51940	0.51540	0.51920
400	0.59560	0.59560	0.59200	0.59540
500	0.65620	0.65600	0.65280	0.65620
750	0.76240	0.76220	0.76060	0.76200
1000	0.83240	0.83220	0.83120	0.83200
2000	0.95000	0.94980	0.94940	0.94960
3000	0.98380	0.98380	0.98340	0.98380
5000	0.99860	0.99860	0.99840	0.99860
7500	0.99980	0.99980	0.99980	0.99980
10000	1.00000	1.00000	1.00000	1.00000

the lowest $P(RL \leq t)$ values for D&R 4. These results imply that, under the assumptions of this simulation, different control chart combinations are preferable depending on the measurement used.

The information in Tables 8.1b-8.4b also indicates that the $P(RL \leq t)$ values when t=1 are reasonably close to the theoretical probability of a false alarm. Assuming independence between the control charts for centering and spread, the theoretical probability of a false alarm (i.e., P(false alarm)) may be calculated using equation (8.2):

$$\text{P(false alarm)} = \alpha_{\text{Cen}} + (\alpha_{\text{SpreadUCL}} + \alpha_{\text{SpreadLCL}}) - \alpha_{\text{Cen}} \cdot (\alpha_{\text{SpreadUCL}} + \alpha_{\text{SpreadLCL}}) \tag{8.2}$$

where

α_{Cen} : P(false alarm) on the control chart for centering

$\alpha_{\text{SpreadUCL}}$: P(false alarm) on the control chart for spread above the upper control limit (UCL)

$\alpha_{\text{SpreadLCL}}$: P(false alarm) on the control chart for spread below the lower control limit (LCL)

For the sample runs of the program, $\alpha_{\text{Cen}} = 0.0027$, $\alpha_{\text{SpreadUCL}} = 0.005$, and $\alpha_{\text{SpreadLCL}} = 0.001$. This means that P(false alarm), as calculated by equation (8.2), is equal to 0.0086838.

For example, the $P(RL \leq t)$ value from Table 8.1b for D&R 1 and t=1 is 0.00940. The fact that this value is reasonably close to the theoretical probability of a false alarm is not surprising. As was mentioned in the P(false alarm), APFL, and SDPFL subsection of the

Measurements section of this chapter, Hillier's (1969) methodology, upon which the two stage short run variables control charts presented in Chapters IV-VII are based, allows for the specification of the desired probability of a false alarm.

In Table 8.5b, each of the $P(RL \leq t)$ values for t=1 are much lower than 0.0086838. The closest one is 60.847% smaller than 0.0086838. However, these lower $P(RL \leq t)$ values for t=1 come at the price of having the lowest ARL for D&R 4 among Tables 8.1a-8.5a. This is an example of the tradeoff mentioned by Del Castillo (1995) between having a low probability of a false alarm and a high probability of detecting a special cause signal inherent with two stage short run control charts.

It should be noted that the information in Tables 8.1a-8.5a also indicates that D&R 1 and D&R 2 are iterating more than once. These multiple iterations seem to create conditions causing replications to be skipped and the chosen D&R procedure to be stopped. Also, if one were to construct confidence intervals using the ARL and SDRL values in Tables 8.1a-8.5a, then, depending on the confidence level chosen, the ARL results in Tables 8.1a-8.5a may not be statistically significantly different.

Sample Runs for an OOC Process in Stage 1 and an IC Process in Stage 2

The next 18 sample runs of the program are for the process being OOC during Stage 1 control charting and IC during Stage 2 control charting. Two stage short run control charting for $(\overline{X}, R)$ charts was simulated using all six D&R procedures for each OOC condition (MN, SD, MS). The results of these simulations appear in Tables 8.6a-8.8b.

As in the previous subsection, since the process is being simulated as IC in Stage 2, it is desirable for the ARL values in Tables 8.6a-8.8a to be as high as possible. Also, it is

Table 8.6a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MN) and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	332.74	833.38	4996 (4)	10
2	314.33	515.14	4996 (4)	10
3	299.30	487.34	5000 (0)	0
4	302.32	492.05	5000 (-----)	-----
5	309.47	508.73	4999 (1)	0
6	303.24	492.75	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 108 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 7 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 626				

Table 8.6b. $P(RL \leq t)$ for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MN) and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.03883	0.03823	0.03860	0.03440	0.03841	0.03640
2	0.06385	0.06485	0.06840	0.06140	0.06601	0.06540
3	0.08527	0.08667	0.09080	0.08220	0.08882	0.08660
4	0.10248	0.10388	0.10960	0.09980	0.10582	0.10440
5	0.11209	0.11629	0.12160	0.10980	0.11522	0.11600
6	0.12830	0.13151	0.13840	0.12620	0.13263	0.13380
8	0.15753	0.15973	0.16660	0.15580	0.16343	0.16420
10	0.17734	0.17974	0.18840	0.17720	0.18344	0.18600
15	0.22778	0.23058	0.24360	0.22980	0.23365	0.23580
20	0.26301	0.26821	0.28000	0.26680	0.26885	0.27440
30	0.30885	0.31405	0.32520	0.31500	0.31546	0.31820
40	0.34788	0.35488	0.36600	0.35640	0.35547	0.35860
50	0.38131	0.39071	0.40180	0.39260	0.38968	0.39560
100	0.49420	0.50420	0.51020	0.50620	0.50050	0.50480
200	0.61489	0.62470	0.62760	0.62480	0.62252	0.62520
300	0.69456	0.69936	0.70520	0.70260	0.70214	0.70540
400	0.75120	0.75600	0.76400	0.76240	0.75995	0.76480
500	0.79223	0.79664	0.80820	0.80660	0.80096	0.80480
750	0.86649	0.87050	0.87960	0.87860	0.87297	0.87700
1000	0.91173	0.91273	0.91920	0.91820	0.91518	0.91820
2000	0.98159	0.98199	0.98480	0.98460	0.98380	0.98420
5000	0.99860	0.99980	0.99980	0.99960	0.99920	0.99980
10000	0.99960	1.00000	1.00000	1.00000	1.00000	1.00000
50000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.7a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (SD) and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	463.12	561.26	5000 (0)	5
2	455.32	549.20	5000 (0)	4
3	453.95	546.51	5000 (0)	0
4	453.07	533.20	5000 (-----)	-----
5	460.32	554.43	5000 (0)	0
6	455.49	549.37	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 68				
# of Times D&R 2 Iterated More Than Once for the R Control Chart: 29				
# of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 196				

Table 8.7b. P(RL ≤ t) for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (SD) and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00260	0.00360	0.00320	0.00200	0.00200	0.00240
2	0.00540	0.00740	0.00580	0.00420	0.00480	0.00480
3	0.01000	0.01340	0.01120	0.00860	0.01000	0.01020
4	0.01420	0.01760	0.01460	0.01220	0.01380	0.01400
5	0.01680	0.02080	0.01740	0.01420	0.01620	0.01640
6	0.02060	0.02400	0.02080	0.01640	0.01940	0.01960
8	0.02740	0.03140	0.02780	0.02240	0.02600	0.02660
10	0.03460	0.03760	0.03400	0.02740	0.03300	0.03280
15	0.04960	0.05260	0.04900	0.04040	0.04780	0.04740
20	0.06260	0.06540	0.06180	0.05300	0.06100	0.06020
30	0.08660	0.09000	0.08720	0.07660	0.08540	0.08520
40	0.11300	0.11700	0.11500	0.10340	0.11320	0.11240
50	0.13860	0.14080	0.13940	0.12720	0.13800	0.13680
100	0.23880	0.24300	0.24300	0.22720	0.23800	0.23980
200	0.40080	0.40600	0.40600	0.39440	0.40000	0.40460
300	0.52000	0.52200	0.52300	0.52000	0.52020	0.52260
400	0.61660	0.62120	0.62060	0.61940	0.61600	0.62080
500	0.69160	0.69600	0.69780	0.69860	0.69260	0.69740
750	0.81100	0.81400	0.81620	0.81600	0.81160	0.81640
1000	0.87980	0.88220	0.88280	0.88600	0.88140	0.88320
2000	0.97400	0.97580	0.97540	0.97600	0.97540	0.97540
3000	0.99220	0.99360	0.99320	0.99400	0.99280	0.99340
5000	0.99920	0.99920	0.99920	0.99940	0.99920	0.99920
7500	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.8a. ARL, SDRL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MS) and Stage 2: IC

D&R Procedure	ARL	SDRL	Replications (Skipped)	Stops
1	431.11	610.46	4992 (8)	9
2	407.63	494.82	4997 (3)	13
3	384.80	469.57	5000 (0)	0
4	401.66	480.23	5000 (-----)	-----
5	407.99	491.72	5000 (0)	0
6	400.00	488.78	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 126 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 29 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 427				

Table 8.8b. $P(RL \leq t)$ for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MS) and Stage 2: IC

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00501	0.00981	0.01240	0.00700	0.00580	0.00840
2	0.01062	0.01701	0.02000	0.01100	0.01180	0.01440
3	0.01643	0.02341	0.02920	0.01760	0.01880	0.02160
4	0.01983	0.02802	0.03440	0.02120	0.02280	0.02620
5	0.02284	0.03262	0.03940	0.02460	0.02680	0.03040
6	0.02704	0.03662	0.04500	0.02880	0.03180	0.03560
8	0.03466	0.04623	0.05580	0.03700	0.03980	0.04500
10	0.03986	0.05483	0.06400	0.04260	0.04680	0.05360
15	0.05569	0.07324	0.08540	0.05880	0.06360	0.07320
20	0.07031	0.08905	0.10100	0.07300	0.07760	0.08900
30	0.10076	0.11847	0.13000	0.09980	0.10700	0.11800
40	0.12881	0.14509	0.15900	0.12800	0.13520	0.14640
50	0.15625	0.17150	0.18720	0.15580	0.16240	0.17340
100	0.26342	0.28177	0.29860	0.27000	0.27200	0.28580
200	0.42808	0.44187	0.45960	0.43540	0.43980	0.45000
300	0.54868	0.56234	0.58080	0.56100	0.55980	0.57060
400	0.64744	0.65799	0.67560	0.65960	0.65640	0.66500
500	0.72135	0.72964	0.74580	0.73360	0.73060	0.73760
750	0.83373	0.83910	0.85300	0.84640	0.84120	0.84520
1000	0.89724	0.90014	0.90960	0.90700	0.90240	0.90380
2000	0.97897	0.98239	0.98560	0.98420	0.98280	0.98260
5000	0.99840	0.99980	0.99980	0.99960	0.99980	0.99980
10000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
20000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

desirable for the P(RL ≤ t) values in Tables 8.6b-8.8b to be as low as possible (since they correspond to probabilities of false alarms within t or less subgroups after starting Stage 2 control charting), especially for small numbers of subgroups (since a short run situation is in effect).

Based on the ARL, Tables 8.6a-8.8a indicate that D&R 1 is the delete and revise procedure of choice, regardless of the OOC condition in Stage 1. However, the SDRL values for D&R 1 are higher than those for the other D&R procedures. The ARL for D&R 1 in Table 8.7a is higher than the ARL values for D&R 1 in Tables 8.6a and 8.8a. The ARL for D&R 1 in Table 8.6a is the lowest of the three. These results imply that, under the assumptions of this simulation, the type of OOC condition in Stage 1 has an affect on the IC ARL in Stage 2. Additionally, the ARL values for each of the six D&R procedures in Table 8.1a are higher than the respective ARL values in Tables 8.6a-8.8a. This result implies that, under the assumptions of this simulation, an OOC condition in Stage 1 causes a reduction in the IC ARL in Stage 2, regardless of the D&R procedure used.

The choice of the appropriate D&R procedure based on the P(RL ≤ t) values in Tables 8.6b-8.8b varies depending on the OOC condition as well as the subgroup number t. In Table 8.6b, D&R 4 results in the lowest P(RL ≤ t) values for shown values of t ≤ 10. For shown values of t > 10, D&R 1 is the delete and revise procedure of choice. In Table 8.7b, D&R 4 again results in the lowest P(RL ≤ t) values, but for shown values of t ≤ 300. For most of the shown values of t ≥ 300, D&R 1 is the delete and revise procedure of choice. In Table 8.8b, D&R 1 results in the lowest P(RL ≤ t) values for each of the shown values of t except t: 30, 40, 50. Since D&R 1 is not the delete and revise procedure of choice in Tables 8.6b and 8.7b for shown values of $t \leq 10$ and $t \leq 200$, respectively, this is an example of how the ARL can be misleading in choosing the appropriate D&R procedure to use in a short run situation.

The results from Tables 8.6b and 8.7b imply that, under the assumptions of this simulation, it is preferable to use subgroups that signal shifts in either the mean or the standard deviation in the construction of second stage control limits. The cost, in terms of the loss in reliability of second stage control limits, is higher by throwing out subgroups that signal shifts in either the mean or the standard deviation than it is by including them in the construction of second stage control limits.

The $P(RL \leq t)$ values for shown values of $t \leq 300$ for D&R 4 and for shown values of $t \geq 300$ for D&R 1 in Table 8.7b are lower than the lowest $P(RL \leq t)$ values in Tables 8.6b and 8.8b. The lowest $P(RL \leq t)$ values in Table 8.6b are higher than those in Tables 8.7b and 8.8b. These results imply that, under the assumptions of this simulation, the type of OOC condition in Stage 1 has an affect on the $P(RL \leq t)$ values in Stage 2. Additionally, the lowest $P(RL \leq t)$ values in Table 8.1b are higher than those in Table 8.7b for shown values of $t \leq 200$ and in Table 8.8b for shown values of $t \leq 100$. These results imply that, under the assumptions of this simulation, having Stage 1 IC does not necessarily result in Stage 2 control limits with the lowest $P(RL \leq t)$ values.

An issue of concern is the $P(RL \leq t)$ values when t=1. In Table 8.6b, each of these values is much larger than 0.0086838, the theoretical probability of a false alarm. The closest one is 396.140% larger than 0.0086838. In Table 8.7b, each of these values is much smaller than 0.0086838. The closest one is 241.217% smaller than 0.0086838. In Table 8.8b, some of these values are reasonably close to 0.0086838, while others are not.

These results are in contrast to the $P(RL \leq t)$ values when t=1 in Table 8.1b. Clearly, under the assumptions of this simulation, an OOC condition as well as the type of OOC condition in Stage 1 has a significant effect on the $P(RL \leq t)$ values when t=1 in Stage 2.

Again, as in the previous subsection, the information in Tables 8.6a-8.8a indicates that D&R 1 and D&R 2 are iterating more than once. These multiple iterations seem to create conditions causing replications to be skipped and the chosen D&R procedure to be stopped. Also, if one were to construct confidence intervals using the ARL and SDRL values in Tables 8.6a-8.8a, then, depending on the confidence level chosen, the ARL results in Tables 8.6a-8.8a may not be statistically significantly different.

Sample Runs for an IC Process in Stage 1 and an OOC Process in Stage 2

The next 18 sample runs of the program are for the process being IC during Stage 1 control charting and OOC during Stage 2 control charting. Two stage short run control charting for $(\overline{X}, R)$ charts was simulated using all six D&R procedures for each OOC condition (MN, SD, MS). The results of these simulations appear in Tables 8.9a-8.11b.

Since the process is being simulated as OOC in Stage 2, it is desirable for the ARL and, as always, the APFL values in Tables 8.9a-8.11a to be as low as possible. Also, it is desirable for the $P(RL \leq t)$ values in Tables 8.9b-8.11b to be as high as possible (since they correspond to probabilities of detecting special causes within t or less subgroups after the shift in Stage 2), especially for small numbers of subgroups (since a short run situation is in effect).

Based on the ARL, D&R 2 (in Tables 8.9a and 8.11a) and D&R 4 (in Table 8.10a) are the delete and revise procedures of choice. The ARL for D&R 2 in Table 8.11a is lower

Table 8.9a. ARL, SDRL, APFL, SDPFL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: IC and Stage 2: OOC (MN)

D&R Procedure	ARL	SDRL	APFL	SDPFL	Replications (Skipped)	Stops
1	95.01	241.02	0.01116	0.05639	5000 (0)	1
2	94.39	240.51	0.01252	0.06003	5000 (0)	1
3	95.08	241.31	0.01098	0.05263	5000 (0)	0
4	95.00	240.54	0.00738	0.03638	5000 (-----)	-----
5	95.01	241.49	0.01064	0.05253	5000 (0)	0
6	94.63	240.54	0.01092	0.05120	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 19 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 10 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 82						

Table 8.9b. $P(RL \le t)$ for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: IC and Stage 2: OOC (MN)

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.14340	0.14800	0.14600	0.13480	0.14320	0.14620
2	0.22360	0.22600	0.22500	0.21380	0.22360	0.22560
3	0.27540	0.27960	0.27940	0.26720	0.27600	0.27900
4	0.31760	0.32120	0.32160	0.31060	0.31800	0.32040
5	0.35140	0.35580	0.35540	0.34480	0.35300	0.35440
6	0.38120	0.38520	0.38500	0.37500	0.38300	0.38380
8	0.42780	0.43200	0.43160	0.42160	0.43040	0.43000
10	0.46400	0.46840	0.46720	0.45820	0.46600	0.46580
15	0.52920	0.53380	0.53160	0.52700	0.53120	0.53140
20	0.57820	0.58260	0.58080	0.57800	0.58000	0.58060
30	0.64700	0.65020	0.64720	0.64600	0.64760	0.64760
40	0.68480	0.68740	0.68480	0.68400	0.68540	0.68540
50	0.71320	0.71500	0.71320	0.71240	0.71360	0.71400
100	0.80120	0.80180	0.80140	0.80180	0.80180	0.80160
200	0.87360	0.87500	0.87340	0.87340	0.87420	0.87380
300	0.91100	0.91200	0.91100	0.91240	0.91120	0.91160
400	0.93520	0.93600	0.93580	0.93580	0.93500	0.93620
500	0.95180	0.95200	0.95180	0.95180	0.95160	0.95220
750	0.97420	0.97400	0.97340	0.97380	0.97360	0.97400
1000	0.98500	0.98540	0.98520	0.98540	0.98500	0.98540
2000	0.99780	0.99780	0.99780	0.99780	0.99780	0.99780
3000	0.99920	0.99920	0.99920	0.99920	0.99920	0.99920
4000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.10a. ARL, SDRL, APFL, SDPFL, Replications, and Stops for Two Stage Short Run $(\bar{X}, R)$ Control Charts with Stage 1: IC and Stage 2: OOC (SD)

D&R Procedure	ARL	SDRL	APFL	SDPFL	Replications (Skipped)	Stops
1	23.24	93.78	0.01100	0.05779	5000 (0)	1
2	22.38	89.05	0.01178	0.05779	5000 (0)	2
3	22.56	89.39	0.01056	0.04953	5000 (0)	0
4	22.16	86.67	0.00736	0.03421	5000 (-----)	-----
5	22.84	92.74	0.00994	0.04787	5000 (0)	0
6	22.57	89.39	0.01052	0.04839	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 28 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 10 # of Times D&R 2 Iterated More Than Once for the $\bar{X}$ Control Chart: 96						

Table 8.10b. $P(RL \leq t)$ for Two Stage Short Run $(\bar{X}, R)$ Control Charts with Stage 1: IC and Stage 2: OOC (SD)

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.18840	0.18860	0.18760	0.17680	0.18860	0.18880
2	0.31400	0.31380	0.31300	0.30000	0.31460	0.31380
3	0.40000	0.39980	0.39960	0.38880	0.40160	0.40040
4	0.46680	0.46780	0.46620	0.45780	0.46800	0.46720
5	0.51900	0.52000	0.51980	0.51160	0.52140	0.52040
6	0.56100	0.56200	0.56140	0.55500	0.56320	0.56200
8	0.62960	0.63080	0.62980	0.62600	0.63100	0.63020
10	0.67980	0.68040	0.67920	0.67500	0.68080	0.67940
15	0.75680	0.75940	0.75940	0.75680	0.75900	0.75920
20	0.80380	0.80800	0.80620	0.80480	0.80480	0.80600
30	0.86120	0.86340	0.86320	0.86060	0.86200	0.86280
40	0.89240	0.89460	0.89440	0.89260	0.89380	0.89420
50	0.91340	0.91640	0.91500	0.91420	0.91460	0.91500
100	0.96120	0.96260	0.96220	0.96220	0.96220	0.96220
200	0.98220	0.98300	0.98280	0.98400	0.98280	0.98280
300	0.98940	0.99000	0.98960	0.99080	0.98980	0.98960
400	0.99280	0.99340	0.99320	0.99400	0.99320	0.99320
500	0.99520	0.99540	0.99540	0.99620	0.99540	0.99540
750	0.99680	0.99720	0.99720	0.99760	0.99700	0.99720
1000	0.99780	0.99800	0.99800	0.99800	0.99780	0.99800
2000	0.99940	0.99940	0.99940	0.99940	0.99940	0.99940
3000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.11a. ARL, SDRL, APFL, SDPFL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: IC and Stage 2: OOC (MS)

D&R Procedure	ARL	SDRL	APFL	SDPFL	Replications (Skipped)	Stops
1	8.88	130.78	0.01072	0.05435	4999 (1)	1
2	6.63	17.56	0.01086	0.05166	5000 (0)	1
3	6.76	18.00	0.01082	0.05077	5000 (0)	0
4	6.64	15.57	0.00724	0.03515	5000 (-----)	-----
5	6.78	17.58	0.01000	0.04863	5000 (0)	0
6	6.75	17.98	0.01052	0.04835	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 20 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 4 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 89						

Table 8.11b. $P(RL \le t)$ for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: IC and Stage 2: OOC (MS)

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.31026	0.31540	0.31680	0.30540	0.31320	0.31520
2	0.47650	0.48620	0.48520	0.47760	0.48140	0.48540
3	0.59492	0.60440	0.60220	0.59960	0.60120	0.60260
4	0.67013	0.67980	0.67720	0.67260	0.67620	0.67760
5	0.72334	0.73500	0.73240	0.72880	0.73120	0.73320
6	0.76215	0.77460	0.77120	0.76960	0.77000	0.77180
8	0.81716	0.82680	0.82380	0.82280	0.82320	0.82400
10	0.85437	0.86500	0.86140	0.86140	0.86100	0.86160
15	0.91158	0.91700	0.91440	0.91340	0.91500	0.91480
20	0.93599	0.94160	0.93980	0.93920	0.93980	0.94020
30	0.96179	0.96540	0.96400	0.96420	0.96340	0.96460
40	0.97680	0.97980	0.97900	0.97960	0.97900	0.97920
50	0.98260	0.98480	0.98440	0.98460	0.98400	0.98440
100	0.99420	0.99560	0.99540	0.99560	0.99500	0.99540
200	0.99760	0.99840	0.99800	0.99840	0.99800	0.99800
300	0.99920	0.99940	0.99940	0.99980	0.99960	0.99940
400	0.99960	0.99960	0.99960	1.00000	0.99980	0.99960
500	0.99960	0.99980	0.99980	1.00000	0.99980	0.99980
750	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
1000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
2000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
5000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
10000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

than the ARL values for D&Rs 2 and 4 in Tables 8.9a and 8.10a, respectively. The ARL for D&R 2 in Table 8.9a is the highest of the three (it is 1423.680% larger than the ARL for D&R 2 in Table 8.11a). These results imply that, under the assumptions of this simulation, the type of OOC condition in Stage 2 has an affect on the OOC ARL in Stage 2. As expected, the ARL values for each of the six D&R procedures in Tables 8.9a-8.11a are much lower than the respective ARL values in Table 8.1a.

Based on the APFL, Tables 8.9a-8.11a indicate that D&R 4 is the delete and revise procedure of choice regardless of the OOC condition in Stage 2. This reaffirms the statement made in the first subsection of this section that, in terms of the APFL, it is preferable to use subgroups that signal false alarms in the construction of second stage control limits. Also, the APFL values for D&R 4 are reasonably close to 0.0086838, the theoretical probability of a false alarm. However, the APFL values for the other D&R procedures are slightly inflated.

The choice of the appropriate D&R procedure based on the $P(RL \leq t)$ values varies depending on the OOC condition as well as the subgroup number t. In Table 8.9b, D&R 2 results in the highest $P(RL \leq t)$ values for shown values of $t \leq 200$ (except t=4). In Table 8.10b, D&Rs 5 (for shown values of $t \leq 10$ (except t=1)), 2 (for shown values of $t \geq 15$ and $t \leq 100$), and 4 (for shown values of $t \geq 200$) result in the highest $P(RL \leq t)$ values. In Table 8.11b, D&Rs 2 (for shown values of $t \leq 200$ (except t=1)) and 4 (for shown values of $t \geq 100$) result in the highest $P(RL \leq t)$ values. Since the ARL value in Table 8.10a is not the lowest for D&R 2 or D&R 5, this is another example of how the ARL can be misleading in choosing the appropriate D&R procedure in a short run situation.

The largest P(RL ≤ t) values in Table 8.11b are larger than the largest P(RL ≤ t) values in Tables 8.9b and 8.10b. The largest P(RL ≤ t) values in Table 8.9b are lower than those in Tables 8.10b and 8.11b. These results imply that, under the assumptions of this simulation, the type of OOC condition in Stage 2 has an affect on the P(RL ≤ t) values in Stage 2. As expected, the P(RL ≤ t) values for each of the six D&R procedures in Tables 8.9b-8.11b are much higher than the respective P(RL ≤ t) values in Table 8.1a.

The information in Tables 8.9a-8.11b presents another example of the tradeoff mentioned by Del Castillo (1995) between having a low probability of a false alarm and a high probability of detecting a special cause signal inherent with two stage short run control charts. While D&R 4 results in the lowest APFL values regardless of the OOC condition in Stage 2, it also results in the lowest P(RL ≤ t) values for many of the shown values of t in Tables 8.9b and 8.10b.

Again, as in the two previous subsections, the information in Tables 8.9a-8.11a indicates that D&R 1 and D&R 2 are iterating more than once. These multiple iterations seem to create conditions causing replications to be skipped and the chosen D&R procedure to be stopped. Also, if one were to construct confidence intervals using the ARL and SDRL values in Tables 8.9a-8.11a, then, depending on the confidence level chosen, the ARL results in Tables 8.9a-8.11a may not be statistically significantly different.

Sample Runs for an OOC Process in Stages 1 and 2

The final 18 sample runs of the program are for the process being OOC during both

Stage 1 and Stage 2 control charting. Two stage short run control charting for $(\overline{X}, R)$ charts was simulated using all six D&R procedures for each OOC condition (MN, SD, MS) in Stage 1 and one OOC condition (MN) in Stage 2. The results of these simulations appear in Tables 8.12a-8.14b.

As in the previous subsection, since the process is being simulated as OOC in Stage 2, it is desirable for the ARL and, as always, the APFL values in Tables 8.12a-8.14a to be as low as possible. Also, it is desirable for the $P(RL \leq t)$ values in Tables 8.12b-8.14b to be as high as possible (since they correspond to probabilities of detecting special causes within t or less subgroups after the shift in Stage 2), especially for small numbers of subgroups (since a short run situation is in effect).

Based on the ARL, D&R 2 (in Tables 8.12a and 8.14a) and D&R 3 (in Table 8.13a) are the delete and revise procedures of choice. The ARL for D&R 3 in Table 8.13a is lower than the ARL values for D&R 2 in Tables 8.12a and 8.14a. The ARL for D&R 2 in Table 8.14a is the highest of the three. These results imply that, under the assumptions of this simulation, the type of OOC condition in Stage 1 has an affect on the OOC (MN) ARL in Stage 2. Additionally, the ARL values for each of the six D&R procedures in Table 8.9a are much lower than the respective ARL values in Tables 8.12a-8.14a. This result implies that, under the assumptions of this simulation, an OOC condition in Stage 1 causes an increase in the OOC (MN) ARL in Stage 2, regardless of the D&R procedure used.

Based on the APFL, Tables 8.12a-8.14a indicate that D&R 4 is the delete and revise procedure of choice regardless of the OOC condition in Stage 1. This implies that, under the assumptions of this simulation, it is preferable to use subgroups that signal shifts in

Table 8.12a. ARL, SDRL, APFL, SDPFL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MN) and Stage 2: OOC (MN)

D&R Procedure	ARL	SDRL	APFL	SDPFL	Replications (Skipped)	Stops
1	464.86	693.88	0.03813	0.11174	4996 (4)	12
2	393.96	584.75	0.03465	0.09819	4995 (5)	11
3	415.52	596.73	0.03844	0.10604	5000 (0)	0
4	422.42	603.49	0.03208	0.08711	5000 (-----)	-----
5	450.38	654.57	0.03823	0.10840	4999 (1)	0
6	425.71	603.89	0.03441	0.09416	4998 (2)	0
# of Times D&R 1 Iterated More Than Once: 111						
# of Times D&R 2 Iterated More Than Once for the R Control Chart: 2						
# of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 644						

Table 8.12b. P(RL ≤ t) for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MN) and Stage 2: OOC (MN)

t	Delete and Revise (D&R) Procedure					
	1	**2**	**3**	**4**	**5**	**6**
1	0.01801	0.03003	0.02220	0.01700	0.01760	0.01741
2	0.03243	0.05005	0.04120	0.03280	0.03181	0.03201
3	0.04724	0.06647	0.05700	0.04660	0.04701	0.04522
4	0.05805	0.08028	0.06860	0.05680	0.05741	0.05482
5	0.06805	0.09329	0.07920	0.06700	0.06841	0.06603
6	0.07686	0.10430	0.08820	0.07640	0.07682	0.07383
8	0.09267	0.12513	0.10860	0.09620	0.09382	0.09284
10	0.10969	0.14234	0.12460	0.11220	0.11082	0.10944
15	0.13491	0.17137	0.15420	0.14100	0.13703	0.13906
20	0.15873	0.20180	0.18520	0.17100	0.16363	0.16847
30	0.20056	0.25185	0.22920	0.21560	0.20664	0.21449
40	0.23259	0.28529	0.26240	0.24940	0.23785	0.24790
50	0.25560	0.31051	0.28600	0.27340	0.26025	0.27231
100	0.35649	0.41622	0.38660	0.37580	0.36067	0.37675
200	0.48679	0.54234	0.51780	0.51100	0.49210	0.50900
300	0.57906	0.63023	0.60820	0.60360	0.58312	0.60124
400	0.65232	0.69530	0.67640	0.67200	0.65673	0.66967
500	0.70136	0.74374	0.72640	0.72160	0.70734	0.71929
750	0.80004	0.82943	0.82000	0.81800	0.80436	0.81453
1000	0.85989	0.88308	0.87720	0.87580	0.86377	0.87275
2000	0.96357	0.97337	0.97160	0.97060	0.96699	0.97099
5000	0.99760	0.99920	0.99920	0.99900	0.99800	0.99920
10000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
20000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.13a. ARL, SDRL, APFL, SDPFL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (SD) and Stage 2: OOC (MN)

D&R Procedure	**ARL**	**SDRL**	**APFL**	**SDPFL**	**Replications (Skipped)**	**Stops**
1	308.94	783.30	0.00468	0.02977	4999 (1)	4
2	288.91	391.09	0.00490	0.02909	5000 (0)	6
3	288.71	389.04	0.00452	0.02675	5000 (0)	0
4	306.79	395.70	0.00298	0.01901	5000 (-----)	-----
5	295.94	391.20	0.00426	0.02668	5000 (0)	0
6	291.88	393.77	0.00374	0.02218	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 85						
# of Times D&R 2 Iterated More Than Once for the R Control Chart: 30						
# of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 192						

Table 8.13b. $P(RL \leq t)$ for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (SD) and Stage 2: OOC (MN)

t	**Delete and Revise (D&R) Procedure**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.03021	0.03240	0.03200	0.01840	0.02800	0.02720
2	0.04921	0.05480	0.05420	0.03200	0.04860	0.04860
3	0.06401	0.06820	0.06660	0.04280	0.06260	0.06160
4	0.07361	0.07840	0.07700	0.05140	0.07220	0.07120
5	0.08382	0.09040	0.08720	0.05940	0.08260	0.08280
6	0.09322	0.09960	0.09640	0.06680	0.09200	0.09140
8	0.10762	0.11840	0.11480	0.08000	0.10720	0.10960
10	0.12102	0.13120	0.12700	0.09100	0.12020	0.12200
15	0.14923	0.16000	0.15600	0.11720	0.14760	0.15220
20	0.17223	0.18380	0.17920	0.13860	0.17100	0.17620
30	0.21264	0.22400	0.22140	0.18060	0.20980	0.21860
40	0.24625	0.25840	0.25600	0.21520	0.24380	0.25280
50	0.27305	0.28680	0.28380	0.24380	0.27100	0.28120
100	0.38908	0.40520	0.40320	0.36740	0.39000	0.40020
200	0.55931	0.57000	0.56940	0.54400	0.56200	0.56640
300	0.66813	0.68120	0.67940	0.65880	0.67080	0.67740
400	0.75195	0.76240	0.76260	0.74580	0.75560	0.76160
500	0.80576	0.81780	0.81900	0.80480	0.80980	0.81560
750	0.89858	0.90520	0.90480	0.89740	0.90100	0.90400
1000	0.94179	0.94420	0.94400	0.94220	0.94280	0.94320
2000	0.99060	0.99120	0.99140	0.99060	0.99140	0.99080
5000	0.99940	0.99980	0.99980	0.99980	0.99980	0.99980
10000	0.99980	1.00000	1.00000	1.00000	1.00000	1.00000
50000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 8.14a. ARL, SDRL, APFL, SDPFL, Replications, and Stops for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MS) and Stage 2: OOC (MN)

D&R Procedure	ARL	SDRL	APFL	SDPFL	Replications (Skipped)	Stops
1	429.83	640.60	0.00615	0.04033	4993 (7)	11
2	405.27	504.02	0.00788	0.04529	4998 (2)	14
3	420.65	511.23	0.01102	0.05815	5000 (0)	0
4	428.56	506.37	0.00580	0.03254	5000 (-----)	-----
5	421.66	529.70	0.00688	0.04451	5000 (0)	0
6	415.90	508.27	0.00716	0.03900	5000 (0)	0
# of Times D&R 1 Iterated More Than Once: 120 # of Times D&R 2 Iterated More Than Once for the R Control Chart: 30 # of Times D&R 2 Iterated More Than Once for the $\overline{X}$ Control Chart: 411						

Table 8.14b. $P(RL \leq t)$ for Two Stage Short Run $(\overline{X}, R)$ Control Charts with Stage 1: OOC (MS) and Stage 2: OOC (MN)

t	Delete and Revise (D&R) Procedure					
	1	2	3	4	5	6
1	0.00841	0.00960	0.00500	0.00240	0.00600	0.00460
2	0.01682	0.01961	0.01160	0.00700	0.01240	0.01140
3	0.02223	0.02601	0.01780	0.01000	0.01900	0.01780
4	0.02704	0.03061	0.02280	0.01400	0.02460	0.02240
5	0.03265	0.03842	0.02940	0.01780	0.02940	0.02900
6	0.03685	0.04462	0.03440	0.02140	0.03380	0.03420
8	0.04506	0.05442	0.04120	0.02660	0.04140	0.04300
10	0.05327	0.06343	0.04920	0.03360	0.04900	0.05140
15	0.07090	0.08123	0.06680	0.05020	0.06560	0.06780
20	0.08412	0.09664	0.08080	0.06260	0.08020	0.08240
30	0.11376	0.12745	0.11160	0.08880	0.10960	0.11340
40	0.14240	0.15606	0.13760	0.11560	0.13880	0.14120
50	0.16663	0.18327	0.16320	0.14040	0.16500	0.16720
100	0.27278	0.29192	0.27060	0.24780	0.26960	0.27440
200	0.43221	0.44798	0.42940	0.41480	0.43340	0.43500
300	0.55257	0.56623	0.54780	0.54080	0.55200	0.55380
400	0.64991	0.66246	0.64780	0.63940	0.65180	0.65380
500	0.71841	0.72929	0.71640	0.71100	0.71760	0.72140
750	0.83457	0.84174	0.83560	0.83180	0.83400	0.83600
1000	0.89625	0.90276	0.89860	0.89540	0.89640	0.90040
2000	0.97877	0.98159	0.97980	0.97940	0.97920	0.98040
5000	0.99840	0.99940	0.99940	0.99960	0.99920	0.99940
10000	0.99960	1.00000	1.00000	1.00000	1.00000	1.00000
20000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

the mean, the standard deviation, or both in the construction of second stage control limits. The cost, in terms of the loss in reliability of second stage control limits, is higher by throwing out subgroups that signal shifts in the mean, the standard deviation, or both than it is by including them in the construction of second stage control limits. Additionally, comparing the APFL results in Table 8.9a with those in Tables 8.12a-8.14a reveals that, under the assumptions of this simulation, an MN in Stage 1 has the effect of increasing the APFL (see Table 8.12a) and an SD in Stage 1 has the effect of decreasing the APFL (see Table 8.13a).

An issue of concern is the differences in the APFL values from 0.0086838, the theoretical probability of a false alarm. The APFL value for D&R 4 in Table 8.12a is 369.424% larger than 0.0086838. The APFL values for D&R 4 in Tables 8.13a and 8.14a are 65.683% and 33.209%, respectively, smaller than 0.0086838. These results are somewhat consistent with those regarding the $P(RL \leq t)$ values when t=1 in Tables 8.6b-8.8b. Clearly, under the assumptions of this simulation, the type of OOC condition in Stage 1 has a significant effect on the APFL values before the shift in Stage 2.

Based on the $P(RL \leq t)$ values, D&R 2 is the appropriate delete and revise procedure for most of the shown values of t regardless of the OOC condition in Stage 1. Since Table 8.13a indicates that D&R 3 is the delete and revise procedure of choice, this is another example of how the ARL can be misleading in choosing the appropriate D&R procedure in a short run situation. The fact that the largest $P(RL \leq t)$ values in Table 8.14b are lower than those in Tables 8.12b and 8.13b for most of the shown values of t implies that, under the assumptions of this simulation, the type of OOC condition in Stage 1 has an affect on the $P(RL \leq t)$ values in Stage 2.

Additionally, the largest P(RL ≤ t) values in Table 8.9b are larger than those in Tables 8.12b-8.14b. This result implies that, under the assumptions of this simulation, an OOC condition in Stage 1 decreases the P(RL ≤ t) values in Stage 2. This is not desirable because of the MN in Stage 2. However, this is desirable for Stage 2 IC as was the case in comparing results in Table 8.1b to those in Tables 8.6b-8.8b earlier. Clearly, under the assumptions of this simulation, when one is interested in detecting MN in Stage 2, it is highly desirable to have the process IC when drawing first stage subgroups.

The information in Tables 8.12a-8.14b presents another example of the tradeoff mentioned by Del Castillo (1995) between having a low probability of a false alarm and a high probability of detecting a special cause signal inherent with two stage short run control charts. While D&R 4 results in the lowest APFL values regardless of the OOC condition in Stage 1, it also results in the lowest P(RL ≤ t) values for many of the shown values of t in Tables 8.13b and 8.14b.

Again, as in the three previous subsections, the information in Tables 8.12a-8.14a indicates that D&R 1 and D&R 2 are iterating more than once. These multiple iterations seem to create conditions causing replications to be skipped and the chosen D&R procedure to be stopped. Also, if one were to construct confidence intervals using the ARL and SDRL values in Tables 8.12a-8.14a, then, depending on the confidence level chosen, the ARL results in Tables 8.12a-8.14a may not be statistically significantly different.

Conclusions from the Sample Runs

The interpretation of the sample runs of the computer program in this section establish the fact that no hard and fast rules can be developed regarding which D&R procedure is appropriate when performing two stage short run variables control charting. Under the assumptions of the simulations performed in this section, the choice of the appropriate D&R procedure varies both among and within measurements, among control chart combinations, among IC and various OOC conditions in both stages, and among numbers of subgroups plotted in Stage 2. It may even be possible that the choice of the appropriate D&R procedure varies among shift sizes and the timing of shifts, though this is not investigated here.

If no decisions can be made regarding values for these variables, then extensive sample runs similar to the ones in this section need to be performed. However, if certain values for these variables are desired, then the process of making sample runs and interpreting their results is much simpler.

Conclusions

This chapter and the methodology it presents make important contributions. For the first time, the appropriate D&R procedure to use when performing two stage short run variables control charting may be determined. The importance of the computer program is evident because the choice of the appropriate D&R procedure varies depending on the values of many variables. Tables would only be able to provide very limited results. Additionally, the computer program can be expanded to include other variable values (e.g., other types of OOC conditions).

CHAPTER IX

SUMMARY

Introduction

This chapter serves three purposes. The first is to briefly summarize Chapters I-VIII of this dissertation in order to provide an overall perspective of the process undertaken to develop and solve the research problem, which is stated in Chapter I and will be restated in this chapter. The second is to provide final conclusions based on the research in Chapters IV-VIII. The third is to present areas for future research within the realm of two stage short run control charting.

Summary of Chapters

Chapter I includes the following: background information on and the statement of the research problem; the research objective, sub-objectives, and tasks; and the research contributions. The research problem has two parts. The first part is that Hillier's (1969) methodology is limited to $(\overline{X}, R)$ control charts (see Hillier (1969)) and to $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ control charts (see Yang and Hillier (1970)). Additionally, limited and in some cases incorrect results are presented in the literature for these charts. The second part is that the process of establishing control in the first stage of the two stage procedure is not clear (see Faltin, Mastrangelo, Runger, and Ryan (1997)).

The research objective, which is a statement of the resolution of the research problem, is to investigate, extend, and generalize a methodology for two stage short run variables control charting. The "investigate" part of the research objective involves the entire process of developing the research problem, the research objective, the five sub-objectives and their respective tasks; learning and applying relevant theory; developing methodologies; examining the results from the implementation of the methodologies; and drawing conclusions based on the results. The "extend" part involves extending Hillier's (1969) two stage short run theory to $(\overline{X}, s)$ and (X, MR) control charts. It also involves extending it to allow for the determination of the appropriate execution of the two stage procedure. The "generalize" part involves the development of the computer programs to calculate two stage short run control chart factors for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts. It also involves the development of the computer program that provides information that one may use to determine which delete and revise (D&R) procedure to use to establish control in the first stage of the two stage procedure.

Chapter II is a literature review of the three main topics that are essential to understanding the development and resolution of the research problem. The first topic is the different approaches to applying $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts to short run situations. The second topic is the different ways of executing the two stage procedure. The third topic is the different metrics used to determine control chart performance in a short run situation.

Chapter III describes the process required to perform two stage short run variables control charting in order to indicate where and how to use the research presented in Chapters IV-VIII in this process. Included in this description are tables that indicate, based on the choice of the two stage short run control chart ($(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, or (X, MR)), the appropriate program to use from Chapters IV-VII, the output to use from these programs, and the equations to use to construct Stage 1 and Stage 2 control limits. Additionally, a table is presented that indicates, based on the choice of the statistic ($\overline{R}$, $\overline{v}$, $\sqrt{\overline{v}}$, $\overline{s}$, or $\overline{MR}$), the appropriate program to use from Chapters IV-VII, the output to use from these programs, and the equations to use to calculate unbiased estimates of the process variance and standard deviation.

The research in Chapter IV accomplishes the tasks associated with research sub-objective 1, which is stated in Chapter I. The *Mathcad* (1998) program in Chapter IV accurately calculates, using exact equations, two stage short run control chart factors for $(\overline{X}, R)$ charts regardless of the subgroup size, number of subgroups, alpha for the $\overline{X}$ control chart, alpha for the R control chart above the upper control limit, and alpha for the R control chart below the lower control limit (alpha is the probability of a Type I error (i.e., the probability of a false alarm)).

The research in Chapter V accomplishes the tasks associated with research sub-objective 2, which is stated in Chapter I. The *Mathcad* (1998) program in Chapter V accurately calculates, using exact equations, two stage short run control chart factors for $(\overline{X}, v)$ and $(\overline{X}, \sqrt{v})$ charts regardless of the subgroup size, number of subgroups, alpha for the $\overline{X}$ control chart, alpha for the v and $\sqrt{v}$ control charts above the upper control limit, and alpha for the v and $\sqrt{v}$ control charts below the lower control limit.

The research in Chapter VI accomplishes the tasks associated with research sub-objective 3, which is stated in Chapter I. The *Mathcad* (1998) program in Chapter VI accurately calculates, using exact equations, two stage short run control chart factors for $(\overline{X}, s)$ charts regardless of the subgroup size, number of subgroups, alpha for the $\overline{X}$

control chart, alpha for the s control chart above the upper control limit, and alpha for the s control chart below the lower control limit.

The research in Chapter VII accomplishes the tasks associated with research sub-objective 4, which is stated in Chapter I. The *Mathcad* (1998) program in Chapter VII accurately calculates, using exact equations, two stage short run control chart factors for (X, MR) charts regardless of the number of subgroups, alpha for the X control chart, alpha for the MR control chart above the upper control limit, and alpha for the MR control chart below the lower control limit.

The research in Chapter VIII accomplishes the tasks associated with research sub-objective 5, which is stated in Chapter I. The FORTRAN (1999) program in Chapter VIII simulates two stage short run control charting for $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts for in-control and various out-of-control conditions in both stages using six different D&R procedures.

The accomplishment of the tasks associated with the five research sub-objectives means that the research objective is met. Consequently, the research problem as stated in Chapter I of this dissertation and restated in this chapter is solved.

Conclusions

The research in this dissertation results in a comprehensive, theoretically sound, easy-to-implement, and effective methodology for two stage short run control charting using $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) charts. The application of this research is immediate because of the computer programs in Chapters IV-VIII that implement the research. Also, the application of this research is not limited because of the inputs accepted by the programs. Additionally, the program in Chapter VIII can be expanded to accept more varied inputs.

As a result of the research and computer programs in Chapters IV-VII, those involved with quality control in industry will, for the first time, be able to use theoretically precise control chart factors to determine control limits for $(\overline{\overline{X}}, R)$, $(\overline{\overline{X}}, v)$, $(\overline{\overline{X}}, \sqrt{v})$, $(\overline{\overline{X}}, s)$, and (X, MR) charts regardless of the subgroup size, number of subgroups, and alpha values. This flexibility is valuable in that process monitoring will no longer have to be adjusted to use the limited, and in some cases incorrect, results previously available in the literature. Also, the programs put an end to the erroneous use of conventional control chart constants when in a short run situation.

It is recommended that the computer programs in Chapters IV, V, and VII replace the use of the tables of two stage short run control chart factors in Hillier (1969), Yang and Hillier (1970), Pyzdek (1993), and Yang (1995, 1999, 2000) because of the limited, and in some cases incorrect, results given in these papers. The corrections provided by the tables in the appendices of this dissertation are given in detail in Chapters IV, V, and VII. Any other corrections can be made by the appropriate program from these chapters.

As a result of the research and computer program in Chapter VIII, a methodology is available that, for the first time, provides information that one may use to determine which D&R procedure is most appropriate to use when performing two stage short run control charting with $(\overline{\overline{X}}, R)$, $(\overline{\overline{X}}, v)$, $(\overline{\overline{X}}, \sqrt{v})$, $(\overline{\overline{X}}, s)$, and (X, MR) charts. The program is important because, based on the sample runs in Chapter VIII, the choice of the appropriate D&R procedure varies depending on the values of many variables.

Concerning academia, Chapters IV, V, VI, and VII provide a valuable reference for anyone interested in anything having to do with $(\overline{\overline{X}}, R)$, $(\overline{\overline{X}}, v)$, $(\overline{\overline{X}}, \sqrt{v})$, $(\overline{\overline{X}}, s)$, and (X, MR) control charts, respectively. Furthermore, the programs in these chapters eliminate the need for the research question of how many subgroups are enough before conventional control chart constants may be used. Also, the research in Chapter VIII advances the study of the control chart revision process.

In addition to the above contributions, the research in Chapters VI and VII provides results that may be useful beyond the realm of quality control. These results are two new equations to calculate unbiased estimates of a process variance based on the statistics $\bar{s}$ (Chapter VI) and $\overline{MR}$ (Chapter VII).

Areas for Future Research

Several areas for future research exist within the realm of two stage short run control charting. One area is to continue developing multivariate counterparts to two stage short run $(\overline{\overline{X}}, R)$, $(\overline{\overline{X}}, v)$, $(\overline{\overline{X}}, \sqrt{v})$, $(\overline{\overline{X}}, s)$, and (X, MR) control charts. This has already been done for Yang and Hillier's (1970) two stage short run $\overline{\overline{X}}$ control chart (see Alt, Goode, and Wadsworth (1976)). This is desirable because situations may exist in which it is beneficial to use multivariate control charting when in a short run situation.

Another area is to continue developing two stage short run attributes control charts. This has already been done for p control charts (see Nedumaran and Leon (1998)), which are based on the Binomial distribution. This is desirable because situations may exist in which it is beneficial to chart classification or count data when in a short run situation.

A third area concerns the updating of Stage 2 control limits when in a short run situation. The issue is what to do with previous in-control subgroups that plot out-of-control after an update. If they are deleted so that they will not be used in the next update, then important information about the process is being thrown away. Since information is already limited in a short run situation, this may result in less reliable Stage 2 control limits. However, keeping these out-of-control subgroups so that they will be used in the next update may also result in less reliable control limits. It is desirable to develop a methodology that will provide information to examine this tradeoff.

A fourth area is to study the performance of two stage short run $(\overline{X}, R)$, $(\overline{X}, v)$, $(\overline{X}, \sqrt{v})$, $(\overline{X}, s)$, and (X, MR) control charts when data obtained from a process are non-normal and/or non-independent. The computer program in Chapter VIII may be modified to do this.

Final areas for future research concern extensions of the computer program in Chapter VIII. One extension is to include the approach by Roes, Does, and Schurink (1993) (see the Stage One Control Limits subsection of The Two Stage Procedure section of Chapter II) for determining out-of-control subgroups in Stage 1. Another extension is to include the option of not deleting false alarms before a shift in Stage 1. A third extension is to include an out-of-control condition caused by a trend in one or both of the population parameters. A fourth extension is to include the option of performing Stage 2 control charting with any desired combination of Nelson's (1984) tests for special causes or runs rules (i.e., the four tests for instability in Western Electric Co., Inc. (1956)).

REFERENCES

Al-Salti, M. and Statham, A. (1994). "The Application of Group Technology Concept for Implementing SPC in Small Batch Manufacture". *International Journal of Quality and Reliability Management*, 11 (4), 64-76.

Alt, F. B., Goode, J. J., and Wadsworth, H. M. (1976). "Small Sample Probability Limits for the Mean of a Multivariate Normal Process". *30th Annual ASQC Technical Conference Transactions - Toronto*, 170-176.

Bain, L. J. and Engelhardt, M. (1992). *Introduction to Probability and Mathematical Statistics*, 2nd Ed. Duxbury Press, Belmont, CA.

Bothe, D. R. (1988). "SPC for Short Production Runs". *Quality*, 27 (12), 58-59.

Bothe, D. R. (1989). "A Powerful New Control Chart for Job Shops". *43rd Annual ASQC Quality Congress Transactions - Toronto*, 265-270.

Burr, I. W. (1969). "Control Charts for Measurements with Varying Sample Sizes". *Journal of Quality Technology*, 1 (3), 163-167.

Burr, I. W. (1976). *Statistical Quality Control Methods*. Marcel Dekker, Inc., New York, NY.

Burr, J. T. (1989). "SPC in the Short Run". *43rd Annual ASQC Quality Congress Transactions - Toronto*, 776-780.

Cadwell, J. H. (1953). "Approximating to the Distributions of Measures of Dispersion by a Power of χ^2". *Biometrika*, 40, 336-346.

Cadwell, J. H. (1954). "The Statistical Treatment of Mean Deviation". *Biometrika*, 41, 12-18.

Case, K. E. (1998). "The Delete and Revise (D&R) Procedure". *Class Notes / Lecture Material*, Oklahoma State University, Stillwater, Oklahoma.

Chan, J. C. S. (1994). "Design of Exponentially Weighted Moving Average Control Charts for Short Production Runs". Master's Thesis, Kansas State University, Manhattan, Kansas.

Crowder, S. V. and Halbleib, L. L. (2000). "Statistical Process Control for Low Volume Manufacturing: A Review". Unpublished Manuscript.

Cryer, J. D. and Ryan, T. P. (1990). "The Estimation of Sigma for an X Chart: $\overline{MR}/d_2$ or S/c_4 ?". *Journal of Quality Technology*, 22 (3), 187-192.

David, H. A. (1951). "Further Applications of Range to the Analysis of Variance". *Biometrika*, 38, 393-409.

Del Castillo, E. and Montgomery, D. C. (1994). "Short-Run Statistical Process Control: Q-Chart Enhancements and Alternative Methods". *Quality and Reliability Engineering International*, 10, 87-97.

Del Castillo, E. (1995). "Discussion". *Journal of Quality Technology*, 27 (4), 316-321.

Del Castillo, E. and Montgomery, D. C. (1996). "A General Model for the Optimal Economic Design of $\overline{X}$ Charts Used to Control Short or Long Run Processes". *IIE Transactions*, 28 (3), 193-201.

Del Castillo, E. (1996a). "Run Length Distributions and Economic Design of $\overline{X}$ Charts with Unknown Process Variance". *Metrika*, 43 (3), 189-201.

Del Castillo, E. (1996b). "An Algorithm and a Graphical Approach for the Economic Design of $\overline{X}$ Charts for Short Run Processes" in *Statistical Applications in Process Control* edited by J. Bert Keats and Douglas C. Montgomery. Marcel Dekker, Inc, New York, NY.

Deming, W. E. (1986). *Out of the Crisis.* Center for Advanced Engineering Study, Massachusetts Institute of Technology, Cambridge, MA.

Doganaksoy, N. and Vandeven, M. (1997). "Process Monitoring with Multiple Products and Production Lines". *Quality Engineering*, 9 (4), 689-702.

Doty, L. A. (1997). *SPC for Short Run Manufacturing.* Hanser Gardner Publications, Cincinnati, OH.

Duncan, A. J. (1955a). "The Use of Ranges in Comparing Variabilities". *Industrial Quality Control*, 11 (5), 18-19, 22.

Duncan, A. J. (1955b). "Clarification and Further Comments on the 'Use of the Range in Comparing Variabilities'". *Industrial Quality Control*, 11 (8), 70.

Duncan, A. J. (1955c). "Errata - Clarification and Further Comments on the 'Use of the Range in Comparing Variabilities'". *Industrial Quality Control*, 12 (2), 36.

Duncan, A. J. (1956). "The Economic Design of $\overline{X}$ Charts Used to Maintain Current Control of a Process". *Journal of the American Statistical Association*, 51 (274), 228-242.

Duncan, A. J. (1958). "Design and Operation of a Double-Limit Variables Sampling Plan". *Journal of the American Statistical Association*, 53 (282), 543-550.

Duncan, A. J. (1974). *Quality Control and Industrial Statistics*, 4th Ed. Richard D. Irwin, Inc., Homewood, IL.

Elam, M. E. and Case, K. E. (2001). "A Computer Program to Calculate Two Stage Short Run Control Chart Factors for $(\overline{X}, R)$ Charts". *Quality Engineering*, 14 (1), To Appear.

Faltin, F. W., Mastrangelo, C. M., Runger, G. C., and Ryan, T. P. (1997). "Considerations in the Monitoring of Autocorrelated and Independent Data". *Journal of Quality Technology*, 29 (2), 131-133.

Farnum, N. R. (1992). "Control Charts for Short Runs: Nonconstant Process and Measurement Error". *Journal of Quality Technology*, 24 (3), 138-144.

FORTRAN - *Essential Lahey FORTRAN 90 Compiler Release 4.00c* (1999). Lahey Computer Systems, Inc., Incline Village, NV.

Foster, G. K. (1988). "Implementing SPC in Low Volume Manufacturing". *42nd Annual ASQC Quality Congress Transactions - Dallas*, 261-267.

Griffith, G. K. (1996). *Statistical Process Control Methods for Long and Short Runs.* ASQC Quality Press, Milwaukee, WI.

Grubbs, F. E. (1946). "The Difference Control Chart with an Example of its Use". *Industrial Quality Control*, 3 (1), 22-25.

Harter, H. L., Clemm, D. S., and Guthrie, E. H. (1959). "The Probability Integrals of the Range and the Studentized Range – Probability Integral and Percentage Points of the Studentized Range; Critical Values for Duncan's New Multiple Range Test". *Wright Air Development Center Technical Report 58-484*, Vol. II; Defense Technical Information Center (DTIC) Document AD Number 231733.

Harter, H. L. (1960). "Tables of Range and Studentized Range". *Annals of Mathematical Statistics*, 31, 1122-1147.

Hartley, H. O. (1944). "Studentization, or the Elimination of the Standard Deviation of the Parent Population from the Random Sample-Distribution of Statistics". *Biometrika*, 33, 173-180.

Hawkins, D. M. (1987). "Self-Starting Cusum Charts for Location and Scale". *Statistician*, 36 (4), 299-316.

Hillier, F. S. (1964). "$\overline{X}$ Chart Control Limits Based on A Small Number of Subgroups". *Industrial Quality Control*, 20 (8), 24-29.

Hillier, F. S. (1967). "Small Sample Probability Limits for the Range Chart". *Journal of the American Statistical Association*, 62 (320), 1488-1493.

Hillier, F. S. (1969). "$\overline{X}$ - and R-Chart Control Limits Based on A Small Number of Subgroups". *Journal of Quality Technology*, 1 (1), 17-26.

Hoel, P. G. (1946). "The Efficiency of the Mean Moving Range". *Annals of Mathematical Statistics*, 17, 475-482.

Irwin, J. O. (1931). "Mathematical Theorems Involved in the Analysis of Variance". *Journal of the Royal Statistical Society*, 94 (Part II), 284-300.

Johnson, N. L. and Welch, B. L. (1939). "On the Calculation of the Cumulants of the χ-distribution". *Biometrika*, 31, 216-218.

Johnson, N. L., Kotz, S., and Balakrishnan, N. (1994). *Continuous Univariate Distributions - Volume 1*, 2nd Ed. John Wiley and Sons, Inc., New York, NY.

King, E. P. (1953). "Estimating the Standard Deviation Of A Normal Population". *Industrial Quality Control*, 10 (2), 30-33.

King, E. P. (1954). "Probability Limits for the Average Chart When Process Standards Are Unspecified". *Industrial Quality Control*, 10 (6), 62-64.

Koons, G. F. and Luner, J. J. (1988). "SPC: Use in Low-Volume Manufacturing Environment". *42nd Annual ASQC Quality Congress Transactions - Dallas,* 541-547.

Koons, G. F. and Luner, J. J. (1991). "SPC in Low-Volume Manufacturing: A Case Study". *Journal of Quality Technology*, 23 (4), 287-295.

Lin, S.-Y., Lai, Y.-J., and Chang, S. I. (1997). "Short-Run Statistical Process Control: Multicriteria Part Family Formation". *Quality and Reliability Engineering International*, 13, 9-24.

Lord, E. (1950). "Power of the Modified *t*-Test (*u*-Test) Based on Range". *Biometrika*, 37, 64-77.

Lucas, J. M. and Saccucci, M. S. (1990). "Exponentially Weighted Moving Average Control Schemes: Properties and Enhancements". *Technometrics*, 32 (1), 1-12.

Marse, K. and Roberts, S. D. (1983). "Implementing a Portable FORTRAN Uniform (0, 1) Generator". *Simulation*, 41 (4), 135-139.

Mathcad 8.03 Professional (1998). MathSoft, Inc., Cambridge, MA.

Mathcad 2000 Professional (1999). MathSoft, Inc., Cambridge, MA.

Mead, R. (1966). "A Quick Method of Estimating the Standard Deviation". *Biometrika*, 53, 559-564.

Montgomery, D. C. (1997). *Introduction to Statistical Quality Control*, 3rd Ed. John Wiley and Sons, Inc., New York, NY.

Nedumaran, G. and Leon, V. J. (1998). "*p*-Chart Control Limits Based on a Small Number of Subgroups". *Quality Engineering*, 11 (1), 1-9.

Nelson, L. S. (1975). "Use of the Range to Estimate Variability". *Journal of Quality Technology*, 7 (1), 46-48.

Nelson, L. S. (1984). "The Shewhart Control Chart - Tests for Special Causes". *Journal of Quality Technology*, 16 (4), 237-239.

Nelson, L. S. (1987). "Upper 10%, 5% and 1% Points of the Maximum F-Ratio". *Journal of Quality Technology*, 19 (3), 165-167.

Nelson, L. S. (1990). "Setting Up a Control Chart Using Subgroups of Varying Sizes". *Journal of Quality Technology*, 22 (3), 245-246.

Ng, C. H. and Case, K. E. (1992). "Control Limits and the ARL: Some Surprises". *First Industrial Engineering Research Conference Proceedings of the Institute of Industrial Engineers*, 127-129.

Numerical Recipes Extension Pack (1997). MathSoft, Inc., Cambridge, MA.

Occasione, J. F. (1956). "Quality Control As Applied to Continuous Processes". *Industrial Quality Control*, 13 (4), 9-13.

Ott, E. R. and Schilling, E. G. (1990). *Process Quality Control*, 2nd Ed. McGraw-Hill, Inc., New York, NY.

Pachares, J. (1959). "Table of the Upper 10% Points of the Studentized Range". *Biometrika*, 46, 461-466.

Palm, A. C. and Wheeler, D. J. (1990). "Equivalent Degrees of Freedom for Estimates of the Process Standard Deviation Based on Shewhart Control Charts". Unpublished Manuscript.

Patnaik, P. B. (1950). "The Use of Mean Range as an Estimator of Variance in Statistical Tests". *Biometrika*, 37, 78-87.

Pearson, E. S. and Hartley, H. O. (1943). "Tables of the Probability Integral of the Studentized Range". *Biometrika*, 33, 89-99.

Pearson, E. S. (1952). "Comparison of Two Approximations to the Distribution of the Range in Small Samples from Normal Populations". *Biometrika*, 39, 130-136.

Pearson, E. S. and Hartley, H. O., Editors. (1962). *Biometrika Tables for Statisticians, Volume 1*, 2nd Ed. Reprint. University Press, Cambridge, Great Britain.

Prescott, P. (1971). "Use of a Simple Range-Type Estimator of σ in Tests of Hypotheses". *Biometrika*, 58 (2), 333-340.

Proschan, F. and Savage, I. R. (1960). "Starting a Control Chart". *Industrial Quality Control*, 17 (3), 12-13.

Pyzdek, T. (1993). "Process Control for Short and Small Runs". *Quality Progress*, 26 (4), 51-60.

Quesenberry, C. P. (1991). "SPC Q Charts for Start-Up Processes and Short or Long Runs". *Journal of Quality Technology*, 23 (3), 213-224.

Quesenberry, C. P. (1993). "The Effect of Sample Size on Estimated Limits for $\overline{X}$ and X Control Charts". *Journal of Quality Technology*, 25 (4), 237-247.

Quesenberry, C. P. (1995a). "On Properties of Q Charts for Variables". *Journal of Quality Technology*, 27 (3), 184-203.

Quesenberry, C. P. (1995b). "Response". *Journal of Quality Technology*, 27 (4), 333-343.

Quesenberry, C. (1998). "Statistical Gymnastics: A critical analysis of one approach to short-run control charts". *Quality Progress*, 31 (9), 77-79.

Roes, K. C. B., Does, R. J. M. M., and Schurink, Y. (1993). "Shewhart-Type Control Charts for Individual Observations". *Journal of Quality Technology*, 25 (3), 188-198.

Ryan, T. P. (1989). *Statistical Methods for Quality Improvement*. John Wiley and Sons, Inc., New York, NY.

Ryan, T. P. (1997). "Efficient Estimation of Control Chart Parameters" in *Frontiers in Statistical Quality Control 5* edited by H.-J. Lenz and P.-Th. Wilrich. Physica-Verlag Heidelberg, Germany.

Sathe, Y. S. and Kamat, A. R. (1957). "Approximations to the Distributions of Some Measures of Dispersion Based on Successive Differences". *Biometrika*, 44, 349-359.

Shewhart, W. A. (1925). "The Application of Statistics as an Aid in Maintaining Quality of a Manufactured Product". *Journal of the American Statistical Association*, 20 (152), 546-548.

Shewhart, W. A. (1926). "Quality Control Charts". *Bell System Technical Journal*, 5 (4), 593-603.

Shewhart, W. A. (1927). "Quality Control". *Bell System Technical Journal*, 6 (4), 722-735.

Shewhart, W. A. (1931). *Economic Control of Quality of Manufactured Product*, 5th Printing. D. Van Nostrand Company, Inc., New York, NY.

Wade, M. R. (1992). "A Study of Q-Charts and Cause-Selecting Charts". Ph.D. Dissertation, University of Alabama, Tuscaloosa, Alabama.

Wasserman, G. S. and Sudjianto, A. (1993). "Short Run SPC Based Upon the Second Order Dynamic Linear Model for Trend Detection". *Communications in Statistics - Simulation and Computation*, 22 (4), 1011-1036.

Wasserman, G. S. (1994). "Short Run SPC Using Dynamic Control Chart". *Computers and Industrial Engineering*, 27, 353-356.

Western Electric Co., Inc. (1956). *Statistical Quality Control Handbook*. Mack Printing Company, Easton, PA.

Wheeler, D. J. (1991). *Short Run SPC*. SPC Press, Inc., Knoxville, TN.

Wheeler, D. J. (1995). *Advanced Topics in Statistical Process Control*. SPC Press, Inc., Knoxville, TN.

Woodall, W. H., Crowder, S. V., and Wade, M. R. (1995). "Discussion". *Journal of Quality Technology*, 27 (4), 328-332.

Yang, C.-H. and Hillier, F. S. (1970). "Mean and Variance Control Chart Limits Based on a Small Number of Subgroups". *Journal of Quality Technology*, 2 (1), 9-16.

Yang, S.-F. (1995). *Control Policy for Small and Short Production Runs*. National Science Council Research Report, Taiwan, ROC.

Yang, S.-F. (1999). "An Approach to Controlling Process Variability for Short Production Runs". *Total Quality Management*, 10 (8), 1123-1129.

Yang, S.-F. (2000). "Statistical Process Control for Short Run Manufacturing Systems". *Process Control and Quality*, 11 (5), 433-439.

APPENDICES

APPENDIX A – Analytical Results for Chapter 2

Show: $E\left(\overline{X} - \overline{\overline{X}}\right) = 0.0$

$$E\left(\overline{X} - \overline{\overline{X}}\right) = E\left(\overline{X}\right) - E\left(\overline{\overline{X}}\right) = E\left(\frac{\sum_{j=1}^{n} X_j}{n}\right) - E\left(\frac{\sum_{i=1}^{m} \overline{X}_i}{m}\right) = \left(\frac{1}{n}\right)\cdot\sum_{j=1}^{n} E\left(X_j\right) - \left(\frac{1}{m}\right)\cdot\sum_{i=1}^{m} E\left(\overline{X}_i\right)$$

$$\Rightarrow E\left(\overline{X} - \overline{\overline{X}}\right) = \left(\frac{1}{n}\right)\cdot\sum_{j=1}^{n}\mu - \left(\frac{1}{m}\right)\cdot\sum_{i=1}^{m} E\left(\frac{\sum_{j=1}^{n} X_{i,j}}{n}\right)$$

$$\Rightarrow E\left(\overline{X} - \overline{\overline{X}}\right) = \left(\frac{1}{n}\right)\cdot(n\cdot\mu) - \left(\frac{1}{m\cdot n}\right)\cdot\sum_{i=1}^{m}\left(\sum_{j=1}^{n} E\left(X_{i,j}\right)\right) = \mu - \left(\frac{1}{m\cdot n}\right)\cdot\sum_{i=1}^{m}\left(\sum_{j=1}^{n}\mu\right)$$

$$\Rightarrow E\left(\overline{X} - \overline{\overline{X}}\right) = \mu - \left(\frac{1}{m\cdot n}\right)\cdot(m\cdot n\cdot\mu) = \mu - \mu = 0.0$$

<u>Show:</u> $\sqrt{Var\left(\overline{X}-\overline{\overline{X}}\right)}=\sqrt{\frac{m+1}{n\cdot m}}\cdot\sigma$

$$Var\left(\overline{X}-\overline{\overline{X}}\right)=Var\left(\overline{X}\right)+Var\left(\overline{\overline{X}}\right)=Var\left(\frac{\sum_{j=1}^{n}X_j}{n}\right)+Var\left(\frac{\sum_{i=1}^{m}\overline{X}_i}{m}\right)$$

$$\Rightarrow Var\left(\overline{X}-\overline{\overline{X}}\right)=\left(\frac{1}{n^2}\right)\cdot\sum_{j=1}^{n}Var\left(X_j\right)+\left(\frac{1}{m^2}\right)\cdot\sum_{i=1}^{m}Var\left(\overline{X}_i\right)$$

since the X_j's and $\overline{X}_i$'s are independent.

$$\Rightarrow Var\left(\overline{X}-\overline{\overline{X}}\right)=\left(\frac{1}{n^2}\right)\cdot\sum_{j=1}^{n}\sigma^2+\left(\frac{1}{m^2}\right)\cdot\sum_{i=1}^{m}Var\left(\frac{\sum_{j=1}^{n}X_{i,j}}{n}\right)$$

$$=\left(\frac{1}{n^2}\right)\cdot\left(n\cdot\sigma^2\right)+\left(\frac{1}{m^2\cdot n^2}\right)\cdot\sum_{i=1}^{m}\left(\sum_{j=1}^{n}Var\left(X_{i,j}\right)\right)$$

since the $X_{i,j}$'s are independent.

$$\Rightarrow Var\left(\overline{X}-\overline{\overline{X}}\right)=\left(\frac{\sigma^2}{n}\right)+\left(\frac{1}{m^2\cdot n^2}\right)\cdot\sum_{i=1}^{m}\left(\sum_{j=1}^{n}\sigma^2\right)=\left(\frac{\sigma^2}{n}\right)+\left(\frac{1}{m^2\cdot n^2}\right)\cdot\left(m\cdot n\cdot\sigma^2\right)$$

$$\Rightarrow Var\left(\overline{X}-\overline{\overline{X}}\right)=\left(\frac{\sigma^2}{n}\right)+\left(\frac{\sigma^2}{m\cdot n}\right)=\left(\frac{m+1}{n\cdot m}\right)\cdot\sigma^2$$

$$\Rightarrow\sqrt{Var\left(\overline{X}-\overline{\overline{X}}\right)}=\sqrt{\frac{m+1}{n\cdot m}}\cdot\sigma$$

Show: $E\left(\overline{X}_k - \overline{\overline{X}}\right) = 0.0$

$$\overline{X}_k - \overline{\overline{X}} = \overline{X}_k - \frac{\sum_{i=1}^{m} \overline{X}_i}{m} = \overline{X}_k - \frac{\overline{X}_k}{m} - \frac{\sum_{\substack{i=1 \\ i \neq k}}^{m} \overline{X}_i}{m} = \left(\frac{m-1}{m}\right) \cdot \overline{X}_k - \frac{\sum_{\substack{i=1 \\ i \neq k}}^{m} \overline{X}_i}{m}$$

$$\Rightarrow E\left(\overline{X}_k - \overline{\overline{X}}\right) = E\left(\left(\frac{m-1}{m}\right) \cdot \overline{X}_k - \frac{\sum_{\substack{i=1 \\ i \neq k}}^{m} \overline{X}_i}{m}\right) = \left(\frac{m-1}{m}\right) \cdot E\left(\overline{X}_k\right) - \left(\frac{1}{m}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} E\left(\overline{X}_i\right)$$

$$\Rightarrow E\left(\overline{X}_k - \overline{\overline{X}}\right) = \left(\frac{m-1}{m}\right) \cdot E\left(\frac{\sum_{j=1}^{n} X_{k,j}}{n}\right) - \left(\frac{1}{m}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} E\left(\frac{\sum_{j=1}^{n} X_{i,j}}{n}\right)$$

$$= \left(\frac{m-1}{m \cdot n}\right) \cdot \sum_{j=1}^{n} E\left(X_{k,j}\right) - \left(\frac{1}{m \cdot n}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} \left(\sum_{j=1}^{n} E\left(X_{i,j}\right)\right)$$

$$= \left(\frac{m-1}{m \cdot n}\right) \cdot \sum_{j=1}^{n} \mu - \left(\frac{1}{m \cdot n}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} \left(\sum_{j=1}^{n} \mu\right)$$

$$= \left(\frac{m-1}{m \cdot n}\right) \cdot (n \cdot \mu) - \left(\frac{1}{m \cdot n}\right) \cdot ((m-1) \cdot n \cdot \mu)$$

$$\Rightarrow E\left(\overline{X}_k - \overline{\overline{X}}\right) = \left(\frac{m-1}{m}\right) \cdot \mu - \left(\frac{m-1}{m}\right) \cdot \mu = 0.0$$

Show: $\sqrt{\operatorname{Var}\left(\overline{X}_k - \overline{\overline{X}}\right)} = \sqrt{\frac{m-1}{n \cdot m}} \cdot \sigma$

$$\operatorname{Var}\left(\overline{X}_k - \overline{\overline{X}}\right) = \operatorname{Var}\left(\left(\frac{m-1}{m}\right) \cdot \overline{X}_k - \frac{\sum_{\substack{i=1 \\ i \neq k}}^{m} \overline{X}_i}{m}\right) = \left(\frac{m-1}{m}\right)^2 \cdot \operatorname{Var}\left(\overline{X}_k\right) + \left(\frac{1}{m^2}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} \operatorname{Var}\left(\overline{X}_i\right)$$

since the $\overline{X}_i$'s are independent.

$$\Rightarrow \operatorname{Var}\left(\overline{X}_k - \overline{\overline{X}}\right) = \left(\frac{m-1}{m}\right)^2 \cdot \operatorname{Var}\left(\frac{\sum_{j=1}^{n} X_{k,j}}{n}\right) + \left(\frac{1}{m^2}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} \operatorname{Var}\left(\frac{\sum_{j=1}^{n} X_{i,j}}{n}\right)$$

$$= \left(\frac{m-1}{m \cdot n}\right)^2 \cdot \sum_{j=1}^{n} \operatorname{Var}\left(X_{k,j}\right) + \left(\frac{1}{m^2 \cdot n^2}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} \left(\sum_{j=1}^{n} \operatorname{Var}\left(X_{i,j}\right)\right)$$

since the $X_{k,j}$'s and the $X_{i,j}$'s are independent.

$$\Rightarrow \operatorname{Var}\left(\overline{X}_k - \overline{\overline{X}}\right) = \left(\frac{m-1}{m \cdot n}\right)^2 \cdot \sum_{j=1}^{n} \sigma^2 + \left(\frac{1}{m^2 \cdot n^2}\right) \cdot \sum_{\substack{i=1 \\ i \neq k}}^{m} \left(\sum_{j=1}^{n} \sigma^2\right)$$

$$= \left(\frac{m-1}{m \cdot n}\right)^2 \cdot \left(n \cdot \sigma^2\right) + \left(\frac{1}{m^2 \cdot n^2}\right) \cdot \left((m-1) \cdot n \cdot \sigma^2\right)$$

$$= \left(\frac{m-1}{m}\right)^2 \cdot \frac{\sigma^2}{n} + \left(\frac{m-1}{m^2}\right) \cdot \frac{\sigma^2}{n}$$

$$\Rightarrow \operatorname{Var}\left(\overline{X}_k - \overline{\overline{X}}\right) = \left(\left(\frac{m-1}{n \cdot m}\right) \cdot \sigma^2\right) \cdot \left(\frac{m-1}{m} + \frac{1}{m}\right) = \left(\left(\frac{m-1}{n \cdot m}\right) \cdot \sigma^2\right) \cdot 1.0 = \left(\frac{m-1}{n \cdot m}\right) \cdot \sigma^2$$

$$\Rightarrow \sqrt{\operatorname{Var}\left(\overline{X}_k - \overline{\overline{X}}\right)} = \sqrt{\frac{m-1}{n \cdot m}} \cdot \sigma$$

APPENDIX B.1 – Analytical Results for Chapter 4

From David (1951), the variance of the mean of m ranges, each based on n observations, is $d3^2/m$, which implies M_n/V_n from Prescott (1971) is equal to:

$$\frac{d2}{\left(\frac{d3^2}{m}\right)} = \frac{d2 \cdot m}{d3^2}$$

(d2 is also the mean of the distribution of the mean range $\overline{R}/\sigma$).

$$\Rightarrow \frac{d2^2 \cdot m}{d3^2} = \frac{\left(\frac{2^{0.5} \cdot \Gamma(0.5 \cdot x + 0.5)}{\Gamma(0.5 \cdot x)}\right)^2}{x - 2 \cdot \left(\frac{\Gamma(0.5 \cdot x + 0.5)}{\Gamma(0.5 \cdot x)}\right)^2} = \frac{\left(\frac{2 \cdot (\Gamma(0.5 \cdot x + 0.5))^2}{(\Gamma(0.5 \cdot x))^2}\right)}{\left(\frac{x \cdot (\Gamma(0.5 \cdot x))^2 - 2 \cdot (\Gamma(0.5 \cdot x + 0.5))^2}{(\Gamma(0.5 \cdot x))^2}\right)}$$

$$\Rightarrow \frac{d2^2 \cdot m}{d3^2} = \frac{2 \cdot (\Gamma(0.5 \cdot x + 0.5))^2}{x \cdot (\Gamma(0.5 \cdot x))^2 - 2 \cdot (\Gamma(0.5 \cdot x + 0.5))^2}$$

$$= \frac{2}{x \cdot \frac{(\Gamma(0.5 \cdot x))^2}{(\Gamma(0.5 \cdot x + 0.5))^2} - 2}$$

$$= \frac{2}{x \cdot e^{\ln\left[\frac{(\Gamma(0.5 \cdot x))^2}{(\Gamma(0.5 \cdot x + 0.5))^2}\right]} - 2}$$

$$= \frac{2}{x \cdot e^{\ln(\Gamma(0.5 \cdot x))^2 - \ln(\Gamma(0.5 \cdot x + 0.5))^2} - 2}$$

$$= \frac{2}{x \cdot e^{2 \cdot \text{gammln}(0.5 \cdot x) - 2 \cdot \text{gammln}(0.5 \cdot x + 0.5)} - 2}$$

$$= \frac{2}{x \cdot e^{2 \cdot (\text{gammln}(0.5 \cdot x) - \text{gammln}(0.5 \cdot x + 0.5))} - 2}$$

$$\Rightarrow \frac{d3^2}{m \cdot d2^2} = \frac{x \cdot e^{2 \cdot (\text{gammln}(0.5 \cdot x) - \text{gammln}(0.5 \cdot x + 0.5))} - 2}{2}$$

From Harter, Clemm, and Guthrie (1959):

$$C(\nu) = \frac{2 \cdot \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} \cdot e^{\frac{-\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}.$$

Let $c\nu = \ln(C(\nu))$.

$$\Rightarrow c\nu = \ln\left[\frac{2 \cdot \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} \cdot e^{\frac{-\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\right]$$

$$= \ln\left[2 \cdot \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} \cdot e^{\frac{-\nu}{2}}\right] - \ln\left(\Gamma\left(\frac{\nu}{2}\right)\right)$$

$$= \ln\left[2 \cdot \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}\right] + \ln\left(e^{\frac{-\nu}{2}}\right) - \text{gammln}\left(\frac{\nu}{2}\right)$$

$$= \ln(2) + \ln\left[\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}\right] + \left(\frac{-\nu}{2}\right) - \text{gammln}\left(\frac{\nu}{2}\right)$$

$$= \ln(2) + \left(\frac{\nu}{2}\right) \cdot \ln\left(\frac{\nu}{2}\right) - \left(\frac{\nu}{2}\right) - \text{gammln}\left(\frac{\nu}{2}\right)$$

APPENDIX B.2 – Computer Program ccfsR.mcd for Chapter 4

Page 1 of program: ccfsR.mcd

ENTER the following 5 values:

(1) alphaMean := 0.0027 — **alphaMean** - alpha for the $\overline{X}$ chart.

(2) alphaRangeUCL := 0.005 — **alphaRangeUCL** - alpha for the R chart above the UCL.

(3) alphaRangeLCL := 0.001 — **alphaRangeLCL** - alpha for the R chart below the LCL *.

(4) m := 5 — **m** - number of subgroups.

(5) n := 5 — **n** - subgroup size for the ($\overline{X}$,R) charts.

*** Note - If no LCL is desired, leave alphaRangeLCL blank (do not enter zero).**

Please PAGE DOWN to begin the program.

(1.1) $TOL := 10^{-10}$

$f(x) := dnorm(x,0,1) \qquad := \left[(2\cdot\pi)^{-0.5}\right]\cdot e^{\frac{-x^2}{2}}$ $\qquad F(x) := pnorm(x,0,1) \qquad := \int_0^x f(t)\,dt$

$$W1 := n\cdot(n-1)\cdot\int_{-\infty}^{\infty}\left[\int_0^{\infty} W\cdot(F(x+W)-F(x))^{n-2}\cdot f(x+W)\,dW\right]\cdot f(x)\,dx$$

$$W2 := n\cdot(n-1)\cdot\int_{-\infty}^{\infty}\left[\int_0^{\infty} W^2\cdot(F(x+W)-F(x))^{n-2}\cdot f(x+W)\,dW\right]\cdot f(x)\,dx$$

$Var := W2 - W1^2 \qquad d2 := W1 \qquad d3 := Var^{0.5}$

Page 2 of program: ccfsR.mcd

(2.1) $P(W) := n \cdot \int_{-\infty}^{\infty} f(x) \cdot (F(x+W) - F(x))^{n-1} dx$

$DUCL(W) := P(W) - (1 - alphaRangeUCL)$

$DLCL(W) := P(W) - alphaRangeLCL$

```
Wseed1(start) := | W0 ← start
                 | W1 ← start + 0.01
                 | A0 ← DUCL(W0)
                 | A1 ← DUCL(W1)
                 | while A0·A1 > 0
                 |     | W0 ← W1
                 |     | W1 ← W1 + 0.01
                 |     | A0 ← A1
                 |     | A1 ← DUCL(W1)
                 | W
```

```
Wseed2(start) := | W0 ← start
                 | W1 ← start + 0.01
                 | A0 ← DLCL(W0)
                 | A1 ← DLCL(W1)
                 | while A0·A1 > 0
                 |     | W0 ← W1
                 |     | W1 ← W1 + 0.01
                 |     | A0 ← A1
                 |     | A1 ← DLCL(W1)
                 | W
```

$seedD4 := Wseed1(0.01)$

$seedD3 := Wseed2(0.001)$

$wD4 := zbrent(DUCL, seedD4_0, seedD4_1, TOL)$

$wD3 := zbrent(DLCL, seedD3_0, seedD3_1, TOL)$

(2.2) $x := \left[-2 + 2 \cdot \left[1 + \left(\frac{2}{m}\right) \cdot \left(\frac{d3}{d2}\right)^2\right]^{0.5}\right]^{-1}$

$r := \frac{d3^2}{m \cdot d2^2}$

$rprevm := \frac{d3^2}{(m-1) \cdot d2^2}$

$xprevm := \left[-2 + 2 \cdot \left[1 + \left(\frac{2}{m-1}\right) \cdot \left(\frac{d3}{d2}\right)^2\right]^{0.5}\right]^{-1}$

$h(x) := \frac{x \cdot e^{2 \cdot (gammln(0.5 \cdot x) - gammln(0.5 \cdot x + 0.5))} - 2}{2}$

$d(x) := h(x) - r$

$dprevm(x) := h(x) - rprevm$

$v := zbrent(d, x - 0.5, x + 0.5, TOL)$

$vprevm := zbrent(dprevm, xprevm - 0.5, xprevm + 0.5, TOL)$

(3.1) $$P1(z) := \int_0^{11} \left[\left(5 \cdot \frac{W}{z}\right) \cdot e^{\frac{z^2 - 25 \cdot W^2}{2 \cdot z^2}} \right]^{\nu - 1} \cdot e^{\frac{z^2 - 25 \cdot W^2}{2 \cdot z^2}} \cdot P(W)\, dW$$

$$P2(z) := \left(\frac{z}{5}\right) \cdot \int_{\frac{55}{z}}^{\infty} \left(x \cdot e^{\frac{1 - x^2}{2}} \right)^{\nu - 1} \cdot e^{\frac{1 - x^2}{2}}\, dx$$

$$c\nu := \ln(2) + \left(\frac{\nu}{2}\right) \cdot \ln\left(\frac{\nu}{2}\right) - \left(\frac{\nu}{2}\right) - \text{gammln}\left(\frac{\nu}{2}\right)$$

$$P3(z) := \left(\frac{5}{z}\right) \cdot e^{c\nu} \cdot (P1(z) + P2(z))$$

(3.2) Zseed1(start) :=

```
Z_0 ← start
Z_1 ← start + 5.0
A_0 ← P3(Z_0)
A_1 ← P3(Z_1)
while A_1 < (1 − alphaRangeUCL)
    Z_0 ← Z_1
    Z_1 ← Z_1 + 5.0
    A_0 ← A_1
    A_1 ← P3(Z_1)
Zguess ← linterp(A, Z, 1 − alphaRangeUCL)
Zguess
```

seed1 := Zseed1(5.0)

D(x) := P3(x) − (1 − alphaRangeUCL)

$$qD4 := \frac{\text{zbrent}(D, \text{seed1} - 5.0, \text{seed1} + 5.0, \text{TOL})}{5}$$

$$\blacksquare := \frac{\text{root}[\,|P3(\text{seed1}) - (1 - \text{alphaRangeUCL})|\,, \text{seed1}]}{5}$$

Page 4 of program: ccfsR.mcd

(4.1) Zseed2(start) :=

```
Zv_0 ← 0.0
Av_0 ← 0.0
Z ← start
while (P3(Z) < alphaRangeLCL)
  Z ← Z + 1.0
for i ∈ 1..6
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3(Zv_i)
for i ∈ 7..20
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3(Zv_i)
Zguess ← linterp(Av, Zv, alphaRangeLCL)
A ← ratint(Zv, Av, Zguess)
Aguess ← A_0
while |Aguess − alphaRangeLCL| > 10^-15
  | if (Aguess − alphaRangeLCL) > 10^-15
  |   | Av_1 ← Aguess
  |   | Zv_1 ← Zguess
  | if (Aguess − alphaRangeLCL) < −10^-15
  |   | Av_0 ← Aguess
  |   | Zv_0 ← Zguess
  | Zguess ← linterp(Av, Zv, alphaRangeLCL)
  | A ← ratint(Zv, Av, Zguess)
  | Aguess ← A_0
Zguess
```

seed2 := Zseed2(1.0)

Monitor Results

$$qD3 := \frac{seed2}{5}$$

qD3 = 0.3584551343

$$qD3 := \frac{root(|P3(seed2) - alphaRangeLCL|, seed2)}{5}$$

qD3 = 0.3584551342

(5.1) $$P1prevm(z) := \int_0^{11} \left[\left(5\cdot\frac{W}{z}\right)\cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}}\right]^{vprevm-1}\cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}}\cdot P(W)\,dW$$

$$P2prevm(z) := \left(\frac{z}{5}\right)\cdot\int_{\frac{55}{z}}^{\infty}\left(x\cdot e^{\frac{1-x^2}{2}}\right)^{vprevm-1}\cdot e^{\frac{1-x^2}{2}}\,dx$$

$$cvprevm := \ln(2) + \left(\frac{vprevm}{2}\right)\cdot\ln\left(\frac{vprevm}{2}\right) - \left(\frac{vprevm}{2}\right) - gammln\left(\frac{vprevm}{2}\right)$$

$$P3prevm(z) := \left(\frac{5}{z}\right)\cdot e^{cvprevm}\cdot(P1prevm(z) + P2prevm(z))$$

(5.2) Zseed3(start) :=

```
Z0 ← start
Z1 ← start + 5.0
A0 ← P3prevm(Z0)
A1 ← P3prevm(Z1)
while A1 < (1 − alphaRangeUCL)
    Z0 ← Z1
    Z1 ← Z1 + 5.0
    A0 ← A1
    A1 ← P3prevm(Z1)
Zguess ← linterp(A, Z, 1 − alphaRangeUCL)
Zguess
```

$seed3 := Zseed3(5.0)$ $\qquad$ $Dprevm(x) := P3prevm(x) - (1 - alphaRangeUCL)$

$$qD4prevm := \frac{zbrent(Dprevm, seed3 - 5.0, seed3 + 5.0, TOL)}{5}$$

$$\blacksquare := \frac{root[\,|P3prevm(seed3) - (1 - alphaRangeUCL)|\,, seed3]}{5}$$

Page 6 of program: ccfsR.mcd

(6.1) Zseed4(start) :=

```
Zv_0 ← 0.0
Av_0 ← 0.0
Z ← start
while (P3prevm(Z) < alphaRangeLCL)
  Z ← Z + 1.0
for i ∈ 1..6
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3prevm(Zv_i)
for i ∈ 7..20
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3prevm(Zv_i)
Zguess ← linterp(Av,Zv,alphaRangeLCL)
A ← ratint(Zv,Av,Zguess)
Aguess ← A_0
while |Aguess − alphaRangeLCL| > 10^-15
  | if (Aguess − alphaRangeLCL) > 10^-15
  |   | Av_1 ← Aguess
  |   | Zv_1 ← Zguess
  | if (Aguess − alphaRangeLCL) < −10^-15
  |   | Av_0 ← Aguess
  |   | Zv_0 ← Zguess
  | Zguess ← linterp(Av,Zv,alphaRangeLCL)
  | A ← ratint(Zv,Av,Zguess)
  | Aguess ← A_0
Zguess
```

seed4 := Zseed4(1.0)

$$qD3prevm := \frac{seed4}{5}$$

$$qD3prevm := \frac{root(|P3prevm(seed4) - alphaRangeLCL|, seed4)}{5}$$

Monitor Results

qD3prevm = 0.3564225553

qD3prevm = 0.3564225551

Page 7 of program: ccfsR.mcd

(7.1) $d2star := \left(d2^2 + \frac{d3^2}{m}\right)^{0.5}$ $adj\_alpha := 1 - \frac{alphaMean}{2}$

$d2starprevm := \left(d2^2 + \frac{d3^2}{m-1}\right)^{0.5}$ $crit\_t := qt(adj\_alpha, \nu)$ $crit\_z := qnorm(adj\_alpha, 0, 1)$

(7.2) $A21 := \left(\frac{crit\_t}{d2star}\right)\cdot\left(\frac{m-1}{n\cdot m}\right)^{0.5}$ $A22 := \left(\frac{crit\_t}{d2star}\right)\cdot\left(\frac{m+1}{n\cdot m}\right)^{0.5}$ $A2 := \frac{crit\_z}{d2\cdot n^{0.5}}$

$D41 := \frac{m\cdot qD4prevm}{d2starprevm\cdot(m-1) + qD4prevm}$ $D42 := \frac{qD4}{d2star}$ $D4 := \frac{wD4}{d2}$

$D31 := \frac{m\cdot qD3prevm}{d2starprevm\cdot(m-1) + qD3prevm}$ $D32 := \frac{qD3}{d2star}$ $D3 := \frac{wD3}{d2}$

FINAL RESULTS:

(1) alphaMean = 0.0027
(2) alphaRangeUCL = 0.005
(3) alphaRangeLCL = 0.001
(4) m = 5
(5) n = 5

Control Chart Factors

First Stage	Second Stage	Conventional
A21 = 0.58784	A22 = 0.71995	A2 = 0.5768149104
D41 = 1.95711	D42 = 2.46759	D4 = 2.1004874391
D31 = 0.18149	D32 = 0.15203	D3 = 0.1579549576

Mean, Stand. Dev., and Variance of the Dist. of the Range	Duncan's (1974) Table D3	Harter, Clemm, and Guthrie's (1959) Table II.2	
	ν = 18.3541743541	qD4 = 5.81811	qD3 = 0.35846
d2 = 2.3259289473	d2star = 2.35781	qD4prevm = 6.08629	qD3prevm = 0.35642
d3 = 0.8640819411	vprevm = 14.72881	wD4 = 4.8855845381	wD3 = 0.3673920082
Var = 0.7466376009	d2starprevm = 2.36571		

APPENDIX B.3 – Tables Generated from ccfsR.mcd

Table B.3.1. Partial Re-creation of Table D3 in the Appendix of Duncan (1974)

n	2		3		4		5		6	
m	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*
1	1.00000	1.41421	1.98463	1.91154	2.92916	2.23887	3.82651	2.48125	4.67716	2.67253
2	1.91952	1.27930	3.83372	1.80538	5.69354	2.15069	7.47105	2.40484	9.16121	2.60439
3	2.81729	1.23105	5.66278	1.76857	8.44146	2.12049	11.10185	2.37883	13.63350	2.58127
4	3.70617	1.20620	7.48535	1.74988	11.18455	2.10522	14.72881	2.36571	18.10259	2.56964
5	4.59060	1.19105	9.30506	1.73857	13.92559	2.09601	18.35417	2.35781	22.57035	2.56263
6	5.47253	1.18083	11.12327	1.73099	16.66558	2.08985	21.97872	2.35253	27.03745	2.55795
7	6.35291	1.17348	12.94060	1.72555	19.40495	2.08543	25.60279	2.34875	31.50415	2.55460
8	7.23227	1.16794	14.75735	1.72146	22.14394	2.08212	29.22657	2.34591	35.97062	2.55209
9	8.11092	1.16361	16.57373	1.71828	24.88267	2.07953	32.85015	2.34369	40.43692	2.55013
10	8.98907	1.16014	18.38984	1.71572	27.62121	2.07747	36.47359	2.34192	44.90311	2.54856
11	9.86684	1.15729	20.20575	1.71363	30.35962	2.07577	40.09692	2.34047	49.36922	2.54728
12	10.74432	1.15490	22.02151	1.71189	33.09793	2.07436	43.72018	2.33927	53.83526	2.54621
13	11.62158	1.15289	23.83716	1.71041	35.83616	2.07316	47.34338	2.33824	58.30126	2.54530
14	12.49866	1.15115	25.65271	1.70914	38.57433	2.07214	50.96654	2.33737	62.76721	2.54453
15	13.37559	1.14965	27.46819	1.70804	41.31245	2.07125	54.58965	2.33660	67.23314	2.54385
16	14.25241	1.14833	29.28362	1.70708	44.05053	2.07047	58.21274	2.33594	71.69904	2.54326
17	15.12913	1.14717	31.09899	1.70623	46.78857	2.06978	61.83580	2.33535	76.16493	2.54274
18	16.00577	1.14613	32.91432	1.70547	49.52659	2.06917	65.45884	2.33483	80.63079	2.54228
19	16.88234	1.14520	34.72962	1.70479	52.26459	2.06862	69.08186	2.33436	85.09664	2.54187
20	17.75886	1.14437	36.54489	1.70419	55.00257	2.06813	72.70487	2.33394	89.56248	2.54150
25	22.14078	1.14119	45.62091	1.70187	68.69224	2.06626	90.81974	2.33234	111.8915	2.54008
30	26.52202	1.13906	54.69660	1.70032	82.38169	2.06501	108.9344	2.33127	134.2205	2.53914
50	44.04420	1.13480	90.99798	1.69723	137.1386	2.06251	181.3926	2.32914	223.5356	2.53725
75	65.94485	1.13266	136.3737	1.69567	205.5840	2.06126	271.9647	2.32807	335.1791	2.53630
100	87.84479	1.13159	181.7490	1.69490	274.0292	2.06063	362.5367	2.32753	446.8224	2.53583
150	131.6440	1.13052	272.4994	1.69412	410.9194	2.06000	543.6805	2.32700	670.1090	2.53536
200	175.4428	1.12999	363.2496	1.69373	547.8094	2.05969	724.8242	2.32673	893.3955	2.53512
250	219.2414	1.12967	453.9998	1.69350	684.6994	2.05950	905.9679	2.32657	1116.682	2.53498
300	263.0400	1.12945	544.7499	1.69335	821.5894	2.05938	1087.112	2.32646	1339.968	2.53489
d_2	1.1283791671		1.6925687506		2.0587507460		2.3259289473		2.5344127212	
d_3	0.8525024664		0.8883680040		0.8798082028		0.8640819411		0.8480396861	
d_3^2 (Var.)	0.7267604553		0.7891977106		0.7740624738		0.7466376009		0.7191713092	

Table B.3.1 continued. Partial Re-creation of Table D3 in the Appendix of Duncan (1974)

n	7		8		10		25		50	
m	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*
1	5.48415	2.82980	6.25123	2.96288	7.68007	3.17905	15.62977	3.99396	24.02990	4.54518
2	10.76747	2.76779	12.29594	2.90562	15.14589	3.12869	31.02740	3.96242	47.82145	4.52172
3	16.04046	2.74681	18.33145	2.88628	22.60405	3.11172	46.42111	3.95185	71.61044	4.51388
4	21.31070	2.73626	24.36452	2.87656	30.06021	3.10320	61.81384	3.94656	95.39878	4.50995
5	26.57981	2.72991	30.39659	2.87071	37.51556	3.09808	77.20616	3.94338	119.1869	4.50759
6	31.84834	2.72567	36.42816	2.86681	44.97049	3.09466	92.59828	3.94126	142.9748	4.50602
7	37.11655	2.72263	42.45944	2.86401	52.42520	3.09222	107.9903	3.93974	166.7627	4.50490
8	42.38454	2.72035	48.49054	2.86192	59.87975	3.09038	123.3822	3.93860	190.5505	4.50405
9	47.65240	2.71858	54.52152	2.86029	67.33421	3.08895	138.7741	3.93772	214.3383	4.50340
10	52.92017	2.71716	60.55241	2.85898	74.78859	3.08781	154.1660	3.93701	238.1261	4.50287
11	58.18786	2.71600	66.58324	2.85791	82.24293	3.08687	169.5578	3.93643	261.9138	4.50244
12	63.45549	2.71503	72.61402	2.85702	89.69723	3.08609	184.9496	3.93595	285.7016	4.50209
13	68.72309	2.71421	78.64477	2.85627	97.15150	3.08543	200.3414	3.93554	309.4893	4.50178
14	73.99066	2.71351	84.67549	2.85562	104.6057	3.08487	215.7332	3.93519	333.2771	4.50152
15	79.25820	2.71290	90.70619	2.85506	112.0600	3.08438	231.1249	3.93488	357.0648	4.50130
16	84.52571	2.71237	96.73687	2.85457	119.5142	3.08395	246.5167	3.93462	380.8525	4.50110
17	89.79321	2.71190	102.7675	2.85414	126.9684	3.08357	261.9085	3.93438	404.6402	4.50093
18	95.06070	2.71148	108.7982	2.85375	134.4226	3.08323	277.3002	3.93417	428.4279	4.50077
19	100.3282	2.71110	114.8288	2.85341	141.8768	3.08293	292.6920	3.93399	452.2156	4.50063
20	105.5956	2.71077	120.8595	2.85310	149.3309	3.08266	308.0837	3.93382	476.0033	4.50051
25	131.9328	2.70949	151.0125	2.85192	186.6017	3.08163	385.0424	3.93318	594.9419	4.50004
30	158.2699	2.70863	181.1655	2.85113	223.8725	3.08094	462.0011	3.93276	713.8803	4.49972
50	263.6178	2.70692	301.7769	2.84956	372.9550	3.07957	769.8356	3.93191	1189.634	4.49909
75	395.3023	2.70607	452.5409	2.84877	559.3079	3.07888	1154.629	3.93148	1784.326	4.49878
100	526.9867	2.70564	603.3047	2.84838	745.6608	3.07854	1539.422	3.93127	2379.019	4.49862
150	790.3554	2.70521	904.8323	2.84799	1118.366	3.07819	2309.007	3.93105	3568.403	4.49846
200	1053.724	2.70500	1206.360	2.84779	1491.072	3.07802	3078.593	3.93095	4757.787	4.49838
250	1317.093	2.70487	1507.887	2.84767	1863.777	3.07792	3848.179	3.93088	5947.172	4.49834
300	1580.461	2.70478	1809.415	2.84759	2236.483	3.07785	4617.765	3.93084	7136.556	4.49830
d_2	2.7043567512		2.8472006121		3.0775054617		3.9306292195		4.4981472588	
d_3	0.8332053356		0.8198314898		0.7970506735		0.7084407659		0.6521425884	
d_3^2 (Var.)	0.6942311313		0.6721236717		0.6352897762		0.5018883188		0.4252899557	

Table B.3.2. Partial Re-creation of Table II.2 for P=0.995 (alphaRangeUCL=0.005) in Harter, Clemm, and Guthrie (1959)

	n				
m	**2**	**3**	**4**	**5**	**6**
1	180.05956	27.42040	15.97331	12.55293	10.99826
2	21.69172	10.21636	8.35496	7.67754	7.35145
3	11.39731	7.55702	6.82575	6.56813	6.45828
4	8.45485	6.54888	6.19062	6.08629	6.05995
5	7.13703	6.02643	5.84535	5.81811	5.83514
6	6.40423	5.70854	5.62895	5.64756	5.69092
7	5.94176	5.49523	5.48079	5.52962	5.59060
8	5.62475	5.34238	5.37305	5.44323	5.51680
9	5.39447	5.22756	5.29121	5.37725	5.46025
10	5.21988	5.13817	5.22694	5.32521	5.41553
11	5.08308	5.06664	5.17514	5.28312	5.37929
12	4.97307	5.00810	5.13251	5.24838	5.34932
13	4.88272	4.95932	5.09681	5.21922	5.32413
14	4.80722	4.91805	5.06648	5.19439	5.30266
15	4.74320	4.88268	5.04040	5.17300	5.28414
16	4.68823	4.85203	5.01772	5.15439	5.26801
17	4.64053	4.82522	4.99784	5.13803	5.25382
18	4.59875	4.80156	4.98025	5.12355	5.24126
19	4.56186	4.78054	4.96459	5.11064	5.23004
20	4.52904	4.76174	4.95055	5.09906	5.21998
25	4.40761	4.69126	4.89771	5.05537	5.18197
30	4.32945	4.64512	4.86292	5.02652	5.15682
50	4.17954	4.55483	4.79437	4.96949	5.10701
75	4.10766	4.51067	4.76061	4.94131	5.08235
100	4.07246	4.48882	4.74385	4.92729	5.07007
150	4.03775	4.46714	4.72718	4.91334	5.05783
200	4.02057	4.45635	4.71888	4.90638	5.05173
250	4.01032	4.44990	4.71390	4.90221	5.04807
300	4.00351	4.44561	4.71059	4.89943	5.04564
∞	3.9697452252	4.4242351777	4.6940874592	4.8855845381	5.0334791352

Table B.3.2 continued. Partial Re-creation of Table II.2 for
P=0.995 (alphaRangeUCL=0.005) in Harter, Clemm, and Guthrie (1959)

	n				
m	**7**	**8**	**10**	**25**	**50**
1	10.13317	9.59128	8.96259	7.99977	7.91156
2	7.17114	7.06337	6.95315	6.95639	7.15747
3	6.40976	6.39095	6.39383	6.63514	6.91715
4	6.06422	6.08197	6.13253	6.47939	6.79913
5	5.86739	5.90480	5.98139	6.38750	6.72903
6	5.74040	5.79002	5.88293	6.32690	6.68261
7	5.65171	5.70964	5.81372	6.28394	6.64959
8	5.58628	5.65022	5.76241	6.25190	6.62492
9	5.53603	5.60451	5.72287	6.22708	6.60578
10	5.49623	5.56826	5.69146	6.20730	6.59050
11	5.46392	5.53882	5.66590	6.19115	6.57801
12	5.43719	5.51442	5.64471	6.17773	6.56763
13	5.41469	5.49388	5.62685	6.16639	6.55885
14	5.39549	5.47634	5.61159	6.15669	6.55133
15	5.37893	5.46120	5.59841	6.14830	6.54482
16	5.36449	5.44800	5.58690	6.14096	6.53913
17	5.35178	5.43638	5.57677	6.13449	6.53411
18	5.34052	5.42607	5.56779	6.12875	6.52966
19	5.33047	5.41688	5.55976	6.12362	6.52567
20	5.32145	5.40861	5.55255	6.11900	6.52208
25	5.28733	5.37736	5.52525	6.10149	6.50847
30	5.26474	5.35664	5.50713	6.08984	6.49941
50	5.21993	5.31551	5.47111	6.06662	6.48132
75	5.19770	5.29509	5.45321	6.05504	6.47229
100	5.18663	5.28492	5.44429	6.04925	6.46778
150	5.17560	5.27477	5.43538	6.04348	6.46328
200	5.17009	5.26971	5.43093	6.04059	6.46102
250	5.16679	5.26667	5.42827	6.03886	6.45967
300	5.16459	5.26465	5.42649	6.03771	6.45877
∞	5.1536133124	5.2545498162	5.4176160146	6.0319395194	6.4542688862

Table B.3.3. Partial Re-creation of Table II.2 for P=0.001 (alphaRangeLCL=0.001) in Harter, Clemm, and Guthrie (1959)

	n				
m	**2**	**3**	**4**	**5**	**6**
1	0.00222	0.06026	0.18632	0.33245	0.47538
2	0.00201	0.06025	0.19194	0.34723	0.50030
3	0.00193	0.06025	0.19418	0.35319	0.51042
4	0.00189	0.06025	0.19539	0.35642	0.51594
5	0.00187	0.06025	0.19614	0.35846	0.51941
6	0.00185	0.06025	0.19666	0.35985	0.52180
7	0.00184	0.06025	0.19704	0.36087	0.52354
8	0.00183	0.06025	0.19733	0.36165	0.52487
9	0.00183	0.06025	0.19755	0.36226	0.52592
10	0.00182	0.06025	0.19773	0.36275	0.52676
11	0.00182	0.06025	0.19789	0.36316	0.52746
12	0.00181	0.06025	0.19801	0.36350	0.52805
13	0.00181	0.06025	0.19812	0.36379	0.52854
14	0.00181	0.06025	0.19821	0.36404	0.52897
15	0.00181	0.06025	0.19829	0.36426	0.52935
16	0.00180	0.06025	0.19836	0.36445	0.52967
17	0.00180	0.06025	0.19842	0.36462	0.52996
18	0.00180	0.06025	0.19848	0.36477	0.53022
19	0.00180	0.06025	0.19853	0.36490	0.53046
20	0.00180	0.06025	0.19857	0.36503	0.53067
25	0.00179	0.06025	0.19875	0.36549	0.53147
30	0.00179	0.06025	0.19886	0.36580	0.53200
50	0.00178	0.06025	0.19909	0.36643	0.53309
75	0.00178	0.06024	0.19921	0.36675	0.53363
100	0.00178	0.06024	0.19927	0.36691	0.53391
150	0.00178	0.06024	0.19933	0.36707	0.53418
200	0.00177	0.06024	0.19936	0.36715	0.53432
250	0.00177	0.06024	0.19938	0.36720	0.53440
300	0.00177	0.06024	0.19939	0.36723	
∞	0.0017724543	0.0602447314	0.1994460628	0.3673920082	0.5347362725

Table B.3.3 continued. Partial Re-creation of Table II.2 for P=0.001 (alphaRangeLCL=0.001) in Harter, Clemm, and Guthrie (1959)

	n				
m	**7**	**8**	**10**	**25**	**50**
1	0.60798	0.72902	0.93957	1.82816	2.46937
2	0.64281	0.77307	0.99964	1.94858	2.62247
3	0.65703	0.79110	1.02434	1.99857	2.68617
4	0.66478	0.80096	1.03788	2.02614	2.72141
5	0.66968	0.80719	1.04644	2.04365	2.74386
6	0.67304	0.81148	1.05234	2.05577	2.75942
7	0.67551	0.81461	1.05666	2.06466	2.77086
8	0.67738	0.81700	1.05996	2.07146	2.77962
9	0.67886	0.81889	1.06256	2.07683	2.78655
10	0.68006	0.82041	1.06467	2.08118	2.79216
11	0.68105	0.82167	1.06640	2.08478	2.79680
12	0.68187	0.82273	1.06786	2.08780	2.80071
13	0.68258	0.82363	1.06910	2.09037	2.80404
14	0.68319	0.82440	1.07017	2.09259	2.80691
15	0.68371	0.82508	1.07110	2.09453	2.80941
16	0.68418	0.82567	1.07192	2.09623	2.81162
17	0.68459	0.82619	1.07265	2.09773	2.81357
18	0.68496	0.82666	1.07329	2.09908	2.81531
19	0.68528	0.82708	1.07387	2.10028	2.81687
20	0.68558	0.82746	1.07439	2.10137	2.81829
25	0.68671	0.82891	1.07639	2.10554	2.82369
30	0.68747	0.82988	1.07774	2.10834	2.82733
50	0.68901	0.83184	1.08045	2.11400	2.83469
75	0.68978	0.83283	1.08182	2.11687	2.83841
100	0.69017	0.83333	1.08251	2.11831	2.84029
150	0.69056	0.83382	1.08320	2.120	2.84217
200	0.6908	0.83407	1.0835	2.120	2.84311
250		0.83422		2.121	2.84368
300				2.121	
∞	0.6913468703	0.8348258291	1.0845826539	2.1226552123	2.8459534386

Table B.3.4. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	2					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	166.72424	127.32134	0.00157
2	8.27583	1.98441	0.00314	14.33417	16.95587	0.00157
3	4.73208	2.68348	0.00235	6.69217	9.25818	0.00157
4	3.62681	3.02106	0.00209	4.68219	7.00946	0.00157
5	3.11850	3.18338	0.00196	3.81937	5.99224	0.00157
6	2.83285	3.27080	0.00188	3.35187	5.42349	0.00157
7	2.65175	3.32336	0.00183	3.06197	5.06335	0.00157
8	2.52736	3.35784	0.00179	2.86575	4.81596	0.00157
9	2.43693	3.38200	0.00177	2.72457	4.63598	0.00157
10	2.36837	3.39981	0.00175	2.61833	4.49937	0.00157
11	2.31466	3.41346	0.00173	2.53558	4.39225	0.00157
12	2.27148	3.42425	0.00171	2.46936	4.30605	0.00157
13	2.23604	3.43300	0.00170	2.41520	4.23522	0.00157
14	2.20643	3.44023	0.00169	2.37009	4.17601	0.00157
15	2.18135	3.44631	0.00168	2.33196	4.12579	0.00157
16	2.15982	3.45150	0.00168	2.29930	4.08265	0.00157
17	2.14114	3.45597	0.00167	2.27102	4.04522	0.00157
18	2.12479	3.45988	0.00166	2.24630	4.01242	0.00157
19	2.11036	3.46331	0.00166	2.22451	3.98345	0.00157
20	2.09753	3.46636	0.00165	2.20516	3.95768	0.00157
25	2.05010	3.47759	0.00164	2.13381	3.86230	0.00157
30	2.01962	3.48479	0.00162	2.08810	3.80088	0.00157
50	1.96128	3.49858	0.00160	2.00090	3.68305	0.00157
75	1.93337	3.50522	0.00159	1.95932	3.62654	0.00157
100	1.91972	3.50849	0.00159	1.93901	3.59887	0.00157
150	1.90627	3.51172	0.00158	1.91902	3.57157	0.00157
200	1.89962	3.51333	0.00158	1.90914	3.55806	0.00157
250	1.89565	3.51429	0.00158	1.90325	3.55000	0.00157
300	1.89302	3.51492	0.00158	1.89934	3.54465	0.00157
∞	1.8799567883	3.5180951058	0.0015707967	1.8799567883	3.5180951058	0.0015707967

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	3					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	8.35221	14.34466	0.03152
2	1.56033	1.86966	0.06112	2.70257	5.65885	0.03337
3	1.35226	2.21659	0.04924	1.91239	4.27295	0.03407
4	1.25601	2.35005	0.04491	1.62151	3.74247	0.03443
5	1.20246	2.41685	0.04267	1.47271	3.46631	0.03465
6	1.16868	2.45655	0.04130	1.38280	3.29785	0.03481
7	1.14551	2.48283	0.04037	1.32272	3.18462	0.03491
8	1.12866	2.50151	0.03970	1.27978	3.10340	0.03500
9	1.11588	2.51550	0.03920	1.24759	3.04233	0.03506
10	1.10584	2.52636	0.03881	1.22255	2.99476	0.03511
11	1.09776	2.53505	0.03849	1.20254	2.95667	0.03516
12	1.09112	2.54215	0.03823	1.18617	2.92549	0.03519
13	1.08556	2.54808	0.03801	1.17254	2.89950	0.03522
14	1.08085	2.55310	0.03783	1.16101	2.87750	0.03525
15	1.07679	2.55740	0.03767	1.15114	2.85864	0.03527
16	1.07327	2.56113	0.03754	1.14258	2.84230	0.03529
17	1.07018	2.56440	0.03741	1.13510	2.82800	0.03531
18	1.06745	2.56728	0.03731	1.12850	2.81539	0.03532
19	1.06503	2.56985	0.03721	1.12264	2.80417	0.03534
20	1.06285	2.57215	0.03713	1.11739	2.79414	0.03535
25	1.05467	2.58078	0.03681	1.09774	2.75653	0.03540
30	1.04930	2.58645	0.03660	1.08487	2.73191	0.03543
50	1.03872	2.59760	0.03619	1.05971	2.68370	0.03550
75	1.03353	2.60309	0.03599	1.04740	2.66010	0.03553
100	1.03095	2.60582	0.03589	1.04132	2.64843	0.03554
150	1.02839	2.60853	0.03579	1.03527	2.63684	0.03556
200	1.02712	2.60988	0.03574	1.03227	2.63108	0.03557
250	1.02636	2.61069	0.03571	1.03047	2.62763	0.03557
300	1.02585	2.61123	0.03569	1.02927	2.62534	0.03558
∞	1.0233188600	2.6139175593	0.0355936687	1.0233188600	2.6139175593	0.0355936687

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	4					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	3.01070	7.13456	0.08322
2	0.83127	1.75414	0.15366	1.43980	3.88477	0.08925
3	0.80653	1.98042	0.12815	1.14060	3.21895	0.09157
4	0.78832	2.07041	0.11848	1.01772	2.94060	0.09281
5	0.77660	2.11840	0.11338	0.95113	2.78880	0.09358
6	0.76860	2.14831	0.11023	0.90943	2.69347	0.09410
7	0.76285	2.16879	0.10809	0.88087	2.62813	0.09448
8	0.75853	2.18371	0.10654	0.86009	2.58057	0.09477
9	0.75517	2.19507	0.10537	0.84430	2.54442	0.09500
10	0.75248	2.20403	0.10445	0.83190	2.51602	0.09518
11	0.75028	2.21126	0.10371	0.82189	2.49312	0.09533
12	0.74845	2.21723	0.10310	0.81365	2.47426	0.09546
13	0.74691	2.22225	0.10260	0.80675	2.45847	0.09556
14	0.74558	2.22652	0.10216	0.80088	2.44505	0.09566
15	0.74444	2.23020	0.10179	0.79584	2.43351	0.09574
16	0.74344	2.23341	0.10147	0.79145	2.42347	0.09581
17	0.74255	2.23623	0.10119	0.78760	2.41467	0.09587
18	0.74177	2.23872	0.10094	0.78419	2.40688	0.09592
19	0.74107	2.24095	0.10071	0.78116	2.39995	0.09597
20	0.74044	2.24295	0.10052	0.77844	2.39373	0.09602
25	0.73805	2.25050	0.09977	0.76819	2.37033	0.09619
30	0.73647	2.25550	0.09927	0.76144	2.35491	0.09630
50	0.73330	2.26540	0.09830	0.74812	2.32453	0.09653
75	0.73173	2.27031	0.09782	0.74155	2.30957	0.09665
100	0.73094	2.27276	0.09758	0.73829	2.30214	0.09670
150	0.73016	2.27520	0.09735	0.73504	2.29474	0.09676
200	0.72977	2.27642	0.09723	0.73342	2.29106	0.09679
250	0.72953	2.27715	0.09716	0.73245	2.28886	0.09681
300	0.72937	2.27764	0.09711	0.73181	2.28739	0.09682
∞	0.7285915982	2.2800659421	0.0968772267	0.7285915982	2.2800659421	0.0968772267

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	5					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	1.76214	5.05912	0.13399
2	0.57850	1.66992	0.23631	1.00199	3.19254	0.14439
3	0.58948	1.84450	0.20200	0.83366	2.76108	0.14847
4	0.58920	1.91706	0.18863	0.76066	2.57271	0.15066
5	0.58784	1.95711	0.18149	0.71995	2.46759	0.15203
6	0.58654	1.98264	0.17705	0.69400	2.40063	0.15296
7	0.58545	2.00038	0.17402	0.67602	2.35428	0.15364
8	0.58455	2.01344	0.17182	0.66282	2.32031	0.15416
9	0.58381	2.02347	0.17015	0.65272	2.29435	0.15457
10	0.58319	2.03141	0.16884	0.64475	2.27386	0.15489
11	0.58267	2.03786	0.16778	0.63829	2.25729	0.15516
12	0.58223	2.04321	0.16692	0.63295	2.24360	0.15539
13	0.58184	2.04771	0.16619	0.62846	2.23211	0.15558
14	0.58151	2.05156	0.16557	0.62464	2.22233	0.15575
15	0.58122	2.05488	0.16504	0.62135	2.21390	0.15589
16	0.58096	2.05778	0.16457	0.61848	2.20656	0.15602
17	0.58073	2.06033	0.16417	0.61596	2.20011	0.15613
18	0.58052	2.06259	0.16381	0.61372	2.19440	0.15623
19	0.58034	2.06461	0.16349	0.61173	2.18931	0.15632
20	0.58017	2.06643	0.16320	0.60994	2.18474	0.15640
25	0.57952	2.07331	0.16212	0.60319	2.16751	0.15671
30	0.57908	2.07787	0.16141	0.59872	2.15613	0.15691
50	0.57819	2.08696	0.16001	0.58987	2.13362	0.15733
75	0.57774	2.09148	0.15932	0.58549	2.12249	0.15753
100	0.57751	2.09374	0.15898	0.58331	2.11696	0.15764
150	0.57728	2.09599	0.15863	0.58114	2.11145	0.15774
200	0.57716	2.09712	0.15846	0.58006	2.10870	0.15780
250	0.57709	2.09779	0.15836	0.57941	2.10705	0.15783
300	0.57705	2.09824	0.15829	0.57897	2.10596	0.15785
∞	0.5768149104	2.1004874391	0.1579549576	0.5768149104	2.1004874391	0.1579549576

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	6					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	1.25023	4.11530	0.17788
2	0.45107	1.60902	0.30203	0.78128	2.82272	0.19210
3	0.47212	1.75589	0.26290	0.66767	2.50197	0.19774
4	0.47776	1.81896	0.24735	0.61679	2.35829	0.20078
5	0.48003	1.85450	0.23898	0.58792	2.27701	0.20269
6	0.48116	1.87743	0.23375	0.56932	2.22480	0.20399
7	0.48180	1.89349	0.23016	0.55634	2.18844	0.20494
8	0.48220	1.90539	0.22755	0.54676	2.16168	0.20566
9	0.48245	1.91456	0.22557	0.53940	2.14117	0.20623
10	0.48263	1.92185	0.22401	0.53357	2.12494	0.20669
11	0.48276	1.92779	0.22275	0.52884	2.11178	0.20707
12	0.48286	1.93272	0.22172	0.52492	2.10090	0.20738
13	0.48293	1.93687	0.22085	0.52162	2.09175	0.20765
14	0.48298	1.94043	0.22011	0.51881	2.08395	0.20789
15	0.48303	1.94350	0.21948	0.51638	2.07722	0.20809
16	0.48306	1.94619	0.21892	0.51426	2.07136	0.20827
17	0.48309	1.94856	0.21844	0.51239	2.06620	0.20842
18	0.48311	1.95066	0.21801	0.51074	2.06163	0.20856
19	0.48313	1.95253	0.21763	0.50927	2.05756	0.20869
20	0.48315	1.95422	0.21728	0.50794	2.05390	0.20880
25	0.48320	1.96063	0.21599	0.50293	2.04008	0.20923
30	0.48322	1.96488	0.21514	0.49961	2.03093	0.20952
50	0.48325	1.97337	0.21346	0.49301	2.01282	0.21010
75	0.48325	1.97761	0.21263	0.48974	2.00384	0.21040
100	0.48325	1.97972	0.21222	0.48811	1.99937	0.21055
150	0.48325	1.98183	0.21181	0.48648	1.99492	0.21069
200	0.48325	1.98289	0.21160	0.48567	1.99270	0.21077
250	0.48325	1.98352	0.21148	0.48519	1.99137	0.21081
300	0.48325	1.98394		0.48486	1.99048	
∞	0.4832423182	1.9860534526	0.2109902101	0.4832423182	1.9860534526	0.2109902101

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	7					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	0.97756	3.58088	0.21485
2	0.37394	1.56340	0.35370	0.64769	2.59093	0.23225
3	0.39800	1.69307	0.31213	0.56286	2.33353	0.23920
4	0.40591	1.75008	0.29538	0.52403	2.21625	0.24295
5	0.40968	1.78262	0.28630	0.50175	2.14930	0.24531
6	0.41184	1.80379	0.28061	0.48730	2.10605	0.24693
7	0.41323	1.81869	0.27670	0.47716	2.07583	0.24811
8	0.41419	1.82976	0.27385	0.46965	2.05351	0.24901
9	0.41490	1.83832	0.27168	0.46387	2.03637	0.24971
10	0.41543	1.84514	0.26997	0.45928	2.02278	0.25028
11	0.41585	1.85070	0.26859	0.45554	2.01175	0.25075
12	0.41619	1.85533	0.26745	0.45245	2.00262	0.25115
13	0.41647	1.85923	0.26650	0.44984	1.99494	0.25148
14	0.41670	1.86257	0.26569	0.44761	1.98838	0.25177
15	0.41690	1.86546	0.26499	0.44569	1.98272	0.25202
16	0.41707	1.86799	0.26438	0.44401	1.97779	0.25224
17	0.41722	1.87022	0.26385	0.44253	1.97345	0.25244
18	0.41735	1.87220	0.26338	0.44122	1.96960	0.25261
19	0.41746	1.87397	0.26296	0.44004	1.96616	0.25277
20	0.41756	1.87556	0.26258	0.43899	1.96308	0.25291
25	0.41794	1.88160	0.26116	0.43500	1.95142	0.25345
30	0.41818	1.88562	0.26022	0.43236	1.94369	0.25381
50	0.41864	1.89365	0.25837	0.42710	1.92836	0.25454
75	0.41886	1.89766	0.25745	0.42448	1.92076	0.25490
100	0.41897	1.89966	0.25700	0.42318	1.91697	0.25509
150	0.41907	1.90167	0.25654	0.42188	1.91319	0.25527
200	0.41913	1.90267	0.2563	0.42123	1.91131	0.2554
250	0.41916	1.90327		0.42084	1.91018	
300	0.41918	1.90367		0.42058	1.90943	
∞	0.4192807486	1.9056706590	0.2556418897	0.4192807486	1.9056706590	0.2556418897

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	8					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	0.80906	3.23715	0.24605
2	0.32197	1.52798	0.39493	0.55767	2.43094	0.26606
3	0.34663	1.64588	0.35223	0.49020	2.21426	0.27409
4	0.35542	1.69862	0.33486	0.45884	2.11432	0.27844
5	0.35983	1.72899	0.32540	0.44070	2.05691	0.28118
6	0.36246	1.74885	0.31945	0.42886	2.01968	0.28306
7	0.36419	1.76288	0.31536	0.42053	1.99358	0.28443
8	0.36543	1.77334	0.31237	0.41435	1.97428	0.28547
9	0.36634	1.78143	0.31009	0.40958	1.95942	0.28630
10	0.36705	1.78789	0.30830	0.40579	1.94764	0.28696
11	0.36761	1.79316	0.30685	0.40270	1.93806	0.28751
12	0.36807	1.79755	0.30566	0.40014	1.93013	0.28797
13	0.36845	1.80125	0.30465	0.39798	1.92345	0.28836
14	0.36878	1.80443	0.30380	0.39613	1.91774	0.28870
15	0.36905	1.80718	0.30307	0.39453	1.91282	0.28899
16	0.36929	1.80958	0.30243	0.39314	1.90852	0.28925
17	0.36949	1.81170	0.30187	0.39191	1.90474	0.28947
18	0.36968	1.81358	0.30137	0.39082	1.90138	0.28968
19	0.36984	1.81527	0.30093	0.38984	1.89839	0.28986
20	0.36998	1.81678	0.30053	0.38897	1.89570	0.29002
25	0.37052	1.82254	0.29903	0.38565	1.88552	0.29065
30	0.37087	1.82637	0.29804	0.38345	1.87878	0.29107
50	0.37155	1.83403	0.29609	0.37906	1.86538	0.29192
75	0.37188	1.83786	0.29512	0.37687	1.85873	0.29235
100	0.37204	1.83977	0.29464	0.37578	1.85541	0.29256
150	0.37221	1.84169	0.29416	0.37470	1.85211	0.29278
200	0.37229	1.84264	0.29392	0.37415	1.85045	0.29288
250	0.37233	1.84322	0.29378	0.37383	1.84947	0.29295
300	0.37237	1.84360		0.37361	1.84881	
∞	0.3725245186	1.8455144305	0.2932093459	0.3725245186	1.8455144305	0.2932093459

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	10					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	0.61168	2.81927	0.29555
2	0.25585	1.47634	0.45626	0.44314	2.22238	0.31951
3	0.27949	1.57900	0.41324	0.39526	2.05476	0.32919
4	0.28856	1.62600	0.39552	0.37253	1.97619	0.33445
5	0.29331	1.65339	0.38581	0.35923	1.93068	0.33777
6	0.29623	1.67142	0.37968	0.35050	1.90099	0.34005
7	0.29820	1.68421	0.37545	0.34433	1.88011	0.34172
8	0.29961	1.69377	0.37236	0.33973	1.86463	0.34299
9	0.30068	1.70120	0.37000	0.33617	1.85269	0.34399
10	0.30152	1.70712	0.36814	0.33334	1.84320	0.34480
11	0.30219	1.71197	0.36663	0.33103	1.83548	0.34546
12	0.30273	1.71601	0.36539	0.32911	1.82908	0.34602
13	0.30319	1.71942	0.36435	0.32748	1.82368	0.34650
14	0.30358	1.72235	0.36347	0.32610	1.81907	0.34691
15	0.30391	1.72488	0.36270	0.32490	1.81508	0.34727
16	0.30420	1.72710	0.36204	0.32385	1.81161	0.34758
17	0.30445	1.72906	0.36145	0.32292	1.80854	0.34786
18	0.30468	1.73080	0.36093	0.32210	1.80583	0.34811
19	0.30488	1.73235	0.36047	0.32137	1.80340	0.34833
20	0.30505	1.73375	0.36006	0.32071	1.80122	0.34853
25	0.30572	1.73908	0.35850	0.31820	1.79296	0.34929
30	0.30616	1.74263	0.35747	0.31654	1.78748	0.34981
50	0.30702	1.74973	0.35543	0.31322	1.77658	0.35084
75	0.30744	1.75328	0.35442	0.31156	1.77117	0.35137
100	0.30764	1.75506	0.35392	0.31074	1.76847	0.35163
150	0.30785	1.75684	0.35342	0.30991	1.76577	0.35189
200	0.30795	1.75772	0.3532	0.30950	1.76442	0.3520
250	0.30802	1.75826		0.30925	1.76362	
300	0.30806	1.75861		0.30909	1.76308	
∞	0.3082613611	1.7603920065	0.3524226577	0.3082613611	1.7603920065	0.3524226577

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	25					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	0.25204	2.00297	0.45773
2	0.11638	1.33399	0.62800	0.20157	1.75559	0.49177
3	0.13099	1.40238	0.59207	0.18524	1.67900	0.50573
4	0.13719	1.43535	0.57703	0.17711	1.64178	0.51339
5	0.14063	1.45502	0.56874	0.17224	1.61980	0.51825
6	0.14282	1.46814	0.56349	0.16898	1.60530	0.52160
7	0.14433	1.47754	0.55986	0.16666	1.59501	0.52406
8	0.14544	1.48459	0.55721	0.16491	1.58734	0.52594
9	0.14629	1.49010	0.55518	0.16356	1.58139	0.52742
10	0.14696	1.49450	0.55358	0.16247	1.57665	0.52862
11	0.14750	1.49812	0.55229	0.16158	1.57278	0.52961
12	0.14795	1.50113	0.55122	0.16084	1.56957	0.53044
13	0.14832	1.50369	0.55032	0.16021	1.56685	0.53115
14	0.14864	1.50588	0.54956	0.15967	1.56452	0.53176
15	0.14892	1.50778	0.54890	0.15920	1.56251	0.53230
16	0.14916	1.50944	0.54833	0.15879	1.56075	0.53276
17	0.14937	1.51091	0.54782	0.15843	1.55920	0.53318
18	0.14956	1.51222	0.54738	0.15811	1.55782	0.53355
19	0.14973	1.51339	0.54698	0.15783	1.55659	0.53388
20	0.14988	1.51445	0.54662	0.15757	1.55549	0.53418
25	0.15044	1.51846	0.54527	0.15658	1.55129	0.53533
30	0.15082	1.52114	0.54438	0.15593	1.54849	0.53610
50	0.15155	1.52651	0.54262	0.15462	1.54292	0.53765
75	0.15192	1.52920	0.54175	0.15396	1.54014	0.53844
100	0.15210	1.53055	0.54132	0.15363	1.53875	0.53884
150	0.15228	1.53190	0.541	0.15330	1.53737	0.539
200	0.15238	1.53257	0.541	0.15314	1.53667	0.539
250	0.15243	1.53298	0.541	0.15304	1.53626	0.540
300	0.15247	1.53325	0.541	0.15297	1.53598	0.540
∞	0.1526461452	1.5345989618	0.5400293677	0.1526461452	1.5345989618	0.5400293677

Table B.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaRangeUCL=0.005, and alphaRangeLCL=0.001

n	50					
m	A21	D41	D31	A22	D42	D32
1	-----	-----	-----	0.14716	1.74065	0.54329
2	0.06999	1.27025	0.70407	0.12122	1.58291	0.57997
3	0.07951	1.32538	0.67439	0.11244	1.53242	0.59509
4	0.08366	1.35241	0.66212	0.10800	1.50758	0.60342
5	0.08599	1.36864	0.65541	0.10531	1.49282	0.60872
6	0.08748	1.37951	0.65119	0.10351	1.48304	0.61239
7	0.08852	1.38731	0.64828	0.10221	1.47608	0.61508
8	0.08928	1.39317	0.64617	0.10124	1.47088	0.61714
9	0.08987	1.39775	0.64456	0.10048	1.46684	0.61877
10	0.09033	1.40142	0.64329	0.09987	1.46362	0.62008
11	0.09071	1.40443	0.64227	0.09937	1.46099	0.62117
12	0.09102	1.40694	0.64142	0.09895	1.45880	0.62209
13	0.09128	1.40907	0.64072	0.09860	1.45694	0.62287
14	0.09151	1.41089	0.64012	0.09829	1.45536	0.62355
15	0.09170	1.41248	0.63960	0.09803	1.45399	0.62413
16	0.09187	1.41387	0.63915	0.09780	1.45279	0.62465
17	0.09201	1.41509	0.63875	0.09760	1.45173	0.62511
18	0.09215	1.41619	0.63840	0.09742	1.45079	0.62552
19	0.09226	1.41716	0.63809	0.09725	1.44994	0.62588
20	0.09237	1.41804	0.63781	0.09711	1.44919	0.62621
25	0.09276	1.42139	0.63676	0.09655	1.44632	0.62748
30	0.09303	1.42363	0.63607	0.09618	1.44440	0.62833
50	0.09355	1.42811	0.63470	0.09544	1.44058	0.63006
75	0.09381	1.43036	0.63403	0.09507	1.43868	0.63093
100	0.09394	1.43149	0.63369	0.09488	1.43773	0.63137
150	0.09406	1.43262	0.63336	0.09469	1.43677	0.63181
200	0.09413	1.43318	0.63319	0.09460	1.43630	0.63203
250	0.09417	1.43352	0.63309	0.09454	1.43601	0.63216
300	0.09419	1.43374		0.09451	1.43582	
∞	0.0943190142	1.4348727409	0.6326945907	0.0943190142	1.4348727409	0.6326945907

APPENDIX C.1 – Analytical Results for Chapter 5

Show: The distribution of the variance v with ν1 degrees of freedom may be represented as follows:

$$p(v) = \left(\frac{1}{\sigma^{\nu 1}}\right) \cdot \left[e^{\left(\frac{\nu 1}{2}\right) \cdot \ln\left(\frac{\nu 1}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right) + \left(\frac{\nu 1}{2} - 1\right) \cdot \ln(v) - \frac{\nu 1 \cdot v}{2 \cdot \sigma^2}} \right]$$

From Pearson and Hartley (1962),

$$p(v) = \left(\frac{\nu 1}{2}\right)^{\frac{\nu 1}{2}} \cdot \left(\Gamma\left(\frac{\nu 1}{2}\right)\right)^{-1} \cdot \sigma^{-\nu 1} \cdot v^{\frac{\nu 1}{2} - 1} \cdot e^{\frac{-\nu 1 \cdot v}{2 \cdot \sigma^2}}$$

$$\Rightarrow p(v) = e^{\ln\left[\left(\frac{\nu 1}{2}\right)^{\frac{\nu 1}{2}} \cdot \left(\Gamma\left(\frac{\nu 1}{2}\right)\right)^{-1} \cdot \sigma^{-\nu 1} \cdot v^{\frac{\nu 1}{2} - 1} \cdot e^{\frac{-\nu 1 \cdot v}{2 \cdot \sigma^2}}\right]}$$

$$= \left(\frac{1}{\sigma^{\nu 1}}\right) \cdot \left[e^{\left(\frac{\nu 1}{2}\right) \cdot \ln\left(\frac{\nu 1}{2}\right) - \ln\left(\Gamma\left(\frac{\nu 1}{2}\right)\right) + \left(\frac{\nu 1}{2} - 1\right) \cdot \ln(v) - \frac{\nu 1 \cdot v}{2 \cdot \sigma^2}} \right]$$

$$= \left(\frac{1}{\sigma^{\nu 1}}\right) \cdot \left[e^{\left(\frac{\nu 1}{2}\right) \cdot \ln\left(\frac{\nu 1}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right) + \left(\frac{\nu 1}{2} - 1\right) \cdot \ln(v) - \frac{\nu 1 \cdot v}{2 \cdot \sigma^2}} \right]$$

<u>Show:</u> The distribution of the studentized variance $f = (v/v')$ with $\nu 1$ degrees of freedom for v and $\nu 2$ degrees of freedom for v' may be represented as follows:

$$p3(f) = e^{p1+p2(f)}$$

where

$$p1 = \mathrm{gammln}\left(\frac{\nu 1+\nu 2}{2}\right) - \mathrm{gammln}\left(\frac{\nu 1}{2}\right) - \mathrm{gammln}\left(\frac{\nu 2}{2}\right)$$

$$p2(f) = \left(\frac{\nu 1}{2}\right)\cdot(\ln(\nu 1) - \ln(\nu 2)) + \left(\frac{\nu 1}{2} - 1\right)\cdot\ln(f) - \left(\frac{\nu 1+\nu 2}{2}\right)\cdot\ln\left(1 + \frac{\nu 1}{\nu 2}\cdot f\right)$$

From Bain and Engelhardt (1992),

$$p3(f) = \frac{\Gamma\left(\frac{\nu 1+\nu 2}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)\cdot\Gamma\left(\frac{\nu 2}{2}\right)}\cdot\left(\frac{\nu 1}{\nu 2}\right)^{\frac{\nu 1}{2}}\cdot f^{\frac{\nu 1}{2}-1}\cdot\left(1+\frac{\nu 1}{\nu 2}\cdot f\right)^{-\frac{\nu 1+\nu 2}{2}}$$

$$\Rightarrow p3(f) = e^{\ln\left[\frac{\Gamma\left(\frac{\nu 1+\nu 2}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)\cdot\Gamma\left(\frac{\nu 2}{2}\right)}\cdot\left(\frac{\nu 1}{\nu 2}\right)^{\frac{\nu 1}{2}}\cdot f^{\frac{\nu 1}{2}-1}\cdot\left(1+\frac{\nu 1}{\nu 2}\cdot f\right)^{-\frac{\nu 1+\nu 2}{2}}\right]}$$

$$= e^{\ln\left(\Gamma\left(\frac{\nu 1+\nu 2}{2}\right)\right) - \ln\left(\Gamma\left(\frac{\nu 1}{2}\right)\right) - \ln\left(\Gamma\left(\frac{\nu 2}{2}\right)\right) + \left(\frac{\nu 1}{2}\right)\cdot\ln\left(\frac{\nu 1}{\nu 2}\right) + \left(\frac{\nu 1}{2}-1\right)\cdot\ln(f) - \left(\frac{\nu 1+\nu 2}{2}\right)\cdot\ln\left(1+\frac{\nu 1}{\nu 2}\cdot f\right)}$$

$$= e^{\mathrm{gammln}\left(\frac{\nu 1+\nu 2}{2}\right) - \mathrm{gammln}\left(\frac{\nu 1}{2}\right) - \mathrm{gammln}\left(\frac{\nu 2}{2}\right) + \left(\frac{\nu 1}{2}\right)\cdot(\ln(\nu 1)-\ln(\nu 2)) + \left(\frac{\nu 1}{2}-1\right)\cdot\ln(f) - \left(\frac{\nu 1+\nu 2}{2}\right)\cdot\ln\left(1+\frac{\nu 1}{\nu 2}\cdot f\right)}$$

Let $p1 = \mathrm{gammln}\left(\frac{\nu 1+\nu 2}{2}\right) - \mathrm{gammln}\left(\frac{\nu 1}{2}\right) - \mathrm{gammln}\left(\frac{\nu 2}{2}\right)$

$$p2(f) = \left(\frac{\nu 1}{2}\right)\cdot(\ln(\nu 1) - \ln(\nu 2)) + \left(\frac{\nu 1}{2} - 1\right)\cdot\ln(f) - \left(\frac{\nu 1+\nu 2}{2}\right)\cdot\ln\left(1 + \frac{\nu 1}{\nu 2}\cdot f\right)$$

$$\Rightarrow p3(f) = e^{p1+p2(f)}$$

Show: $\bar{v}$ is an unbiased estimate of σ^2; i.e., show $E(\bar{v}) = \sigma^2$

$$E(\bar{v}) = E\left(\frac{\sum_{i=1}^{m} v_i}{m}\right) = \left(\frac{1}{m}\right)\cdot E\left(\sum_{i=1}^{m} v_i\right) = \left(\frac{1}{m}\right)\cdot \sum_{i=1}^{m} E(v_i) = \left(\frac{1}{m}\right)\cdot \sum_{i=1}^{m} \sigma^2$$

since $E(v) = \sigma^2$.

$$\Rightarrow E(\bar{v}) = \left(\frac{1}{m}\right)\cdot\left(m\cdot\sigma^2\right) = \sigma^2$$

Show: $\sqrt{\bar{v}} = s_p$, where s_p is the pooled standard deviation

From Burr (1969) and Nelson (1990), $s_p = \sqrt{\dfrac{\sum_{i=1}^{m}\left[(n_i - 1)\cdot s_i^2\right]}{\sum_{i=1}^{m}(n_i) - m}}$

Since the subgroup size n is the same for each of the m subgroups,

$$s_p = \sqrt{\frac{\sum_{i=1}^{m}\left[(n-1)\cdot s_i^2\right]}{\sum_{i=1}^{m}(n) - m}} = \sqrt{\frac{(n-1)\cdot \sum_{i=1}^{m} s_i^2}{(m\cdot n) - m}} = \sqrt{\frac{(n-1)\cdot \sum_{i=1}^{m} s_i^2}{m\cdot (n-1)}} = \sqrt{\frac{\sum_{i=1}^{m} v_i}{m}}$$

since $v_i = s_i^2$.

$$\Rightarrow s_p = \sqrt{\bar{v}}$$

Show: The mean of the distribution of the standard deviation s with (x-1) degrees of freedom may be represented as follows:

$$c4(x) = \sigma \cdot \left(\frac{2}{x-1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{x}{2}\right) - \mathrm{gammln}\left(\frac{x-1}{2}\right)}\right)$$

From Mead (1966),

$$E(s) = c_4 = \sigma \cdot \left(\frac{2}{n-1}\right)^{0.5} \cdot \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

where n is the size of the subgroup from which the statistic that is used to estimate σ is calculated.

$$\Rightarrow c4 = \sigma \cdot \left(\frac{2}{n-1}\right)^{0.5} \cdot \left(\frac{e^{\ln\left(\Gamma\left(\frac{n}{2}\right)\right)}}{e^{\ln\left(\Gamma\left(\frac{n-1}{2}\right)\right)}}\right)$$

where $c4 \equiv c_4$.

$$\Rightarrow c4 = \sigma \cdot \left(\frac{2}{n-1}\right)^{0.5} \cdot \left(\frac{e^{\mathrm{gammln}\left(\frac{n}{2}\right)}}{e^{\mathrm{gammln}\left(\frac{n-1}{2}\right)}}\right) = \sigma \cdot \left(\frac{2}{n-1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{n}{2}\right) - \mathrm{gammln}\left(\frac{n-1}{2}\right)}\right)$$

$$\Rightarrow c4(x) = \sigma \cdot \left(\frac{2}{x-1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{x}{2}\right) - \mathrm{gammln}\left(\frac{x-1}{2}\right)}\right)$$

Show: $p(v) = c\left(\frac{v1 \cdot v}{\sigma^2}\right) \cdot \frac{v1}{\sigma^2}$, where p(v) is the distribution of the variance with v1 degrees of freedom and c is the χ^2 distribution with v1 degrees of freedom.

Bain and Engelhardt (1992) give the χ^2 distribution as follows:

$$c(x) = \frac{1}{2^{\frac{v1}{2}} \cdot \Gamma\left(\frac{v1}{2}\right)} \cdot x^{\frac{v1}{2}-1} \cdot e^{\frac{-x}{2}}$$

Let $x = \frac{v1 \cdot v}{\sigma^2}$

$$\Rightarrow dx = \frac{v1}{\sigma^2} dv \Rightarrow c(x)\, dx = c\left(\frac{v1 \cdot v}{\sigma^2}\right) \cdot \frac{v1}{\sigma^2} dv$$

$$\Rightarrow c\left(\frac{v1 \cdot v}{\sigma^2}\right) \cdot \frac{v1}{\sigma^2} dv = \frac{1}{2^{\frac{v1}{2}} \cdot \Gamma\left(\frac{v1}{2}\right)} \cdot \left(\frac{v1 \cdot v}{\sigma^2}\right)^{\frac{v1}{2}-1} \cdot e^{\frac{-\left(\frac{v1 \cdot v}{\sigma^2}\right)}{2}} \cdot \frac{v1}{\sigma^2} dv$$

$$= \frac{v1^{\frac{v1}{2}-1} \cdot v1}{2^{\frac{v1}{2}} \cdot \Gamma\left(\frac{v1}{2}\right) \cdot \left(\sigma^2\right)^{\frac{v1}{2}-1} \cdot \sigma^2} \cdot v^{\frac{v1}{2}-1} \cdot e^{\frac{-v1 \cdot v}{2 \cdot \sigma^2}} dv$$

$$= \frac{v1^{\frac{v1}{2}} \cdot 2^{\frac{-v1}{2}} \cdot \left(\Gamma\left(\frac{v1}{2}\right)\right)^{-1}}{\sigma^{v1}} \cdot v^{\frac{v1}{2}-1} \cdot e^{\frac{-v1 \cdot v}{2 \cdot \sigma^2}} dv$$

$$= v1^{\frac{v1}{2}} \cdot \left(\frac{1}{2}\right)^{\frac{v1}{2}} \cdot \left(\Gamma\left(\frac{v1}{2}\right)\right)^{-1} \cdot \sigma^{-v1} \cdot v^{\frac{v1}{2}-1} \cdot e^{\frac{-v1 \cdot v}{2 \cdot \sigma^2}} dv$$

$$= \left(\frac{v1}{2}\right)^{\frac{v1}{2}} \cdot \left(\Gamma\left(\frac{v1}{2}\right)\right)^{-1} \cdot \sigma^{-v1} \cdot v^{\frac{v1}{2}-1} \cdot e^{\frac{-v1 \cdot v}{2 \cdot \sigma^2}} dv$$

$$= p(v)\, dv$$

APPENDIX C.2 – Computer Program ccfsv.mcd for Chapter 5

Page 1 of program: ccfsv.mcd

ENTER the following 5 values:

(1) alphaMean := 0.0027 — **alphaMean** - alpha for the $\overline{X}$ chart.

(2) alphaVarUCL := 0.005 — **alphaVarUCL** - alpha for the v or $\sqrt{v}$ chart above the UCL.

(3) alphaVarLCL := 0.001 — **alphaVarLCL** - alpha for the v or $\sqrt{v}$ chart below the LCL *.

(4) m := 5 — **m** - number of subgroups.

(5) n := 5 — **n** - subgroup size for the $(\overline{X}, v)$ or $(\overline{X}, \sqrt{v})$ charts.

*** Note - If no LCL is desired, leave alphaVarLCL blank (do not enter zero).**

Please PAGE DOWN to begin the program.

(1.1) $TOL := 10^{-12}$ $\sigma := 1.0$ $v1 := n - 1$

$$c4(x) := \sigma \cdot \left(\frac{2}{x-1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{x}{2}\right) - \mathrm{gammln}\left(\frac{x-1}{2}\right)}\right)$$

Page 2 of program: ccfsv.mcd

(2.1) $p(v) := \left(\frac{1}{\sigma^{v1}}\right)\cdot\left[e^{\left(\frac{v1}{2}\right)\cdot\ln\left(\frac{v1}{2}\right)-\text{gammln}\left(\frac{v1}{2}\right)+\left(\frac{v1}{2}-1\right)\cdot\ln(v)-\frac{v1\cdot v}{2\cdot\sigma^2}}\right]$

$P(V) := \int_0^V p(v)\,dv$

(2.2) $DUCL(V) := P(V) - (1 - alphaVarUCL)$ $\qquad$ $DLCL(V) := P(V) - alphaVarLCL$

```
Vseed1(start) := | V0 ← start
                 | V1 ← start + 0.01
                 | A0 ← DUCL(V0)
                 | A1 ← DUCL(V1)
                 | while A0·A1 > 0
                 |    | V0 ← V1
                 |    | V1 ← V1 + 0.01
                 |    | A0 ← A1
                 |    | A1 ← DUCL(V1)
                 | V

Vseed2(start) := | V0 ← start
                 | V1 ← start + 0.01
                 | A0 ← DLCL(V0)
                 | A1 ← DLCL(V1)
                 | while A0·A1 > 0
                 |    | V0 ← V1
                 |    | V1 ← V1 + 0.01
                 |    | A0 ← A1
                 |    | A1 ← DLCL(V1)
                 | V
```

$seedB8 := Vseed1(0.01)$ $\qquad$ $seedB7 := Vseed2(0.000001)$

$vB8 := zbrent(DUCL, seedB8_0, seedB8_1, TOL)$ $\qquad$ $vB7 := zbrent(DLCL, seedB7_0, seedB7_1, TOL)$

(2.3) $v2 := m\cdot(n - 1)$

$v2prevm := (m - 1)\cdot(n - 1)$

(3.1) $p1 := \mathrm{gammln}\left(\frac{v1+v2}{2}\right) - \mathrm{gammln}\left(\frac{v1}{2}\right) - \mathrm{gammln}\left(\frac{v2}{2}\right)$

$$p2(f) := \left(\frac{v1}{2}\right)\cdot(\ln(v1) - \ln(v2)) + \left(\frac{v1}{2} - 1\right)\cdot\ln(f) - \left(\frac{v1+v2}{2}\right)\cdot\ln\left(1 + \frac{v1}{v2}\cdot f\right)$$

$$p3(f) := e^{p1+p2(f)}$$

$$P3(F) := \int_0^F p3(f)\,df$$

(3.2)

```
Fseed1(start,delta1) := | F0 ← start
                        | F1 ← start + delta1
                        | A0 ← P3(F0)
                        | A1 ← P3(F1)
                        | while A1 < (1 − alphaVarUCL)
                        |     | F0 ← F1
                        |     | F1 ← F1 + delta1
                        |     | A0 ← A1
                        |     | A1 ← P3(F1)
                        | Fguess ← linterp(A,F,1 − alphaVarUCL)
                        | Fguess
```

```
seed1 := Fseed1(0.1,delta1)

delta1 := | 100.0 if (n = 2)·(m = 1)
          | 0.1   otherwise

D1(x) := P3(x) − (1 − alphaVarUCL)

fB8 := zbrent(D1,seed1 − delta1,seed1 + delta1,TOL)

■ := root[ |P3(seed1) − (1 − alphaVarUCL)| ,seed1]
```

```
(4.1)   Fseed2(start,delta2) := | F0 ← start
                                | F1 ← start + delta2
                                | A0 ← P3(F0)
                                | A1 ← P3(F1)
                                | while A1 < alphaVarLCL
                                |    | F0 ← F1
                                |    | F1 ← F1 + delta2
                                |    | A0 ← A1
                                |    | A1 ← P3(F1)
                                | Fguess ← linterp(A,F,alphaVarLCL)
                                | Fguess

        seed2 := Fseed2(0.000001,delta2)        delta2 := | 0.0000001  if (n = 2)
                                                          | 0.001   otherwise

        D2(x) := P3(x) − alphaVarLCL

        fB7 := zbrent(D2,seed2 − delta2,seed2 + delta2,TOL)

        ▪ := root(|P3(seed2) − alphaVarLCL|,seed2)
```

Page 5 of program: ccfsv.mcd

(5.1) $p1prevm := gammln\left(\frac{\nu 1 + \nu 2prevm}{2}\right) - gammln\left(\frac{\nu 1}{2}\right) - gammln\left(\frac{\nu 2prevm}{2}\right)$

$$p2prevm(f) := \left(\frac{\nu 1}{2}\right)\cdot(\ln(\nu 1) - \ln(\nu 2prevm)) + \left(\frac{\nu 1}{2} - 1\right)\cdot\ln(f) - \left(\frac{\nu 1 + \nu 2prevm}{2}\right)\cdot\ln\left(1 + \frac{\nu 1}{\nu 2prevm}\cdot f\right)$$

$$p3prevm(f) := e^{p1prevm + p2prevm(f)}$$

$$P3prevm(F) := \int_0^F p3prevm(f)\, df$$

(5.2) Fseed3(start, delta3) :=

$F_0 \leftarrow start$
$F_1 \leftarrow start + delta3$
$A_0 \leftarrow P3prevm(F_0)$
$A_1 \leftarrow P3prevm(F_1)$
while $A_1 < (1 - alphaVarUCL)$
  $F_0 \leftarrow F_1$
  $F_1 \leftarrow F_1 + delta3$
  $A_0 \leftarrow A_1$
  $A_1 \leftarrow P3prevm(F_1)$
$Fguess \leftarrow linterp(A, F, 1 - alphaVarUCL)$
Fguess

seed3 := Fseed3(0.1, delta3)

delta3 := 100.0 if $(n = 2)\cdot(m \le 2)$
0.1 otherwise

D1prevm(x) := P3prevm(x) − (1 − alphaVarUCL)

fB8prevm := zbrent(D1prevm, seed3 − delta3, seed3 + delta3, TOL)

▮ := root[|P3prevm(seed3) − (1 − alphaVarUCL)| , seed3]

Page 6 of program: ccfsv.mcd

(6.1) $Fseed4(start, delta4) :=$

```
F0 ← start
F1 ← start + delta4
A0 ← P3prevm(F0)
A1 ← P3prevm(F1)
while A1 < alphaVarLCL
    F0 ← F1
    F1 ← F1 + delta4
    A0 ← A1
    A1 ← P3prevm(F1)
Fguess ← linterp(A, F, alphaVarLCL)
Fguess
```

$seed4 := Fseed4(0.000001, delta4)$

$delta4 := \begin{cases} 0.0000001 & \text{if } (n = 2) \\ 0.001 & \text{otherwise} \end{cases}$

$D2prevm(x) := P3prevm(x) - alphaVarLCL$

$fB7prevm := zbrent(D2prevm, seed4 - delta4, seed4 + delta4, TOL)$

$\blacksquare := root(|P3prevm(seed4) - alphaVarLCL|, seed4)$

Page 7 of program: ccfsv.mcd

(7.1) $\text{adj\_alpha} := 1 - \frac{\text{alphaMean}}{2}$ $\text{crit\_t} := \text{qt}(\text{adj\_alpha}, v2)$ $\text{crit\_z} := \text{qnorm}(\text{adj\_alpha}, 0, 1)$

(7.2) $A41 := \left(\frac{\text{crit\_t}}{c4(v2+1)}\right)\cdot\left(\frac{m-1}{n\cdot m}\right)^{0.5}$ $A42 := \left(\frac{\text{crit\_t}}{c4(v2+1)}\right)\cdot\left(\frac{m+1}{n\cdot m}\right)^{0.5}$ $A4 := \frac{\text{crit\_z}}{n^{0.5}}$

$B81 := \frac{m\cdot fB8prevm}{m-1+fB8prevm}$ $B82 := fB8$ $B8 := vB8$

$B71 := \frac{m\cdot fB7prevm}{m-1+fB7prevm}$ $B72 := fB7$ $B7 := vB7$

$B81sqrt := \frac{B81^{0.5}}{c4(v2prevm+1)}$ $B82sqrt := \frac{B82^{0.5}}{c4(v2+1)}$ $B8sqrt := B8^{0.5}$

$B71sqrt := \frac{B71^{0.5}}{c4(v2prevm+1)}$ $B72sqrt := \frac{B72^{0.5}}{c4(v2+1)}$ $B7sqrt := B7^{0.5}$

FINAL RESULTS:

(1) alphaMean = 0.0027
(2) alphaVarUCL = 0.005
(3) alphaVarLCL = 0.001
(4) m = 5
(5) n = 5

Control Chart Factors

First Stage	Second Stage	Conventional
A41 = 1.38606	A42 = 1.69757	A4 = 1.3416304973
B81 = 2.92485	B82 = 5.17428	B8 = 3.7150647501
B71 = 0.0266826472	B72 = 0.0216918527	B7 = 0.0227010089
B81sqrt = 1.73713	B82sqrt = 2.3033	B8sqrt = 1.9274503237
B71sqrt = 0.16592	B72sqrt = 0.14913	B7sqrt = 0.1506685398

v1 = 4

v2 = 20

c4(v2 + 1) = 0.98758

v2prevm = 16

c4(v2prevm + 1) = 0.98451

(1 - alphaVarUCL) and alphaVarLCL Percentage Points of the Distributions of the Studentized Variance and the Variance

fB8 = 5.17428	fB8prevm = 5.63785	vB8 = 3.7150647501
fB7 = 0.0216918527	fB7prevm = 0.0214606431	vB7 = 0.0227010089

APPENDIX C.3 – Tables Generated from ccfsv.mcd

Table C.3.1. ν2 (Degrees of Freedom) and $c_4(\nu2+1)$ Values ($\nu2 = m \cdot (n - 1)$)

n	2		3		4		5		6	
m	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$
1	1.0	0.79788	2.0	0.88623	3.0	0.92132	4.0	0.93999	5.0	0.95153
2	2.0	0.88623	4.0	0.93999	6.0	0.95937	8.0	0.96931	10.0	0.97535
3	3.0	0.92132	6.0	0.95937	9.0	0.97266	12.0	0.97941	15.0	0.98348
4	4.0	0.93999	8.0	0.96931	12.0	0.97941	16.0	0.98451	20.0	0.98758
5	5.0	0.95153	10.0	0.97535	15.0	0.98348	20.0	0.98758	25.0	0.99005
6	6.0	0.95937	12.0	0.97941	18.0	0.98621	24.0	0.98964	30.0	0.99170
7	7.0	0.96503	14.0	0.98232	21.0	0.98817	28.0	0.99111	35.0	0.99288
8	8.0	0.96931	16.0	0.98451	24.0	0.98964	32.0	0.99222	40.0	0.99377
9	9.0	0.97266	18.0	0.98621	27.0	0.99079	36.0	0.99308	45.0	0.99446
10	10.0	0.97535	20.0	0.98758	30.0	0.99170	40.0	0.99377	50.0	0.99501
11	11.0	0.97756	22.0	0.98870	33.0	0.99245	44.0	0.99433	55.0	0.99547
12	12.0	0.97941	24.0	0.98964	36.0	0.99308	48.0	0.99481	60.0	0.99584
13	13.0	0.98097	26.0	0.99043	39.0	0.99361	52.0	0.99520	65.0	0.99616
14	14.0	0.98232	28.0	0.99111	42.0	0.99407	56.0	0.99555	70.0	0.99644
15	15.0	0.98348	30.0	0.99170	45.0	0.99446	60.0	0.99584	75.0	0.99667
16	16.0	0.98451	32.0	0.99222	48.0	0.99481	64.0	0.99610	80.0	0.99688
17	17.0	0.98541	34.0	0.99268	51.0	0.99511	68.0	0.99633	85.0	0.99706
18	18.0	0.98621	36.0	0.99308	54.0	0.99538	72.0	0.99653	90.0	0.99723
19	19.0	0.98693	38.0	0.99344	57.0	0.99562	76.0	0.99672	95.0	0.99737
20	20.0	0.98758	40.0	0.99377	60.0	0.99584	80.0	0.99688	100.0	0.99750
25	25.0	0.99005	50.0	0.99501	75.0	0.99667	100.0	0.99750	125.0	0.99800
30	30.0	0.99170	60.0	0.99584	90.0	0.99723	120.0	0.99792	150.0	0.99833
50	50.0	0.99501	100.0	0.99750	150.0	0.99833	200.0	0.99875	250.0	0.99900
75	75.0	0.99667	150.0	0.99833	225.0	0.99889	300.0	0.99917	375.0	0.99933
100	100.0	0.99750	200.0	0.99875	300.0	0.99917	400.0	0.99938	500.0	0.99950
150	150.0	0.99833	300.0	0.99917	450.0	0.99944	600.0	0.99958	750.0	0.99967
200	200.0	0.99875	400.0	0.99938	600.0	0.99958	800.0	0.99969	1000.0	0.99975
250	250.0	0.99900	500.0	0.99950	750.0	0.99967	1000.0	0.99975	1250.0	0.99980
300	300.0	0.99917	600.0	0.99958	900.0	0.99972	1200.0	0.99979	1500.0	0.99983
$c_4(\infty)$	1.00000		1.00000		1.00000		1.00000		1.00000	

Table C.3.1 continued. ν2 (Degrees of Freedom) and $c_4(\nu2+1)$ Values ($\nu2 = m \cdot (n - 1)$)

n	7		8		10		25		50	
m	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$	ν2	$c_4(\nu2+1)$
1	6.0	0.95937	7.0	0.96503	9.0	0.97266	24.0	0.98964	49.0	0.99491
2	12.0	0.97941	14.0	0.98232	18.0	0.98621	48.0	0.99481	98.0	0.99745
3	18.0	0.98621	21.0	0.98817	27.0	0.99079	72.0	0.99653	147.0	0.99830
4	24.0	0.98964	28.0	0.99111	36.0	0.99308	96.0	0.99740	196.0	0.99873
5	30.0	0.99170	35.0	0.99288	45.0	0.99446	120.0	0.99792	245.0	0.99898
6	36.0	0.99308	42.0	0.99407	54.0	0.99538	144.0	0.99827	294.0	0.99915
7	42.0	0.99407	49.0	0.99491	63.0	0.99604	168.0	0.99851	343.0	0.99927
8	48.0	0.99481	56.0	0.99555	72.0	0.99653	192.0	0.99870	392.0	0.99936
9	54.0	0.99538	63.0	0.99604	81.0	0.99692	216.0	0.99884	441.0	0.99943
10	60.0	0.99584	70.0	0.99644	90.0	0.99723	240.0	0.99896	490.0	0.99949
11	66.0	0.99622	77.0	0.99676	99.0	0.99748	264.0	0.99905	539.0	0.99954
12	72.0	0.99653	84.0	0.99703	108.0	0.99769	288.0	0.99913	588.0	0.99957
13	78.0	0.99680	91.0	0.99726	117.0	0.99787	312.0	0.99920	637.0	0.99961
14	84.0	0.99703	98.0	0.99745	126.0	0.99802	336.0	0.99926	686.0	0.99964
15	90.0	0.99723	105.0	0.99762	135.0	0.99815	360.0	0.99931	735.0	0.99966
16	96.0	0.99740	112.0	0.99777	144.0	0.99827	384.0	0.99935	784.0	0.99968
17	102.0	0.99755	119.0	0.99790	153.0	0.99837	408.0	0.99939	833.0	0.99970
18	108.0	0.99769	126.0	0.99802	162.0	0.99846	432.0	0.99942	882.0	0.99972
19	114.0	0.99781	133.0	0.99812	171.0	0.99854	456.0	0.99945	931.0	0.99973
20	120.0	0.99792	140.0	0.99822	180.0	0.99861	480.0	0.99948	980.0	0.99974
25	150.0	0.99833	175.0	0.99857	225.0	0.99889	600.0	0.99958	1225.0	0.99980
30	180.0	0.99861	210.0	0.99881	270.0	0.99907	720.0	0.99965	1470.0	0.99983
50	300.0	0.99917	350.0	0.99929	450.0	0.99944	1200.0	0.99979	2450.0	0.99990
75	450.0	0.99944	525.0	0.99952	675.0	0.99963	1800.0	0.99986	3675.0	0.99993
100	600.0	0.99958	700.0	0.99964	900.0	0.99972	2400.0	0.99990	4900.0	0.99995
150	900.0	0.99972	1050.0	0.99976	1350.0	0.99981	3600.0	0.99993	7350.0	0.99997
200	1200.0	0.99979	1400.0	0.99982	1800.0	0.99986	4800.0	0.99995	9800.0	0.99997
250	1500.0	0.99983	1750.0	0.99986	2250.0	0.99989	6000.0	0.99996	12250.0	0.99998
300	1800.0	0.99986	2100.0	0.99988	2700.0	0.99991	7200.0	0.99997	14700.0	0.99998
$c_4(\infty)$	1.00000		1.00000		1.00000		1.00000		1.00000	

Table C.3.2. (1 - alphaVarUCL) Percentage
Points of the Studentized Variance (alphaVarUCL = 0.005)

	n				
m	**2**	**3**	**4**	**5**	**6**
1	16210.72272	199.00000	47.46723	23.15450	14.93961
2	198.50125	26.28427	12.91660	8.80513	6.87237
3	55.55196	14.54411	8.71706	6.52114	5.37214
4	31.33277	11.04241	7.22576	5.63785	4.76157
5	22.78478	9.42700	6.47604	5.17428	4.43267
6	18.63500	8.50963	6.02777	4.88978	4.22758
7	16.23556	7.92164	5.73039	4.69771	4.08760
8	14.68820	7.51382	5.51900	4.55943	3.98605
9	13.61361	7.21483	5.36113	4.45517	3.90902
10	12.82647	6.98646	5.23879	4.37378	3.84860
11	12.22631	6.80645	5.14124	4.30848	3.79996
12	11.75423	6.66095	5.06165	4.25494	3.75995
13	11.37354	6.54095	4.99548	4.21025	3.72647
14	11.06025	6.44030	4.93962	4.17239	3.69803
15	10.79805	6.35469	4.89182	4.13989	3.67359
16	10.57546	6.28098	4.85047	4.11171	3.65236
17	10.38418	6.21687	4.81434	4.08703	3.63373
18	10.21809	6.16059	4.78251	4.06524	3.61727
19	10.07253	6.11079	4.75425	4.04586	3.60261
20	9.94393	6.06643	4.72899	4.02851	3.58947
25	9.47531	5.90162	4.63452	3.96338	3.54005
30	9.17968	5.79499	4.57284	3.92065	3.50753
50	8.62576	5.58922	4.45252	3.83683	3.44350
75	8.36627	5.48995	4.39385	3.79572	3.41198
100	8.24064	5.44119	4.36488	3.77536	3.39634
150	8.11767	5.39300	4.33614	3.75513	3.38079
200	8.05716	5.36912	4.32187	3.74507	3.37304
250	8.02116	5.35486	4.31333	3.73905	3.36840
300	7.99729	5.34538	4.30765	3.73504	3.36531
∞	7.8794385766	5.2983173665	4.2793854889	3.7150647501	3.3499204687

Table C.3.2 continued. (1 - alphaVarUCL) Percentage
Points of the Studentized Variance (alphaVarUCL = 0.005)

	n				
m	**7**	**8**	**10**	**25**	**50**
1	11.07304	8.88539	6.54109	2.96674	2.11305
2	5.75703	5.03134	4.14098	2.39439	1.85121
3	4.66274	4.17893	3.55707	2.22167	1.76595
4	4.20189	3.81099	3.29645	2.13823	1.72354
5	3.94921	3.60665	3.14915	2.08904	1.69813
6	3.78993	3.47681	3.05454	2.05660	1.68121
7	3.68042	3.38706	2.98864	2.03359	1.66912
8	3.60053	3.32133	2.94013	2.01642	1.66005
9	3.53970	3.27113	2.90292	2.00312	1.65300
10	3.49183	3.23154	2.87348	1.99251	1.64736
11	3.45319	3.19951	2.84960	1.98385	1.64274
12	3.42134	3.17308	2.82985	1.97665	1.63890
13	3.39464	3.15089	2.81324	1.97057	1.63564
14	3.37194	3.13200	2.79908	1.96536	1.63285
15	3.35239	3.11572	2.78686	1.96085	1.63043
16	3.33539	3.10155	2.77621	1.95691	1.62832
17	3.32046	3.08910	2.76685	1.95344	1.62645
18	3.30726	3.07808	2.75855	1.95036	1.62479
19	3.29549	3.06825	2.75114	1.94760	1.62330
20	3.28494	3.05943	2.74449	1.94512	1.62197
25	3.24518	3.02617	2.71937	1.93571	1.61688
30	3.21896	3.00420	2.70274	1.92944	1.61350
50	3.16721	2.96076	2.66978	1.91695	1.60672
75	3.14167	2.93929	2.65344	1.91071	1.60333
100	3.12899	2.92861	2.64530	1.90760	1.60163
150	3.11636	2.91797	2.63719	1.90449	1.59994
200	3.11006	2.91267	2.63314	1.90293	1.59909
250	3.10629	2.90949	2.63072	1.90200	1.59858
300	3.10378	2.90738	2.62910	1.90138	1.59824
∞	3.0912640298	2.8968199821	2.6210389757	1.8982713307	1.5965450633

Table C.3.3. alphaVarLCL Percentage Points
of the Studentized Variance (alphaVarLCL = 0.001)

	n				
m	**2**	**3**	**4**	**5**	**6**
1	0.00000247	0.00100100	0.00709	0.01871	0.03361
2	0.00000200	0.00100075	0.00753	0.02041	0.03715
3	0.00000185	0.00100067	0.00770	0.02109	0.03859
4	0.00000178	0.00100063	0.00779	0.02146	0.03938
5	0.00000173	0.00100060	0.00785	0.02169	0.03987
6	0.00000171	0.00100058	0.00789	0.02185	0.04021
7	0.00000169	0.00100057	0.00792	0.02197	0.04046
8	0.00000167	0.00100056	0.00794	0.02205	0.04065
9	0.00000166	0.00100056	0.00796	0.02212	0.04080
10	0.00000165	0.00100055	0.00797	0.02218	0.04092
11	0.00000164	0.00100055	0.00798	0.02222	0.04101
12	0.00000164	0.00100054	0.00799	0.02226	0.04110
13	0.00000163	0.00100054	0.00800	0.02230	0.04117
14	0.00000163	0.00100054	0.00801	0.02232	0.04123
15	0.00000162	0.00100053	0.00801	0.02235	0.04128
16	0.00000162	0.00100053	0.00802	0.02237	0.04133
17	0.00000162	0.00100053	0.00802	0.02239	0.04137
18	0.00000162	0.00100053	0.00803	0.02241	0.04141
19	0.00000161	0.00100053	0.00803	0.02242	0.04144
20	0.00000161	0.00100053	0.00803	0.02244	0.04147
25	0.00000160	0.00100052	0.00805	0.02249	0.04158
30	0.00000160	0.00100052	0.00806	0.02252	0.04166
50	0.00000159	0.00100051	0.00807	0.02259	0.04181
75	0.00000158	0.00100051	0.00808	0.02263	0.04189
100	0.00000158	0.00100051	0.00809	0.02265	0.04193
150	0.00000158	0.00100050	0.00809	0.02266	0.04196
200	0.00000157	0.00100050	0.00809	0.02267	0.04198
250	0.00000157	0.00100050	0.00809	0.02268	0.04200
300	0.00000157	0.00100050	0.00809	0.02268	0.04200
∞	0.0000015708	0.0010005003	0.0080991953	0.0227010089	0.0420425205

Table C.3.3 continued. alphaVarLCL Percentage
Points of the Studentized Variance (alphaVarLCL = 0.001)

	n				
m	**7**	**8**	**10**	**25**	**50**
1	0.04993	0.06658	0.09894	0.26771	0.40576
2	0.05559	0.07444	0.11096	0.29660	0.44132
3	0.05791	0.07767	0.11593	0.30841	0.45558
4	0.05918	0.07944	0.11864	0.31484	0.46331
5	0.05998	0.08055	0.12036	0.31890	0.46816
6	0.06053	0.08132	0.12155	0.32170	0.47149
7	0.06093	0.08189	0.12241	0.32374	0.47392
8	0.06124	0.08231	0.12307	0.32530	0.47577
9	0.06148	0.08265	0.12359	0.32652	0.47722
10	0.06167	0.08292	0.12402	0.32752	0.47840
11	0.06183	0.08315	0.12436	0.32833	0.47937
12	0.06197	0.08334	0.12466	0.32902	0.48018
13	0.06208	0.08350	0.12490	0.32960	0.48087
14	0.06218	0.08364	0.12512	0.33011	0.48147
15	0.06227	0.08376	0.12530	0.33055	0.48198
16	0.06235	0.08386	0.12547	0.33093	0.48244
17	0.06241	0.08396	0.12561	0.33127	0.48284
18	0.06247	0.08404	0.12574	0.33157	0.48320
19	0.06253	0.08412	0.12586	0.33185	0.48352
20	0.06257	0.08418	0.12596	0.33209	0.48381
25	0.06276	0.08444	0.12636	0.33303	0.48492
30	0.06288	0.08462	0.12663	0.33366	0.48567
50	0.06313	0.08497	0.12717	0.33493	0.48716
75	0.06326	0.08514	0.12744	0.33558	0.48792
100	0.06332	0.08523	0.12758	0.33590	0.48830
150	0.06338	0.08532	0.12772	0.33622	0.48868
200	0.06342	0.08537	0.12779	0.33638	0.48887
250	0.06343	0.08539	0.12783	0.33648	0.48899
300	0.06345	0.08541	0.12786	0.33655	0.48906
∞	0.0635111259	0.0854991075	0.1279943940	0.3368700659	0.4894454026

Table C.3.4. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	2					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	295.51103	16210.72272	0.00000247
2	10.83583	1.99988	0.00000493	18.76822	198.50125	0.00000200
3	5.77696	2.97008	0.00000300	8.16986	55.55196	0.00000185
4	4.31278	3.79505	0.00000247	5.56777	31.33277	0.00000178
5	3.66033	4.43395	0.00000222	4.48297	22.78478	0.00000173
6	3.29958	4.92027	0.00000208	3.90411	18.63500	0.00000171
7	3.07298	5.29511	0.00000199	3.54838	16.23556	0.00000169
8	2.91825	5.58990	0.00000193	3.30898	14.68820	0.00000167
9	2.80619	5.82654	0.00000188	3.13742	13.61361	0.00000166
10	2.72145	6.02010	0.00000184	3.00867	12.82647	0.00000165
11	2.65518	6.18103	0.00000182	2.90861	12.22631	0.00000164
12	2.60199	6.31679	0.00000179	2.82866	11.75423	0.00000164
13	2.55836	6.43275	0.00000177	2.76335	11.37354	0.00000163
14	2.52195	6.53289	0.00000176	2.70901	11.06025	0.00000163
15	2.49111	6.62020	0.00000174	2.66311	10.79805	0.00000162
16	2.46466	6.69697	0.00000173	2.62383	10.57546	0.00000162
17	2.44172	6.76499	0.00000172	2.58984	10.38418	0.00000162
18	2.42165	6.82567	0.00000171	2.56014	10.21809	0.00000162
19	2.40393	6.88011	0.00000170	2.53397	10.07253	0.00000161
20	2.38819	6.92924	0.00000170	2.51074	9.94393	0.00000161
25	2.33000	7.11692	0.00000167	2.42514	9.47531	0.00000160
30	2.29261	7.24284	0.00000165	2.37035	9.17968	0.00000160
50	2.22106	7.49628	0.00000162	2.26594	8.62576	0.00000159
75	2.18683	7.62366	0.00000160	2.21619	8.36627	0.00000158
100	2.17009	7.68749	0.00000159	2.19190	8.24064	0.00000158
150	2.15359	7.75141	0.00000159	2.16800	8.11767	0.00000158
200	2.14543	7.78339	0.00000158	2.15618	8.05716	0.00000157
250	2.14056	7.80259	0.00000158	2.14914	8.02116	0.00000157
300	2.13733	7.81539	0.00000158	2.14447	7.99729	0.00000157
∞	2.1213040749	7.8794385766	0.0000015708	2.1213040749	7.8794385766	0.0000015708

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	2					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	295.51103	159.57363	0.00197
2	10.83583	1.77240	0.00278	18.76822	15.89779	0.00160
3	5.77696	1.94464	0.00195	8.16986	8.08985	0.00148
4	4.31278	2.11446	0.00170	5.56777	5.95495	0.00142
5	3.66033	2.24014	0.00159	4.48297	5.01647	0.00138
6	3.29958	2.33115	0.00152	3.90411	4.49965	0.00136
7	3.07298	2.39857	0.00147	3.54838	4.17535	0.00135
8	2.91825	2.44997	0.00144	3.30898	3.95386	0.00133
9	2.80619	2.49025	0.00141	3.13742	3.79338	0.00132
10	2.72145	2.52256	0.00140	3.00867	3.67192	0.00132
11	2.65518	2.54900	0.00138	2.90861	3.57688	0.00131
12	2.60199	2.57102	0.00137	2.82866	3.50054	0.00131
13	2.55836	2.58962	0.00136	2.76335	3.43789	0.00130
14	2.52195	2.60553	0.00135	2.70901	3.38557	0.00130
15	2.49111	2.61929	0.00134	2.66311	3.34122	0.00130
16	2.46466	2.63131	0.00134	2.62383	3.30317	0.00129
17	2.44172	2.64189	0.00133	2.58984	3.27016	0.00129
18	2.42165	2.65128	0.00133	2.56014	3.24126	0.00129
19	2.40393	2.65966	0.00132	2.53397	3.21574	0.00129
20	2.38819	2.66719	0.00132	2.51074	3.19305	0.00129
25	2.33000	2.69568	0.00131	2.42514	3.10913	0.00128
30	2.29261	2.71455	0.00130	2.37035	3.05515	0.00127
50	2.22106	2.75194	0.00128	2.26594	2.95168	0.00127
75	2.18683	2.77044	0.00127	2.21619	2.90211	0.00126
100	2.17009	2.77964	0.00127	2.19190	2.87784	0.00126
150	2.15359	2.78881	0.00126	2.16800	2.85390	0.00126
200	2.14543	2.79338	0.00126	2.15618	2.84206	0.00126
250	2.14056	2.79612	0.00126	2.14914	2.83500	0.00126
300	2.13733	2.79794	0.00126	2.14447	2.83031	0.00126
∞	2.1213040749	2.8070337683	0.0012533145	2.1213040749	2.8070337683	0.0012533145

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	3					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	17.69484	199.00000	0.00100100
2	2.87519	1.99000	0.00200000	4.97997	26.28427	0.00100075
3	2.40967	2.78787	0.00150038	3.40779	14.54411	0.00100067
4	2.20599	3.31601	0.00133378	2.84792	11.04241	0.00100063
5	2.09497	3.67043	0.00125047	2.56580	9.42700	0.00100060
6	2.02564	3.92057	0.00120048	2.39677	8.50963	0.00100058
7	1.97838	4.10537	0.00116715	2.28444	7.92164	0.00100057
8	1.94415	4.24706	0.00114335	2.20446	7.51382	0.00100056
9	1.91823	4.35898	0.00112549	2.14465	7.21483	0.00100056
10	1.89794	4.44953	0.00111161	2.09825	6.98646	0.00100055
11	1.88162	4.52426	0.00110050	2.06121	6.80645	0.00100055
12	1.86822	4.58695	0.00109141	2.03097	6.66095	0.00100054
13	1.85702	4.64030	0.00108383	2.00581	6.54095	0.00100054
14	1.84751	4.68623	0.00107742	1.98455	6.44030	0.00100054
15	1.83935	4.72618	0.00107193	1.96635	6.35469	0.00100053
16	1.83226	4.76125	0.00106716	1.95059	6.28098	0.00100053
17	1.82605	4.79228	0.00106300	1.93682	6.21687	0.00100053
18	1.82057	4.81993	0.00105932	1.92468	6.16059	0.00100053
19	1.81569	4.84472	0.00105605	1.91390	6.11079	0.00100053
20	1.81132	4.86707	0.00105313	1.90426	6.06643	0.00100053
25	1.79489	4.95234	0.00104217	1.86818	5.90162	0.00100052
30	1.78410	5.00947	0.00103498	1.84459	5.79499	0.00100052
50	1.76290	5.12441	0.00102091	1.79852	5.58922	0.00100051
75	1.75249	5.18218	0.00101401	1.77601	5.48995	0.00100051
100	1.74733	5.21115	0.00101060	1.76489	5.44119	0.00100051
150	1.74220	5.24016	0.00100721	1.75386	5.39300	0.00100050
200	1.73965	5.25468	0.00100553	1.74837	5.36912	0.00100050
250	1.73812	5.26340	0.00100452	1.74509	5.35486	0.00100050
300	1.73710	5.26921	0.00100384	1.74290	5.34538	0.00100050
∞	1.7320375243	5.2983173665	0.0010005003	1.7320375243	5.2983173665	0.0010005003

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	3					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	17.69484	15.91775	0.03570
2	2.87519	1.59177	0.05046	4.97997	5.45415	0.03365
3	2.40967	1.77629	0.04121	3.40779	3.97519	0.03297
4	2.20599	1.89811	0.03807	2.84792	3.42822	0.03263
5	2.09497	1.97649	0.03648	2.56580	3.14794	0.03243
6	2.02564	2.03008	0.03552	2.39677	2.97847	0.03230
7	1.97838	2.06878	0.03488	2.28444	2.86521	0.03220
8	1.94415	2.09794	0.03442	2.20446	2.78427	0.03213
9	1.91823	2.12067	0.03408	2.14465	2.72359	0.03207
10	1.89794	2.13888	0.03381	2.09825	2.67643	0.03203
11	1.88162	2.15377	0.03359	2.06121	2.63872	0.03199
12	1.86822	2.16619	0.03341	2.03097	2.60790	0.03196
13	1.85702	2.17668	0.03327	2.00581	2.58223	0.03194
14	1.84751	2.18568	0.03314	1.98455	2.56053	0.03191
15	1.83935	2.19347	0.03303	1.96635	2.54194	0.03190
16	1.83226	2.20029	0.03294	1.95059	2.52584	0.03188
17	1.82605	2.20629	0.03286	1.93682	2.51176	0.03186
18	1.82057	2.21163	0.03279	1.92468	2.49935	0.03185
19	1.81569	2.21641	0.03272	1.91390	2.48832	0.03184
20	1.81132	2.22070	0.03267	1.90426	2.47845	0.03183
25	1.79489	2.23700	0.03245	1.86818	2.44150	0.03179
30	1.78410	2.24785	0.03231	1.84459	2.41733	0.03176
50	1.76290	2.26950	0.03203	1.79852	2.37007	0.03171
75	1.75249	2.28029	0.03190	1.77601	2.34697	0.03168
100	1.74733	2.28568	0.03183	1.76489	2.33555	0.03167
150	1.74220	2.29106	0.03176	1.75386	2.32422	0.03166
200	1.73965	2.29375	0.03173	1.74837	2.31859	0.03165
250	1.73812	2.29536	0.03171	1.74509	2.31521	0.03165
300	1.73710	2.29644	0.03170	1.74290	2.31297	0.03164
∞	1.7320375243	2.3018074130	0.0316306866	1.7320375243	2.3018074130	0.0316306866

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	4					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	7.07531	47.46723	0.00709
2	1.80725	1.95874	0.01407	3.13025	12.91660	0.00753
3	1.71844	2.59776	0.01125	2.43023	8.71706	0.00770
4	1.66424	2.97585	0.01024	2.14852	7.22576	0.00779
5	1.63082	3.21838	0.00972	1.99733	6.47604	0.00785
6	1.60849	3.38586	0.00941	1.90319	6.02777	0.00789
7	1.59259	3.50808	0.00919	1.83897	5.73039	0.00792
8	1.58073	3.60108	0.00904	1.79237	5.51900	0.00794
9	1.57154	3.67416	0.00892	1.75703	5.36113	0.00796
10	1.56422	3.73308	0.00883	1.72931	5.23879	0.00797
11	1.55825	3.78158	0.00876	1.70698	5.14124	0.00798
12	1.55330	3.82219	0.00870	1.68862	5.06165	0.00799
13	1.54912	3.85669	0.00865	1.67324	4.99548	0.00800
14	1.54555	3.88635	0.00861	1.66018	4.93962	0.00801
15	1.54246	3.91213	0.00857	1.64896	4.89182	0.00801
16	1.53976	3.93474	0.00854	1.63920	4.85047	0.00802
17	1.53738	3.95473	0.00852	1.63064	4.81434	0.00802
18	1.53527	3.97253	0.00849	1.62307	4.78251	0.00803
19	1.53339	3.98849	0.00847	1.61634	4.75425	0.00803
20	1.53170	4.00286	0.00845	1.61030	4.72899	0.00803
25	1.52528	4.05766	0.00838	1.58757	4.63452	0.00805
30	1.52103	4.09434	0.00833	1.57260	4.57284	0.00806
50	1.51256	4.16804	0.00824	1.54312	4.45252	0.00807
75	1.50836	4.20505	0.00819	1.52860	4.39385	0.00808
100	1.50626	4.22360	0.00817	1.52140	4.36488	0.00809
150	1.50416	4.24217	0.00814	1.51422	4.33614	0.00809
200	1.50312	4.25146	0.00813	1.51065	4.32187	0.00809
250	1.50249	4.25704	0.00813	1.50851	4.31333	0.00809
300	1.50207	4.26076	0.00812	1.50709	4.30765	0.00809
∞	1.4999884964	4.2793854889	0.0080991953	1.4999884964	4.2793854889	0.0080991953

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	4					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	7.07531	7.47804	0.09137
2	1.80725	1.51907	0.12876	3.13025	3.74618	0.09044
3	1.71844	1.68002	0.11055	2.43023	3.03546	0.09022
4	1.66424	1.77356	0.10404	2.14852	2.74460	0.09014
5	1.63082	1.83171	0.10068	1.99733	2.58754	0.09009
6	1.60849	1.87097	0.09862	1.90319	2.48947	0.09007
7	1.59259	1.89917	0.09722	1.83897	2.42248	0.09005
8	1.58073	1.92037	0.09622	1.79237	2.37385	0.09004
9	1.57154	1.93688	0.09546	1.75703	2.33694	0.09004
10	1.56422	1.95009	0.09486	1.72931	2.30799	0.09003
11	1.55825	1.96090	0.09439	1.70698	2.28467	0.09003
12	1.55330	1.96991	0.09399	1.68862	2.26549	0.09002
13	1.54912	1.97753	0.09367	1.67324	2.24943	0.09002
14	1.54555	1.98406	0.09339	1.66018	2.23579	0.09002
15	1.54246	1.98972	0.09315	1.64896	2.22407	0.09001
16	1.53976	1.99467	0.09294	1.63920	2.21388	0.09001
17	1.53738	1.99903	0.09276	1.63064	2.20494	0.09001
18	1.53527	2.00292	0.09260	1.62307	2.19704	0.09001
19	1.53339	2.00639	0.09246	1.61634	2.19001	0.09001
20	1.53170	2.00951	0.09233	1.61030	2.18370	0.09001
25	1.52528	2.02137	0.09185	1.58757	2.15998	0.09001
30	1.52103	2.02927	0.09153	1.57260	2.14437	0.09000
50	1.51256	2.04505	0.09091	1.54312	2.11362	0.09000
75	1.50836	2.05293	0.09060	1.52860	2.09848	0.09000
100	1.50626	2.05687	0.09045	1.52140	2.09097	0.09000
150	1.50416	2.06080	0.09030	1.51422	2.08350	0.09000
200	1.50312	2.06277	0.09022	1.51065	2.07978	0.09000
250	1.50249	2.06395	0.09018	1.50851	2.07755	0.09000
300	1.50207	2.06474	0.09015	1.50709	2.07606	0.09000
∞	1.4999884964	2.0686675636	0.0899955292	1.4999884964	2.0686675636	0.0899955292

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	5					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	4.45422	23.15450	0.01871
2	1.39519	1.91720	0.03674	2.41654	8.80513	0.02041
3	1.40341	2.44471	0.03031	1.98472	6.52114	0.02109
4	1.39422	2.73965	0.02793	1.79993	5.63785	0.02146
5	1.38606	2.92485	0.02668	1.69757	5.17428	0.02169
6	1.37977	3.05139	0.02592	1.63257	4.88978	0.02185
7	1.37493	3.14317	0.02540	1.58764	4.69771	0.02197
8	1.37114	3.21274	0.02503	1.55473	4.55943	0.02205
9	1.36810	3.26726	0.02474	1.52958	4.45517	0.02212
10	1.36562	3.31112	0.02452	1.50975	4.37378	0.02218
11	1.36355	3.34717	0.02434	1.49370	4.30848	0.02222
12	1.36181	3.37733	0.02420	1.48045	4.25494	0.02226
13	1.36033	3.40292	0.02407	1.46932	4.21025	0.02230
14	1.35904	3.42491	0.02397	1.45984	4.17239	0.02232
15	1.35792	3.44400	0.02388	1.45168	4.13989	0.02235
16	1.35694	3.46075	0.02380	1.44457	4.11171	0.02237
17	1.35606	3.47554	0.02374	1.43832	4.08703	0.02239
18	1.35528	3.48871	0.02367	1.43279	4.06524	0.02241
19	1.35458	3.50051	0.02362	1.42785	4.04586	0.02242
20	1.35395	3.51113	0.02357	1.42343	4.02851	0.02244
25	1.35153	3.55162	0.02339	1.40672	3.96338	0.02249
30	1.34990	3.57870	0.02327	1.39568	3.92065	0.02252
50	1.34662	3.63305	0.02304	1.37383	3.83683	0.02259
75	1.34497	3.66033	0.02293	1.36302	3.79572	0.02263
100	1.34414	3.67399	0.02287	1.35765	3.77536	0.02265
150	1.34330	3.68767	0.02281	1.35229	3.75513	0.02266
200	1.34289	3.69451	0.02279	1.34962	3.74507	0.02267
250	1.34264	3.69862	0.02277	1.34802	3.73905	0.02268
300	1.34247	3.70136	0.02276	1.34695	3.73504	0.02268
∞	1.3416304973	3.7150647501	0.0227010089	1.3416304973	3.7150647501	0.0227010089

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	5					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	4.45422	5.11913	0.14553
2	1.39519	1.47303	0.20392	2.41654	3.06129	0.14739
3	1.40341	1.61306	0.17960	1.98472	2.60735	0.14828
4	1.39422	1.68999	0.17062	1.79993	2.41178	0.14880
5	1.38606	1.73713	0.16592	1.69757	2.30330	0.14913
6	1.37977	1.76879	0.16301	1.63257	2.23443	0.14937
7	1.37493	1.79146	0.16104	1.58764	2.18685	0.14954
8	1.37114	1.80848	0.15961	1.55473	2.15203	0.14967
9	1.36810	1.82173	0.15853	1.52958	2.12543	0.14977
10	1.36562	1.83233	0.15768	1.50975	2.10447	0.14986
11	1.36355	1.84100	0.15700	1.49370	2.08751	0.14993
12	1.36181	1.84822	0.15644	1.48045	2.07352	0.14999
13	1.36033	1.85433	0.15597	1.46932	2.06178	0.15004
14	1.35904	1.85957	0.15557	1.45984	2.05178	0.15008
15	1.35792	1.86411	0.15522	1.45168	2.04317	0.15012
16	1.35694	1.86807	0.15493	1.44457	2.03567	0.15015
17	1.35606	1.87158	0.15466	1.43832	2.02909	0.15018
18	1.35528	1.87469	0.15443	1.43279	2.02326	0.15021
19	1.35458	1.87747	0.15423	1.42785	2.01806	0.15023
20	1.35395	1.87998	0.15404	1.42343	2.01340	0.15025
25	1.35153	1.88949	0.15335	1.40672	1.99581	0.15033
30	1.34990	1.89583	0.15289	1.39568	1.98419	0.15039
50	1.34662	1.90849	0.15199	1.37383	1.96123	0.15050
75	1.34497	1.91481	0.15154	1.36302	1.94989	0.15056
100	1.34414	1.91798	0.15132	1.35765	1.94424	0.15058
150	1.34330	1.92114	0.15110	1.35229	1.93862	0.15061
200	1.34289	1.92271	0.15100	1.34962	1.93582	0.15063
250	1.34264	1.92366	0.15093	1.34802	1.93414	0.15063
300	1.34247	1.92429	0.15089	1.34695	1.93303	0.15064
∞	1.3416304973	1.9274503237	0.1506685398	1.3416304973	1.9274503237	0.1506685398

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	6					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	3.34141	14.93961	0.03361
2	1.17112	1.87453	0.06504	2.02845	6.87237	0.03715
3	1.21554	2.32374	0.05471	1.71903	5.37214	0.03859
4	1.22511	2.56667	0.05080	1.58162	4.76157	0.03938
5	1.22802	2.71731	0.04874	1.50401	4.43267	0.03987
6	1.22897	2.81957	0.04747	1.45413	4.22758	0.04021
7	1.22922	2.89346	0.04660	1.41938	4.08760	0.04046
8	1.22920	2.94932	0.04597	1.39378	3.98605	0.04065
9	1.22906	2.99301	0.04550	1.37413	3.90902	0.04080
10	1.22888	3.02813	0.04512	1.35857	3.84860	0.04092
11	1.22868	3.05696	0.04482	1.34595	3.79996	0.04101
12	1.22849	3.08106	0.04458	1.33551	3.75995	0.04110
13	1.22830	3.10149	0.04437	1.32672	3.72647	0.04117
14	1.22813	3.11904	0.04419	1.31923	3.69803	0.04123
15	1.22797	3.13428	0.04404	1.31276	3.67359	0.04128
16	1.22782	3.14763	0.04391	1.30712	3.65236	0.04133
17	1.22768	3.15942	0.04380	1.30216	3.63373	0.04137
18	1.22756	3.16992	0.04370	1.29776	3.61727	0.04141
19	1.22744	3.17931	0.04361	1.29383	3.60261	0.04144
20	1.22733	3.18778	0.04352	1.29031	3.58947	0.04147
25	1.22689	3.22002	0.04322	1.27699	3.54005	0.04158
30	1.22657	3.24156	0.04302	1.26816	3.50753	0.04166
50	1.22589	3.28478	0.04262	1.25066	3.44350	0.04181
75	1.22552	3.30645	0.04243	1.24197	3.41198	0.04189
100	1.22533	3.31731	0.04233	1.23765	3.39634	0.04193
150	1.22514	3.32817	0.04223	1.23333	3.38079	0.04196
200	1.22504	3.33360	0.04219	1.23118	3.37304	0.04198
250	1.22498	3.33686	0.04216	1.22989	3.36840	0.04200
300	1.22494	3.33904	0.04214	1.22903	3.36531	0.04200
∞	1.2247354787	3.3499204687	0.0420425205	1.2247354787	3.3499204687	0.0420425205

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	6					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	3.34141	4.06205	0.19267
2	1.17112	1.43887	0.26801	2.02845	2.68777	0.19762
3	1.21554	1.56291	0.23982	1.71903	2.35671	0.19975
4	1.22511	1.62899	0.22918	1.58162	2.20954	0.20093
5	1.22802	1.66915	0.22355	1.50401	2.12655	0.20169
6	1.22897	1.69603	0.22006	1.45413	2.07331	0.20220
7	1.22922	1.71525	0.21768	1.41938	2.03627	0.20258
8	1.22920	1.72967	0.21595	1.39378	2.00902	0.20288
9	1.22906	1.74088	0.21464	1.37413	1.98814	0.20310
10	1.22888	1.74985	0.21361	1.35857	1.97162	0.20329
11	1.22868	1.75718	0.21278	1.34595	1.95823	0.20344
12	1.22849	1.76329	0.21209	1.33551	1.94715	0.20357
13	1.22830	1.76846	0.21152	1.32672	1.93784	0.20368
14	1.22813	1.77289	0.21103	1.31923	1.92991	0.20377
15	1.22797	1.77672	0.21062	1.31276	1.92306	0.20386
16	1.22782	1.78008	0.21025	1.30712	1.91710	0.20393
17	1.22768	1.78304	0.20993	1.30216	1.91185	0.20399
18	1.22756	1.78567	0.20965	1.29776	1.90720	0.20405
19	1.22744	1.78802	0.20940	1.29383	1.90306	0.20410
20	1.22733	1.79014	0.20917	1.29031	1.89933	0.20415
25	1.22689	1.79818	0.20833	1.27699	1.88527	0.20432
30	1.22657	1.80354	0.20777	1.26816	1.87596	0.20444
50	1.22589	1.81425	0.20666	1.25066	1.85752	0.20468
75	1.22552	1.81959	0.20612	1.24197	1.84839	0.20480
100	1.22533	1.82227	0.20585	1.23765	1.84384	0.20486
150	1.22514	1.82494	0.20558	1.23333	1.83930	0.20492
200	1.22504	1.82627	0.20544	1.23118	1.83704	0.20495
250	1.22498	1.82708	0.20536	1.22989	1.83569	0.20497
300	1.22494	1.82761	0.20531	1.22903	1.83478	0.20498
∞	1.2247354787	1.8302787954	0.2050427285	1.2247354787	1.8302787954	0.2050427285

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	7					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	2.73231	11.07304	0.04993
2	1.02719	1.83434	0.09510	1.77914	5.75703	0.05559
3	1.08754	2.22651	0.08113	1.53801	4.66274	0.05791
4	1.10628	2.43398	0.07575	1.42820	4.20189	0.05918
5	1.11481	2.56154	0.07290	1.36536	3.94921	0.05998
6	1.11954	2.64775	0.07112	1.32466	3.78993	0.06053
7	1.12249	2.70988	0.06991	1.29614	3.68042	0.06093
8	1.12448	2.75676	0.06904	1.27504	3.60053	0.06124
9	1.12591	2.79339	0.06837	1.25880	3.53970	0.06148
10	1.12697	2.82279	0.06785	1.24592	3.49183	0.06167
11	1.12780	2.84692	0.06743	1.23544	3.45319	0.06183
12	1.12845	2.86707	0.06708	1.22676	3.42134	0.06197
13	1.12898	2.88415	0.06679	1.21944	3.39464	0.06208
14	1.12942	2.89881	0.06654	1.21319	3.37194	0.06218
15	1.12979	2.91154	0.06633	1.20780	3.35239	0.06227
16	1.13011	2.92268	0.06615	1.20309	3.33539	0.06235
17	1.13038	2.93253	0.06598	1.19895	3.32046	0.06241
18	1.13061	2.94129	0.06584	1.19527	3.30726	0.06247
19	1.13082	2.94913	0.06571	1.19199	3.29549	0.06253
20	1.13100	2.95620	0.06560	1.18904	3.28494	0.06257
25	1.13166	2.98308	0.06517	1.17787	3.24518	0.06276
30	1.13208	3.00104	0.06489	1.17047	3.21896	0.06288
50	1.13286	3.03705	0.06433	1.15575	3.16721	0.06313
75	1.13322	3.05509	0.06405	1.14843	3.14167	0.06326
100	1.13339	3.06412	0.06392	1.14478	3.12899	0.06332
150	1.13356	3.07316	0.06378	1.14114	3.11636	0.06338
200	1.13364	3.07769	0.06371	1.13933	3.11006	0.06342
250	1.13369	3.08040	0.06367	1.13824	3.10629	0.06343
300	1.13372	3.08221	0.06365	1.13751	3.10378	0.06345
∞	1.1338847231	3.0912640298	0.0635111259	1.1338847231	3.0912640298	0.0635111259

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	7					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	2.73231	3.46855	0.23290
2	1.02719	1.41174	0.32145	1.77914	2.44983	0.24073
3	1.08754	1.52352	0.29082	1.53801	2.18952	0.24401
4	1.10628	1.58193	0.27908	1.42820	2.07131	0.24582
5	1.11481	1.61723	0.27282	1.36536	2.00389	0.24696
6	1.11954	1.64080	0.26892	1.32466	1.96034	0.24774
7	1.12249	1.65764	0.26625	1.29614	1.92989	0.24832
8	1.12448	1.67026	0.26431	1.27504	1.90741	0.24876
9	1.12591	1.68007	0.26284	1.25880	1.89014	0.24910
10	1.12697	1.68791	0.26168	1.24592	1.87645	0.24938
11	1.12780	1.69433	0.26075	1.23544	1.86533	0.24961
12	1.12845	1.69967	0.25998	1.22676	1.85612	0.24980
13	1.12898	1.70418	0.25933	1.21944	1.84837	0.24997
14	1.12942	1.70806	0.25879	1.21319	1.84176	0.25011
15	1.12979	1.71141	0.25831	1.20780	1.83605	0.25023
16	1.13011	1.71434	0.25790	1.20309	1.83107	0.25034
17	1.13038	1.71693	0.25754	1.19895	1.82669	0.25044
18	1.13061	1.71923	0.25723	1.19527	1.82280	0.25052
19	1.13082	1.72128	0.25694	1.19199	1.81933	0.25060
20	1.13100	1.72313	0.25669	1.18904	1.81622	0.25067
25	1.13166	1.73016	0.25573	1.17787	1.80444	0.25093
30	1.13208	1.73484	0.25510	1.17047	1.79664	0.25111
50	1.13286	1.74419	0.25385	1.15575	1.78115	0.25147
75	1.13322	1.74887	0.25323	1.14843	1.77346	0.25165
100	1.13339	1.75120	0.25292	1.14478	1.76963	0.25174
150	1.13356	1.75353	0.25262	1.14114	1.76581	0.25183
200	1.13364	1.75470	0.25247	1.13933	1.76390	0.25188
250	1.13369	1.75540	0.25238	1.13824	1.76276	0.25190
300	1.13372	1.75587	0.25232	1.13751	1.76200	0.25192
∞	1.1338847231	1.7581990871	0.2520141382	1.1338847231	1.7581990871	0.2520141382

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	8					
m	**A41**	**B81**	**B71**	**A42**	**B82**	**B72**
1	-----	-----	-----	2.34703	8.88539	0.06658
2	0.92530	1.79768	0.12486	1.60267	5.03134	0.07444
3	0.99316	2.14668	0.10765	1.40453	4.17893	0.07767
4	1.01678	2.32844	0.10095	1.31265	3.81099	0.07944
5	1.02844	2.43950	0.09737	1.25958	3.60665	0.08055
6	1.03531	2.51432	0.09513	1.22499	3.47681	0.08132
7	1.03980	2.56813	0.09361	1.20066	3.38706	0.08189
8	1.04296	2.60868	0.09250	1.18261	3.32133	0.08231
9	1.04530	2.64032	0.09166	1.16868	3.27113	0.08265
10	1.04710	2.66571	0.09100	1.15762	3.23154	0.08292
11	1.04853	2.68653	0.09047	1.14860	3.19951	0.08315
12	1.04968	2.70391	0.09003	1.14113	3.17308	0.08334
13	1.05064	2.71863	0.08966	1.13482	3.15089	0.08350
14	1.05144	2.73127	0.08935	1.12943	3.13200	0.08364
15	1.05213	2.74223	0.08908	1.12477	3.11572	0.08376
16	1.05272	2.75184	0.08885	1.12071	3.10155	0.08386
17	1.05323	2.76032	0.08864	1.11712	3.08910	0.08396
18	1.05369	2.76786	0.08846	1.11395	3.07808	0.08404
19	1.05409	2.77461	0.08830	1.11111	3.06825	0.08412
20	1.05444	2.78069	0.08815	1.10855	3.05943	0.08418
25	1.05577	2.80383	0.08761	1.09888	3.02617	0.08444
30	1.05663	2.81928	0.08725	1.09246	3.00420	0.08462
50	1.05830	2.85024	0.08654	1.07968	2.96076	0.08497
75	1.05910	2.86574	0.08619	1.07332	2.93929	0.08514
100	1.05950	2.87351	0.08602	1.07014	2.92861	0.08523
150	1.05989	2.88127	0.08584	1.06697	2.91797	0.08532
200	1.06008	2.88516	0.08576	1.06539	2.91267	0.08537
250	1.06019	2.88749	0.08570	1.06444	2.90949	0.08539
300	1.06027	2.88904	0.08567	1.06381	2.90738	0.08541
∞	1.0606520375	2.8968199821	0.0854991075	1.0606520375	2.8968199821	0.0854991075

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	8					
m	**A41**	**B81sqrt**	**B71sqrt**	**A42**	**B82sqrt**	**B72sqrt**
1	-----	-----	-----	2.34703	3.08885	0.26739
2	0.92530	1.38936	0.36615	1.60267	2.28344	0.27774
3	0.99316	1.49153	0.33401	1.40453	2.06872	0.28203
4	1.01678	1.54419	0.32153	1.31265	1.96968	0.28438
5	1.02844	1.57590	0.31483	1.25958	1.91273	0.28586
6	1.03531	1.59703	0.31065	1.22499	1.87575	0.28688
7	1.03980	1.61210	0.30778	1.20066	1.84981	0.28762
8	1.04296	1.62340	0.30570	1.18261	1.83061	0.28819
9	1.04530	1.63218	0.30411	1.16868	1.81582	0.28863
10	1.04710	1.63919	0.30286	1.15762	1.80408	0.28900
11	1.04853	1.64493	0.30185	1.14860	1.79453	0.28929
12	1.04968	1.64970	0.30102	1.14113	1.78662	0.28954
13	1.05064	1.65374	0.30033	1.13482	1.77996	0.28976
14	1.05144	1.65720	0.29973	1.12943	1.77426	0.28994
15	1.05213	1.66020	0.29922	1.12477	1.76935	0.29010
16	1.05272	1.66282	0.29878	1.12071	1.76506	0.29024
17	1.05323	1.66513	0.29839	1.11712	1.76128	0.29036
18	1.05369	1.66719	0.29805	1.11395	1.75793	0.29047
19	1.05409	1.66902	0.29774	1.11111	1.75494	0.29057
20	1.05444	1.67068	0.29746	1.10855	1.75225	0.29066
25	1.05577	1.67696	0.29643	1.09888	1.74208	0.29101
30	1.05663	1.68114	0.29574	1.09246	1.73533	0.29123
50	1.05830	1.68950	0.29439	1.07968	1.72192	0.29170
75	1.05910	1.69367	0.29372	1.07332	1.71525	0.29193
100	1.05950	1.69575	0.29339	1.07014	1.71193	0.29205
150	1.05989	1.69784	0.29306	1.06697	1.70861	0.29217
200	1.06008	1.69888	0.29289	1.06539	1.70696	0.29223
250	1.06019	1.69951	0.29280	1.06444	1.70597	0.29226
300	1.06027	1.69992	0.29273	1.06381	1.70531	0.29228
∞	1.0606520375	1.7020046951	0.2924023042	1.0606520375	1.7020046951	0.2924023042

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	10					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	1.88245	6.54109	0.09894
2	0.78799	1.73479	0.18007	1.36485	4.14098	0.11096
3	0.86077	2.02296	0.15769	1.21731	3.55707	0.11593
4	0.88861	2.16991	0.14882	1.14719	3.29645	0.11864
5	0.90317	2.25894	0.14403	1.10615	3.14915	0.12036
6	0.91209	2.31864	0.14104	1.07920	3.05454	0.12155
7	0.91810	2.36144	0.13899	1.06013	2.98864	0.12241
8	0.92242	2.39363	0.13750	1.04593	2.94013	0.12307
9	0.92568	2.41872	0.13636	1.03494	2.90292	0.12359
10	0.92822	2.43883	0.13547	1.02618	2.87348	0.12402
11	0.93025	2.45530	0.13475	1.01904	2.84960	0.12436
12	0.93192	2.46904	0.13415	1.01311	2.82985	0.12466
13	0.93331	2.48068	0.13365	1.00809	2.81324	0.12490
14	0.93449	2.49066	0.13323	1.00381	2.79908	0.12512
15	0.93550	2.49932	0.13287	1.00010	2.78686	0.12530
16	0.93638	2.50689	0.13255	0.99685	2.77621	0.12547
17	0.93715	2.51358	0.13227	0.99400	2.76685	0.12561
18	0.93783	2.51953	0.13203	0.99146	2.75855	0.12574
19	0.93843	2.52486	0.13181	0.98919	2.75114	0.12586
20	0.93897	2.52965	0.13161	0.98715	2.74449	0.12596
25	0.94099	2.54789	0.13087	0.97941	2.71937	0.12636
30	0.94231	2.56005	0.13038	0.97427	2.70274	0.12663
50	0.94491	2.58442	0.12941	0.96400	2.66978	0.12717
75	0.94618	2.59662	0.12894	0.95888	2.65344	0.12744
100	0.94681	2.60272	0.12870	0.95633	2.64530	0.12758
150	0.94744	2.60882	0.12846	0.95378	2.63719	0.12772
200	0.94775	2.61188	0.12835	0.95250	2.63314	0.12779
250	0.94794	2.61371	0.12828	0.95173	2.63072	0.12783
300	0.94806	2.61493	0.12823	0.95122	2.62910	0.12786
∞	0.9486760225	2.6210389757	0.1279943940	0.9486760225	2.6210389757	0.1279943940

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	10					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	1.88245	2.62945	0.32340
2	0.78799	1.35414	0.43628	1.36485	2.06339	0.33777
3	0.86077	1.44219	0.40266	1.21731	1.90356	0.34364
4	0.88861	1.48676	0.38936	1.14719	1.82826	0.34685
5	0.90317	1.51345	0.38216	1.10615	1.78447	0.34887
6	0.91209	1.53119	0.37765	1.07920	1.75583	0.35025
7	0.91810	1.54383	0.37454	1.06013	1.73564	0.35127
8	0.92242	1.55329	0.37228	1.04593	1.72064	0.35204
9	0.92568	1.56063	0.37055	1.03494	1.70906	0.35265
10	0.92822	1.56650	0.36920	1.02618	1.69985	0.35314
11	0.93025	1.57130	0.36810	1.01904	1.69234	0.35354
12	0.93192	1.57529	0.36719	1.01311	1.68611	0.35388
13	0.93331	1.57867	0.36644	1.00809	1.68086	0.35417
14	0.93449	1.58156	0.36579	1.00381	1.67637	0.35442
15	0.93550	1.58406	0.36523	1.00010	1.67248	0.35464
16	0.93638	1.58625	0.36475	0.99685	1.66909	0.35483
17	0.93715	1.58818	0.36432	0.99400	1.66610	0.35500
18	0.93783	1.58990	0.36395	0.99146	1.66345	0.35515
19	0.93843	1.59143	0.36361	0.98919	1.66108	0.35528
20	0.93897	1.59282	0.36331	0.98715	1.65895	0.35540
25	0.94099	1.59806	0.36218	0.97941	1.65088	0.35587
30	0.94231	1.60155	0.36143	0.97427	1.64552	0.35618
50	0.94491	1.60852	0.35994	0.96400	1.63485	0.35681
75	0.94618	1.61201	0.35921	0.95888	1.62954	0.35712
100	0.94681	1.61375	0.35885	0.95633	1.62689	0.35728
150	0.94744	1.61549	0.35848	0.95378	1.62424	0.35744
200	0.94775	1.61636	0.35830	0.95250	1.62292	0.35752
250	0.94794	1.61688	0.35820	0.95173	1.62213	0.35757
300	0.94806	1.61722	0.35812	0.95122	1.62160	0.35760
∞	0.9486760225	1.6189623145	0.3577630417	0.9486760225	1.6189623145	0.3577630417

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	25					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	0.95593	2.96674	0.26771
2	0.44979	1.49581	0.42235	0.77906	2.39439	0.29660
3	0.50923	1.63462	0.38745	0.72015	2.22167	0.30841
4	0.53486	1.70188	0.37288	0.69051	2.13823	0.31484
5	0.54919	1.74173	0.36484	0.67262	2.08904	0.31890
6	0.55835	1.76812	0.35974	0.66065	2.05660	0.32170
7	0.56471	1.78688	0.35622	0.65207	2.03359	0.32374
8	0.56938	1.80091	0.35363	0.64562	2.01642	0.32530
9	0.57296	1.81180	0.35166	0.64059	2.00312	0.32652
10	0.57579	1.82050	0.35010	0.63656	1.99251	0.32752
11	0.57809	1.82761	0.34884	0.63326	1.98385	0.32833
12	0.57998	1.83353	0.34780	0.63051	1.97665	0.32902
13	0.58158	1.83853	0.34693	0.62817	1.97057	0.32960
14	0.58294	1.84281	0.34618	0.62617	1.96536	0.33011
15	0.58411	1.84652	0.34554	0.62444	1.96085	0.33055
16	0.58513	1.84977	0.34498	0.62292	1.95691	0.33093
17	0.58603	1.85263	0.34449	0.62158	1.95344	0.33127
18	0.58682	1.85517	0.34405	0.62038	1.95036	0.33157
19	0.58753	1.85745	0.34366	0.61931	1.94760	0.33185
20	0.58817	1.85950	0.34332	0.61835	1.94512	0.33209
25	0.59058	1.86727	0.34200	0.61469	1.93571	0.33303
30	0.59217	1.87244	0.34113	0.61225	1.92944	0.33366
50	0.59533	1.88279	0.33941	0.60736	1.91695	0.33493
75	0.59689	1.88795	0.33856	0.60491	1.91071	0.33558
100	0.59767	1.89053	0.33813	0.60368	1.90760	0.33590
150	0.59845	1.89311	0.33771	0.60245	1.90449	0.33622
200	0.59884	1.89440	0.33750	0.60184	1.90293	0.33638
250	0.59907	1.89518	0.33737	0.60147	1.90200	0.33648
300	0.59922	1.89569	0.33729	0.60122	1.90138	0.33655
∞	0.5999953985	1.8982713307	0.3368700659	0.5999953985	1.8982713307	0.3368700659

Table C.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	25					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	0.95593	1.74045	0.52282
2	0.44979	1.23584	0.65669	0.77906	1.55546	0.54746
3	0.50923	1.28520	0.62570	0.72015	1.49571	0.55727
4	0.53486	1.30910	0.61276	0.69051	1.46608	0.56257
5	0.54919	1.32319	0.60559	0.67262	1.44837	0.56589
6	0.55835	1.33248	0.60103	0.66065	1.43658	0.56817
7	0.56471	1.33907	0.59788	0.65207	1.42816	0.56983
8	0.56938	1.34398	0.59556	0.64562	1.42186	0.57109
9	0.57296	1.34779	0.59378	0.64059	1.41696	0.57208
10	0.57579	1.35082	0.59238	0.63656	1.41303	0.57289
11	0.57809	1.35330	0.59124	0.63326	1.40983	0.57355
12	0.57998	1.35536	0.59031	0.63051	1.40716	0.57410
13	0.58158	1.35710	0.58952	0.62817	1.40489	0.57457
14	0.58294	1.35859	0.58884	0.62617	1.40296	0.57498
15	0.58411	1.35988	0.58826	0.62444	1.40128	0.57533
16	0.58513	1.36101	0.58776	0.62292	1.39981	0.57564
17	0.58603	1.36200	0.58731	0.62158	1.39851	0.57591
18	0.58682	1.36288	0.58692	0.62038	1.39736	0.57616
19	0.58753	1.36367	0.58657	0.61931	1.39633	0.57638
20	0.58817	1.36438	0.58625	0.61835	1.39540	0.57658
25	0.59058	1.36707	0.58506	0.61469	1.39188	0.57733
30	0.59217	1.36886	0.58427	0.61225	1.38953	0.57784
50	0.59533	1.37244	0.58271	0.60736	1.38483	0.57886
75	0.59689	1.37422	0.58194	0.60491	1.38248	0.57937
100	0.59767	1.37511	0.58155	0.60368	1.38130	0.57963
150	0.59845	1.37600	0.58117	0.60245	1.38013	0.57989
200	0.59884	1.37645	0.58098	0.60184	1.37954	0.58002
250	0.59907	1.37671	0.58086	0.60147	1.37919	0.58009
300	0.59922	1.37689	0.58079	0.60122	1.37895	0.58015
∞	0.5999953985	1.3777776783	0.5804050877	0.5999953985	1.3777776783	0.5804050877

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	50					
m	A41	B81	B71	A42	B82	B72
1	-----	-----	-----	0.63533	2.11305	0.40576
2	0.30862	1.35754	0.57728	0.53455	1.85121	0.44132
3	0.35299	1.44205	0.54231	0.49921	1.76595	0.45558
4	0.37264	1.48214	0.52736	0.48107	1.72354	0.46331
5	0.38377	1.50566	0.51902	0.47002	1.69813	0.46816
6	0.39095	1.52114	0.51369	0.46257	1.68121	0.47149
7	0.39596	1.53211	0.50999	0.45722	1.66912	0.47392
8	0.39966	1.54029	0.50728	0.45317	1.66005	0.47577
9	0.40250	1.54662	0.50519	0.45001	1.65300	0.47722
10	0.40476	1.55168	0.50355	0.44748	1.64736	0.47840
11	0.40659	1.55580	0.50221	0.44540	1.64274	0.47937
12	0.40811	1.55923	0.50111	0.44366	1.63890	0.48018
13	0.40938	1.56213	0.50018	0.44218	1.63564	0.48087
14	0.41047	1.56460	0.49939	0.44092	1.63285	0.48147
15	0.41141	1.56675	0.49871	0.43982	1.63043	0.48198
16	0.41223	1.56863	0.49811	0.43886	1.62832	0.48244
17	0.41296	1.57028	0.49759	0.43801	1.62645	0.48284
18	0.41360	1.57175	0.49712	0.43725	1.62479	0.48320
19	0.41417	1.57306	0.49671	0.43657	1.62330	0.48352
20	0.41468	1.57424	0.49634	0.43596	1.62197	0.48381
25	0.41662	1.57872	0.49494	0.43364	1.61688	0.48492
30	0.41791	1.58170	0.49401	0.43208	1.61350	0.48567
50	0.42047	1.58765	0.49217	0.42896	1.60672	0.48716
75	0.42174	1.59062	0.49125	0.42740	1.60333	0.48792
100	0.42237	1.59210	0.49080	0.42662	1.60163	0.48830
150	0.42300	1.59359	0.49035	0.42583	1.59994	0.48868
200	0.42332	1.59433	0.49012	0.42544	1.59909	0.48887
250	0.42351	1.59477	0.48999	0.42520	1.59858	0.48899
300	0.42363	1.59507	0.48990	0.42505	1.59824	0.48906
∞	0.4242608150	1.5965450633	0.4894454026	0.4242608150	1.5965450633	0.4894454026

Table C.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaVarUCL=0.005, and alphaVarLCL=0.001

n	50					
m	A41	B81sqrt	B71sqrt	A42	B82sqrt	B72sqrt
1	-----	-----	-----	0.63533	1.46107	0.64025
2	0.30862	1.17110	0.76368	0.53455	1.36407	0.66602
3	0.35299	1.20392	0.73830	0.49921	1.33115	0.67612
4	0.37264	1.21950	0.72743	0.48107	1.31451	0.68154
5	0.38377	1.22862	0.72135	0.47002	1.30445	0.68492
6	0.39095	1.23460	0.71746	0.46257	1.29772	0.68723
7	0.39596	1.23884	0.71475	0.45722	1.29289	0.68892
8	0.39966	1.24199	0.71275	0.45317	1.28925	0.69020
9	0.40250	1.24443	0.71122	0.45001	1.28642	0.69120
10	0.40476	1.24637	0.71001	0.44748	1.28415	0.69202
11	0.40659	1.24795	0.70903	0.44540	1.28229	0.69268
12	0.40811	1.24927	0.70822	0.44366	1.28074	0.69324
13	0.40938	1.25038	0.70753	0.44218	1.27942	0.69372
14	0.41047	1.25133	0.70695	0.44092	1.27830	0.69413
15	0.41141	1.25216	0.70645	0.43982	1.27732	0.69449
16	0.41223	1.25287	0.70601	0.43886	1.27646	0.69480
17	0.41296	1.25351	0.70562	0.43801	1.27571	0.69508
18	0.41360	1.25407	0.70528	0.43725	1.27503	0.69532
19	0.41417	1.25457	0.70498	0.43657	1.27443	0.69554
20	0.41468	1.25502	0.70470	0.43596	1.27389	0.69574
25	0.41662	1.25674	0.70367	0.43364	1.27183	0.69650
30	0.41791	1.25788	0.70298	0.43208	1.27045	0.69702
50	0.42047	1.26015	0.70162	0.42896	1.26769	0.69804
75	0.42174	1.26129	0.70094	0.42740	1.26631	0.69856
100	0.42237	1.26185	0.70061	0.42662	1.26562	0.69882
150	0.42300	1.26242	0.70027	0.42583	1.26493	0.69908
200	0.42332	1.26270	0.70010	0.42544	1.26458	0.69921
250	0.42351	1.26287	0.70000	0.42520	1.26438	0.69929
300	0.42363	1.26298	0.69994	0.42505	1.26424	0.69934
∞	0.4242608150	1.2635446424	0.6996037468	0.4242608150	1.2635446424	0.6996037468

APPENDIX D.1 – Analytical Results for Chapter 6

Show: The distribution of the standard deviation s with ν1 degrees of freedom may be represented as follows:

$$p(s) = \left(\frac{1}{\sigma^{\nu 1}}\right)\cdot\left[e^{\left(\frac{\nu 1}{2}\right)\cdot\ln(\nu 1)-\left(\frac{\nu 1}{2}-1\right)\cdot\ln(2)-\mathrm{gammln}\left(\frac{\nu 1}{2}\right)+(\nu 1-1)\cdot\ln(s)-\frac{\nu 1\cdot s^2}{2\cdot\sigma^2}}\right]$$

From Lord (1950), $p(s) = \dfrac{\nu 1^{\frac{\nu 1}{2}}}{2^{\frac{\nu 1}{2}-1}\cdot\Gamma\left(\frac{\nu 1}{2}\right)\cdot\sigma^{\nu 1}}\cdot s^{\nu 1-1}\cdot e^{\frac{-\nu 1\cdot s^2}{2\cdot\sigma^2}}$

$$\Rightarrow p(s) = e^{\ln\left(\frac{\nu 1^{\frac{\nu 1}{2}}}{2^{\frac{\nu 1}{2}-1}\cdot\Gamma\left(\frac{\nu 1}{2}\right)\cdot\sigma^{\nu 1}}\cdot s^{\nu 1-1}\cdot e^{\frac{-\nu 1\cdot s^2}{2\cdot\sigma^2}}\right)}$$

$$= \left(\frac{1}{\sigma^{\nu 1}}\right)\cdot\left[e^{\left(\frac{\nu 1}{2}\right)\cdot\ln(\nu 1)-\left(\frac{\nu 1}{2}-1\right)\cdot\ln(2)-\ln\left(\Gamma\left(\frac{\nu 1}{2}\right)\right)+(\nu 1-1)\cdot\ln(s)-\frac{\nu 1\cdot s^2}{2\cdot\sigma^2}}\right]$$

$$= \left(\frac{1}{\sigma^{\nu 1}}\right)\cdot\left[e^{\left(\frac{\nu 1}{2}\right)\cdot\ln(\nu 1)-\left(\frac{\nu 1}{2}-1\right)\cdot\ln(2)-\mathrm{gammln}\left(\frac{\nu 1}{2}\right)+(\nu 1-1)\cdot\ln(s)-\frac{\nu 1\cdot s^2}{2\cdot\sigma^2}}\right]$$

Show: The mean of the distribution of the standard deviation s with $\nu 1$ degrees of freedom may be represented as follows:

$$c4 = \sigma \cdot \left(\frac{2}{\nu 1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{\nu 1+1}{2}\right) - \mathrm{gammln}\left(\frac{\nu 1}{2}\right)}\right)$$

From Mead (1966), $E(s) = c4 = \sigma \cdot \left(\frac{2}{\nu 1}\right)^{0.5} \cdot \frac{\Gamma\left(\frac{\nu 1+1}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)}$

$$\Rightarrow c4 = \sigma \cdot \left(\frac{2}{\nu 1}\right)^{0.5} \cdot \left(\frac{e^{\ln\left(\Gamma\left(\frac{\nu 1+1}{2}\right)\right)}}{e^{\ln\left(\Gamma\left(\frac{\nu 1}{2}\right)\right)}}\right)$$

$$= \sigma \cdot \left(\frac{2}{\nu 1}\right)^{0.5} \cdot \left(\frac{e^{\mathrm{gammln}\left(\frac{\nu 1+1}{2}\right)}}{e^{\mathrm{gammln}\left(\frac{\nu 1}{2}\right)}}\right)$$

$$= \sigma \cdot \left(\frac{2}{\nu 1}\right)^{0.5} \cdot \left(e^{\mathrm{gammln}\left(\frac{\nu 1+1}{2}\right) - \mathrm{gammln}\left(\frac{\nu 1}{2}\right)}\right)$$

Show: The standard deviation of the distribution of the standard deviation s with ν1 degrees of freedom may be represented as follows:

$$c5 = \sigma \cdot \left[\left(\frac{2}{\nu 1}\right) \cdot \left[e^{\text{gammln}\left(\frac{\nu 1+2}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right)} - e^{2 \cdot \left(\text{gammln}\left(\frac{\nu 1+1}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right)\right)}\right]\right]^{0.5}$$

From Mead (1966), $\text{var}(s) = c5^2 = \left(\frac{2 \cdot \sigma^2}{\nu 1}\right) \cdot \left[\frac{\Gamma\left(\frac{\nu 1+2}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)} - \left(\frac{\Gamma\left(\frac{\nu 1+1}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)}\right)^2\right]$

$$\Rightarrow c5 = \sigma \cdot \left[\left(\frac{2}{\nu 1}\right) \cdot \left[\left(\frac{e^{\ln\left(\Gamma\left(\frac{\nu 1+2}{2}\right)\right)}}{e^{\ln\left(\Gamma\left(\frac{\nu 1}{2}\right)\right)}}\right) - e^{\ln\left[\left(\frac{\Gamma\left(\frac{\nu 1+1}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right)}\right)^2\right]}\right]\right]^{0.5}$$

$$= \sigma \cdot \left[\left(\frac{2}{\nu 1}\right) \cdot \left[\left(\frac{e^{\text{gammln}\left(\frac{\nu 1+2}{2}\right)}}{e^{\text{gammln}\left(\frac{\nu 1}{2}\right)}}\right) - e^{2 \cdot \left(\ln\left(\Gamma\left(\frac{\nu 1+1}{2}\right)\right) - \ln\left(\Gamma\left(\frac{\nu 1}{2}\right)\right)\right)}\right]\right]^{0.5}$$

$$= \sigma \cdot \left[\left(\frac{2}{\nu 1}\right) \cdot \left[e^{\text{gammln}\left(\frac{\nu 1+2}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right)} - e^{2 \cdot \left(\text{gammln}\left(\frac{\nu 1+1}{2}\right) - \text{gammln}\left(\frac{\nu 1}{2}\right)\right)}\right]\right]^{0.5}$$

Show: The distribution of the studentized standard deviation $t = (s/s')$ with $\nu1$ degrees of freedom for s and $\nu2$ degrees of freedom for s' may be represented as follows:

$$p3(t) = e^{p1(t)-p2(t)}$$

where

$$p1(t) = \ln(2) + \left(\frac{\nu1}{2}\right)\cdot\ln(\nu1) + \left(\frac{\nu2}{2}\right)\cdot\ln(\nu2) + \text{gammln}\left(\frac{\nu1+\nu2}{2}\right) + (\nu1-1)\cdot\ln(t)$$

$$p2(t) = \text{gammln}\left(\frac{\nu1}{2}\right) + \text{gammln}\left(\frac{\nu2}{2}\right) + \left(\frac{\nu1+\nu2}{2}\right)\cdot\ln\left(\nu1\cdot t^2+\nu2\right)$$

From Irwin (1931), $p3(t) = \dfrac{2\cdot\nu1^{\frac{\nu1}{2}}\cdot\nu2^{\frac{\nu2}{2}}\cdot\Gamma\left(\frac{\nu1+\nu2}{2}\right)\cdot t^{\nu1-1}}{\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)\cdot\left(\nu1\cdot t^2+\nu2\right)^{\frac{\nu1+\nu2}{2}}}$

$$\Rightarrow p3(t) = e^{\ln\left[\frac{2\cdot\nu1^{\frac{\nu1}{2}}\cdot\nu2^{\frac{\nu2}{2}}\cdot\Gamma\left(\frac{\nu1+\nu2}{2}\right)\cdot t^{\nu1-1}}{\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)\cdot\left(\nu1\cdot t^2+\nu2\right)^{\frac{\nu1+\nu2}{2}}}\right]}$$

$$= e^{\ln\left(2\cdot\nu1^{\frac{\nu1}{2}}\cdot\nu2^{\frac{\nu2}{2}}\cdot\Gamma\left(\frac{\nu1+\nu2}{2}\right)\cdot t^{\nu1-1}\right)-\ln\left[\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)\cdot\left(\nu1\cdot t^2+\nu2\right)^{\frac{\nu1+\nu2}{2}}\right]}$$

Let $p1(t) = \ln\left(2\cdot\nu1^{\frac{\nu1}{2}}\cdot\nu2^{\frac{\nu2}{2}}\cdot\Gamma\left(\frac{\nu1+\nu2}{2}\right)\cdot t^{\nu1-1}\right)$

$$p2(t) = \ln\left[\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)\cdot\left(\nu1\cdot t^2+\nu2\right)^{\frac{\nu1+\nu2}{2}}\right]$$

(continued on the next page)

(continued from the previous page)

$$\Rightarrow p1(t) = \ln(2) + \left(\frac{\nu1}{2}\right)\cdot\ln(\nu1) + \left(\frac{\nu2}{2}\right)\cdot\ln(\nu2) + \ln\left(\Gamma\left(\frac{\nu1+\nu2}{2}\right)\right) + (\nu1-1)\cdot\ln(t)$$

$$p2(t) = \ln\left(\Gamma\left(\frac{\nu1}{2}\right)\right) + \ln\left(\Gamma\left(\frac{\nu2}{2}\right)\right) + \left(\frac{\nu1+\nu2}{2}\right)\cdot\ln\left(\nu1\cdot t^2+\nu2\right)$$

$$\Rightarrow p1(t) = \ln(2) + \left(\frac{\nu1}{2}\right)\cdot\ln(\nu1) + \left(\frac{\nu2}{2}\right)\cdot\ln(\nu2) + \text{gammln}\left(\frac{\nu1+\nu2}{2}\right) + (\nu1-1)\cdot\ln(t)$$

$$p2(t) = \text{gammln}\left(\frac{\nu1}{2}\right) + \text{gammln}\left(\frac{\nu2}{2}\right) + \left(\frac{\nu1+\nu2}{2}\right)\cdot\ln\left(\nu1\cdot t^2+\nu2\right)$$

$$\Rightarrow p3(t) = e^{p1(t)-p2(t)}$$

Derive: $c4star = \left(c4^2 + \frac{c5^2}{m} \right)^{0.5}$

We first need to determine the mean and variance of the distribution of the mean standard deviation $\bar{s}/\sigma$.

Note: By definition, $E\left(\frac{s}{\sigma}\right) = c4$

$$\Rightarrow \left(\frac{1}{\sigma}\right) \cdot E(s) = c4 \Rightarrow E(s) = c4 \cdot \sigma$$

$$E\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma}\right) \cdot E(\bar{s}) = \left(\frac{1}{\sigma}\right) \cdot E\left(\frac{\sum_{i=1}^{m} s_i}{m}\right) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m}\right) \cdot E\left(\sum_{i=1}^{m} s_i\right)$$

$$\Rightarrow E\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m}\right) \cdot \sum_{i=1}^{m} E(s_i) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m}\right) \cdot \sum_{i=1}^{m} (c4 \cdot \sigma)$$

since $E(s) = c4 \cdot \sigma$.

$$\Rightarrow E\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m}\right) \cdot (m \cdot c4 \cdot \sigma) = c4$$

(continued on the next page)

(continued from the previous page)

Note: By definition, $\mathrm{Var}\left(\frac{s}{\sigma}\right) = c5^2$

$$\Rightarrow \left(\frac{1}{\sigma^2}\right) \cdot \mathrm{Var}(s) = c5^2 \Rightarrow \mathrm{Var}(s) = c5^2 \cdot \sigma^2$$

$$\mathrm{Var}\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma^2}\right) \cdot \mathrm{Var}(\bar{s}) = \left(\frac{1}{\sigma^2}\right) \cdot \mathrm{Var}\left(\frac{\sum_{i=1}^{m} s_i}{m}\right) = \left(\frac{1}{\sigma^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \mathrm{Var}\left(\sum_{i=1}^{m} s_i\right)$$

$$\Rightarrow \mathrm{Var}\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \sum_{i=1}^{m} \mathrm{Var}(s_i)$$

since the s_i's are independent.

$$\Rightarrow \mathrm{Var}\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \sum_{i=1}^{m} \left(c5^2 \cdot \sigma^2\right)$$

since $\mathrm{Var}(s) = c5^2 \cdot \sigma^2$.

$$\Rightarrow \mathrm{Var}\left(\frac{\bar{s}}{\sigma}\right) = \left(\frac{1}{\sigma^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \left(m \cdot c5^2 \cdot \sigma^2\right) = \frac{c5^2}{m}$$

(continued on the next page)

(continued from the previous page)

Derive: $c4star = \left(c4^2 + \frac{c5^2}{m} \right)^{0.5}$

According to Johnson and Welch (1939), the mean of the χ distribution with $\nu 2$ degrees of freedom is calculated using the following equation (with some modifications in notation):

$$E(\chi) = \sqrt{2} \cdot \frac{\Gamma(0.5 \cdot \nu 2 + 0.5)}{\Gamma(0.5 \cdot \nu 2)}$$

$$\Rightarrow E\left(\frac{\chi \cdot c4star}{\sqrt{\nu 2}} \right) = \left(\frac{c4star}{\sqrt{\nu 2}} \right) \cdot E(\chi) = \sqrt{2} \cdot \left(\frac{c4star}{\sqrt{\nu 2}} \right) \cdot \left(\frac{\Gamma(0.5 \cdot \nu 2 + 0.5)}{\Gamma(0.5 \cdot \nu 2)} \right)$$

Equating the squared means of the distribution of the mean standard deviation $\bar{s}/\sigma$ and the $(\chi \cdot c4star)/\sqrt{\nu 2}$ distribution with $\nu 2$ degrees of freedom results in the following:

$$c4^2 = 2 \cdot \left(\frac{c4star^2}{\nu 2} \right) \cdot \left(\frac{\Gamma(0.5 \cdot \nu 2 + 0.5)}{\Gamma(0.5 \cdot \nu 2)} \right)^2$$

$$\Rightarrow c4star^2 = c4^2 \cdot \left(\frac{\nu 2}{2} \right) \cdot \left(\frac{\Gamma(0.5 \cdot \nu 2)}{\Gamma(0.5 \cdot \nu 2 + 0.5)} \right)^2$$

(continued on the next page)

(continued from the previous page)

Using results obtained from Johnson and Welch (1939) (with some modifications in notation), the equation to calculate the variance of the χ distribution with v2 degrees of freedom may be determined as follows:

$$\mathrm{Var}(\chi) = \mathrm{E}(\chi^2) - (\mathrm{E}(\chi))^2 = 2\cdot\frac{\Gamma(0.5\cdot v2+1)}{\Gamma(0.5\cdot v2)} - \left(\sqrt{2}\cdot\frac{\Gamma(0.5\cdot v2+0.5)}{\Gamma(0.5\cdot v2)}\right)^2$$

$$\Rightarrow \mathrm{Var}(\chi) = 2\cdot\frac{(0.5\cdot v2)\cdot\Gamma(0.5\cdot v2)}{\Gamma(0.5\cdot v2)} - 2\cdot\left(\frac{\Gamma(0.5\cdot v2+0.5)}{\Gamma(0.5\cdot v2)}\right)^2 = v2 - 2\cdot\left(\frac{\Gamma(0.5\cdot v2+0.5)}{\Gamma(0.5\cdot v2)}\right)^2$$

$$\Rightarrow \mathrm{Var}\left(\frac{\chi\cdot c4star}{\sqrt{v2}}\right) = \left(\frac{c4star^2}{v2}\right)\cdot\mathrm{Var}(\chi) = \left(\frac{c4star^2}{v2}\right)\cdot\left[v2 - 2\cdot\left(\frac{\Gamma(0.5\cdot v2+0.5)}{\Gamma(0.5\cdot v2)}\right)^2\right]$$

Equating the variances of the distribution of the mean standard deviation $\bar{s}/\sigma$ and the $(\chi\cdot c4star)/\sqrt{v2}$ distribution with v2 degrees of freedom results in the following:

$$\frac{c5^2}{m} = \left(\frac{c4star^2}{v2}\right)\cdot\left[v2 - 2\cdot\left(\frac{\Gamma(0.5\cdot v2+0.5)}{\Gamma(0.5\cdot v2)}\right)^2\right]$$

$$\Rightarrow \left(\frac{\Gamma(0.5\cdot v2+0.5)}{\Gamma(0.5\cdot v2)}\right)^2 = \frac{\dfrac{c5^2\cdot v2}{m\cdot c4star^2} - v2}{-2}$$

$$\Rightarrow \left(\frac{\Gamma(0.5\cdot v2)}{\Gamma(0.5\cdot v2+0.5)}\right)^2 = \frac{2}{v2\cdot\left(1 - \dfrac{c5^2}{m\cdot c4star^2}\right)}$$

(continued on the next page)

(continued from the previous page)

Substituting

$$\left(\frac{\Gamma(0.5\cdot\nu2)}{\Gamma(0.5\cdot\nu2+0.5)}\right)^2=\frac{2}{\nu2\cdot\left(1-\frac{c5^2}{m\cdot c4star^2}\right)}$$

into

$$c4star^2=c4^2\cdot\left(\frac{\nu2}{2}\right)\cdot\left(\frac{\Gamma(0.5\cdot\nu2)}{\Gamma(0.5\cdot\nu2+0.5)}\right)^2$$

gives the following equation:

$$c4star^2=c4^2\cdot\left(\frac{\nu2}{2}\right)\cdot\left[\frac{2}{\nu2\cdot\left(1-\frac{c5^2}{m\cdot c4star^2}\right)}\right]$$

$$\Rightarrow c4star^2=\frac{c4^2}{1-\frac{c5^2}{m\cdot c4star^2}}=\frac{c4star^2\cdot c4^2}{c4star^2-\frac{c5^2}{m}}$$

$$\Rightarrow 1=\frac{c4^2}{c4star^2-\frac{c5^2}{m}}\Rightarrow c4star^2=c4^2+\frac{c5^2}{m}$$

$$\Rightarrow c4star=\left(c4^2+\frac{c5^2}{m}\right)^{0.5}$$

Show: $\bar{s}/c4$ is an unbiased estimate of σ; i.e., show $E(\bar{s}/c4) = \sigma$

$$E\left(\frac{\bar{s}}{c4}\right) = \left(\frac{1}{c4}\right) \cdot E(\bar{s}) = \left(\frac{1}{c4}\right) \cdot E\left(\frac{\sum_{i=1}^{m} s_i}{m}\right) = \left(\frac{1}{c4}\right) \cdot \left(\frac{1}{m}\right) \cdot E\left(\sum_{i=1}^{m} s_i\right)$$

$$\Rightarrow E\left(\frac{\bar{s}}{c4}\right) = \left(\frac{1}{c4}\right) \cdot \left(\frac{1}{m}\right) \cdot \sum_{i=1}^{m} E(s_i) = \left(\frac{1}{c4}\right) \cdot \left(\frac{1}{m}\right) \cdot \sum_{i=1}^{m} (c4 \cdot \sigma)$$

since $E(s) = c4 \cdot \sigma$ (a result shown earlier in this appendix (Appendix D.1)).

$$\Rightarrow E\left(\frac{\bar{s}}{c4}\right) = \left(\frac{1}{c4}\right) \cdot \left(\frac{1}{m}\right) \cdot (m \cdot c4 \cdot \sigma) = \sigma$$

Note: This result may also be obtained as follows. It is shown earlier in this appendix (Appendix D.1) that the following holds:

$$E\left(\frac{\bar{s}}{\sigma}\right) = c4$$

$$\Rightarrow \left(\frac{1}{\sigma}\right) \cdot E(\bar{s}) = c4 \Rightarrow \left(\frac{1}{c4}\right) \cdot E(\bar{s}) = \sigma \Rightarrow E\left(\frac{\bar{s}}{c4}\right) = \sigma$$

Derive: $B42 = (tB4/c4star)$, where tB4 is the (1-alphaStandUCL) percentage point of the distribution of the studentized standard deviation $t = (s/s')$ with $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s' (alphaStandUCL is the probability of a Type I error on the s chart above the upper control limit).

Notes: The ensuing derivation is based on the derivation of D_4^* in the appendix of Hillier (1969). The value s denotes the standard deviation of a subgroup drawn while in the second stage of the two stage procedure.

We need to determine the value B42 such that the following holds:

$$P(s \le B42 \cdot \bar{s}) = 1 - \text{alphaStandUCL}$$

$$\Rightarrow P\left(\frac{s}{\bar{s}} \le B42\right) = 1 - \text{alphaStandUCL}$$

We know s/σ is the statistic for the distribution of the standard deviation s with $\nu 1$ degrees of freedom. We now need an independent estimate of σ, denoted by s', based on $\bar{s}$. Replacing σ with this independent estimate results in the statistic for the distribution of the studentized standard deviation $t = (s/s')$, which has $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s'. The equation to calculate $\nu 2$ is based on the fact that we have applied the Patnaik (1950) approximation to the distribution of the mean standard deviation. If we were to use $\bar{s}/c4$ (which is an unbiased estimate of σ, a result

(continued on the next page)

(continued from the previous page)

shown earlier in this appendix (Appendix D.1)) as this independent estimate, then we would not have the appropriate equation for ν2. As a result, we need to use $\bar{s}/\text{c4star}$.

$$\Rightarrow \frac{s}{\sigma} = \frac{s}{\left(\dfrac{\bar{s}}{\text{c4star}}\right)} = \frac{s \cdot \text{c4star}}{\bar{s}}$$

where $(s \cdot \text{c4star})/\bar{s}$ is the statistic for the distribution of the studentized standard deviation $t = (s/s')$ with ν1 degrees of freedom for s and ν2 degrees of freedom for s'.

$$\Rightarrow 1 - \text{alphaStandUCL} = P\left(\frac{s \cdot \text{c4star}}{\bar{s}} \leq \text{tB4}\right) = P\left(\frac{s}{\bar{s}} \leq \frac{\text{tB4}}{\text{c4star}}\right)$$

where tB4 is defined above.

Setting $\text{B42} = \dfrac{\text{tB4}}{\text{c4star}} \Rightarrow 1 - \text{alphaStandUCL} = P\left(\frac{s}{\bar{s}} \leq \text{B42}\right) = P\left(s \leq \text{B42} \cdot \bar{s}\right)$

Show: $p3(t) = f\left(t^2\right)\cdot 2\cdot t$, where p3(t) is the distribution of the studentized standard deviation $t = (s/s')$ with $\nu 1$ degrees of freedom for s and $\nu 2$ degrees of freedom for s' and f is the F distribution with $\nu 1$ numerator degrees of freedom and $\nu 2$ denominator degrees of freedom.

Bain and Engelhardt (1992) give the F distribution as follows:

$$f(x) = \frac{\Gamma\left(\frac{\nu1+\nu2}{2}\right)}{\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)}\cdot\left(\frac{\nu1}{\nu2}\right)^{\frac{\nu1}{2}}\cdot x^{\frac{\nu1}{2}-1}\cdot\left(1+\frac{\nu1}{\nu2}\cdot x\right)^{-\frac{\nu1+\nu2}{2}}$$

Let $x = t^2$

$\Rightarrow dx = 2\cdot t\,dt \Rightarrow f(x)\,dx = f(t^2)\cdot 2\cdot t\,dt$

$$\Rightarrow f\left(t^2\right)\cdot 2\cdot t\,dt = \frac{\Gamma\left(\frac{\nu1+\nu2}{2}\right)}{\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)}\cdot\left(\frac{\nu1}{\nu2}\right)^{\frac{\nu1}{2}}\cdot\left(t^2\right)^{\frac{\nu1}{2}-1}\cdot\left(1+\frac{\nu1}{\nu2}\cdot t^2\right)^{-\frac{\nu1+\nu2}{2}}\cdot 2\cdot t\,dt$$

$$= \frac{2\cdot \nu1^{\frac{\nu1}{2}}\cdot \nu2^{\frac{-\nu1}{2}}\cdot\Gamma\left(\frac{\nu1+\nu2}{2}\right)}{\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)}\cdot t^{\nu1-1}\cdot\left[\left(\frac{1}{\nu2}\right)\cdot\left(\nu2+\nu1\cdot t^2\right)\right]^{-\frac{\nu1+\nu2}{2}}\,dt$$

$$= \frac{2\cdot \nu1^{\frac{\nu1}{2}}\cdot \nu2^{\frac{-\nu1}{2}}\cdot \nu2^{\left(\frac{\nu1+\nu2}{2}\right)}\cdot\Gamma\left(\frac{\nu1+\nu2}{2}\right)}{\Gamma\left(\frac{\nu1}{2}\right)\cdot\Gamma\left(\frac{\nu2}{2}\right)}\cdot\frac{t^{\nu1-1}}{\left(\nu2+\nu1\cdot t^2\right)^{\frac{\nu1+\nu2}{2}}}\,dt$$

(continued on the next page)

(continued from the previous page)

$$= \frac{2 \cdot \nu 1^{\frac{\nu 1}{2}} \cdot \nu 2^{\frac{\nu 2}{2}} \cdot \Gamma\left(\frac{\nu 1 + \nu 2}{2}\right)}{\Gamma\left(\frac{\nu 1}{2}\right) \cdot \Gamma\left(\frac{\nu 2}{2}\right)} \cdot \frac{t^{\nu 1 - 1}}{\left(\nu 1 \cdot t^2 + \nu 2\right)^{\frac{\nu 1 + \nu 2}{2}}} dt$$

$$= p3(t)\, dt$$

$$\Rightarrow p3(t) = f\left(t^2\right) \cdot 2 \cdot t$$

<u>Show:</u> $p(s) = c\left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right) \cdot \frac{2 \cdot \nu 1 \cdot s}{\sigma^2}$, where p(s) is the distribution of the standard deviation s

with ν1 degrees of freedom and c is the χ^2 distribution with ν1 degrees of freedom.

Bain and Engelhardt (1992) give the χ^2 distribution as follows:

$$c(x) = \frac{1}{2^{\frac{\nu 1}{2}} \cdot \Gamma\left(\frac{\nu 1}{2}\right)} \cdot x^{\frac{\nu 1}{2}-1} \cdot e^{\frac{-x}{2}}$$

Let $x = \frac{\nu 1 \cdot s^2}{\sigma^2}$

$$\Rightarrow dx = \frac{2 \cdot \nu 1 \cdot s}{\sigma^2} ds \Rightarrow c(x)\, dx = c\left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right) \cdot \frac{2 \cdot \nu 1 \cdot s}{\sigma^2} ds$$

$$\Rightarrow c\left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right) \cdot \frac{2 \cdot \nu 1 \cdot s}{\sigma^2} ds = \frac{1}{2^{\frac{\nu 1}{2}} \cdot \Gamma\left(\frac{\nu 1}{2}\right)} \cdot \left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right)^{\frac{\nu 1}{2}-1} \cdot e^{\frac{-\left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right)}{2}} \cdot \frac{2 \cdot \nu 1 \cdot s}{\sigma^2} ds$$

$$= \frac{\nu 1^{\frac{\nu 1}{2}-1} \cdot \nu 1}{2^{\frac{\nu 1}{2}} \cdot 2^{-1} \cdot \Gamma\left(\frac{\nu 1}{2}\right) \cdot \left(\sigma^2\right)^{\frac{\nu 1}{2}-1} \cdot \sigma^2} \cdot \left(s^2\right)^{\frac{\nu 1}{2}-1} \cdot s \cdot e^{\frac{-\nu 1 \cdot s^2}{2 \cdot \sigma^2}} ds$$

$$= \frac{\nu 1^{\frac{\nu 1}{2}}}{2^{\frac{\nu 1}{2}-1} \cdot \Gamma\left(\frac{\nu 1}{2}\right) \cdot \sigma^{\nu 1}} \cdot s^{\nu 1 - 1} \cdot e^{\frac{-\nu 1 \cdot s^2}{2 \cdot \sigma^2}} ds$$

$$= p(s)\, ds$$

$$\Rightarrow p(s) = c\left(\frac{\nu 1 \cdot s^2}{\sigma^2}\right) \cdot \frac{2 \cdot \nu 1 \cdot s}{\sigma^2}$$

Show: $\left(\bar{s}/c_4^*\right)^2$ is an unbiased estimate of σ^2; i.e., show $E\left[\left(\bar{s}/c_4^*\right)^2\right] = \sigma^2$

$$E\left[\left(\frac{\bar{s}}{c_4^*}\right)^2\right] = \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot E\left[\left(\bar{s}\right)^2\right] = \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot E\left[\left(\frac{\sum_{i=1}^{m} s_i}{m}\right)^2\right]$$

$$\Rightarrow E\left[\left(\frac{\bar{s}}{c_4^*}\right)^2\right] = \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot E\left[\left(\sum_{i=1}^{m} s_i\right)^2\right]$$

$$= \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \left[Var\left(\sum_{i=1}^{m} s_i\right) + \left[E\left(\sum_{i=1}^{m} s_i\right)\right]^2\right]$$

$$= \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \left[\sum_{i=1}^{m} Var(s_i) + \left(\sum_{i=1}^{m} E(s_i)\right)^2\right]$$

since the s_i's are independent.

$$\Rightarrow E\left[\left(\frac{\bar{s}}{c_4^*}\right)^2\right] = \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \left[\sum_{i=1}^{m} \left(c_5^2 \cdot \sigma^2\right) + \left[\sum_{i=1}^{m} \left(c_4 \cdot \sigma\right)\right]^2\right]$$

since $Var(s) = c_5^2 \cdot \sigma^2$ and $E(s) = c_4 \cdot \sigma$

(results shown earlier in this appendix (Appendix D.1)).

$$\Rightarrow E\left[\left(\frac{\bar{s}}{c_4^*}\right)^2\right] = \left(\frac{1}{\left(c_4^*\right)^2}\right) \cdot \left(\frac{1}{m^2}\right) \cdot \left[m \cdot c_5^2 \cdot \sigma^2 + \left(m \cdot c_4 \cdot \sigma\right)^2\right]$$

(continued on the next page)

(continued from the previous page)

$$\Rightarrow E\left[\left(\frac{\bar{s}}{c_4^*}\right)^2\right] = \left(\frac{1}{(c_4^*)^2}\right)\cdot\left(\frac{1}{m^2}\right)\cdot\left(m\cdot c_5^2\cdot\sigma^2 + m^2\cdot c_4^2\cdot\sigma^2\right)$$

$$= \left(\frac{c_5^2\cdot\sigma^2}{m\cdot(c_4^*)^2}\right) + \left(\frac{c_4^2\cdot\sigma^2}{(c_4^*)^2}\right)$$

$$= \sigma^2\cdot\left(\frac{c_4^2 + \frac{c_5^2}{m}}{(c_4^*)^2}\right)$$

$$= \sigma^2\cdot\left(\frac{(c_4^*)^2}{(c_4^*)^2}\right)$$

since $c_4^* = \left(c_4^2 + \frac{c_5^2}{m}\right)^{0.5}$ (a result shown earlier in this appendix (Appendix D.1)).

$$\Rightarrow E\left[\left(\frac{\bar{s}}{c_4^*}\right)^2\right] = \sigma^2\cdot(1) = \sigma^2$$

APPENDIX D.2 – Computer Program ccfss.mcd for Chapter 6

Page 1 of program: ccfss.mcd

ENTER the following 5 values:

(1) alphaMean := 0.0027 — **alphaMean** - alpha for the $\overline{X}$ chart.

(2) alphaStandUCL := 0.005 — **alphaStandUCL** - alpha for the s chart above the UCL.

(3) alphaStandLCL := 0.001 — **alphaStandLCL** - alpha for the s chart below the LCL *.

(4) m := 5 — **m** - number of subgroups.

(5) n := 5 — **n** - subgroup size for the $(\overline{X}, s)$ charts.

* Note - If no LCL is desired, leave alphaStandLCL blank (do not enter zero).

Please PAGE DOWN to begin the program.

(1.1) $TOL := 10^{-12}$ $\sigma := 1.0$ $v1 := n - 1$

$$p(s) := \left(\frac{1}{\sigma^{v1}}\right)\cdot\left[e^{\left(\frac{v1}{2}\right)\cdot\ln(v1)-\left(\frac{v1}{2}-1\right)\cdot\ln(2)-\text{gammln}\left(\frac{v1}{2}\right)+(v1-1)\cdot\ln(s)-\frac{v1\cdot s^2}{2\cdot\sigma^2}}\right]$$

$$c4 := \sigma\cdot\left(\frac{2}{v1}\right)^{0.5}\cdot\left(e^{\text{gammln}\left(\frac{v1+1}{2}\right)-\text{gammln}\left(\frac{v1}{2}\right)}\right)$$

$$c5 := \sigma\cdot\left[\left(\frac{2}{v1}\right)\cdot\left[e^{\text{gammln}\left(\frac{v1+2}{2}\right)-\text{gammln}\left(\frac{v1}{2}\right)} - e^{2\cdot\left(\text{gammln}\left(\frac{v1+1}{2}\right)-\text{gammln}\left(\frac{v1}{2}\right)\right)}\right]\right]^{0.5}$$

Page 2 of program: ccfss.mcd

(2.1) $P(S) := \int_0^S p(s)\,ds$

$DUCL(S) := P(S) - (1 - alphaStandUCL)$

$DLCL(S) := P(S) - alphaStandLCL$

```
Sseed1(start) := | S0 ← start
                 | S1 ← start + 0.01
                 | A0 ← DUCL(S0)
                 | A1 ← DUCL(S1)
                 | while A0·A1 > 0
                 |    | S0 ← S1
                 |    | S1 ← S1 + 0.01
                 |    | A0 ← A1
                 |    | A1 ← DUCL(S1)
                 | S
```

```
Sseed2(start) := | S0 ← start
                 | S1 ← start + 0.01
                 | A0 ← DLCL(S0)
                 | A1 ← DLCL(S1)
                 | while A0·A1 > 0
                 |    | S0 ← S1
                 |    | S1 ← S1 + 0.01
                 |    | A0 ← A1
                 |    | A1 ← DLCL(S1)
                 | S
```

$seedB4 := Sseed1(0.01)$

$sB4 := zbrent(DUCL, seedB4_0, seedB4_1, TOL)$

$seedB3 := Sseed2(0.001)$

$sB3 := zbrent(DLCL, seedB3_0, seedB3_1, TOL)$

(2.2) $h(x) := \dfrac{x \cdot e^{2\cdot(gammln(0.5\cdot x) - gammln(0.5\cdot x + 0.5))} - 2}{2}$

$r := \dfrac{c5^2}{m \cdot c4^2}$

$rprevm := \dfrac{c5^2}{(m-1)\cdot c4^2}$

$v(A) := A^{-1} + \left(\frac{1}{4}\right) - \left(\frac{3}{16}\right)\cdot A + \left(\frac{3}{64}\right)\cdot A^2 + \left(\frac{33}{256}\right)\cdot A^3 - \left(\frac{1255}{4096}\right)\cdot A^4$

$d(x) := h(x) - r$

$v2 := zbrent\left[d, v\left[\left(\frac{2}{m}\right)\cdot\left(\frac{c5}{c4}\right)^2\right] - 0.5, v\left[\left(\frac{2}{m}\right)\cdot\left(\frac{c5}{c4}\right)^2\right] + 0.5, TOL\right]$

$dprevm(x) := h(x) - rprevm$

$v2prevm := zbrent\left[dprevm, v\left[\left(\frac{2}{m-1}\right)\cdot\left(\frac{c5}{c4}\right)^2\right] - 0.5, v\left[\left(\frac{2}{m-1}\right)\cdot\left(\frac{c5}{c4}\right)^2\right] + 0.5, TOL\right]$

(3.1) $p1(t) := \ln(2) + \left(\frac{v1}{2}\right)\cdot\ln(v1) + \left(\frac{v2}{2}\right)\cdot\ln(v2) + \mathrm{gammln}\left(\frac{v1 + v2}{2}\right) + (v1 - 1)\cdot\ln(t)$

$p2(t) := \mathrm{gammln}\left(\frac{v1}{2}\right) + \mathrm{gammln}\left(\frac{v2}{2}\right) + \left(\frac{v1 + v2}{2}\right)\cdot\ln\left(v1\cdot t^2 + v2\right)$

$p3(t) := e^{p1(t)-p2(t)}$

$P3(T) := \int_0^T p3(t)\, dt$

(3.2) Tseed1(start) :=

```
T0 ← start
T1 ← start + 0.1
A0 ← P3(T0)
A1 ← P3(T1)
while A1 < (1 − alphaStandUCL)
    T0 ← T1
    T1 ← T1 + 0.1
    A0 ← A1
    A1 ← P3(T1)
Tguess ← linterp(A, T, 1 − alphaStandUCL)
Tguess
```

seed1 := Tseed1(0.1)

D1(x) := P3(x) − (1 − alphaStandUCL)

tB4 := zbrent(D1, seed1 − 0.1, seed1 + 0.1, TOL)

▪ := root[|P3(seed1) − (1 − alphaStandUCL)| , seed1]

Page 4 of program: ccfss.mcd

(4.1) Tseed2(start) :=

```
T0 ← start
T1 ← start + 0.001
A0 ← P3(T0)
A1 ← P3(T1)
while A1 < alphaStandLCL
    T0 ← T1
    T1 ← T1 + 0.001
    A0 ← A1
    A1 ← P3(T1)
Tguess ← linterp(A, T, alphaStandLCL)
Tguess
```

seed2 := Tseed2(0.00001)

D2(x) := P3(x) − alphaStandLCL

tB3 := zbrent(D2, seed2 − 0.001, seed2 + 0.001, TOL)

▮ := root(|P3(seed2) − alphaStandLCL| , seed2)

(5.1) $p1prevm(t) := \ln(2) + \left(\frac{\nu1}{2}\right)\cdot\ln(\nu1) + \left(\frac{\nu2prevm}{2}\right)\cdot\ln(\nu2prevm) + gammln\left(\frac{\nu1 + \nu2prevm}{2}\right) + (\nu1 - 1)\cdot\ln(t)$

$p2prevm(t) := gammln\left(\frac{\nu1}{2}\right) + gammln\left(\frac{\nu2prevm}{2}\right) + \left(\frac{\nu1 + \nu2prevm}{2}\right)\cdot\ln\left(\nu1\cdot t^2 + \nu2prevm\right)$

$p3prevm(t) := e^{p1prevm(t) - p2prevm(t)}$

$P3prevm(T) := \int_0^T p3prevm(t)\, dt$

(5.2) Tseed3(start) :=

```
T0 ← start
T1 ← start + 0.1
A0 ← P3prevm(T0)
A1 ← P3prevm(T1)
while  A1 < (1 − alphaStandUCL)
    T0 ← T1
    T1 ← T1 + 0.1
    A0 ← A1
    A1 ← P3prevm(T1)
Tguess ← linterp(A,T,1 − alphaStandUCL)
Tguess
```

seed3 := Tseed3(0.1)

D1prevm(x) := P3prevm(x) − (1 − alphaStandUCL)

tB4prevm := zbrent(D1prevm, seed3 − 0.1, seed3 + 0.1, TOL)

▮ := root[|P3prevm(seed3) − (1 − alphaStandUCL)| , seed3]

Page 6 of program: ccfss.mcd

(6.1) Tseed4(start) :=

```
T_0 ← start
T_1 ← start + 0.001
A_0 ← P3prevm(T_0)
A_1 ← P3prevm(T_1)
while A_1 < alphaStandLCL
    T_0 ← T_1
    T_1 ← T_1 + 0.001
    A_0 ← A_1
    A_1 ← P3prevm(T_1)
Tguess ← linterp(A, T, alphaStandLCL)
Tguess
```

seed4 := Tseed4(0.00001)

D2prevm(x) := P3prevm(x) − alphaStandLCL

tB3prevm := zbrent(D2prevm, seed4 − 0.001, seed4 + 0.001, TOL)

■ := root(|P3prevm(seed4) − alphaStandLCL| , seed4)

Page 7 of program: ccfss.mcd

(7.1) $c4star := \left(c4^2 + \frac{c5^2}{m}\right)^{0.5}$ $adj\_alpha := 1 - \frac{alphaMean}{2}$

$c4starprevm := \left(c4^2 + \frac{c5^2}{m-1}\right)^{0.5}$ $crit\_t := qt(adj\_alpha, v2)$ $crit\_z := qnorm(adj\_alpha, 0, 1)$

(7.2) $A31 := \left(\frac{crit\_t}{c4star}\right)\cdot\left(\frac{m-1}{n\cdot m}\right)^{0.5}$ $A32 := \left(\frac{crit\_t}{c4star}\right)\cdot\left(\frac{m+1}{n\cdot m}\right)^{0.5}$ $A3 := \frac{crit\_z}{c4\cdot n^{0.5}}$

$B41 := \frac{m\cdot tB4prevm}{c4starprevm\cdot(m-1) + tB4prevm}$ $B42 := \frac{tB4}{c4star}$ $B4 := \frac{sB4}{c4}$

$B31 := \frac{m\cdot tB3prevm}{c4starprevm\cdot(m-1) + tB3prevm}$ $B32 := \frac{tB3}{c4star}$ $B3 := \frac{sB3}{c4}$

FINAL RESULTS:

(1) alphaMean = 0.0027
(2) alphaStandUCL = 0.005
(3) alphaStandLCL = 0.001
(4) m = 5
(5) n = 5

Control Chart Factors

First Stage	Second Stage	Conventional
A31 = 1.44561	A32 = 1.77051	A3 = 1.4272883468
B41 = 1.92584	B42 = 2.40542	B4 = 2.0505104733
B31 = 0.18442	B32 = 0.15452	B3 = 0.1602881356

Mean, Stand. Dev., and Variance of the Dist. of the Stand. Dev.	v1 = 4 / v2 = 19.2129357766	(1 - alphaStandUCL) and alphaStandLCL Percentage Points of the Distributions of the Studentized Stand. Dev. and the Stand. Dev.	
c4 = 0.939985603	c4star = 0.95229	tB4 = 2.29066	tB3 = 0.14715
c5 = 0.3412141061	v2prevm = 15.41602	tB4prevm = 2.39394	tB3prevm = 0.14635
$c5^2$ = 0.1164270662	c4starprevm = 0.95534	sB4 = 1.9274503237	sB3 = 0.1506685398

APPENDIX D.3 – Tables Generated from ccfss.mcd

Table D.3.1. ν2 (Degrees of Freedom) and c_4^* (c4star) Values

n	2		3		4		5		6	
m	ν2	c_4^*	ν2	c_4^*	ν2	c_4^*	ν2	c_4^*	ν2	c_4^*
1	1.00000	1.00000	2.00000	1.00000	3.00000	1.00000	4.00000	1.00000	5.00000	1.00000
2	1.91952	0.90460	3.86384	0.94483	5.83358	0.96146	7.81543	0.97046	9.80353	0.97607
3	2.81729	0.87049	5.70771	0.92571	8.65095	0.94827	11.61757	0.96041	14.59593	0.96796
4	3.70617	0.85292	7.54512	0.91600	11.46358	0.94160	15.41602	0.95534	19.38531	0.96388
5	4.59060	0.84220	9.37970	0.91012	14.27420	0.93758	19.21294	0.95229	24.17345	0.96142
6	5.47253	0.83497	11.21278	0.90618	17.08379	0.93489	23.00907	0.95025	28.96096	0.95978
7	6.35291	0.82978	13.04498	0.90336	19.89278	0.93296	26.80475	0.94879	33.74812	0.95861
8	7.23227	0.82586	14.87662	0.90123	22.70140	0.93152	30.60015	0.94770	38.53505	0.95773
9	8.11092	0.82280	16.70788	0.89958	25.50976	0.93039	34.39536	0.94684	43.32182	0.95704
10	8.98907	0.82034	18.53888	0.89825	28.31794	0.92949	38.19043	0.94616	48.10849	0.95649
11	9.86684	0.81832	20.36967	0.89717	31.12599	0.92875	41.98541	0.94560	52.89508	0.95604
12	10.74432	0.81664	22.20032	0.89626	33.93394	0.92813	45.78031	0.94513	57.68161	0.95567
13	11.62158	0.81521	24.03086	0.89549	36.74182	0.92761	49.57516	0.94474	62.46810	0.95535
14	12.49866	0.81399	25.86131	0.89483	39.54963	0.92716	53.36996	0.94440	67.25455	0.95508
15	13.37559	0.81292	27.69168	0.89426	42.35740	0.92677	57.16473	0.94411	72.04098	0.95484
16	14.25241	0.81199	29.52199	0.89376	45.16513	0.92643	60.95947	0.94385	76.82738	0.95463
17	15.12913	0.81117	31.35226	0.89332	47.97283	0.92613	64.75418	0.94362	81.61376	0.95445
18	16.00577	0.81044	33.18249	0.89293	50.78050	0.92586	68.54888	0.94342	86.40012	0.95429
19	16.88234	0.80978	35.01268	0.89258	53.58815	0.92563	72.34356	0.94324	91.18648	0.95415
20	17.75886	0.80919	36.84284	0.89226	56.39578	0.92541	76.13822	0.94308	95.97282	0.95401
25	22.14078	0.80694	45.99333	0.89106	70.43371	0.92459	95.11138	0.94246	119.9044	0.95352
30	26.52202	0.80544	55.14349	0.89025	84.47143	0.92405	114.0844	0.94205	143.8359	0.95319
50	44.04420	0.80243	91.74277	0.88865	140.6214	0.92296	189.9757	0.94122	239.5611	0.95253
75	65.94485	0.80092	137.4909	0.88784	210.8082	0.92241	284.8394	0.94081	359.2174	0.95220
100	87.84479	0.80016	183.2386	0.88744	280.9948	0.92214	379.7029	0.94060	478.8735	0.95203
150	131.6440	0.79940	274.7337	0.88703	421.3678	0.92186	569.4298	0.94040	718.1855	0.95186
200	175.4428	0.79902	366.2287	0.88683	561.7407	0.92173	759.1566	0.94030	957.4975	0.95178
250	219.2414	0.79879	457.7236	0.88671	702.1135	0.92165	948.8833	0.94023	1196.809	0.95173
300	263.0400	0.79864	549.2185	0.88663	842.4863	0.92159	1138.610	0.94019	1436.121	0.95170
c_4	0.7978845608		0.8862269255		0.9213177319		0.9399856030		0.9515328619	
c_5	0.6028102750		0.4632513752		0.3888105411		0.3412141061		0.3075470901	
c_5^2 (Var.)	0.3633802276		0.2146018366		0.1511736368		0.1164270662		0.0945852126	

Table D.3.1 continued. ν2 (Degrees of Freedom) and c_4^* (c4star) Values

n	7		8		10		25		50	
m	ν2	c_4^*	ν2	c_4^*	ν2	c_4^*	ν2	c_4^*	ν2	c_4^*
1	6.00000	1.00000	7.00000	1.00000	9.00000	1.00000	24.00000	1.00000	49.00000	1.00000
2	11.79520	0.97990	13.78907	0.98267	17.78069	0.98642	47.76168	0.99483	97.75573	0.99746
3	17.58086	0.97310	20.56981	0.97683	26.55475	0.98186	71.52078	0.99311	146.5102	0.99661
4	23.36398	0.96969	27.34836	0.97389	35.32710	0.97957	95.27923	0.99224	195.2643	0.99619
5	29.14606	0.96763	34.12602	0.97213	44.09875	0.97819	119.0374	0.99172	244.0184	0.99593
6	34.92762	0.96626	40.90323	0.97095	52.87006	0.97727	142.7955	0.99137	292.7723	0.99576
7	40.70888	0.96528	47.68019	0.97010	61.64117	0.97661	166.5535	0.99113	341.5262	0.99564
8	46.48995	0.96454	54.45698	0.96947	70.41214	0.97612	190.3114	0.99094	390.2801	0.99555
9	52.27089	0.96397	61.23366	0.96898	79.18304	0.97573	214.0693	0.99080	439.0340	0.99548
10	58.05175	0.96351	68.01027	0.96858	87.95388	0.97543	237.8272	0.99068	487.7879	0.99542
11	63.83254	0.96313	74.78682	0.96826	96.72467	0.97518	261.5851	0.99059	536.5417	0.99537
12	69.61328	0.96282	81.56333	0.96799	105.4954	0.97497	285.3429	0.99051	585.2956	0.99534
13	75.39398	0.96256	88.33981	0.96777	114.2662	0.97479	309.1008	0.99044	634.0494	0.99530
14	81.17466	0.96233	95.11627	0.96757	123.0369	0.97464	332.8586	0.99038	682.8033	0.99528
15	86.95531	0.96213	101.8927	0.96740	131.8076	0.97451	356.6165	0.99033	731.5571	0.99525
16	92.73594	0.96196	108.6691	0.96725	140.5783	0.97439	380.3743	0.99029	780.3110	0.99523
17	98.51655	0.96181	115.4455	0.96712	149.3490	0.97429	404.1321	0.99025	829.0648	0.99521
18	104.2972	0.96167	122.2219	0.96701	158.1196	0.97420	427.8900	0.99022	877.8186	0.99519
19	110.0777	0.96155	128.9983	0.96690	166.8903	0.97412	451.6478	0.99019	926.5725	0.99518
20	115.8583	0.96144	135.7747	0.96681	175.6610	0.97404	475.4056	0.99016	975.3263	0.99517
25	144.7611	0.96103	169.6565	0.96645	219.5142	0.97377	594.1947	0.99006	1219.095	0.99512
30	173.6638	0.96075	203.5382	0.96622	263.3673	0.97358	712.9837	0.98999	1462.865	0.99508
50	289.2742	0.96020	339.0646	0.96574	438.7796	0.97321	1188.140	0.98985	2437.941	0.99501
75	433.7869	0.95992	508.4723	0.96551	658.0448	0.97303	1782.085	0.98978	3656.787	0.99498
100	578.2994	0.95978	677.8800	0.96539	877.3099	0.97294	2376.030	0.98974	4875.632	0.99496
150	867.3244	0.95965	1016.695	0.96527	1315.840	0.97284	3563.920	0.98971	7313.324	0.99495
200	1156.349	0.95958	1355.510	0.96521	1754.370	0.97280	4751.810	0.98969	9751.014	0.99494
250	1445.374	0.95953	1694.326	0.96517	2192.900	0.97277	5939.700	0.98968	12188.71	0.99493
300	1734.399	0.95951	2033.141	0.96515	2631.430	0.97275	7127.590	0.98968	14626.39	0.99493
c_4	0.9593687887		0.9650304561		0.9726592741		0.9896403756		0.9949113047	
c_5	0.2821551475		0.2621377857		0.2322368112		0.1435685446		0.1007546319	
c_5^2 (Var.)	0.0796115273		0.0687162187		0.0539339365		0.0206119270		0.0101514958	

Table D.3.2. (1 - alphaStandUCL) Percentage Points
of the Studentized Standard Deviation (alphaStandUCL = 0.005)

	n				
m	**2**	**3**	**4**	**5**	**6**
1	127.32134	14.10674	6.88965	4.81191	3.86518
2	15.33836	5.29746	3.65688	2.99975	2.64128
3	8.05912	3.92624	2.99837	2.57854	2.33344
4	5.97848	3.40499	2.72320	2.39394	2.19458
5	5.04664	3.13442	2.57307	2.29066	2.11568
6	4.52848	2.96960	2.47875	2.22474	2.06484
7	4.20146	2.85892	2.41406	2.17904	2.02936
8	3.97730	2.77957	2.36696	2.14550	2.00320
9	3.81447	2.71993	2.33114	2.11985	1.98311
10	3.69101	2.67348	2.30299	2.09959	1.96720
11	3.59428	2.63629	2.28029	2.08319	1.95429
12	3.51649	2.60586	2.26159	2.06964	1.94360
13	3.45261	2.58049	2.24592	2.05825	1.93461
14	3.39922	2.55902	2.23261	2.04856	1.92694
15	3.35395	2.54062	2.22116	2.04020	1.92032
16	3.31508	2.52467	2.21120	2.03292	1.91455
17	3.28135	2.51072	2.20246	2.02653	1.90947
18	3.25181	2.49841	2.19473	2.02086	1.90497
19	3.22572	2.48747	2.18784	2.01581	1.90096
20	3.20252	2.47768	2.18167	2.01127	1.89735
25	3.11665	2.44098	2.15842	1.99416	1.88372
30	3.06138	2.41695	2.14311	1.98285	1.87469
50	2.95538	2.36990	2.11291	1.96046	1.85678
75	2.90455	2.34688	2.09802	1.94938	1.84790
100	2.87966	2.33549	2.09063	1.94387	1.84348
150	2.85512	2.32418	2.08327	1.93838	1.83907
200	2.84297	2.31856	2.07961	1.93564	1.83686
250	2.83572	2.31519	2.07742	1.93400	1.83555
300	2.83091	2.31295	2.07595	1.93290	1.83467
∞	2.8070337683	2.3018074130	2.0686675636	1.9274503237	1.8302787954

Table D.3.2 continued. (1 - alphaStandUCL) Percentage Points of the Studentized Standard Deviation (alphaStandUCL = 0.005)

	n				
m	**7**	**8**	**10**	**25**	**50**
1	3.32762	2.98084	2.55756	1.72242	1.45363
2	2.41271	2.25269	2.04067	1.54824	1.36083
3	2.17013	2.05216	1.89083	1.49129	1.32910
4	2.05855	1.95860	1.81957	1.46291	1.31302
5	1.99448	1.90448	1.77791	1.44590	1.30328
6	1.95293	1.86921	1.75058	1.43456	1.29675
7	1.92380	1.84440	1.73127	1.42646	1.29206
8	1.90224	1.82600	1.71690	1.42038	1.28854
9	1.88565	1.81181	1.70579	1.41565	1.28579
10	1.87249	1.80053	1.69694	1.41187	1.28359
11	1.86179	1.79136	1.68973	1.40878	1.28178
12	1.85292	1.78374	1.68375	1.40620	1.28027
13	1.84545	1.77733	1.67869	1.40401	1.27899
14	1.83907	1.77184	1.67437	1.40214	1.27790
15	1.83356	1.76710	1.67063	1.40052	1.27695
16	1.82876	1.76297	1.66736	1.39910	1.27611
17	1.82453	1.75933	1.66448	1.39785	1.27538
18	1.82078	1.75609	1.66193	1.39673	1.27472
19	1.81743	1.75321	1.65965	1.39574	1.27414
20	1.81442	1.75061	1.65759	1.39484	1.27361
25	1.80303	1.74079	1.64981	1.39143	1.27161
30	1.79547	1.73427	1.64464	1.38915	1.27027
50	1.78047	1.72129	1.63433	1.38461	1.26758
75	1.77301	1.71484	1.62919	1.38233	1.26624
100	1.76930	1.71162	1.62663	1.38119	1.26557
150	1.76559	1.70841	1.62407	1.38005	1.26489
200	1.76374	1.70681	1.62279	1.37949	1.26456
250	1.76263	1.70585	1.62203	1.37914	1.26435
300	1.76189	1.70521	1.62152	1.37892	1.26422
∞	1.7581990871	1.7020046951	1.6189623145	1.3777776783	1.2635446424

Table D.3.3. alphaStandLCL Percentage Points of
the Studentized Standard Deviation (alphaStandLCL = 0.001)

	n				
m	**2**	**3**	**4**	**5**	**6**
1	0.00157	0.03164	0.08418	0.13680	0.18333
2	0.00142	0.03163	0.08668	0.14270	0.19254
3	0.00137	0.03163	0.08767	0.14507	0.19624
4	0.00134	0.03163	0.08820	0.14635	0.19825
5	0.00132	0.03163	0.08854	0.14715	0.19951
6	0.00131	0.03163	0.08877	0.14770	0.20037
7	0.00130	0.03163	0.08893	0.14810	0.20101
8	0.00130	0.03163	0.08906	0.14841	0.20149
9	0.00129	0.03163	0.08916	0.14865	0.20186
10	0.00129	0.03163	0.08924	0.14884	0.20217
11	0.00129	0.03163	0.08931	0.14900	0.20242
12	0.00128	0.03163	0.08936	0.14914	0.20263
13	0.00128	0.03163	0.08941	0.14925	0.20281
14	0.00128	0.03163	0.08945	0.14935	0.20297
15	0.00128	0.03163	0.08949	0.14944	0.20310
16	0.00128	0.03163	0.08952	0.14951	0.20322
17	0.00127	0.03163	0.08955	0.14958	0.20333
18	0.00127	0.03163	0.08957	0.14964	0.20342
19	0.00127	0.03163	0.08959	0.14969	0.20350
20	0.00127	0.03163	0.08961	0.14974	0.20358
25	0.00127	0.03163	0.08969	0.14992	0.20387
30	0.00127	0.03163	0.08974	0.15004	0.20406
50	0.00126	0.03163	0.08984	0.15029	0.20445
75	0.00126	0.03163	0.08989	0.15042	0.20465
100	0.00126	0.03163	0.08992	0.15048	0.20475
150	0.00126	0.03163	0.08994	0.15054	0.20484
200	0.00126	0.03163	0.08996	0.15057	0.20489
250	0.00125	0.03163	0.08996	0.15059	0.20492
300	0.00125	0.03163	0.08997	0.15061	0.20494
∞	0.0012533145	0.0316306866	0.0899955292	0.1506685398	0.2050427285

Table D.3.3 continued. alphaStandLCL Percentage Points of the Studentized Standard Deviation (alphaStandLCL = 0.001)

	n				
m	**7**	**8**	**10**	**25**	**50**
1	0.22344	0.25804	0.31456	0.51741	0.63699
2	0.23553	0.27258	0.33285	0.54446	0.66424
3	0.24041	0.27844	0.34022	0.55519	0.67490
4	0.24305	0.28162	0.34422	0.56098	0.68060
5	0.24471	0.28362	0.34673	0.56460	0.68416
6	0.24585	0.28499	0.34845	0.56708	0.68660
7	0.24669	0.28599	0.34971	0.56889	0.68837
8	0.24732	0.28675	0.35067	0.57026	0.68972
9	0.24782	0.28736	0.35142	0.57135	0.69077
10	0.24822	0.28784	0.35203	0.57222	0.69163
11	0.24856	0.28824	0.35254	0.57294	0.69233
12	0.24884	0.28858	0.35296	0.57354	0.69292
13	0.24907	0.28886	0.35332	0.57405	0.69342
14	0.24928	0.28911	0.35362	0.57450	0.69385
15	0.24945	0.28932	0.35389	0.57488	0.69423
16	0.24961	0.28951	0.35413	0.57522	0.69455
17	0.24975	0.28968	0.35434	0.57552	0.69485
18	0.24987	0.28982	0.35452	0.57578	0.69511
19	0.24998	0.28996	0.35469	0.57602	0.69534
20	0.25008	0.29008	0.35484	0.57624	0.69555
25	0.25046	0.29053	0.35542	0.57706	0.69635
30	0.25072	0.29084	0.35580	0.57761	0.69688
50	0.25123	0.29146	0.35658	0.57872	0.69796
75	0.25149	0.29177	0.35697	0.57928	0.69851
100	0.25162	0.29193	0.35717	0.57956	0.69878
150	0.25175	0.29209	0.35737	0.57984	0.69905
200	0.25182	0.29217	0.35747	0.57998	0.69919
250	0.25186	0.29221	0.35752	0.58007	0.69927
300	0.25188	0.29224	0.35756	0.58012	0.69933
∞	0.2520141382	0.2924023042	0.3577630417	0.5804050877	0.6996037468

Table D.3.4. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	2					
m	A31	B41	B31	A32	B42	B32
1	-----	-----	-----	235.78369	127.32134	0.00157
2	11.70380	1.98441	0.00314	20.27157	16.95587	0.00157
3	6.69217	2.68348	0.00235	9.46416	9.25818	0.00157
4	5.12908	3.02106	0.00209	6.62162	7.00946	0.00157
5	4.41023	3.18338	0.00196	5.40140	5.99224	0.00157
6	4.00626	3.27080	0.00188	4.74027	5.42349	0.00157
7	3.75013	3.32336	0.00183	4.33028	5.06335	0.00157
8	3.57422	3.35784	0.00179	4.05278	4.81596	0.00157
9	3.44634	3.38200	0.00177	3.85313	4.63598	0.00157
10	3.34938	3.39981	0.00175	3.70287	4.49937	0.00157
11	3.27342	3.41346	0.00173	3.58585	4.39225	0.00157
12	3.21236	3.42425	0.00171	3.49220	4.30605	0.00157
13	3.16224	3.43300	0.00170	3.41560	4.23522	0.00157
14	3.12037	3.44023	0.00169	3.35182	4.17601	0.00157
15	3.08489	3.44631	0.00168	3.29788	4.12579	0.00157
16	3.05444	3.45150	0.00168	3.25170	4.08265	0.00157
17	3.02803	3.45597	0.00167	3.21171	4.04522	0.00157
18	3.00491	3.45988	0.00166	3.17675	4.01242	0.00157
19	2.98450	3.46331	0.00166	3.14594	3.98345	0.00157
20	2.96635	3.46636	0.00165	3.11857	3.95768	0.00157
25	2.89928	3.47759	0.00164	3.01766	3.86230	0.00157
30	2.85617	3.48479	0.00162	2.95302	3.80088	0.00157
50	2.77366	3.49858	0.00160	2.82970	3.68305	0.00157
75	2.73419	3.50522	0.00159	2.77090	3.62654	0.00157
100	2.71489	3.50849	0.00159	2.74218	3.59887	0.00157
150	2.69587	3.51172	0.00158	2.71390	3.57157	0.00157
200	2.68646	3.51333	0.00158	2.69993	3.55806	0.00157
250	2.68085	3.51429	0.00158	2.69160	3.55000	0.00157
300	2.67713	3.51492	0.00158	2.68607	3.54465	0.00157
∞	2.6586603867	3.5180951058	0.0015707967	2.6586603867	3.5180951058	0.0015707967

Table D.3.4 continued. Two Stage Short Run Control Chart Factors for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	3					
m	A31	B41	B31	A32	B42	B32
1	-----	-----	-----	15.68165	14.10674	0.03164
2	2.95828	1.86761	0.06134	5.12390	5.60680	0.03348
3	2.57119	2.21123	0.04940	3.63621	4.24135	0.03417
4	2.39128	2.34285	0.04505	3.08713	3.71725	0.03453
5	2.29099	2.40840	0.04280	2.80588	3.44396	0.03476
6	2.22764	2.44716	0.04142	2.63578	3.27705	0.03491
7	2.18416	2.47270	0.04049	2.52205	3.16478	0.03502
8	2.15253	2.49078	0.03982	2.44074	3.08418	0.03510
9	2.12851	2.50426	0.03931	2.37975	3.02355	0.03516
10	2.10966	2.51469	0.03892	2.33232	2.97631	0.03521
11	2.09448	2.52301	0.03860	2.29438	2.93847	0.03526
12	2.08199	2.52981	0.03834	2.26336	2.90748	0.03529
13	2.07154	2.53545	0.03812	2.23752	2.88164	0.03532
14	2.06267	2.54023	0.03794	2.21566	2.85977	0.03535
15	2.05504	2.54432	0.03778	2.19693	2.84102	0.03537
16	2.04842	2.54786	0.03764	2.18071	2.82477	0.03539
17	2.04261	2.55095	0.03752	2.16652	2.81055	0.03541
18	2.03748	2.55368	0.03741	2.15400	2.79799	0.03542
19	2.03291	2.55611	0.03732	2.14288	2.78684	0.03544
20	2.02882	2.55828	0.03723	2.13293	2.77685	0.03545
25	2.01343	2.56641	0.03691	2.09564	2.73942	0.03550
30	2.00331	2.57173	0.03670	2.07124	2.71490	0.03553
50	1.98341	2.58216	0.03629	2.02348	2.66687	0.03559
75	1.97363	2.58728	0.03609	2.00012	2.64336	0.03563
100	1.96878	2.58981	0.03599	1.98856	2.63173	0.03564
150	1.96396	2.59233	0.03589	1.97709	2.62017	0.03566
200	1.96156	2.59358	0.03584	1.97139	2.61443	0.03567
250	1.96012	2.59433	0.03581	1.96797	2.61099	0.03567
300	1.95916	2.59483	0.03579	1.96570	2.60870	0.03568
∞	1.9543950590	2.5973115315	0.0356914078	1.9543950590	2.5973115315	0.0356914078

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	4					
m	A31	B41	B31	A32	B42	B32
1	-----	-----	-----	6.51861	6.88965	0.08418
2	1.83276	1.74650	0.15529	3.17444	3.80345	0.09015
3	1.78740	1.96613	0.12940	2.52776	3.16194	0.09245
4	1.75114	2.05256	0.11958	2.26072	2.89208	0.09367
5	1.72737	2.09812	0.11441	2.11558	2.74437	0.09443
6	1.71103	2.12622	0.11122	2.02452	2.65138	0.09495
7	1.69922	2.14528	0.10905	1.96209	2.58752	0.09532
8	1.69032	2.15907	0.10748	1.91664	2.54097	0.09561
9	1.68337	2.16951	0.10629	1.88207	2.50555	0.09583
10	1.67781	2.17769	0.10536	1.85489	2.47770	0.09601
11	1.67326	2.18427	0.10461	1.83297	2.45523	0.09616
12	1.66947	2.18969	0.10399	1.81491	2.43672	0.09628
13	1.66627	2.19422	0.10348	1.79977	2.42120	0.09639
14	1.66352	2.19807	0.10304	1.78691	2.40801	0.09648
15	1.66114	2.20137	0.10266	1.77583	2.39666	0.09656
16	1.65906	2.20425	0.10234	1.76620	2.38679	0.09663
17	1.65723	2.20677	0.10205	1.75775	2.37813	0.09669
18	1.65560	2.20900	0.10180	1.75028	2.37046	0.09674
19	1.65414	2.21099	0.10157	1.74361	2.36364	0.09679
20	1.65283	2.21277	0.10137	1.73764	2.35752	0.09683
25	1.64785	2.21947	0.10061	1.71514	2.33446	0.09700
30	1.64454	2.22388	0.10011	1.70031	2.31926	0.09711
50	1.63794	2.23258	0.09912	1.67103	2.28928	0.09734
75	1.63465	2.23687	0.09864	1.65659	2.27450	0.09745
100	1.63301	2.23900	0.09840	1.64942	2.26716	0.09751
150	1.63137	2.24112	0.09816	1.64228	2.25985	0.09757
200	1.63055	2.24218	0.09804	1.63872	2.25621	0.09760
250	1.63005	2.24281	0.09797	1.63659	2.25403	0.09761
300	1.62973	2.24323	0.09792	1.63517	2.25258	0.09762
∞	1.6280903367	2.2453356665	0.0976813167	1.6280903367	2.2453356665	0.0976813167

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	5					
m	A31	B41	B31	A32	B42	B32
1	-----	-----	-----	4.18690	4.81191	0.13680
2	1.40670	1.65588	0.24067	2.43647	3.09107	0.14705
3	1.44282	1.82147	0.20546	2.04046	2.68484	0.15105
4	1.44648	1.88912	0.19174	1.86740	2.50585	0.15319
5	1.44561	1.92584	0.18442	1.77051	2.40542	0.15452
6	1.44401	1.94891	0.17987	1.70858	2.34121	0.15543
7	1.44244	1.96476	0.17676	1.66559	2.29665	0.15610
8	1.44105	1.97632	0.17451	1.63400	2.26392	0.15660
9	1.43985	1.98513	0.17279	1.60980	2.23886	0.15700
10	1.43882	1.99207	0.17145	1.59068	2.21907	0.15731
11	1.43794	1.99768	0.17037	1.57518	2.20303	0.15758
12	1.43717	2.00230	0.16947	1.56237	2.18979	0.15780
13	1.43651	2.00618	0.16873	1.55161	2.17865	0.15798
14	1.43592	2.00948	0.16809	1.54243	2.16917	0.15814
15	1.43541	2.01232	0.16755	1.53451	2.16099	0.15828
16	1.43495	2.01479	0.16707	1.52762	2.15387	0.15841
17	1.43453	2.01697	0.16666	1.52155	2.14760	0.15852
18	1.43416	2.01889	0.16629	1.51618	2.14206	0.15861
19	1.43383	2.02060	0.16596	1.51139	2.13711	0.15870
20	1.43352	2.02214	0.16567	1.50709	2.13267	0.15878
25	1.43234	2.02794	0.16456	1.49083	2.11591	0.15908
30	1.43154	2.03178	0.16383	1.48008	2.10482	0.15928
50	1.42988	2.03935	0.16239	1.45877	2.08288	0.15968
75	1.42903	2.04310	0.16169	1.44821	2.07202	0.15988
100	1.42860	2.04496	0.16133	1.44296	2.06661	0.15998
150	1.42817	2.04682	0.16098	1.43772	2.06123	0.16008
200	1.42795	2.04774	0.16081	1.43511	2.05854	0.16013
250	1.42782	2.04830	0.16070	1.43354	2.05693	0.16017
300	1.42773	2.04867	0.16064	1.43250	2.05586	0.16019
∞	1.4272883468	2.0505104733	0.1602881356	1.4272883468	2.0505104733	0.1602881356

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	6					
m	A31	B41	B31	A32	B42	B32
1	-----	-----	-----	3.17946	3.86518	0.18333
2	1.17743	1.58892	0.30986	2.03936	2.70604	0.19726
3	1.24158	1.72504	0.26932	1.75586	2.41068	0.20274
4	1.26077	1.78217	0.25320	1.62765	2.27682	0.20568
5	1.26929	1.81368	0.24452	1.55456	2.20058	0.20751
6	1.27392	1.83367	0.23909	1.50732	2.15137	0.20877
7	1.27676	1.84749	0.23538	1.47428	2.11699	0.20968
8	1.27866	1.85762	0.23267	1.44986	2.09162	0.21038
9	1.28000	1.86537	0.23061	1.43108	2.07213	0.21093
10	1.28099	1.87148	0.22900	1.41619	2.05668	0.21137
11	1.28176	1.87643	0.22769	1.40409	2.04415	0.21173
12	1.28236	1.88052	0.22662	1.39407	2.03376	0.21203
13	1.28284	1.88395	0.22572	1.38563	2.02503	0.21229
14	1.28324	1.88688	0.22495	1.37842	2.01758	0.21252
15	1.28358	1.88940	0.22429	1.37220	2.01114	0.21271
16	1.28386	1.89160	0.22372	1.36677	2.00553	0.21288
17	1.28410	1.89353	0.22321	1.36199	2.00060	0.21303
18	1.28431	1.89524	0.22277	1.35776	1.99622	0.21316
19	1.28449	1.89677	0.22237	1.35397	1.99231	0.21328
20	1.28465	1.89814	0.22202	1.35058	1.98881	0.21339
25	1.28524	1.90331	0.22068	1.33772	1.97554	0.21380
30	1.28560	1.90673	0.21979	1.32919	1.96676	0.21408
50	1.28626	1.91350	0.21805	1.31225	1.94932	0.21464
75	1.28657	1.91686	0.21719	1.30384	1.94067	0.21492
100	1.28671	1.91853	0.21676	1.29964	1.93636	0.21506
150	1.28685	1.92019	0.21633	1.29546	1.93207	0.21520
200	1.28692	1.92102	0.21612	1.29337	1.92992	0.21527
250	1.28696	1.92152	0.21599	1.29212	1.92864	0.21532
300	1.28699	1.92185	0.21591	1.29128	1.92778	0.21534
∞	1.2871184251	1.9235056072	0.2154867548	1.2871184251	1.9235056072	0.2154867548

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	7					
m	A31	B41	B31	A32	B42	B32
1	-----	-----	-----	2.62129	3.32762	0.22344
2	1.03107	1.53785	0.36527	1.78587	2.46221	0.24037
3	1.10629	1.65538	0.32187	1.56453	2.23012	0.24705
4	1.13254	1.70560	0.30434	1.46210	2.12290	0.25065
5	1.14554	1.73357	0.29484	1.40300	2.06120	0.25290
6	1.15322	1.75143	0.28887	1.36451	2.02112	0.25444
7	1.15826	1.76382	0.28477	1.33745	1.99300	0.25556
8	1.16182	1.77293	0.28178	1.31737	1.97217	0.25641
9	1.16445	1.77991	0.27951	1.30189	1.95614	0.25708
10	1.16648	1.78543	0.27772	1.28959	1.94341	0.25762
11	1.16809	1.78990	0.27627	1.27958	1.93305	0.25807
12	1.16940	1.79359	0.27508	1.27127	1.92447	0.25844
13	1.17048	1.79670	0.27408	1.26426	1.91724	0.25876
14	1.17139	1.79935	0.27323	1.25828	1.91107	0.25903
15	1.17217	1.80164	0.27250	1.25310	1.90573	0.25927
16	1.17284	1.80363	0.27186	1.24858	1.90108	0.25948
17	1.17342	1.80538	0.27130	1.24461	1.89698	0.25967
18	1.17394	1.80694	0.27080	1.24107	1.89335	0.25983
19	1.17439	1.80832	0.27036	1.23792	1.89010	0.25998
20	1.17480	1.80956	0.26997	1.23509	1.88718	0.26011
25	1.17631	1.81426	0.26848	1.22435	1.87614	0.26062
30	1.17730	1.81737	0.26749	1.21722	1.86882	0.26096
50	1.17920	1.82354	0.26555	1.20303	1.85427	0.26165
75	1.18012	1.82660	0.26459	1.19596	1.84704	0.26199
100	1.18058	1.82812	0.26411	1.19244	1.84343	0.26216
150	1.18102	1.82964	0.26363	1.18892	1.83984	0.26234
200	1.18125	1.83040	0.26340	1.18717	1.83804	0.26243
250	1.18138	1.83085	0.26325	1.18611	1.83696	0.26248
300	1.18147	1.83115	0.26316	1.18541	1.83625	0.26251
∞	1.1819070377	1.8326623794	0.2626874474	1.1819070377	1.8326623794	0.2626874474

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	8					
m	**A31**	**B41**	**B31**	**A32**	**B42**	**B32**
1	-----	-----	-----	2.26496	2.98084	0.25804
2	0.92789	1.49759	0.41022	1.60716	2.29241	0.27738
3	1.00745	1.60218	0.36540	1.42475	2.10084	0.28504
4	1.03713	1.64745	0.34708	1.33893	2.01111	0.28917
5	1.05244	1.67283	0.33709	1.28897	1.95909	0.29175
6	1.06174	1.68909	0.33080	1.25627	1.92514	0.29352
7	1.06797	1.70041	0.32647	1.23318	1.90124	0.29481
8	1.07242	1.70874	0.32330	1.21601	1.88350	0.29578
9	1.07577	1.71513	0.32089	1.20275	1.86981	0.29656
10	1.07837	1.72019	0.31899	1.19218	1.85893	0.29718
11	1.08045	1.72429	0.31746	1.18358	1.85007	0.29769
12	1.08216	1.72769	0.31619	1.17643	1.84272	0.29812
13	1.08357	1.73054	0.31513	1.17039	1.83653	0.29848
14	1.08477	1.73298	0.31423	1.16523	1.83123	0.29880
15	1.08580	1.73508	0.31345	1.16077	1.82665	0.29907
16	1.08669	1.73691	0.31277	1.15687	1.82265	0.29931
17	1.08747	1.73852	0.31218	1.15344	1.81913	0.29952
18	1.08816	1.73995	0.31165	1.15039	1.81601	0.29971
19	1.08877	1.74123	0.31118	1.14766	1.81322	0.29988
20	1.08931	1.74237	0.31076	1.14521	1.81071	0.30003
25	1.09136	1.74670	0.30917	1.13592	1.80121	0.30062
30	1.09269	1.74957	0.30812	1.12975	1.79490	0.30101
50	1.09531	1.75526	0.30605	1.11744	1.78235	0.30180
75	1.09659	1.75808	0.30502	1.11131	1.77611	0.30220
100	1.09722	1.75948	0.30451	1.10825	1.77299	0.30240
150	1.09785	1.76089	0.30401	1.10519	1.76988	0.30260
200	1.09816	1.76159	0.30375	1.10366	1.76833	0.30270
250	1.09834	1.76201	0.30360	1.10275	1.76740	0.30276
300	1.09847	1.76229	0.30350	1.10214	1.76678	0.30280
∞	1.0990865943	1.7636797722	0.3029980062	1.0990865943	1.7636797722	0.3029980062

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	10					
m	**A31**	**B41**	**B31**	**A32**	**B42**	**B32**
1	-----	-----	-----	1.83098	2.55756	0.31456
2	0.78934	1.43782	0.47857	1.36718	2.06875	0.33743
3	0.87005	1.52535	0.43308	1.23044	1.92577	0.34651
4	0.90213	1.56383	0.41417	1.16465	1.85752	0.35140
5	0.91929	1.58559	0.40377	1.12589	1.81755	0.35446
6	0.92995	1.59959	0.39719	1.10033	1.79129	0.35656
7	0.93721	1.60937	0.39265	1.08220	1.77273	0.35809
8	0.94247	1.61658	0.38932	1.06867	1.75890	0.35925
9	0.94646	1.62212	0.38679	1.05818	1.74821	0.36016
10	0.94959	1.62651	0.38478	1.04981	1.73969	0.36090
11	0.95211	1.63008	0.38316	1.04298	1.73275	0.36151
12	0.95418	1.63303	0.38183	1.03730	1.72698	0.36202
13	0.95591	1.63552	0.38070	1.03250	1.72211	0.36245
14	0.95738	1.63764	0.37975	1.02839	1.71794	0.36283
15	0.95864	1.63947	0.37892	1.02483	1.71433	0.36315
16	0.95974	1.64107	0.37820	1.02172	1.71118	0.36344
17	0.96070	1.64247	0.37757	1.01897	1.70841	0.36369
18	0.96155	1.64372	0.37702	1.01654	1.70595	0.36391
19	0.96230	1.64483	0.37652	1.01436	1.70374	0.36411
20	0.96298	1.64583	0.37607	1.01240	1.70176	0.36430
25	0.96553	1.64961	0.37439	1.00496	1.69425	0.36499
30	0.96721	1.65212	0.37327	1.00001	1.68926	0.36546
50	0.97052	1.65709	0.37107	0.99013	1.67931	0.36639
75	0.97214	1.65956	0.36998	0.98519	1.67435	0.36687
100	0.97295	1.66079	0.36944	0.98273	1.67188	0.36710
150	0.97375	1.66202	0.36889	0.98026	1.66941	0.36734
200	0.97415	1.66264	0.36863	0.97903	1.66817	0.36746
250	0.97439	1.66300	0.36846	0.97830	1.66743	0.36753
300	0.97455	1.66325	0.36836	0.97780	1.66694	0.36758
∞	0.9753425971	1.6644701362	0.3678194937	0.9753425971	1.6644701362	0.3678194937

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	25					
m	**A31**	**B41**	**B31**	**A32**	**B42**	**B32**
1	-----	-----	-----	0.94603	1.72242	0.51741
2	0.44990	1.26536	0.68196	0.77925	1.55628	0.54729
3	0.51111	1.31284	0.64455	0.72281	1.50164	0.55905
4	0.53775	1.33430	0.62831	0.69424	1.47435	0.56536
5	0.55272	1.34660	0.61919	0.67694	1.45797	0.56931
6	0.56231	1.35458	0.61334	0.66534	1.44704	0.57201
7	0.56899	1.36018	0.60926	0.65701	1.43923	0.57398
8	0.57391	1.36432	0.60627	0.65075	1.43337	0.57548
9	0.57768	1.36752	0.60397	0.64586	1.42880	0.57665
10	0.58066	1.37006	0.60214	0.64194	1.42515	0.57760
11	0.58308	1.37212	0.60067	0.63873	1.42216	0.57838
12	0.58508	1.37383	0.59944	0.63605	1.41967	0.57904
13	0.58676	1.37527	0.59842	0.63378	1.41756	0.57959
14	0.58820	1.37651	0.59754	0.63183	1.41576	0.58007
15	0.58944	1.37757	0.59678	0.63014	1.41419	0.58049
16	0.59052	1.37850	0.59612	0.62865	1.41282	0.58086
17	0.59147	1.37932	0.59554	0.62735	1.41161	0.58118
18	0.59231	1.38005	0.59503	0.62618	1.41053	0.58147
19	0.59306	1.38070	0.59457	0.62514	1.40957	0.58173
20	0.59374	1.38128	0.59415	0.62420	1.40870	0.58196
25	0.59628	1.38349	0.59260	0.62063	1.40540	0.58285
30	0.59797	1.38495	0.59156	0.61825	1.40320	0.58345
50	0.60132	1.38787	0.58951	0.61347	1.39881	0.58465
75	0.60298	1.38932	0.58850	0.61108	1.39660	0.58526
100	0.60381	1.39004	0.58799	0.60988	1.39550	0.58556
150	0.60463	1.39076	0.58749	0.60868	1.39440	0.58587
200	0.60505	1.39112	0.58723	0.60808	1.39385	0.58602
250	0.60529	1.39134	0.58708	0.60772	1.39352	0.58611
300	0.60546	1.39148	0.58698	0.60748	1.39330	0.58617
∞	0.6062761922	1.3922003510	0.5864808086	0.6062761922	1.3922003510	0.5864808086

Table D.3.4 continued. Two Stage Short Run Control Chart Factors
for alphaMean=0.0027, alphaStandUCL=0.005, and alphaStandLCL=0.001

n	50					
m	**A31**	**B41**	**B31**	**A32**	**B42**	**B32**
1	-----	-----	-----	0.63210	1.45363	0.63699
2	0.30864	1.18488	0.77825	0.53458	1.36429	0.66594
3	0.35361	1.21656	0.74938	0.50008	1.33362	0.67719
4	0.37361	1.23096	0.73664	0.48232	1.31805	0.68321
5	0.38496	1.23922	0.72942	0.47148	1.30861	0.68696
6	0.39229	1.24459	0.72477	0.46417	1.30227	0.68952
7	0.39742	1.24836	0.72152	0.45890	1.29772	0.69139
8	0.40120	1.25116	0.71913	0.45492	1.29430	0.69280
9	0.40411	1.25332	0.71728	0.45181	1.29163	0.69391
10	0.40642	1.25503	0.71582	0.44932	1.28949	0.69481
11	0.40830	1.25642	0.71464	0.44727	1.28773	0.69555
12	0.40985	1.25758	0.71365	0.44556	1.28627	0.69617
13	0.41116	1.25856	0.71283	0.44410	1.28503	0.69669
14	0.41228	1.25939	0.71212	0.44286	1.28396	0.69714
15	0.41324	1.26011	0.71151	0.44177	1.28304	0.69754
16	0.41408	1.26074	0.71098	0.44083	1.28223	0.69788
17	0.41482	1.26129	0.71051	0.43999	1.28152	0.69819
18	0.41548	1.26178	0.71010	0.43924	1.28088	0.69846
19	0.41607	1.26222	0.70973	0.43857	1.28031	0.69871
20	0.41659	1.26261	0.70939	0.43797	1.27980	0.69893
25	0.41859	1.26411	0.70813	0.43568	1.27785	0.69977
30	0.41991	1.26510	0.70730	0.43415	1.27655	0.70033
50	0.42253	1.26707	0.70564	0.43107	1.27394	0.70146
75	0.42384	1.26805	0.70482	0.42953	1.27263	0.70203
100	0.42449	1.26854	0.70441	0.42876	1.27197	0.70232
150	0.42514	1.26903	0.70400	0.42798	1.27132	0.70261
200	0.42546	1.26928	0.70379	0.42759	1.27099	0.70275
250	0.42566	1.26942	0.70367	0.42736	1.27079	0.70284
300	0.42578	1.26952	0.70359	0.42721	1.27066	0.70289
∞	0.4264307914	1.2700073227	0.7031820259	0.4264307914	1.2700073227	0.7031820259

APPENDIX E.1 – Analytical Results for Chapter 7

Derive: $\text{d2starMR} = \left(\text{d2}^2 + \text{d2}^2 \cdot r\right)^{0.5}$

We first need to determine the mean and variance of the distribution of the mean moving range $\overline{MR}/\sigma$.

Note: By definition, $E\left(\frac{MR}{\sigma}\right) = d2$

$$\Rightarrow \left(\frac{1}{\sigma}\right) \cdot E(MR) = d2 \Rightarrow E(MR) = d2 \cdot \sigma$$

$$E\left(\frac{\overline{MR}}{\sigma}\right) = \left(\frac{1}{\sigma}\right) \cdot E\left(\overline{MR}\right) = \left(\frac{1}{\sigma}\right) \cdot E\left(\frac{\sum_{i=1}^{m-1} MR_i}{m-1}\right) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m-1}\right) \cdot E\left(\sum_{i=1}^{m-1} MR_i\right)$$

$$\Rightarrow E\left(\frac{\overline{MR}}{\sigma}\right) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m-1}\right) \cdot \sum_{i=1}^{m-1} E(MR_i) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m-1}\right) \cdot \sum_{i=1}^{m-1} (d2 \cdot \sigma)$$

since $E(MR) = d2 \cdot \sigma$.

$$\Rightarrow E\left(\frac{\overline{MR}}{\sigma}\right) = \left(\frac{1}{\sigma}\right) \cdot \left(\frac{1}{m-1}\right) \cdot \left((m-1) \cdot d2 \cdot \sigma\right) = d2$$

(continued on the next page)

(continued from the previous page)

$$\mathrm{Var}\left(\frac{\overline{MR}}{\sigma}\right)=\left(\frac{1}{\sigma^2}\right)\cdot \mathrm{Var}\left(\overline{MR}\right)$$

From Palm and Wheeler (1990), $\mathrm{Var}\left(\overline{MR}/d2\right)=\sigma^2\cdot r$

where

$$r=\frac{b\cdot(m-1)-c}{(m-1)^2}$$

with

$$b=\frac{2\cdot\pi}{3}-3+\sqrt{3}$$

$$c=\frac{\pi}{6}-2+\sqrt{3}$$

$$\Rightarrow r=\left(\frac{1}{\sigma^2}\right)\cdot \mathrm{Var}\left(\frac{\overline{MR}}{d2}\right)=\left(\frac{1}{\sigma^2}\right)\cdot\left(\frac{1}{d2^2}\right)\cdot \mathrm{Var}\left(\overline{MR}\right)$$

$$\Rightarrow d2^2\cdot r=\left(\frac{1}{\sigma^2}\right)\cdot \mathrm{Var}\left(\overline{MR}\right)$$

$$\Rightarrow \mathrm{Var}\left(\frac{\overline{MR}}{\sigma}\right)=d2^2\cdot r$$

(continued on the next page)

(continued from the previous page)

<u>Derive:</u> $\text{d2starMR} = \left(\text{d2}^2 + \text{d2}^2 \cdot r\right)^{0.5}$

According to Johnson and Welch (1939), the mean of the χ distribution with ν degrees of freedom is calculated using the following equation (with some modifications in notation):

$$E(\chi) = \sqrt{2} \cdot \frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)}$$

$$\Rightarrow E\left(\frac{\chi \cdot \text{d2starMR}}{\sqrt{\nu}}\right) = \left(\frac{\text{d2starMR}}{\sqrt{\nu}}\right) \cdot E(\chi) = \sqrt{2} \cdot \left(\frac{\text{d2starMR}}{\sqrt{\nu}}\right) \cdot \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)}\right)$$

Equating the squared means of the distribution of the mean moving range $\overline{\text{MR}}/\sigma$ and the $(\chi \cdot \text{d2starMR})/\sqrt{\nu}$ distribution with ν degrees of freedom results in the following:

$$\text{d2}^2 = 2 \cdot \left(\frac{\text{d2starMR}^2}{\nu}\right) \cdot \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)}\right)^2$$

$$\Rightarrow \text{d2starMR}^2 = \text{d2}^2 \cdot \left(\frac{\nu}{2}\right) \cdot \left(\frac{\Gamma(0.5 \cdot \nu)}{\Gamma(0.5 \cdot \nu + 0.5)}\right)^2$$

(continued on the next page)

(continued from the previous page)

Using results obtained from Johnson and Welch (1939) (with some modifications in notation), the equation to calculate the variance of the χ distribution with ν degrees of freedom may be determined as follows:

$$\mathrm{Var}(\chi) = \mathrm{E}(\chi^2) - (\mathrm{E}(\chi))^2 = 2 \cdot \frac{\Gamma(0.5 \cdot \nu + 1)}{\Gamma(0.5 \cdot \nu)} - \left(\sqrt{2} \cdot \frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)} \right)^2$$

$$\Rightarrow \mathrm{Var}(\chi) = 2 \cdot \frac{(0.5 \cdot \nu) \cdot \Gamma(0.5 \cdot \nu)}{\Gamma(0.5 \cdot \nu)} - 2 \cdot \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)} \right)^2 = \nu - 2 \cdot \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)} \right)^2$$

$$\Rightarrow \mathrm{Var}\left(\frac{\chi \cdot \mathrm{d2starMR}}{\sqrt{\nu}} \right) = \left(\frac{\mathrm{d2starMR}^2}{\nu} \right) \cdot \mathrm{Var}(\chi)$$

$$\Rightarrow \mathrm{Var}\left(\frac{\chi \cdot \mathrm{d2starMR}}{\sqrt{\nu}} \right) = \left(\frac{\mathrm{d2starMR}^2}{\nu} \right) \cdot \left[\nu - 2 \cdot \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)} \right)^2 \right]$$

Equating the variances of the distribution of the mean moving range $\overline{\mathrm{MR}}/\sigma$ and the $(\chi \cdot \mathrm{d2starMR})/\sqrt{\nu}$ distribution with ν degrees of freedom results in the following:

$$\mathrm{d2}^2 \cdot r = \left(\frac{\mathrm{d2starMR}^2}{\nu} \right) \cdot \left[\nu - 2 \cdot \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)} \right)^2 \right]$$

$$\Rightarrow \left(\frac{\Gamma(0.5 \cdot \nu + 0.5)}{\Gamma(0.5 \cdot \nu)} \right)^2 = \frac{\dfrac{\mathrm{d2}^2 \cdot r \cdot \nu}{\mathrm{d2starMR}^2} - \nu}{-2}$$

(continued on the next page)

(continued from the previous page)

$$\Rightarrow\left(\frac{\Gamma(0.5\cdot\nu)}{\Gamma(0.5\cdot\nu+0.5)}\right)^2=\frac{2}{\nu\cdot\left(1-\frac{d2^2\cdot r}{d2starMR^2}\right)}$$

Substituting

$$\left(\frac{\Gamma(0.5\cdot\nu)}{\Gamma(0.5\cdot\nu+0.5)}\right)^2=\frac{2}{\nu\cdot\left(1-\frac{d2^2\cdot r}{d2starMR^2}\right)}$$

into

$$d2starMR^2=d2^2\cdot\left(\frac{\nu}{2}\right)\cdot\left(\frac{\Gamma(0.5\cdot\nu)}{\Gamma(0.5\cdot\nu+0.5)}\right)^2$$

gives the following equation:

$$d2starMR^2=d2^2\cdot\left(\frac{\nu}{2}\right)\cdot\left[\frac{2}{\nu\cdot\left(1-\frac{d2^2\cdot r}{d2starMR^2}\right)}\right]$$

$$\Rightarrow d2starMR^2=\frac{d2^2}{1-\frac{d2^2\cdot r}{d2starMR^2}}=\frac{d2starMR^2\cdot d2^2}{d2starMR^2-d2^2\cdot r}$$

$$\Rightarrow 1=\frac{d2^2}{d2starMR^2-d2^2\cdot r}\Rightarrow d2starMR^2=d2^2+d2^2\cdot r$$

$$\Rightarrow d2starMR=\left(d2^2+d2^2\cdot r\right)^{0.5}$$

Show: $\overline{MR}/d2$ is an unbiased estimate of σ; i.e., show $E(\overline{MR}/d2) = \sigma$

$$E\left(\frac{\overline{MR}}{d2}\right) = \left(\frac{1}{d2}\right)\cdot E(\overline{MR}) = \left(\frac{1}{d2}\right)\cdot E\left(\frac{\sum_{i=1}^{m-1} MR_i}{m-1}\right) = \left(\frac{1}{d2}\right)\cdot\left(\frac{1}{m-1}\right)\cdot E\left(\sum_{i=1}^{m-1} MR_i\right)$$

$$\Rightarrow E\left(\frac{\overline{MR}}{d2}\right) = \left(\frac{1}{d2}\right)\cdot\left(\frac{1}{m-1}\right)\cdot\sum_{i=1}^{m-1} E(MR_i) = \left(\frac{1}{d2}\right)\cdot\left(\frac{1}{m-1}\right)\cdot\sum_{i=1}^{m-1}(d2\cdot\sigma)$$

since $E(MR) = d2\cdot\sigma$ (a result shown earlier in this appendix (Appendix E.1)).

$$\Rightarrow E\left(\frac{\overline{MR}}{d2}\right) = \left(\frac{1}{d2}\right)\cdot\left(\frac{1}{m-1}\right)\cdot((m-1)\cdot d2\cdot\sigma) = \sigma$$

Note: This result may also be obtained as follows. It is shown earlier in this appendix (Appendix E.1) that the following holds:

$$E\left(\frac{\overline{MR}}{\sigma}\right) = d2$$

$$\Rightarrow \left(\frac{1}{\sigma}\right)\cdot E(\overline{MR}) = d2 \Rightarrow \left(\frac{1}{d2}\right)\cdot E(\overline{MR}) = \sigma \Rightarrow E\left(\frac{\overline{MR}}{d2}\right) = \sigma$$

Derive: $D42 = (qD4/d2starMR)$, where qD4 is the (1-alphaMRUCL) percentage point of the distribution of the studentized range $Q = (w/s)$ for subgroup size two with ν degrees of freedom (alphaMRUCL is the probability of a Type I error on the MR chart above the upper control limit).

Notes: The ensuing derivation is based on the derivation of D_4^* in the appendix of Hillier (1969). The value MR denotes the moving range of a subgroup of size two drawn while in the second stage of the two stage procedure.

We need to determine the value D42 such that the following holds:

$$P\left(MR \leq D42 \cdot \overline{MR}\right) = 1 - alphaMRUCL$$

$$\Rightarrow P\left(\frac{MR}{\overline{MR}} \leq D42\right) = 1 - alphaMRUCL$$

We know MR/σ is the statistic for the distribution of the range $W = (w/\sigma)$ for subgroup size two. We now need an independent estimate of σ based on $\overline{MR}$. Replacing σ with this independent estimate results in the statistic for the distribution of the studentized range $Q = (w/s)$ for subgroup size two, which has ν degrees of freedom. The equation to calculate ν is based on the fact that we have applied the Patnaik (1950) approximation to the distribution of the mean moving range. If we were to use $\overline{MR}/d2$ (which is an unbiased estimate of σ, a result shown earlier in this appendix (Appendix E.1)) as this

(continued on the next page)

(continued from the previous page)

independent estimate, then we would not have the appropriate equation for ν. As a result, we need to use $\overline{MR}/\text{d2starMR}$.

$$\Rightarrow \frac{MR}{\sigma} = \frac{MR}{\left(\dfrac{\overline{MR}}{\text{d2starMR}}\right)} = \frac{MR \cdot \text{d2starMR}}{\overline{MR}}$$

where $(MR \cdot \text{d2starMR})/\overline{MR}$ is the statistic for the distribution of the studentized range $Q = (w/s)$ for subgroup size two with ν degrees of freedom.

$$\Rightarrow 1 - \text{alphaMRUCL} = P\left(\frac{MR \cdot \text{d2starMR}}{\overline{MR}} \le \text{qD4}\right) = P\left(\frac{MR}{\overline{MR}} \le \frac{\text{qD4}}{\text{d2starMR}}\right)$$

where qD4 is defined above.

Setting $\text{D42} = \dfrac{\text{qD4}}{\text{d2starMR}} \Rightarrow 1 - \text{alphaMRUCL} = P\left(\dfrac{MR}{\overline{MR}} \le \text{D42}\right) = P\left(MR \le \text{D42} \cdot \overline{MR}\right)$

Show: $\left(\overline{MR}/d_2^*(MR)\right)^2$ is an unbiased estimate of σ^2; i.e., show $E\left[\left(\overline{MR}/d_2^*(MR)\right)^2\right] = \sigma^2$

$$E\left[\left(\frac{\overline{MR}}{d_2^*(MR)}\right)^2\right] = \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot E\left[\left(\overline{MR}\right)^2\right] = \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot\left[Var\left(\overline{MR}\right) + \left(E\left(\overline{MR}\right)\right)^2\right]$$

$$\Rightarrow E\left[\left(\frac{\overline{MR}}{d_2^*(MR)}\right)^2\right] = \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot\left[d_2^2\cdot r\cdot\sigma^2 + \left[E\left(\frac{\sum_{i=1}^{m-1} MR_i}{m-1}\right)\right]^2\right]$$

since $Var\left(\overline{MR}/\sigma\right) = d_2^2\cdot r \Rightarrow \left(1/\sigma^2\right)\cdot Var\left(\overline{MR}\right) = d_2^2\cdot r \Rightarrow Var\left(\overline{MR}\right) = d_2^2\cdot r\cdot\sigma^2$

(the fact that $Var\left(\overline{MR}/\sigma\right) = d_2^2\cdot r$ is shown earlier in this appendix (Appendix E.1)).

$$\Rightarrow E\left[\left(\frac{\overline{MR}}{d_2^*(MR)}\right)^2\right] = \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot\left[d_2^2\cdot r\cdot\sigma^2 + \left(\frac{1}{(m-1)^2}\right)\cdot\left[E\left(\sum_{i=1}^{m-1} MR_i\right)\right]^2\right]$$

$$= \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot\left[d_2^2\cdot r\cdot\sigma^2 + \left(\frac{1}{(m-1)^2}\right)\cdot\left(\sum_{i=1}^{m-1} E(MR_i)\right)^2\right]$$

$$= \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot\left[d_2^2\cdot r\cdot\sigma^2 + \left(\frac{1}{(m-1)^2}\right)\cdot\left[\sum_{i=1}^{m-1}(d_2\cdot\sigma)\right]^2\right]$$

since $E(MR) = d_2\cdot\sigma$ (a result shown earlier in this appendix (Appendix E.1)).

$$\Rightarrow E\left[\left(\frac{\overline{MR}}{d_2^*(MR)}\right)^2\right] = \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right)\cdot\left[d_2^2\cdot r\cdot\sigma^2 + \left(\frac{1}{(m-1)^2}\right)\cdot((m-1)\cdot d_2\cdot\sigma)^2\right]$$

(continued on the next page)

(continued from the previous page)

$$\Rightarrow E\left[\left(\frac{\overline{MR}}{d_2^*(MR)}\right)^2\right] = \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right) \cdot \left(d_2^2 \cdot r \cdot \sigma^2 + d_2^2 \cdot \sigma^2\right)$$

$$= \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right) \cdot \sigma^2 \cdot \left(d_2^2 + d_2^2 \cdot r\right)$$

$$= \left(\frac{1}{\left(d_2^*(MR)\right)^2}\right) \cdot \sigma^2 \cdot \left(d_2^*(MR)\right)^2$$

since $d_2^*(MR) = \left(d_2^2 + d_2^2 \cdot r\right)^{0.5}$ (a result shown earlier in this appendix (Appendix E.1)).

$$\Rightarrow E\left[\left(\frac{\overline{MR}}{d_2^*(MR)}\right)^2\right] = \sigma^2 \cdot (1) = \sigma^2$$

APPENDIX E.2 – Computer Program ccfsMR.mcd for Chapter 7

Page 1 of program: ccfsMR.mcd

ENTER the following 4 values:

(1) alphaInd := 0.0027 — **alphaInd - alpha for the X chart.**

(2) alphaMRUCL := 0.005 — **alphaMRUCL - alpha for the MR chart above the UCL.**

(3) alphaMRLCL := 0.001 — **alphaMRLCL - alpha for the MR chart below the LCL *.**

(4) m := 5 — **m - number of subgroups (i.e., the number of MRs plus one).**

*** Note - If no LCL is desired, leave alphaMRLCL blank (do not enter zero).**

Please PAGE DOWN to begin the program.

(1.1) $TOL := 10^{-10}$

$f(x) := \mathrm{dnorm}(x,0,1) \quad := \left[(2\cdot\pi)^{-0.5}\right]\cdot e^{\frac{-x^2}{2}}$ $\qquad F(x) := \mathrm{pnorm}(x,0,1) \quad := \int_0^x f(t)\,dt$

$$P(W) := 2\cdot\int_{-\infty}^{\infty} f(x)\cdot(F(x+W)-F(x))\,dx$$

$$d2 := \frac{2}{\pi^{0.5}}$$

(2.1) $DUCL(W) := P(W) - (1 - alphaMRUCL)$

$DLCL(W) := P(W) - alphaMRLCL$

```
Wseed1(start) := | W_0 ← start
                 | W_1 ← start + 0.01
                 | A_0 ← DUCL(W_0)
                 | A_1 ← DUCL(W_1)
                 | while A_0·A_1 > 0
                 |     | W_0 ← W_1
                 |     | W_1 ← W_1 + 0.01
                 |     | A_0 ← A_1
                 |     | A_1 ← DUCL(W_1)
                 | W
```

```
Wseed2(start) := | W_0 ← start
                 | W_1 ← start + 0.01
                 | A_0 ← DLCL(W_0)
                 | A_1 ← DLCL(W_1)
                 | while A_0·A_1 > 0
                 |     | W_0 ← W_1
                 |     | W_1 ← W_1 + 0.01
                 |     | A_0 ← A_1
                 |     | A_1 ← DLCL(W_1)
                 | W
```

$seedD4 := Wseed1(0.01)$

$wD4 := zbrent(DUCL, seedD4_0, seedD4_1, TOL)$

$seedD3 := Wseed2(0.001)$

$wD3 := zbrent(DLCL, seedD3_0, seedD3_1, TOL)$

(2.2) $$h(x) := \frac{x \cdot e^{2\cdot(gammln(0.5\cdot x) - gammln(0.5\cdot x + 0.5))} - 2}{2}$$

$$b := \frac{2\cdot\pi}{3} - 3 + 3^{0.5} \qquad c := \frac{\pi}{6} - 2 + 3^{0.5}$$

```
dfseed(y) := | df_0 ← 0.9
             | df_1 ← 1.1
             | A_0 ← y(df_0)
             | A_1 ← y(df_1)
             | while A_0·A_1 > 0
             |     | df_0 ← df_1
             |     | df_1 ← df_1 + 0.5
             |     | A_0 ← A_1
             |     | A_1 ← y(df_1)
             | df
```

$$r := \frac{b\cdot(m-1) - c}{(m-1)^2} \qquad rprevm := \frac{b\cdot(m-2) - c}{(m-2)^2}$$

$d(x) := h(x) - r$

$dprevm(x) := h(x) - rprevm$

$seedv := dfseed(d)$

$seedvprevm := dfseed(dprevm)$

$v := zbrent(d, seedv_0, seedv_1, TOL)$

$vprevm := zbrent(dprevm, seedvprevm_0, seedvprevm_1, TOL)$

(3.1) $$P1(z) := \int_0^{11} \left[\left(5\cdot\frac{W}{z}\right)\cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}} \right]^{\nu-1} \cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}} \cdot P(W)\, dW$$

$$P2(z) := \left(\frac{z}{5}\right)\cdot \int_{\frac{55}{z}}^{\infty} \left(x\cdot e^{\frac{1-x^2}{2}}\right)^{\nu-1} \cdot e^{\frac{1-x^2}{2}}\, dx$$

$$c\nu := \ln(2) + \left(\frac{\nu}{2}\right)\cdot\ln\left(\frac{\nu}{2}\right) - \left(\frac{\nu}{2}\right) - \mathrm{gammln}\left(\frac{\nu}{2}\right)$$

$$P3(z) := \left(\frac{5}{z}\right)\cdot e^{c\nu}\cdot(P1(z) + P2(z))$$

(3.2) Zseed1(start) :=

```
Z0 ← start
Z1 ← start + 5.0
A0 ← P3(Z0)
A1 ← P3(Z1)
while A1 < (1 − alphaMRUCL)
    Z0 ← Z1
    Z1 ← Z1 + 5.0
    A0 ← A1
    A1 ← P3(Z1)
Zguess ← linterp(A, Z, 1 − alphaMRUCL)
Zguess
```

seed1 := Zseed1(5.0)

$D(x) := P3(x) - (1 - \mathrm{alphaMRUCL})$

$$qD4 := \frac{\mathrm{zbrent}(D, \mathrm{seed1} - 5.0, \mathrm{seed1} + 5.0, \mathrm{TOL})}{5}$$

$$\blacksquare := \frac{\mathrm{root}\left[\left|P3(\mathrm{seed1}) - (1 - \mathrm{alphaMRUCL})\right|, \mathrm{seed1}\right]}{5}$$

Page 4 of program: ccfsMR.mcd

(4.1) Zseed2(start) :=

```
Zv_0 ← 0.0
Av_0 ← 0.0
Z ← start
while (P3(Z) < alphaMRLCL)
  Z ← Z + 1.0
for i ∈ 1..6
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3(Zv_i)
for i ∈ 7..20
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3(Zv_i)
Zguess ← linterp(Av,Zv,alphaMRLCL)
A ← ratint(Zv,Av,Zguess)
Aguess ← A_0
while |Aguess − alphaMRLCL| > 10^-15
  | if (Aguess − alphaMRLCL) > 10^-15
  |   | Av_1 ← Aguess
  |   | Zv_1 ← Zguess
  | if (Aguess − alphaMRLCL) < −10^-15
  |   | Av_0 ← Aguess
  |   | Zv_0 ← Zguess
  | Zguess ← linterp(Av,Zv,alphaMRLCL)
  | A ← ratint(Zv,Av,Zguess)
  | Aguess ← A_0
Zguess
```

seed2 := Zseed2(1.0)

$$qD3 := \frac{seed2}{5}$$

$$qD3 := \frac{root(|P3(seed2) - alphaMRLCL|, seed2)}{5}$$

Monitor Results

$qD3 = 1.9340341866 \times 10^{-3}$

$qD3 = 1.9340341866 \times 10^{-3}$

(5.1) $$P1prevm(z) := \int_0^{11} \left[\left(5\cdot\frac{W}{z}\right)\cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}}\right]^{vprevm-1}\cdot e^{\frac{z^2-25\cdot W^2}{2\cdot z^2}}\cdot P(W)\,dW$$

$$P2prevm(z) := \left(\frac{z}{5}\right)\cdot\int_{\frac{55}{z}}^{\infty}\left(x\cdot e^{\frac{1-x^2}{2}}\right)^{vprevm-1}\cdot e^{\frac{1-x^2}{2}}\,dx$$

$$cvprevm := \ln(2) + \left(\frac{vprevm}{2}\right)\cdot\ln\left(\frac{vprevm}{2}\right) - \left(\frac{vprevm}{2}\right) - gammln\left(\frac{vprevm}{2}\right)$$

$$P3prevm(z) := \left(\frac{5}{z}\right)\cdot e^{cvprevm}\cdot(P1prevm(z) + P2prevm(z))$$

(5.2) Zseed3(start) :=

```
Z0 ← start
Z1 ← start + 5.0
A0 ← P3prevm(Z0)
A1 ← P3prevm(Z1)
while A1 < (1 − alphaMRUCL)
    Z0 ← Z1
    Z1 ← Z1 + 5.0
    A0 ← A1
    A1 ← P3prevm(Z1)
Zguess ← linterp(A, Z, 1 − alphaMRUCL)
Zguess
```

seed3 := Zseed3(5.0)

Dprevm(x) := P3prevm(x) − (1 − alphaMRUCL)

$$qD4prevm := \frac{zbrent(Dprevm, seed3 - 5.0, seed3 + 5.0, TOL)}{5}$$

$$\blacksquare := \frac{root[\,|P3prevm(seed3) - (1 - alphaMRUCL)|\,, seed3]}{5}$$

Page 6 of program: ccfsMR.mcd

(6.1) Zseed4(start) :=

```
Zv_0 ← 0.0
Av_0 ← 0.0
Z ← start
while (P3prevm(Z) < alphaMRLCL)
  Z ← Z + 1.0
for i ∈ 1..6
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3prevm(Zv_i)
for i ∈ 7..20
  | Zv_i ← Z + (1.0)·(i − 1)
  | Av_i ← P3prevm(Zv_i)
Zguess ← linterp(Av,Zv,alphaMRLCL)
A ← ratint(Zv,Av,Zguess)
Aguess ← A_0
while |Aguess − alphaMRLCL| > 10^−15
  | if (Aguess − alphaMRLCL) > 10^−15
  |   | Av_1 ← Aguess
  |   | Zv_1 ← Zguess
  | if (Aguess − alphaMRLCL) < −10^−15
  |   | Av_0 ← Aguess
  |   | Zv_0 ← Zguess
  | Zguess ← linterp(Av,Zv,alphaMRLCL)
  | A ← ratint(Zv,Av,Zguess)
  | Aguess ← A_0
Zguess
```

seed4 := Zseed4(1.0)

$$qD3prevm := \frac{seed4}{5}$$

$$qD3prevm := \frac{root(|P3prevm(seed4) - alphaMRLCL|, seed4)}{5}$$

Monitor Results

$qD3prevm = 1.9793483369 \times 10^{-3}$

$qD3prevm = 1.9793483369 \times 10^{-3}$

(7.1) $d2starMR := \left(d2^2 + d2^2 \cdot r\right)^{0.5}$ $adj\_alpha := 1 - \frac{alphaInd}{2}$

$d2starMRprevm := \left(d2^2 + d2^2 \cdot rprevm\right)^{0.5}$ $crit\_t := qt(adj\_alpha, v)$ $crit\_z := qnorm(adj\_alpha, 0, 1)$

(7.2) $E21 := \left(\frac{crit\_t}{d2starMR}\right) \cdot \left(\frac{m-1}{m}\right)^{0.5}$ $E22 := \left(\frac{crit\_t}{d2starMR}\right) \cdot \left(\frac{m+1}{m}\right)^{0.5}$ $E2 := \frac{crit\_z}{d2}$

$D41 := \frac{m \cdot qD4prevm}{d2starMRprevm \cdot (m-1) + qD4prevm}$ $D42 := \frac{qD4}{d2starMR}$ $D4 := \frac{wD4}{d2}$

$D31 := \frac{m \cdot qD3prevm}{d2starMRprevm \cdot (m-1) + qD3prevm}$ $D32 := \frac{qD3}{d2starMR}$ $D3 := \frac{wD3}{d2}$

FINAL RESULTS:

(1) alphaInd = 0.0027

(2) alphaMRUCL = 0.005

(3) alphaMRLCL = 0.001

(4) m = 5

Control Chart Factors

First Stage	Second Stage	Conventional
E21 = 7.34996	E22 = 9.00182	E2 = 2.6586603867
D41 = 3.83736	D42 = 9.2788	D4 = 3.5180951058
D31 = 0.00196	D32 = 0.00157	D3 = 0.0015707967

For:

(# of MRs) = m − 1 = 4

ν = 2.8121232012

d2starMR = 1.23124

Mean of the Distribution of the Range for Subgroup Size Two and the Variance of the Distribution of the Mean Moving Range

d2 = 1.1283791671 $d2^2 \cdot r = 0.2427219561$

(# of MRs) = (m − 1) − 1 = 3

νprevm = 2.19944

d2starMRprevm = 1.26009

Harter, Clemm, and Guthrie's (1959) Table II.2 Results for n=2

qD4 = 11.42447	qD4prevm = 16.63594	wD4 = 3.9697452252
qD3 = 0.00193	qD3prevm = 0.00198	wD3 = 0.0017724543

APPENDIX E.3 – Tables Generated from ccfsMR.mcd

Table E.3.1. ν (Degrees of Freedom) and $d_2^*(MR)$ (d2starMR) Values

m	ν	$d_2^*(MR)$	m	ν	$d_2^*(MR)$
2	1.00000	1.41421	**16**	9.49655	1.15842
3	1.58682	1.31072	**17**	10.10245	1.15660
4	2.19944	1.26009	**18**	10.70825	1.15499
5	2.81212	1.23124	**19**	11.31397	1.15356
6	3.42328	1.21271	**20**	11.91962	1.15227
7	4.03312	1.19982	**25**	14.94711	1.14740
8	4.64196	1.19034	**30**	17.97377	1.14418
9	5.25006	1.18308	**50**	30.07712	1.13780
10	5.85761	1.17734	**75**	45.20381	1.13464
11	6.46473	1.17269	**100**	60.32965	1.13306
12	7.07152	1.16885	**150**	90.58051	1.13150
13	7.67805	1.16562	**200**	120.83094	1.13072
14	8.28438	1.16287	**250**	151.08121	1.13025
15	8.89053	1.16049	**300**	181.33139	1.12994
			d_2	1.1283791671	

Table E.3.2. Partial Re-creation of Table II.2
for P=0.995 (alphaMRUCL=0.005) and P=0.001
(alphaMRLCL=0.001) in Harter, Clemm, and Guthrie (1959)

m	qD4	qD3	m	qD4	qD3
2	180.05956	0.00222	16	5.13700	0.00182
3	34.23460	0.00206	17	5.05126	0.00182
4	16.63594	0.00198	18	4.97717	0.00181
5	11.42447	0.00193	19	4.91251	0.00181
6	9.12057	0.00190	20	4.85560	0.00181
7	7.86303	0.00188	25	4.64991	0.00180
8	7.08300	0.00187	30	4.52154	0.00180
9	6.55624	0.00186	50	4.28392	0.00179
10	6.17842	0.00185	75	4.17390	0.00178
11	5.89503	0.00184	100	4.12094	0.00178
12	5.67501	0.00184	150	4.06929	0.00178
13	5.49947	0.00183	200	4.04394	0.00178
14	5.35628	0.00183	250	4.02888	0.00178
15	5.23734	0.00182	300	4.01890	0.00177
			∞	3.9697452252	0.0017724543

Table E.3.3. Two Stage Short Run Control Chart Factors for alphaInd=0.0027, alphaMRUCL=0.005, and alphaMRLCL=0.001

m	E21	D41	D31	E22	D42	D32
2	117.89184	----	----	204.19466	127.32134	0.00157
3	22.24670	2.95360	0.00235	31.46159	26.11886	0.00157
4	10.72641	3.58790	0.00209	13.84773	13.20218	0.00157
5	7.34996	3.83736	0.00196	9.00182	9.27880	0.00157
6	5.87022	3.89898	0.00188	6.94574	7.52080	0.00157
7	5.06862	3.89368	0.00183	5.85274	6.55349	0.00157
8	4.57470	3.86822	0.00179	5.18723	5.95038	0.00157
9	4.24308	3.83885	0.00177	4.74391	5.54166	0.00157
10	4.00644	3.81088	0.00175	4.42928	5.24776	0.00157
11	3.82972	3.78583	0.00173	4.19525	5.02691	0.00157
12	3.69307	3.76385	0.00171	4.01479	4.85521	0.00157
13	3.58441	3.74470	0.00170	3.87161	4.71806	0.00157
14	3.49606	3.72800	0.00169	3.75537	4.60610	0.00157
15	3.42287	3.71338	0.00168	3.65920	4.51303	0.00157
16	3.36128	3.70053	0.00168	3.57836	4.43448	0.00157
17	3.30877	3.68916	0.00167	3.50948	4.36732	0.00157
18	3.26348	3.67906	0.00166	3.45012	4.30926	0.00157
19	3.22404	3.67004	0.00166	3.39843	4.25857	0.00157
20	3.18937	3.66194	0.00165	3.35304	4.21395	0.00157
25	3.06459	3.63141	0.00164	3.18972	4.05258	0.00157
30	2.98713	3.61141	0.00162	3.08841	3.95179	0.00157
50	2.84471	3.57258	0.00160	2.90218	3.76510	0.00157
75	2.77924	3.55387	0.00159	2.81655	3.67863	0.00157
100	2.74785	3.54471	0.00159	2.77546	3.63699	0.00157
150	2.71730	3.53570	0.00158	2.73548	3.59637	0.00157
200	2.70234	3.53124	0.00158	2.71588	3.57644	0.00157
250	2.69346	3.52859	0.00158	2.70425	3.56460	0.00157
300	2.68758	3.52682	0.00158	2.69655	3.55675	0.00157
∞	2.6586603867	3.5180951058	0.0015707967	2.6586603867	3.5180951058	0.0015707967

APPENDIX F.1 – Simulation Program cc.f90 for Chapter 8

```
!     Last change:  C    23 Apr 2001   10:13 pm
!
module random_mod
!
! *****************************************************************
! ***** This module contains the subroutine that generates    *****
! ***** Uniform (0, 1) random variates using the Marse-Roberts *****
! ***** code (see Marse and Roberts (1983))                    *****
! *****************************************************************
!
  implicit none
!
  contains
!
!
!
!
!
  subroutine random(uniran, seed)
!
! ****************************************************
! ***** This subroutine generates Uniform (0, 1)    *****
! ***** random variates using the Marse-Roberts code *****
! ****************************************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    REAL(KIND=DOUBLE), INTENT(OUT) :: uniran
    INTEGER, INTENT(IN OUT) :: seed
    INTEGER :: hi15, hi31, low15, lowprd, ovflow
    INTEGER, PARAMETER :: mult1 = 24112, mult2 = 26143, &
                          b2e15 = 32768, b2e16 = 65536, &
                          modlus = 2147483647
!
    hi15 = seed / b2e16
    lowprd = (seed - hi15 * b2e16) * mult1
    low15 = lowprd / b2e16
    hi31 = hi15 * mult1 + low15
    ovflow = hi31 / b2e15
    seed = (((lowprd - low15 * b2e16) - modlus) + &
           (hi31 - ovflow * b2e15) * b2e16) + ovflow
!
    if (seed < 0) seed = seed + modlus
!
    hi15 = seed / b2e16
    lowprd = (seed - hi15 * b2e16) * mult2
    low15 = lowprd / b2e16
    hi31 = hi15 * mult2 + low15
    ovflow = hi31 / b2e15
    seed = (((lowprd - low15 * b2e16) - modlus) + &
           (hi31 - ovflow * b2e15) * b2e16) + ovflow
!
    if (seed < 0) seed = seed + modlus
!
    uniran = (2 * (seed / 256) + 1) / 16777216.0
!
    return
```

```
  end subroutine random
!
!
!
!
!
end module random_mod
!
!
!
!
!
!
!
!
!
!
module Stage_2
!
! **************************************************
! ***** This module contains the subroutines  *****
! ***** that perform Stage 2 control charting *****
! ***** for each control chart combination    *****
! **************************************************
!
  USE random_mod
  implicit none
!
  contains
!
!
!
!
!
  subroutine Xbar_R_2(mean, sd, n, m_Xbar, m_R, Xbar2, Range2, &
                      answer2, shifttype2, shiftsize2mean, &
                      shiftsize2sd, shifttime2, falsealarm, RL, seed)
!
! *******************************************************
! ***** Stage 2 Control Charting for (Xbar, R) Charts *****
! *******************************************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    INTEGER :: i, j, subgroup
    INTEGER, INTENT(IN) :: n, m_Xbar, m_R, shifttime2
    INTEGER, INTENT(IN OUT) :: seed
    REAL(KIND=DOUBLE) :: UCCFR2, LCCFR2, CCFXbar2, pi
    REAL(KIND=DOUBLE) :: Xbarsum, Rsum, Xbarbar, Rbar
    REAL(KIND=DOUBLE) :: UCLR2, LCLR2, UCLXbar2, LCLXbar2
    REAL(KIND=DOUBLE) :: Xsum, r1, r2, X, large, small, Xbar, R
    REAL(KIND=DOUBLE) :: temp1, temp2, temp3, temp4, temp5
    REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
    REAL(KIND=DOUBLE), INTENT(IN) :: Xbar2(m_Xbar), Range2(m_R)
    REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize2mean, shiftsize2sd
    REAL(KIND=DOUBLE), INTENT(OUT) :: falsealarm, RL
    CHARACTER(LEN=1), INTENT(IN) :: answer2
```

```
    CHARACTER(LEN=2), INTENT(IN) :: shifttype2
!
    REWIND(1)
    falsealarm = 0
    subgroup = 0
    Xbarsum = 0
    Rsum = 0
!
! Read second stage short run control chart factors from input file
!
    do i = 1, (m_R - 1)
      READ(1, *)
    end do
!
    READ(1, *) temp1, temp2, temp3, temp4, UCCFR2, LCCFR2
!
    REWIND(1)
!
    do i = 1, (m_Xbar - 1)
      READ(1, *)
    end do

    READ(1, *) temp1, temp2, temp3, CCFXbar2, temp4, temp5
!
    temp1 = temp1 * temp2 * temp3 * temp4 * temp5
    pi = ACOS(-1.0)
!
! Construct second stage control limits
!
    do i = 1, m_Xbar
      Xbarsum = Xbarsum + Xbar2(i)
    end do
!
    do i = 1, m_R
      Rsum = Rsum + Range2(i)
    end do
!
    Xbarbar = Xbarsum / m_Xbar
    Rbar = Rsum / m_R
    UCLR2 = UCCFR2 * Rbar
    LCLR2 = LCCFR2 * Rbar
    UCLXbar2 = Xbarbar + CCFXbar2 * Rbar
    LCLXbar2 = Xbarbar - CCFXbar2 * Rbar
!
! If a shift occurs in Stage 2, then determine the
! number of false alarms before the shift occurs
!
    if (answer2 == 'Y') then
!
      do i = 1, (shifttime2 - 1)
        Xsum = 0
!
        do j = 1, n
          call random(r1, seed)
          call random(r2, seed)
!
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
```

```
          Xsum = Xsum + X
!
          if (j == 1) then
            large = X
            small = X
          else
!
            if (X > large) large = X
!
            if (X < small) small = X
!
          end if
!
        end do
!
        Xbar = Xsum / n
        R = large - small
!
        if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
            ((R > UCLR2) .or. (R < LCLR2))) &
           falsealarm = falsealarm + 1
!
      end do
!
    end if
!
! Determine run length (RL)
!
    do
      Xsum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if (answer2 == 'Y') then
!
          if (shifttype2 == 'MN') then
            X = (mean + shiftsize2mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'SD') then
            X = mean + (sd + shiftsize2sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'MS') then
            X = (mean + shiftsize2mean) + (sd + shiftsize2sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
        end if
!
        Xsum = Xsum + X
!
        if (j == 1) then
          large = X
```

```
          small = X
        else
!
          if (X > large) large = X
!
          if (X < small) small = X
!
        end if
!
      end do
!
      subgroup = subgroup + 1
      Xbar = Xsum / n
      R = large - small
!
      if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
          ((R > UCLR2) .or. (R < LCLR2))) then
        RL = subgroup
        exit
      end if
!
    end do
!
    return
  end subroutine Xbar_R_2
!
!
!
!
!
  subroutine Xbar_v_2(mean, sd, n, m_Xbar, m_v, Xbar2, v2, &
                      answer2, shifttype2, shiftsize2mean, &
                      shiftsize2sd, shifttime2, falsealarm, RL, seed)
!
! *********************************************************
! ***** Stage 2 Control Charting for (Xbar, v) Charts *****
! *********************************************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    INTEGER :: i, j, subgroup
    INTEGER, INTENT(IN) :: n, m_Xbar, m_v, shifttime2
    INTEGER, INTENT(IN OUT) :: seed
    REAL(KIND=DOUBLE) :: UCCFv2, LCCFv2, CCFXbar2, pi
    REAL(KIND=DOUBLE) :: Xbarsum, vsum, Xbarbar, vbar
    REAL(KIND=DOUBLE) :: UCLv2, LCLv2, UCLXbar2, LCLXbar2
    REAL(KIND=DOUBLE) :: Xsum, X2sum, r1, r2, X, Xbar, v
    REAL(KIND=DOUBLE) :: temp1, temp2, temp3, temp4, temp5
    REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
    REAL(KIND=DOUBLE), INTENT(IN) :: Xbar2(m_Xbar), v2(m_v)
    REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize2mean, shiftsize2sd
    REAL(KIND=DOUBLE), INTENT(OUT) :: falsealarm, RL
    CHARACTER(LEN=1), INTENT(IN) :: answer2
    CHARACTER(LEN=2), INTENT(IN) :: shifttype2
!
    REWIND(1)
    falsealarm = 0
```

```
    subgroup = 0
    Xbarsum = 0
    vsum = 0
!
! Read second stage short run control chart factors from input file
!
    do i = 1, (m_v - 1)
      READ(1, *)
    end do
!
    READ(1, *) temp1, temp2, temp3, temp4, UCCFv2, LCCFv2
!
    REWIND(1)
!
    do i = 1, (m_Xbar - 1)
      READ(1, *)
    end do

    READ(1, *) temp1, temp2, temp3, CCFXbar2, temp4, temp5
!
    temp1 = temp1 * temp2 * temp3 * temp4 * temp5
    pi = ACOS(-1.0)
!
! Construct second stage control limits
!
    do i = 1, m_Xbar
      Xbarsum = Xbarsum + Xbar2(i)
    end do
!
    do i = 1, m_v
      vsum = vsum + v2(i)
    end do
!
    Xbarbar = Xbarsum / m_Xbar
    vbar = vsum / m_v
    UCLv2 = UCCFv2 * vbar
    LCLv2 = LCCFv2 * vbar
    UCLXbar2 = Xbarbar + CCFXbar2 * SQRT(vbar)
    LCLXbar2 = Xbarbar - CCFXbar2 * SQRT(vbar)
!
! If a shift occurs in Stage 2, then determine the
! number of false alarms before the shift occurs
!
    if (answer2 == 'Y') then
!
      do i = 1, (shifttime2 - 1)
        Xsum = 0
        X2sum = 0
!
        do j = 1, n
          call random(r1, seed)
          call random(r2, seed)
!
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          Xsum = Xsum + X
          X2sum = X2sum + (X**2)
        end do
```

```
!
        Xbar = Xsum / n
        v = (n * X2sum - (Xsum**2)) / (n * (n - 1.))
!
        if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
            ((v > UCLv2) .or. (v < LCLv2))) &
           falsealarm = falsealarm + 1
!
      end do
!
    end if
!
! Determine run length (RL)
!
    do
      Xsum = 0
      X2sum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if (answer2 == 'Y') then
!
          if (shifttype2 == 'MN') then
            X = (mean + shiftsize2mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'SD') then
            X = mean + (sd + shiftsize2sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'MS') then
            X = (mean + shiftsize2mean) + (sd + shiftsize2sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
        end if
!
        Xsum = Xsum + X
        X2sum = X2sum + (X**2)
      end do
!
      subgroup = subgroup + 1
      Xbar = Xsum / n
      v = (n * X2sum - (Xsum**2)) / (n * (n - 1.))
!
      if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
          ((v > UCLv2) .or. (v < LCLv2))) then
        RL = subgroup
        exit
      end if
!
    end do
!
    return
```

```
 end subroutine Xbar_v_2
!
!
!
!
!
 subroutine Xbar_sqrtv_2(mean, sd, n, m_Xbar, m_v, Xbar2, v2, &
                         answer2, shifttype2, shiftsize2mean, &
                         shiftsize2sd, shifttime2, falsealarm, RL, &
                         seed)
!
! ***********************************************************
! ***** Stage 2 Control Charting for (Xbar, sqrtv) Charts *****
! ***********************************************************
!
   implicit none
   INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
   INTEGER :: i, j, subgroup
   INTEGER, INTENT(IN) :: n, m_Xbar, m_v, shifttime2
   INTEGER, INTENT(IN OUT) :: seed
   REAL(KIND=DOUBLE) :: UCCFsqrtv2, LCCFsqrtv2, CCFXbar2, pi
   REAL(KIND=DOUBLE) :: Xbarsum, vsum, Xbarbar, vbar
   REAL(KIND=DOUBLE) :: UCLsqrtv2, LCLsqrtv2, UCLXbar2, LCLXbar2
   REAL(KIND=DOUBLE) :: Xsum, X2sum, r1, r2, X, Xbar, sqrtv
   REAL(KIND=DOUBLE) :: temp1, temp2, temp3, temp4, temp5
   REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
   REAL(KIND=DOUBLE), INTENT(IN) :: Xbar2(m_Xbar), v2(m_v)
   REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize2mean, shiftsize2sd
   REAL(KIND=DOUBLE), INTENT(OUT) :: falsealarm, RL
   CHARACTER(LEN=1), INTENT(IN) :: answer2
   CHARACTER(LEN=2), INTENT(IN) :: shifttype2
!
   REWIND(1)
   falsealarm = 0
   subgroup = 0
   Xbarsum = 0
   vsum = 0
!
! Read second stage short run control chart factors from input file
!
   do i = 1, (m_v - 1)
     READ(1, *)
   end do
!
   READ(1, *) temp1, temp2, temp3, temp4, UCCFsqrtv2, LCCFsqrtv2
!
   REWIND(1)
!
   do i = 1, (m_Xbar - 1)
     READ(1, *)
   end do

   READ(1, *) temp1, temp2, temp3, CCFXbar2, temp4, temp5
!
   temp1 = temp1 * temp2 * temp3 * temp4 * temp5
   pi = ACOS(-1.0)
!
```

```
! Construct second stage control limits
!
    do i = 1, m_Xbar
      Xbarsum = Xbarsum + Xbar2(i)
    end do
!
    do i = 1, m_v
      vsum = vsum + v2(i)
    end do
!
    Xbarbar = Xbarsum / m_Xbar
    vbar = vsum / m_v
    UCLsqrtv2 = UCCFsqrtv2 * SQRT(vbar)
    LCLsqrtv2 = LCCFsqrtv2 * SQRT(vbar)
    UCLXbar2 = Xbarbar + CCFXbar2 * SQRT(vbar)
    LCLXbar2 = Xbarbar - CCFXbar2 * SQRT(vbar)
!
! If a shift occurs in Stage 2, then determine the
! number of false alarms before the shift occurs
!
    if (answer2 == 'Y') then
!
      do i = 1, (shifttime2 - 1)
        Xsum = 0
        X2sum = 0
!
        do j = 1, n
          call random(r1, seed)
          call random(r2, seed)
!
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          Xsum = Xsum + X
          X2sum = X2sum + (X**2)
        end do
!
        Xbar = Xsum / n
        sqrtv = SQRT((n * X2sum - (Xsum**2)) / (n * (n - 1.)))
!
        if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
            ((sqrtv > UCLsqrtv2) .or. (sqrtv < LCLsqrtv2))) &
           falsealarm = falsealarm + 1
!
      end do
!
    end if
!
! Determine run length (RL)
!
    do
      Xsum = 0
      X2sum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if (answer2 == 'Y') then
```

```
!
        if (shifttype2 == 'MN') then
          X = (mean + shiftsize2mean) + sd * &
              ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        else if (shifttype2 == 'SD') then
          X = mean + (sd + shiftsize2sd) * &
              ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        else if (shifttype2 == 'MS') then
          X = (mean + shiftsize2mean) + (sd + shiftsize2sd) * &
              ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        end if
!
      else
        X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                         (COS(2. * pi * r2)))
      end if
!
      Xsum = Xsum + X
      X2sum = X2sum + (X**2)
    end do
!
    subgroup = subgroup + 1
    Xbar = Xsum / n
    sqrtv = SQRT((n * X2sum - (Xsum**2)) / (n * (n - 1.)))
!
    if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
        ((sqrtv > UCLsqrtv2) .or. (sqrtv < LCLsqrtv2))) then
      RL = subgroup
      exit
    end if
!
  end do
!
  return
end subroutine Xbar_sqrtv_2
!
!
!
!
!
subroutine Xbar_s_2(mean, sd, n, m_Xbar, m_s, Xbar2, s2, &
                    answer2, shifttype2, shiftsize2mean, &
                    shiftsize2sd, shifttime2, falsealarm, RL, seed)
!
! *********************************************************
! ***** Stage 2 Control Charting for (Xbar, s) Charts *****
! *********************************************************
!
  implicit none
  INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
  INTEGER :: i, j, subgroup
  INTEGER, INTENT(IN) :: n, m_Xbar, m_s, shifttime2
  INTEGER, INTENT(IN OUT) :: seed
  REAL(KIND=DOUBLE) :: UCCFs2, LCCFs2, CCFXbar2, pi
  REAL(KIND=DOUBLE) :: Xbarsum, ssum, Xbarbar, sbar
  REAL(KIND=DOUBLE) :: UCLs2, LCLs2, UCLXbar2, LCLXbar2
  REAL(KIND=DOUBLE) :: Xsum, X2sum, r1, r2, X, Xbar, s
```

```
    REAL(KIND=DOUBLE) :: temp1, temp2, temp3, temp4, temp5
    REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
    REAL(KIND=DOUBLE), INTENT(IN) :: Xbar2(m_Xbar), s2(m_s)
    REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize2mean, shiftsize2sd
    REAL(KIND=DOUBLE), INTENT(OUT) :: falsealarm, RL
    CHARACTER(LEN=1), INTENT(IN) :: answer2
    CHARACTER(LEN=2), INTENT(IN) :: shifttype2
!
    REWIND(1)
    falsealarm = 0
    subgroup = 0
    Xbarsum = 0
    ssum = 0
!
! Read second stage short run control chart factors from input file
!
    do i = 1, (m_s - 1)
      READ(1, *)
    end do
!
    READ(1, *) temp1, temp2, temp3, temp4, UCCFs2, LCCFs2
!
    REWIND(1)
!
    do i = 1, (m_Xbar - 1)
      READ(1, *)
    end do

    READ(1, *) temp1, temp2, temp3, CCFXbar2, temp4, temp5
!
    temp1 = temp1 * temp2 * temp3 * temp4 * temp5
    pi = ACOS(-1.0)
!
! Construct second stage control limits
!
    do i = 1, m_Xbar
      Xbarsum = Xbarsum + Xbar2(i)
    end do
!
    do i = 1, m_s
      ssum = ssum + s2(i)
    end do
!
    Xbarbar = Xbarsum / m_Xbar
    sbar = ssum / m_s
    UCLs2 = UCCFs2 * sbar
    LCLs2 = LCCFs2 * sbar
    UCLXbar2 = Xbarbar + CCFXbar2 * sbar
    LCLXbar2 = Xbarbar - CCFXbar2 * sbar
!
! If a shift occurs in Stage 2, then determine the
! number of false alarms before the shift occurs
!
    if (answer2 == 'Y') then
!
      do i = 1, (shifttime2 - 1)
        Xsum = 0
```

```
        X2sum = 0
!
        do j = 1, n
          call random(r1, seed)
          call random(r2, seed)
!
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          Xsum = Xsum + X
          X2sum = X2sum + (X**2)
        end do
!
        Xbar = Xsum / n
        s = SQRT((n * X2sum - (Xsum**2)) / (n * (n - 1.)))
!
        if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
            ((s > UCLs2) .or. (s < LCLs2))) &
             falsealarm = falsealarm + 1
!
      end do
!
    end if
!
! Determine run length (RL)
!
    do
      Xsum = 0
      X2sum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if (answer2 == 'Y') then
!
          if (shifttype2 == 'MN') then
            X = (mean + shiftsize2mean) + sd * &
                 ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'SD') then
            X = mean + (sd + shiftsize2sd) * &
                 ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'MS') then
            X = (mean + shiftsize2mean) + (sd + shiftsize2sd) * &
                 ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
        end if
!
        Xsum = Xsum + X
        X2sum = X2sum + (X**2)
      end do
!
      subgroup = subgroup + 1
      Xbar = Xsum / n
      s = SQRT((n * X2sum - (Xsum**2)) / (n * (n - 1.)))
```

```
!
     if (((Xbar > UCLXbar2) .or. (Xbar < LCLXbar2)) .or. &
         ((s > UCLs2) .or. (s < LCLs2))) then
       RL = subgroup
       exit
     end if
!
   end do
!
   return
 end subroutine Xbar_s_2
!
!
!
!
!
 subroutine X_MR_2(mean, sd, m_X, m_MR, X2, MR2, &
                  answer2, shifttype2, shiftsize2mean, &
                  shiftsize2sd, shifttime2, falsealarm, RL, seed)
!
! *******************************************************
! ***** Stage 2 Control Charting for (X, MR) Charts *****
! *******************************************************
!
! Note: m_MR IS THE NUMBER OF SUBGROUPS, NOT THE NUMBER OF MRs
!
   implicit none
   INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
   INTEGER :: i, flag, subgroup
   INTEGER, INTENT(IN) :: m_X, m_MR, shifttime2
   INTEGER, INTENT(IN OUT) :: seed
   REAL(KIND=DOUBLE) :: UCCFMR2, LCCFMR2, CCFX2, pi
   REAL(KIND=DOUBLE) :: Xsum, MRsum, Xbar, MRbar
   REAL(KIND=DOUBLE) :: UCLMR2, LCLMR2, UCLX2, LCLX2
   REAL(KIND=DOUBLE) :: r1, r2, X_1, X_2, MR
   REAL(KIND=DOUBLE) :: temp1, temp2, temp3, temp4, temp5
   REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
   REAL(KIND=DOUBLE), INTENT(IN) :: X2(m_X), MR2(m_MR - 1)
   REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize2mean, shiftsize2sd
   REAL(KIND=DOUBLE), INTENT(OUT) :: falsealarm, RL
   CHARACTER(LEN=1), INTENT(IN) :: answer2
   CHARACTER(LEN=2), INTENT(IN) :: shifttype2
!
   REWIND(1)
   falsealarm = 0
   subgroup = 0
   Xsum = 0
   MRsum = 0
   flag = 0
!
! Read second stage short run control chart factors from input file
!
   do i = 2, (m_MR - 1)
     READ(1, *)
   end do
!
   READ(1, *) temp1, temp2, temp3, temp4, UCCFMR2, LCCFMR2
```

```
!
   REWIND(1)
!
   do i = 2, (m_X - 1)
     READ(1, *)
   end do

   READ(1, *) temp1, temp2, temp3, CCFX2, temp4, temp5
!
   temp1 = temp1 * temp2 * temp3 * temp4 * temp5
   pi = ACOS(-1.0)
!
! Construct second stage control limits
!
   do i = 1, m_X
     Xsum = Xsum + X2(i)
   end do
!
   do i = 1, (m_MR - 1)
     MRsum = MRsum + MR2(i)
   end do
!
   Xbar = Xsum / m_X
   MRbar = MRsum / (m_MR - 1)
   UCLMR2 = UCCFMR2 * MRbar
   LCLMR2 = LCCFMR2 * MRbar
   UCLX2 = Xbar + CCFX2 * MRbar
   LCLX2 = Xbar - CCFX2 * MRbar
!
! If a shift occurs in Stage 2, then determine the
! number of false alarms before the shift occurs
!
   if ((answer2 == 'Y') .and. (shifttime2 == 2)) then
     call random(r1, seed)
     call random(r2, seed)
!
     X_1 = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                        (COS(2. * pi * r2)))
!
     if ((X_1 > UCLX2) .or. (X_1 < LCLX2)) &
        falsealarm = falsealarm + 1
!
     flag = 1
   end if
!
   if ((answer2 == 'Y') .and. (shifttime2 > 2)) then
!
     do i = 1, (shifttime2 - 2)
!
       if (i == 1) then
         call random(r1, seed)
         call random(r2, seed)
!
         X_1 = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                            (COS(2. * pi * r2)))
!
         if ((X_1 > UCLX2) .or. (X_1 < LCLX2)) &
```

```
          falsealarm = falsealarm + 1
!
        end if
!
        call random(r1, seed)
        call random(r2, seed)
!
        X_2 = mean + sd * ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        MR = ABS(X_2 - X_1)
!
        if (((X_2 > UCLX2) .or. (X_2 < LCLX2)) .or. &
            ((MR > UCLMR2) .or. (MR < LCLMR2))) &
           falsealarm = falsealarm + 1
!
        X_1 = X_2
        flag = 1
      end do
!
    end if
!
! Determine run length (RL)
!
    do
!
      if (flag == 0) then
        call random(r1, seed)
        call random(r2, seed)
!
        if (answer2 == 'Y') then
!
          if (shifttype2 == 'MN') then
            X_1 = (mean + shiftsize2mean) + sd * &
                  ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'SD') then
            X_1 = mean + (sd + shiftsize2sd) * &
                  ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype2 == 'MS') then
            X_1 = (mean + shiftsize2mean) + (sd + shiftsize2sd) * &
                  ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X_1 = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                             (COS(2. * pi * r2)))
        end if
!
        subgroup = subgroup + 1
        flag = 1
!
        if ((X_1 > UCLX2) .or. (X_1 < LCLX2)) then
          RL = subgroup
          exit
        end if
!
      end if
!
      call random(r1, seed)
```

```
      call random(r2, seed)
!
      if (answer2 == 'Y') then
!
        if (shifttype2 == 'MN') then
          X_2 = (mean + shiftsize2mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        else if (shifttype2 == 'SD') then
          X_2 = mean + (sd + shiftsize2sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        else if (shifttype2 == 'MS') then
          X_2 = (mean + shiftsize2mean) + (sd + shiftsize2sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
        end if
!
      else
        X_2 = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
      end if
!
      subgroup = subgroup + 1
      MR = ABS(X_2 - X_1)
!
      if (((X_2 > UCLX2) .or. (X_2 < LCLX2)) .or. &
          ((MR > UCLMR2) .or. (MR < LCLMR2))) then
        RL = subgroup
        exit
      end if
!
      X_1 = X_2
    end do
!
    return
  end subroutine X_MR_2
!
!
!
!
!
end module Stage_2
!
!
!
!
!
!
!
!
!
!
module D_and_R
!
! **************************************************************
! ***** This module contains the subroutines that perform  *****
! ***** each of the six Delete and Revise (D&R) procedures *****
! **************************************************************
!
```

```
  implicit none
!
  contains
!
!
!
!
!
  subroutine D_and_R_1(m, save_m, choice1, Cen1, Spread1, &
                       Cen1status, Spread1status, new_m, &
                       Cen2, Spread2, count1, stops)
!
! ***************************
! ***** D&R Procedure 1 *****
! ***************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    INTEGER :: i, flag
    INTEGER, INTENT(IN) :: save_m, choice1
    INTEGER, INTENT(OUT) :: new_m, count1, stops
    INTEGER, INTENT(IN OUT) :: m
    REAL(KIND=DOUBLE) :: Spread1temp(save_m), Cen1temp(save_m)
    REAL(KIND=DOUBLE) :: Spread1sum, Cen1sum, Spread1bar, Cen1bar
    REAL(KIND=DOUBLE) :: CCFCen1, UCCFSpread1, LCCFSpread1
    REAL(KIND=DOUBLE) :: UCLSpread1, LCLSpread1, UCLCen1, LCLCen1
    REAL(KIND=DOUBLE), INTENT(OUT) :: Spread2(save_m), Cen2(save_m)
    REAL(KIND=DOUBLE), INTENT(IN OUT) :: Spread1(save_m), Cen1(save_m)
    CHARACTER(LEN=1) :: Spread1statustemp(save_m)
    CHARACTER(LEN=1) :: Cen1statustemp(save_m)
    CHARACTER(LEN=1), INTENT(IN OUT) :: Spread1status(save_m)
    CHARACTER(LEN=1), INTENT(IN OUT) :: Cen1status(save_m)
!
    m = save_m
    count1 = 0
!
    do
      REWIND(1)
      new_m = 0
      Spread1temp = 0
      Cen1temp = 0
      Spread1statustemp = ' '
      Cen1statustemp = ' '
      Spread1sum = 0
      Cen1sum = 0
      flag = 0
!
! Delete out-of-control (OOC) subgroups
!
      do i = 1, m
!
        if ((Spread1status(i) == 'I') .and. &
            (Cen1status(i) == 'I')) then
          new_m = new_m + 1
          Spread1temp(new_m) = Spread1(i)
          Spread1sum = Spread1sum + Spread1temp(new_m)
          Cen1temp(new_m) = Cen1(i)
```

```
          Cen1sum = Cen1sum + Cen1temp(new_m)
        else
          cycle
        end if
!
      end do
!
      if (new_m == 0) then
        WRITE(*, *)
        WRITE(*, *) "(# of subgroups) = 0 in D&R procedure 1"
        WRITE(*, *) "- replication does not count"
        return
      end if
!
      if (new_m == m) exit
!
      if (new_m == 1) then
        WRITE(*, *)
        WRITE(*, *) "D&R procedure 1 stopped"
        WRITE(*, *) "- (# of subgroups) = 1"
        stops = stops + 1
        exit
      end if
!
! Read first stage short run control chart factors from input file
!
      do i = 1, (new_m - 1)
        READ(1, *)
      end do
!
      READ(1, *) CCFCen1, UCCFSpread1, LCCFSpread1
!
! Construct first stage control limits
!
      Cen1bar = Cen1sum / new_m
      Spread1bar = Spread1sum / new_m
!
      if (choice1 == 2) then
        UCLSpread1 = UCCFSpread1 * Spread1bar
        LCLSpread1 = LCCFSpread1 * Spread1bar
        UCLCen1 = Cen1bar + CCFCen1 * SQRT(Spread1bar)
        LCLCen1 = Cen1bar - CCFCen1 * SQRT(Spread1bar)
      else if (choice1 == 3) then
        UCLSpread1 = UCCFSpread1 * SQRT(Spread1bar)
        LCLSpread1 = LCCFSpread1 * SQRT(Spread1bar)
        UCLCen1 = Cen1bar + CCFCen1 * SQRT(Spread1bar)
        LCLCen1 = Cen1bar - CCFCen1 * SQRT(Spread1bar)
      else
        UCLSpread1 = UCCFSpread1 * Spread1bar
        LCLSpread1 = LCCFSpread1 * Spread1bar
        UCLCen1 = Cen1bar + CCFCen1 * Spread1bar
        LCLCen1 = Cen1bar - CCFCen1 * Spread1bar
      end if
!
! Determine out-of-control (OOC) subgroups
!
      do i = 1, new_m
```

```
!
        if ((Spread1temp(i) > UCLSpread1) .or. &
            (Spread1temp(i) < LCLSpread1)) then
          Spread1statustemp(i) = 'O'
          flag = 1
        else
          Spread1statustemp(i) = 'I'
        end if
!
        if ((Cen1temp(i) > UCLCen1) .or. &
            (Cen1temp(i) < LCLCen1)) then
          Cen1statustemp(i) = 'O'
          flag = 1
        else
          Cen1statustemp(i) = 'I'
        end if
!
      end do
!
      if (flag == 0) exit
!
      m = new_m
      Spread1 = 0
      Cen1 = 0
      Spread1 = Spread1temp
      Cen1 = Cen1temp
      Spread1status = ' '
      Cen1status = ' '
      Spread1status = Spread1statustemp
      Cen1status = Cen1statustemp
      count1 = 1
    end do
!
    Cen2 = 0
    Spread2 = 0
    Cen2 = Cen1temp
    Spread2 = Spread1temp
!
    return
  end subroutine D_and_R_1
!
!
!
!
!
  subroutine D_and_R_2(m, save_m, choice1, Cen1, Spread1, &
                       Spread1status, mCen, mSpread, Cen2, &
                       Spread2, count2Spread, count2Cen, stops)
!
! **************************
! ***** D&R Procedure 2 *****
! **************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    INTEGER :: i, flag
    INTEGER, INTENT(IN) :: save_m, choice1
```

```
    INTEGER, INTENT(OUT) :: mSpread, mCen
    INTEGER, INTENT(OUT) :: count2Spread, count2Cen, stops
    INTEGER, INTENT(IN OUT) :: m
    REAL(KIND=DOUBLE) :: Spread1temp(save_m), Cen1temp(save_m)
    REAL(KIND=DOUBLE) :: Spread1sum, Cen1sum, Spread1bar, Cen1bar
    REAL(KIND=DOUBLE) :: CCFCen1, UCCFSpread1, LCCFSpread1, temp1
    REAL(KIND=DOUBLE) :: UCLSpread1, LCLSpread1, UCLCen1, LCLCen1
    REAL(KIND=DOUBLE), INTENT(OUT) :: Spread2(save_m), Cen2(save_m)
    REAL(KIND=DOUBLE), INTENT(IN OUT) :: Spread1(save_m), Cen1(save_m)
    CHARACTER(LEN=1) :: Spread1statustemp(save_m)
    CHARACTER(LEN=1), INTENT(IN OUT) :: Spread1status(save_m)
!
! D&R procedure 2 for the control chart for spread
!
    m = save_m
    count2Spread = 0
!
    do
      REWIND(1)
      mSpread = 0
      Spread1temp = 0
      Spread1statustemp = ' '
      Spread1sum = 0
      flag = 0
!
! Delete out-of-control (OOC) subgroups
!
      if (choice1 /= 5) then
!
        do i = 1, m
!
          if (Spread1status(i) == 'I') then
            mSpread = mSpread + 1
            Spread1temp(mSpread) = Spread1(i)
            Spread1sum = Spread1sum + Spread1temp(mSpread)
          else
            cycle
          end if
!
        end do
!
      else if (choice1 == 5) then
!
        do i = 1, (m - 1)
!
          if (Spread1status(i) == 'I') then
            mSpread = mSpread + 1
            Spread1temp(mSpread) = Spread1(i)
            Spread1sum = Spread1sum + Spread1temp(mSpread)
          else
            cycle
          end if
!
        end do
!
      end if
!
```

```
      if (mSpread == 0) then
        WRITE(*, *)
        WRITE(*, *) "(# of subgroups for the control chart"
        WRITE(*, *) " for spread) = 0 in D&R procedure 2"
        WRITE(*, *) "- replication does not count"
        return
      end if
!
      Spread1bar = Spread1sum / mSpread
!
      if (choice1 == 5) mSpread = mSpread + 1
!
      if (mSpread == m) exit
!
      if ((choice1 /= 5) .and. (mSpread == 1)) then
        WRITE(*, *)
        WRITE(*, *) "D&R procedure 2 for the control"
        WRITE(*, *) "chart for spread stopped"
        WRITE(*, *) "- (# of subgroups) = 1"
        stops = stops + 1
        exit
      end if
!
      if ((choice1 == 5) .and. (mSpread == 2)) then
        WRITE(*, *)
        WRITE(*, *) "D&R procedure 2 for the control"
        WRITE(*, *) "chart for spread stopped"
        WRITE(*, *) "- (# of subgroups) = 2"
        stops = stops + 1
        exit
      end if
!
! Read first stage short run control chart factors from input file
!
      if (choice1 /= 5) then
!
        do i = 1, (mSpread - 1)
          READ(1, *)
        end do
!
      else if (choice1 == 5) then
!
        do i = 2, (mSpread - 1)
          READ(1, *)
        end do
!
      end if
!
      READ(1, *) temp1, UCCFSpread1, LCCFSpread1
!
! Construct first stage control limits
!
      temp1 = temp1 * 1
!
      if (choice1 == 3) then
        UCLSpread1 = UCCFSpread1 * SQRT(Spread1bar)
        LCLSpread1 = LCCFSpread1 * SQRT(Spread1bar)
```

```
      else
        UCLSpread1 = UCCFSpread1 * Spread1bar
        LCLSpread1 = LCCFSpread1 * Spread1bar
      end if
!
      if (choice1 == 5) mSpread = mSpread - 1
!
! Determine out-of-control (OOC) subgroups
!
      do i = 1, mSpread
!
        if ((Spread1temp(i) > UCLSpread1) .or. &
            (Spread1temp(i) < LCLSpread1)) then
          Spread1statustemp(i) = 'O'
          flag = 1
        else
          Spread1statustemp(i) = 'I'
        end if
!
      end do
!
      if (choice1 == 5) mSpread = mSpread + 1
!
      if (flag == 0) exit
!
      m = mSpread
      Spread1 = 0
      Spread1 = Spread1temp
      Spread1status = ' '
      Spread1status = Spread1statustemp
      count2Spread = 1
    end do
!
    Spread2 = 0
    Spread2 = Spread1temp
!
! D&R procedure 2 for the control chart for centering
!
    m = save_m
    count2Cen = 0
!
    do
      REWIND(1)
      mCen = 0
      Cen1temp = 0
      Cen1sum = 0
!
      do i = 1, m
        Cen1sum = Cen1sum + Cen1(i)
      end do
!
! Read first stage short run control chart factor from input file
!
      if (choice1 /= 5) then
!
        do i = 1, (m - 1)
          READ(1, *)
```

```
        end do
!
      else if (choice1 == 5) then
!
        do i = 2, (m - 1)
          READ(1, *)
        end do
!
      end if
!
      READ(1, *) CCFCen1
!
! Construct first stage control limits
!
      Cen1bar = Cen1sum / m
!
      if ((choice1 == 2) .or. (choice1 == 3)) then
        UCLCen1 = Cen1bar + CCFCen1 * SQRT(Spread1bar)
        LCLCen1 = Cen1bar - CCFCen1 * SQRT(Spread1bar)
      else
        UCLCen1 = Cen1bar + CCFCen1 * Spread1bar
        LCLCen1 = Cen1bar - CCFCen1 * Spread1bar
      end if
!
! Delete out-of-control (OOC) subgroups
!
      do i = 1, m
!
        if ((Cen1(i) > UCLCen1) .or. (Cen1(i) < LCLCen1)) cycle
!
        mCen = mCen + 1
        Cen1temp(mCen) = Cen1(i)
      end do
!
      if (mCen == 0) then
        WRITE(*, *)
        WRITE(*, *) "(# of subgroups for the control chart"
        WRITE(*, *) " for centering) = 0 in D&R procedure 2"
        WRITE(*, *) "- replication does not count"
        return
      end if
!
      if ((choice1 == 5) .and. (mCen == 1)) then
        WRITE(*, *)
        WRITE(*, *) "(# of subgroups for the control chart"
        WRITE(*, *) " for centering) = 1 in D&R procedure 2"
        WRITE(*, *) "- replication does not count"
        return
      end if
!
      if (mCen == m) exit
!
      if ((choice1 /= 5) .and. (mCen == 1)) then
        WRITE(*, *)
        WRITE(*, *) "D&R procedure 2 for the control"
        WRITE(*, *) "chart for centering stopped"
        WRITE(*, *) "- (# of subgroups) = 1"
```

```
        stops = stops + 1
        exit
      end if
!
      if ((choice1 == 5) .and. (mCen == 2)) then
        WRITE(*, *)
        WRITE(*, *) "D&R procedure 2 for the control"
        WRITE(*, *) "chart for centering stopped"
        WRITE(*, *) "- (# of subgroups) = 2"
        stops = stops + 1
        exit
      end if
!
      m = mCen
      Cen1 = 0
      Cen1 = Cen1temp
      count2Cen = 1
    end do
!
    Cen2 = 0
    Cen2 = Cen1temp
!
    return
  end subroutine D_and_R_2
!
!
!
!
!
  subroutine D_and_R_3(m, choice1, Cen1, Spread1, Spread1status, &
                       mCen, mSpread, Cen2, Spread2)
!
! **************************
! ***** D&R Procedure 3 *****
! **************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    INTEGER :: i
    INTEGER, INTENT(IN) :: m, choice1
    INTEGER, INTENT(OUT) :: mCen, mSpread
    REAL(KIND=DOUBLE), INTENT(IN) :: Cen1(m), Spread1(m)
    REAL(KIND=DOUBLE), INTENT(OUT) :: Cen2(m), Spread2(m)
    CHARACTER(LEN=1), INTENT(IN) :: Spread1status(m)
!
    mSpread = 0
    mCen = m
    Spread2 = 0
    Cen2 = Cen1
!
! Delete out-of-control (OOC) subgroups
!
    if (choice1 /= 5) then
!
      do i = 1, m
!
        if (Spread1status(i) == 'I') then
```

```
        mSpread = mSpread + 1
        Spread2(mSpread) = Spread1(i)
      else
        cycle
      end if
!
    end do
!
  else if (choice1 == 5) then
!
    do i = 1, (m - 1)
!
      if (Spread1status(i) == 'I') then
        mSpread = mSpread + 1
        Spread2(mSpread) = Spread1(i)
      else
        cycle
      end if
!
    end do
!
  end if
!
  if (mSpread == 0) then
    WRITE(*, *)
    WRITE(*, *) "(# of subgroups for the control chart"
    WRITE(*, *) " for spread) = 0 in D&R procedure 3"
    WRITE(*, *) "- replication does not count"
    return
  end if
!
  if (choice1 == 5) mSpread = mSpread + 1
!
  return
end subroutine D_and_R_3
!
!
!
!
!
subroutine D_and_R_5(m, Cen1, Spread1, Cen1status, Spread1status, &
                     new_m, Cen2, Spread2)
!
! **************************
! ***** D&R Procedure 5 *****
! **************************
!
  implicit none
  INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
  INTEGER :: i
  INTEGER, INTENT(IN) :: m
  INTEGER, INTENT(OUT) :: new_m
  REAL(KIND=DOUBLE), INTENT(IN) :: Cen1(m), Spread1(m)
  REAL(KIND=DOUBLE), INTENT(OUT) :: Cen2(m), Spread2(m)
  CHARACTER(LEN=1), INTENT(IN) :: Cen1status(m), Spread1status(m)
!
  new_m = 0
```

```
    Spread2 = 0
    Cen2 = 0
!
! Delete out-of-control (OOC) subgroups
!
    do i = 1, m
!
      if ((Spread1status(i) == 'I') .and. (Cen1status(i) == 'I')) then
        new_m = new_m + 1
        Spread2(new_m) = Spread1(i)
        Cen2(new_m) = Cen1(i)
      else
        cycle
      end if
!
    end do
!
    if (new_m == 0) then
      WRITE(*, *)
      WRITE(*, *) "(# of subgroups) = 0 in D&R procedure 5"
      WRITE(*, *) "- replication does not count"
      return
    end if
!
    return
  end subroutine D_and_R_5
!
!
!
!
!
  subroutine D_and_R_6(m, choice1, Cen1, Spread1, Spread1status, &
                       mCen, mSpread, Cen2, Spread2)
!
! **************************
! ***** D&R Procedure 6 *****
! **************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p=15)
    INTEGER :: i
    INTEGER, INTENT(IN) :: m, choice1
    INTEGER, INTENT(OUT) :: mCen, mSpread
    REAL(KIND=DOUBLE) :: Spread2sum, Cen1sum, Spread2bar, Cen1bar
    REAL(KIND=DOUBLE) :: CCFCen1, UCLCen1, LCLCen1
    REAL(KIND=DOUBLE), INTENT(IN) :: Cen1(m), Spread1(m)
    REAL(KIND=DOUBLE), INTENT(OUT) :: Cen2(m), Spread2(m)
    CHARACTER(LEN=1), INTENT(IN) :: Spread1status(m)
!
! D&R procedure 6 for the control chart for spread
!
    REWIND(1)
    mSpread = 0
    mCen = 0
    Spread2 = 0
    Cen2 = 0
    Spread2sum = 0
```

```
   Cen1sum = 0
!
! Delete out-of-control (OOC) subgroups
!
   if (choice1 /= 5) then
!
     do i = 1, m
!
       if (Spread1status(i) == 'I') then
         mSpread = mSpread + 1
         Spread2(mSpread) = Spread1(i)
         Spread2sum = Spread2sum + Spread2(mSpread)
       else
         cycle
       end if
!
     end do
!
   else if (choice1 == 5) then
!
     do i = 1, (m - 1)
!
       if (Spread1status(i) == 'I') then
         mSpread = mSpread + 1
         Spread2(mSpread) = Spread1(i)
         Spread2sum = Spread2sum + Spread2(mSpread)
       else
         cycle
       end if
!
     end do
!
   end if
!
   if (mSpread == 0) then
     WRITE(*, *)
     WRITE(*, *) "(# of subgroups for the control chart"
     WRITE(*, *) " for spread) = 0 in D&R procedure 6"
     WRITE(*, *) "- replication does not count"
     return
   end if
!
! D&R procedure 6 for the control chart for centering
!
! Read first stage short run control chart factor from input file
!
   if (choice1 /= 5) then
!
     do i = 1, (m - 1)
       READ(1, *)
     end do
!
   else if (choice1 == 5) then
!
     do i = 2, (m - 1)
       READ(1, *)
     end do
```

```
!
    end if
!
    READ(1, *) CCFCen1
!
! Construct first stage control limits
!
    do i = 1, m
      Cen1sum = Cen1sum + Cen1(i)
    end do
!
    Spread2bar = Spread2sum / mSpread
!
    if (choice1 == 5) mSpread = mSpread + 1
!
    Cen1bar = Cen1sum / m
!
    if ((choice1 == 2) .or. (choice1 == 3)) then
      UCLCen1 = Cen1bar + CCFCen1 * SQRT(Spread2bar)
      LCLCen1 = Cen1bar - CCFCen1 * SQRT(Spread2bar)
    else
      UCLCen1 = Cen1bar + CCFCen1 * Spread2bar
      LCLCen1 = Cen1bar - CCFCen1 * Spread2bar
    end if
!
! Delete out-of-control (OOC) subgroups
!
    do i = 1, m
!
      if ((Cen1(i) > UCLCen1) .or. (Cen1(i) < LCLCen1)) cycle
!
      mCen = mCen + 1
      Cen2(mCen) = Cen1(i)
    end do
!
    if (mCen == 0) then
      WRITE(*, *)
      WRITE(*, *) "(# of subgroups for the control chart"
      WRITE(*, *) " for centering) = 0 in D&R procedure 6"
      WRITE(*, *) "- replication does not count"
      return
    end if
!
    if ((choice1 == 5) .and. (mCen == 1)) then
      WRITE(*, *)
      WRITE(*, *) "(# of subgroups for the control chart"
      WRITE(*, *) " for centering) = 1 in D&R procedure 6"
      WRITE(*, *) "- replication does not count"
      return
    end if
!
    return
  end subroutine D_and_R_6
!
!
!
!
```

```
!
end module D_and_R
!
!
!
!
!
!
!
!
!
!
module Stage_1
!
! ******************************************************************
! ***** This module contains the subroutines that perform Stage 1 *****
! ***** control charting for each control chart combination       *****
! ******************************************************************
!
  USE random_mod
  implicit none
!
  contains
!
!
!
!
!
  subroutine Xbar_R_1(mean, sd, n, m, answer1, shifttype1, &
                      shiftsize1mean, shiftsize1sd, shifttime1, &
                      Xbar, R, Xbarstatus, Rstatus, seed)
!
! **********************************************************
! ***** Stage 1 Control Charting for (Xbar, R) Charts *****
! **********************************************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p = 15)
    INTEGER :: i, j
    INTEGER, INTENT(IN) :: n, m, shifttime1
    INTEGER, INTENT(IN OUT) :: seed
    REAL(KIND=DOUBLE) :: UCCFR1, LCCFR1, CCFXbar1, pi
    REAL(KIND=DOUBLE) :: Xsum, r1, r2, X, large, small
    REAL(KIND=DOUBLE) :: Xbarsum, Rsum, Xbarbar, Rbar
    REAL(KIND=DOUBLE) :: UCLR1, LCLR1, UCLXbar1, LCLXbar1
    REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
    REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize1mean, shiftsize1sd
    REAL(KIND=DOUBLE), INTENT(OUT) :: Xbar(m), R(m)
    CHARACTER(LEN=1), INTENT(IN) :: answer1
    CHARACTER(LEN=2), INTENT(IN) :: shifttype1
    CHARACTER(LEN=1), INTENT(OUT) :: Xbarstatus(m), Rstatus(m)
!
    REWIND(1)
    Xbarsum = 0
    Rsum = 0
!
! Read first stage short run control chart factors from input file
```

```
!
    do i = 1, (m - 1)
      READ(1, *)
    end do
!
    READ(1, *) CCFXbar1, UCCFR1, LCCFR1
!
    pi = ACOS(-1.0)
!
! Generate first stage subgroups
!
    do i = 1, m
      Xsum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if ((answer1 == 'Y') .and. (i >= shifttime1)) then
!
          if (shifttype1 == 'MN') then
            X = (mean + shiftsize1mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype1 == 'SD') then
            X = mean + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype1 == 'MS') then
            X = (mean + shiftsize1mean) + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
        end if
!
        Xsum = Xsum + X
!
        if (j == 1) then
          large = X
          small = X
        else
!
          if (X > large) large = X
!
          if (X < small) small = X
!
        end if
!
      end do
!
      Xbar(i) = Xsum / n
      R(i) = large - small
      Xbarsum = Xbarsum + Xbar(i)
      Rsum = Rsum + R(i)
    end do
!
```

```
! Construct first stage control limits
!
    Xbarbar = Xbarsum / m
    Rbar = Rsum / m
    UCLR1 = UCCFR1 * Rbar
    LCLR1 = LCCFR1 * Rbar
    UCLXbar1 = Xbarbar + CCFXbar1 * Rbar
    LCLXbar1 = Xbarbar - CCFXbar1 * Rbar
!
! Determine out-of-control (OOC) subgroups
!
    do i = 1, m
!
      if ((R(i) > UCLR1) .or. (R(i) < LCLR1)) then
        Rstatus(i) = 'O'
      else
        Rstatus(i) = 'I'
      end if
!
      if ((Xbar(i) > UCLXbar1) .or. (Xbar(i) < LCLXbar1)) then
        Xbarstatus(i) = 'O'
      else
        Xbarstatus(i) = 'I'
      end if
!
    end do
!
    return
  end subroutine Xbar_R_1
!
!
!
!
!
  subroutine Xbar_v_1(mean, sd, n, m, answer1, shifttype1, &
                      shiftsize1mean, shiftsize1sd, shifttime1, &
                      Xbar, v, Xbarstatus, vstatus, seed)
!
! ********************************************************
! ***** Stage 1 Control Charting for (Xbar, v) Charts *****
! ********************************************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p = 15)
    INTEGER :: i, j
    INTEGER, INTENT(IN) :: n, m, shifttime1
    INTEGER, INTENT(IN OUT) :: seed
    REAL(KIND=DOUBLE) :: UCCFv1, LCCFv1, CCFXbar1, pi
    REAL(KIND=DOUBLE) :: Xsum, X2sum, r1, r2, X
    REAL(KIND=DOUBLE) :: Xbarsum, vsum, Xbarbar, vbar
    REAL(KIND=DOUBLE) :: UCLv1, LCLv1, UCLXbar1, LCLXbar1
    REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
    REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize1mean, shiftsize1sd
    REAL(KIND=DOUBLE), INTENT(OUT) :: Xbar(m), v(m)
    CHARACTER(LEN=1), INTENT(IN) :: answer1
    CHARACTER(LEN=2), INTENT(IN) :: shifttype1
    CHARACTER(LEN=1), INTENT(OUT) :: Xbarstatus(m), vstatus(m)
```

```
!
    REWIND(1)
    Xbarsum = 0
    vsum = 0
!
! Read first stage short run control chart factors from input file
!
    do i = 1, (m - 1)
      READ(1, *)
    end do
!
    READ(1, *) CCFXbar1, UCCFv1, LCCFv1
!
    pi = ACOS(-1.0)
!
! Generate first stage subgroups
!
    do i = 1, m
      Xsum = 0
      X2sum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if ((answer1 == 'Y') .and. (i >= shifttime1)) then
!
          if (shifttype1 == 'MN') then
            X = (mean + shiftsize1mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype1 == 'SD') then
            X = mean + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype1 == 'MS') then
            X = (mean + shiftsize1mean) + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
        end if
!
        Xsum = Xsum + X
        X2sum = X2sum + (X**2)
      end do
!
      Xbar(i) = Xsum / n
      v(i) = (n * X2sum - (Xsum**2)) / (n * (n - 1.))
      Xbarsum = Xbarsum + Xbar(i)
      vsum = vsum + v(i)
    end do
!
! Construct first stage control limits
!
    Xbarbar = Xbarsum / m
    vbar = vsum / m
```

```
      UCLv1 = UCCFv1 * vbar
      LCLv1 = LCCFv1 * vbar
      UCLXbar1 = Xbarbar + CCFXbar1 * SQRT(vbar)
      LCLXbar1 = Xbarbar - CCFXbar1 * SQRT(vbar)
!
! Determine out-of-control (OOC) subgroups
!
      do i = 1, m
!
        if ((v(i) > UCLv1) .or. (v(i) < LCLv1)) then
          vstatus(i) = 'O'
        else
          vstatus(i) = 'I'
        end if
!
        if ((Xbar(i) > UCLXbar1) .or. (Xbar(i) < LCLXbar1)) then
          Xbarstatus(i) = 'O'
        else
          Xbarstatus(i) = 'I'
        end if
!
      end do
!
      return
    end subroutine Xbar_v_1
!
!
!
!
!
    subroutine Xbar_sqrtv_1(mean, sd, n, m, answer1, shifttype1, &
                            shiftsize1mean, shiftsize1sd, shifttime1, &
                            Xbar, v, Xbarstatus, sqrtvstatus, seed)
!
! ************************************************************
! ***** Stage 1 Control Charting for (Xbar, v^0.5) Charts *****
! ************************************************************
!
      implicit none
      INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p = 15)
      INTEGER :: i, j
      INTEGER, INTENT(IN) :: n, m, shifttime1
      INTEGER, INTENT(IN OUT) :: seed
      REAL(KIND=DOUBLE) :: UCCFsqrtv1, LCCFsqrtv1, CCFXbar1, pi
      REAL(KIND=DOUBLE) :: Xsum, X2sum, r1, r2, X
      REAL(KIND=DOUBLE) :: Xbarsum, vsum, Xbarbar, vbar
      REAL(KIND=DOUBLE) :: UCLsqrtv1, LCLsqrtv1, UCLXbar1, LCLXbar1
      REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
      REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize1mean, shiftsize1sd
      REAL(KIND=DOUBLE), INTENT(OUT) :: Xbar(m), v(m)
      CHARACTER(LEN=1), INTENT(IN) :: answer1
      CHARACTER(LEN=2), INTENT(IN) :: shifttype1
      CHARACTER(LEN=1), INTENT(OUT) :: Xbarstatus(m), sqrtvstatus(m)
!
      REWIND(1)
      Xbarsum = 0
      vsum = 0
```

```
!
! Read first stage short run control chart factors from input file
!
   do i = 1, (m - 1)
     READ(1, *)
   end do
!
   READ(1, *) CCFXbar1, UCCFsqrtv1, LCCFsqrtv1
!
   pi = ACOS(-1.0)
!
! Generate first stage subgroups
!
   do i = 1, m
     Xsum = 0
     X2sum = 0
!
     do j = 1, n
       call random(r1, seed)
       call random(r2, seed)
!
       if ((answer1 == 'Y') .and. (i >= shifttime1)) then
!
         if (shifttype1 == 'MN') then
           X = (mean + shiftsize1mean) + sd * &
               ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
         else if (shifttype1 == 'SD') then
           X = mean + (sd + shiftsize1sd) * &
               ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
         else if (shifttype1 == 'MS') then
           X = (mean + shiftsize1mean) + (sd + shiftsize1sd) * &
               ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
         end if
!
       else
         X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                          (COS(2. * pi * r2)))
       end if
!
       Xsum = Xsum + X
       X2sum = X2sum + (X**2)
     end do
!
     Xbar(i) = Xsum / n
     v(i) = (n * X2sum - (Xsum**2)) / (n * (n - 1.))
     Xbarsum = Xbarsum + Xbar(i)
     vsum = vsum + v(i)
   end do
!
! Construct first stage control limits
!
   Xbarbar = Xbarsum / m
   vbar = vsum / m
   UCLsqrtv1 = UCCFsqrtv1 * SQRT(vbar)
   LCLsqrtv1 = LCCFsqrtv1 * SQRT(vbar)
   UCLXbar1 = Xbarbar + CCFXbar1 * SQRT(vbar)
   LCLXbar1 = Xbarbar - CCFXbar1 * SQRT(vbar)
```

```
!
! Determine out-of-control (OOC) subgroups
!
   do i = 1, m
!
     if ((SQRT(v(i)) > UCLsqrtv1) .or. (SQRT(v(i)) < LCLsqrtv1)) then
       sqrtvstatus(i) = 'O'
     else
       sqrtvstatus(i) = 'I'
     end if
!
     if ((Xbar(i) > UCLXbar1) .or. (Xbar(i) < LCLXbar1)) then
       Xbarstatus(i) = 'O'
     else
       Xbarstatus(i) = 'I'
     end if
!
   end do
!
   return
 end subroutine Xbar_sqrtv_1
!
!
!
!
!
 subroutine Xbar_s_1(mean, sd, n, m, answer1, shifttype1, &
                     shiftsize1mean, shiftsize1sd, shifttime1, &
                     Xbar, s, Xbarstatus, sstatus, seed)
!
! *********************************************************
! ***** Stage 1 Control Charting for (Xbar, s) Charts *****
! *********************************************************
!
   implicit none
   INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p = 15)
   INTEGER :: i, j
   INTEGER, INTENT(IN) :: n, m, shifttime1
   INTEGER, INTENT(IN OUT) :: seed
   REAL(KIND=DOUBLE) :: UCCFs1, LCCFs1, CCFXbar1, pi
   REAL(KIND=DOUBLE) :: Xsum, X2sum, r1, r2, X
   REAL(KIND=DOUBLE) :: Xbarsum, ssum, Xbarbar, sbar
   REAL(KIND=DOUBLE) :: UCLs1, LCLs1, UCLXbar1, LCLXbar1
   REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
   REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize1mean, shiftsize1sd
   REAL(KIND=DOUBLE), INTENT(OUT) :: Xbar(m), s(m)
   CHARACTER(LEN=1), INTENT(IN) :: answer1
   CHARACTER(LEN=2), INTENT(IN) :: shifttype1
   CHARACTER(LEN=1), INTENT(OUT) :: Xbarstatus(m), sstatus(m)
!
   REWIND(1)
   Xbarsum = 0
   ssum = 0
!
! Read first stage short run control chart factors from input file
!
   do i = 1, (m - 1)
```

```
      READ(1, *)
    end do
!
    READ(1, *) CCFXbar1, UCCFs1, LCCFs1
!
    pi = ACOS(-1.0)
!
! Generate first stage subgroups
!
    do i = 1, m
      Xsum = 0
      X2sum = 0
!
      do j = 1, n
        call random(r1, seed)
        call random(r2, seed)
!
        if ((answer1 == 'Y') .and. (i >= shifttime1)) then
!
          if (shifttype1 == 'MN') then
            X = (mean + shiftsize1mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype1 == 'SD') then
            X = mean + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          else if (shifttype1 == 'MS') then
            X = (mean + shiftsize1mean) + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
          end if
!
        else
          X = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                          (COS(2. * pi * r2)))
        end if
!
        Xsum = Xsum + X
        X2sum = X2sum + (X**2)
      end do
!
      Xbar(i) = Xsum / n
      s(i) = SQRT((n * X2sum - (Xsum**2)) / (n * (n - 1.)))
      Xbarsum = Xbarsum + Xbar(i)
      ssum = ssum + s(i)
    end do
!
! Construct first stage control limits
!
    Xbarbar = Xbarsum / m
    sbar = ssum / m
    UCLs1 = UCCFs1 * sbar
    LCLs1 = LCCFs1 * sbar
    UCLXbar1 = Xbarbar + CCFXbar1 * sbar
    LCLXbar1 = Xbarbar - CCFXbar1 * sbar
!
! Determine out-of-control (OOC) subgroups
!
    do i = 1, m
```

```
!
      if ((s(i) > UCLs1) .or. (s(i) < LCLs1)) then
        sstatus(i) = 'O'
      else
        sstatus(i) = 'I'
      end if
!
      if ((Xbar(i) > UCLXbar1) .or. (Xbar(i) < LCLXbar1)) then
        Xbarstatus(i) = 'O'
      else
        Xbarstatus(i) = 'I'
      end if
!
    end do
!
    return
  end subroutine Xbar_s_1
!
!
!
!
!
  subroutine X_MR_1(mean, sd, m, answer1, shifttype1, &
                    shiftsize1mean, shiftsize1sd, shifttime1, &
                    X, MR, Xstatus, MRstatus, seed)
!
! *******************************************************
! ***** Stage 1 Control Charting for (X, MR) Charts *****
! *******************************************************
!
    implicit none
    INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p = 15)
    INTEGER :: i
    INTEGER, INTENT(IN) :: m, shifttime1
    INTEGER, INTENT(IN OUT) :: seed
    REAL(KIND=DOUBLE) :: UCCFMR1, LCCFMR1, CCFX1, pi
    REAL(KIND=DOUBLE) :: r1, r2
    REAL(KIND=DOUBLE) :: Xsum, MRsum, Xbar, MRbar
    REAL(KIND=DOUBLE) :: UCLMR1, LCLMR1, UCLX1, LCLX1
    REAL(KIND=DOUBLE), INTENT(IN) :: mean, sd
    REAL(KIND=DOUBLE), INTENT(IN) :: shiftsize1mean, shiftsize1sd
    REAL(KIND=DOUBLE), INTENT(OUT) :: X(m), MR(m - 1)
    CHARACTER(LEN=1), INTENT(IN) :: answer1
    CHARACTER(LEN=2), INTENT(IN) :: shifttype1
    CHARACTER(LEN=1), INTENT(OUT) :: Xstatus(m), MRstatus(m - 1)
!
    REWIND(1)
    Xsum = 0
    MRsum = 0
!
! Read first stage short run control chart factors from input file
!
    do i = 2, (m - 1)
      READ(1, *)
    end do
!
    READ(1, *) CCFX1, UCCFMR1, LCCFMR1
```

```
!
   pi = ACOS(-1.0)
!
! Generate first stage subgroups
!
   call random(r1, seed)
   call random(r2, seed)
!
   if ((answer1 == 'Y') .and. (shifttime1 == 1)) then
!
     if (shifttype1 == 'MN') then
       X(1) = (mean + shiftsize1mean) + sd * &
              ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
     else if (shifttype1 == 'SD') then
       X(1) = mean + (sd + shiftsize1sd) * &
              ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
     else if (shifttype1 == 'MS') then
       X(1) = (mean + shiftsize1mean) + (sd + shiftsize1sd) * &
              ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
     end if
!
   else
     X(1) = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                         (COS(2. * pi * r2)))
   end if
!
   Xsum = Xsum + X(1)
!
   do i = 2, m
     call random(r1, seed)
     call random(r2, seed)
!
     if ((answer1 == 'Y') .and. (i >= shifttime1)) then
!
       if (shifttype1 == 'MN') then
         X(i) = (mean + shiftsize1mean) + sd * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
       else if (shifttype1 == 'SD') then
         X(i) = mean + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
       else if (shifttype1 == 'MS') then
         X(i) = (mean + shiftsize1mean) + (sd + shiftsize1sd) * &
                ((SQRT(-2. * LOG(r1))) * (COS(2. * pi * r2)))
       end if
!
     else
       X(i) = mean + sd * ((SQRT(-2. * LOG(r1))) * &
                           (COS(2. * pi * r2)))
     end if
!
     MR(i - 1) = ABS(X(i) - X(i - 1))
     Xsum = Xsum + X(i)
     MRsum = MRsum + MR(i - 1)
   end do
!
! Construct first stage control limits
!
```

```
    Xbar = Xsum / m
    MRbar = MRsum / (m - 1)
    UCLMR1 = UCCFMR1 * MRbar
    LCLMR1 = LCCFMR1 * MRbar
    UCLX1 = Xbar + CCFX1 * MRbar
    LCLX1 = Xbar - CCFX1 * MRbar
!
! Determine out-of-control (OOC) subgroups
!
    do i = 1, (m - 1)
!
      if ((MR(i) > UCLMR1) .or. (MR(i) < LCLMR1)) then
        MRstatus(i) = 'O'
      else
        MRstatus(i) = 'I'
      end if
!
    end do
!
    do i = 1, m
!
      if ((X(i) > UCLX1) .or. (X(i) < LCLX1)) then
        Xstatus(i) = 'O'
      else
        Xstatus(i) = 'I'
      end if
!
    end do
!
    return
  end subroutine X_MR_1
!
!
!
!
!
end module Stage_1
!
!
!
!
!
!
!
!
!
!
program cc
!
! **********************************************************
! ***** Two Stage Short Run Variables Control Charting *****
! **********************************************************
!
  USE Stage_1
  USE D_and_R
  USE Stage_2
  implicit none
```

```
  INTEGER, parameter :: DOUBLE=SELECTED_REAL_KIND(p = 15)
  INTEGER :: k, l, rep, n, m, save_m, new_m, mCen, mSpread
  INTEGER :: choice1, choice2, shifttime1, shifttime2
  INTEGER :: seed = 1973272912, maxRL = 50000
  INTEGER :: count1, count2Spread, count2Cen, skips = 0, stops = 0
  INTEGER :: sumcount1 = 0, sumcount2Spread = 0, sumcount2Cen = 0
  REAL(KIND=DOUBLE) :: mean, sd
  REAL(KIND=DOUBLE) :: shiftsize1mean = 0, shiftsize1sd = 0
  REAL(KIND=DOUBLE) :: shiftsize2mean = 0, shiftsize2sd = 0
  REAL(KIND=DOUBLE) :: RL, sumRL = 0, sumRL2 = 0, ARL, SDRL
  REAL(KIND=DOUBLE) :: falsealarm, Pfalsealarm, APFL, SDPFL
  REAL(KIND=DOUBLE) :: sumPfalsealarm = 0, sumPfalsealarm2 = 0
  REAL, ALLOCATABLE, DIMENSION(:) :: RunL, RLnum
  REAL(KIND=DOUBLE), ALLOCATABLE, DIMENSION(:) :: Cen1, Spread1
  REAL(KIND=DOUBLE), ALLOCATABLE, DIMENSION(:) :: Cen2, Spread2
  CHARACTER(LEN=13) :: text
  CHARACTER(LEN=1) :: answer1, answer2, answer3
  CHARACTER(LEN=2) :: shifttype1, shifttype2
  CHARACTER(LEN=50) :: filenamein, filenameout
  CHARACTER(LEN=1), ALLOCATABLE, DIMENSION (:) :: Cen1status
  CHARACTER(LEN=1), ALLOCATABLE, DIMENSION (:) :: Spread1status
!
  WRITE(*, *) "Enter mean --> "
  READ(*, *) mean
  WRITE(*, *) "Enter standard deviation --> "
  READ(*, *) sd
  WRITE(*, *) "Enter number of times to replicate two stage"
  WRITE(*, *) "  short run control charting procedure --> "
  READ(*, *) rep
!
! Enter control chart combination choice
!
  WRITE(*, *) "-------------------------------------------"
  WRITE(*, *) "      Enter 1, 2, 3, 4, or 5 for the"
  WRITE(*, *) "control chart combination you wish to use:"
  WRITE(*, *) "-------------------------------------------"
  WRITE(*, *) "1. (Xbar, R)"
  WRITE(*, *) "2. (Xbar, v)"
  WRITE(*, *) "3. (Xbar, v^0.5)"
  WRITE(*, *) "4. (Xbar, s)"
  WRITE(*, *) "5. (X, MR)"
  WRITE(*, *)
  WRITE(*, *) "Enter choice --> "
  READ(*, *) choice1
!
  do
!
    if ((choice1 == 1) .or. (choice1 == 2) .or. (choice1 == 3) .or. &
        (choice1 == 4) .or. (choice1 == 5)) exit
!
    WRITE(*, *) "Invalid choice - please enter 1, 2, 3, 4, or 5 --> "
    READ(*, *) choice1
!
  end do
!
  if (choice1 == 1) then
    text = "(Xbar, R)"
```

```
  else if (choice1 == 2) then
    text = "(Xbar, v)"
  else if (choice1 == 3) then
    text = "(Xbar, v^0.5)"
  else if (choice1 == 4) then
    text = "(Xbar, s)"
  else if (choice1 == 5) then
    text = "(X, MR)"
  end if
!
  if (choice1 /= 5) then
    WRITE(*, *) "Enter n, the subgroup size --> "
    READ(*, *) n
  end if
!
! Enter data for Stage 1
!
  WRITE(*, *) "Enter m, the number of subgroups, for Stage 1:"
  WRITE(*, *)
!
  if (choice1 /= 5) then
    WRITE(*, *) "  (Note: m cannot be smaller than 2 for ", TRIM(text)
  else if (choice1 == 5) then
    WRITE(*, *) "  (Note: m cannot be smaller than 3 for ", TRIM(text)
  end if
!
  WRITE(*, *) "   control charts.)"
  WRITE(*, *)
  WRITE(*, *) "  Enter m --> "
  READ(*, *) m
!
  do
!
    if (((choice1 /= 5) .and. (m >= 2)) .or. &
        ((choice1 == 5) .and. (m >= 3))) exit
!
    WRITE(*, *) "The value for m, the number of subgroups,"
    WRITE(*, *) "is too small."
    WRITE(*, *)
    WRITE(*, *) "Enter a value for m --> "
    READ(*, *) m
  end do
!
  save_m = m
!
  ALLOCATE(Cen1(m), Spread1(m))
  ALLOCATE(Cen2(m), Spread2(m))
  ALLOCATE(Cen1status(m), Spread1status(m))
  ALLOCATE(RunL(rep))
  ALLOCATE(RLnum(maxRL))
!
  RunL = 0
  RLnum = 0
!
  WRITE(*, *) "Would you like to force a sustained shift"
  WRITE(*, *) "  in the mean, the standard deviation, or"
  WRITE(*, *) "  both in Stage 1 (Y or N)? --> "
```

```
  READ(*, *) answer1
!
  do
!
    if ((answer1 == 'Y') .or. (answer1 == 'N')) exit
!
    WRITE(*, *) "Invalid choice - please enter Y or N --> "
    READ(*, *) answer1
    cycle
  end do
!
  if (answer1 == 'Y') then
    WRITE(*, *) "Enter MN for a sustained shift in the mean,"
    WRITE(*, *) "  SD for a sustained shift in the standard"
    WRITE(*, *) "  deviation, or MS for a sustained shift"
    WRITE(*, *) "  in both in Stage 1 --> "
    READ(*, *) shifttype1
!
    do
!
      if ((shifttype1 == 'MN') .or. (shifttype1 == 'SD') .or. &
          (shifttype1 == 'MS')) exit
!
      WRITE(*, *) "Invalid choice - please enter MN, SD, or MS --> "
      READ(*, *) shifttype1
      cycle
    end do
!
    if (shifttype1 == 'MN') then
      WRITE(*, *) "Enter shift size in mean using the same"
      WRITE(*, *) "  units as the mean --> "
      READ(*, *) shiftsize1mean
    else if (shifttype1 == 'SD') then
      WRITE(*, *) "Enter shift size in standard deviation using the"
      WRITE(*, *) "  same units as the standard deviation --> "
      READ(*, *) shiftsize1sd
    else if (shifttype1 == 'MS') then
      WRITE(*, *) "Enter shift size in mean using the same"
      WRITE(*, *) "  units as the mean --> "
      READ(*, *) shiftsize1mean
      WRITE(*, *) "Enter shift size in standard deviation using the"
      WRITE(*, *) "  same units as the standard deviation --> "
      READ(*, *) shiftsize1sd
    end if
!
    WRITE(*, *) "Enter the number of the first subgroup after the"
    WRITE(*, *) "  shift in Stage 1 --> "
    READ(*, *) shifttime1
!
  end if
!
! Enter data for Stage 2
!
  WRITE(*, *) "Would you like to force a sustained shift"
  WRITE(*, *) "  in the mean, the standard deviation, or"
  WRITE(*, *) "  both in Stage 2 (Y or N)? --> "
  READ(*, *) answer2
```

```
!
  do
!
    if ((answer2 == 'Y') .or. (answer2 == 'N')) exit
!
    WRITE(*, *) "Invalid choice - please enter Y or N --> "
    READ(*, *) answer2
    cycle
  end do
!
  if (answer2 == 'Y') then
    WRITE(*, *) "Enter MN for a sustained shift in the mean,"
    WRITE(*, *) "  SD for a sustained shift in the standard"
    WRITE(*, *) "  deviation, or MS for a sustained shift"
    WRITE(*, *) "  in both in Stage 2 --> "
    READ(*, *) shifttype2
!
    do
!
      if ((shifttype2 == 'MN') .or. (shifttype2 == 'SD') .or. &
          (shifttype2 == 'MS')) exit
!
      WRITE(*, *) "Invalid choice - please enter MN, SD, or MS --> "
      READ(*, *) shifttype2
      cycle
    end do
!
    if (shifttype2 == 'MN') then
      WRITE(*, *) "Enter shift size in mean using the same"
      WRITE(*, *) "  units as the mean --> "
      READ(*, *) shiftsize2mean
    else if (shifttype2 == 'SD') then
      WRITE(*, *) "Enter shift size in standard deviation using the"
      WRITE(*, *) "  same units as the standard deviation --> "
      READ(*, *) shiftsize2sd
    else if (shifttype2 == 'MS') then
      WRITE(*, *) "Enter shift size in mean using the same"
      WRITE(*, *) "  units as the mean --> "
      READ(*, *) shiftsize2mean
      WRITE(*, *) "Enter shift size in standard deviation using the"
      WRITE(*, *) "  same units as the standard deviation --> "
      READ(*, *) shiftsize2sd
    end if
!
    WRITE(*, *) "Enter the number of the first subgroup after the"
    WRITE(*, *) "  shift in Stage 2 (the first subgroup drawn in"
    WRITE(*, *) "  Stage 2 is subgroup number one) --> "
    READ(*, *) shifttime2
  end if
!
  WRITE(*, *) "Would you like to use a different starting value"
  WRITE(*, *) "  for seed (Y or N)? --> "
  READ(*, *) answer3
!
  do
!
    if ((answer3 == 'Y') .or. (answer3 == 'N')) exit
```

```
!
    WRITE(*, *) "Invalid choice - please enter Y or N --> "
    READ(*, *) answer3
    cycle
  end do
!
  if (answer3 == 'Y') then
    WRITE(*, *) "Enter a value for seed --> "
    READ(*, *) seed
  end if
!
! Enter D&R procedure choice
!
  WRITE(*, *) "-------------------------------------------------------"
!
  if (choice1 /= 5) then
    WRITE(*, *) "            Enter 1, 2, 3, 4, 5, or 6 for the"
  else
    WRITE(*, *) "               Enter 2, 3, 4, or 6 for the"
  end if
!
  WRITE(*, *) "  Delete and Revise (D&R) procedure you wish to use:"
  WRITE(*, *) "-------------------------------------------------------"
!
  if (choice1 /= 5) then
    WRITE(*, *) "1. (i)    Deletes out-of-control (OOC) initial"
    WRITE(*, *) "          subgroups on either the control chart for"
    WRITE(*, *) "          centering or spread entirely (i.e., if a"
    WRITE(*, *) "          subgroup shows OOC on either control chart,"
    WRITE(*, *) "          it is deleted from both charts)."
    WRITE(*, *) "   (ii)   Recalculates the control limits for both"
    WRITE(*, *) "          charts using the subgroups remaining after"
    WRITE(*, *) "          step (i)."
    WRITE(*, *) "   (iii)  Determines OOC subgroups."
    WRITE(*, *) "   (iv)   Repeats steps (i)-(iii) until no initial"
    WRITE(*, *) "          subgroups show OOC on either chart."
    WRITE(*, *)
    WRITE(*, *) "Press the Enter key to continue..."
    READ(*, *)
  end if
!
  WRITE(*, *) "2. (i)    Deletes out-of-control (OOC) initial"
  WRITE(*, *) "          subgroups on the control chart for spread."
  WRITE(*, *) "   (ii)   Recalculates the control limits for the"
  WRITE(*, *) "          control chart for spread using the subgroups"
  WRITE(*, *) "          remaining after step (i)."
  WRITE(*, *) "   (iii)  Determines OOC subgroups."
  WRITE(*, *) "   (iv)   Repeats steps (i)-(iii) until no initial"
  WRITE(*, *) "          subgroups show OOC on the control chart for"
  WRITE(*, *) "          spread."
  WRITE(*, *) "   (v)    Determines the control limits for the chart"
  WRITE(*, *) "          for centering using the parameter estimate"
  WRITE(*, *) "          for spread obtained after completing steps"
  WRITE(*, *) "          (i)-(iv) and the overall average obtained"
  WRITE(*, *) "          from all of the initial subgroups."
  WRITE(*, *) "   (vi)   Repeats steps (i)-(ii) for the control chart"
  WRITE(*, *) "          for centering until no initial subgroups"
```

```
  WRITE(*, *) "         show OOC."
  WRITE(*, *)
  WRITE(*, *) "Press the Enter key to continue..."
  READ(*, *)
  WRITE(*, *) "3. Deletes out-of-control (OOC) initial subgroups on"
  WRITE(*, *) "   the control chart for spread just once.  No D&R is"
  WRITE(*, *) "   performed on the control chart for centering."
  WRITE(*, *)
  WRITE(*, *) "Press the Enter key to continue..."
  READ(*, *)
  WRITE(*, *) "4. Does not perform D&R.  This means all of the"
  WRITE(*, *) "   initial subgroups will be used to determine second"
  WRITE(*, *) "   stage control limits for both the control charts"
  WRITE(*, *) "   for centering and spread."
  WRITE(*, *)
  WRITE(*, *) "Press the Enter key to continue..."
  READ(*, *)
!
  if (choice1 /= 5) then
    WRITE(*, *) "5. Deletes out-of-control (OOC) initial subgroups"
    WRITE(*, *) "   on either the control chart for centering or"
    WRITE(*, *) "   spread entirely (i.e., if a subgroup shows OOC"
    WRITE(*, *) "   on either control chart, it is deleted from both"
    WRITE(*, *) "   charts).  D&R is performed just once."
    WRITE(*, *)
    WRITE(*, *) "Press the Enter key to continue..."
    READ(*, *)
  end if
!
  WRITE(*, *) "6. (i)   Deletes out-of-control (OOC) initial"
  WRITE(*, *) "         subgroups on the control chart for spread"
  WRITE(*, *) "         just once."
  WRITE(*, *) "   (ii)  Determines the control limits for the chart"
  WRITE(*, *) "         for centering using the parameter estimate"
  WRITE(*, *) "         for spread obtained after completing step i"
  WRITE(*, *) "         and the overall average obtained from all of"
  WRITE(*, *) "         the initial subgroups."
  WRITE(*, *) "   (iii) Performs step (i) for the control chart for"
  WRITE(*, *) "         centering."
  WRITE(*, *)
!
  if (choice1 /= 5) then
    WRITE(*, *) "Enter 1, 2, 3, 4, 5, or 6 --> "
  else
    WRITE(*, *) "(Note: D&R procedures 1 and 5 are not valid for"
    WRITE(*, *) " (X, MR) control charts)"
    WRITE(*, *)
    WRITE(*, *) "Enter 2, 3, 4, or 6 --> "
  end if
!
  READ(*, *) choice2
!
  do
!
    if ((choice1 == 5) .or. ((choice2 == 1) .or. (choice2 == 2) .or. &
        (choice2 == 3) .or. (choice2 == 4) .or. (choice2 == 5) .or. &
        (choice2 == 6))) exit
```

```
!
    WRITE(*, *) "Invalid choice - please enter"
    WRITE(*, *) "  1, 2, 3, 4, 5, or 6 --> "
    READ(*, *) choice2
  end do
!
  do
!
    if ((choice1 /= 5) .or. ((choice2 == 2) .or. (choice2 == 3) .or. &
        (choice2 == 4) .or. (choice2 == 6))) exit
!
    if ((choice2 == 1) .or. (choice2 == 5)) then
      WRITE(*, *) "Invalid D&R procedure for (X, MR) control charts."
      WRITE(*, *)
      WRITE(*, *) "Enter 2, 3, 4, or 6 --> "
      READ(*, *) choice2
    else
      WRITE(*, *) "Invalid choice - please enter 2, 3, 4, or 6 --> "
      READ(*, *) choice2
    end if
!
  end do
!
! Enter input file name
!
  WRITE(*, *) "---------------------------------------------"
  WRITE(*, *) "Enter the name (including the location) of the"
  WRITE(*, *) "  text file (extension .txt) that has the two"
  WRITE(*, *) "  stage short run control chart factors for"
!
  if (choice1 /=5) then
    WRITE(*, 10) TRIM(text), " charts for n = ", n, ":"
  else
    WRITE(*, *) "  ", TRIM(text), " charts:"
  end if
!
  WRITE(*, *)
  WRITE(*, *) "  (Note 1: the file should have at least the"
  WRITE(*, *) "   factors for all values of m up to and"
  WRITE(*, 20) "   including m = ", m, ".)"
  WRITE(*, *)
  WRITE(*, *) "  (Note 2: the name (including the location)"
  WRITE(*, *) "   of the text file must be no longer than"
  WRITE(*, *) "   50 characters.)"
  WRITE(*, *)
  WRITE(*, *) "  Enter file name --> "
  READ(*, *) filenamein
  WRITE(*, *) "---------------------------------------------"
  WRITE(*, *)
!
  OPEN(UNIT=1, FILE=TRIM(filenamein), STATUS="old", ACTION="read")
!
! Enter output file name
!
  WRITE(*, *) "---------------------------------------------"
  WRITE(*, *) "Enter the name (including the location) of the"
  WRITE(*, *) "  text file (extension .txt) that will store"
```

```
  WRITE(*, *) "  the results from this program:"
  WRITE(*, *)
  WRITE(*, *) "  (Note: the name (including the location) of"
  WRITE(*, *) "   the text file must be no longer than 50"
  WRITE(*, *) "   characters.)"
  WRITE(*, *)
  WRITE(*, *) "  Enter file name --> "
  READ(*, *) filenameout
  WRITE(*, *) "-------------------------------------------------"
  WRITE(*, *)
!
  OPEN(UNIT=2, FILE=TRIM(filenameout), STATUS="unknown", &
       ACTION="write")
!
  WRITE(*, *) "The program is running..."
!
  do k = 1, rep
!
! Subroutines for Stage 1 control charting
!
    if (choice1 == 1) then
      call Xbar_R_1(mean, sd, n, m, answer1, shifttype1, &
                    shiftsize1mean, shiftsize1sd, shifttime1, &
                    Cen1, Spread1, Cen1status, Spread1status, seed)
    else if (choice1 == 2) then
      call Xbar_v_1(mean, sd, n, m, answer1, shifttype1, &
                    shiftsize1mean, shiftsize1sd, shifttime1, &
                    Cen1, Spread1, Cen1status, Spread1status, seed)
    else if (choice1 == 3) then
      call Xbar_sqrtv_1(mean, sd, n, m, answer1, shifttype1, &
                        shiftsize1mean, shiftsize1sd, shifttime1, &
                        Cen1, Spread1, Cen1status, Spread1status, seed)
    else if (choice1 == 4) then
      call Xbar_s_1(mean, sd, n, m, answer1, shifttype1, &
                    shiftsize1mean, shiftsize1sd, shifttime1, &
                    Cen1, Spread1, Cen1status, Spread1status, seed)
    else if (choice1 == 5) then
      call X_MR_1(mean, sd, m, answer1, shifttype1, &
                  shiftsize1mean, shiftsize1sd, shifttime1, &
                  Cen1, Spread1, Cen1status, Spread1status, seed)
    end if
!
! Subroutines for Delete and Revise (D&R) procedures
!
    if (choice2 == 1) then
      call D_and_R_1(m, save_m, choice1, Cen1, Spread1, &
                     Cen1status, Spread1status, new_m, &
                     Cen2, Spread2, count1, stops)
!
      if (new_m == 0) then
        skips = skips + 1
        cycle
      end if
!
      mCen = new_m
      mSpread = new_m
    else if (choice2 == 2) then
```

```
      call D_and_R_2(m, save_m, choice1, Cen1, Spread1, &
                     Spread1status, mCen, mSpread, Cen2, &
                     Spread2, count2Spread, count2Cen, stops)
!
      if ((mSpread == 0) .or. (mCen == 0)) then
        skips = skips + 1
        cycle
      else if ((choice1 == 5) .and. (mCen == 1)) then
        skips = skips + 1
        cycle
      end if
!
    else if (choice2 == 3) then
      call D_and_R_3(m, choice1, Cen1, Spread1, Spread1status, &
                     mCen, mSpread, Cen2, Spread2)
!
      if (mSpread == 0) then
        skips = skips + 1
        cycle
      end if
!
    else if (choice2 == 4) then
      mCen = m
      mSpread = m
      Cen2 = Cen1
      Spread2 = Spread1
    else if (choice2 == 5) then
      call D_and_R_5(m, Cen1, Spread1, Cen1status, Spread1status, &
                     new_m, Cen2, Spread2)
!
      if (new_m == 0) then
        skips = skips + 1
        cycle
      end if
!
      mCen = new_m
      mSpread = new_m
    else if (choice2 == 6) then
      call D_and_R_6(m, choice1, Cen1, Spread1, Spread1status, &
                     mCen, mSpread, Cen2, Spread2)
!
      if ((mSpread == 0) .or. (mCen == 0)) then
        skips = skips + 1
        cycle
      else if ((choice1 == 5) .and. (mCen == 1)) then
        skips = skips + 1
        cycle
      end if
!
    end if
!
! Subroutines for Stage 2 control charting
!
    if (choice1 == 1) then
      call Xbar_R_2(mean, sd, n, mCen, mSpread, Cen2, Spread2, &
                    answer2, shifttype2, shiftsize2mean, &
                    shiftsize2sd, shifttime2, falsealarm, RL, seed)
```

```
    else if (choice1 == 2) then
      call Xbar_v_2(mean, sd, n, mCen, mSpread, Cen2, Spread2, &
                    answer2, shifttype2, shiftsize2mean, &
                    shiftsize2sd, shifttime2, falsealarm, RL, seed)
    else if (choice1 == 3) then
      call Xbar_sqrtv_2(mean, sd, n, mCen, mSpread, Cen2, Spread2, &
                        answer2, shifttype2, shiftsize2mean, &
                        shiftsize2sd, shifttime2, falsealarm, RL, seed)
    else if (choice1 == 4) then
      call Xbar_s_2(mean, sd, n, mCen, mSpread, Cen2, Spread2, &
                    answer2, shifttype2, shiftsize2mean, &
                    shiftsize2sd, shifttime2, falsealarm, RL, seed)
    else if (choice1 == 5) then
!
! Note: mSpread IS THE NUMBER OF SUBGROUPS, NOT THE NUMBER OF MRs
!
      call X_MR_2(mean, sd, mCen, mSpread, Cen2, Spread2, &
                  answer2, shifttype2, shiftsize2mean, &
                  shiftsize2sd, shifttime2, falsealarm, RL, seed)
    end if
!
! Store run length (RL) results to a vector
! and calculate appropriate sums
!
    RunL(k) = RL
    sumRL = sumRL + RL
    sumRL2 = sumRL2 + (RL**2)
!
! Determine counts for POD calculations
!
    do l = 1, maxRL
!
      if (RunL(k) <= l) then
        RLnum(l) = RLnum(l) + 1
      end if
!
    end do
!
! Calculate applicable sums
!
    if ((answer2 == 'Y') .and. (shifttime2 > 1)) then
      Pfalsealarm = falsealarm / (shifttime2 - 1)
      sumPfalsealarm = sumPfalsealarm + Pfalsealarm
      sumPfalsealarm2 = sumPfalsealarm2 + (Pfalsealarm ** 2)
    end if
!
    if (choice2 == 1) sumcount1 = sumcount1 + count1
!
    if (choice2 == 2) then
      sumcount2Spread = sumcount2Spread + count2Spread
      sumcount2Cen = sumcount2Cen + count2Cen
    end if
!
  end do
!
! Write input information to output file
!
```

```
  WRITE(2, *) "----------------------------------------"
  WRITE(2, 30) "mean: .................... ", mean
  WRITE(2, 30) "standard deviation: ...... ", sd
  WRITE(2, *) "# of replications of"
  WRITE(2, 40) "  two stage procedure: ... ", (rep - skips)
  WRITE(2, *) "Control chart combination: ", TRIM(text)
!
  if (choice1 /= 5) then
    WRITE(2, 40) "n: ....................... ", n
  end if
!
  WRITE(2, 40) "m (Stage 1): ............. ", save_m
  WRITE(2, 50) "D&R procedure: ...........    ", choice2
  WRITE(2, *) "----------------------------------------"
!
! Write Stage 1 input information to output file
!
  if (answer1 == 'Y') then
!
    WRITE(2, *)
    WRITE(2, *) "----------------------------------------------"
!
    if (shifttype1 == 'MN') then
      WRITE(2, 60) "Stage 1: shift size of ", shiftsize1mean, &
                   " (same"
      WRITE(2, *) "         units as the mean) in the mean"
      WRITE(2, 70) "          between subgroups ", (shifttime1 - 1), &
                   " and ", shifttime1, "."
    else if (shifttype1 == 'SD') then
      WRITE(2, 60) "Stage 1: shift size of ", shiftsize1sd, &
                   " (same"
      WRITE(2, *) "         units as the standard deviation)"
      WRITE(2, *) "         in the standard deviation between"
      WRITE(2, 70) "          subgroups ", (shifttime1 - 1), " and ", &
                   shifttime1, "."
    else if (shifttype1 == 'MS') then
      WRITE(2, 60) "Stage 1: shift size of ", shiftsize1mean, &
                   " (same"
      WRITE(2, *) "         units as the mean) in the mean"
      WRITE(2, 80) "          and a shift size of", shiftsize1sd
      WRITE(2, *) "         (same units as the standard"
      WRITE(2, *) "         deviation) in the standard deviation"
      WRITE(2, 70) "          between subgroups ", (shifttime1 - 1), &
                   " and ", shifttime1, "."
    end if
!
  else
    WRITE(2, *)
    WRITE(2, *) "----------------------------------------------"
    WRITE(2, *) "Stage 1: No shifts in either the mean or the"
    WRITE(2, *) "         standard deviation."
  end if
!
! Write Stage 2 input information to output file
!
  if (answer2 == 'Y') then
!
```

```
    WRITE(2, *)
!
    if (shifttype2 == 'MN') then
      WRITE(2, 60) "Stage 2: shift size of ", shiftsize2mean, &
                   " (same"
      WRITE(2, *) "         units as the mean) in the mean"
      WRITE(2, 70) "          between subgroups ", (shifttime2 - 1), &
                   " and ", shifttime2, "."
    else if (shifttype2 == 'SD') then
      WRITE(2, 60) "Stage 2: shift size of ", shiftsize2sd, &
                   " (same"
      WRITE(2, *) "         units as the standard deviation)"
      WRITE(2, *) "         in the standard deviation between"
      WRITE(2, 70) "          subgroups ", (shifttime2 - 1), " and ", &
                   shifttime2, "."
    else if (shifttype2 == 'MS') then
      WRITE(2, 60) "Stage 2: shift size of ", shiftsize2mean, &
                   " (same"
      WRITE(2, *) "         units as the mean) in the mean"
      WRITE(2, 80) "          and a shift size of", shiftsize2sd
      WRITE(2, *) "         (same units as the standard"
      WRITE(2, *) "         deviation) in the standard deviation"
      WRITE(2, 70) "          between subgroups ", (shifttime2 - 1), &
                   " and ", shifttime2, "."
    end if
!
    WRITE(2, *) "----------------------------------------------"
!
  else
    WRITE(2, *)
    WRITE(2, *) "Stage 2: No shifts in either the mean or the"
    WRITE(2, *) "         standard deviation."
    WRITE(2, *) "----------------------------------------------"
  end if
!
! Write ARL and SDRL results to output file
!
  WRITE(2, *)
  WRITE(2, *) "----------------------------------------------------"
!
  if (answer2 == 'Y') then
    WRITE(2, *) "Out-of-Control (OOC) Average Run Length (ARL) and"
  else
    WRITE(2, *) "In-Control (IC) Average Run Length (ARL) and"
  end if
!
  WRITE(2, *) "Standard Deviation of the Run Length (SDRL) results"
  WRITE(2, *) "----------------------------------------------------"
!
  ARL = sumRL / (rep - skips)
  SDRL = SQRT(((rep - skips) * sumRL2 - (sumRL**2)) / &
             ((rep - skips) * ((rep - skips) - 1)))
!
  WRITE(2, 80) "ARL (in number of subgroups):  ", ARL
  WRITE(2, 80) "SDRL (in number of subgroups): ", SDRL
  WRITE(2, *) "----------------------------------------------------"
  WRITE(2, *)
```

```
!
! Write APFL and SDPFL results to output file
!
  if ((answer2 == 'Y') .and. (shifttime2 > 1)) then
    WRITE(2, *) "-------------------------------------------------"
    WRITE(2, *) "The Average Probability of a False Alarm (APFL)"
    WRITE(2, *) "and the Standard Deviation of the Probability of"
!
    if (shifttime2 == 2) then
      WRITE(2, *) "a False Alarm (SDPFL) on the subgroup before the"
      WRITE(2, *) "shift in Stage 2:"
    else if (shifttime2 > 2) then
      WRITE(2, 90) "a False Alarm (SDPFL) in the first ", &
                   (shifttime2 - 1), " subgroups"
      WRITE(2, *) "before the shift in Stage 2:"
    end if
!
    WRITE(2, *) "-------------------------------------------------"
!
    APFL = sumPfalsealarm / (rep - skips)
    SDPFL = SQRT(((rep - skips) * sumPfalsealarm2 - &
                              (sumPfalsealarm**2)) / &
                  ((rep - skips) * ((rep - skips) - 1)))
    WRITE(2, 100) "APFL:  ", APFL
    WRITE(2, 100) "SDPFL: ", SDPFL
    WRITE(2, *) "-------------------------------------------------"
    WRITE(2, *)
  end if
!
! Write POD results to output file
!
  if (answer2 == 'Y') then
    WRITE(2, *) "-------------------------------------------"
    WRITE(2, 90) "     Starting at subgroup ", shifttime2, &
                 " in Stage 2:"
    WRITE(2, *) "-------------------------------------------"
  else
    WRITE(2, *) "-------------------------------------------"
    WRITE(2, *) "     Starting at subgroup 1 in Stage 2:"
    WRITE(2, *) "-------------------------------------------"
  end if
!
  WRITE(2, *) "  t       Number of RLs <= t     P(RL <= t)"
  WRITE(2, *) "-----     ------------------     ----------"
!
  do l = 1, 10
    WRITE(2, 110) l, INT(RLnum(l)), RLnum(l) / (rep - skips)
  end do
!
  WRITE(2, 110) 15, INT(RLnum(15)), RLnum(15) / (rep - skips)
!
  do l = 20, 50, 10
    WRITE(2, 110) l, INT(RLnum(l)), RLnum(l) / (rep - skips)
  end do
!
  WRITE(2, 110) 75, INT(RLnum(75)), RLnum(75) / (rep - skips)
!
```

```
  do l = 100, 500, 100
    WRITE(2, 110) l, INT(RLnum(l)), RLnum(l) / (rep - skips)
  end do
!
  WRITE(2, 110) 750, INT(RLnum(750)), RLnum(750) / (rep - skips)
!
  do l = 1000, 5000, 1000
    WRITE(2, 110) l, INT(RLnum(l)), RLnum(l) / (rep - skips)
  end do
!
  WRITE(2, 110) 7500, INT(RLnum(7500)), RLnum(7500) / (rep - skips)
!
  do l = 10000, 50000, 10000
    WRITE(2, 110) l, INT(RLnum(l)), RLnum(l) / (rep - skips)
  end do
!
  WRITE(2, *) "--------------------------------------------"
!
! Write applicable counts to output file
!
  if (choice2 == 1) then
    WRITE(2, *)
    WRITE(2, *) "The first D&R procedure iterated more than"
    WRITE(2, 90) "  once a total of ", sumcount1, " time(s)."
  end if
!
  if (choice2 == 2) then
    WRITE(2, *)
    WRITE(2, *) "The second D&R procedure iterated more than"
    WRITE(2, 90) "  once a total of ", sumcount2Spread, &
                 " time(s) for the"
    WRITE(2, *) "  control chart for spread and a total of "
    WRITE(2, 120) sumcount2Cen, " time(s) for the control chart for"
    WRITE(2, *) "  centering."
  end if
!
  if (skips > 0) then
    WRITE(2, *)
    WRITE(2, 90) "Replications skipped ", skips, " time(s)"
    WRITE(2, *) "  because the number of subgroups dropped"
!
    if (choice1 /= 5) then
      WRITE(2, *) "  to zero after out-of-control (OOC)"
      WRITE(2, *) "  subgroups were deleted."
    else if (choice1 == 5) then
      WRITE(2, *) "  to zero or to one after out-of-control"
      WRITE(2, *) "  (OOC) subgroups were deleted."
    end if
!
  end if
!
  if (stops > 0) then
    WRITE(2, *)
    WRITE(2, 130) "D&R procedure ", choice2, " stopped ", stops, &
               " time(s)"
    WRITE(2, *) "  because the number of subgroups dropped"
!
```

```
    if (choice1 /= 5) then
      WRITE(2, *) "  to one after out-of-control (OOC)"
    else if (choice1 == 5) then
      WRITE(2, *) "  to two after out-of-control (OOC)"
    end if
!
    WRITE(2, *) "  subgroups were deleted."
  end if
!
10 FORMAT(T4, A, A, I3, A)
20 FORMAT(T2, A, I4, A)
30 FORMAT(A, F9.5)
40 FORMAT(A, I4)
50 FORMAT(A, I1)
60 FORMAT(A, F11.5, A)
70 FORMAT(A, I3, A, I3, A)
80 FORMAT(A, F12.5)
90 FORMAT(A, I3, A)
100 FORMAT(A, F7.5)
110 FORMAT(I5, I16, F21.5)
120 FORMAT(T3, I3, A)
130 FORMAT(A, I1, A, I3, A)
!
  stop
end program cc
```

APPENDIX F.2 – Sample Input Files for cc.f90

Sample Input File Containing First and Second Stage Short

Run Control Chart Factors for $(\overline{X}, R)$ Charts for n=3 and m: 1-5

0.00000	0.00000	0.00000	8.35221	14.34466	0.03152
1.56033	1.86966	0.06112	2.70257	5.65885	0.03337
1.35226	2.21659	0.04924	1.91239	4.27295	0.03407
1.25601	2.35005	0.04491	1.62151	3.74247	0.03443
1.20246	2.41685	0.04267	1.47271	3.46631	0.03465

Sample Input File Containing First and Second Stage Short

Run Control Chart Factors for $(\overline{X}, v)$ Charts for n=3 and m: 1-5

0.00000	0.00000	0.00000	17.69484	199.00000	0.00100100
2.87519	1.99000	0.00200000	4.97997	26.28427	0.00100075
2.40967	2.78787	0.00150038	3.40779	14.54411	0.00100067
2.20599	3.31601	0.00133378	2.84792	11.04241	0.00100063
2.09497	3.67043	0.00125047	2.56580	9.42700	0.00100060

Sample Input File Containing First and Second Stage Short

Run Control Chart Factors for $(\overline{X}, \sqrt{v})$ Charts for n=3 and m: 1-5

0.00000	0.00000	0.00000	17.69484	15.91775	0.03570
2.87519	1.59177	0.05046	4.97997	5.45415	0.03365
2.40967	1.77629	0.04121	3.40779	3.97519	0.03297
2.20599	1.89811	0.03807	2.84792	3.42822	0.03263
2.09497	1.97649	0.03648	2.56580	3.14794	0.03243

Sample Input File Containing First and Second Stage Short

Run Control Chart Factors for $(\overline{X}, s)$ Charts for n=3 and m: 1-5

0.00000	0.00000	0.00000	15.68165	14.10674	0.03164
2.95828	1.86761	0.06134	5.12390	5.60680	0.03348
2.57119	2.21123	0.04940	3.63621	4.24135	0.03417
2.39128	2.34285	0.04505	3.08713	3.71725	0.03453
2.29099	2.40840	0.04280	2.80588	3.44396	0.03476

Sample Input File Containing First and Second Stage

Short Run Control Chart Factors for (X, MR) Charts for m: 2-15

0.00000	0.00000	0.00000	204.19466	127.32134	0.00157
22.24670	2.95360	0.00235	31.46159	26.11886	0.00157
10.72641	3.58790	0.00209	13.84773	13.20218	0.00157
7.34996	3.83736	0.00196	9.00182	9.27880	0.00157
5.87022	3.89898	0.00188	6.94574	7.52080	0.00157
5.06862	3.89368	0.00183	5.85274	6.55349	0.00157
4.57470	3.86822	0.00179	5.18723	5.95038	0.00157
4.24308	3.83885	0.00177	4.74391	5.54166	0.00157
4.00644	3.81088	0.00175	4.42928	5.24776	0.00157
3.82972	3.78583	0.00173	4.19525	5.02691	0.00157
3.69307	3.76385	0.00171	4.01479	4.85521	0.00157
3.58441	3.74470	0.00170	3.87161	4.71806	0.00157
3.49606	3.72800	0.00169	3.75537	4.60610	0.00157
3.42287	3.71338	0.00168	3.65920	4.51303	0.00157

APPENDIX F.3 – Sample Output Files from cc.f90

Sample Output File #1

```
----------------------------------------
mean: ....................   0.00000
standard deviation: ......   1.00000
# of replications of
  two stage procedure: ... 4996
Control chart combination: (Xbar, R)
n: ......................    3
m (Stage 1): ............    5
D&R procedure: ..........    1
----------------------------------------

---------------------------------------------
Stage 1: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups   2 and   3.

Stage 2: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups  10 and  11.
---------------------------------------------

---------------------------------------------------
Out-of-Control (OOC) Average Run Length (ARL) and
Standard Deviation of the Run Length (SDRL) results
---------------------------------------------------
ARL (in number of subgroups):     464.85809
SDRL (in number of subgroups):    693.88171
---------------------------------------------------

------------------------------------------------
The Average Probability of a False Alarm (APFL)
and the Standard Deviation of the Probability of
a False Alarm (SDPFL) in the first  10 subgroups
before the shift in Stage 2:
------------------------------------------------
APFL:  0.03813
SDPFL: 0.11174
------------------------------------------------

-------------------------------------------
    Starting at subgroup  11 in Stage 2:
-------------------------------------------
  t       Number of RLs <= t      P(RL <= t)
-----     ------------------      ----------
    1             90               0.01801
    2            162               0.03243
    3            236               0.04724
    4            290               0.05805
    5            340               0.06805
    6            384               0.07686
    7            422               0.08447
    8            463               0.09267
    9            508               0.10168
   10            548               0.10969
   15            674               0.13491
   20            793               0.15873
   30           1002               0.20056
```

40	1162	0.23259
50	1277	0.25560
75	1550	0.31025
100	1781	0.35649
200	2432	0.48679
300	2893	0.57906
400	3259	0.65232
500	3504	0.70136
750	3997	0.80004
1000	4296	0.85989
2000	4814	0.96357
3000	4934	0.98759
4000	4973	0.99540
5000	4984	0.99760
7500	4994	0.99960
10000	4995	0.99980
20000	4996	1.00000
30000	4996	1.00000
40000	4996	1.00000
50000	4996	1.00000

The first D&R procedure iterated more than once a total of 111 time(s).

Replications skipped 4 time(s) because the number of subgroups dropped to zero after out-of-control (OOC) subgroups were deleted.

D&R procedure 1 stopped 12 time(s) because the number of subgroups dropped to one after out-of-control (OOC) subgroups were deleted.

Sample Output File #2

```
----------------------------------------
mean: ....................   0.00000
standard deviation: ......   1.00000
# of replications of
  two stage procedure: ... 4995
Control chart combination: (Xbar, R)
n: .......................     3
m (Stage 1): .............     5
D&R procedure: ...........     2
----------------------------------------

---------------------------------------------
Stage 1: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups   2 and   3.

Stage 2: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups  10 and  11.
---------------------------------------------

---------------------------------------------------
Out-of-Control (OOC) Average Run Length (ARL) and
Standard Deviation of the Run Length (SDRL) results
---------------------------------------------------
ARL (in number of subgroups):      393.95576
SDRL (in number of subgroups):     584.75096
---------------------------------------------------

------------------------------------------------
The Average Probability of a False Alarm (APFL)
and the Standard Deviation of the Probability of
a False Alarm (SDPFL) in the first  10 subgroups
before the shift in Stage 2:
------------------------------------------------
APFL:  0.03465
SDPFL: 0.09819
------------------------------------------------

-----------------------------------------
    Starting at subgroup  11 in Stage 2:
-----------------------------------------
  t       Number of RLs <= t     P(RL <= t)
-----     ------------------     ----------
    1            150               0.03003
    2            250               0.05005
    3            332               0.06647
    4            401               0.08028
    5            466               0.09329
    6            521               0.10430
    7            573               0.11471
    8            625               0.12513
    9            672               0.13453
   10            711               0.14234
   15            856               0.17137
   20           1008               0.20180
   30           1258               0.25185
```

40	1425	0.28529
50	1551	0.31051
75	1836	0.36757
100	2079	0.41622
200	2709	0.54234
300	3148	0.63023
400	3473	0.69530
500	3715	0.74374
750	4143	0.82943
1000	4411	0.88308
2000	4862	0.97337
3000	4954	0.99179
4000	4984	0.99780
5000	4991	0.99920
7500	4995	1.00000
10000	4995	1.00000
20000	4995	1.00000
30000	4995	1.00000
40000	4995	1.00000
50000	4995	1.00000

The second D&R procedure iterated more than once a total of 2 time(s) for the control chart for spread and a total of 644 time(s) for the control chart for centering.

Replications skipped 5 time(s) because the number of subgroups dropped to zero after out-of-control (OOC) subgroups were deleted.

D&R procedure 2 stopped 11 time(s) because the number of subgroups dropped to one after out-of-control (OOC) subgroups were deleted.

Sample Output File #3

```
---------------------------------------
mean: ...................   0.00000
standard deviation: ......   1.00000
# of replications of
  two stage procedure: .... 5000
Control chart combination: (Xbar, R)
n: ......................    3
m (Stage 1): ............    5
D&R procedure: ...........    3
---------------------------------------

--------------------------------------------
Stage 1: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups   2 and   3.

Stage 2: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups  10 and  11.
--------------------------------------------

--------------------------------------------------
Out-of-Control (OOC) Average Run Length (ARL) and
Standard Deviation of the Run Length (SDRL) results
--------------------------------------------------
ARL (in number of subgroups):     415.51700
SDRL (in number of subgroups):    596.72832
--------------------------------------------------

-----------------------------------------------
The Average Probability of a False Alarm (APFL)
and the Standard Deviation of the Probability of
a False Alarm (SDPFL) in the first  10 subgroups
before the shift in Stage 2:
-----------------------------------------------
APFL:  0.03844
SDPFL: 0.10604
-----------------------------------------------

-------------------------------------------
    Starting at subgroup  11 in Stage 2:
-------------------------------------------
```

t	Number of RLs <= t	P(RL <= t)
1	111	0.02220
2	206	0.04120
3	285	0.05700
4	343	0.06860
5	396	0.07920
6	441	0.08820
7	490	0.09800
8	543	0.10860
9	589	0.11780
10	623	0.12460
15	771	0.15420
20	926	0.18520
30	1146	0.22920

40	1312	0.26240
50	1430	0.28600
75	1706	0.34120
100	1933	0.38660
200	2589	0.51780
300	3041	0.60820
400	3382	0.67640
500	3632	0.72640
750	4100	0.82000
1000	4386	0.87720
2000	4858	0.97160
3000	4958	0.99160
4000	4989	0.99780
5000	4996	0.99920
7500	5000	1.00000
10000	5000	1.00000
20000	5000	1.00000
30000	5000	1.00000
40000	5000	1.00000
50000	5000	1.00000

Sample Output File #4

```
----------------------------------------
mean: ....................   0.00000
standard deviation: ......   1.00000
# of replications of
  two stage procedure: ... 5000
Control chart combination: (Xbar, R)
n: ......................    3
m (Stage 1): ............    5
D&R procedure: ..........    4
----------------------------------------

---------------------------------------------
Stage 1: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups   2 and   3.

Stage 2: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups  10 and  11.
---------------------------------------------

---------------------------------------------------
Out-of-Control (OOC) Average Run Length (ARL) and
Standard Deviation of the Run Length (SDRL) results
---------------------------------------------------
ARL (in number of subgroups):      422.41960
SDRL (in number of subgroups):     603.47804
---------------------------------------------------

------------------------------------------------
The Average Probability of a False Alarm (APFL)
and the Standard Deviation of the Probability of
a False Alarm (SDPFL) in the first  10 subgroups
before the shift in Stage 2:
------------------------------------------------
APFL:  0.03208
SDPFL: 0.08711
------------------------------------------------

-------------------------------------------
    Starting at subgroup  11 in Stage 2:
-------------------------------------------
  t       Number of RLs <= t     P(RL <= t)
-----     ------------------     ----------
    1             85               0.01700
    2            164               0.03280
    3            233               0.04660
    4            284               0.05680
    5            335               0.06700
    6            382               0.07640
    7            427               0.08540
    8            481               0.09620
    9            523               0.10460
   10            561               0.11220
   15            705               0.14100
   20            855               0.17100
   30           1078               0.21560
```

40	1247	0.24940
50	1367	0.27340
75	1647	0.32940
100	1879	0.37580
200	2555	0.51100
300	3018	0.60360
400	3360	0.67200
500	3608	0.72160
750	4090	0.81800
1000	4379	0.87580
2000	4853	0.97060
3000	4956	0.99120
4000	4986	0.99720
5000	4995	0.99900
7500	5000	1.00000
10000	5000	1.00000
20000	5000	1.00000
30000	5000	1.00000
40000	5000	1.00000
50000	5000	1.00000

Sample Output File #5

```
----------------------------------------
mean: ...................   0.00000
standard deviation: ......   1.00000
# of replications of
 two stage procedure: ... 4999
Control chart combination: (Xbar, R)
n: .......................    3
m (Stage 1): .............    5
D&R procedure: ...........    5
----------------------------------------

---------------------------------------------
Stage 1: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups   2 and   3.

Stage 2: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups  10 and  11.
---------------------------------------------

---------------------------------------------------
Out-of-Control (OOC) Average Run Length (ARL) and
Standard Deviation of the Run Length (SDRL) results
---------------------------------------------------
ARL (in number of subgroups):     450.38248
SDRL (in number of subgroups):    654.56502
---------------------------------------------------

------------------------------------------------
The Average Probability of a False Alarm (APFL)
and the Standard Deviation of the Probability of
a False Alarm (SDPFL) in the first  10 subgroups
before the shift in Stage 2:
------------------------------------------------
APFL:  0.03823
SDPFL: 0.10840
------------------------------------------------

-------------------------------------------
    Starting at subgroup  11 in Stage 2:
-------------------------------------------
  t       Number of RLs <= t     P(RL <= t)
-----     ------------------     ----------
    1              88              0.01760
    2             159              0.03181
    3             235              0.04701
    4             287              0.05741
    5             342              0.06841
    6             384              0.07682
    7             423              0.08462
    8             469              0.09382
    9             516              0.10322
   10             554              0.11082
   15             685              0.13703
   20             818              0.16363
   30            1033              0.20664
```

40	1189	0.23785
50	1301	0.26025
75	1580	0.31606
100	1803	0.36067
200	2460	0.49210
300	2915	0.58312
400	3283	0.65673
500	3536	0.70734
750	4021	0.80436
1000	4318	0.86377
2000	4834	0.96699
3000	4945	0.98920
4000	4980	0.99620
5000	4989	0.99800
7500	4998	0.99980
10000	4999	1.00000
20000	4999	1.00000
30000	4999	1.00000
40000	4999	1.00000
50000	4999	1.00000

Replications skipped 1 time(s)
because the number of subgroups dropped
to zero after out-of-control (OOC)
subgroups were deleted.

Sample Output File #6

```
----------------------------------------
mean: ....................   0.00000
standard deviation: ......   1.00000
# of replications of
  two stage procedure: ... 4998
Control chart combination: (Xbar, R)
n: ........................    3
m (Stage 1): .............     5
D&R procedure: ...........     6
----------------------------------------

---------------------------------------------
Stage 1: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups   2 and   3.

Stage 2: shift size of     1.50000 (same
         units as the mean) in the mean
         between subgroups  10 and  11.
---------------------------------------------

---------------------------------------------------
Out-of-Control (OOC) Average Run Length (ARL) and
Standard Deviation of the Run Length (SDRL) results
---------------------------------------------------
ARL (in number of subgroups):     425.71108
SDRL (in number of subgroups):    603.88839
---------------------------------------------------

------------------------------------------------
The Average Probability of a False Alarm (APFL)
and the Standard Deviation of the Probability of
a False Alarm (SDPFL) in the first  10 subgroups
before the shift in Stage 2:
------------------------------------------------
APFL:  0.03441
SDPFL: 0.09416
------------------------------------------------

-------------------------------------------
    Starting at subgroup  11 in Stage 2:
-------------------------------------------
  t       Number of RLs <= t     P(RL <= t)
-----     ------------------     ----------
    1              87              0.01741
    2             160              0.03201
    3             226              0.04522
    4             274              0.05482
    5             330              0.06603
    6             369              0.07383
    7             416              0.08323
    8             464              0.09284
    9             508              0.10164
   10             547              0.10944
   15             695              0.13906
   20             842              0.16847
   30            1072              0.21449
```

40	1239	0.24790
50	1361	0.27231
75	1641	0.32833
100	1883	0.37675
200	2544	0.50900
300	3005	0.60124
400	3347	0.66967
500	3595	0.71929
750	4071	0.81453
1000	4362	0.87275
2000	4853	0.97099
3000	4952	0.99080
4000	4986	0.99760
5000	4994	0.99920
7500	4998	1.00000
10000	4998	1.00000
20000	4998	1.00000
30000	4998	1.00000
40000	4998	1.00000
50000	4998	1.00000

Replications skipped 2 time(s)
because the number of subgroups dropped
to zero after out-of-control (OOC)
subgroups were deleted.

2

VITA

Matthew E. Elam

Candidate for the Degree of

Doctor of Philosophy

Thesis: INVESTIGATION, EXTENSION, AND GENERALIZATION OF A METHODOLOGY FOR TWO STAGE SHORT RUN VARIABLES CONTROL CHARTING

Major Field: Industrial Engineering and Management

Biographical:

Personal Data: Born on January 11, 1969, the son of Dr. Jim B. and Mary Alice Elam.

Education: Received Bachelor of Science degree in Mathematics from the University of Texas at Tyler, Tyler, Texas in August 1991. Received Master of Science degree in Mathematics from the University of Texas at Tyler in August 1994. Completed the requirements for the Doctor of Philosophy degree with a major in Industrial Engineering and Management at Oklahoma State University in May 2001.

Experience: Developmental Mathematics Lab Coordinator, Kilgore College, Kilgore, Texas, August 1993 - May 1994. Mathematics Instructor, East Texas Baptist University, Marshall, Texas, July 1994 - August 1995. Mathematics Instructor, Austin Community College, Austin, Texas, August 1995 - May 1996. Graduate Teaching Assistant, Department of Mathematics, Oklahoma State University, August 1996 - May 1997. Graduate Teaching Assistant, School of Industrial Engineering and Management, Oklahoma State University, August 1997 - May 2001.

Professional Memberships: Tau Beta Pi, American Society for Quality, Alpha Pi Mu, Institute of Industrial Engineers, Alpha Chi National College Honor Scholarship Society.