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Sree Ram Valluri

J.W. Mielniczuk

Farrukh Chishtie

D. Lamm

S. Auddy

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S. R. Valluri,[1,2]★ J. W. Mielniczuk,[3] F. Chishtie,[1]★ D. Lamm[1,4] and S. Auddy[1]

[1]*Department of Physics and Astronomy, The University of Western Ontario, London, ON N6A 3K7, Canada*

[2]*King's University College, London, ON N6A 2M3, Canada*

[3]*82 Brick Kiln Road Apt 1-301, Chelmsford, MA 01824, USA*

[4]*Georgia Institute of Technology, Atlanta, GA 30332, USA*

ABSTRACT

We analyse the spectrum of the Hamiltonian of a photon propagating in a strong magnetic field $B \sim B_{cr}$, where $B_{cr} = \frac{m^2}{e} \simeq 4.4 \times 10^{13}$ G is the Schwinger critical field. We show that the anomalous magnetic moment of a photon in the one-loop approximation is a non-decreasing function of the magnetic field B in the range $0 \leq B \leq 30\,B_{cr}$. We provide a numerical representation of the expression for the anomalous magnetic moment in terms of special functions. We find that the anomalous magnetic moment μ_γ of a photon for $B = 30\,B_{cr}$ is 8/3 of the anomalous magnetic moment of a photon for $B = 1/2B_{cr}$. Based on the recent observational evidence for vacuum birefringence from the neutron star RX J1856.5−3764 by Mignani et al., we suggest vacuum birefringence, the anomalous magnetic moment of the photon and the Faraday rotation angle as key observables for future experiments and measurements.

Key words: astroparticle physics – magnetic fields – scattering – instrumentation: polarimeters – stars: neutron – pulsars: general.

1 INTRODUCTION

The effective interaction that results due to the corrections from the virtual excitations of the charged quantum fields, such as electron e^- and positron e^+, leads to well-known interesting effects (Baier & Breitenlohner 1967; Dittrich & Gies 2000). More recently, other interesting aspects of the quantum vacuum have been explored by Shabad & Usov (2011), Villalba-Chávez & Shabad (2012) and Altschul (2008) to name but a few. In the case of electromagnetic fields that vary slowly with respect to the Compton wavelength, i.e. frequencies much less than the pair creation threshold, the one-loop quantum electrodynamic effective Heisenberg–Euler Lagrangian (HEL) describes the dominant physical effects (Heisenberg & Euler 1936; Dunne 2005; Shabad & Usov 2011; Villalba-Chávez & Shabad 2012). The HEL is known to all orders in electromagnetic fields. It is well known that electrons acquire an anomalous magnetic moment due to the radiative corrections in quantum electrodynamics (QED) with the $e^- - e^+$ pairs and virtual photons in the background (Schwinger 1951). It is also of great fundamental interest that there is an anomalous photon magnetic moment μ_γ due to the interaction with the external magnetic field in the environment of the virtual $e^- - e^+$

quanta of the vacuum. The last couple of decades has seen a resurgence of interest in quantum vacuum physics (Mielniczuk, Lamm & Valluri 1988; Baring 1995; Heyl & Hernquist 1997; Dunne 2009, 2012). The promise of high-intensity experimental facilities ($\sim 10^{15}$ W) has stimulated immense interest to investigate the non-linear quantum vacuum in practical optical experiments (Marklund & Shukla 2006; Dunne 2009; Della Valle et al. 2013, 2014). Recently, direct evidence of vacuum birefringence via optical polarimetry has been reported by Mignani et al. (2017), where they have studied the radio-quiet neutron star J1856.5−3754. This opens up exciting possibilities, via various astrophysical sources, of further measurements of observables in non-linear QED, such as the anomalous magnetic moment of the photon as well as Faraday rotation angle, for which we derive new results for various ranges of magnetic field values, in this work. In Section 2, we outline and discuss the analytic calculations on the anomalous magnetic moment of the photon. We present and discuss the results. Section 3 presents the conclusions.

2 ANOMALOUS MAGNETIC MOMENT OF A PHOTON

Villalba-Chávez (2010), Villalba-Chávez & Shabad (2012) and Rojas & Querts (2006, 2007) have discussed the notion of the anomalous magnetic moment of a photon. The photon anomalous magnetic moment and its paramagnetic properties that have

★ E-mail: valluri@uwo.ca (SRV); fchishti@uwo.ca (FC)

been studied by Pérez Rojas & Rodríguez Querts (2014) and Rojas & Querts (2006, 2007) have provided values of μ_γ in the two extreme limits of $B \ll B_{cr}$ and $B \gg B_{cr}$. The purpose of this paper is to provide numerical values and an analytic formula for the range $B \sim B_{cr}$. Our results for the photon anomalous magnetic moment are applicable in the range $0 \leq B \leq 30\,B_{cr}$.

At one-loop order, the Heisenberg–Euler effective Lagrangian in constant external electromagnetic fields (Heisenberg & Euler 1936; Karbstein & Shaisultanov 2015), describing the effective non-linear interactions between the electromagnetic fields mediated by electron–positron fluctuations in the vacuum, can be represented concisely in terms of the following proper time integral (Schwinger 1951),

$$\mathcal{L} = \frac{\alpha}{2\pi} \int_0^\infty \frac{ds}{s} e^{-i\frac{m^2}{e}s} \left[ab \coth(as) \cot(bs) - \frac{a^2 - b^2}{3} - \frac{1}{s^2} \right] \quad (1)$$

with the prescription $m^2 \rightarrow m^2 - i0^+$, and the proper time integration contour assumed to lie slightly below the real positive s-axis. Here, m is the electron mass, e is the elementary charge, $\alpha = \frac{e^2}{4\pi}$ is the fine structure constant, and $a = (\sqrt{\mathcal{F}^2 + \mathcal{G}^2} - \mathcal{F})^{1/2}$ and $b = (\sqrt{\mathcal{F}^2 + \mathcal{G}^2} + \mathcal{F})^{1/2}$ are the secular invariants made up of the gauge and Lorentz invariants of the electromagnetic field: $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\boldsymbol{B}^2 - \boldsymbol{E}^2)$ and $\mathcal{G} = \frac{1}{4}F^*_{\mu\nu}F^{\mu\nu} = -\boldsymbol{E} \cdot \boldsymbol{B}$, with $^*F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ denoting the dual field strength tensor; $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor, fulfilling $\epsilon^{0123} = 1$. Our metric convention is $\boldsymbol{g}_{\mu\nu} = diag(-1, +1, +1, +1)$, and we use the units where $c = \hbar = 1$. To keep notations compact, we moreover employ the shorthand notations $\int_x \equiv \int d^4x$ and $\int_k \equiv \int \frac{d^4k}{(2\pi)^4}$ for the integration over the position and the momentum space, respectively.

The seminal paper of Schwinger (1951) on gauge invariance and vacuum polarization has used the proper time parameter formulation to the solution of the equation of motion of a particle. Thereby, the effective Lagrangian (Karbstein & Shaisultanov 2015) is finite, gauge and Lorentz invariant. Equation (1) is also applicable for slowly varying inhomogeneous fields fulfilling $\frac{v}{m} \ll 1$, or in other words for inhomogeneities whose typical spatial (temporal) scales of variation are much larger than the Compton wavelength (time) $\sim \frac{1}{m}$ of the virtual charged particle. The electron Compton wavelength is $\lambda_c = 3.86 \times 10^{-13}$ m and the Compton time is $\tau_c = 1.29 \times 10^{-21}$ s. In turn, many electromagnetic fields available in the laboratory, e.g. the electromagnetic field pulses generated by optical high-intensity lasers, featuring wavelengths of $\mathcal{O}(\mu m)$ and pulse durations of $\mathcal{O}(fs)$, are compatible with this requirement.

In the absence of an external electric field, the partial derivatives of the effective action in the one-loop approximation are (Lundin 2009, 2010)

$$\gamma_{\mathcal{F}} = \frac{\partial \mathcal{L}}{\partial \mathcal{F}}, \quad \gamma_{\mathcal{F}\mathcal{F}} = \frac{\partial^2 \mathcal{L}}{\partial \mathcal{F}^2}, \quad \gamma_{\mathcal{G}\mathcal{G}} = \frac{\partial^2 \mathcal{L}}{\partial \mathcal{G}^2}. \quad (2)$$

Expressions such as $\gamma_\mathcal{G}$, $\gamma_{\mathcal{F}\mathcal{G}}$ are zero for zero electric field. Further,

$$\gamma_{\mathcal{F}} = -1 - \frac{\alpha}{2\pi}\left[\frac{1}{3} + 2h^2 - 8\zeta'(-1, h) + 4h \ln \Gamma(h) \right.$$
$$\left. - 2h \ln h + \frac{2}{3}\ln h - 2h \ln 2\pi \right] \quad (2a)$$

$$\gamma_{\mathcal{F}\mathcal{F}} = \frac{\alpha}{2\pi B^2}\left[\frac{2}{3} + 4h^2\psi(1 + h) - 2h - 4h^2 - 4h \ln \Gamma(h) \right.$$
$$\left. + 2h \ln 2\pi - 2h \ln h \right] \quad (2b)$$

$$\gamma_{\mathcal{G}\mathcal{G}} = \frac{\alpha}{2\pi B^2}\left[-\frac{1}{3} - \frac{2}{3}\psi(1 + h) - 2h^2 + (3h)^{-1} + 8\zeta'(-1, h) \right.$$
$$\left. - 4h \ln \Gamma(h) + 2h \ln(2\pi) + 2h \ln h \right], \quad (2c)$$

where ψ is the digamma function, Γ is the gamma function and $h = \frac{1}{2}\frac{B_{cr}}{B}$,

$$\gamma_{\mathcal{G}\mathcal{G}} = \left.\frac{\partial^2 \mathcal{L}}{\partial \mathcal{G}^2}\right|_{\mathcal{F} = \frac{1}{2}B^2}^{\mathcal{G}=0}. \quad (3)$$

Also

$$\zeta'(s, h) = \partial_s \zeta(s, h), \quad (4)$$

where $\zeta(s, h)$ is the Hurwitz zeta function, for $s = -1$ given by Adamchik (2004) and $h \gg 1$ (Dittrich, Tsai & Zimmermann 1979)

$$\zeta'(-1, h) \cong \frac{1}{12} - \frac{h^2}{4} + \frac{\ln h}{2}\left(h^2 - h + \frac{1}{6} \right)$$
$$+ \int_0^\infty \frac{e^{-hx}}{x^2}\left(\frac{1}{1 - e^{-x}} - \frac{1}{x} - \frac{1}{2} - \frac{x}{12} \right) dx,$$
$$Re(h) > 0 \quad (5)$$

$$\zeta'(-1, h) \cong \frac{1}{12} - \frac{h^2}{4} + \frac{\ln h}{2}B_2(h) + \frac{1}{720}\frac{1}{h^2}, \quad (6)$$

where $B_2(h) = h^2 - h + \frac{1}{6}$ is the second Bernoulli polynomial (Olver et al. 2010). The integral above is convergent (Adamchik 2004).

The refractive indices for perpendicular and parallel polarized photons are of particular interest in this context. It is worth noting that

$$\frac{4\pi}{\alpha}(n_\perp - 1) = \frac{2\pi B^2}{\alpha}\gamma_{\mathcal{G}\mathcal{G}}, \quad (7)$$

where $\gamma_{\mathcal{G}\mathcal{G}}$ has been defined in equation (2c). For the weak field case, $n_\perp$ in terms of h is given by the expression (Heyl & Hernquist 1997), where $h = \frac{B}{B_{cr}}$ and $h > \frac{1}{2}$,

$$n_\perp = 1 + \frac{\alpha}{4\pi}\sin^2\theta\left[\frac{7}{90h^2} \right.$$
$$\left. - \frac{1}{3}\sum_{j=2}^\infty \frac{2^{2j}(6B_{2(j+1)} - (2j+1)B_{2j})}{j(2j+1)}(2h)^{-2j} \right]$$
$$+ \mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2 \right], \quad (8)$$

α is the fine structure constant, B_{2j} and B_{2j+1} are the Bernoulli numbers.

In the strong-field limit $h < 1$, we obtain

$$n_\perp = 1 + \frac{\alpha}{4\pi}\sin^2\theta\left[\frac{1}{3h} - \left(8 \ln A - \frac{1}{3} - \frac{2}{3}\gamma \right) \right.$$
$$- 2h\left(\ln \pi + \frac{1}{18}\pi^2 - 2 + \ln(2h) \right) - 4h^2\left(-\frac{1}{2} - \frac{1}{6}\zeta(3) \right)$$
$$\left. - \sum_{j=3}^\infty \frac{(-1)^{j-1}}{2^{j-2}}\left[\frac{j-2}{j(j-1)}\zeta(j-1) + \frac{1}{6}\zeta(j+1)(2h)^j \right] \right.$$

$$+ \mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2\right]$$

$$= 1 + \frac{\alpha h^2 \sin^2(\theta)}{18\pi}\left[\frac{3}{h}\left\{\frac{1}{2h}\left(12\ln A + \ln\left(\frac{\pi^3}{2}\right)\right)\right.\right.$$

$$\left.-3\ln(2\pi h)\right\} + h^{-1}\left((3h^{-1} - 33)(2h)^{-1} + \pi^2 + 18\right)$$

$$\left.-3h^{-2}\psi(1+h) + 9\right] + \mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2\right], \tag{9}$$

where we have derived and used the following relation,

$$\sum_{j=3}^{\infty} \frac{(-1)^{j-1}}{2^{j-2}}\left[\frac{j-2}{j(j-1)}\zeta(j-1) + \frac{1}{6}\zeta(j+1)(2h)^j\right]$$

$$= \frac{2h^2}{9}\left[3\left(h^{-2}(6 - 18\ln(A)) + \gamma + \frac{1}{4}\left\{\ln\left(\frac{4}{\pi^6}\right) + \zeta(3)\right\}\right)\right.$$

$$\left. + 3h^{-2}\psi(1+h) - \pi^2(2h)^{-1}\right]. \tag{10}$$

For parallel polarizations, the refractive index is given by Tsai & Erber (1975)

$$n_\parallel = 1 + \frac{\alpha}{4\pi}\sin^2\theta\left[-\frac{1}{3} - \frac{2}{3}\psi(1+h) + 8\zeta'(-1, h) - 2h^2\right.$$

$$\left.+ \frac{1}{3h} - 4h\ln\Gamma(h) + 2h\ln(2\pi h)\right], \tag{11}$$

which is valid for all $B \leq \frac{\pi}{\alpha}B_{\text{cr}}$.

We define a quantity $\Delta n_{\perp,\parallel} = n_\perp - n_\parallel$ by using equations (10) and (11), and derive the expression below:

$$\Delta n_{\perp,\parallel} = \frac{\alpha\sin^2\theta}{360\pi h^2}\left[20h^2\{18(\ln(A) - 1) + h(27h + \pi^2 + 18)\}\right.$$

$$+ 10h^2\left\{-6\ln(h) + 18\,h\left((1 - 2\,h)\ln(h) - \ln(4\pi^2 h)\right.\right.$$

$$\left.\left.+ 2\Gamma(h)\right) + \ln\left(\frac{\pi^9}{8}\right)\right\} - 1\right]. \tag{12}$$

Here, ζ is the Riemann zeta function, $\zeta(3) \cong 1.202$, θ is the angle between the magnetic field $\boldsymbol{B}$ and the vector $\boldsymbol{k}$, $\gamma \cong 0.577$ is the Euler–Mascheroni constant and $A \cong 1.282\,427\,12...$ is the Glaisher–Kinkelin constant (Olver et al. 2010).

An important physical variable related to $\Delta n_{\perp,\parallel}$ is the Faraday rotation angle, where $\chi = k(n_\perp - n_\parallel)l$. Here, k is the magnitude of the photon wave vector and l can be viewed as the path distance of the photon in the magnetic field. The Faraday rotation can, in principle, be observable for appreciable values of k and l. As a rough estimate, the Faraday rotation of a radiowave of a few hundred megahertz traversing a path length of around hundred metres can be of the order of a few radians or more for strong magnetic fields around a neutron star.

We briefly analyse the properties of a photon propagating in a strong magnetic field $\boldsymbol{B}$. The quantum expectation value for the Hamiltonian of a perpendicularly polarized photon is given by Bialynicki-Birula & Bialynicka-Birula (2012) and Bialynicka-
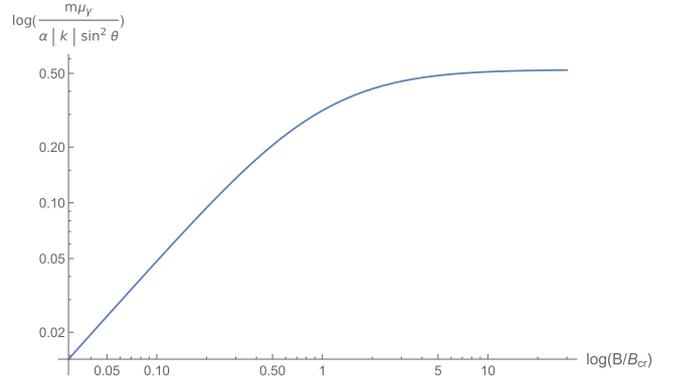

Figure 1. The log–log plot of the photon anomalous magnetic moment $\frac{m\mu_\gamma}{\alpha|\boldsymbol{k}|\sin^2\theta}$ as a function of the magnetic field B/B_{cr}.

Birula & Bialynicki-Birula (2014):

$$\langle H(B)\rangle \cong \langle H(0)\rangle - \frac{1}{2}B^2\gamma_{\mathcal{GG}}, \tag{13}$$

$\langle\rangle$ denotes the quantum expectation value. From the linearity in the term proportional to the magnetic moment μ_γ of the Hamiltonian (Villalba-Chávez & Shabad 2012; Pérez Rojas & Rodríguez Querts 2014), one has

$$\mu_\gamma = -\frac{\text{d}\langle\hat{H}(\boldsymbol{B})\rangle}{\text{d}B}. \tag{14}$$

Following Bialynicki-Birula & Bialynicka-Birula (2012) and Bialynicka-Birula & Bialynicki-Birula (2014), we will call the mode perpendicular if the magnetic field of the photon is in the plane formed by the vectors $\boldsymbol{B}$ and $\boldsymbol{k}$, where $\boldsymbol{k}$ is the wave vector of the photon.

From equations (13) and (14), we derive the following expression for μ_γ. From the fact that $\mu_\gamma(0) = 0$, we find that the photon magnetic moment of a perpendicularly polarized photon for $B \leq 30B_{\text{cr}}$ is

$$\mu_\gamma(B) = \frac{\alpha}{4\pi}\left\{\frac{2}{3} + \frac{1}{B^3}\left[\frac{1}{3}B\psi'\left(1 + \frac{1}{2B}\right) + \psi\left(\frac{1}{2B}\right)\right.\right.$$

$$\left.\left.-2B\ln\Gamma\left(\frac{1}{2B}\right) + B\ln(4\pi B) + B - 1\right]\right\}\left(\frac{|\boldsymbol{k}|}{m}\right)\sin^2\theta, \tag{15}$$

where ψ is the digamma function and Γ is the Euler gamma function. From equation (15), one observes that the photon magnetic moment contributes to both the external field strength as well as the photon energy through its momentum. The anomalous magnetic moment is plotted as a function of magnetic field in Fig. 1.

For $B > \frac{1}{2}B_{\text{cr}}$, we can approximate

$$\mu_\gamma(B) \cong \frac{\alpha}{4\pi}\left[\frac{2}{3} + \left(\ln(\pi) + \frac{\pi^2}{18} - 1\right)B^{-2}\right.$$

$$\left.-\frac{\ln B}{B^2}\right]\left(\frac{|\boldsymbol{k}|}{m}\sin^2\theta\right), \tag{16}$$

where $\boldsymbol{k}$ is the photon wave vector.

For $0 \leq B \leq 0.44B_{\text{cr}}$

$$\mu_\gamma(B) \cong \frac{\alpha}{4\pi}\frac{28}{45}\left(B - \frac{52}{49}B^3\right)\left(\frac{|\boldsymbol{k}|}{m}\sin^2\theta\right). \tag{17}$$

For a perpendicularly polarized photon, we note that equation (17) can be replaced by the inequality

$$\mu_\gamma(B) \geq \frac{\alpha}{4\pi} \frac{28}{45} \left(B - \frac{52}{49} B^3 \right) \left(\frac{|\boldsymbol{k}|}{m} \sin^2 \theta \right). \qquad (18)$$

We restrict equation (15) to $0 \leq B \leq 30 B_{cr}$. Using equation (16), we obtain that $\mu_\gamma(B = 30 B_{cr})$ is only 3 per cent smaller than the asymptotic value $\alpha/(3\pi)$ of the Bohr magneton. It is approximately 10^{-3} of the Bohr magneton for $|\boldsymbol{k}| \sim m$. $\mu_\gamma(B)$ grows from the value of

$$\frac{\alpha}{4\pi} \frac{28}{45} \frac{3}{4} \frac{1}{2} \frac{|\boldsymbol{k}|}{m} \sin^2 \theta,$$

for $B = \frac{1}{2} B_{cr}$, to the value very close to

$$\frac{\alpha}{4\pi} \frac{2}{3} \frac{|\boldsymbol{k}|}{m} \sin^2 \theta,$$

for $B = 30 B_{cr}$, thus the growth is only by a factor of ≈ 3. Formally, our equation is applicable only when

$$\frac{|\boldsymbol{k}|}{m} \ll 1. \qquad (19)$$

Equation (16) is the generalization of equation 157 of Villalba-Chávez & Shabad (2012), who state that

$$\mu_\gamma(B) \sim \frac{\alpha}{3\pi} \left(\frac{1}{2} \frac{e}{m} \right), \qquad (20)$$

for large values of the magnetic field $\boldsymbol{B}$. This suggests that the one-loop approximation provides a good estimate of μ_γ in the low-frequency case. Here, e denotes the electron charge and m is the corresponding mass. At low and high photon frequency, Villalba-Chávez & Pérez-Rojas (2006) have shown that the photon magnetic moment shows a paramagnetic behaviour as is also true for the vacuum embedded in a strong external magnetic field (Mielniczuk et al. 1988). Our equation (17) is similar to equation 18 of Pérez Rojas & Rodríguez Querts (2014) except that our numerical factor $\frac{28}{45}$ is twice as large as their corresponding factor $\frac{14}{45}$. Equations (15) and (16) are the main results of our paper. The paramagnetic behaviour is a physical effect due to the effect of the external magnetic field on the virtual $e^- - e^+$ pairs.

3 CONCLUSIONS

We have shown that the anomalous magnetic moment of a photon for $B = 30 B_{cr}$ is 8/3 of the anomalous magnetic moment of a photon for $B = \frac{1}{2} B_{cr}$. At low and high photon frequencies, the photon magnetic moment shows a paramagnetic behaviour. We find that the one-loop Lagrangian is a good approximation in the range of magnetic fields considered. We have shown that the anomalous magnetic moment of a photon is a non-decreasing function of the magnetic field B for $0 \leq B \leq 30 B_{cr}$.

Light propagation in the magnetized vacuum is analogous to the dispersion of light in an anisotropic medium. The reason for the anisotropy is due to the breaking of symmetry due to the choice of B along a preferred direction. The magnetic moment of the photon might have both astrophysical and cosmological consequences. In the presence of magnetic fields around astrophysical objects such as magnetars, magnetic lensing may be a strong observable effect.

Earlier works (Heyl & Hemquist 2005; Wang & Lai 2009) claim that QED non-linear effects are detectable. The recent observational evidence for vacuum birefringence by Mignani et al. (2017) by using optical polarimetry of RX J1856.5−3754, an isolated, radio-quiet neutron star in the 'Magnificent Seven' (M7) group, provides an exciting avenue for further tests of non-linear QED. This neutron star has an inferred magnetic field of $B \approx 10^{13}$ G, which makes it amenable for our predictions. Hence, based on this work, we suggest that anomalous magnetic moment of the photon and the Faraday rotation angle as key observables for future experiments utilizing pulsar observations. Photons that go by a strongly magnetized star would undergo a deflection besides the well-known gravitational shift caused by the stellar mass (Villalba-Chávez & Pérez-Rojas 2006). The cosmic microwave background (CMB) spectrum shows a substantial polarization-dependent field in the vicinity of magnetars (Bialynicka-Birula & Bialynicki-Birula 2014). Bialynicka-Birula & Bialynicki-Birula (2014) have estimated the polarization-dependent heating of the CMB radiation due to strong magnetic fields. Although the large magnetic fields around the region of magnetars are appreciable, the estimated distortion of the CMB due to the increase in temperature T cannot be detected with the current detector sensitivity. Efforts to build an X-ray polarimeter are on the way. Soffitta et al. (2013) have shown the influence of magnetic vacuum birefringence on the polarization of magnetic neutron stars. Magnetars should provide an avenue for measurement through astroparticle physics in the large frequency limit. It is also possible that further improvements in estimated angular resolutions as well as in the precision of the temperature fluctuation measurements and experimental facilities such as the Large Hadron Collider will make such effects as well as those of the photon anomalous magnetic moment observable.

There has been a surge of interest to investigate quantum non-linearity in state of the art optical experimental setups (Marklund & Shukla 2006). The QED vacuum in an external field will reveal further interesting insights into processes such as electrogravitational conversion (Papini & Valluri 1977). Some of the strongest magnetic fields in the Universe are expected to exist around magnetars (Bassa et al. 2008; Olausen & Kaspi 2014; Olausen & Kaspi 2014). A strong magnetic field exists around the centre of the Galaxy where a supermassive black hole exists (Eatough et al. 2013). Recently, a pulsar PSR J1745-2900, with an unusually large Faraday rotation, was discovered close to the Galactic Centre (Eatough et al. 2013 and see references 9–12 therein). These objects with such strong magnetic fields, although contained in regions small relative to the cosmos, can still provide us with possibilities of observing non-linear effects such as birefringence that can provide a handle to estimate physical quantities such as the photon anomalous magnetic moment and Faraday rotation. Proposals have been given to search for birefringence with the use of the time varying electromagnetic fields and high-precision interferometry (Zavattini & Calloni 2009; Grote 2015).

More refined ground-based experimental observations of vacuum birefringence may facilitate a measurement of the photon anomalous magnetic moment. The Polarization of the Vacuum with Laser (PVLAS) experiment aims to measure the birefringence of the external magnetic field in the vacuum (Bregant et al. 2008; Cantatore et al. 2008; Della Valle et al. 2016). The BMV experiment (Berceau et al. 2010; Cadène et al. 2014) is also working on the vacuum birefringence measurements. An appreciable signal of the Faraday rotation angle χ for the magnetized vacuum would be a new signature of fundamental physics. Zavattini et al. (2008) have reported upper limits for the vacuum dichroism that has not yet been experimentally verified.

Direct measurement through other astronomical sources and ground-based experiments such as PVLAS and BMV would indeed be strong experimental proof of the vacuum polarization effects for strong macroscopic electromagnetic fields.

REFERENCES

Adamchik V. S., 2004, Comput. Phys. Commun., 157, 181
Altschul B., 2008, Astropart. Phys., 29, 290
Baier R., Breitenlohner P., 1967, Il Nuovo Cimento B, 47, 117
Baring M. G., 1995, ApJ, 440, L69
Bassa C., Wang Z., Cumming A., Kaspi V., 2008, AIP Conf. Proc. Vol. 983, 40 Years of Pulsars: Millisecond Pulsars, Magnetars, and More. Springer, Berlin
Berceau P., Battesti R., Fouché M., Rizzo C., 2010, Can. J. Phys., 89, 153
Bialynicki-Birula I., Bialynicka-Birula Z., 2012, Phys. Rev. A, 86, 022118
Bialynicka-Birula Z., Bialynicka-Birula I., 2014, Phys. Rev. D, 90, 127303
Bregant M. et al., 2008, Phys. Rev. D, 78, 032066
Cadène A., Berceau P., Fouché M., Battesti R., Rizzo C., 2014, Eur. Phys. J. D, 68, 1
Cantatore G. et al., 2008, in Kuster M., Raffelt G., Beltrán B., eds, Lecture Notes Physics, 741, Recent Results from the PVLAS Experiment on the Magnetized Vacuum. Springer, p. 157
Della Valle F., Gastaldi U., Messineo G., Milotti E., Pengo R., Piemontese L., Ruoso G., Zavattini G., 2013, New J. Phys., 15, 053026
Della Valle F. et al., 2014, Phys. Rev. D, 90, 092003
Della Valle F., Ejilli A., Gastaldi U., Messineo G., Milloti E., Pengo R., Ruoso G., Zavattini G., 2016, Eur. Phys. J. C, 76, 1
Dittrich W., Gies H., 2000, Probing the Quantum Vacuum, Vol. 166. Springer-Verlag, Berlin
Dittrich W., Tsai W., Zimmermann K., 1979, Phys. Rev. D, 19, 2929
Dunne G. V., 2005, From Fields to Strings: Circumnavigating Theoretical Physics. World Scientific Press, Singapore
Dunne G. V., 2009, Eur. Phys. J. D, 55, 327
Dunne G. V., 2012, Int. J. Mod. Phys. A, 27, 1260004
Eatough R. et al., 2013, Nature, 501, 391
Grote H., 2015, Phys. Rev. D, 91, 022002
Heisenberg W., Euler H., 1936, Z. Phys., 98, 714
Heyl J., Hemquist L., 2005, ApJ, 618, 463
Heyl J. S., Hernquist L., 1997, Phys. Rev. D, 55, 2449
Karbstein F., Shaisultanov R., 2015, Phys. Rev. D, 91, 085027
Lundin J., 2009, Europhys. Lett., 87, 31001
Lundin J., 2010, PhD thesis, Umeå Universitet
Marklund M., Shukla P. K., 2006, Rev. Mod. Phys., 78, 591
Mielniczuk W. J., Lamm D. R., Valluri S. R., 1988, Can. J. Phys., 66, 692
Mignani R. P., Testa V., González Caniulef D., Taverna R., Turolla R., Zane S., Wu K., 2017, MNRAS, 465, 492
Olausen S. A., Kaspi V. M., 2014, ApJS, 212, 6
Olver F. W., Lozier D. W., Boisvert R. F., Clark C. W., 2010, NIST Handbook of Mathematical Functions Hardback. Cambridge Univ. Press, Cambridge
Papini G., Valluri S. R., 1977, Phys. Rep., 33, 51
Pérez Rojas H., Rodríguez Querts E., 2014, Eur. Phys. J. C, 74, 2899
Rojas H. P., Querts E. R., 2006, in Novello M., Perez Bergliaffa S., Ruffini R., eds, The Tenth Marcel Grossmann Meeting Part A. World Scientific Press, Singapore, p. 2241
Rojas H. P., Querts E. R., 2007, Int. J. Mod. Phys. D, 16, 165
Schwinger J., 1951, Phys. Rev., 82, 664
Shabad A. E., Usov V. V., 2011, Phys. Rev. D, 83, 105006
Soffitta P. et al., 2013, Exp. Astron., 36, 523
Tsai W.-Y., Erber T., 1975, Phys. Rev. D, 12, 1132
Villalba-Chávez S., 2010, Phys. Rev. D, 81, 105019
Villalba-Chávez S., Pérez-Rojas H., 2006, preprint (hep-th/0604059)
Villalba-Chávez S., Shabad A. E., 2012, Phys. Rev. D, 86, 105040
Wang C., Lai D., 2009, MNRAS, 398, 515
Zavattini E. et al. (PVLAS Collaboration), 2008, Phys. Rev. D, 77, 032006

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