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Three Essays on Irregular Entries to the End-Customer Market

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Abstract

I study irregular entries to the end-customer market and the impact of such entries on suppliers, buyers, and customers. I am particularly interested in the irregularities of supplier encroachment and counterfeiting problems. This dissertation addresses these issues and proposes solutions in the form of three essays. In the first essay, I study a supply chain, consisting of a supplier and a buyer where the supplier can encroach on the end-customer market and keeps private information on its own production capacity. The supplier can decide on its capacity allocation and the buyer can order strategically, hoarding the supply capacity, to remove the competition. I find that the supplier is worse off, and the buyer is better off, when the supplier keeps its capacity information private. Further, I demonstrate that the supplier may no longer encroach on the end-customer market when it has more capacity. The second and third essays are inspired by the counterfeiting problem on online e-commerce platforms. In the second essay, I develop an algorithm that analyzes customers' reviews on an online platform and provides an authenticity score for the products. I trained context-specific word embedding based on a large corpus of Amazon customer reviews to show that my unsupervised methodology provides good predictive power. Next, I study the effect of customers' reviews on an e-commerce platform's anti-counterfeiting strategy against third-party sellers. The platform can provide a tool for customers that analyzes just the product reviews or a more advanced tool that analyzes both the product and seller reviews to help customers determine if products are fake or genuine. On the seller's side, it can choose to reveal its fake products by charging a lower separating price based on its profit under these two options. I demonstrate that even when the tools are free, the platform does not provide the advanced tool if the seller sells products with a low authenticity score (fake products), and it provides the basic tool if and only if the demand of the genuine product is sufficiently high. Together, these papers provide solutions on how to maximize profits by making informed decisions in the face of market irregularities for the supplier, the buyer, and the customer.

Keywords: Strategic Supply Chain Management, Game Theory, Counterfeit Detection, Fuzzy c-Means, Natural Language Processing.

Summary for Lay Audience

In this dissertation, I study abnormalities in the end-customer's market and how they impact suppliers, buyers, and customers. In particular, I am interested in factory-direct selling and counterfeiting problems on online e-commerce platforms. In this dissertation, I study a situation where a supplier decides to sell directly to the customers and compete with its buying firm. Because the supplier has a limited amount of product, it must allocate it between its own selling channel and the buyer. Because the buyer does not know how much product the supplier has, the buyer may aggressively over-buy the product to control the competition in the end-customer market. I show that contrary to the common belief, keeping its product amount private hurts the supplier's profit because it must signal its production capacity to the buyer. Moreover, having excess product hurts the profit of both the supplier and the buyer because, in this case, the firms have to compete in the end-customer market.

I also study counterfeiting problems on online e-commerce platforms. First, I use machine learning techniques in text analysis to develop a counterfeit detection algorithm. My algorithm analyzes previous customers' reviews for indications of fake or genuine products on online platforms and provides an authenticity score for each product. I test the accuracy of my algorithm by manually assigning a label (i.e., fake/genuine) to the reviews using human coders. Second, I study the strategic behavior of online e-commerce platforms and third-party sellers with regard to using platform-provided anti-counterfeiting tools. I demonstrate that even when the tool is free, the platform does not provide an advanced counterfeiting tool if the seller sells products with a low authenticity score (fake products). Moreover, I show that the platform provides a basic counterfeiting tool if and only if the demand of the genuine product is sufficiently high.

Co-Authorship Statement

I hereby declare that this thesis incorporates some material that is a result of joint research. Essay 1 was co-authored by Dr. Hubert Pun and Dr. Salar Ghamat. Essays 2 and 3 were co-authored by Dr. Rasha Kashef, Dr. Hubert Pun and Dr. Salar Ghamat. As the first author, I was in charge of all aspects of the projects including formulating research questions, literature review, research design, model formulation and analysis, and preparing the first and the following complete versions of the manuscripts. With the above exceptions, I certify that this dissertation and the research to which it refers, is fully a product of my own work. Overall, this dissertation includes three original papers, with the first essay currently under review by *Omega: The International Journal of Management Science*.

Essay 1 Status: Under review.

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Chapter 1

1 Introduction

In this dissertation, I study different irregularities in the end-customer market. In the supply chain setting, the supplying firm may decide to enter the end-customer market and compete with its supply chain partner. For instance, Franz Inc. supplies home décor accessories to original brand manufacturers (OBM) such as Enesco and Lenox and started to sell its own product in 2002. Franz had limited production capacity and decided to prioritize its own brand over the OBMs' (Ghamat et al., 2018). On the other hand, the OBMs may acquire the supplies aggressively to control the competition in the final products' market. For instance, Apple acquire its components aggressively to have strategic dominance over its competitors (Oliver, 2011). To more complicate these dynamics, the supplying firm usually has private information on its production capacity (Ye et al., 2013; Scola, 2012).

Another anomaly in the market is the counterfeiting problem by third party sellers on online e-commerce platforms. Online platforms provide a storefront for third-party sellers and these sellers are the main contributor to the platforms' profit. In 2020, Amazon's gross merchandise value (GMV) was about $490 billion from which $300 billion was from third-party sellers (Danziger, 2021). As a result, platforms keep the barriers to entry low for third-party sellers and such low entry costs expose millions of customers to counterfeit sellers (Cantrell, 2021). Platforms have taken various steps to combat this problem; however, counterfeit products are still abundant on online platforms (Greene, 2019).

1.1 Overview of Thesis and Specific Essays

In Chapter 2, I study supplier encroachment, where a capacitated supplier of products can enter the end-customer market and compete with its buying firm. There are many studies that examine supplier encroachment (Venkatesh et al., 2006; Xu et al., 2010; Ghamat et al., 2018, Yang et al 2018). However, in all these studies, either the capacity is unlimited or both parties have the perfect information, whereas in my study, I extend the literature to consider information asymmetry and production capacity at the same time.

Accordingly, I consider a signaling game in which the supplier has private information on its own capacity. The buyer decides on the order quantity and the supplier decides whether to encroach on the end-customer market. The buyer can order strategically to curb the competition in the end-customer market and the supplier with high capacity can pretend to have low capacity so it can take advantage of the best of both worlds—not only enjoying a higher wholesale price, but also selling to the end customers. Surprisingly, I find that the supplier winds up worse off while the buyer winds up better off when the supplier keeps its capacity information private because the supplier may incur the signalling costs. Therefore, it is beneficial to the supplier to find ways to disclose its capacity information credibly (e.g., by using Electronic Data Interchange (EDI) or linking its database with the buyer) while the buyer should be cautious when adopting these technologies. Moreover, due to information asymmetry, a supplier may encroach on the end-customer market when it has less capacity. I also show that capacity withholding is less likely when information is asymmetrical. Finally, I find that both firms can simultaneously benefit from the supplier's capacity constraint. Furthermore, not having information about supplier capacity not only increases the possibility of supplier encroachment, but also makes strategic inventory (capacity withholding) less important.

In Chapters 3 and 4, I consider the counterfeiting problem of third-party sellers on online e-commerce platforms. More precisely, in Chapter 3, I develop an algorithm that analyzes the sentiment of customer reviews of a product and the similarity of their embedding to that of the word "Fake" as an input and provides an authenticity score using an unsupervised fuzzy clustering algorithm as an output. I tested the algorithm on a scraped data set from Amazon and I showed that my unsupervised clustering algorithm can separate the products into meaningful clusters. Moreover, I tested the robustness of the algorithm by using human coders to label the dataset, namely, the customer reviews, and the algorithm was seen to provide an AUC of 0.680.

In Chapter 4, I study the effect of such a counterfeit detection tool on the online e-commerce platform and third-party seller's strategic behavior. My analytical studies consider deceptive counterfeit deterrence strategies and also the effect of customers' reviews (Cho et al., 2015; Gao, 2018; Zhang et al., 2021; Fang, 2021; Vana and Lambrecht, 2021). I extend the literature by investigating the effect of product reviews and seller reviews on the platform's anti-counterfeiting strategy and third-party sellers' pricing decisions. I consider a system with a third-party seller who sells two (potentially fake) products on an online e-commerce platform. The platform can provide a tool for the customers that analyzes just the product reviews or a more advanced tool that analyzes both the product reviews and the seller reviews. The third-party seller can choose to reveal its fake products by charging a lower separating price or conceal it by charging the same price as the genuine product. I show that even when the tools are free, the platform does not provide the advanced tool if the seller sells any fake products. Moreover, I show that the platform provides the basic tool if and only if the demand of the genuine product is sufficiently high.

In the final chapter, I present an overview of the main results.

Chapter 2

2 Information Liability: A Capacitated Supplier Encroaches on a Less Informed Buyer

In today's complex business environment, conflicting relationships among firms are becoming the norm. Firms can be supply chain partners, but at the same time, they can be competitors. Moreover, capacity is often limited. Prior research has examined this problem in a perfect information setting, but in reality, a supplier often has private information on its own capacity. I consider a signaling game in which the supplier has private information on its own capacity. The supplier first sets the wholesale price. The buyer then decides on the order quantity and the supplier decides whether or not to encroach on the end-customer market. I find that the supplier can be worse off while the buyer can be better off from the supplier's private information on capacity. Moreover, due to information asymmetry, a supplier may encroach on the end-customer market when it has less capacity. My paper also shows that capacity withholding is less likely when information is asymmetrical. Finally, I find that both firms can simultaneously benefit from the supplier's capacity constraint. My paper demonstrates that keeping information on the capacity level private can be harmful, so the supplier should find ways to disclose its capacity information credibly (e.g., by using Electronic Data Interchange (EDI) or linking its database with the buyer). However, the buyer should be cautious when adopting these technologies. Furthermore, not having information about supplier capacity not only increases the possibility of supplier encroachment, but also makes strategic inventory (capacity withholding) less important.

Keywords: supply chain management; limited capacity; signaling; supplier encroachment; withholding

2.1 Introduction

Samsung is one of Apple's fiercest competitors in the consumer electronics industry, but at the same time, it is also one of its major component suppliers. In particular, Samsung is Apple's only supplier of OLED panels, a crucial component to both Apple's iPhone X and Samsung's Galaxy smartphone. However, there is a shortage of OLED panels, forcing Samsung to make the difficult choice between supplying these parts to Apple or using the panels for its own products (McKevitt, 2017). Further complicating this dynamic is the fact that information concerning capacity level is not easily available (Ye et al., 2013; Scola, 2012). Given Apple's dependence on Samsung's supply, analysts have noted, "It would be bad for Apple if Samsung were forced to choose between Apple and itself in case of a supply shortage at its factories" (Forbes, 2013)—a situation that is, in fact, already in place.

Growing demand often leads to a capacity shortage at supply sources. On the one hand, the supplier may be reluctant to supply to its buyer. For example, Franz Inc. is a supplier of home decoration accessories for original brand manufacturers such as Enesco and Lenox. Franz started selling products under its own brand in 2002, and its production shortly met the capacity limit. In 2005, Franz decided to prioritize its own brand over the original brand manufacturers (Ghamat et al., 2018). On the other hand, the buyer may purchase the supply aggressively or even withhold some capacity. For example, Apple's operating strategy is to order components from its suppliers aggressively, "leaving competitors out while products like the iPhone [dominate] components" (Oliver, 2011). It has hoarded 60 percent of Samsung's touchscreens. Apple has also bought holiday air freight space to limit its competitors' ability to get products to the market (Satariano and Burrows, 2011). Another example is the competitive relationship between TPV Technology (TPV) and Philips. TPV is a large computer monitor producer that supplies monitors to Philips, which then sells the monitors under its own brand. TPV also sells monitors to customers directly under its own brands, AOC and Envision. When demand for these monitors rose beyond TPV's capacity limit, the company had to reduce the production of its own brands to satisfy the outsourcing orders from Philips (Wang, 2008).

In this competitive environment with asymmetric information on capacity, firms face several challenges. First, the supplier must decide whether or not (and how) to share its capacity information with the buyer. Moreover, it must choose whether to use the capacity to encroach on the end-customer market or to supply to the buyer. Second, the buyer needs to decide how to mitigate competition in the end-customer market by limiting how much capacity the supplier can have if it does encroach, such as by aggressive supply acquisition or capacity withholding. However, since the buyer does not know the supplier's capacity level, it is uncertain about the effectiveness of such effort, which may be fruitless if the supplier turns out to have high capacity. My paper addresses the following research question: How does the supplier's private information about its capacity affect the supply chain?

To examine this question, I consider a market with a buyer and a capacitated supplier. The supplier has private information about whether it has a high or low capacity level (type), and I model this scenario using a signaling game. In this game, the supplier (sender) first sets the wholesale price (signal). The buyer (receiver) uses this wholesale price to update its belief on the supplier's type, and then it selects the order quantity. Lastly, the two firms decide how much quantity to sell to end customers.

I show that the supplier may be worse off from having more information, and, at the same time, the buyer can be better off from not knowing the supplier's capacity. This translates to the following managerial implications. Conventional wisdom suggests that maintaining a company's internal information is important to the success of a firm. However, my paper demonstrates that sometimes keeping capacity level information private can be harmful. To combat this harm, the supplier should find ways to credibly disclose its capacity information to the buyer. For example, many suppliers such as Intel and Japan Display make official announcements about their capacity levels (Cheng and Lauly, 2018). Sony is one of the main battery suppliers for Dell's laptops, and the two companies link their information systems to establish credible information sharing. Moreover, Cisco uses two-way information sharing with its supply chain partners to share production capacity information (Zhou and Benton Jr., 2007). However, buyers should be cautious about adopting these technologies because my results show that buyers can benefit from the presence of asymmetric information.

I also show that the supplier may stop encroaching on the end-customer market when it has more capacity, for the following reasons. The buyer cannot eliminate the threat of supplier encroachment because it does not know the supplier's capacity. When the capacity level of a low-capacity supplier is small, the supplier has minimal capacity to encroach, so competition from encroachment is not intense. Consequently, the buyer would not buy extra quantities to eliminate encroachment. However, competition from supplier encroachment becomes more intense when the low-capacity supplier has more capacity. In this scenario, avoiding competition becomes a priority, so the buyer will buy extra to eliminate the possibility of encroachment from a low-capacity supplier. This implies that the supplier may not encroach when it has more capacity. This insight is unique to the asymmetric information game. Moreover, I find that capacity withholding is less prevalent under the asymmetric information game because the purpose of withholding is to eliminate supplier encroachment. Yet, since the buyer does not know the supplier's capacity level, there is always a threat that the supplier will turn out to have high capacity. Hence, this threat makes withholding less attractive.

Finally, I show that both the buyer and the supplier can benefit from the supplier having less capacity: the buyer is better off because it is cheaper to buy the entire capacity of the supplier, while the supplier is better off because the buyer will pass along some of the gains of being a monopoly. Nonetheless, because of the lower wholesale prices, the supplier's possible gain from capacity constraint is lower when compared to the full-information setting. Hence, this is the extra motivation for the supplier to credibly signal its capacity level.

2.2 Literature Review

This research relates to three separate literature streams. The first is on the topic of supplier encroachment. There are several papers that examine the direct selling of a supplier to the end-customer market, either in an original equipment manufacturer setting (e.g., Venkatesh et al., 2006; Xu et al., 2010) or in a retail setting (e.g., Chiang et al., 2003; Tsay and Agrawal 2004; Cattani et al., 2006; Arya et al., 2007; Dumrongsiri et al., 2008; Pun, 2013; Guan et al., 2019). These papers examine three possible strategies for the supplier: supplier only, supplier encroachment (direct selling), or sole seller to end customers. The papers show that supply chains may benefit from supplier encroachment and illustrate the conditions for such outcomes in different settings. I extend this literature by considering supplier limitations in terms of capacity. This capacity constraint has a non-trivial impact on the supplier's encroachment decision and the buyer's ordering behavior because the buyer can order strategically to manipulate the degree of competition in the end-customer market.

Another related literature stream studies the effect of capacity management on supply chains. Gupta and Wang (2007) examine the capacity allocation problem of a contract manufacturer between contractual orders and one-time transactional orders. Similarly, Yu et al. (2015) study a firm's capacity allocation problem where the firm decides how much to allocate for advanced selling and how much to reserve for future sales. Nazerzadeh and Perakis (2016) consider two capacitated suppliers selling through a common retailer using an incomplete information game where the suppliers have private information about their capacity level. Guo and Wu (2018) examine the optimal capacity sharing among competitors; the capacity sharing price can be determined either before or after the retail price is set. These papers do not consider supplier encroachment.

Ghamat et al. (2018) model a supply chain in which an original brand manufacturer must decide whether to outsource to a third-party supplier or a capacitated contract manufacturer that can produce a competing product. Yang et al. (2018) model a supply chain where a proprietary component supplier can sell a product through a retailer, its own channel, or both. Both papers show that in the capacitated system, the buyer may aggressively acquire supply to control the market. However, when the capacity information is private, the strategy to manipulate the degree of competition in the end-customer market may be challenging for the buyer. This is because in the asymmetric information setting, the buyer may not be able to limit the market output and may end up competing with a high-capacity supplier.

I also contribute to the stream of literature on information asymmetry in a supply chain. Jiang et al. (2016) consider a scenario where the supplier receives a private forecast about the market size; this supplier might pretend to have received a low market forecast to incentivize a lower retail price and higher unit sales by the buyer. Li et al. (2015) investigate the alternative setting, where the buyer has private information about the market size. They demonstrate that when the supplier can offer a menu of contracts, the supplier's possibility of encroachment may cause the buyer to up-distort or down-distort its order quantity.

Li et al. (2014) and Gao et al. (2021) are the only papers in the supplier encroachment literature that consider a signaling game. The asymmetric information is on market size (Li et al. 2014) or selling cost (Gao et al. 2021). Furthermore, Li et al. (2014) assume that the buyer has perfect information about the supplier, while in my model, the supplier has private information. These two papers assume that capacity is unlimited, and pooling equilibrium does not exist. In real life, suppliers have a capacity constraint. Capacity limitation introduces a constraint to the optimization problem of the supplier, such that the supplier and the buyer may not be able to sell a quantity that corresponds to the interior solution. In addition, given that the capacity limit is the supplier's private information, the buyer can find out the supplier's capacity by placing an order that is larger than the capacity of the low-capacity supplier. Li et al. (2014) and Gao et al. (2021) do not have this constraint. This leads to a fundamental difference in the structural property of the equilibrium solution: Pooling equilibrium does not exist under Li et al. (2014) and Gao et al. (2021), but in my model, the optimal solution is pooling equilibrium under a large range of parameter setting. Note that Lai et al. (2012)

conjecture in footnote 5 (p.1937) that incorporating a supply constraint may lead to the existence of a pooling equilibrium, and my paper confirms their intuition. The managerial insights of my paper, such as a supplier may encroach on the end-customer market when it has less capacity and capacity withholding is less likely when information is asymmetrical, are derived within pooling equilibrium. Consequently, by considering asymmetric information on capacity limit, there are lots of new results that are insightful and relevant in my work that have not been studied before.

2.3 The Model

Consider a market that consists of a buyer (B) and a capacitated supplier (S). The buyer orders from the supplier and sells the product to the end customers. The supplier has the option of encroaching on the market. For mathematical tractability, I normalize the production cost to zero. The per-unit selling cost of the buyer and the supplier are C_B and C_S, respectively. The buyer and the supplier decide the amount sold to the end customers, q_B and q_S, respectively. I use an inverse demand function that is widely used in the literature on supplier encroachment (e.g., Arya et al., 2007; Li et al., 2014; Yang et al., 2018). In particular, the retail (market clearing) price is $P = \max\big(0, D - (q_B + q_S)\big)$, where D is the market size.

The supplier has a capacity limit for producing the product. More specifically, the supplier's capacity level K_i, $i \in \{H, L\}$ can either be high or low, where $K_H > K_L$. I will present conditions on K_i in Definition 2.1. The supplier has private information about whether it is high- or low-type. The buyer knows the parameter value K_i, but it only has a prior belief that the supplier has a high capacity with probability $0 < \lambda_0 < 1$ and low capacity with probability $1 - \lambda_0$. The buyer orders quantity Ω from the supplier. Throughout the analysis, I make the standard assumption in papers that consider capacity constraint (e.g., Ghamat et al. 2018; Yang et al. 2018): the supplier would fulfil up to its capacity and the leftover order will not be met, i.e., a type-i supplier would deliver a quantity $Q = \min[\Omega, K_i]$. This also implies that the buyer can infer the supplier's type if the supplier does not have sufficient capacity to fulfil the buyer's order quantity. Specifically, consider the situation where the buyer orders a quantity Ω that is larger than K_L; then only a high-type supplier is able to fulfill this order quantity.

Note that the type-i supplier cannot sell a quantity that is more than its leftover quantity $(K_i - Q)$ to end customers. Similarly, the buyer cannot sell more than its received quantity from the supplier ($Q \geq q_B$); withholding occurs when the buyer has leftover quantity after selling to the end customers ($Q > q_B$). The supplier's realized profit is $\pi = (P - C_S)q_S + wQ$, and the buyer's realized profit is $\Pi = (P - C_B)q_B - wQ$. These two profits comprise the sales to the end-customer market, and the transfer payment for the quantity that the supplier delivers to the buyer.

The game sequence is as follows. In step 1, the supplier sets the wholesale price, w. In step 2, the buyer decides on its order quantity, Ω. At the end of step 2, the buyer receives the quantity, $Q = \min[\Omega, K_i]$, from a type-i supplier. In step 3, the buyer decides how much quantity q_B to sell to the end customers. In step 4, the supplier decides how much quantity q_S to sell. The main objective of this paper is to study the implications of private information on capacity in the business context of supplier encroachment. Therefore, I extend Yang et al. (2018) from a perfect information setting to an asymmetric information setting, and I structure my model such that it closely follows Yang et al. (2018). Specifically, under Yang et al. (2018) and my model, the supplier sets the wholesale price in the first step of the game, and then the buyer decides how much to buy in the second step. Therefore, I do not consider the case where the supplier explicitly sets capacity allocation in the first step.[1] With that being said, the supplier can use the wholesale price (w) as an indirect mechanism to manipulate the capacity allocation to the buyer. To illustrate, if the supplier has a high capacity level, it can set a low wholesale price to incentivize the buyer to order more. However, if the supplier has a low capacity level or if it wants to prioritize its selling quantity to the end customers over its selling quantity to the buyer, it can set a high wholesale price to discourage the buyer from ordering a large quantity.

2.4 Equilibrium Analysis

A higher selling cost for the supplier (compared to the selling cost of an incumbent retailer) is ubiquitous in the supplier encroachment literature and can be attributed to the supplier's lack of experience in direct selling (Guan et al., 2019). Thus, assuming $C_S > C_B$, without loss of generality, I normalize $C_B = 0$ and $C_S = C$. In order to focus on the interesting regions, I

[1] Under the main model of Yang et al. (2018), the supplier decides the wholesale price in the first step of the game. See Equation 8 of Section 2.4 (p.2205). Then they extend the model in Section 7 (p.2215) such that the supplier decides both wholesale price and capacity allocation in the first step. They show in Proposition 9 that "the supplier does not restrict the buyer's capacity allocation; that is, the optimal maximum capacity allocated to the buyer is $Q^* = K$". Therefore, they conclude that whether or not the supplier decides capacity allocation in the first step has no impact on the final solution. We apply the same logic in our paper.

consider the parameter settings $\overline{D} > D > 2C > 0$. The definitions of all thresholds and the proof of all results are given in the online appendix. Throughout the paper, I use the comparison in a "weakly" sense.

2.4.1 Full Information Game

As a benchmark scenario, I first examine a game with perfect information in which the buyer knows the supplier's capacity level, K. Lemma 2.1 highlights the market structure of the full information game; the proof of Lemma 2.1 and Table A3 presents the optimal solutions (w^F, q_S^F, q_B^F and Q^F); I use the superscript F to denote this scenario. Figure 2.1 illustrates the market structure; the x-axis is the market size (D) and the y-axis is the capacity level (K). Throughout the paper, I use a two-letter nomenclature to name a region: the first letter denotes whether the market structure is supplier only (S) or dual channel (D), and the second letter denotes whether the buyer withholds some of the supplier's capacity (W) or not (N). (It can easily be shown that the market structure where the supplier is a monopoly in the end-customer market is not optimal.) The regions (DN^F, SW^F and SN^F) are illustrated in Figure 2.1 and the thresholds of the regions are analytically defined in the proof of Lemma 2.1.

Lemma 2.1: *The buyer's ordered and received quantity is always the same, i.e., $\Omega^F = Q^F$. Moreover, the market structure of the full information game is as follows.*

a. *Dual products and no withholding: $\Omega^F = q_B^F > 0$, $q_S^F > 0$ and $q_B^F + q_S^F < K$ in Region DN^F;*
b. *Single-product-by-the-buyer and withholding: $\Omega^F = K > q_B^F > 0$ and $q_S^F = 0$ in Region SW^F; and*
c. *Single-product-by-the-buyer and no withholding: $\Omega^F = K = q_B^F > 0$ and $q_S^F = 0$ in Region SN^F.*

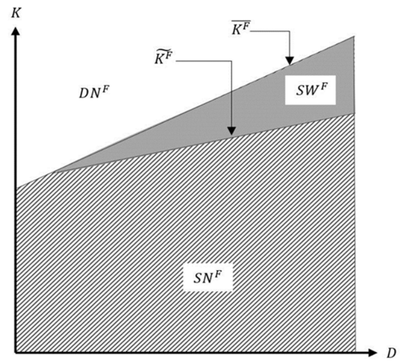

Figure 2-1: Optimal market structure for the full information game

At the one extreme, when the supplier has a lot of capacity, the capacity constraint is not binding (cf. Lemma 2.1a; Region DN^F). The optimal solutions (w^F, q_S^F, q_B^F and Q^F) are the same as in an uncapacitated system and are independent of the capacity level K. In other words, the buyer would sell all the purchased quantities from the supplier to the end customers ($Q^F = q_B^F > 0$); the supplier would encroach on the end-customer market ($q_S^F > 0$); and there is leftover capacity ($q_B^F + q_S^F < K$). At the other extreme, when the supplier has very limited capacity (Region SN^F), the buyer would order the supplier's entire capacity and sell all purchased quantities to the end customers (cf. Lemma 2.1c). The supplier does not have any leftover capacity to encroach on the market. Lastly, Lemma 2.1b shows that the benefit of having a monopoly in the market justifies the withholding cost (the cost of purchasing units from the supplier but not selling them to end customers) when the supplier's capacity is intermediate (cf. Region SW^F). Thus, in this scenario, the buyer would buy the supplier's entire capacity to prevent the supplier from encroaching on the market, and it would sell a portion of all purchased items to the end customers (withholding).

Lemma 2.1a (Region DN^F) implies that the capacity constraint is redundant because the supplier always has leftover capacity. Moreover, Lemmas 2.1b and 2.1c imply that when the supplier's capacity is sufficiently small (Regions SW^F and SN^F), the buyer can choose the order quantity strategically to manipulate the supplier's encroachment decision. Therefore, when I consider the asymmetric information game in the following subsection, I use the following definition:

Definition 2.1: *For the asymmetric information game, a high-type supplier is one with a capacity level such that the capacity constraint is not binding under the full information game ($K_H \in$ Region DN^F). Otherwise, the supplier is of low-type ($K_L \in$ Regions SW^F and SN^F).*

2.4.2 Asymmetric Information Game

In this subsection, I analyze the asymmetric information game where the buyer does not know whether the supplier is high-type or low-type. I use backward induction to ensure subgame perfection for the Perfect Bayesian Equilibrium.

At step 4 of the game, a type-i supplier decides the selling quantity q_{Si} to optimize its profit π_i, and the selling quantity q_{Si} cannot exceed the remaining capacity, $K_i - Q$. The optimization problem of a type-$i \in \{H, L\}$ supplier is as follows:

$$\begin{aligned} & q_{Si} = arg\max_{q_{Si}} \pi_i \\ \text{s.t.} \quad & q_{Si} \leq K_i - Q \end{aligned} \tag{2.1}$$

At steps 2 and 3 of the game, the buyer decides the order quantity (Ω) and the selling quantity (q_B). The buyer does not know the supplier's type, so its objective function is the expected profit $\mathrm{E}[\Pi(\Omega, q_B)]$, anticipating the best response of the type-i supplier's selling quantity q_{Si} at step 4 of the game. In the signaling game literature (e.g., Li et al., 2014; Gao et al., 2021), the supplier's type is only revealed if it uses two different wholesale prices (separating). In my paper, the supplier's type is also revealed if the buyer orders a quantity that is larger than the low-type supplier's capacity. The buyer uses the information revealed from the supplier's wholesale price signal (after step 1 of the game) or the delivered quantity (after step 2 of the game) to update its prior belief (λ_0) regarding the type of the supplier. Define λ as the posterior belief of the buyer.

At step 1 of the game, the supplier decides the optimal wholesale price. Since the supplier anticipates the response of the buyer to its wholesale price, it may choose to signal its capacity information to the buyer by manipulating the wholesale price.

Before deriving the optimal wholesale price under the asymmetric information game, Lemma 2.2 presents the supplier's distorted incentive if it uses the wholesale prices of the full information game, w_H^F and w_L^F. The profit function $\pi_i(w_j^F)$, $i \neq j \in \{H, L\}$ means that the type-i supplier sets w_j^F at step 1 of the game. The buyer naively believes that this is a type-j

supplier *without* considering the possibility that the supplier may misrepresent its type; it chooses the order and selling quantities Ω_j^F and q_{Bj}^F, respectively. For example, $\pi_H(w_L^F)$ refers to the profit function for a high-type supplier that sets w_L^F and the buyer believes that this is a low-type supplier. At step 4 of the game, the supplier would decide the optimal q_S that maximizes its profit (there is no need for the supplier to misrepresent itself at step 4).

Lemma 2.2:

a. $\pi_H(w_H^F) < \pi_H(w_L^F) \Leftrightarrow K_L > \underline{K}$

b. $\pi_L(w_L^F) \geq \pi_L(w_H^F)$

A high-type supplier has an incentive to pretend to be a low-type unless the capacity level of the low-type supplier is very low (cf. Lemma 2.1a). This is because if the buyer believes that the high-type supplier is actually low-type, it may pay a higher wholesale price to buy all capacity. Yet, this high-type supplier would have some leftover capacity to encroach on the end-customer market, which implies that a high-type supplier gets the best of both worlds—not only enjoying a higher wholesale price, but also selling to the end customers. However, when the low-type supplier has very little capacity (i.e., $K_L < \underline{K}$), the high-type supplier has no incentive to pretend to be a low-type because it would earn very little from selling minimal quantities to the buyer. On the other hand, a low-type supplier does not want to pretend to be high-type (Lemma 2.2b). This is because the buyer may order more than its maximum capacity, immediately revealing its type. Moreover, the wholesale price of a low-type supplier is higher than that of a high-type supplier except when K_L is very high. The buyer's order quantity to a low-type supplier is also larger unless K_L is small. Therefore, the profit of a low-type supplier is higher when it uses w_L^F than when it uses w_H^F.

Two types of equilibria are possible: separating and pooling. When facing a pooling wholesale price, the buyer has the option of ordering a quantity that is larger than the low-type supplier's capacity (i.e., $\Omega > K_L$) to find out the supplier's type: only a high-type supplier can satisfy this order. I will present this mechanism as a subcase in the subsection on pooling equilibrium. Following Jiang et al. (2016) and Piccolo et al. (2017), I assume that the buyer's prior belief is $\lambda_0 = 0.5$ for mathematical tractability. To motivate, if the buyer has very little information about the supplier, then it might set $\lambda_0 = 0.5$. However, my results are robust to other values of λ_0.

2.4.2.1 Separating Equilibrium

Under a separating equilibrium, the two types of suppliers would set a different wholesale price at step 1 of the game, i.e., $w_H^{SE} \neq w_L^{SE}$. I use the superscript SE to denote the solution for separating equilibrium and the subscript H or L to represent the supplier's type. The buyer can infer the capacity of the supplier from the wholesale price. Definition 2.2 establishes the belief system of a separating equilibrium in which the buyer's posterior belief is that the supplier is of low-type if the wholesale price is w_L^{SE}, and high-type otherwise.

Definition 2.2: *The belief system of a separating equilibrium is* $\lambda_{w_L^{SE}} = 0$ *and* $\lambda_{w \neq w_L^{SE}} = 1$.

The wholesale price of a high-type supplier (w_H^{SE}) is given as follows. The objective function $\pi_H^{SE}(w)$ is the profit of the high-type supplier under the separating equilibrium.

$$w_H^{SE} = arg\max_{w} \pi_H^{SE}(w) \tag{2.2}$$

As discussed in Lemma 2.2b, a low-type supplier has no incentive to pretend to be a high-type, so the high-type supplier can charge the same wholesale price as the full information game, i.e., $w_H^{SE} = w_H^F$. Then the buyer's order quantity and selling quantity would be the same as the ones under the full information game, i.e., $\Omega_H^{SE} = \Omega_H^F$ and $q_H^{SE} = q_H^F$.

On the other hand, a high-type supplier has an incentive to pretend to be a low-type when the low-type supplier's capacity level is not very small (i.e., $K > \underline{K}$; cf. Lemma 2.2a). Therefore, the low-type supplier needs to down-distort the wholesale price (as compared to the full information game) to deter the high-type supplier from mimicking it, i.e., $w_L^{SE} \leq w_L^F$. When observing the wholesale price w_L^{SE}, the buyer would update its belief about the supplier's capacity level (i.e., $\lambda_{w_L^{SE}} = 0$). As a result, it would order a quantity that corresponds to a low-type in step 2 ($\Omega = \Omega_L^{SE}$). The objective function $\pi_L^{SE}(w)$ is the profit of the low-type supplier under the separating equilibrium.

$$\begin{aligned} & w_L^{SE} = arg\max_{w} \pi_L^{SE}(w) \\ \text{s.t.} \quad & \max_{w \neq w_L^{SE}} \pi_H^{SE}(w) \geq \pi_H^{SE}(w_L^{SE}) \end{aligned} \tag{2.3}$$

The constraint in Equation 2.3 is the incentive compatibility constraint for the high-type supplier not to choose the wholesale price of a low-type supplier. In particular, the left-hand side of the constraint, $\max_{w \neq w_L^{SE}} \pi_H^{SE}(w)$, indicates that a high-type supplier would choose a wholesale price that differs from w_L^{SE}. The buyer would then update its belief that the supplier is of high-type and set an order quantity accordingly, i.e., $\Omega = \Omega_H^{SE}$. The right-hand side of the constraint, $\pi_H^{SE}(w_L^{SE})$, is the situation where a high-type supplier pretends to be low-type by charging w_L^{SE}. The buyer would set the order quantity corresponding to a low-type supplier, i.e., $\Omega = \Omega_L^{SE}$.

Section B.2 in the online appendix presents the solutions to the separating equilibrium (cf. Equations 2.2 and 2.3). Lemma 2.3 illustrates some characteristics of the separating wholesale prices.

Lemma 2.3: $w_H^{SE} = w_H^F$, $w_L^{SE} \leq w_L^F$, *and there exists a unique* K_{SE} *such that* $w_H^{SE} < w_L^{SE} \Leftrightarrow K_L < K_{SE}$.

A low-type supplier has no incentive to pretend to be a high-type, but the opposite is not true. Therefore, the first two comparisons illustrate that the high-type supplier can charge the same wholesale price as in the full information game, but the low-type supplier needs to down-distort the wholesale price to deter the high-type supplier from mimicking it. Moreover, w_L^{SE} is larger than w_H^{SE} as long as $K_L < K_{SE}$. This is because a low-type supplier sells fewer items, and the buyer is willing to pay more to buy all the capacity and be a monopoly (single-product-by-the-buyer).

2.4.2.2 Pooling Equilibrium

Under a pooling equilibrium, the two types of suppliers set the same wholesale price, w^{PE}. This common wholesale price does not reveal any additional information to the buyer about the supplier's type. Therefore, after step 1 of the game, the buyer's posterior belief is the same as the prior belief, i.e., $\lambda_{w^{PE}} = \lambda_0$. Given that a high-type supplier wants to pretend to be a low-type but the opposite is not true (cf. Lemma 2.2), Definition 2.3 presents the belief system of a pooling equilibrium after step 1 in which the buyer assumes that the supplier that gives a wholesale price different from w^{PE} is a high-type. This definition follows the common approach in the literature (e.g., Li et al., 2014; Jiang et al., 2016):

Definition 2.3: *The belief system of a pooling equilibrium after step 1 is* $\lambda_{w^{PE}} = \lambda_0$ *and* $\lambda_{w \neq w^{PE}} = 1$.

As discussed earlier, the buyer has the option of ordering a quantity that is larger than the low-type supplier's capacity to reveal the supplier's type. Therefore, I consider two different possibilities. First, I present the case where the buyer orders less than or equal to the low-type supplier's capacity, i.e., $\Omega \leq K_L$. Then, I examine the other scenario where the buyer orders more than the low-type supplier's capacity, i.e., $\Omega > K_L$. Finally, I present the buyer's optimal order quantity and the supplier's pooling wholesale price.

2.4.2.2.1 Case 1 – Pooling Equilibrium with $\boldsymbol{\Omega \leq K_L}$

First, I consider the case where the buyer's order quantity is not larger than the capacity of a low-type supplier. The analysis is the same as the typical pooling equilibrium presented in the literature. At step 4 of the game, the optimization problem of a type-i supplier can be written as follows (cf. Equation 2.1). Note that $D - (q_B + q_S)$ is the retail price.

$$q_{Si} = arg\max_{q_S} \pi_i = (D - (q_B + q_S) - C)q_S + wQ$$

$$\text{s.t.} \quad q_{Si} \leq K_i - Q \tag{2.4}$$

Since the right-hand side of the constraint includes the supplier's capacity limit K_i, the supplier's optimal selling quantity differs depending on whether it is a low-type or a high-type supplier. In other words, $q_{SH} \neq q_{SL}$ can be true, even though both types of suppliers use the same (pooling) wholesale price in step 1 of the game.

At step 2, the buyer's order quantity satisfies $\Omega \leq K_L$, so it does not find out the supplier's type. There is no further updating of the buyer's belief after step 2, so the buyer's posterior belief after step 2 is the same as the prior belief according to Definition 2.3 ($\lambda = \lambda_0$). The buyer's optimization problem in steps 2 and 3 of the game is as follows. Denote the order quantity that solves Equation 2.5 as Ω^{PE1}.

$$\max_{\Omega, q_B} \mathrm{E}[\Pi(\Omega, q_B)] = [\lambda_0(D - q_B - q_{SH}) + (1 - \lambda_0)(D - q_B - q_{SL})]q_B - wQ$$

$$\text{s.t.} \quad \Omega \leq K_L$$

$$Q = \Omega$$

$$q_B \leq Q \tag{2.5}$$

The first part of the buyer's expected profit is the expected revenue from selling q_B based on its posterior belief on the supplier's type. This is composed of the retail price when the supplier is of high-type ($D - q_B - q_{SH}$) with probability λ_0, and the retail price when the supplier is of low-type ($D - q_B - q_{SL}$) with probability $1 - \lambda_0$. Note that the buyer would have a pooling selling quantity (q_B) and not a type-dependent selling quantity because it does not know the supplier's type when making the selling quantity decision at step 3. The last part is the wholesale price paid to the supplier, and it is independent of the supplier's type because this is a pooling equilibrium. The buyer receives a quantity that is equal to its order quantity ($Q = \Omega$) because it does not order more than the low-type supplier's capacity ($\Omega \leq K_L$). The buyer's selling quantity cannot exceed the quantity that it receives from the supplier, $q_B \leq Q$.

2.4.2.2.2 Case 2 –Pooling Equilibrium with $\boldsymbol{\Omega > K_L}$

As the next case, I examine the situation where the buyer orders a quantity that is larger than the low-type supplier's capacity. The buyer can find out whether the supplier is high-type or low-type after step 2 of the game from the quantity that the supplier delivers. In this case, the buyer's belief system can be updated after the second step of the game. Definition 2.4 updates the belief system given in Definition 2.3, and retains the key feature of Definition 2.3, i.e., the buyer assumes that the supplier that gives a wholesale price different from w^{PE} is a high-type.

Definition 2.4: *The belief system of a pooling equilibrium after step 2 if* $\Omega > K_L$ *is* $\lambda_{w^{PE},Q=K_L} = 0$*,* $\lambda_{w^{PE},Q=\Omega} = 1$ *and* $\lambda_{w \neq w^{PE}} = 1$.

Since the buyer knows the supplier's type after step 2, the buyer and the type-i supplier make the selling quantity decisions q_{Bi} and q_{Si} at steps 3 and 4 similar to the way they would in a separating equilibrium.

The buyer's optimization problem in step 2 is as follows. Denote the order quantity that solves Equation 2.6 as Ω^{PE2}.

$$\max_{\Omega} \mathrm{E}[\Pi(\Omega)] = \lambda_0\big((D - q_{BH} - q_{SH})q_{BH} - w\Omega\big) + (1 - \lambda_0)\big((D - q_{BL} - q_{SL})q_{BL} - wK_L\big)$$

$$\text{s.t.} \quad \Omega > K_L \tag{2.6}$$

The buyer does not know the supplier's type from the wholesale price w^{PE}, so it uses the belief λ_0 (cf. Definition 2.3) to derive its expected profit. However, unlike Equation 2.5, the buyer anticipates that it will know the supplier's type at step 3 based on the received quantity, so it uses a separating selling quantity q_{BH} and q_{BL} (instead of a pooling selling quantity q_B) when computing its expected profit. Moreover, the buyer receives a quantity $Q = \min[\Omega, K_i]$ from a type-i supplier. Hence, the transfer payment is $w\Omega$ if the supplier is a high-type and is wK_L if the supplier is a low-type.

2.4.2.2.3 Optimal Pooling Equilibrium

Finally, the buyer decides the optimal order quantity that maximizes its profit as follows:

$$\Omega^{PE} = \begin{cases} \Omega^{PE1} & \text{if } \mathrm{E}[\Pi(\Omega^{PE1})] > \mathrm{E}[\Pi(\Omega^{PE2})] \\ \Omega^{PE2} & \text{otherwise} \end{cases} \tag{2.7}$$

As the supplier anticipates what will happen at step 4 when setting the wholesale price at step 1 (cf. discussion around Equation 2.4), the pooling wholesale price that maximizes the profit for each of the two types of suppliers will differ. Given the asymmetrical distorted incentive between the two types of suppliers (cf. Lemma 2.2), the optimal pooling wholesale price is the one that maximizes the profit of the low-type supplier. The reason is as follows. Denote the pooling wholesale price for a type-i supplier without considering the incentive of the other type of supplier to be $\widehat{w}_i^{PE}$. If I use $\widehat{w}_H^{PE}$ as the pooling wholesale price, the low-type supplier will always deviate to an off-equilibrium wholesale price because $\widehat{w}_H^{PE}$ is not a profit maximizer for the low-type supplier. On the other hand, if I use $\widehat{w}_L^{PE}$ as the pooling wholesale price, the high-type supplier may not have an incentive to deviate because mimicking the low-type supplier can be profitable. Therefore, $\widehat{w}_L^{PE}$ is a candidate solution for a pooling wholesale price, but $\widehat{w}_H^{PE}$ is not.

Mathematically, the pooling wholesale price w^{PE} is defined as follows:

$$\begin{aligned} & w^{PE} = arg\max_{w} \pi_L^{PE}(w) \\ \text{s.t.} \quad & \pi_H^{PE}(w^{PE}) \geq \max_{w \neq w^{PE}} \pi_H^{SE}(w) \end{aligned} \tag{2.8}$$

The objective function is the profit of the low-type supplier $\pi_L^{PE}(w)$, anticipating that the buyer's best response function of order quantity in step 2 is Ω^{PE}. The constraint is an incentive compatibility constraint for the high-type supplier to choose the wholesale price of a pooling equilibrium. Specifically, the left-hand side of the constraint, $\pi_H^{PE}(w^{PE})$, indicates that a high-type supplier uses w^{PE}. Then the equilibrium is pooling, so the buyer's order quantity in step 2 is Ω^{PE}. The right-hand side of the constraint, $\max_{w \neq w^{PE}} \pi_H^{SE}(w)$, states that the high-type supplier would deviate and choose a wholesale price that differs from w^{PE}. This price reveals the supplier to be a high-type according to the belief system Definition 2.3, $\lambda_{w \neq w^{PE}} = 1$. This leads to a separating equilibrium (the superscript SE in π_H^{SE}) where the buyer's order quantity is that of a separating equilibrium for a high-type supplier, Ω_H^{SE}. Section B.3 in the online appendix presents the solutions to the pooling equilibrium.

2.4.3 Equilibrium Analysis of the Asymmetric Information Game

I use the lexicographically maximum sequential equilibrium (LMSE; Mailath et al., 1993), which is a widely used technique in the literature (e.g., Jiang et al., 2016; Guo and Jiang, 2016; Jiang and Yang, 2019), to refine the equilibria. This is because Schmidt and Buell (2017) demonstrate that a real-life operations management situation can often be explained accurately by a pooling equilibrium. LMSE allows the possibility where pooling equilibrium is chosen. Therefore, they conclude that "the undefeated refinement [LMSE] predicts the outcome that yields the highest equilibrium payoff for each type of informed player" (Schmidt and Buell 2017). Similarly, Jiang et al. (2016) state that "the pooling PBE, which l-dominates [LMSE-dominates] the separating PBE, would seem more plausible since the separating PBE must be based on some unreasonable off-equilibrium belief that forces the l-type firm to separate rather than pool." Based on the pooling equilibrium's ability in explaining real-life situations, I use LMSE because this allows us to have the flexibility of selecting the most efficient equilibrium (either pooling or separating) based on the type of supplier that wants to reveal the type (low-type supplier; cf. Lemma 2.2). Specifically, the optimal wholesale price for a type-i supplier is defined as follows. I use the superscript * to represent the optimal solution under the asymmetric information game.

$$w_i^* = \begin{cases} w_i^{SE} & \text{if } \pi_L^{SE}(w_L^{SE}) > \pi_L^{PE}(w^{PE}) \\ w^{PE} & \text{otherwise} \end{cases} \tag{2.9}$$

Equation 2.9 states that the type-i supplier would choose wholesale price w_i^{SE} if the profit of the low-type supplier under the separating equilibrium, $\pi_L^{SE}(w_L^{SE})$, is higher than that under the pooling equilibrium, $\pi_L^{PE}(w^{PE})$. The buyer would update its belief about the supplier's type according to Definition 2 (i.e., $\lambda_{w_L^{SE}} = 0$ and $\lambda_{w \neq w_L^{SE}} = 1$) and set the optimal order quantity corresponding to the separating equilibrium in step 2 (Ω_i^{SE}). This implies that the equilibrium is separating. Otherwise, the type-i supplier would choose wholesale price w^{PE}; this leads to a pooling equilibrium. The buyer would then update its belief based on Definition 3 (i.e., $\lambda_{w^{PE}} = \lambda_0$ and $\lambda_{w \neq w^{PE}} = 1$); it may also order more than the low-type capacity to reveal the supplier's type according to Definition 2.4.

Proposition 2.1 and Figure 2.2 present the optimal market structure for the asymmetric information game. The white space on the top left corner of Figure 2.2 is undefined because, according to Definition 1, K_L must be within the parameter setting of a capacitated system under the full information game (Regions SW^F and SN^F in Figure 2.1). The thresholds of the regions in Figure 2.2 are analytically defined in the proof of Proposition 2.1.

Proposition 2.1: $q_{SH}^* > 0$. *The optimal market structure for the low-type supplier and the buyer are:*

a. *Pooling equilibrium; dual products and no withholding:* $\Omega^* = Q^* = q_B^* > 0$, $q_{SL}^* > 0$. *Moreover,*
   (i) $q_B^* + q_{SL}^* < K_L$ *in Region* DN^*.
   (ii) $q_B^* + q_{SL}^* = K_L$ *in Region* $\overline{DN}^*$.
b. *Pooling equilibrium; single-product-by-the-buyer and withholding:* $\Omega^* > Q_L^* = K_L > q_{BL}^* > 0$ *and* $q_{SL}^* = 0$ *in Region* SW^*.
c. *Pooling equilibrium; single-product-by-the-buyer and no withholding:* $\Omega^* = Q^* = q_B^* = K_L > 0$ *and* $q_{SL}^* = 0$ *in Region* SN^* *and* $K_L > \dot{K}$.

d. *Separating equilibrium; single-product-by-the-buyer and no withholding:* $\Omega_L^* = Q_L^* = q_{BL}^* = K_L > 0$ *and* $q_{SL}^* = 0$ *in Region* SN^* *and* $K_L \leq \dot{K}$.

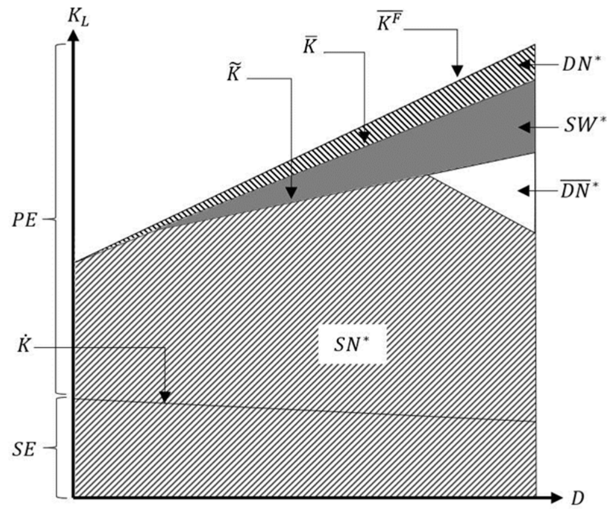

Figure 2-2: Optimal market structure for the asymmetric information game

The regions (DN^*, SW^*, and SN^*) also exist in the full information game (cf. Lemma 2.1), but Region $\overline{DN}^*$ is unique to the asymmetric information game. First, when the low-type supplier has a sufficiently large capacity (Region DN^*), buying all of this capacity from the supplier is too costly. Hence, the buyer would not withhold, and the supplier would encroach on the market; this is one of the non-trivial impacts of information asymmetry on the market structure. Specifically, at this capacity level, the buyer buys all capacity from the supplier under a full information setting. Under an asymmetric information setting, it will not buy all capacity but will instead compete with both supplier types in the end-customer market.

Second, when the capacity of the low-type supplier is small (cf. Region SN^*), the buyer would order $\Omega^* = K_L$ and sell all purchased quantities to the end customers. Region SN^* is further separated into two sub-regions. When the capacity is very small (Region SN^* and $K_L \leq \dot{K}$), the supplier uses the wholesale price to truthfully signal its type (separating equilibrium). However, when the capacity is intermediate (Region SN^* and $K_L > \dot{K}$), the supplier does not reveal its type (pooling equilibrium).

Third, consider Region SW^*. This is the parameter setting where the supplier sets a pooling wholesale price, and the buyer would order more than the low-type supplier's capacity to reveal the supplier's type. This is the new mechanism of revealing the supplier's type with the order quantity under a pooling equilibrium, which is one of the contributions of this paper to the literature. Specifically, the buyer's expected profit is decreasing in the order quantity; hence, it would order $\Omega^* = K_L + \varepsilon$, where ε is a small positive number. This is the lowest cost that the buyer needs to incur to reveal the supplier's type through the order quantity. If the supplier turns out to be low-type, then the buyer receives $Q = K_L$ and sells a quantity $q_{BL} < Q$, implying that there is withholding. The supplier has no capacity to encroach, so $q_{SL} = 0$. On the other hand, if the supplier turns out to be high-type, the buyer receives $Q = \Omega^*$ and sells $q_{BH} = Q$ to customers. The supplier has capacity to encroach, so $q_{SH} > 0$. Under this setting, buying all capacity from a low-type supplier is not too costly. The benefit of having the monopoly in the market (single-product-by-the-buyer) justifies the withholding costs, so the buyer withholds, and the supplier does not encroach on the market. Revealing the supplier's type (despite facing a pooling wholesale price) is beneficial for the buyer since it can choose the optimal selling quantity that depends on whether the supplier turns out to be a high-type or low-type.

Fourth, when the capacity of the low-type supplier is in Region $\overline{DN}^*$ (a new region that is unique to the asymmetric information game), the supplier has sufficient leftover capacity to encroach on the market, and the buyer does not withhold. However, unlike the market structure in Region DN^*, the buyer and the low-type supplier use up all capacity to sell to end customers, i.e., $q_B^* + q_{SL}^* = K_L$. This region is defined by an upper and a lower capacity threshold. First, below the lower threshold, the buyer purchases the entire capacity (Region SN^*). Next, when the capacity level is between the two thresholds, the cost of buying the entire capacity of a low-type supplier increases. Moreover, the capacity of the low-type supplier is low enough, such that ordering more than the low-type capacity may not provide any value to the buyer. Unlike in the full information game where the buyer can guarantee its monopoly in the end-customer market by purchasing the supplier's excess amount, the buyer does not know the supplier's type in the asymmetric information game. Thus, there is a possibility that the supplier is high-type and has plenty of capacity to encroach on the market. In this case, the buyer would buy a limited quantity that would simply cap the output of the low-type supplier to reduce competition (Region $\overline{DN}^*$). Lastly, when the capacity level (K_L) is larger than the upper threshold, the buyer would once again buy all low-type capacity (Region SW^*). Avoiding competition can be beneficial for both firms in a capacitated supplier encroachment scenario. Consequently, above this upper threshold, the supplier will set its wholesale price such that the buyer buys a quantity equal to the low-type capacity, leading to the following result:

Corollary 2.1: *The low-type supplier may no longer encroach when it has more capacity.*

2.5 Impacts of Capacity Constraint and Private Information

The two key features that I contribute to the literature on supplier encroachment are the concepts of considering a capacitated system and studying the value of information. In this section, I investigate how these two characteristics affect the optimal solutions.

2.5.1 How Private Information Affects the Impact of Capacity on Firms

In this subsection, I examine the impact of capacity on firms. Specifically, I consider a benchmark where the supplier is uncapacitated. I use the superscript U to denote the solution to this benchmark. It can easily be shown that the high-type supplier is never worse off from the low-type one being capacitated ($\pi_H^* \geq \pi_H^U$). Therefore, Proposition 2.2 focuses on the impact of K_L on the low-type supplier and the buyer.

Proposition 2.2: *a.* $\pi_L^* \geq \pi_L^U \Leftrightarrow K_L \geq \widehat{K}$*, i.e., the low-type supplier is better off from its capacity constraint if and only if* $K_L \geq \widehat{K}$*. Moreover,* $\pi_L^* - \pi_L^U \leq \pi_L^F - \pi_L^U$ *when* $K_L \geq \widehat{K}$*, i.e., the low-type supplier benefits less from capacity constraint when there is information asymmetry if and only if* $K_L \geq \widehat{K}$*.*

b. $E[\Pi^*] \geq \Pi^U \Leftrightarrow K_L > \breve{K}$*, i.e., the buyer is better off from the supplier's capacity constraint if and only if* $K_L > \breve{K}$*. Moreover,* $E[\Pi^*] - \Pi^U \geq \Pi^F - \Pi^U$ *when* $\breve{K} \leq K_L \leq \overline{K}$*, i.e., the buyer benefits more from capacity constraint when there is information asymmetry if and only if* $\breve{K} \leq K_L \leq \overline{K}$*.*

Both firms are better off as long as the supplier's capacity constraint satisfies $K_L > \max[\breve{K}, \widehat{K}]$. First, the firms can be better off at Regions SN^* and SW^* because the supplier always encroaches on the end-customer market when it has no capacity limit. Yet, in a capacitated system, the buyer has a monopoly in the end-customer market at these two regions, so it is better off. The supplier can also be better off from its capacity constraint because the buyer may use some of the gain resulting from being in the monopoly position to incentivize the supplier not to encroach. Second, the firms can also be better off under the capacitated system at Region $\overline{DN}^*$ because, compared to the uncapacitated system, the buyer would order much more from the supplier ($\Omega^* \gg \Omega^U$); this implies that the supplier would be left with minimal capacity to sell to end customers ($q_S^* \ll q_S^U$). Consequently, the capacity constraint decreases the degree of competition in the end-customer market, so the two firms are better off under the capacitated system. Lastly, Region DN^* represents the uncapacitated scenario in the asymmetric information game, so the firms' profits are the same between the uncapacitated benchmark and the main math model.

When compared to the impact of a capacitated system under a full information game, a capacitated system under an asymmetric information game shifts the benefit of limited capacity from the supplier to the buyer in Regions $\overline{DN}^*$, SW^*, and SN^*. This is because the wholesale price under the asymmetric information setting is smaller than the wholesale price under the full information setting ($w_i^* \le w_i^F$). Therefore, a lower wholesale price reduces the supplier's profit and increases the buyer's profit from the wholesale market. Moreover, both the supplier and the buyer no longer benefit from the capacity constraint under the asymmetric information setting in Region DN^*.

2.5.2 Value of Information

In this subsection, I study the impact of information asymmetry by comparing between the full information (cf. Lemma 2.1) and the asymmetric information games (cf. Proposition 2.1). Proposition 2.3 presents findings of how private information affects firms.

Proposition 2.3:

a. $\pi_L^* \le \pi_L^F$, i.e., the low-type supplier, is always worse off from having private information.

b. $E[\Pi^*] \ge \Pi^F \Leftrightarrow K_L \le \overline{K}$, i.e., the buyer, is better off from having less information if and only if $K_L \le \overline{K}$.

One might expect that the supplier can benefit from having private information about its capacity; however, one important result of my paper is that the low-type supplier is worse off from having such private information. There are three reasons for this result. First, when the capacity is very small, the supplier uses the wholesale price to truthfully signal its type, and the wholesale price of the high-type differs from that of the low-type. To achieve a separating equilibrium, the low-type supplier must incur a signaling cost by distorting its wholesale price downward so that the high-type supplier has no incentive to mimic it ($w_L^{SE} \leq w_L^F$). Second, when the capacity is intermediate, the wholesale price of the two types is the same. This pooling wholesale price is lower than the low-type supplier's wholesale price under full information ($w^{PE} \leq w_L^F$). Third, there are parameter settings where the supplier would not encroach under perfect information (Region SN^F or SW^F) but would encroach under asymmetric information (Region DN^*). Therefore, the supplier is worse off from having private information because this leads to a competitive end-customer market.

Another interesting result is that the buyer can be better off from having less information about the supplier's capacity. The main driver for this insight is that the wholesale price is lower under the asymmetric information game ($w^* < w^F$), either due to the signaling cost or due to the threat of a competitive market. First, consider the setting where the supplier does not encroach (Regions SN^* and SW^*). Under both asymmetric and full information games, the buyer orders the supplier's entire capacity ($\Omega^* = \Omega^F = K_L$) and sells the same quantity to the end-customer market ($q_B^* = q_B^F$). The inverse demand function $P = D - (q_B + q_S)$ implies that the retail price is the same under both asymmetric and full information settings ($P^* = P^F$), so the buyer's retail profit does not change. In these circumstances, the buyer is better off under the asymmetric information game because the wholesale price is lower.

Second, consider the region where the capacity is intermediate, and the supplier encroaches (Region $\overline{DN}^*$). Under either the asymmetric or full information games, the total amount sold to end customers is the supplier's capacity level, so the retail price is the same under both games. In the asymmetric information game, in this region, the leftover capacity for the supplier is very small, and thus the effect on the buyer's retail profit is minimal. However, the wholesale price under the asymmetric information game is significantly lower than that under the full information game, so the buyer is better off from not knowing the supplier's capacity.

Next, Proposition 2.4 illustrates how private information affects the withholding decision.

Proposition 2.4: *$SW^* \subseteq SW^F$, i.e., the range of parameters under which withholding occurs is smaller under the asymmetric information game than under the full information game.*

Withholding is less likely when information is asymmetrical. In particular, the upper limit where withholding is beneficial is lower under the asymmetric game, as the buyer's purpose for withholding is to eliminate the possibility of a supplier having leftover capacity to encroach on the market. If the buyer does not know the supplier's capacity, then it is possible that the supplier is high-type, which means that the withholding cost that the buyer incurs may turn out to be ineffective. Therefore, the buyer is willing to pay a lower wholesale price for the whole capacity, compared to the full information game. Consequently, after a certain threshold, the supplier does not sell its whole capacity and competes with the buyer (Region DN^*).

2.6 Conclusion

I use a signaling game to examine a competitive supply chain in which the supplier may choose to signal its capacity level to the buyer and, given this signal, the buyer may use its order quantity to deter supplier encroachment or alter the output of the supplier to the end-customer market.

I show that private information may negatively affect the supplier's profit because of adjustments that it must make to its wholesale price to credibly signal its type in a separating equilibrium, and also to disguise its capacity level in a pooling equilibrium. Moreover, the buyer can be better off from not knowing the supplier's capacity level due to lower wholesale prices in the presence of asymmetric information. Consequently, my paper provides strategic insights for practicing managers on potential information-sharing strategies in a competitive supply chain. Specifically, I illustrate the benefits available to the supplier if its capacity information is shared credibly with the buyer. This credibility can be achieved by using information sharing technologies, through official announcements, installing video cameras at the supplier's warehouses or by digitally tokenizing the capacity with the blockchain. Yet, the buyer should be cautious when communicating with its supplier via these technologies.

I also find that a capacitated supplier would encroach on the market more often when its capacity level is private information. Specifically, when the buyer cannot distinguish between a high- and low-capacity supplier, its ability to control the market is limited. In this situation, even a low-capacity supplier can encroach on the market and may have some unused capacity. Furthermore, I characterize the conditions under which a low-capacity supplier encroaches on the market, and I show that this encroachment does not necessarily happen at higher capacity levels because of the intense competition in the end-customer market. This insight is unique to the asymmetric information game.

One of the major implications of a capacitated supply chain is the aggressive supply acquisition of the buyer to control the market (Ghamat et al., 2018; Guan et al., 2019). In this paper, I show that strategic capacity withholding is less prevalent in the presence of information asymmetry; this is because when the buyer does not know the supplier's capacity level, there is always a risk that the supplier may turn out to have high capacity, which makes aggressive supply acquisition less attractive. Finally, I find that both the buyer and the supplier can benefit from limited supply as (1) it is cheaper for the buyer to buy the entire capacity of a more capacitated supplier and become a monopoly, and (2) the buyer will share some of its gains with the supplier through higher wholesale prices.

I used a stylized model to study the dynamics of firms' optimal decisions but extending some of my assumptions may lead to interesting future work. I consider an inverse demand function where the market size is public information. However, market information may be private information to a buyer that has an established selling channel and so it has more information about end customers. Therefore, a possible avenue for future research would be to analytically study the impact of two-sided information asymmetry between the buyer and the supplier in a competitive setting. Moreover, my model considered only one buyer for the supplier's limited capacity. It would be worthwhile to consider multiple strategic buyers, where the supplier can allocate some of its capacity among the less informed buyers and its own in case of encroachment.

Chapter 3

3 Counterfeiting Detection Using a Sentiment Analysis and Machine Learning Approach

Counterfeit products are abundant on online platforms and the platforms' effort to fight this problem turns out to be mostly fruitless (Shepard, 2017). Because of this situation, customer reviews provide a powerful tool for other customers to detect counterfeit products. In this chapter, I propose a novel counterfeit detection algorithm that uses sentiment analysis and word embedding to analyze the reviews and utilizes unsupervised fuzzy clustering to give the products an authenticity score. I also trained context-specific word embedding based on a large corpus of customer reviews to capture the information content of the reviews. I used a data set from Amazon to test the performance of my algorithm with both the product and seller reviews and again with just the product reviews. I show that the centers of the clusters are meaningfully apart in both cases. In order to analyze the accuracy of my model, I used two independent human coders to read the reviews and identify whether a product is real or fake. I show that with or without the seller reviews, the algorithm performs well with AUC-ROC = 0.602 if I incorporate the seller reviews and AUC-ROC=0.623 if I use just the product reviews. This result shows that the product reviews have more impact on customers' perceptions of the products than the seller reviews and because of that, the seller reviews do not help with classifying the fake products.

Keywords: online platforms; third-party seller; customers' reviews; NLP, word2vec, sentiment analysis.

3.1 Introduction

Amid the pandemic, when total US retail sales declined by 10.5%, Amazon's net sales increased by 26%, reaching $75.5 billion in the first quarter of 2020 (Perez, 2020). At the same time, Alibaba recorded more than 700 million active buyers (Alibaba.com, 2020). Online platforms have the largest market share in e-commerce sales. For example, Alibaba occupies 56% of the e-commerce market share in China (Hanbury, 2019), while Amazon has more than 38% of the e-commerce market share in the US (Bloomberg, 2019). With the increases in internet penetration, e-commerce is expected to grow even faster (Young, 2019). For example, Alibaba states that more than 70% of their new customers are coming from low-tier cities, which previously had low internet access (Bloomberg, 2019).

However, this rise in sales is still far from its potential because of customer trust in online shopping (Nielsen, 2018). For example, many customers are still making their purchases in brick-and-mortar stores because of the potential counterfeit products sold on online platforms (John, 2019; Nicasio, 2020). E-commerce platforms have low entry barriers and provide counterfeit sellers with an effortless storefront with millions of potential consumers (Khan, 2019). Although there are cases where customers may intentionally buy a counterfeit product because of the lower price (non-deceptive counterfeits), many of the customers on e-commerce platforms presume that they are buying authentic products at full price, but instead, are duped into purchasing counterfeit goods (deceptive counterfeits) (Khan, 2019). According to the US Government Accountability Office, about 40% of a sample of goods bought on Amazon, Walmart, eBay, Sears Marketplace, and Newegg were fake, while all were advertised as new, brand-name items sold by third-party sellers (Erikson, 2018). An original brand manufacturer of electronic goods found that, from the products sold under its brand name by third-party sellers on Amazon, over 70% were actually fake (DH Anticounterfeit, 2017). Besides monetary loss, purchasing deceptive counterfeit products can pose a threat to the health and safety of consumers. For instance, counterfeit cosmetics have been found by federal agencies to contain hazardous substances, including cyanide and arsenic (CBS, 2017). Moreover, according to a report by BCG, "Aside from lost revenues, brands can suffer reputational harm when customers buy what they believe are genuine parts but are disappointed when the inferior substitutes don't work as expected" (Bhatia et al., 2019).

E-commerce platforms have made efforts to fight the problem of counterfeit products. For example, eBay Authenticate enables sellers and buyers to have merchandise authenticated by human inspectors (Ismael, 2019) and Amazon launched Project Zero as a counterfeit removal tool for brands that sell on Amazon's Marketplace (O'Shea, 2019). This latter project uses machine learning and image processing techniques to automatically detect counterfeit products based on the brands' logos (Gartenberg, 2019). However, although the platform owners claim that they have "zero tolerance" for fake products, counterfeit products are still prevalent in online marketplaces (Shepard, 2017). Amazon admits that they may be unable to eradicate the counterfeit problem (Bain, 2019). This is partly because the majority of anti-counterfeiting measures on platforms such as eBay are still manual (Greene, 2019). Moreover, detecting counterfeit products using the information provided by a firm selling genuine products on an online platform is not reliable because the counterfeiter can duplicate this information easily. For instance, many counterfeit sellers are using official images of the products, making it more difficult for the computer algorithms to detect them (Chopra, 2020).

Product reviews, then, provide a valuable source for detecting deceptive counterfeits (Suthivarakom, 2020). Even sophisticated fake products can be identified by the reviews posted by previous customers. Greene (2019) reported that the counterfeit version of Hermès’ Clic H Bracelet, sold on Amazon with the same price as the original product, had all the details of a Hermès “H” logo. However, the counterfeit seller was revealed because reviews posted by previous customers mentioned that this product was fake. Moreover, on many e-commerce platforms such as Amazon, customers can provide feedback about sellers. A seller with a bad reputation is more likely to sell fake products, so the seller reviews can be used to identify if a seller has sold counterfeit products (Hosch, 2020, Suthivarakom, 2020). Figure 3-1 shows an example of reviews indicating the product is fake. However, in order to benefit from these reviews for counterfeit detection, customers and platforms must process a large volume of product and seller reviews, which can be very time consuming (Greene, 2019).

Figure 3-1: Example of reviews for suspicious products

In this chapter, I propose a novel counterfeit detection algorithm which analyzes the customers' reviews and provides an authenticity score for the products. In particular, I used a sentiment analysis technique, VADER, which is customized for user-generated content. This helped me to capture the context-related aspects of the reviews, such as negation; although negative or positive sentiment in a review does not always indicate a counterfeit or authentic product, respectively, a counterfeit product will likely have reviews with negative sentiment. Consequently, the sentiment of the reviews can provide an additional tool to detect counterfeit products.

I also trained an artificial neural network, Word2Vec, to detect potentially fake products. Word2Vec gives us a reliable measure of similarity between the reviews and a common fake product/seller review such as "This is fake" or "Sells fake products". While word embedding is often pretrained on very large generic text data without any human supervision, directly applying these word embeddings to customers' reviews may not be appropriate because words and expressions used by customers in customer reviews are very different from those in general text, for example, news articles or financial reports (Loughran and McDonald 2011, Yang et al 2021). Because of that, I trained my context-specific word embedding based on a large corpus of customers' reviews. Once I obtained high-quality review-specific word embeddings, I could better understand the information content of the reviews, which subsequently improved counterfeit detection.

My algorithm, then, takes the sentiment of the reviews and the similarity of their embedding to that of the word "Fake" as an input and provides an authenticity score using an unsupervised fuzzy clustering algorithm as an output. In order to evaluate my proposed algorithm, I scraped 8,000 reviews from Amazon, which included 600 unique products and 117 unique sellers. Because my algorithm was unsupervised, I used human coders to then label my data set — namely, the product and seller reviews — as fake or genuine to evaluate the accuracy of my methodology.

The contributions of my research are as follows: First, I contribute to the counterfeit detection literature by providing a decision support system that uses seller and product reviews to detect counterfeit products on online platforms. Second, I propose a novel hybrid Natural Language Processing (NLP) methodology to detect fake products from unstructured data, namely, user reviews. Moreover, I build my own domain-specific word representation based on the Word2Vec framework. These new learned word embeddings could be of broad interest to researchers (Yang et al. 2018). Finally, I show that seller reviews do not help with classifying the fake products; product reviews have more impact on customers' perceptions of the authenticity of products.

This study is organized as follows: Section 3.2 provides the literature review, followed by the theoretical framework and proposed algorithm in Section 3.3. Section 3.4 provides the experimental analysis and performance evaluation of my methodology. In Section 3.5, I provide the conclusion and future research directions.

3.2 Literature Review

My work relates to two streams of literature. Firstly, it relates to the literature that studies the detection of deceptive counterfeits. For example, Blankenburg et al. (2015) use different technologies, including image processing, to detect counterfeit products by comparing them with genuine products' inherent characteristics such as olfactory, texture and shape features. In contrast, Sharma et al. (2017) use a bag of visual words and convolutional neural networks (CNN) to detect fake products using a special device to capture the microscopic image of a large area of a physical object. However, neither of these papers investigates counterfeit detection problems on online platforms. In an online platform setting, Arnold et al. (2016) propose a semi-automatic approach to identifying counterfeit products. They use hierarchical bottom-up clustering to cluster similar product offers and propose a scoring mechanism based on the price and seller rating to detect counterfeit products. In another study, using a ResNet pretrained network weight, Cheung et al. (2019) propose a methodology to detect online counterfeit sellers by comparing images that a seller uses with images from known counterfeit sellers. Finally, Wang et al. (2018) study the effect of copycat mobile applications for original applications. In order to identify copycats, they use image-matching analysis and NLP to show that high-quality, non-deceptive counterfeits have a negative impact on the demand for an original application and low-quality, deceptive counterfeits have a positive impact on the demand for an original application. The efficiency of the methodology used by these papers, however, mostly depends on the characteristics of a brand's product that are publicly available and replicable by counterfeiters. In contrast, I rely on methodologies that do not depend on product characteristics.

Secondly, my paper relates to the stream of literature that studies sentiment analysis and document embedding to detect deceptive actions. Few papers use NLP to detect fake news articles (Anjali et al. 2019; Bhutani 2019; Reza and Ding 2022) or fake reviews (Barbado et al. 2019; Kaufman et al. 2019; Barushka et al. 2019, Hajek et al. 2020) on social platforms. Instead, most studies use supervised machine learning algorithms. They extract features of text such as sentiment score or document embedding to build a fake detection classifier such as SVM, naive Bayes or Neural Network. In contrast to news and reviews, however, there is no reliable labeled data set for counterfeit products, so for my study, I develop an unsupervised methodology to flag potentially counterfeit products sold on online platforms. In a related paper, Wimmer and Yoon (2017) propose a lexicon-based counterfeit detection algorithm. To calculate the counterfeit score, using WordNet lexicon, their algorithm counts the occurrence of synonyms of the word "Fake" in product reviews. My methodology is different from their approach in four aspects. First, their research question is how providing authenticity scores affects the purchasing behavior of customers instead of how customers' reviews affect online platforms' anti-counterfeiting strategy, and they do not check the accuracy of their algorithm. Second, in addition to product reviews, I consider seller reviews to detect a counterfeit product. Third, my algorithm is sensitive to context-related adjustments. Fourth, I use a two-tier approach with document embedding to detect similar reviews.

3.3 Theoretical Framework and Proposed Algorithm

In this section, I describe my model which incorporates sentiment analysis and word embedding to detect potential counterfeit products. Before explaining the algorithm, I first review the theoretical background around clustering analysis, sentiment analysis and word embeddings.

3.3.1 Clustering Analysis

Clustering is an unsupervised machine learning technique which seeks to amass unlabeled data into classes or groups such that items inside a group have higher homogeneous attributes compared to the items that have been placed with other groups. Clustering algorithms can be classified as hard clustering or soft clustering. Hard clustering computes a hard assignment for each item in the data set and the cluster membership is Boolean, while in soft clustering, each item is assigned to a cluster based on a high probability of its belonging there; Please note that the products that are available on online platforms are not proven to be fake, otherwise they would have been delisted by the platforms. Because of that I use a soft clustering algorithm to provide the probability of the authenticity of the product. Fuzzy c-means (FCM) is a method of a soft clustering algorithm which was developed by Dunn in 1973 and improved by Bezdek in 1981. The FCM algorithm tries to find the best fuzzy clustering of n observations into k clusters by solving the following optimization problem:

$$\min J = \sum_{i=1}^{n} \sum_{g=1}^{k} u_{ig}^{2} d(x_i, h_g)$$

$$st\ u_{ig} \in [0,1], \sum_{g=1}^{k} u_{ig} = 1 \tag{3.1}$$

where $d(x_i, h_g)$ is a distance measure between a data point x_i and a cluster center h_g, and u_{ig} is the membership degree of the data point x_i to the cluster g (Bezdak, 1981). The validity index for FCM is the fuzzy partition coefficient (FPC) which is proposed by Bezdek (1981). This index is calculated as follows:

$$FPC = \frac{\sum_{i=1}^{n} \sum_{g=1}^{k} u_{ig}^{2}}{n} \in [0,1] \tag{3.2}$$

where $FPC = 1$ indicates the best clustering quality.

3.3.2 Sentiment Analysis

Sentiment analysis or opinion mining is the field of study that uses NLP techniques to extract and classify emotion, sentiment, and attitude from unstructured text data (Liu 2015). Sentiment analysis algorithms follow two main approaches: 1) The lexicon-based approach that calculates the sentiment of a document from the semantic orientation or intensity of words or phrases in the document, and 2) The machine-learning approach that relies on labeled training data (Liu 2015). The latter approach uses the extracted features of the text, such as the TF-IDF matrix, and the associated labels from the training set to train a classifier such as naive Bayes (NB), Maximum Entropy, SVM and deep learning. Then, this classifier is used to predict the sentiment orientation of a new text. The quality of the classifier depends significantly on the quality and quantity of the training data. Machine learning approaches require huge training data sets with validated labels as ground truth. These data sets are computationally expensive and not easy to acquire.

VADER (Valence Aware Dictionary for Sentiment Reasoner) is a parsimonious lexicon- and rule-based sentiment analysis tool developed by Hutto and Gilbert (2014). VADER's dictionary includes all the other well-established sentiment lexicons such as LIWC and GI. Additionally, it incorporates a list of western-style emoticons such as ":-)", acronyms such as “LOL” and slang such as “meh”. These features make VADER suitable for sentiment analysis in social media and user-generated content (UGC) such as Amazon reviews (Hutto and Gilbert, 2014). For each token in the dictionary, Hutto and Gilbert use the wisdom-of-the-crowd (WotC) approach (Surowiecki, 2004) to obtain the intensity of sentiment. Ten independent, pre-trained and pre-assessed human raters rate each word from -4, extremely negative, to 4, extremely positive, and the average of these ratings are considered the sentiment valence of each token.

VADER methodology captures sentiment not only by words and phrases but also by context. It also includes some generalizable context-related rules to adjust sentiment intensity in the text. In particular, exclamation points, “!”, increase the intensity of the words without changing the valence orientation. Further, writing the words in all-caps increases the sentiment intensity of the tokens without changing the polarity. Degree adverbs such as “extremely” or “marginally” may increase or decrease the sentiment intensity of the proceeding words. The coordinating conjunction “but” signals a shift in valence with the sentiment of the text following the conjunction defining the overall sentiment of the sentence.

The VADER algorithm returns a “Compound Score” as a measure of sentiment intensity of the text. This score is calculated by adding the context-adjusted sentiment score of each word in the text and then normalizing the sum to a score between -1 and +1. Equation 1 illustrates the calculation of the compound score in which x is the sum of the context-adjusted sentiment scores of the words, and α is a constant usually chosen as 15 (Adarsh et al., 2019).

$$Compund\ Score = \frac{x}{\sqrt{x^2+\alpha}} \tag{3.3}$$

3.3.3 Word Embedding

In order to perform analysis on text data, I need to convert text to numerical presentation, so it is understandable by the machine. In the bag-of-words model, text is mainly modeled via a one-hot encoding strategy which suffers from the dimensionality problem and in which many latent and deep semantics among words are usually ignored. The other approach is word embeddings, in which each word is represented by a real-valued vector with pre-defined size (embedding vectors) (Brownlee, 2018). In other words, word embedding enables us to represent text in a more intense way and, consequently, eliminates the high-dimensionality problem in the bag-of-words method. One of the most efficient ways to calculate the word embedding vectors is the Word2Vec model, which was developed by Mikolov et al. (2013) (Brownlee, 2018). Word2Vec is a shallow artificial neural network consisting of the input layer, the output layer, and a hidden layer (with the size of embedding vectors). The embeddings are learned by determining the probability that each word is surrounded by other words. There are two main approaches to train the Word2Vec neural network: 1) The Continuous Bag-of-Words or CBOW model that learns the embedding by predicting the current word based on the neighboring words, and 2) The Continuous Skip-Gram Model, or SG, that learns the embedding by predicting the surrounding words using the current word. Both models learn about words by considering their context where the context is defined by a window of surrounding words. Consequently, similar words tend to have similar vector values and are grouped in the same block (Mikolov et al., 2013). Figure 3-2 illustrates these two algorithms.

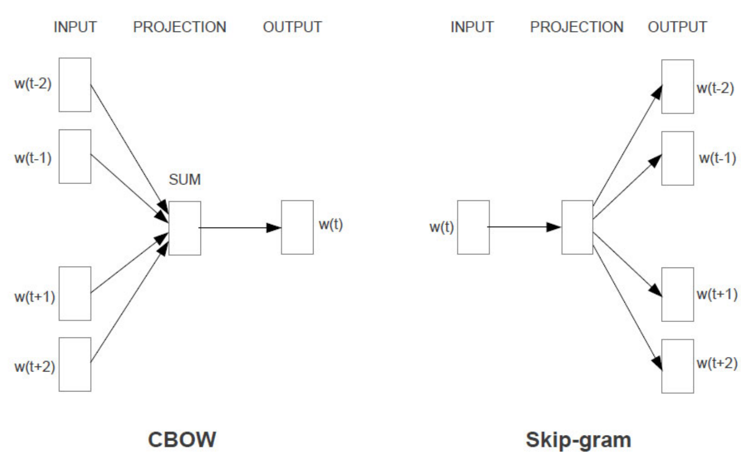

Figure 3-2: CBOW (on the left side) vs Skip-Gram (on the right side) (Mikolov et al., 2013)

In this study, I build my own domain-specific word representation based on the Word2Vec framework. SG and CBOW optimize very similar objective functions and yield very similar results (Yang et al., 2018). In this study, for the sake of simplicity, I used the SG model.

In the SG model, the objective is to maximize the probability of each word, given the surrounding words. Consequently, the objective function can be written as:

$$L = \frac{1}{|W|} \sum_{w_t \in W} \log\left(P(w_{t-n}, \dots, w_{t+n} | w_t)\right) \quad (3.4)$$

where W is the set of all words, w_t is the word for which I am calculating the word embedding and $w_{t-n}, \dots, w_{t+n}$ are the surrounding words. Each word has two roles in Equation 2: it can be treated as the word itself or the context (surrounding) word for the other words. Because of that, I have two types of word embeddings. Let $\boldsymbol{w}$ function as the word embedding for the word w when it is treated as the word itself, and $\boldsymbol{w}'$ function as the word embedding for the word w when it is treated as one of the surrounding words. The probability in Equation 3.4 can be estimated via a SoftMax function.[2]

$$P(w_{t+i}|w_t) = \frac{\exp(\boldsymbol{w}'_{t+i} . \boldsymbol{w}_t)}{\sum_{\boldsymbol{w}'_{t+i} \in V} \exp(\boldsymbol{w}'_{t+i} . \boldsymbol{w}_t)} \quad (3.5)$$

A common document-embedding method in NLP is the term average over embeddings of all words (Arora et al., 2017). This method for document-embedding is proven to be very effective in preserving document semantics (Yang et al., 2018). I use this strategy on the customer reviews. Given a customer's review for a product or a seller which contains n words, the vector representation of the reviews x_i is calculated as:

$$x_i = \frac{1}{n} \sum_{k=1}^{n} \boldsymbol{w_k} \quad (3.6)$$

3.3.4. Proposed Algorithm

I propose a novel counterfeit detection algorithm that can analyze the customer's review and provide an authenticity score for each product on online platforms. The VADER score can help identify the sentiment of customer reviews; products with negative reviews are more likely to be fake. So, I extract the sentiment of the reviews of the products and use it as a feature in my clustering analysis. Moreover, the distance between the vector representation of the word "Fake" in the customer reviews' context and the vector representation of the reviews (cf. Equation 4) can provide a good measure of how previous customers are talking about the authenticity of the products. Cosine similarity measures the cosine of the angle between two

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[2] For a detailed explanation of Word2Vec training please refer to Markolov et al., 2013.

vectors projected in a multi-dimensional space and is widely used in NLP (Han et al., 2012). In this case, I compare the vector representation of text with each other to understand whether the reviews denote a fake or genuine product. In my proposed algorithm, first, for each pair of product and seller I calculate a set of four scores: the VADER score of product reviews (V_p) , the cosine similarity of product reviews to the word embedding of the word "Fake" (C_P) , the VADER score of seller reviews (V_{SP}), and the cosine similarity of seller reviews to the word embedding of the word "Fake" (C_{SP}). The higher the VADER score is, the more positive the sentiment of the reviews are while the lower the cosine similarity score is, the less the reviews are pointing to the fakeness of the products. Then, the products are clustered using FC-means and the percentage belonging to the good cluster can be interpreted as an authenticity score (A_p). The proposed counterfeit detection algorithm is summarized in Algorithm 3.1.

Algorithm 3.1

Input: The products reviews' data set, X, and the seller reviews' data set Y

Output: An authenticity score set A_P

Initialize: P= Set of products, S=Set of sellers, n=Number of products,

m=Number of sellers

For each product $p \in \{1, \dots, n\}$ in the set P and seller $s \in \{1, \dots, m\}$ in the set S

R_p = All customer reviews of the product p

R_{sp} = All customer reviews of the seller s who sells product p

$$V_P = VADER\ (R_p)$$

$$V_{sp} = VADER\ (R_{sp})$$

C_p= Similarity (R_p, "Fake")

C_{sp}= Similarity (R_{sp}, "Fake")

$$G_p = (V_p, V_{sp}, C_p, C_{sp})$$

End

For each product $p \in \{1, \dots, n\}$ in the set P

A_p = Fuzzy Cluster (G_p)

Return A_p

End

Figure 3-3: Counterfeit detection algorithm with the seller reviews

3.4 Experimental Analysis

In this section, I test the performance of my proposed algorithm in detecting counterfeit products.

3.4.1 Data Sets

In order to train the word embedding, I used a publicly available data set (McAuley et al., 2015) which contains 142.8 million reviews from Amazon spanning May 1996 to October 2018. This data set includes various product categories, from books and groceries to digital music and cell phone accessories. In order to focus on the most counterfeited products, I chose the categories of Electronics, Clothing-Shoes-Jewelry, Cellphones and Accessories, and Beauty (Anderson, 2018) which contain 19,158,760 unique product reviews. To improve the quality of my data, I performed several preprocessing operations, such as punctuation and stop-word removal and lower casing. I used Gensim as a specialized Python package for NLP (Řehůřek, 209) to train the word embedding.

In order to validate my proposed algorithm, I used a Python scraper to get the total of 8,000 product reviews and 813 seller reviews from the Amazon website. I then scraped the data from Electronics, Clothing-Shoes-Jewelry, Cellphones and Accessories, and Beauty; the four categories for which I trained my word embedding. Then, after removing the non-English reviews, I combined the reviews for unique products and sellers. Table 3.1 shows the configuration of my data set. For each review I calculated two scores. The first score is the VADER score which is calculated using a Python package, VADER sentiment. Since the VADER algorithm is sensitive to the context, I did not perform any preprocessing on the reviews' text. The second score is the cosine similarity of the reviews as a document with the word "Fake" using Equation 3.6. In order not to dilute the contextual meaning, I removed the punctuation and stop-words. Also, in order to be comparable with my trained word embedding, I lower-cased the reviews.

Table 3-1: Data Set Configuration

Number of products	600
Number of sellers	117
Average number of products per seller	5.1
Average number of reviews per product	14.1
Average number of reviews per seller	4.9

3.4.2 Validation Metrics

After calculating the four measures (V_p, V_{sP}, C_P, C_{sP}) for the data set, I performed a FCM clustering analysis for each product based on four scores. It is common to scale data prior to fitting a machine learning model. The aim of feature scaling is to give all parameters the same importance in clustering (Hull, 2021). Robust Scaler algorithms scale features such that the clustering algorithms are robust to outliers. Please note that the aim of this algorithm is not to detect outliers in each feature but to cluster based on the combination of the features. Robust scaling may change the range of variables, but it does not let a feature dominate the clustering because of the outliers, and it is widely used in NLP (Brownlee, 2018). For every instance of features, x_i, the scaled version, $x_i{}'$ is calculated by the following formula:

$$x_i' = \frac{x_i - Q_1(x)}{Q_3(x) - Q_1(x)} \quad (3.7)$$

where $Q_1(x)$ and $Q_3(x)$ are the first and third quartile of the feature x, respectively.

After robust scaling the data, I used the Elbow method (Thorndike, 1953), a way of deciding the most effective way to visualize the data or, in this case, to choose the best number of clusters to show the data. Figure 3-4 illustrates the FPC score (cf. Equation 3.2) for different numbers of clusters where $n = 2$ provides the best clustering outcome. Consequently, I chose two clusters.

Table 3-2 and Figure 3-5 show the results of clustering the data set into two clusters. We can observe that the centers of the two clusters are meaningfully apart from each other. The center of cluster 1 has a lower VADER Score and a higher cosine similarity to the word “Fake” for both the product and the seller reviews. The center of cluster 2 has a higher VADER Score and a lower cosine similarity to the word “Fake” for both the product and the seller reviews. So, I consider cluster 1 the fake cluster and cluster 2 the genuine cluster, and I define the probability of belonging to this latter cluster is the “Authenticity Score”. I assigned the products into clusters based on the arbitrary cut-off threshold of 50% for the probability of belonging to a specific cluster; however, note that the cut-off threshold for assigning the products to a class is an arbitrary decision since increasing or decreasing the threshold would change the false positive and true positive rate in a labeled data set. In Section 3.4, I use human coders to label the data set in order to evaluate both the accuracy of my model and the effect of change in the cut-off threshold.

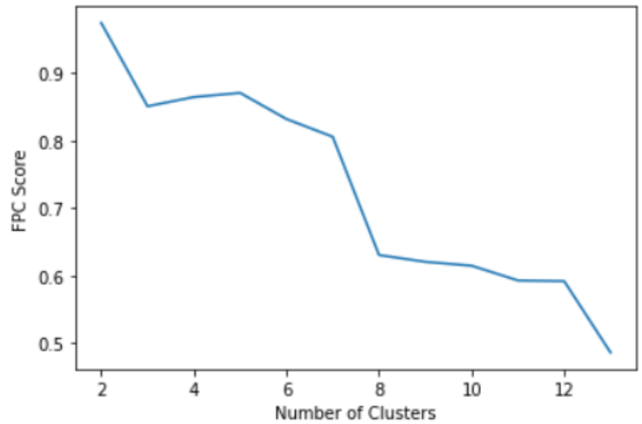

Figure 3-4: FPC score for different numbers of clusters in Algorithm 3.1

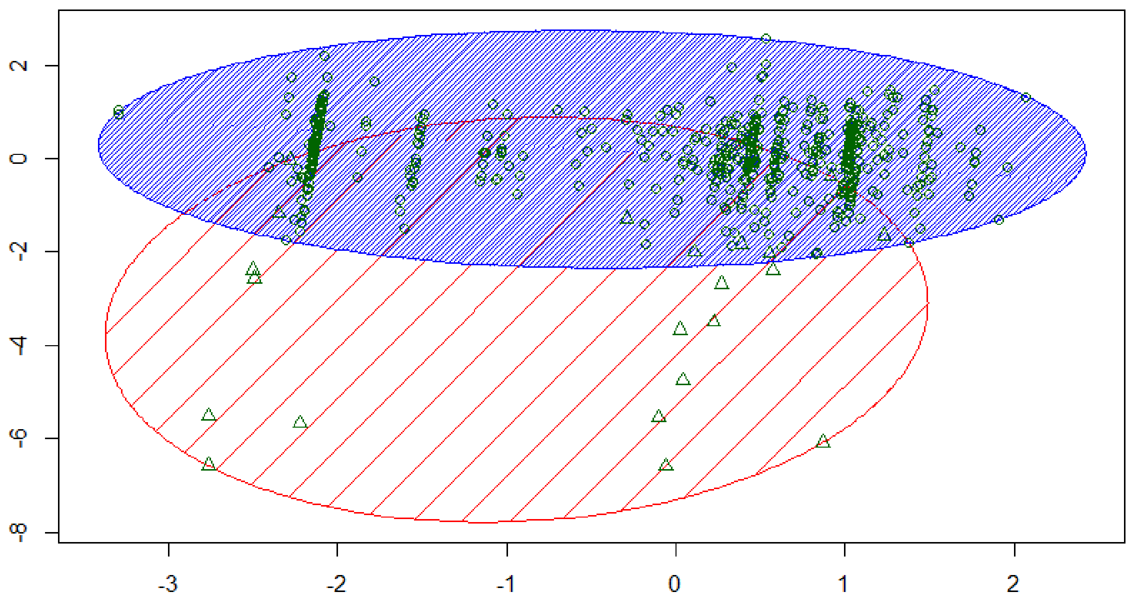

Figure 3-5: Product clusters with Algorithm 3.1

Table 3-2: Clusters Descriptions in Algorithm 3.1

	$number\ of\ members$	V_P	C_P	V_{SP}	C_{SP}
Center 1	22	-0.325	0.490	-0.143	0.169
Center 2	578	0.823	-0.141	0.627	.004

3.4.3 Performance Evaluation

3.4.3.1 Human Coding

In order to assess the validity of my algorithm, I manually coded a sample of the product and seller reviews. To that end, I employed two independent research assistants. These assistants were asked to read the exact same product and seller reviews that were processed by the algorithm. Both coders received training on the coding task before starting the process. Their main task was to identify whether a product was real or fake. After reading the product and seller review for each product, each coder labeled the product as real (value of 1), or fake (value of 0). Coders were in spontaneous agreement for 86% of the cases; the remaining 14% were resolved through discussion.

3.4.3.2 Performance Measure

To measure the performance of my proposed algorithm, I used the labeled data set as the ground truth. However, Because the data set was imbalanced, using accuracy as a measure of performance does not provide a good comparison (only 4.83% of the data set was labeled as fake, so labeling all the products as genuine gave us a base accuracy of 95.17%). In order to evaluate the performance of my proposed algorithm, I used evaluation metrics for the imbalanced data sets. The two measures are widely used in Precision and Recall. Precision is a measure that quantifies the number of correctly predicted minority (positive) classes. More specifically, out of all the times the algorithm identified a review as describing a fake product, how many times was it correct? This is calculated using the following formula:

$$Precision = \frac{True\ Positive}{True\ Positive + False\ Positive} \tag{3.8}$$

Recall is a measure that quantifies the number of correctly predicted minority classes out of the total of minority classes in the data set. In other words, out of all the reviews that described a fake product, how many did the algorithm identify?

$$Recall = \frac{True\ Positive}{True\ Positive + False\ Negative} \tag{3.9}$$

However, changing the cut-off threshold to increase the number of correctly identified fake products, i.e., increase in recall, often results in increase in the number of products falsy identified as fake, i.e., decrease in precision. The two measures that are more robust to the cut-off threshold are Area Under the Curve (AUC) of Receiver Operating Characteristic Curve (ROC) and F1-Score. The ROC curve is a plot of true positive rate against false positive rate at different cut-off thresholds and AUC-ROC can be interpreted as diagnostic accuray of the model where AUC-ROC=0.5 indicates no classification power while AUC-ROC=1 indicates perfect classification. However, AUC-ROC does not consider the imbalanced dataset (He and Ma, 2013). Because of that, the measure, which is widely used in the performance evaluation of highly imbalanced data sets and is robust to the cut-off threshold, is the F1-Score, which combines both precision and recall into a single metric (Brownlee, 2018):

$$F1 - Score = 2\frac{Precision * Recall}{Precision + Recall} \tag{3.10}$$

The F1-Score of a classification algorithm, can be compared to the F1-Score of a random classifier (He and Ma, 2013).

For calculation of precision, recall and accuracy, I used the 50% cut-off for assigning the products to either the fake or the genuine cluster. Table 3-3 shows the confusion matrix and Table 3-4 shows the result of the performance evaluation of the proposed algorithm. It can be seen that 64% of the products which were identified as fake by the algorithm were also coded as fake by the human coders. Moreover, out of the products that were identified as fake by the human coders, my algorithm was able to identify 52% of them. The F1-score of 0.57 is significantly higher than the F1-score of a random classifier (0.086). Moreover, the AUC-ROC=.602 shows reasonable classification power (Mandrekar, 2010). Although compared to the supervised text classification literature, the metrics are lower (F1-Score≥0.75, AUC≥ 0.82)

(Zhang and Zang, 2016, Dang et al., 2020), my unsupervised algorithm provides predictive power with an unlabeled data set, a decided disadvantage.

Table 3-3: Confusion matrix for Algorithm 3.1

	Labelled		
Predicted	N=600	Fake	Geniune
	Fake	14	8
	Geniune	13	565

Table 3-4: Cluster Analysis Evaluation with Algorithm 3.1

Measure	
Precision	0.636
Recall	0.517
F1-Score	0.571
AUC-ROC	0.602

3.4.3.3 Product-Only Reviews

In order to identify the effect of incorporating the seller reviews, I performed the same analysis as Algorithm 3.1, but without the seller reviews. Algorithm 3.2 illustrates this proposed analysis. Figure 3.7 illustrates the FPC-score for a different number of clusters where $n = 2$ provides the best clustering outcome. Consequently, again I chose two as the optimal number of clusters. Tables 3.5, 3.6 and 3.7 show the results of clustering the data set into two clusters. We can observe in Table 3.4 that the center of cluster 2 has a higher VADER Score and a lower cosine similarity to the word "Fake" for the product reviews. So, I consider cluster 2 the genuine cluster and the probability of belonging to this cluster the authenticity score.

Algorithm 3.2

Input: The products reviews' data set, X

Output: An authenticity score set A_P

Initialize: P= Set of products, n=Number of products,

For each product $p \in \{1, \dots, n\}$ in the set

R_p = All customer reviews of the product p

$V_P = VADER\ (R_p)$

C_p= Similarity (R_p, "fake")

$G_p = (V_p, C_p)$

End

$G = (G_1, G_2, \dots, G_n)$

For each product $p \in \{1, \dots, n\}$ in the set P

A_p = Fuzzy Cluster (G_p)

Return A_p

End

Figure 3-6: Counterfeit detection algorithm without the seller reviews

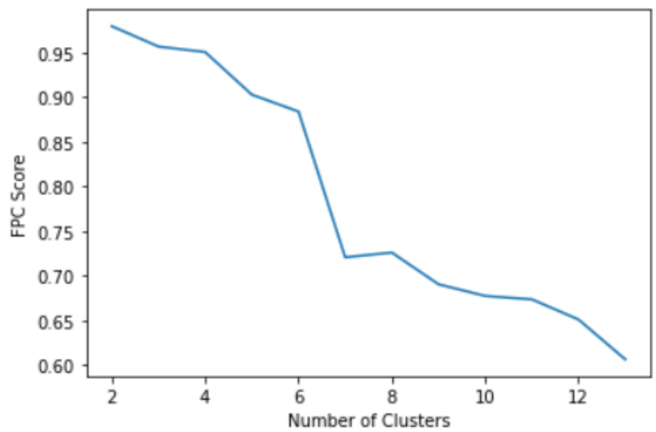

Figure 3-7: FPC score for different numbers of clusters in Algorithm 3.2

Table 3-5: Cluster Descriptions for Algorithm 3.2

	number of members	V_P	C_P
Center 1	23	-0.253	0.394
Center 2	577	0.724	-0.127

Table 3-6: Confusion matrix for Algorithm 3.2

	Labelled		
Predicted	N=600	Fake	Geniune
	Fake	15	8
	Geniune	12	565

Table 3-7: Cluster Analysis Evaluation for Algorithm 3.2

Measure	

Precision	0.652
Recall	0.555
F1-Score	0.600
AUC	0.623

Table 3-5 demonstrates that without the seller reviews, the precision, recall, and AUC is slightly higher compared to the model with the seller reviews. One possible explanation for this result is that, as I show in Sections 3.4.3.2 and 3.4.4, the centers of the clusters are separated in the characteristics of the seller reviews and the product reviews in Algorithm 3.1, and in the product reviews in Algorithm 3.2. However, the average number of reviews per product is higher than the average number of reviews per seller (14.1 to 4.9). Because of that, when the products are on the two sides of the clustering spectrum (high or low authenticity scores), the seller reviews have an augmentation effect on the coders' perception of the products. On the other hand, when the products are in the middle spectrum, the seller and product reviews may have contradicting results. However, since the product reviews are longer (14.1 reviews per product) and are talking specifically about the product under investigation, they have more influence on the coders' perception of the product. Because of that, Algorithm 3.2 has slightly better metrics compared to Algorithm 3.1. Although the seller reviews provide an additional source of information for the customers, the product reviews provide a more prominent impact on the customers' perception of the products.

3.5 Conclusion and Future Directions

In this chapter I proposed a novel unsupervised machine-learning approach to analyzing unstructured text data of customers' reviews on an online platform for detecting potential fake products. My methodology incorporates recent advanced feature learning techniques in NLP as well as well-established lexicon-based sentiment analysis to extract the features from the reviews' text data. First, I trained my own domain-specific word representation based on the Word2Vec framework using 19,158,760 unique product reviews from Amazon. This specialized word representation gave me a measure of similarity between the reviews and a common fake product/seller review. Moreover, a counterfeit product will likely have reviews with negative sentiment. Because of that I used VADER, a lexicon- based sentiment analysis algorithm which was customized for user-generated content, to provide me with a measure sentiment of the reviews. In order to test my proposed algorithm, I scraped data from Amazon to get a total of 8,000 product reviews and 813 seller reviews. I showed that when performing my algorithm with and without the seller reviews on this unlabeled data set, the centers of the clusters are meaningfully apart from each other. In order to check the accuracy of my model, I employed two independent human coders to read the reviews and identify whether a product was real or fake. Both algorithms performed reasonably well in detecting the fake products. However, adding the seller reviews decreased the accuracy of the algorithm slightly because product reviews have more of an effect on the purchasing decisions of customers than seller reviews. Also, when a seller sells some fake products and some genuine products, while a fake product may have a negative review, some of the seller reviews could still be positive.

One possible extension is to achieve a large, labeled data set of customers' reviews and to train a supervised classification algorithm on the features of the reviews. Doing so would not only increase the accuracy of the model but would also let us incorporate many other extracted features of the text, such as TF-IDF, in classifying the products. Another extension to my model would be to train specialized word-embeddings for different categories of products on different platforms. Doing so would help to capture the context-related word representation for different product categories and for different customers using online platforms.

Chapter 4

4 Effect of Customers' Reviews on Online Platforms' Anti-Counterfeiting Strategy

Counterfeit products are a serious problem for online platforms. Online platforms in this context can be defined as a website that provides a storefront for third-party sellers where the customers can post reviews about the products and the seller who sells that product. In order to study the effect of the customers' reviews on an online platform, I use a stylized model that considers a third-party seller who sells two (potentially fake) products. The platform can provide a tool for the customers that analyzes just the product reviews or a more advanced tool that analyzes both the product reviews and the seller reviews. The third-party seller can choose to reveal its fake products by charging a lower separating price based on its profit under these two options. I show that because the profits of the seller and platforms are aligned, even when the tools are free, the platform does not provide the advanced tool if the seller sells any fake products. Moreover, I show that the platform provides the basic tool if and only if the demand of the genuine product is sufficiently high.

Keywords: Anti-counterfeiting strategy; online platforms; third-party seller; customers' reviews.

4.1 Introduction

Third-party sellers are the most significant contributors to online platforms' profit. In 2020, Amazon's gross merchandise value (GMV) was about $490 billion from which $300 billion was from third-party sellers (Danziger, 2021). As a result, platforms keep the barriers to entry low for third-party sellers and such low entry costs expose millions of customers to counterfeit sellers (Cantrell, 2021). According to the Department of Homeland Security report, "Many consumers are … unaware of the significant probabilities they face of being defrauded by counterfeiters when they shop on e-commerce platforms" (Homeland Security, 2020). Exacerbating the problem is that many of the counterfeit products are advertised as authentic and are sold at almost full prices as if the product is authentic (Khan, 2019). In an investigation by CBC, it was found that half of the well-known products ordered from popular e-commerce platforms were fake while all had legitimate advertisements and were sold at full price (Cowley et al., 2020). A manufacturer of electronic products found that over 70% of the products claiming to be authentic and sold by third parties on Amazon were actually counterfeit (DH Anticounterfeit, 2017).

The platform owners understand this threat to legitimate commerce and have taken various actions to address the counterfeiting problem. For instance, Amazon spent over $500 with 8,000 employees to fight counterfeit products and launched the Counterfeit Crimes Unit and Project Zero as their special anti-counterfeiting task forces (Palmer, 2020). Alibaba formed the Alibaba Anti-counterfeiting Alliance (AACA) with more than 450 brands. However, because of the vast number of deceptive counterfeits being sold on platforms, these efforts may not be able to resolve the problem (Bain, 2019). In this situation, product reviews, written by customers who have seen and used these products, provide a valuable source for detecting counterfeits (Suthivarakom, 2020).

In the previous chapter, I proposed a methodology to analyze the customers' reviews and to give an authenticity score to a product. More precisely, I proposed an algorithm which takes the product reviews or the product and seller reviews as an input and returned an authenticity score between 0 and 1. In this chapter, I analytically investigate the effects of the presence of anti-counterfeiting tools on the dynamics of an online e-commerce platform. More precisely, I investigate the effect of different tools on the platform's anti-counterfeiting strategy and third-party sellers' pricing decisions.

To examine this question, I developed an analytical model that considers a third-party seller who sells two products on an online platform. Either of these products can be genuine or fake. First, the platform chooses to provide a tool that analyzes the product reviews (basic tool), a tool that analyzes the product reviews and the seller reviews (advanced tool), or no tool at all based on the prices and authenticity scores of the sellers and their products. Then, the seller decides on its pricing strategy. In particular, in the situation where the seller sells a fake product, the seller can disguise its fake products by charging the same pooling price as a genuine product. Alternatively, the seller can choose a lower separating price to distinguish its fake products from genuine products.

I show that the third-party seller never reveals its fake products by charging a lower price. This is because by charging the same price as the genuine products, the customers will have higher expected quality of the product. I also show that online platforms do not provide the advanced tool if the seller sells fake products. This result is true even when there is no implementation cost for the platform. Moreover, the platform provides the basic tool if, and only if, one product is genuine and one product is fake and the increase in demand for the real product is sufficiently high. The main reason for this result is that the platform is commission-based; the profit of the platform and the seller are aligned, and both depend on the demand and the price of the products.

This chapter of the dissertation is organized as follows: In Section 4.2, I review the related literature. I then describe the math model in Section 4.3 and present the optimal solution in Section 4.4. In Section 4.5, I show that my main findings are robust to an alternative game sequence and in Section 6, I make concluding remarks and managerial insights.

4.2 Literature Review

My work relates to two streams of literature. Firstly, it relates to the analytical papers that study deterrence mechanisms for deceptive counterfeits. Cho et al. (2015) examine how an original brand manufacturer can use quality and pricing decisions as an entry-deterrence mechanism for counterfeit products. They show that increasing the quality and reducing the price may be inefficient against deceptive counterfeits. Qian et al. (2015) study a scenario where a copycat firm chooses whether to enter with a copycat. They consider two dimensions of quality: experiential quality (unobservable) and searchable quality (observable). They show that when the number of informed customers in the market increases, then the original brand manufacturer has more incentive to invest in experiential quality while when the number of informed customers decreases, the manufacturer has more incentive to invest in searchable quality. Gao (2018) studies a pharmaceutical manufacturer's use of overt anti-counterfeiting technologies (OACTs) as a deterrence strategy for deceptive counterfeits and illegitimate products. He shows that overuse of OACTs may increase the magnitude of counterfeit medicine purchases because more patients use dubious sources instead of reliable sources due to the increased chance of obtaining a genuine drug. None of these studies consider the online e-commerce platform's anti-counterfeiting strategy and they do not consider content analysis of customers' reviews. Zhang et al. (2021) investigate the effect of the online platform's revenue models on its anti-counterfeiting efforts. They show that the platform exerts anti-counterfeiting effort when they adapt the brokerage model and the competition in the market is less intense. However, my research investigates the effect of product reviews and seller reviews on the platform's anti-counterfeiting strategy and third-party sellers' pricing decisions.

Secondly, my work relates to the effect of customers' reviews on the platforms' and the sellers' strategic behavior. Fang (2021) studies the effect of online reviews on restaurants' revenue, showing that although reviews do have a strong impact on restaurants' revenue, this effect diminishes with the restaurant's age. Vana and Lambrecht (2021) study how the content of product reviews affects consumers' purchasing decisions, finding that reviews have a significant effect on the purchasing decisions of the customers, even when controlling for the product's average rating. Mankad et al. (2016) analyze online reviews for hotels and show that negative reviews focus on a smaller number of topics compared to positive reviews. Zhao et al. (2013) investigate the effect of online product reviews on the purchase of experiential goods, concluding that customers learn more about book titles from online reviews compared to their own experience with books in the same genre. Pavlov and Dimoka (2006) examine the content of reviews and its role in building the buyer's trust in a seller; the seller with more positive reviews can charge a higher price for the same product. However, none of these studies acknowledge that customers buy deceptive counterfeits from online platforms, a significant problem, one that requires further discussion, and the topic of which is further studied in Section 4.3 of this chapter.

4.3 Model

I consider a system in which a third-party seller sells two non-competing products ($i \in \{\alpha, \beta\}$) on an online platform. Product i can either be genuine or fake. Without loss of generality, I normalize the quality of a genuine product to one and the quality of a fake product i is $x_i < 1$.

Customers do not know whether product i is genuine or fake. They establish a belief that product i is genuine with probability λ_i. Given the product description, pictures and reviews, customers have a prior belief of $\lambda_i = \lambda_i^0$. The reviews collected from the previous customers about the products and the seller can be used by the platform to inform customers about the authenticity of the products and update customers' beliefs accordingly. As shown in the previous chapter, the product and seller reviews can be utilized to estimate the probability of a product being genuine (authenticity score). Additionally, in order to help customers to detect fake products, the platform can provide two types of tools that analyze reviews and yield an authenticity score for products sold on the platform.

The first tool (basic tool) analyzes only product reviews. When product reviews are positive (they yield a high authenticity score), which is more likely if a product is genuine, the customer's posterior belief about product i being genuine will update to $\lambda_i = \lambda_i^1$, where $\lambda_i^1 > \lambda_i^0$. On the other hand, when product reviews are negative (they yield a low authenticity score), which is more likely if a product is fake, the customer's posterior belief will update to $\lambda_i = \lambda_i^{1'}$, where $\lambda_i^{1'} < \lambda_i^0$.

The second tool (advanced tool) is based on both product reviews and seller reviews. Recall that the customers can post seller reviews about whether the seller is selling genuine or fake products. Therefore, seller reviews have an augmentation effect on the customer's posterior belief about product i being genuine. The reviews of all products sold by a seller are reflected in the seller reviews. There are three possible cases of how an advanced tool can affect the posterior belief of the customer. First, the seller receives positive seller reviews if both products that they sell are genuine; the positive seller reviews positively augment the positive product reviews, so the advanced tool gives an authenticity score that improves the customer's posterior belief about product i being genuine to $\lambda_i = \lambda_i^3$, where $\lambda_i^3 > \lambda_i^1$. Second, the seller receives negative seller reviews if both products are fake; the negative seller reviews negatively augment the negative product reviews, so the customer's posterior belief is $\lambda_i = \lambda_i^{3'}$, where $\lambda_i^{3'} < \lambda_i^{1'}$. Third, the seller receives mixed (some positive and some negative) seller reviews if it sells one real and one fake product. Without loss of generality, I assume that product α is genuine and product β is fake. The positive product reviews for product α are negatively affected by mixed seller reviews, so the posterior belief is $\lambda_i = \lambda_i^2$, where $\lambda_i^0 < \lambda_i^2 < \lambda_i^1$. The negative product reviews for product β are positively affected from mixed seller reviews, so the posterior belief is $\lambda_i = \lambda_i^{2'}$, where $\lambda_i^0 > \lambda_i^{2'} > \lambda_i^{1'}$.

Tables 4.1 and 4.2 summarize the effects of the basic and advanced tools on the customer's posterior belief (the belief is λ_i^0 for all cases if the platform provides no tool). I assume that there is no cost to implement either authentication tool; the directional effect for that tool is more pronounced when the implementation cost favors this tool.

Table 4-1: Posterior Belief Under Basic Tool

Basic Tool	Product β

		Fake	Genuine
α Product	Fake	$\lambda_\alpha^{1\prime}, \lambda_\beta^{1\prime}$	$\lambda_\alpha^{1\prime}, \lambda_\beta^{1}$
	Genuine	$\lambda_\alpha^{1}, \lambda_\beta^{1\prime}$	$\lambda_\alpha^{1}, \lambda_\beta^{1}$

Table 4-2: Posterior Belief Under Advanced Tool

Advanced Tool		Product β	
		Fake	Genuine
α Product	Fake	$\lambda_\alpha^{3\prime}, \lambda_\beta^{3\prime}$	$\lambda_\alpha^{2\prime}, \lambda_\beta^{2}$
	Genuine	$\lambda_\alpha^{2}, \lambda_\beta^{2\prime}$	$\lambda_\alpha^{3}, \lambda_\beta^{3}$

The demand for product i increases in its quality and decreases in its price P_i. Recall that customers do not know whether the product is genuine or fake. In this case, even when the seller sells a genuine product i, customers expect the quality is $\lambda_i * 1 + (1 - \lambda_i)x_i$. The demand for a (genuine) product i is as follows, where v is a constant:

$$D_i = v(\lambda_i + (1 - \lambda_i)x_i) - P_i \tag{4.1}$$

When the seller sells a fake product i, they have two options. First, they can disguise their fake product by setting the price as if the product is genuine (pooling price). Since the customers cannot distinguish the fake product, the demand for a (fake) product i is the same as Equation 4.1.

Second, the seller can charge a lower selling price for their fake product compared to the price of a genuine product (separating price). In this case, the customers know that product i is fake (i.e., $\lambda_i = 0$). The demand for a (fake) product i depends on the quality x_i:

$$D_i = v x_i - P_i \tag{4.2}$$

Without loss of generality, I normalize the production cost to zero. The platform charges a commission rate, θ, that is applied to the selling price of each product sold; the remaining $1-\theta$ belongs to the seller. Consequently, the platform's profit is $\pi = \theta \sum_i D_i P_i$ and the seller's profit is $\Pi = (1-\theta) \sum_i D_i P_i$. The commission rate is an exogenous parameter because it does not change frequently (Sun et al., 2020; Liu et al., 2021). For instance, Amazon charges 16% commission rate for all watches sold on its platform and this rate has not been changed since 2017 (Lakes, 2020).

The game sequence is as follows: In the beginning of the game, nature decides on the seller's type. There are three scenarios: (1) two genuine products, (2) two fake products, and (3) one genuine and one fake product. At step 1, the platform decides whether to provide no authentication tool, the basic tool, or the advanced tool. At step 2, the seller decides the prices for their products. If the seller sells a fake product, they can choose to reveal the fake nature of their product by charging a separating price, or to disguise its fake nature by setting a pooling price. This game sequence makes sense because in reality, the platform's anti-counterfeiting strategy is permanent and is difficult to change; however, the seller's pricing decision is an operational short-term decision that can be easily adjusted. In Section 5, I show that my results are robust to an alternative game sequence where the seller decides on its pricing strategy before the platform's authentication tool strategy decision.

4.4 Equilibrium Analysis

I used backward induction to derive the equilibrium solution. For mathematical tractability, I set $\nu = 1$. Moreover, to reduce the number of parameters while preserving the ranking for the posterior beliefs ($\lambda_i^{3'} < \lambda_i^{1'} < \lambda_i^{2'} < \lambda_i^0 < \lambda_i^2 < \lambda_i^1 < \lambda_i^3$), I assumed that $\lambda_i^0 = \frac{1}{2}$, $\lambda_i^{1'} = 1 - \lambda_i^1$, $\lambda_i^{2'} = 1 - \lambda_i^2$ and $\lambda_i^{3'} = 1 - \lambda_i^3$.

4.4.1 Seller Pricing Strategy (Step 2)

At this stage, the seller decides on its pricing strategy.

If the seller uses a pooling price for product i (whether product i is real or fake), the demand is Equation 4.1, which depends on the platform's strategy. On the other hand, if the seller that sells a fake product uses a separating price, the demand is Equation 4.2, which is independent of the platform's authentication tool strategy. I used C to denote a Common pooling price and S to denote a Separating price. If the seller sets a pooling price for product i, its optimal pooling price would be $P_i^C = argmax((1-\theta)(\lambda_i + (1-\lambda_i)x_i - P_i)P_i)$. On the other hand, if the seller sets a separating price for its fake product i, its optimal separating price would be $P_i^S = argmax((1-\theta)(x_i - P_i)P_i)$. The seller chooses between the optimal pooling price and the optimal separating price for the fake product. See Figure 4-1 for the seller's decision tree. Lemma 4.1 presents the subgame perfect Bayesian equilibrium for the seller's pricing strategy.

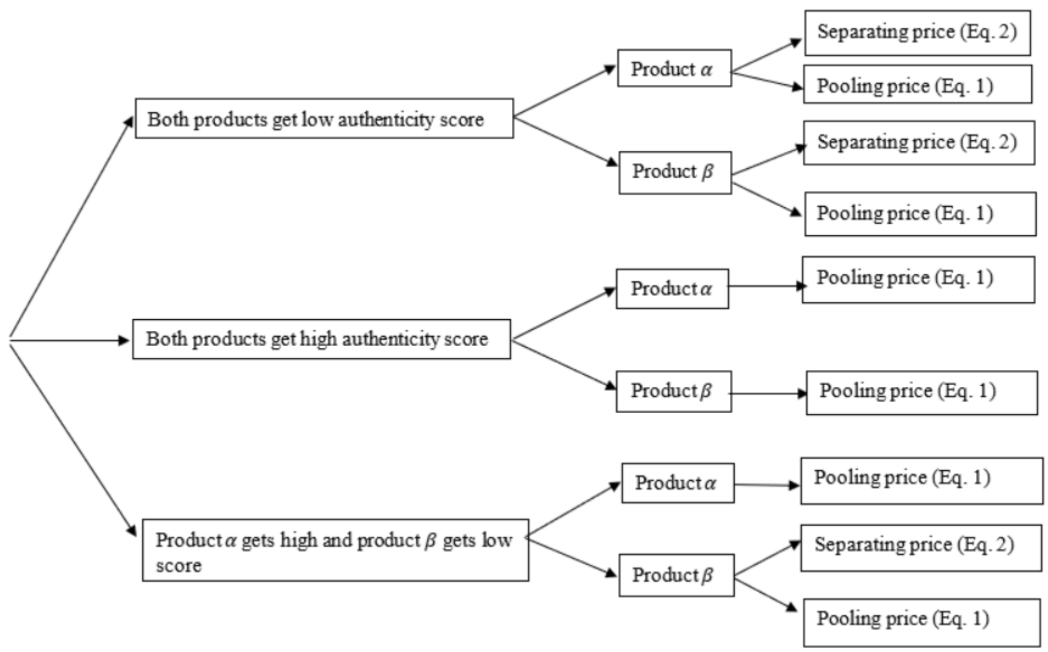

Figure 4-1: Seller's decision tree

Lemma 4.1: *The seller never sets a separating price for the fake products and always sets* $P_i = P_i^C = \frac{\lambda_i + (1-\lambda_i)x_i}{2}$.

For the fake product, if the seller sets the separating price, the customers will know that the product is fake and ascribe the low quality x_i to it. On the other hand, if the seller sets the pooling price, the customers will falsely expect that the product quality is 1 with probability of λ_i. Setting the pooling price will increase customers' expected quality of the product which in turn increases the demand. Hence, the seller will never set a separating price for their fake products.

4.4.2 Platform Authentication Tool Strategy (Step 1)

At this stage, the platform has three possible strategies: providing no tool; providing the basic tool, which only analyzes product reviews; or providing the advanced tool, which analyzes both product and seller reviews. Anticipating the seller's pricing decision, the platform decides which of the three strategies to adopt. Thus, to choose the optimal strategy, the platform has to evaluate the outcomes of the following three scenarios.

1) Both products are fake (i.e., both products get a low authenticity score): In this case, if the platform provides no tool, customers' posterior belief regarding the authenticity of the products is $\lambda_i = \lambda_i^0$. If the platform provides the basic tool, the posterior belief is $\lambda_i = \lambda_i^{1'}$, and finally, if the platform provides the advanced tool, the posterior belief is $\lambda_i = \lambda_i^{3'}$.

2) Both products are genuine (i.e., both products get a high authenticity score): In this case, if the platform provides no tool, then $\lambda_i = \lambda_i^0$. If the platform provides the basic tool, the posterior belief is $\lambda_i = \lambda_i^1$, and finally, if the platform provides the advanced tool, the posterior belief is $\lambda_i = \lambda_i^3$.

3) One product is genuine (i.e., gets a high authenticity score) and one product is fake (i.e., gets a low authenticity score): In this case, if the platform does not provide any tool, then $\lambda_i = \lambda_i^0$. If the platform provides the basic tool, then $\lambda_i = \lambda_i^1$ for the high-authenticity score product and $\lambda_i = \lambda_i^{1\prime}$ for the low-authenticity score product. If the platform provides the advanced tool, then $\lambda_i = \lambda_i^2$ for the high-authenticity score product and $\lambda_i = \lambda_i^{2\prime}$ for the low-authenticity score product.

Proposition 4.1 presents the optimal strategy. The analytical expression of threshold $\bar{x}$ is given in the appendix.

Proposition 4.1:

a) *If the seller sells two fake products, the platform does not provide any tool.*
b) *If the seller sells two genuine products, the platform provides the advanced tool.*
c) *If the seller sells one real (α) and one fake (β) product, then:*
   1. *The platform does not provide any tool if and only if $x_\alpha > \bar{x}$.*
   2. *The platform provides the basic tool otherwise.*

First, if the seller sells two fake products (Proposition 4.1a), both products will get a low authenticity score, and the platform will not provide any tool. This is because providing authenticity scores will decrease the demand for both products and thus the platform does not have an incentive to provide any tool.

If the seller sells two genuine products (Proposition 4.1b), the platform will provide the advanced tool which increases the customer's expected quality of the products. This is because positive product reviews and seller reviews will have the most magnifying effect on demand. The pooling price is high because the customer's expected quality for both products increase with the high authenticity score; hence, the seller can charge higher prices.

Finally, if the seller sells one genuine and one fake product (Proposition 4.1c), providing authenticity scores will have a different effect on the real and fake products' demand. Note that if the seller uses a pooling price, the customer's expected quality of the product i would be $\lambda_i * 1 + (1 - \lambda_i)x_i$. In this case, while the genuine product α has a quality of 1, the customer expects with a probability of $(1 - \lambda_\alpha)$ that the product quality is x_α. Similarly, while the fake product has a quality of x_β, the customer expects that the product quality is 1 with a probability of λ_β. When $x_\alpha > \bar{x}$, the demand for the real product is not affected significantly by the customer's belief, because with probability $(1 - \lambda_\alpha)$, the customers believe that the real product is a fake product but with reasonably high quality. In this case, the increase in profit from the real product does not justify the loss of profit from the fake product when the basic tool is provided; therefore, the platform does not provide any tool. On the other hand, when $x_\alpha < \bar{x}$, the demand for the real product highly depends on the customer's belief, because with probability $(1 - \lambda_\alpha)$, they believe that the real product is a fake product with low quality. In order to increase the demand for the real product, the platform provides the basic tool. In this case, the increase in profit from the real product justifies the loss of profit from the fake product when the basic tool is provided. Note that in the case where the seller sells one real and one fake product, the platform never provides the advanced tool because the positive product reviews for product α are negatively affected by mixed seller reviews which, in turn, will reduce the authenticity score of product α. Figure 2 shows the strategy map for this scenario when $\lambda_\alpha^1 = \lambda_\beta^1 = 0.75$.

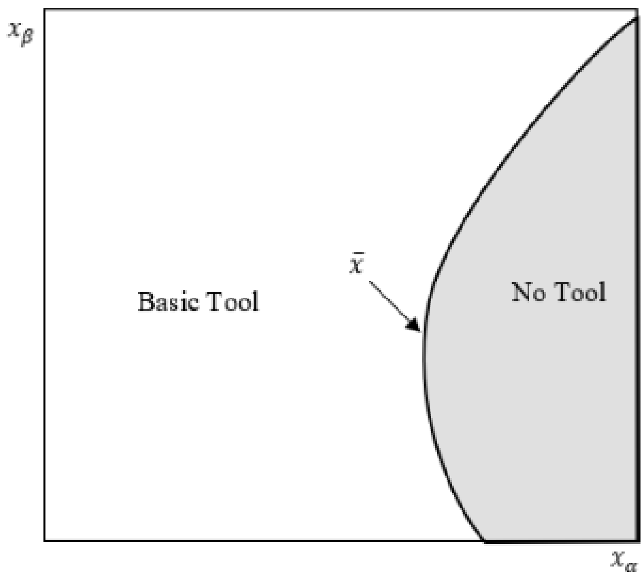

Figure 4-2: Strategy map when the product α is genuine, and the product β is fake

4.5 Extension

In this section I consider an alternative game sequence where steps 1 and 2 are switched. At step 1, the seller decides on their pricing strategy and whether to reveal their fake products and at step 2, the platform decides whether to provide no authentication tool, the basic tool, or the advanced tool.

4.5.1 Platform Authentication Tool Strategy (Step 2)

The platform decides whether to provide no tool, the basic tool, or the advanced tool in this stage. There are two drivers that affect this decision. First, it depends on whether the product has a high or low authenticity score. Second, if the authenticity score is low (implying that the product is likely fake), it depends whether the seller charges a pooling or a separating price. Lemma 4.2 presents the subgame equilibrium for the platform's strategy. The analytical expression of the threshold $\bar{\gamma}$ is given in Appendix B.

Lemma 4.2:

a. *The platform provides the advanced tool if and only if both products get high authenticity scores.*
b. *The platform provides the basic tool if and only if product α gets a high authenticity score and product β gets a low authenticity score and one of the following conditions is satisfied:*

1. *The seller sets a separating price for product β.*
2. *The seller sets a pooling price for product β and the price of product α is sufficiently higher than the price of product β, i.e.,* $\frac{p_\alpha}{p_\beta} > \bar{\gamma}$.

c. *The platform does not provide any tool otherwise.*

If both products get a high authenticity score (implying that the seller sells two genuine products), then the platform always provides the advanced tool (Lemma 4.2a). This is because positive product reviews and seller reviews will have the most magnifying effect on demand.

If product α gets a high authenticity score and product β gets a low authenticity score, the platform's strategy depends on the seller's pricing decision. First, if the seller sets a separating price for product β, the platform always provides the basic tool (Lemma 4.2b.1). This is because the demand for product β is independent of the customer's belief and the platform can increase the demand for product α by using the basic tool. Second, if the seller sets a pooling price for product β, the platform will provide the basic tool only when the price for product α is sufficiently higher than the price for product β (Lemma 4.2b.2). This is because the platform wants to increase the demand for the more expensive product (α). With the basic tool, the increase in profit of product α justifies the loss from product β.

For the remaining setting, the platform does not provide any tool (Lemma 4.2c). This is because providing authenticity scores will either have an adverse effect on the demand for the more expensive product or decrease the demand for both products.

4.5.2 Seller Pricing Strategy (Step 1)

At this stage, the seller knows whether its products are real or fake and can anticipate the platform's strategy. I denote the platform's strategy with $j \in \{N, B, A\}$ where $j = N$ refers to the case where the platform provides no tool, and $j = B$ ($j = A$) refers to the basic (advanced) tool. Moreover, I define Π_k^j as the maximum profit that the seller can achieve for its pricing strategy for the two products where $k \in \{SS, SC, CS, CC\}$ is the case for separating/pooling prices. Based on whether the products are genuine or fake, there are three possibilities:

1) If both products are fake, the platform's best response would be not to provide any tool (cf. Lemma 4.2c). If the seller uses separating prices for both products, its profit is:

$$\Pi_{SS}^{N} = \underset{P_\alpha, P_\beta}{Max}(1-\theta)\sum_i (x_i - P_i)P_i$$

If the seller uses pooling prices for both products, the belief that product i is genuine is $\lambda_i = \lambda_i^0$. So, the seller's profit is

$$\Pi_{CC}^{N} = \underset{P_\alpha, P_\beta}{Max}\,(1-\theta)\sum_i (\lambda_i^0 + (1-\lambda_i^0)x_i - P_i)P_i$$

If the seller uses a pooling price for product α and a separating price for product β, its profit is

$$\Pi_{CS}^{N} = \underset{P_\alpha, P_\beta}{Max}\,(1-\theta)\left((\lambda_\alpha^0 + (1-\lambda_\alpha^0)x_\alpha - P_\alpha)P_\alpha + \left(x_\beta - P_\beta\right)P_\beta\right)$$

Lastly, Π_{SC}^{N} can also be derived similarly. In summary, a seller that sells two fake products derives its optimal pricing strategy by comparing between these four cases:

$$P_\alpha^*, P_\beta^* = \left(P_\alpha, P_\beta \middle| \Pi(P_\alpha, P_\beta) = Max(\Pi_{CC}^{N}, \Pi_{SS}^{N}, \Pi_{SC}^{N}, \Pi_{CS}^{N})\right) \quad (4.3)$$

2) If both products are genuine, the seller sets a pooling price (cf. Equation 4.1) for its products. The platform's best response would be to provide the advanced tool (cf. Lemma 4.2a). So, the seller's profit is

$$\Pi_{CC}^{A} = \underset{P_\alpha, P_\beta}{Max}\,(1-\theta)\sum_i (\lambda_i^3 + (1-\lambda_i^3)x_i - P_i)P_i$$

A seller selling two genuine products sets the selling price according to Equation 4.4.

$$P_\alpha^*, P_\beta^* = \left(P_\alpha, P_\beta \middle| \Pi\left(P_\alpha, P_\beta\right) = \Pi_{CC}^{A}\right) \quad (4.4)$$

3) Consider the case where product α is real and product β is fake. The seller sets a pooling price for product α. If the seller uses a separating price for product β, the platform provides the basic tool (cf. Lemma 2b.1). The posterior belief that product α is genuine is $\lambda_\alpha = \lambda_\alpha^1$, so the seller's maximization problem is

$$\Pi_{CS}^{B} = \underset{P_\alpha,P_\beta}{Max}\,(1-\theta)\left((\lambda_\alpha^1 + (1-\lambda_\alpha^1)x_\alpha - P_\alpha)P_\alpha + \left(x_\beta - P_\beta\right)P_\beta\right)$$

If the seller sets a pooling price for product β and $\frac{P_\alpha}{P_\beta} > \bar{\gamma}$ (cf. Lemma 2b.2), the platform provides the basic tool. The posterior belief that product β is genuine is $\lambda_\beta = \lambda_\beta^{1\prime}$, so the seller's maximization problem is

$$\Pi_{CC}^{B} = \underset{P_\alpha,P_\beta}{Max}\,(1-\theta)\left((\lambda_\alpha^1 + (1-\lambda_\alpha^1)x_\alpha - P_\alpha)P_\alpha + \left(\lambda_\beta^{1\prime} + \left(1-\lambda_\beta^{1\prime}\right)x_\beta - P_\beta\right)P_\beta\right)$$

$$\text{s.t. } \frac{P_\alpha}{P_\beta} > \bar{\gamma}$$

If the seller sets a pooling price for product β and $\frac{P_\alpha}{P_\beta} \leq \gamma$ (cf. Lemma 2b.2), the platform does not provide any tool. The posterior belief is $\lambda_i = \lambda_i^0$, so the seller's maximization problem is

$$\Pi_{CC}^{N} = \underset{P_\alpha,P_\beta}{Max}\,(1-\theta)\sum_i(\lambda_i^0 + (1-\lambda_i^0)x_i - P_i)P_i$$

$$\text{s.t. } \frac{P_\alpha}{P_\beta} \leq \bar{\gamma}$$

The seller solves the following maximization problem if product α is real and product β is fake:

$$P_\alpha^*, P_\beta^* = \left(P_\alpha, P_\beta \middle| \Pi\left(P_\alpha, P_\beta\right) = Max(\Pi_{CC}^B, \Pi_{CS}^B, \Pi_{CC}^N)\right) \tag{4.5}$$

Proposition 4.2 presents the optimal strategy.

Proposition 4.2: *The seller never sets a separating price for fake products. Moreover, the seller sets the price such that the platform provides the basic tool if and only if* $x_\alpha > \bar{x}$.

First, if the seller sells two fake products (Proposition 4.2a), they anticipate that both products will get a low authenticity score, and the platform will not provide any tool. Using a separating price for either product would have an adverse effect on the customer's expected quality of the product, which in turn decreases the profit of the seller. This is because when providing an authenticity tool has a deteriorating effect on the sales of a product, the platform does not have an incentive to provide any tool, thus the seller sells its fake products with a pooling price.

If the seller sells two genuine products (Proposition 4.2b), the platform will provide the advanced tool which increases the customer's expected quality of the products. The pooling price is high because the customer's expected quality for both products increases; hence, the seller can charge higher prices.

Finally, if the seller sells one genuine and one fake product (Proposition 4.2c), they anticipate that one product will get a high authenticity score and the other will get a low authenticity score. When $x_\alpha > \bar{x}$, with probability $(1 - \lambda_\alpha)$, the customer falsely believes that the real product is fake but with a reasonably high quality. If the platform provides the basic tool, the customer's expected quality of product α improves modestly, but their expected quality of product β decreases significantly. The increase in the profit from product α does not justify the loss from product β, so the seller sets prices such that the platform does not provide any tool. On the other hand, when $x_\alpha < \bar{x}$, the seller wants to increase the customer's expected quality of product α because the increase in the profit from product α justifies the loss from product β when the basic tool is provided. So, the seller sets prices such that the platform provides the basic tool.

Irrespective of whether the platform or the seller has the first mover advantage, strategic decisions of the platform and the seller (cf. Section 2.4 and Section 2.5) are identical. This is because the platform and the seller both maximize the same profit function which depends on the demand and the price of the products. Therefore, irrespective of the game sequence, the final equilibrium maximizes the total market profit.

4.6 Conclusion

In this chapter, I use an analytical model to examine an online e-commerce platform where a third-party seller may sell fake products. Conversely, the customers' reviews about the products and the seller can be used to detect possible fake products. I study what affects the platform's decision to provide consumers with anti-counterfeiting tools. They can provide a basic anti-counterfeiting tool based on the product reviews, an advanced anti-counterfeiting tool based on both the product and seller reviews, or they can provide no tool at all. Further, given the platform's decision, the seller can either reveal their fake products by choosing a lower price or disguise them by choosing a higher price that is in line with the price of the genuine products.

I show that the platform does not provide an advanced tool if the seller sells any fake products and provides the basic tool if only one product is genuine and the increase in demand for the real product justifies the decrease in demand for the fake product. This is because the profits of the platform and the seller are aligned, and both depend on the customers' expected quality of the products. I also show that the seller never uses a lower price to reveal its fake products because using the lower price has a negative effect on the products' demand. Although I show in the previous chapter that a tool that analyzes the customers' reviews provides counterfeit detection power, the platform may have no incentive to implement such a tool, even this tool is provided for free.

I use a stylized model to study the use of counterfeit detection tools on online platforms. Extending some of my assumptions may lead to interesting future work. First, I assume that the demand of the products does not depend on the customers' trust in the e-commerce platforms. However, providing an anti-counterfeiting tool may increase the base demand for the online platforms. Especially in the multi-channel distribution environment where there is a direct selling channel, this may be used as a competing strategy for online platforms. Moreover, my model considered only one seller. It would be worthwhile to consider multiple sellers with different types of products, and the effect of the platform's strategy on these sellers' behavior.

Chapter 5

5 Conclusion

In this thesis, I examine irregularities in market entry in different environments, such as a decentralized supply chain and deceptive counterfeits in an online e-commerce platform. In the supply chain, I extend the literature by considering the effect of information asymmetry and production capacity in a cooperative environment where the supplier has private information on its production capacity. I used a signaling game in a theoretical setting to study the dynamics of the supplier and buyer's strategic behavior under different capacity levels considering the information asymmetry.

In Chapters 3 and 4, I study the counterfeiting problem by third-party sellers on an online e-commerce platform. In Chapter 3, I develop an unsupervised classification algorithm which takes the sentiment of the customers' reviews and their similarities to the word embedding of "Fake" and returns an authenticity score. I trained a context-specific word embedding based on customers' reviews on online e-commerce platforms. I then tested my algorithm using an unlabeled data set from Amazon and examined the accuracy of my model by comparing it with the same data set that had been labeled by human coders. In Chapter 4, I studied the effect of such a counterfeit detection tool on the platform's anti-counterfeiting strategy and the third-party seller's pricing decisions. I used a game theoretical model to explain when the platform provides which anti-counterfeiting tool to customers to determine the authenticity of third-party sellers' products so that it still obtains maximum profits.

Together, my studies highlight very real irregularities in market entry, including in the supplier and buyer chain, and in online e-commerce. The indications that these problems and my resulting studies seem to point to are that the efforts to manage them are complex but not impossible. For example, in the realm of supplier encroachment, the supplier benefits greatly from disclosing its product capacity to its buyer. On the other hand, the buyer benefits and protects its own interests by refraining from product hoarding. For both parties in this situation, therefore, transparency and collaboration are beneficial. When it comes to e-commerce fraud, managing it is well within the ability of customers and online platforms. Machine learning, even when unsupervised, is capable of highly skilled counterfeit detection and, as noted in chapter 4, e-commerce platforms have such counterfeit detection tools. It is accountability that they lack, a much larger problem and one that calls for further research and action on a larger scale, perhaps by the World Trade Organization.

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Appendices

Appendix-A:

Throughout the analysis, I consider the parameters setting $C_S = C > 0$ and $C_B = 0$ and $2C < D < \overline{D} = C + \frac{4\sqrt{2}C}{3}$. I will formally define threshold $\overline{D}$ in Part B.3.2.1.2.

(A) Full information game

In this part, I provide the proof of the final equilibrium when the capacity of the supplier is public information. Details are presented in the proof of Lemma 2.1.

Proof of Lemma 2.1: I use backward induction to find the equilibrium solution for the full information case.

(A.1) *Stage 4 - direct selling quantity of the supplier.* The supplier solves: $\max_{0 \le q_S \le K-Q} \pi = (D - q_S - q_B - C)\, q_S + w\, Q$. Using KKT conditions, there are three optimal selling quantities: **1)** non-binding: $q_S = \frac{1}{2}(D - C - q_B)$ if $Q \le K$ and $2Q + D - 2K - C \le q_B \le D - C$; **2)** binding: $q_S = K - Q$ if $Q \le K$ and $q_B \le 2Q + D - 2K - C$; **3)** non-negativity constraint: $q_S = 0$ if $q_B \ge D - C$.

(A.2) *Stage 3 - direct selling quantity of the buyer.* The buyer solves: $\max_{0 \le q_B \le Q} \Pi = P\, q_B - w\, Q$ for each of the three cases of *Stage 4*. For example, in a non-binding solution, i.e., $q_S = \frac{1}{2}(D - C - q_B)$, using KKT conditions, only two solutions are feasible when $2C < D < \overline{D}$: **1)** $q_B = Q$ if $\left(\frac{D-C}{2} < K \le D - C \text{ and } 0 \le Q \le C + 2K - D\right)$ or $(K > D - C$ and $0 \le Q \le D - C)$; **2)** $q_B = D - C$ if $K > D - C$ and $D - C \le Q < K$. The calculations for other cases are similar and are not presented to avoid redundancy. The best response of the buyer when $2C < D < \overline{D}$ is presented in Table A1.

(A.3) *Stage 2 - order quantity of the buyer.* Given the supplier's capacity level, the buyer's optimal order quantity will fall in one of the five cases presented in Table A1. Note that in the full-information setting, the capacity of the supplier is public information, so $\Omega \le K$ and $\Omega = Q$. I only present calculations for Case 2 (i.e., $\frac{1}{2}(D - C) < K \le \frac{D}{2}$) to avoid redundancy. When $\frac{1}{2}(D - C) < K \le \frac{D}{2}$ and $0 \le \Omega \le C + 2K - D$, I have $\frac{d^2\,\Pi}{d\,\Omega^2} < 0$, so the FOC gives $\Omega = \frac{1}{2}(C + D - 2w)$ if $0 \le \frac{1}{2}(C + D - 2w) \le C + 2K - D \Leftrightarrow \frac{1}{2}(3D - C - 4K) \le w \le \frac{C+D}{2}$. If $w > \frac{C+D}{2}$, the lower bound $\Omega = 0$ and if $w < \frac{1}{2}(3D - C - 4K)$, the upper bound

$\Omega = C + 2K - D$ is binding. When $\frac{1}{2}(D - C) < K \leq \frac{D}{2}$ and $C + 2K - D < \Omega \leq K$, I have $\frac{d^2\,\Pi}{d\,\Omega^2} = 0$, so $\Omega = K$ if $\frac{d\,\Pi}{d\,\Omega} \geq 0 \Leftrightarrow w \leq D - K$ and $\Omega = C + 2K - D$ otherwise. The buyer can set the order quantity to choose either of these two cases within $\frac{1}{2}(D - C) < K \leq \frac{D}{2}$. I can show that $\Omega = C + 2K - D$ is never optimal because $D - K > \frac{1}{2}(3D - C - 4K)$ and $\Pi(q_S = K - Q, q_B = Q, \Omega = K) > \Pi\left(q_S = \frac{1}{2}(D - C - q_B), q_B = Q, \Omega = C + 2K - D\right)$. The buyer sets $\Omega = K$ if $\Pi(q_S = K - Q, q_B = Q, \Omega = K) \geq \Pi\left(q_S = \frac{1}{2}(D - C - q_B), q_B = Q, \Omega = \frac{1}{2}(C + D - 2w)\right) \Leftrightarrow w \leq \frac{1}{2}(C + D - 2K) + \sqrt{(D - C)K - K^2}$. Therefore, $\Omega = \frac{1}{2}(C + D - 2w)$ if $\frac{1}{2}(C + D - 2K) + \sqrt{DK - CK - K^2} < w \leq \frac{C+D}{2}$ and $\Omega = 0$ when $w > \frac{C+D}{2}$. Table A2 summarizes the best response of the buyer in *Stage 2*.

(A.4) *Stage 1 – wholesale price of the supplier*. For each of the cases presented in Table A2, the supplier solves: $\max_{w \geq 0} \pi = (P - C_S)\, q_S + w\, Q$. I only present calculations for Case 1 (i.e., $0 < K \leq \frac{1}{2}(D - C)$) to avoid redundancy. When $0 < K \leq \frac{1}{2}(D - C)$ and $w \leq D - K$, I have $\pi = Kw$, so $w = D - K$ and $\pi = K(D - K)$. When $0 < K \leq \frac{1}{2}(D - C)$ and $w > D - K$, I have $\pi = (D - C - K)K$, that is always less than $\pi = K(D - K)$. Thus, when $0 < K \leq \frac{1}{2}(D - C)$, I have $w = D - K$. Table A3 shows the final equilibrium outcome for the full information game. I define $\overline{K^F} = \begin{cases} K_1 & \text{if } 2C < D \leq \widehat{D} \\ K_2 & \text{if } \widehat{D} < D < \overline{D} \end{cases}$ such that when $K > \overline{K^F}$, the capacity constraint of the supplier is non-binding. I also define capacity level $\widetilde{K^F} = \frac{D}{2}$ such that when $\widetilde{K^F} < K_L \leq \overline{K^F}$, the buyer withholds some of its order quantity. $\widehat{D}$ is the market size that solve $36D^3 - 123CD^2 + 126C^2D - 49C^3 = 0$.

K_1 is the capacity level that solves $288K^4 - 288DK^3 + (204C^2 - 72CD + 108D^2)K^2 - (84C^3 + 12C^2D - 36CD^2 + 36D^3)K + 49C^4 + 9D^4 + 78C^2D^2 - 84C^3D - 36CD^3 = 0$.

$$K_2 = \frac{7C^3 + C^2D - 3CD^2 + 3D^3}{2(17C^2 - 6CD + 3D^2)} + \frac{\sqrt{\frac{54CD^5 - 279C^2D^4 + 720C^3D^3 - 1050C^4D^2 + 882C^5D - 343C^6}{(17C^2 - 6CD + 3D^2)^2}}}{\sqrt{6}}. \ \square$$

(B) Asymmetric information game

In this section, I provide the proof of the final equilibrium when the capacity of the supplier is private information. I present two possible perfect Bayesian Equilibria (PBE) for the signaling game. The roadmap of the solution approach is as follows: (B.1) First, I establish the supplier's distorted incentive which may result in misrepresentation of its type (Lemma 2.2). (B.2) I derive the equilibrium wholesale prices for the separating equilibrium (Lemma 2.3). (B.3) I characterize the pooling equilibrium. To find the subgame equilibrium of each stage, I use a similar logic as in the full information game. (B.3.1) I find the direct selling quantity of the type-i supplier to the end-customer market. (B.3.2.1) I find the selling (B.3.2.1.1) and order quantity (B.3.2.1.2) of the buyer when $\Omega \leq K_L$. (B.3.2.2) I find the selling (B.3.2.2.1) and order quantity (B.3.2.2.2) of the buyer when $\Omega > K_L$. (B.3.3) I find the final order and selling quantity of the buyer. (B.3.4) I find the wholesale price of the supplier. (B.3.5) I further refine the pooling equilibrium based on the incentive compatibility constraint of the high-type supplier. (B.4) Finally, I use the lexicographically maximum sequential equilibrium (LMSE) concept from Mailath et al. (1993) to refine the equilibrium outcomes for the asymmetric information game (Proposition 2.1).

Proof of Lemma 2.2:

(B.1) Distorted incentives. a) From Table A3 and Definition 1 I know that a high-type supplier's profit in the full-information setting is $\pi_H^F = \frac{1}{12}(7C^2 - 6CD + 3D^2)$. Based on the definition of $\pi_i(w_j^F)$ in Section 4.2 and the full-information equilibrium solution from Table A3, I can show that a high-type supplier has no incentive to mimic a low-type supplier if $\pi_H(w_L^F) \leq \pi_H(w_H^F) \Leftrightarrow K \leq \underline{K} = \frac{C+D}{3} - \frac{1}{3}\sqrt{-3C^2 + 2CD + D^2}$. b) From the full information equilibrium solution in Table A3, I can show that $\pi_L(w_L^F) \geq \pi_L(w_H^F)$. □

Proof of Lemma 2.3:

(B.2) Separating Equilibrium. From Lemma 2.2 I know that, if $K_L \leq \underline{K}$, a high-type supplier has no incentive to mimic a low-type supplier. Hence, if $\underline{K} < K_L \leq \overline{K^F}$, the low-type supplier has to down-distort its wholesale price such that $\max_{w \geq 0} \pi_H^{SE}(w \neq w_L^{SE} | Q_H^{SE}) \geq \pi_H^{SE}(w_L^{SE} | Q_L^{SE})$. That is, $w_L^{SE} < \frac{4C^2 - 6CK_L + 6DK_L - 3K_L^2}{12K_L}$. Then, given the belief system of the buyer (Definition 2), in the separating equilibrium, $w_L^{SE} = \begin{cases} D - K_L & if \quad K_L < \underline{K} \\ \frac{4C^2 - 6CK_L + 6DK_L - 3K_L^2}{12K_L} & if \;\; \underline{K} < K_L \leq \overline{K^F} \end{cases}$ and $w_H^{SE} = \frac{1}{6}(3D - C)$.

$w_H^{SE} = w_H^F$, $w_L^{SE} \leq w_L^F$ is a direct result of comparing the separating wholesale prices with the full information wholesale prices from Table A3. I can show that $w_H^{SE} < w_L^{SE} \Leftrightarrow K_L < K_{SE} = \frac{2C}{3}$. □

Proof of Proposition 2.1: Proposition 2.1 presents the equilibrium solution to (B) the Asymmetric information game (including parts B.1 and B.2).

(B.3) Pooling Equilibrium. In the pooling equilibrium, to find the subgame equilibrium of each stage, I use a similar logic as in the full information game.

(B.3.1) *Stage 4 - direct selling quantity of the supplier.* A low-type supplier solves: $\max_{0 \leq q_{SL} \leq K_L - Q} \pi_L = (D - q_{SL} - q_B - C)\, q_{SL} + w\, Q$. The results are the same as *Stage 4* of the full information game. As mentioned in Definition 1, when the supplier is high-type, the capacity constraint is redundant. So, a high-type supplier solves: $\max_{q_{SH} \geq 0} \pi_H = (D - q_{SH} - q_B - C)\, q_{SH} + w\, Q$. That gives $q_{SH} = \frac{1}{2}(D - C - q_B)$ when $q_B \leq D - C$ and $q_{SH} = 0$, otherwise.

(B.3.2.1) $PE1$: Pooling Equilibrium with $\Omega \leq K_L$. At *Stage 2* of the game, the buyer's order quantity satisfies $\Omega \leq K_L$, so it does not reveal the supplier's type to use in step 3. The belief system of the buyer for this case is defined in Definition 2. 3.

(B.3.2.1.1) *Stage 3 – direct selling quantity of the buyer.* The buyer maximizes its expected profit: $\max_{0 \leq q_B \leq Q} \mathrm{E}[\Pi(q_B)] = [\lambda(D - q_B - q_{SH}) + (1-\lambda)(D - q_B - q_{SL})]q_B - w\, Q$ for each of the three cases of *Stage 4.* For example, when $q_{SL} = K_L - Q, q_{SH} = \frac{1}{2}(D - C - q_B)$, the buyer's problem becomes $\max_{0 \leq q_B \leq \min\,[Q, 2Q + D - 2K_L - C]} \mathrm{E}[\Pi(q_B)]$. Using KKT conditions, I derive two optimal direct selling quantities for this case. (Note that $q_B = 0$ cannot be optimal because $Q \geq 0$ and $q_B = 0$ results in $\Pi \leq 0$.) **1)** Non-binding: $q_B = \frac{1}{6}(C + 3D + 2Q - 2K_L)$ which requires $\left(\frac{7C}{3} < D < \frac{1}{3}\left(3C + 4\sqrt{2}C\right) \text{ and } \left(\left(\frac{1}{6}(C + 3D) < K_L < \frac{1}{10}(7D - 3C) \text{ and } \frac{1}{4}(C + 3D - 2K_L) \leq Q \leq K_L\right) \text{ or } \left(K_L > \frac{1}{10}(7D - 3C) \text{ and } \frac{1}{10}(7C - 3D + 10K) < Q \leq K_L\right)\right)\right)$; **2)** Binding: $q_B = Q$ which requires $\left(2C < D \leq \frac{7C}{3} \text{ and } \left(\left(0 < K_L \leq \frac{1}{2}(D - C) \text{ and } 0 \leq Q \leq K_L\right) \text{ or } \left(\frac{1}{2}(D - C) < K_L < D - C \text{ and } C - D + 2K_L < Q \leq K_L\right)\right)\right)$ or $\left(\frac{7C}{3} < D < \bar{D} \text{ and } \left(\left(0 < K_L \leq \frac{1}{2}(D - C) \text{ and } 0 \leq Q \leq K_L\right) \text{ or } \left(\frac{1}{2}(D - C) <\right.\right.\right.$

$K_L < \frac{1}{6}(C+3D)$ and $C - D + 2K_L < Q \leq K_L\Big)$ or $\Big(\frac{1}{6}(C+3D) < K_L < \frac{1}{10}(7D - 3C)$ and $C - D + 2K_L < Q < \frac{1}{4}(C+3D-2K_L)\Big)\Big)\Big)$. The calculations for other cases are similar and are not presented to avoid redundancy. The best response of the buyer when $K_L \leq \overline{K^F}$ and $2C < D < \overline{D}$ is presented in Table A4.

(B.3.2.1.2) *Stage 2 - order quantity of the buyer.* Given the supplier's capacity level and the market size, the buyer's optimal order quantity will fall in one of the cases presented in Table A4. In this case, $\Omega \leq K_L$, thus I have $\Omega = Q$. I only present calculations for Case 1 (i.e., $0 < K_L \leq \frac{1}{2}(D-C)$) to avoid redundancy. In this case, $\frac{d^2\,\Pi}{d\,\Omega^2} < 0$, so FOC gives $\Omega = \frac{1}{2}(C+3D-2K_L-4w)$. In order to satisfy the boundary conditions, $0 \leq \frac{1}{2}(C+3D-2K_L-4w) \leq K_L \Leftrightarrow \frac{1}{4}(C+3D-4K_L) \leq w < \frac{1}{4}(C+3D-2K_L)$. For $w \geq \frac{1}{4}(C+3D-2K_L)$, the lower bound $\Omega = 0$ and for $w < \frac{1}{4}(C+3D-4K_L)$ the upper bound $\Omega = K_L$ is binding. The best response of the buyer to different wholesale prices of the supplier in the pooling equilibrium with $\Omega \leq K_L$ is presented in Table A5. Note that the threshold $\overline{D} = C + \frac{4\sqrt{2}C}{3}$ is the market size such that the upper bound threshold of Case 2 and the lower bound threshold of Case 4 intersect, i.e., $\frac{1}{4}(2+\sqrt{2})(D-C) \geq \frac{1}{6}(C+3D)$.

(B.3.2.2) $PE2$: Pooling Equilibrium with $\Omega > K_L$: At step 2 of the game, the buyer's order quantity satisfies $\Omega > K_L$, so the buyer reveals the supplier's type to use in step 3. The belief system of the buyer is defined in Definition 2.4.

(B.3.2.2.1) *Stage 3 - selling quantity of the buyer.* At this stage, the buyer knows the supplier's type, so it maximizes the realized profit, anticipating the supplier's best response function at *Stage 4*. The steps of calculation are similar to *Stage 3* of the full information game (cf. Lemma 1). In this case, since $\Omega > K_L$, the low-type supplier has no leftover capacity to encroach, $q_{SL} = 0$. The best response function of the buyer when the supplier turns out to be low-type is $q_{BL} = Q$ if $Q \leq \frac{D}{2}$ and $q_{BL} = \frac{D}{2}$ if $Q > \frac{D}{2}$. When the supplier turns out to be the high-type, $\Omega = Q > K_L$ and $q_{SH} = \frac{1}{2}(D - C - q_B)$. Then, the best response function of the buyer when the supplier turns out to be high-type becomes $q_{BH} = Q$ if $Q \leq D - C$ and $q_{BH} = D - C$ if $Q > D - C$. The best response of the buyer is presented in Table A6.

(B.3.2.2.2) *Stage 2 - order quantity of the buyer.* Given the supplier's capacity level and the market size, the buyer's optimal order quantity will fall into one of the cases derived in *Stage 3*, Part B.3.2.2.1. The buyer derives the optimal order quantity by solving Equation 2.6. I only present calculations for the case when $0 < K_L \leq \frac{D}{2}, w \leq \frac{1}{4}(3c - D) \rightarrow \Omega = D - C$, if $\frac{1}{2}(3C - D) \leq w \leq \frac{1}{2}(C + D - 2K_L) \rightarrow \Omega = \frac{1}{2}(C + D - 2w)$, and if $\frac{1}{2}(C + D - 2K_L) \leq w \rightarrow \Omega = K_L + \epsilon$ where $\epsilon \rightarrow 0$. The best response of the buyer to different wholesale prices of the supplier in the pooling equilibrium with $\Omega > K_L$ is presented in Table A7.

(B.3.3) Optimal order quantity of the buyer in pooling equilibrium. At *Stage 2* of the game, the buyer compares its profit in parts B.3.2.1 and B.3.2.2 and derives the optimal pooling order quantity and the type of pooling by solving Equation 2.7. For example, when $0 < K_L \leq \frac{1}{2}(D - C)$, the two pooling equilibrium types overlap: **1)** $\boldsymbol{\Omega \leq K_L}$. In this case, $q_B = Q$. When $w \leq \frac{1}{4}(C + 3D - 4K_L) \rightarrow \Omega = K_L$; when $\frac{1}{4}(C + 3D - 4K_L) \leq w \leq \frac{1}{4}(C + 3D - 2K_L) \rightarrow \Omega = \frac{1}{2}\left(C + 3D - 2(K_L + 2w)\right)$; and when $\frac{1}{2}(C + 3D - 2K_L) \leq w \rightarrow \Omega = 0$; **2)** $\boldsymbol{\Omega > K_L}$: In this case $q_{BL} = q_{BH} = Q$ and when $w \leq \frac{1}{4}(3c - D) \rightarrow \Omega = D - C$; when $\frac{1}{2}(3C - D) \leq w \leq \frac{1}{2}(C + D - 2K_L) \rightarrow \Omega = \frac{1}{2}(C + D - 2w)$; and when $\frac{1}{2}(C + D - 2K_L) \leq w \rightarrow \Omega = K_L + \epsilon$ where $\epsilon \rightarrow 0$. I can show that when $w \leq \frac{1}{4}(3c - D) \rightarrow \Omega = D - C > K_L$; when $\frac{1}{2}(3C - D) \leq w \leq \frac{1}{2}(C + D - 2K_L) \rightarrow \Omega = \frac{1}{2}(C + D - 2w) > K_L$; when $\frac{1}{2}(C + D - 2K_L) < w \leq \frac{1}{4}(C + 3D - 4K_L) \rightarrow \Omega = K_L$; when $\frac{1}{4}(C + 3D - 4K_L) \leq w \leq \frac{1}{4}(C + 3D - 2K_L) \rightarrow \Omega = \frac{1}{2}\left(C + 3D - 2(K_L + 2w)\right) \leq K_L$; and when $\frac{1}{2}(C + 3D - 2K_L) \leq w \rightarrow \Omega = 0$. The calculations for other cases are similar and are not presented to avoid redundancy. Table A8 summarizes the best response function of the buyer to different wholesale prices of the supplier in the pooling equilibrium. Based on the best response function of the buyer at this stage, I define a threshold $\ddot{K} = Max[\frac{D}{2}, \frac{-C-\sqrt{2}C+D+2\sqrt{2}D}{4\sqrt{2}}]$ such that for $\ddot{K} < K_L < Max[\overline{K_F}, D - C]$ it is never optimal for the buyer to order all the capacity of the low-type supplier under $PE1$. In this range, when $w \leq \frac{1}{2}(C + D - 2K_L) + \frac{1}{2}\sqrt{\mathrm{D} - 2CK_L - 2DK_L + 2K_L^2}$, I have $\Omega > K_L$, $q_{BL} = \frac{D}{2}$ and $q_{BH} = Q$; when $\frac{1}{2}(C + D - 2K_L) + \frac{1}{2}\sqrt{\mathrm{D} - 2CK_L - 2DK_L + 2K_L^2}\frac{1}{2}(C + D - 2K_L) + \frac{1}{2}\sqrt{\mathrm{D} - 2CK_L - 2DK_L + 2K_L^2} < w < \frac{C+D}{2}$, I have $\Omega = \frac{C+D-2w}{2} < K_L$, $q_{BL} = q_{BH} = Q$; and when $\frac{C+D}{2} \leq w$, I have $\Omega = 0$.

(B.3.4) *Stage 1 – wholesale price of the supplier.* The optimal pooling wholesale price is the one that maximizes the profit of the low-type supplier (Section 4.2.2.3). The low-type supplier maximizes its pooling profit by solving $\max_{w\geq 0} \pi_L = (P - C)\, q_{SL} + w\, Q$ in different cases defined in Table A8. I only present calculations for Case 1 (i.e., $0 < K_L \leq \frac{1}{2}(D - C)$) to avoid redundancy. When $w \leq \frac{1}{4}(C + 3D - 4K_L)$, the profit function of the low-type supplier is $\pi_L = K_L w$, so $w = \frac{1}{4}(C + 3D - 4K_L)$ and $\pi_L = K_L(\frac{1}{4}(C + 3D - 4K_L))$. When $\frac{1}{4}(C + 3D - 4K_L) \leq w < \frac{1}{4}(C + 3D - 2K_L)$, using KKT conditions, only a lower bound solution is feasible when $2C < D < \overline{D}$. When $w \geq \frac{1}{4}(C + 3D - 2K_L)$, the profit function of the low-type supplier is $\pi_L = (D - C - K_L)K_L$, which is less than its profit when $w = \frac{1}{4}(C + 3D - 4K_L)$. So, the low-type supplier sets $w = \frac{1}{4}(C + 3D - 4K_L)$ and $\pi_L = K_L(\frac{1}{4}(C + 3D - 4K_L))$.

(B.3.5) I further refine the pooling equilibrium by considering the incentive compatibility constraint of the high-type supplier in Equation 2.8 Therefore, the pooling equilibrium is defined if and only if $\pi_H^{PE}(w^{PE}|Q^{PE}) \geq \max_{w\geq 0} \pi_H^{SE}(w \neq w^{PE}|Q_H^{SE}) \Leftrightarrow \frac{1}{6}(3C + D) - \frac{1}{6}\sqrt{D^2 - 7C^2 + 6CD} = \dot{K} < K_L < \overline{K^F}$.

(B.4) I further refine the final equilibrium using lexicographically maximum sequential equilibrium (LMSE). The optimal wholesale price is derived using Equation 2.9 in the paper. It can be shown that whenever the pooling equilibrium is defined, $\pi_L^{SE} < \pi_L^{PE}$. The final equilbrium and definition of the regions in the asymmetric information game are presented in Table A9. □

Proof of Corollary 2.1: Corollary 2.1 is the direct result of market structure in Proposition 2.1.

Proof of Proposition 2.2: a.i) From Lemma 2.1, $\pi_L^U = \frac{1}{12}(7C^2 + 3D^2 - 6CD)$. Then, knowing π_L^* from Table A9, I have $\pi_L^U \leq \pi_L^* \Leftrightarrow K_L \geq \widehat{K} = \frac{1}{8}(C + 3D) - \frac{\sqrt{-109C^2+114CD-21D^2}}{8\sqrt{3}}$. a.ii) Knowing π^U and π^F from Lemma 2.1 and π_L^* from Table A9, I can show that $\pi_L^* - \pi_L^U \leq \pi^F - \pi_L^U \Leftrightarrow K_L > \widehat{K}$. b.i) From Lemma 2.1, I have $\Pi^U = \frac{2C^2}{9}$. Then, knowing Π^* from Table A9, I have $\Pi^U \leq \Pi^* \Leftrightarrow K_L \geq \breve{K} = \frac{2}{3}\sqrt{2}C$. b.ii) Knowing Π^U and Π^F from Lemma 2.1 and Π^* from Table A9, I can show that $\Pi^* - \Pi^U \geq \Pi^F - \Pi^U \Leftrightarrow \breve{K} \leq K_L \leq \overline{K}$. □

Proof of Proposition 2.3: This proposition is an immediate result of comparing the profits of the low-type supplier and the buyer in the full information game in Table A3 with their respective profits in the asymmetric information settings in Table A9. □

Proof of Proposition 2.4: From the proof of Lemma 2.1, I know that the supplier withholds capacity in the full information setting when $\widetilde{K^F} \leq K < \overline{K^F}$. From Table A9, I know that the supplier withholds capacity in the asymmetric information setting when $\widetilde{K} \leq K_L \leq \overline{K}$. I can show that $\widetilde{K} \geq \widetilde{K^F}$ and $\overline{K} < \overline{K^F}$. □

Table A1: Sub-Game Equilibrium for Quantities Sold to the End-Customer Market (Full Information)

Case	K	Q	q_B	q_S
1	$0 < K \leq \frac{1}{2}(D-C)$	$0 \leq Q \leq K$	Q	$K-Q$
2	$\frac{1}{2}(D-C) < K \leq \frac{D}{2}$	$0 \leq Q \leq C+2K-D$	Q	$\frac{1}{2}(D-C-q_B)$
		$C+2K-D < Q \leq K$	Q	$K-Q$
3	$\frac{D}{2} < K \leq \frac{1}{3}(2D-C)$	$0 \leq Q \leq C+2K-D$	Q	$\frac{1}{2}(D-C-q_B)$
		$C+2K-D < Q < D-K$	Q	$K-Q$
		$D-K \leq Q \leq K$	$\frac{1}{2}(D+Q-K)$	$K-Q$
4	$\frac{1}{3}(2D-C) < K \leq 2D-C-2\sqrt{CD-C^2}$	$0 < Q < \frac{C+K}{3} + \frac{1}{3}\sqrt{C^2+2CK+6DK-3D^2-2K^2}$	Q	$\frac{1}{2}(D-C-q_B)$
		$\frac{C+K}{3} + \frac{1}{3}\sqrt{C^2+2CK+6DK-3D^2-2K^2} < Q \leq K$	$\frac{1}{2}(D+Q-K)$	$K-Q$
5	$2D-C-2\sqrt{CD-C^2} < K$	$0 \leq Q \leq D-C$	Q	$\frac{1}{2}(D-C-q_B)$
		$D-C \leq Q < K+2\sqrt{CD-C^2}-D$	$D-C$	$\frac{1}{2}(D-C-q_B)$
		$K+2\sqrt{CD-C^2}-D < Q \leq K$	$\frac{1}{2}(D+Q-K)$	$K-Q$

Table A2: Sub-Game Equilibrium for Order Quantity of the Buyer (Full Information)

Case	K	w	$\Omega = Q$	q_B	q_S
1	$0 < K \leq \frac{1}{2}(D-C)$	$w \leq D-K$	K	Q	$K-Q$
		$w > D-K$	0	Q	$K-Q$
2	$\frac{1}{2}(D-C) < K \leq \frac{D}{2}$	$w \leq \frac{1}{2}(C+D-2K) + \sqrt{DK-CK-K^2}$	K	Q	$K-Q$
		$\frac{1}{2}(C+D-2K) + \sqrt{DK-CK-K^2} < w \leq \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Q	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{C+D}{2}$	0	Q	$\frac{1}{2}(D-C-q_B)$
3	$\frac{D}{2} < K \leq \frac{4CD-2C^2-D^2}{6C-2D}$	$w \leq \frac{1}{2}(C+D-2K) + \frac{\sqrt{D^2-2CK-2DK+2K^2}}{\sqrt{2}}$	K	$\frac{1}{2}(D+Q-K)$	$K-Q$
		$\frac{1}{2}(C+D-2K) + \frac{\sqrt{D^2-2CK-2DK+2K^2}}{\sqrt{2}} < w \leq \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Q	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{C+D}{2}$	0	Q	$\frac{1}{2}(D-C-q_B)$
4	$K > \frac{4CD-2C^2-D^2}{6C-2D}$	$0 < w < \frac{1}{2}(3C-D)$	$D-C$	$D-C$	$\frac{1}{2}(D-C-q_B)$

		$\frac{1}{2}(3C-D) < w \leq \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Q	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{C+D}{2}$	0	Q	$\frac{1}{2}(D-C-q_B)$

Table A3: Equilibrium Solution and the Definition of the Regions in the Full Information Game

Region	K	w^F	Ω^F	q_B^F	q_S^F	Π^F	π^F
DN^F	$K \geq \overline{K^F}$	$\frac{1}{6}(3D-C)$	$\frac{2C}{3}$	Q	$\frac{1}{6}(3D-5C)$	$\frac{2C^2}{9}$	$\frac{1}{12}(7C^2-6CD+3D^2)$
SW^F	$\widetilde{K^F} \leq K < \overline{K^F}$	$\frac{1}{2}(C+D-2K) + \frac{\sqrt{D^2-2CK-2DK+2K}}{\sqrt{2}}$	K	$\frac{D}{2}$	0	$\frac{1}{4}(D^2-2K(C+D-2K+\sqrt{2}\sqrt{D^2-2DK+2K(K-C)})$	$K(\frac{1}{2}(C+D-2K)+\frac{\sqrt{D^2-2CK-2DK+2K^2}}{\sqrt{2}}$
SN^F	$\frac{1}{2}(D-C) \leq K < \min[\widetilde{K^F}, \overline{K^F}]$	$\frac{1}{2}(C+D-2K)+\sqrt{DK-CK-K^2}$	K	Q	0	$\frac{1}{2}K(D-C+2\sqrt{K(D-C-K)})$	$K(\frac{1}{2}(C+D-2K)+\sqrt{DK-CK-K^2})$
SN^F	$K < \frac{1}{2}(D-C)$	$D-K$	K	Q	0	0	$K(D-K)$

Table A4: Sub-Game Equilibrium for Quantities Sold to the End-Customer Market (PE1: $\boldsymbol{\Omega \leq K_L}$)

Case	K	Q	q_B	q_{SL}	q_{SH}
1	$0 < K_L \leq \frac{1}{2}(D-C)$	$0 \leq Q \leq K_L$	Q	$K_L - Q$	$\frac{1}{2}(D-C-q_B)$
2	$\frac{1}{2}(D-C) < K_L \leq \min[\overline{K^F}, \frac{1}{6}(C+3D)]$	$0 \leq Q \leq C+2K_L-D$	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
		$C+2K-D < Q \leq K_L$	Q	$K_L - Q$	$\frac{1}{2}(D-C-q_B)$
3	$\frac{1}{6}(C+3D) < K_L \leq \min[\overline{K^F}, \frac{1}{10}(7D-3C)]$	$0 \leq Q \leq C+2K_L-D$	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
		$C+2K_L-D < Q < \frac{1}{4}(C+3D-2K_L)$	Q	$K_L - Q$	$\frac{1}{2}(D-C-q_B)$
		$\frac{1}{4}(C+3D-2K_L) < Q \leq K_L$	$\frac{1}{6}(C+3D-2K_L+2Q)$	$K_L - Q$	$\frac{1}{2}(D-C-q_B)$
4	$\frac{1}{10}(7D-3C) < K_L \leq \overline{K^F}$	$0 \leq Q \leq \frac{1}{14}(5C+3D+2K_L)+\frac{1}{7}\sqrt{\frac{3}{2}}\sqrt{3C^2-2CD-9D^2+8CK_L+16DK_L-4K_L^2}$	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
		$\frac{1}{14}(5C+3D+2K_L)+\frac{1}{7}\sqrt{\frac{3}{2}}\sqrt{3C^2-2CD-9D^2+8CK_L+16DK_L-4K_L^2} < Q \leq K_L$	$\frac{1}{6}(C+3D-2K_L+2Q)$	$K_L - Q$	$\frac{1}{2}(D-C-q_B)$

Table A5: Sub-Game Equilibrium for Order Quantity of the Buyer (PE1: $\boldsymbol{\Omega \leq K_L}$)

Case	K	w	$\Omega = Q$	q_B	q_{SL}	q_{SH}
1	$0 < K_L \leq \frac{1}{2}(D-C)$	$w \leq \frac{1}{4}(C+3D-4K_L)$	K_L	Q	$K_L - Q$	$\frac{1}{2}(D-C-q_B)$

		$\frac{1}{4}(C+3D-4K_L) \le w < \frac{1}{4}(C+3D-2K_L)$	$\frac{1}{2}(C+3D-2K_L-4w)$	Q	K_L-Q	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{1}{4}(C+3D-2K_L)$	0	Q	K_L-Q	$\frac{1}{2}(D-C-q_B)$
2	$\frac{1}{2}(D-C) < K_L \le \frac{1}{4}(2+\sqrt{2})(D-C)$	$w \le \frac{1}{4}(C+3D-4K_L)$	K_L	Q	K_L-Q	$\frac{1}{2}(D-C-q_B)$
		$\frac{1}{4}(C+3D-4K_L) \le w < D-K_L-\frac{\sqrt{C^2-2CD+D^2+4CK_L-4DK_L+4K_L^2}}{2\sqrt{2}}$	$\frac{1}{2}(C+3D-2K_L-4w)$	Q	K_L-Q	$\frac{1}{2}(D-C-q_B)$
		$D-K_L-\frac{\sqrt{C^2-2CD+D^2+4CK_L-4DK_L+4K_L^2}}{2\sqrt{2}} < w \le \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{C+D}{2}$	0	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
3	$\frac{1}{4}(2+\sqrt{2})(D-C) < K_L \le \min[\overline{K^F}, \frac{1}{6}(C+3D)]$	$w \le \frac{1}{2}(C+D-2K_L)+\frac{\sqrt{-CK_L+DK_L-K_L^2}}{\sqrt{2}}$	K_L	Q	K_L-Q	$\frac{1}{2}(D-C-q_B)$
		$\frac{1}{2}(C+D-2K_L)+\frac{\sqrt{-CK_L+DK_L-K_L^2}}{\sqrt{2}} < w \le \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{C+D}{2}$	0	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
4	$\frac{1}{6}(C+3D) \le K_L < \overline{K^F}$	$w \le \frac{1}{2}(C+D-2K_L)+\frac{\sqrt{C^2+6CD+9D^2-24CK_L-24DK_L+24K_L^2}}{2\sqrt{6}}$	K_L	$\frac{1}{6}(C+3D-2K_L+2Q)$	K_L-Q	$\frac{1}{2}(D-C-q_B)$
		$\frac{1}{2}(C+D-2K_L)+\frac{\sqrt{C^2+6CD+9D^2-24CK_L-24DK_L+24K_L^2}}{2\sqrt{6}} < w < \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$
		$w > \frac{C+D}{2}$	0	Q	$\frac{1}{2}(D-C-q_B)$	$\frac{1}{2}(D-C-q_B)$

Table A6: Sub-Game Equilibrium for Quantities Sold to the End-Customer Market (PE2: $\boldsymbol{\Omega > K_L}$)

Case	Q	q_{BL}	q_{BH}	q_{SL}	q_{SH}
1	$0 < Q \le \frac{D}{2}$	Q	Q	0	$\frac{1}{2}(D-C-q_B)$
2	$\frac{D}{2} < Q \le D-C$	$\frac{D}{2}$	Q	0	$\frac{1}{2}(D-C-q_B)$
3	$Q > D-C$	$\frac{D}{2}$	$D-C$	0	0

Table A7: Sub-Game Equilibrium for Order Quantity of the Buyer (PE2: $\boldsymbol{\Omega > K_L}$)

Case	K	w	Ω	q_{BL}	q_{BH}	q_{SL}	q_{SH}
1	$0 < K_L \le \frac{D}{2}$	$w \le \frac{1}{2}(3C-D)$	$D-C$	K_L	Ω	0	0
		$\frac{1}{2}(3C-D) \le w \le \frac{1}{2}(C+D-2K_L)$	$\frac{1}{2}(C+D-2w)$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$
		$w > \frac{1}{4}(C+3D-2K_L)$	K_L	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$

		$w \le \frac{1}{2}(3C - D)$	$D - C$	$\frac{D}{2}$	Ω	0	0
2	$\frac{D}{2} < K_L \le D - C$	$\frac{1}{2}(3C - D) \le w \le \frac{1}{2}(C + D - 2K_L)$	$\frac{1}{2}(C + D - 2w)$	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D - C - q_{BH})$
		$w > \frac{1}{4}(C + 3D - 2K_L)$	K_L	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D - C - q_{BH})$
3	$K_L > D - C$	$w \ge 0$	K_L	$\frac{D}{2}$	$D - C$	0	0

Table A8: Sub-Game Equilibrium for Order Quantity of the Buyer in Pooling Equilibrium

Case	K	w	Ω	q_{BL}	q_{BH}	q_{SL}	q_{SH}	$\Omega > K_L$
1	$0 < K_L \le \frac{1}{2}(D-C)$	$w \le \frac{1}{2}(3C-D)$	$D-C$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(3C-D) \le w \le \frac{1}{2}(C+D-2K_L)$	$\frac{1}{2}(C+D-2w)$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L) < w < \frac{1}{4}(C+3D-4K_L)$	K_L	Ω	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	N
		$\frac{1}{4}(C+3D-4K_L) \le w < \frac{1}{4}(C+3D-2K_L)$	$\frac{1}{2}(C+3D-2K_L-4w)$	Ω	Ω	$K_L-\Omega$	$\frac{1}{2}(D-C-q_{BH})$	N
		$w > \frac{1}{4}(C+3D-2K_L)$	0	Ω	Ω	$K_L-\Omega$	$\frac{1}{2}(D-C-q_{BH})$	N
2	$\frac{1}{2}(D-C) < K_L \le Min(\frac{1}{4}(2+\sqrt{2})(D-C), \frac{D}{2})$	$w \le \frac{1}{2}(3C-D)$	$D-C$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(3C-D) \le w \le \frac{1}{2}(C+D-2K_L)$	$\frac{1}{2}(C+D-2w)$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L) < w < \frac{1}{4}(C+3D-4K_L)$	K_L	Ω	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	N
		$\frac{1}{4}(C+3D-4K_L) \le w < D-K_L-\frac{\sqrt{C^2-2CD+D^2+4CK_L-4DK_L+4K_L^2}}{2\sqrt{2}}$	$\frac{1}{2}(C+3D-2K_L-4w)$	Ω	Ω	$K_L-\Omega$	$\frac{1}{2}(D-C-q_{BH})$	N
		$D-K_L-\frac{\sqrt{C^2-2CD+D^2+4CK_L-4DK_L+4K_L^2}}{2\sqrt{2}} < w \le \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
		$w > \frac{C+D}{2}$	0	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
3	$\frac{1}{4}(2+\sqrt{2})(D-C) < K_L \le Min\,[\overline{K^F}, \frac{D}{2}]$	$w \le \frac{1}{2}(3C-D)$	$D-C$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(3C-D) \le w \le \frac{1}{2}(C+D-2K_L)$	$\frac{1}{2}(C+D-2w)$	K_L	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L) < w \le \frac{1}{2}(C+D-2K_L)+\frac{\sqrt{-CK_L+DK_L-K_L^2}}{\sqrt{2}}$	K_L	Ω	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	N
		$\frac{1}{2}(C+D-2K_L)+\frac{\sqrt{-CK_L+DK_L-K_L^2}}{\sqrt{2}} < w \le \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
		$w > \frac{C+D}{2}$	0	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N

4	$\frac{D}{2} < K_L \le \frac{-C-\sqrt{2}C+D+2\sqrt{2}D}{4\sqrt{2}}$	$w \le \frac{1}{2}(3C-D)$	$D-C$	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(3C-D) \le w \le \frac{1}{2}(C+D-2K_L)$	$\frac{1}{2}(C+D-2w)$	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L) < w \le \frac{1}{4}(c+3D-4K_L)+\frac{\sqrt{D^2-4DK_L+4K_L^2}}{2\sqrt{2}}$	K_L	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{4}(c+3D-4K_L)+\frac{\sqrt{D^2-4DK_L+4K_L^2}}{2\sqrt{2}} < w \le D-K_L-\frac{\sqrt{C^2-2CD+D^2+4CK_L-4DK_L+4K_L^2}}{2\sqrt{2}}$	$\frac{1}{2}(C+3D-2K_L-4w)$	Ω	Ω	$K_L-\Omega$	$\frac{1}{2}(D-C-q_{BH})$	N
		$D-K_L-\frac{\sqrt{C^2-2CD+D^2+4CK_L-4DK_L+4K_L^2}}{2\sqrt{2}} < w \le \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
		$w > \frac{C+D}{2}$	0	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
5	$Max(\frac{D}{2},\frac{-C-\sqrt{2}C+D+2\sqrt{2}D}{4\sqrt{2}}) \le K_L < Min(\overline{K^F}, D-C)$	$w \le \frac{1}{2}(3C-D)$	$D-C$	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(3C-D) \le w \le \frac{1}{2}(C+D-2K_L)$	$\frac{1}{2}(C+D-2w)$	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L) < w \le \frac{1}{2}(C+D-2K_L)+\frac{1}{2}\sqrt{D^2-2CK_L-2DK_L+2K_L^2}$	K_L	$\frac{D}{2}$	Ω	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L)+\frac{1}{2}\sqrt{D^2-2CK_L-2DK_L+2K_L^2} < w < \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
		$w > \frac{C+D}{2}$	0	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
6	$D-C \le K_L < \overline{K^F}$	$w < \frac{1}{2}(C+D-2K_L)+\frac{1}{2}\sqrt{-4C^2+4CD+D^2-4CK_L-4DK_L+4K_L^2}$	K_L	$\frac{D}{2}$	$D-C$	0	$\frac{1}{2}(D-C-q_{BH})$	Y
		$\frac{1}{2}(C+D-2K_L)+\frac{1}{2}\sqrt{-4C^2+4CD+D^2-4CK_L-4DK_L+4K_L^2} < w < \frac{C+D}{2}$	$\frac{1}{2}(C+D-2w)$	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N
		$w > \frac{C+D}{2}$	0	Ω	Ω	$\frac{1}{2}(D-C-q_{BL})$	$\frac{1}{2}(D-C-q_{BH})$	N

Table A9.1: Equilibrium Solution (Decision Variables)

<table>
<tr><th colspan="10">Pooling Equilibrium</th></tr>
<tr><th>Region</th><th>K_L</th><th colspan="2">w^*</th><th colspan="2">Ω^*</th><th>q_{BL}^*</th><th>q_{BH}^*</th><th>q_{SL}^*</th><th>q_{SH}^*</th></tr>
<tr><td>DN^*</td><td>$\bar{K} < K_L < \overline{K^F}$</td><td colspan="2">$\frac{1}{6}(3D - C)$</td><td colspan="2">$\frac{2C}{3}$</td><td colspan="2">$\frac{2C}{3}$</td><td>$\frac{1}{6}(3D - 5C)$</td><td>$\frac{1}{6}(3D - 5C)$</td></tr>
<tr><td>SW^*</td><td>$\ddot{K} \le K_L \le \bar{K}$</td><td colspan="2">$\frac{1}{2}(C + D - 2K_L) + \frac{1}{2}\sqrt{D^2 - 2CK_L - 2DK_L + 2K_L^2}$</td><td colspan="2">$K_L$</td><td>$\frac{D}{2}$</td><td>$K_L$</td><td>0</td><td>$\frac{1}{2}(D - C - K_L)$</td></tr>
<tr><td>SW^*</td><td>$\widetilde{K} < K_L < \ddot{K}$</td><td colspan="2">$\frac{1}{4}(c + 3D - 4K_L) + \frac{\sqrt{D^2 - 4DK_L + 4K_L^2}}{2\sqrt{2}}$</td><td colspan="2">$K_L$</td><td>$\frac{D}{2}$</td><td>$K_L$</td><td>0</td><td>$\frac{1}{2}(D - C - K_L)$</td></tr>
<tr><td>$\overline{DN}^*$</td><td>$\frac{1}{2}(5C - D) \le K_L \le \widetilde{K}$</td><td colspan="2">$\frac{1}{8}(7D - 3C - 6K_L)$</td><td colspan="2">$\frac{1}{4}(5C - D + 2K_L)$</td><td colspan="2">$\frac{1}{4}(5C - D + 2K_L)$</td><td>$\frac{1}{4}(D - 5C + 2K_L)$</td><td>$\frac{1}{8}(5D - 9C - 2K_L)$</td></tr>
<tr><td>SN^*</td><td>$(\frac{1}{4}(2 + \sqrt{2})(D - C) \le K_L \le \min[\bar{K}, \ddot{K}]$</td><td colspan="2">$\frac{1}{2}(C + D - 2K_L) + \frac{\sqrt{DK_L - CK_L - K_L^2}}{\sqrt{2}}$</td><td colspan="2">$K_L$</td><td colspan="2">$K_L$</td><td>0</td><td>$\frac{1}{2}(D - C - K_L)$</td></tr>
<tr><td>SN^*</td><td>$\frac{1}{6}(3C + D) - \frac{1}{6}\sqrt{D^2 - 7C^2 + 6CD} \le K_L \le \text{Min}[(\frac{1}{4}(2 + \sqrt{2})(D - C), \frac{1}{2}(5C - D), \ddot{K}]$</td><td colspan="2">$\frac{1}{4}(C + 3D - 4K_L)$</td><td colspan="2">$K_L$</td><td colspan="2">$K_L$</td><td>0</td><td>$\frac{1}{2}(D - C - K_L)$</td></tr>
<tr><th colspan="10">Separating Equilibrium</th></tr>
<tr><th>Region</th><th>K_L</th><th>w_L^*</th><th>w_H^*</th><th>$\Omega^*|w_L^*$</th><th>$\Omega^*|w_H^*$</th><th>q_{BL}^*</th><th>q_{BH}^*</th><th>q_{SL}^*</th><th>q_{SH}^*</th></tr>
<tr><td>SN^*</td><td>$\frac{C+D}{3} - \frac{1}{3}\sqrt{-3C^2 + 2CD + D^2} \le K_L \le \frac{1}{6}(3C + D) - \frac{1}{6}\sqrt{D^2 - 7C^2 + 6CD}$</td><td>$\frac{4C^2 - 6CK_L + 6DK_L - 3K_L^2}{12K_L}$</td><td>$\frac{1}{6}(3D - C)$</td><td>$K_L$</td><td>$\frac{2C}{3}$</td><td>$K_L$</td><td>$\frac{2C}{3}$</td><td>0</td><td>$\frac{1}{6}(3D - 5C)$</td></tr>
<tr><td>SN^*</td><td>$0 \le K_L \le \frac{C + D}{3} - \frac{1}{3}\sqrt{-3C^2 + 2CD + D^2}$</td><td>$D - K_L$</td><td>$\frac{1}{6}(3D - C)$</td><td>$K_L$</td><td>$\frac{2C}{3}$</td><td>$K_L$</td><td>$\frac{2C}{3}$</td><td>0</td><td>$\frac{1}{6}(3D - 5C)$</td></tr>
</table>

I define $\widehat{D}^A$ as a market size such that $\frac{D}{2} \ge K_{L1}$. Further, I define the capacity level $\bar{K}$ as $\bar{K} = \begin{cases} K_{L1} & \text{if } 2C < D \le \widehat{D}^A \\ K_{L2} & \text{if } \widehat{D}^A < D < \bar{D} \end{cases}$ such that whenever $\bar{K} < K_L \le \overline{K_F}$, the low-type supplier's optimal strategy is the same as the high-type supplier. Finally, I define a capacity level $\widetilde{K}$=$\begin{cases} \frac{D}{2} & \text{if } 2C < D \le \frac{5C}{2} \\ \widetilde{K}_1 & \text{if } \frac{5C}{2} < D < \bar{D} \end{cases}$ such that when $\widetilde{K} < K_L \le \bar{K}$, the buyer challenges the pooling equilibrium and withholds some of its order quantity if the supplier turns out to be low-type.

$\widetilde{K}_1 = \frac{-5C+D+2\sqrt{2}D+2\sqrt{25\sqrt{2}C^2-15\sqrt{2}CD+2(1+\sqrt{2})D^2}}{-2+8\sqrt{2}}$. K_{L1} is the capacity level that solves the following equation:

$$49C^4 - 84C^3D + 78C^2D^2 - 36CD^3 + 9D^4 + (-84C^3 - 12C^2D + 36CD^2 - 36D^3)K_L + (204C^2 - 72CD + 108D^2)K_L^2 + (-72C - 216D)K_L^3 + 216K_L^4 = 0$$

K_{L2} is the capacity level that solves the following equation:

$$49C^4 - 84C^3D + 78C^2D^2 - 36CD^3 + 9D^4 + (-84C^3 - 12C^2D + 36CD^2 - 36D^3)K_L + (204C^2 - 72CD + 72D^2)K_L^2 + (-72C - 72D)K_L^3 + 72K_L^4 = 0$$

Table A9.2: Equilibrium Solution (Profits)

<table>
<tr><th colspan="5">Pooling Equilibrium</th></tr>
<tr><th>Region</th><th colspan="2">$\boldsymbol{E}[\Pi^*]$</th><th>$\boldsymbol{\pi_L^*}$</th><th>$\boldsymbol{\pi_H^*}$</th></tr>
<tr><td>DN^*</td><td colspan="2">$\frac{2C^2}{9}$</td><td>$\frac{1}{12}(7C^2 + 3D^2 - 6CD)$</td><td>$\frac{1}{12}(7C^2 + 3D^2 - 6CD)$</td></tr>
<tr><td>SW^*</td><td colspan="2">$\frac{1}{8}(D^2 - 2DK_L + 2K_L(-C + 3K_L - 2\sqrt{D^2 - 2DK_L + 2K_L(-C + K_L)}))$</td><td>$(\frac{1}{2}(C + D - 2K_L) + \frac{1}{2}\sqrt{D^2 - 2CK_L - 2DK_L + 2K_L^2})K_L$</td><td>$\frac{1}{4}(C^2 - 2CD + D^2 + 4CK_L - 3K_L^2 + 2K_L\sqrt{D^2 - 2DK_L + 2K_L(-C + K_L)})$</td></tr>
<tr><td>SW^*</td><td colspan="2">$\frac{1}{8}(D^2 - 4DK_L - 2\sqrt{2}\sqrt{(D - 2K_L)^2}K_L + 6K_L^2)$</td><td>$(\frac{1}{4}(c + 3D - 4K_L) + \frac{\sqrt{D^2 - 4DK_L + 4K_L^2}}{2\sqrt{2}})K_L$</td><td>$\frac{1}{4}(C^2 - 2CD + D^2 + 3CK_L + DK_L + \sqrt{2}\sqrt{(D - 2K_L)^2}K_L - 3K_L^2)$</td></tr>
<tr><td>$\overline{DN}^*$</td><td colspan="2">$\frac{1}{64}(D - 5C - 2K_L)^2$</td><td>$\frac{1}{32}(25C^2 + D^2 + 28DK_L - 28K_L^2 - 2c(5D + 6K_L))$</td><td>$\frac{1}{64}(51C^2 + 11D^2 + 20DK_L - 20K_L^2 - 2C(7D + 18K_L))$</td></tr>
<tr><td>SN^*</td><td colspan="2">$\frac{1}{4}K_L(D - C + K_L - 2\sqrt{2}\sqrt{K_L(D - C - K_L)})$</td><td>$K_L(\frac{1}{2}(C + D - 2K_L) + \frac{\sqrt{-CK_L+DK_L-K_L^2}}{\sqrt{2}})$</td><td>$\frac{1}{4}(C^2 - 2CD + D^2 + 4CK_L - 3K_L^2 + 2\sqrt{2}K_L\sqrt{K_L(D - C - K_L)})$</td></tr>
<tr><td>SN^*</td><td colspan="2">$\frac{K_L^2}{4}$</td><td>$\frac{1}{4}(C + 3D - 4K_L)K_L$</td><td>$\frac{1}{4}(C^2 - 2CD + D^2 + 3CK_L + DK_L - 3K_L^2)$</td></tr>
<tr><th colspan="5">Separating Equilibrium</th></tr>
<tr><th>Region</th><th>$\Pi^*|w_H^*$</th><th>$\Pi^*|w_L^*$</th><th>$\boldsymbol{\pi_L^*}$</th><th>$\boldsymbol{\pi_H^*}$</th></tr>
<tr><td>SN^*</td><td>$\frac{2C^2}{9}$</td><td>$\frac{1}{12}(6CK_L - 4C^2 + 6DK_L - 9K_L^2)$</td><td>$K_L(\frac{4C^2 - 6CK_L + 6DK_L - 3K_L^2}{12K_L})$</td><td>$\frac{1}{12}(7C^2 + 3D^2 - 6CD)$</td></tr>
<tr><td>SN^*</td><td>$\frac{2C^2}{9}$</td><td>0</td><td>$K_L(D - K_L)$</td><td>$\frac{1}{12}(7C^2 + 3D^2 - 6CD)$</td></tr>
</table>

Appendix-B:

In the proof, to reduce the number of parameters while preserving the ranking for posterior beliefs ($\lambda_i^{3'} < \lambda_i^{1'} < \lambda_i^{2'} < \lambda_i^0 < \lambda_i^2 < \lambda_i^1 < \lambda_i^3$), I assume that $\lambda_i^0 = \frac{1}{2}$, $\lambda_i^{1'} = 1 - \lambda_i^1$, $\lambda_i^{2'} = 1 - \lambda_i^2$ and $\lambda_i^{3'} = 1 - \lambda_i^3$.

Proof of Lemma 4.1: If the seller uses the same price for fake and genuine products, the demand for the product i would be $D_i = v(\lambda_i + (1 - \lambda_i)x_i) - P_i$ and the seller maximizes $\Pi_{\mathrm{i}}^C = (1 - \theta)(\lambda_i + (1 - \lambda_i)x_i - P_i)P_i$. In this case, the seller derives the optimal selling price using the first order condition as $P_i^C = \frac{1}{2}(\lambda_i + x_i - \lambda_i x_i)$. If the seller uses a lower separating price for their fake product, the demand for the product i would be $D_i v x_i - P_i$ and the seller maximizes $\Pi_{\mathrm{i}}^S = (1 - \theta)(x_i - P_i)P_i$. Using the first order condition, the seller derives the optimal selling price as $P_i^S = \frac{x_i}{2}$. For the genuine product, the seller always sets $P_i^C = \frac{1}{2}(\lambda_i + x_i - \lambda_i x_i)$. If the seller sells a fake product, they can choose their pricing strategy by comparing Π_{i}^C and Π_{i}^S. One can show that $\Pi_{\mathrm{i}}^C > \Pi_{\mathrm{i}}^S$ for the range of my parameters.

Proof of Proposition 4.1: At this stage, the platform has three possible strategies: providing no tool; providing the basic tool, which only analyzes product reviews; or providing the advanced tool, which analyzes both product and seller reviews. I use the superscript $j \in \{N, B, A\}$ to denote the platform's strategy where $j = N$ refers to the case where the platform provides no tool, and $j = B$ ($j = A$) refers to the basic (advanced) tool. There are three possible cases:

1- If the seller sells two fake products and the platform provides no tool, the customer's posterior belief regarding the authenticity of the products is $\lambda_i = \lambda_i^0$ and the platform's profit would be $\pi^N = \theta \sum_i \left(\lambda_i^0 + (1 - \lambda_i^0)x_i - \frac{1}{2}(\lambda_i^0 + x_i - \lambda_i^0 x_i)\right)\frac{1}{2}(\lambda_i^0 + x_i - \lambda_i^0 x_i)$. If the platform provides the basic tool, the posterior belief is $\lambda_i = \lambda_i^{1'}$ and the platform's profit would be $\pi^B = \theta \sum_i \left(\lambda_i^{1'} + \left(1 - \lambda_i^{1'}\right)x_i - \frac{1}{2}(\lambda_i^{1'} + x_i - \lambda_i^{1'} x_i)\right)\frac{1}{2}(\lambda_i^{1'} + x_i - \lambda_i^{1'} x_i)$. Finally, if the platform provides the advanced tool, the posterior belief is $\lambda_i =$

$\lambda_i^{3'}$ and the platform's profit would be $\pi^A = \theta \sum_i \left(\lambda_i^{3'} + \left(1 - \lambda_i^{3'}\right)x_i - \frac{1}{2}(\lambda_i^{3'} + x_i - \lambda_i^{3'}x_i)\right)\frac{1}{2}(\lambda_i^{3'} + x_i - \lambda_i^{3'}x_i)$. The platform decides on its optimal strategy by comparing π^N, π^B and π^A. One can show that in this case, $\pi^N > \pi^B > \pi^A$, so the seller provides no tool.

2- If the seller sells two genuine products and the platform provides no tool, the customer's posterior belief regarding the authenticity of the products is $\lambda_i = \lambda_i^0$ and the platform's profit would be $\pi^N = \theta \sum_i \left(\lambda_i^0 + (1 - \lambda_i^0)x_i - \frac{1}{2}(\lambda_i^0 + x_i - \lambda_i^0 x_i)\right)\frac{1}{2}(\lambda_i^0 + x_i - \lambda_i^0 x_i)$. If the platform provides the basic tool, the posterior belief is $\lambda_i = \lambda_i^1$ and the platform's profit would be $\pi^B = \theta \sum_i \left(\lambda_i^1 + (1 - \lambda_i^1)x_i - \frac{1}{2}(\lambda_i^1 + x_i - \lambda_i^1 x_i)\right)\frac{1}{2}(\lambda_i^1 + x_i - \lambda_i^1 x_i)$. Finally, if the platform provides the advanced tool, the posterior belief is $\lambda_i = \lambda_i^{3'}$ and the platform's profit would be $\pi^A = \theta \sum_i \left(\lambda_i^3 + (1 - \lambda_i^3)x_i - \frac{1}{2}(\lambda_i^3 + x_i - \lambda_i^3 x_i)\right)\frac{1}{2}(\lambda_i^3 + x_i - \lambda_i^3 x_i)$. The platform decides on its optimal strategy by comparing π^N, π^B and π^A. One can show that in this case, $\pi^A > \pi^B > \pi^N$, so the seller provides the advanced tool.

3- If the seller sells one real (α) and one fake (β) product and the platform provides no tool, the customer's posterior belief regarding the authenticity of the products is $\lambda_i = \lambda_i^0$ and the platform's profit would be $\pi^N = \theta \sum_i \left(\lambda_i^0 + (1 - \lambda_i^0)x_i - \frac{1}{2}(\lambda_i^0 + x_i - \lambda_i^0 x_i)\right)\frac{1}{2}(\lambda_i^0 + x_i - \lambda_i^0 x_i)$. If the platform provides the basic tool, the posterior belief is $\lambda_\alpha = \lambda_\alpha^1$ and $\lambda_\beta = \lambda_\beta^{1'}$ and the platform's profit would be $\pi^B = \theta\left(\left(\lambda_\alpha^1 + (1 - \lambda_\alpha^1)x_\alpha - \frac{1}{2}(\lambda_\alpha^1 + x_\alpha - \lambda_\alpha^1 x_\alpha)\right)\frac{1}{2}(\lambda_\alpha^1 + x_\alpha - \lambda_\alpha^1 x_i) + \left(\lambda_\beta^{1'} + \left(1 - \lambda_\beta^{1'}\right)x_\beta - \frac{1}{2}(\lambda_\beta^{1'} + x_\beta - \lambda_\beta^{1'} x_\alpha)\right)\frac{1}{2}(\lambda_\beta^{1'} + x_\beta - \lambda_\beta^{1'} x_\beta)\right)$. Finally, if the platform provides the advanced tool, the posterior belief is $\lambda_\alpha = \lambda_\alpha^2$ and $\lambda_\beta = \lambda_\beta^{2'}$ and the platform's profit would be $\pi^A = \theta\left(\left(\lambda_\alpha^2 + (1 - \lambda_\alpha^2)x_\alpha - \frac{1}{2}(\lambda_\alpha^2 + x_\alpha - \lambda_\alpha^2 x_\alpha)\right)\frac{1}{2}(\lambda_\alpha^2 + x_\alpha - \lambda_\alpha^2 x_i) + \left(\lambda_\beta^{2'} + (1 -\right.\right.$

$\lambda_{\beta}^{2\prime})x_{\beta} - \frac{1}{2}(\lambda_{\beta}^{2} + x_{\beta} - \lambda_{\beta}^{2}x_{\alpha})\Big)\frac{1}{2}(\lambda_{\beta}^{2\prime} + x_{\beta} - \lambda_{\beta}^{1^2}x_{\beta})\Big)$. The platform decides on its optimal strategy by comparing π^N, π^B and π^A. One can show that it is never the case that ($\pi^A > \pi^N$ and $\pi^A > \pi^B$) and $\pi^N > \pi^B$ if and only if $x_{\alpha} > 1 - x_{\beta} + x_{\beta}{}^2 = \bar{x}$.

Proof of Lemma 4.2: In this stage, the platform decides whether to provide no tool, the basic tool, or the advanced tool. Based on the seller's type and pricing strategy there are several scenarios. Here, I present the proof for the most complicated case; the proof for all other cases can be achieved in the same way.

If the seller sells one real (α) and one fake (β) product and uses the separating price on the fake product, then the platform's profit would be $\pi^N = (\theta)(\lambda_{\alpha}^{0} + (1 - \lambda_{\alpha}^{0})x_{\alpha} - P_{\alpha})P_{\alpha} + (x_{\beta} - P_{\beta})P_{\beta}$ if it provides no tool, $\pi^B = (\theta)(\lambda_{\alpha}^{1} + (1 - \lambda_{\alpha}^{1})x_{\alpha} - P_{\alpha})P_{\alpha} + (x_{\beta} - P_{\beta})P_{\beta}$ if its provides the basic tool, and $\pi^A = (\theta)(\lambda_{\alpha}^{2} + (1 - \lambda_{\alpha}^{2})x_{\alpha} - P_{\alpha})P_{\alpha} + (x_{\beta} - P_{\beta})P_{\beta}$ if its provides the advanced tool. The platform compares its profit under these three strategies. One can show that for any P_i, π^B is the dominant strategy.

If the seller sells one real (α) and one fake (β) product and uses the pooling price on the fake product, then the platform's profit would be $\pi^N = (\theta)\sum_i (\lambda_i^0 + (1 - \lambda_i^0)x_i - P_i)P_i$ if it provides no tool, $\pi^B = (\theta)(\lambda_{\alpha}^{1} + (1 - \lambda_{\alpha}^{1})x_{\alpha} - P_{\alpha})P_{\alpha} + \left(\lambda_{\beta}^{1\prime} + \left(1 - \lambda_{\beta}^{1\prime}\right)x_{\beta} - P_{\beta}\right)P_{\beta}$ if its provides the basic tool, and $\pi^A = (\theta)(\lambda_{\alpha}^{2} + (1 - \lambda_{\alpha}^{2})x_{\alpha} - P_{\alpha})P_{\alpha} + \left(\lambda_{\beta}^{2\prime} + \left(1 - \lambda_{\beta}^{2\prime}\right)x_{\beta} - P_{\beta}\right)P_{\beta}$ if its provides the advanced tool. The platform compares its profit under these three strategies. One can show that for any P_i, π^A is always dominated by the other two strategies and $\pi^B > \pi^N$ if and only if $\frac{P_{\alpha}}{P_{\beta}} > \frac{1 - x_{\beta}}{1 - x_{\alpha}} = \bar{\gamma}$.

Proof for other cases follows the same logic.

Proof of Proposition 4.2: Proof of proposition 2 is the direct consequence of solving Equations 4.3, 4.4 and 4.5 in the body.

Curriculum Vitae

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