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COMMENTARY PAPER

Promises and potential pitfalls of a 'cognitive neuroscience of mathematics learning'

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Abstract The present commentary discusses the papers of the special issue on 'cognitive neuroscience and mathematics learning' with respect to methodological and theoretical constraints of using neuroscientific methods to study educationally relevant processes associated with mathematics learning. A special focus is laid on the relevance of subject populations, methodological limitations of current neuroimaging methods and theoretical questions concerning the relationship between the well-studied neural correlates of numerical magnitude processing and the less-investigated neural processes underlying higher level mathematical skills, such as algebraic reasoning.

1 Introduction

Following the invention of novel methods to non-invasively measure human brain structure and function, the last 20 years have seen an unprecedented surge in the study of how the human brain enables complex cognitive functions such as language, reasoning, reading and mathematics. In view of these advances, the burgeoning field of cognitive neuroscience has recently started making transdisciplinary links with other fields, such as economics and education. As part of this effort to connect cognitive neuroscience with other fields of inquiry and application, growing attention has been paid to building bridges between, on the one hand, the cognitive neuroscience of numeracy and mathematics and the empirical study of mathematics learning and education, on the other (De Smedt et al. 2010).

The study of the brain mechanisms involved in numerical and mathematical processing has provided significant insights into the neural processes that underlie the ability to represent and process numerical magnitude (the total number of items in a set). Convergent evidence from neuropsychology, single-cell neurophysiology and functional neuroimaging has identified the intraparietal sulcus (IPS) of the brain as a critical substrate for the representation of numerical magnitude (Nieder and Dehaene 2009).

While impressive progress has been made in understanding the brain mechanisms underlying basic numerical processes, comparatively little is known about the neural basis of higher level mathematical skills that are fundamental to mathematics learning in the context of formal schooling (see also the review article by Menon 2010). It was this apparent knowledge gap that provided the motivation for this special issue of ZDM, which presents both empirical and theoretical contributions that seek to enhance our understanding of the cerebral mechanisms that enable higher level mathematical learning.

A distinctive feature of the contributions presented in this special issue is that all of them study the neural processes associated with mathematical processing with a high degree of ecological validity. That is, experimental paradigms were applied that most closely resemble how a particular task would be presented in the mathematics classroom and that elicit the cognitive processes thought to occur when learning in school takes place. This approach is innovative as, in contrast to educational researchers who have a long tradition of conducting ecologically valid

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research, experimental psychologists and cognitive neuroscientists have placed relatively more emphasis on the control of potentially confounding variables, often at the expense of sacrificing ecological validity.

While the pursuit of high ecological validity in cognitive neuroscience investigations is important, especially given the efforts to connect cognitive neuroscience with education, it also poses considerable challenges to the researchers. The contributions in this special issue illustrate several of these challenges, and how to overcome them, and highlight the strengths and limitations of different experimental designs used to gain insights into the cognitive neuroscience of mathematics learning.

In what follows we provide an overview of, what are in our view, the key challenges that face investigators seeking to understand the brain mechanisms underlying mathematics learning. We refer to pathways through which some of these challenges may be overcome and outline a few of the many open questions and future challenges. Furthermore, we discuss how an interdisciplinary, collaborative research, exemplified in the studies of the present special issue, provides promising first steps toward a cognitive neuroscience of mathematics learning.

2 Are we testing the right populations of participants?

The first challenge in cognitive neuroscience studies on mathematical cognition lies in the selection of a study population with a high ecological validity. The straightforward approach of investigating school-related mathematics learning in students of the age or grade in which the learning processes of interest take place can often not be pursued due to restrictions of the applied neuroimaging method. This is particularly true for the widely used functional magnetic resonance imaging (fMRI). In most of the fMRI studies on mathematical cognition, as is the case for the studies presented in the present special issue, adults rather than school children are tested, even if this entails a lower ecological validity of the obtained results. There are two main reasons for this compromise. First, since fMRI data acquisition is severely prone to motion artifacts, it is essential that the participants keep their heads still over a time period of several minutes. Children often have great difficulty exerting control over their head motion. Second, although the ethical considerations are largely the same for adults and children, many cognitive neuroscientists have experienced that ethics review committees are sometimes more concerned about fMRI studies of the latter population, thus further complicating cognitive neuroscience investigations with younger populations.

For some research questions related to mathematics learning, the investigation of adult populations appears legitimate. For example, in comparing the neural correlates of schematic and symbolic strategies for solving algebraic word problems, Lee et al. (2010) deliberately selected young adults who were equally proficient in applying both strategies. In this vein, they avoided the potential problem that adolescents may exhibit different proficiency levels. Such differences in task performance may have arisen if younger participants had been tested, as the symbolic strategy was introduced in school more recently than the schematic strategy. Likewise, starting from behavioral experiments in adolescents and adults showing similar performance patterns, Stavy and Babai (2010) studied the brain correlates of intuitive interference in geometry only in adults. However, in any case, the generalizability of neuroimaging findings from adult samples to children's or adolescents' school learning needs to be scrutinized against the background of developmental changes in the functional (and structural) architecture of the brain (Giedd et al. 1999). There is a growing body of evidence for dynamic age- and competence-related changes in brain activation patterns during numerical and mathematical thinking (e.g., Ansari et al. 2005; Rivera et al. 2005). For example, Rivera et al. (2005) investigated brain activity in a sample of 8–19-year olds and provided compelling evidence of an increasing functional specialization of parietal brain areas for arithmetic problem solving. This result presumably reflects the developmental transition from effortful procedural strategies (such as counting) to the automatic retrieval of facts in mental arithmetic (see also Grabner, Ansari, et al. 2009). Given these dramatic changes in the neural correlates of higher level cognitive functions over the course of development, observations made by studying the fully developed brain of adults cannot be used to characterize the neural correlates of these functions in children. Furthermore, when adults are studied, there are often problems related to the representativeness of the sample. Most frequently, adult samples consist of undergraduate students from middle-class socio-economic (SES) backgrounds and are thus hardly representative of the general adult population, especially given the recent research revealing that brain mechanisms underlying cognitive processes are modulated by factors such as SES (Raizada and Kishiyama 2010).

Administering neuroimaging methods other than fMRI is another way to overcome age-related restrictions. This special issue also comprises investigations using transcranial near-infrared spectroscopy (NIRS) and pupillometry (Bornemann et al. 2010; Landgraf et al. 2010; Obersteiner et al. 2010), which are both easily applicable in younger age groups. NIRS, on the one hand, measures cortical activity by detecting activation-related changes in the absorption and reflection of near-infrared light that is emitted into the scalp. This neuroimaging technique is considered to be a very promising candidate for future educational neuroscience research, as it is comparably insensitive to motion artifacts, ethically quite unobjectionable and, since it is non-stationary, can even be applied in the school or classroom. These advantages, however, are complemented by a poor spatial resolution (in the range of cm) and the fact that only surface areas of the cortex can be measured, thus not providing insights into the subcortical correlates of mathematical learning. Moreover, NIRS measurements are restricted to the investigation of regions of interest, since there are currently no devices that cover the entire scalp, thus not allowing for the recording of the responses from multiple and spatially distal brain regions within the same session. In their paper published as part of the present special issue, Obersteiner et al. (2010) applied NIRS in a sample of fourth and eighth graders and, against the background of their results, provided an elaborate discussion on the potential and limitations of this method. This discussion provides an important roadmap of the challenges that this research methodology poses for investigators interested in pursuing NIRS as a way of gaining greater insights into the neural mechanisms underlying mathematics learning.

Pupillometry, on the other hand, does not measure brain activity, but instead provides a measure of pupil dilation, which may have the potential to index neuronal activity related to cognitive resource allocation in mathematical cognition. The contributions by Bornemann et al. (2010) as well as Landgraf et al. (2010) provide first evidence of its sensitivity to mathematical processes and the link to regional cortical activity. This inexpensive methodology may also help to provide some biological constraints on existing cognitive accounts of mathematics learning. However, for researchers who are interested in understanding the brain networks involved in particular aspects of mathematics learning, this method, on its own, may not be appropriate.

In general, the relative advantages and disadvantages of the methods used in the papers of this special issue highlight the importance of considering methodological approaches that include the application of multiple methods to, for example, harness the spatial resolution of fMRI, while at the same time benefitting from the temporal resolution of EEG.

3 What experimental design can be applied?

Another critical challenge in investigating the brain correlates of higher order mathematics learning concerns the design of the experimental task. First, the signals measured using functional neuroimaging methods contain a large error of measurement. Consequently, several trials (problems) of each task condition need to be averaged to reduce this measurement error and to obtain reliable data. This further necessitates that the duration of each trial is comparably short so that many trials can be presented in one test session. Most of the current neuroimaging studies include at least about 20–30 trials per task conditions; if a high error rate is expected, even more (see, e.g., Stavy and Babai 2010). The constraint that each problem needs to be solved within a few seconds has a strong impact on the task complexity. Computing single-digit and double-digit arithmetic problems fall within the desired complexity range, but when algebraic word problems should be solved, different representations of mathematical functions be compared, or features of geometric objects be evaluated, the task demands need to be reduced. The contributions by Lee et al. (2010), Stavy and Babai (2010) and Thomas et al. (2010) provide good examples for feasible levels of task complexity.

Second, the ways of responding to the task are limited, either due to the requirement of holding still or due to technical constraints such as loud fMRI scanner noise, making the recording of verbal responses difficult. Therefore, in most mathematical neuroimaging studies, participants respond by button press, verifying a given answer (e.g., Bornemann, et al. 2010; Preusse et al. 2010) or choosing between different answer options (e.g., Landgraf, et al. 2010; Lee, et al. 2010; Obersteiner, et al. 2010). An objection that is frequently raised concerning the use of verification tasks is that they may engage different cognitive processes compared to tasks in which the answer has to be actively produced and thereby use a response modality that is not ecologically valid. This point is elaborated and illustrated in the contribution by Menon (2010). But, independently of whether one or more response options are presented, good distractor items (incorrect solutions) need to be created that avoid the use of shortcut strategies such as focusing on the unit position in verifying multiplication equations.

Third, mathematical tasks with higher ecological validity typically involve multiple cognitive processes that occur at various points within the processing stream, but cannot easily be dissociated from one another using currently available neuroimaging methods. In other words, the brain imaging method captures brain activation related to all aspects of processing a particular problem, such as cognitive and emotional processing of the stimuli, the preparation and execution of a motor response, etc. The more processes are involved, the more difficult does it become to disentangle and associate them with specific brain areas. Very frequently, researchers will reveal a large-scale network of activation associated with their task that could be attributable to a multitude of cognitive processes (see, e.g., Thomas, et al. 2010). In fMRI, a wellestablished way to overcome this problem is the use of a subtraction design. This means that control conditions are added that differ from the experimental conditions only in one cognitive process. The subtraction of the activation pattern during the control condition from that during the experimental condition yields the activation that is related to the cognitive process of interest. For instance, Zago et al. (2010) has successfully applied this subtraction procedure to studying the brain mechanisms of counting small and large numerosities. However, the subtraction logic relies on the assumption that processes within a task are additive and that variables of interest can be isolated by subtracting out activation related to other task-related processes (also referred to as the 'pure insertion' assumption). This assumption has been heavily criticized, and parametric variations or adaptation designs have been put forward as solutions. In the first, different levels of complexity of a process of interest are employed to reveal brain regions, the activity of which is correlated with this complexity and hence is likely to be critical for this process. In the latter, a particular stimulus variable is repeated while other attributes vary, and areas in which activity decreases as a function of repetition are measured with the assumption that repetition-related suppression of activation is related to the representation of the stimulus variable that is being repeated.

4 How is basic number processing related to higher order mathematical skills?

The papers in this special issue illustrate the multiple levels of description at which research into the cognitive neuroscience of mathematics learning is currently being undertaken. The issues tackled by researchers in the current issue range from the study of the neural correlates of large numerosity counting (Zago, et al. 2010) to the role of graphical and algebraic representations in student's understanding of functions (Thomas, et al. 2010). One of the key issues that researchers face on the cognitive neuroscience of mathematics learning is finding ways to bridge these different levels of description and to understand how basic numerical and mathematical abilities constrain the acquisition of higher level skills.

The review paper by Butterworth and Laurillard (2010) strongly argues for a link between the processing of numerosity (sets of items) and the development of arithmetic skills and posits that developmental dyscalculia is caused by a low-level impairment in numerosity processing, which impedes the acquisition of arithmetic skills and thus leads to mathematical difficulties in the classroom. While there is behavioral evidence to support this link between basic numerosity processing and arithmetic, understanding of the neural mechanisms and dynamic changes within them that allow for the utilization of early developing numerosity representations in the learning of arithmetic is currently lacking.

In this context, it is important to point out that finding activation in brain areas during the processing of higher level mathematical tasks that have previously been associated with basic number processing does not imply that the higher level mathematical task engages the same neurocognitive processes that were found to be correlated with the basic processes. To put this more concretely, activation of the IPS during both algebraic processing and dot counting does not imply equivalence of processing. It is possible that separate populations of neurons within the same regions of the IPS, subserving completely different processes, lead to the activation of the same regions. Generally, it is problematic to infer functions from brain activations by referring to previous findings (an interpretation approach referred to as 'reverse inference'; for a discussion, see Poldrack 2006).

One avenue for pursuing a better understanding of how different levels of numerical and mathematical learning are linked to one another in the brain is to study the overlap of their neuronal correlates within the same subjects (for example, see Simon et al. 2002). While potentially fruitful, the demonstration of overlap (or lack thereof) of different levels of numerical and mathematical processing and learning does not provide constraints on the mechanisms that bridge different levels of numerical and mathematical learning. Only through studies that directly assess learning, such as those that investigate how brain mechanisms underlying calculation changes as a function of learning (Delazer et al. 2003; Grabner, Ischebeck, et al. 2009; Ischebeck et al. 2006), such insights can be obtained. Training studies in adults as well as longitudinal studies with children that track, for example, the transition from mathematics instruction focused on whole numbers to the teaching of fractions are required.

5 Conclusions and future directions

The contributions of this special issue do not only exemplify current neuroscientific approaches to elucidate brain mechanisms supporting mathematics learning, but also illustrate the tension between educational relevance and methodological constraints imposed by current cognitive neuroimaging methods. Neuroimaging methods open up a new level of analysis, and their application has the potential to provide insights into cognitive processes that cannot be obtained by behavioral studies alone. However, similar to every other research method, the full potential of neuroimaging techniques can only be tapped if their requirements and constraints are carefully considered. Striving for a high ecological validity, sometimes methodological compromises are made that result in unreliable or ambiguous data.

The resolution of this tension requires the collaboration between, on the one hand, educators and educational researchers and, on the other, cognitive neuroscientists (Ansari and Coch 2006; De Smedt et al. 2010). Such interdisciplinary collaborations will enable educational researchers to alert cognitive neuroscientists to educationally relevant research questions and paradigms that tap into cognitive processes most closely resembling those that students engage in mathematics learning in school. Furthermore, educational researchers possess invaluable knowledge of the extraneous variables that influence mathematics learning, such as SES, emotional processes and the role of different ways of instruction and problem presentation. These contributions will enrich cognitive neuroscience research on mathematics learning through the formulation of novel questions, which address problem domains that have previously not been part of neuroscientific investigations. Furthermore, educational researchers often have far more experience about the sequence of learning and the interrelationships between more 'basic' and more 'higher level' mathematical skills. This can facilitate studies on how different mathematical competences are related to each other over the course of learning and, eventually, lead to more answers concerning the relationship between basic and higher level skills.

On the other hand, cognitive neuroscientists have extensive experience in designing experiments to isolate variables of interest through the use of neuroimaging methodologies and can thus assist educational researchers in the design of tightly controlled experimental paradigms to address their questions of interest. Beyond bringing methodological expertise to the table, cognitive neuroscientists are aware of the constraints placed on the interpretation of neuroimaging data and can also help to adequately interpret the results of neuroimaging findings, preventing mis- or over-interpretation.

It is important to emphasize that cognitive neuroscience methods must not be placed above traditional educational research methodologies in terms of explanatory value or power. Rather, results from neuroimaging studies should always be considered in the context of traditional behavioral studies conducted by educational researchers and cognitive psychologists. We contend that only through the mutually constraining explanatory power of experiments conducted using both behavioral and brain imaging methods, greater insights into mathematics learning will be gained.

In future, interdisciplinary training will play an increasingly important role. Students trained in both educational research and cognitive neuroscience will be aware of the chances and limitations of both research approaches and thus be best equipped to walk the tightrope between educational relevance and methodological feasibility.

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