Summer 1999

A Model, Secondary Level, Mathematics Curriculum Developed in Alignment with Washington State Essential Academic Learning Requirements, Easton School District

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A MODEL, SECONDARY LEVEL, MATHEMATICS CURRICULUM

DEVELOPED IN ALIGNMENT WITH

WASHINGTON STATE ESSENTIAL ACADEMIC LEARNING REQUIREMENTS,

EASTON SCHOOL DISTRICT

by

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August, 1999

The purpose of this project was to design and develop a model secondary level mathematics curriculum, in alignment with Washington State Essential Academic Learning Requirements, for the Easton School District in Washington. To accomplish this purpose, a review of current research and literature regarding Washington State Essential Academic Learning Requirements related to secondary mathematics was conducted. In addition, related information from selected sources was obtained and analyzed.

ACKNOWLEDGMENTS

This work is dedicated to my wife Koreann, my daughter Roya, friends and colleagues for their continued support, encouragement, and perseverance.

I would also like to express my appreciation and gratitude to Dr. Jack McPherson for his inspiration, support, guidance, and assistance in the preparation of this paper, during my internship and throughout my entire course of study. In addition, I would also like to thank Dr. Susan Madley and Dr. Linda Beath for their participation as members of my committee.

Finally, a special appreciation is given to Mr. Darell Cain, Superintendent of Easton School District, for his assistance with this project.

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CHAPTER 1

BACKGROUND OF THE PROJECT

Introduction

> Mathematics continues to grow at a rapid rate, spreading into new fields and creating new applications, in its open-ended search for patterns. Several factors . . . growth of technology, increased applications, impact of computers, and expansion of mathematics itself, have combined in the past century to extend greatly both the scope and the application of the mathematical sciences. The change must be reflected in the schools if our students are to be well prepared for tomorrow's world. (Washington State Commission on Student Learning, 1998, Preface)

In the above statement, the Washington State Commission on Student Learning has emphasized the importance of mathematics' rapid growth and necessity of changes in which our schools have to adapt to prepare students for tomorrow's world.

According to RCW 28A.150.210, Basic Education Act of 1998, the goal of the Basic Education Act for Washington State schools shall be to "provide students with the opportunity to become responsible citizens, to contribute to their own economic well-being and to that of their families and communities, and to enjoy productive and satisfying lives." To meet these goals concerning mathematics, students need to "know and apply the core concepts and principles of mathematics." In addition, students should be able to "think analytically, logically, and creatively, and to integrate experience and knowledge to form reasoned judgments and solve problems."

The Mathematical Sciences Education Board (1997) has contended that more than at any other time in history, society is placing demands on citizens to interpret and use mathematics to make sense of information and complex situations. Computers and other technologies have increased our capacity for dealing with numbers and collecting, organizing, representing, and analyzing data.

Research conducted by the National Council of Teachers of Mathematics (1998) has contended that to be well informed as adults and to have access to desirable jobs, students today require beyond what was needed by students in the past. All students must develop and sharpen their skills, deepen their understanding of mathematical concepts and processes, and hone their problem-solving skills, reasoning, and communication abilities while using mathematics to make sense of and to solve compelling problems.

Finally, with the current emphasis in educational reform and Washington State Essential Academic Learning Requirements, all students need a deep understanding of mathematics; for this to occur, rigorous mathematical content must be recognized, taught and assessed in a problem-solving environment. For students to develop this deeper level of understanding, they must connect their knowledge to a variety of ideas and skills across topic areas and grade levels in mathematics, to other subjects taught in school, and to situations outside the classroom.

Purpose of the Project

The purpose of this project was to design and develop a model secondary level mathematics curriculum developed in alignment with Washington State Essential Academic Learning Requirements for Easton School District #28, Easton, Washington. To accomplish this purpose, a review of current research and literature regarding curriculum and instruction related to secondary mathematics and Washington State standards for the Essential Academic Learning Requirements was conducted. In addition, related information from selected sources was obtained and analyzed.

Limitations of the Project

For this project, setting the following limitations was necessary:

1. Research: the preponderance of research and literature reviewed was limited to the past 10 years.
2. Scope: the model secondary level mathematics was designed for implementation in the Easton High School, Easton School District #28, Easton, Washington.
3. Target Population: the model program has been designed for students in grades 8 through 10.

Definition of Terms

Significant terms used in the content of this project have been defined as follows:

1. Algebraic Sense - one of the five concept areas of the Essential Academic Learning Requirements for Mathematics that involves the use of patterns and relations
2. Assessment - the process of gathering information about students, instructional practices, and school programs related to specified outcomes and purposes to understand them better
3. Benchmarks - examples of performances used to measure student progress at different points in time or at different grade levels
4. Cooperative Learning - an environment that encourages a complete and total involvement in the learning process. It is this kind of exposure that lends itself to the improvement in the learning process. It is this kind of exposure that lends itself to the improvement of a student's attitude and outlook with regard to his/her
5. Curricular and Instructional Reform - the development and implementation of content, instructional activities and strategies, and assessment practices in schools that are meaningful, relevant, and accessible to a diverse population of students
6. Essential Academic Learning Requirements for Mathematics - specific statements that outline what a student should know and can do in the state of Washington to achieve a basic level of competency in mathematics for the 21st century
7. Geometric Sense - one of the five concept areas of the Essential Academic learning Requirements for Mathematics that involves

understanding and applying the properties of geometric relationships, shapes, and transformations

8. <u>Integration</u> - the process of bringing together two or more curricular areas in a way that they do not stand alone, but are interconnected

9. <u>Learning Styles</u> - individual perceptual preferences for learning, such as visual, auditory, tactile and/or kinesthetic modes of learning

10. <u>Mathematics</u> - the science treating the exact relations existing between quantities or magnitudes and operations, and of the methods by which, in accordance with these relations, quantities sought are deductible from others known or supposed.

11. <u>Mathematical Concepts</u> - these are the five content areas of mathematics that students are required to master according to the Essential Academic Learning Requirements for Mathematics for Washington State that include number sense, measurement, geometric sense, probability and statistics, and algebraic sense

12. <u>Mathematical Connections</u> - one of the four mathematical processes of the Essential Academic Learning Requirements for Mathematics that involves relating ideas and procedures within mathematics, and to other disciplines and real-life situations

13. <u>Mathematical Processes</u> - these are the four mathematical processes that students are required to use and master according to the Essential Academic Learning Requirements for Mathematics for Washington State that include mathematical problem solving,

mathematical reasoning, mathematical communication, and mathematical connections

14. Mathematical Reasoning - one of the four mathematical processes of the Essential academic Learning Requirements for Mathematics that involves analyzing information, predicting results, making inferences, drawing conclusions, and verifying results

15. Measurement - one of the five concept areas of the Essential Academic Learning Requirements for Mathematics that involves understanding and applying concepts and procedures from measurement, including systems of measurement, dimensions, approximation, and tools

16. Multiple Intelligences Theory - (MI theory) a theory put forth by Howard Gardner that suggests the existence of seven comprehensive abilities or "intelligences," which include verbal/linguistic, logical/mathematical, visual/spatial, bodily/kinesthetic, musical, interpersonal, and intrapersonal

17. Number Sense - one of the five concept areas of the Essential Academic Learning Requirements for Mathematics that involves understanding and applying concepts and procedures of numbers and numeration, computation, and estimation

18. Performance Assessment - the direct, systematic observation of an actual student performance on tasks to show knowledge or skills, and the rating of that performance according to preestablished criteria

19. Performance Criteria - a description of the characteristics on which students will be judged for a task in a performance-based assessment: these may be expressed as a rubric or scoring guide with benchmark performances being used to identify each level of competency in the rubric or scoring guide

20. Problem Solving - one of the four mathematical processes of the Essential Academic Learning Requirements for Mathematics that involves investigating situations, formulating questions and defining problems, and constructing solutions

21. Standards - agreed-upon statements regarding the desired level of student achievement or performance on tasks for performance-based assessment

CHAPTER 2

REVIEW OF RELATED LITERATURE

Introduction

The review of research and literature summarized in Chapter 2 has been organized to address the following topics:

1. The Importance of Mathematics in Today's World
2. Mathematics as an Essential Learning Skill
3. Instructional Methodologies in Mathematics, three selected approaches . . .
   - Multiple Intelligences Theory
   - Problem-solving
   - Cooperative Learning
4. School Reform and Assessment in Mathematics
5. Summary

Data and research current within the last 15 years were identified through an Educational Resources Information Center (ERIC) computer search. A hand search of various other sources was also conducted.

The Importance of Mathematics in Today's World

Communication has created a world economy in which smarter working is more important than merely working harder. Jobs that contribute to

> this world economy require workers who are prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns, and to solve unconventional problems. It is these needs, not just the need for calculation, that make mathematics a prerequisite to so many jobs. (Mathematical Science Education Board, 1997, p. 15)

According to Zaslavsky (1994), skill in mathematics is needed by everyone in today's world. Increasingly, jobs in tomorrow's economy will require a knowledge of mathematics. New technologies will call for the ability to apply mathematics and science in practical ways, and rapid changes will demand that workers learn new skills throughout their lives (p. 28). Although not every job requires specific mathematical skills, it helps to have some facility with math and some confidence in your ability to handle problems as they arise (Zaslavsky, p. 31).

According to the Washington State Commission on Student Learning (1998), children are growing up in a world that has changed dramatically since the days of our own youth. Technology and other forces are rapidly transforming the ways we live and work. The forces of change are also reshaping what it means to have the knowledge and skills necessary to lead a successful life now and in the 21st century.

The Mathematical Sciences Education Board (1997) explains mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological community.

The recent economic notion in which factory employees work the same jobs to produce the same goods in the same manner for decades was a throwback to our past industrial age. Today, economic survival and growth have become dependent on new factories established to produce complex products and services with a very short market cycle. It is a literal reality that before the first products are sold, new replacements are being designed for an ever-changing market. Simultaneously, the research division is at work developing new ideas to feed to the design groups to meet the continuous demand for new products that are, in turn, channeled into the production arena. Traditional notions of basic mathematical competence have been outstripped by ever high expectations of the skills and knowledge of workers; new methods of production demand a technologically competent workforce. The U.S. Congressional Office of Technology Assessment (1988) claims that employees must be prepared to understand the complexities and technologies of communication, to ask questions, to assimilate unfamiliar information, and to work cooperatively in teams. Businesses no longer seek workers with strong backs, clever hands, and "shopkeeper" arithmetic skills. In fact, it is claimed that the "most significant growth in new jobs between now and the year 2000 will be in fields requiring the most education" (Lewis, 1988, p. 464).

Oaxaca and Reynolds (1989) contended that to be more competitive with global competencies in mathematics, educators might consider having higher-level mathematics not as an elective subject, but as a continuum of our current core curriculum in high school. Unfortunately, U.S. students drop out of mathematics at alarming rates, averaging about 50% each year after mathematics becomes an elective subject. Blacks, Hispanics, and other minorities drop out at even greater rates.

Research conducted by Lapointe et al. (1989) cited serious deficiencies in mathematical performance of U.S. students. Compared with other nations, the United States ranks low in mathematical performance along with falling below their own expectations. Although basic computational skills are not ranked low, only 1 in 20 high school graduates can deal competently with problems requiring several successive steps.

Pollak (1987), a noted industrial mathematician, recently summarized the mathematical expectations for new employees in the industry as:

- The ability to set up problems with the appropriate operations
- Knowledge of a variety of techniques to approach and work on problems
- Understanding of the underlying mathematical features of a problem
- The ability to work with others on problems
- The ability to see the applicability of mathematical ideas to common and complex problems
- Preparation for open problem situations, since most real problems are not well formulated
- Belief in the utility and value of mathematics

As suggested by Pollak, the difference between the skills and training inherent in these expectations and those acquired by students working independently to solve explicit sets of drills and practice exercises. Although mathematics is not taught in schools so students can specifically obtain jobs, experiences do reflect to some extent the mathematical needs of today's workplace. This is especially true given that the availability of such broadly educated workers will be a major factor in determining how businesses respond to today's changing economic conditions.

"Since 1900, the growth of mathematical sciences in scope and in application has been explosive" ("Everybody Counts," 1997). The last 40 years have been especially productive, as advances in high speed computing have opened new lines of research and new ways mathematics can be applied. Problems in economics, social sciences, and life sciences, and large scale problems in natural sciences and engineering, used to be unapproachable through mathematics. Suddenly, with the aid of computers and new tools provided by research, many of these problems have become accessible to mathematical analysis (MSEB, 1997, (p. 17).

According to the Mathematics Science Education Board (1997), applications derived from data analysis and statistics and discrete mathematics, and information theory and computing have greatly extended the definition and reach of Mathematical Sciences.

Over the years, professionals concerned with mathematics education have developed a coherent view of what mathematics is, despite some disagreements along the way. Recently, in its Curriculum and Evaluation Standards for School Mathematics, the National Council of Teachers of Mathematics (1998) contended that the traditional sequence of mathematics courses leading to Calculus is inadequate:

> Students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture validity. (NCTM, 1998)

NCTM standards delineate the mathematical skills and knowledge students need to learn under various headings, some familiar (e.g., measurement, algebra, probability, problem-solving), some less familiar (e.g., communication, special sense, discrete mathematics).

NCTM standards emphasize mathematical power and outline experiences designed to help all students gain that power. Particularly important have been the processes of mathematical thinking by which students learn problem-solving, communicating, reasoning, and making connections.

Ernest (1991) argues that mathematics cannot be described by a single unique hierarchical structure and that mathematics cannot be represented as a set of discrete knowledge components. The mathematician William Thurston (1990) uses the metaphor of a tree to describe mathematics: "Mathematics isn't a palm tree, with a single long straight trunk covered with scratchy formulas. It's a banyan tree, with many interconnected trunks and branches, a banyan tree that has grown to the size of a forest, inviting us to climb and explore" (p. 7).

As Romberg, Zarinnia, and Collis (1990) noted, the values and forces that dominated mathematics education for the past century (e.g., Behaviorism) are embedded in the theoretical structures of prevailing methods of assessment tests built on behavioral objectives and a content-by-process matrix are based on Behaviorist ideas about learning--that content can be broken down into small segments to be mastered by the learner in a linear sequential fashion.

A substantial body of evidence from cognitive psychology shows this hierarchical model of learning to be obsolete. The metaphor of the learner as a passive absorber of linearly ordered bits of information is contradicted by research findings from psychology. Ernest (1991) has shown that the uniqueness of learning hierarchies in mathematics is not confirmed theoretically nor empirically. Furthermore, he argues against the notion that concepts in mathematics can be either "possessed" or "lacking" in a learner.

Pressure to change mathematics education reflects society's disappointment with the lack of interest and accomplishment of so many students in today's school. In the background of public debate is the steady criticism that school mathematics is out of step with today's world and is neither well taught nor well learned. Unfortunately, these pressures often suggest inconsistent courses of action, with standard-based curriculum and instruction moving in one direction while mandated tests remain aimed in another direction, at an older, more traditional target. Too often, teachers are caught in the middle. NTCM (1989) explains that to be effective, mathematics education must be rooted in the practice of mathematics, in the art of teaching, and in the needs of society. These pivotal forces drive the current movement to improve mathematics education:

- A more comprehensive view of mathematics and its role in society; mathematics is no longer just a prerequisite subject for prospective scientists and engineers, but is a fundamental aspect of literacy for the 21st century.
- A recommitment to the traditional wisdom that mathematics must be made meaningful to students if it is to be learned, retained, and used.
- The growing recognition that in this technological era, all students should learn more and better mathematics.

A long history of tradition has grown up around what is meant by a good mathematics teacher and a good mathematics student. As many educators recognize, however, those traditions have little in common with mathematics of the 1990s. Mathematics as it is used in the real world is not about the memorization of theorems or rote procedures for getting right answers. It is not about doing well on multiple choice or short answer tests under time limitations. The challenge for mathematics educators is to align the culture of school mathematics with the culture of mathematics in the real world. As stated by Richardson and Salked (1999), "At the heart of mathematics is the search for sense and meaning, order and predictability. Mathematics is the study of patterns and relationships." (p. 21)

Mathematics as an Essential Learning Skill

In 1993, the Washington State Legislature enacted comprehensive school change legislation that has as its primary goal the improvement of teaching and learning (RCW 28.A.630.885). As part of that effort, the Legislature created the Commission on Student Learning and charged it with three important tasks:

- To establish Essential Academic Learning Requirements that define what all students need to know and be able to do in eight subject areas,
- To develop a performance-based assessment system to measure student progress toward achieving the Essential Academic Learning Requirements,
- To recommend an accountability system that will recognize and reward schools in which students are achieving the performance standards and provide support and assistance to those schools in which students are not reaching the standards.

The educational reform movement in Washington State resulting from adoption of the new Basic Education Act (RCW 28A.150.210) in that state in 1993 directed each school district in Washington State to involve parents and community members in providing opportunities for all students to develop knowledge and skills essential to:

1. Read with comprehension, write with skill, and communicate effectively and responsibly in a variety of ways and settings;
2. Know and apply the core concepts and principles of mathematics, social, physical, and life sciences; civics and history; geography, arts and health; and fitness;
3. Think analytically, logically, and creatively, and to integrate experiences and knowledge to form reasoned judgments and solve problems; and

4. Understand the importance of work and how performance, effort, and decisions directly affect career and educational opportunities.

It was further observed that two of the four essential skills for Washington State students are directly related to mathematics (i.e., #2, "Know and apply the core concepts and principles of mathematics," and #3, "Think analytically, logically, and creatively, and to integrate experience and knowledge to form reasoned judgments and solve problems").

Specific essential academic learning in mathematics has been detailed by the Washington State Commission on Student Learning publication entitled, The Essential Academic Learning Requirements in Mathematics (1998, see Appendix A)) and include the following:

1. The student understands the basic concepts and procedures of mathematics, how to use them, and why they work;
2. The student uses mathematics to define and solve problems;
3. The student uses mathematical reasoning;
4. The student communicates mathematical ideas in mathematical and everyday language;
5. The student understands how mathematical ideas connect to other subject areas, real-life situations, and career goals.

The Washington State Commission on Student Learning states its mission as to "update and elevate the standards of academic achievement and improve student performance in Washington State to prepare our young people for living, learning, and working successfully in the 21st century" (Essential Academic Learning Requirements Technical Manual, Draft, January 1997, p. 4). This body was directed to carry out the primary goals of the state's Educational Reform Act of 1993, goals intended to raise standards and student achievement. The math learning goals and accompanying benchmarks, or Essential Academic Learnings for Mathematics, designed by the Commission on Student Learning and modeled after recommendations from the Curriculum and Evaluation Standards for School Mathematics (NTCM, 1989) are available to school districts to identify what students in the state of Washington should know, understand, and be able to do in order to become mathematically literate, responsible, and productive citizens. Rubrics, or scoring guides, that describe the performance criteria for each of the Essential Academic Learning Requirements for Mathematics have been written and are available to help teachers and students focus on the specific concepts and processes to be learned.

The benchmarks, or developmental indicators, used to measure student progress are part of the Essential Academic Learning Requirements for Mathematics. The assessments that monitor students' attainment of the benchmarks are set at grades 4, 7, and 10. Benchmark One relates to K to 4th grades, Benchmark Two relates to the 5th to 7th grades, and Benchmark Three relates to 8th to 10th grades. By grade 10, students must show a mastery of the performance requirements through assessment at Benchmark Three to be eligible to receive the Certificate of Mastery. The law calls for full implementation of the standards and assessment by school districts in Washington State by the year 2000.

Instructional Methodology in Mathematics,
Three Selected Approaches

Current research obtained from Northwest Regional Educational Laboratory (1994) focused attention on instructional options available for boosting standard achievement in mathematics, including Multiple Intelligences Theory, problem solving, and cooperative learning. Accordingly, more detailed research was conducted on each of these instructional methodologies in mathematics. Three instructional methods proven successful in boosting standard achievement in mathematics are Multiple Intelligences Theory, problem solving, and cooperative learning.

Multiple Intelligences Theory.

In an era when criticism of our school system abounds and many Americans are focused on the ideal of providing a meaningful education to a diverse student population, it is important for educators to turn their attention to curricular and instructional reform. Teachers are increasingly faced with meeting the needs of students from a variety of ethnic, linguistic, and economic backgrounds who have striking differences in family structure, lifestyle, health, and physical and mental ability. When considering the improvement of pedagogy, it seems that a critical element

> . . . stresses the importance of teachers finding ways to make subject matter relevant to students, to involve students in setting their own goals, to vary the ways of learning, to use approaches that employ all of the senses, and to be sure that there are opportunities for relating the knowledge to experience or actually using it. (Goodlad, 1984, p. 231).

Multiple Intelligences Theory (MI) challenges the traditional idea of intelligence as a single general capacity found in varying degrees in all individuals that can be measured by IQ tests. Instead, Howard Gardner suggests human cognitive competence can be better described as a set of abilities, talents, or mental skills, which he calls "intelligences." "An intelligence entails the ability to solve problems or fashion products that are of consequence in a particular cultural setting or community" (Gardner, 1993, p. 102). Gardner groups the broad range of capabilities that humans possess into seven comprehensive categories:

Linguistic Intelligence: The capacity to use words effectively, whether orally or in writing.

Logical-Mathematical Intelligence: The capacity to use numbers effectively.

Spatial Intelligence: The ability to perceive the visual-spatial world accurately and to perform transformations upon those perceptions.

Bodily-Kinesthetic Intelligence: Expertise in using one's whole body to express ideas and feelings, and facility in using one's hands to produce or transfer things.

Musical Intelligence: The capacity to perceive and make distinctions in the moods, intentions, motivations, and feelings of other people.

Intrapersonal Intelligence: Self-knowledge and the ability to act adaptively based on that knowledge. (Armstrong, 1994)

These seven intelligences are present to varying degrees in all of us, and most people can develop each intelligence to an adequate level of competency. Furthermore, the intelligences usually work together in complex ways, with each individual combining and using them in unique ways. Gardner's MI theory provides a much broader and interactive approach to learning, since there are many ways to be "smart" within each category and since there is no standard set of attributes that one must have to be considered intelligent in a specific area (Quigley, 1994).

The kind of mathematics curriculum for today's technological society that the NCTM 1989 report recommends is congruent to MI theory. The multiple intelligences as presented by Gardner can enhance students' opportunities and options for learning mathematics by allowing them to solve problems, communicate their understanding, reason, and make connections between the various stands of mathematics, and between mathematics and other disciplines.

Gary and Viens (1994) write that it is necessary to appreciate the many differences characteristic of today's school population in order to give students meaningful learning experiences, as both multi-cultural and individualized perspectives emphasize. "Complementing multi-cultural and individualized approaches to education, MI theory offers a framework for understanding cognitive diversity in school" (Gary & Viens, p. 22). A pluralistic view of intelligence allows teachers to provide an educational environment that can better nurture the full range of their students' abilities and interests.

The National Council of Teachers of Mathematics (NCTM) discusses why and how we should change the way mathematics is taught and learned in the United States, and what the mathematics curriculum for grades K-12 should be, as well as how to assess the progress of students. This document encourages educators to make mathematics accessible to all students, in addition to making reference to what and how students learn. The goals put forth in the report state that in our modern technological society, students should regard mathematics as a powerful problem-solving tool, they should become confident in their ability to do math, and they should learn to communicate and reason.

Gardner's MI theory (1983) offers an expanded definition of intelligence. In Frames of Mind (1985), Gardner defines intelligence as more than "short answers to short questions--answers that predict academic success" (p. 4). He states that

> A human intellectual competence must entail a set of skills of problem solving--enabling the individual to resolve genuine problems or difficulties that he or she encounters and, when appropriate, to create an effective product--and must also entail the potential for finding or creating new problems--thereby laying the ground for the acquisition of new knowledge. (pp. 60-61)

The kind of mathematics curriculum for today's technological society that the NCTM 1989 report recommends is congruent to MI theory. The multiple intelligences as presented by Gardner can enhance students' opportunities and options for learning mathematics by allowing them to solve problems, communicate their understanding, reason, and make connections between the various strands of mathematics, and between mathematics and other disciplines. The use of manipulatives to teach mathematics, and the incorporation of MI theory in the classroom, can help accomplish needed curricular and instructional reform in mathematics education.

Problem Solving.

Problem-solving should permeate the mathematics curriculum so that students can apply, with increasing confidence, mathematical problem-solving strategies to solve problems within and outside mathematics (NCTM, 1989).

So reads, in part, the first of the new standards for the teaching of mathematics published by the National Teachers of Mathematics in 1989.

The importance of the school-age populace learning the skills of problem-solving has never been greater. In the Washington Education Association task force paper (1989), John Naisbitt is quoted as saying, "we are drowning in information, but starved for understanding" (p. 22). Furthermore, Theodore Roszak is quoted as saying,

> Information is not knowledge. You can mass produce raw data and incredible quantities of facts and figures. You cannot mass-produce knowledge, which is created by individual minds, drawing on individual experience, separating the significant from the irrelevant, making value judgments (p. 38).

Technology has begun to lessen the importance of the so-called "basics" and has made the "new basics" the ability to reason and to problem-solve. But as important as it is, the likelihood of failure looms even greater.

In the 1960s, the new math movement floundered because it failed to reach one million elementary school teachers of mathematics (Fiske, 1990). Problem solving is destined for the same fate at all levels of the curriculum unless the mathematics community quickly realizes the need to educate its members about how to use and teach problem-solving skills.

While the best American students compare favorably with the very best from other countries, the vast majority are far outperformed. In part, this reflects the deficiency of American students in the basic skills. However, of greater concern is the inability to reason with mathematics. House refers to Alfred North Whitefield's 1929 statement referring to the "inert ideas," or ideas received without being utilized, tested, or thrown into fresh combinations (House, 1988, p. 13). Such learning results in the possession of knowledge, but rarely in the use of knowledge.

This lack of using newly learned concepts in fresh combinations is a serious matter. Moses (1988) states that to help a student remember a new concept, traditionally a relationship has been shown between the new concept and the one previously-learned piece of information. An even better approach is to relate the new concept to not only one element, but to a complex of interrelated information. This network approach emphasizes that the more relationships the student establishes, the better the concept has been learned.

Simon (1986) states that students who are exploring and discovering new concepts for themselves have the opportunity to do mathematics rather than passively learn about mathematics. The discovery approach challenges students to think more deeply about the concept and to create representations and explanations of it that connect with their experience in a personally meaningful way.

Recent theory and research from cognitive psychology suggest that knowledge is stored in the learner's head as a network of concepts and constructs: the mind of the learner is like a construction of tinker toys. Learning is the making of connections between new information and the learner's existing network of knowledge (Peterson, 1988).

Teaching students to learn to reason is a challenge of teaching any subject. A great deal of planning and creativity are required and it is only by focusing on the need and on the task at hand that we math teachers of America will begin to see our students being successful solving extensive and complex problems.

Learning is the making of connections between new information and the learner's existing network of knowledge--the construction of knowledge by the learner--and instruction should simplify these connections. Teachers are being encouraged to teach according to these cognitive strategies.

Although there does exist a substantive knowledge base regarding the psychology of children's classroom learning of mathematics, experienced teachers do not have this knowledge.

However, if teachers are given access to this knowledge, they can enhance their understanding of children's classroom learning in mathematics and improve their classroom instruction.

With this knowledge, teachers can organize instruction so that children can actively construct their own mathematical knowledge (Peterson, 1988).

Students should become creators of their own knowledge--utilizing the power of mathematics to solve a variety of problems, developing their own techniques (algorithms) for various situations, and constructing convincing arguments to validate their own generalizations (NCTM Standards, 1989). A study conducted by Lee V. Stiff (1989) states that the data clearly show that the greater the student's RK (Relevant Knowledge), the better the student learns the concept.

As to teaching strategy, it takes considerable creativity to use time and space effectively, to assure that every minute in your classroom is used productively to engage the students in your subject (House, 1988). Finding the right problem which is challenging, yet possible, engaging and extensive, yet not too time consuming is the great challenge.

Soledad Ulep suggests an approach to teaching linear programming, other than the usual rote procedure, is by having students realize how their ability in solving systems of linear inequalities are useful (Ulep, 1990).

Larry Askins (1989) disagrees about guided practice and coaching. He speaks of teachers of mathematics assigning many routine problems without an intriguing, challenging problem in sight. He believes teachers should take their cue from coaches and put the relevant problems up front. He speaks of a teacher who was a master at involving students and holding up students' ideas for all to see and admire. He paused in all the right places. He encouraged with "Nudge it, push it, let it roll out in front of you" (p. 57). Students leaving his classroom were eager to try out the ideas discovered in class.

The authors of the Standards have assumed that mathematics teachers are:

1. concerned with problem solving,
2. willing to restructure their form of instruction, and

3. desire to use alternative methods of assessment (NCTM, 1989).

Implicit in these assumptions is another and that is that teachers have the time to implement the desired practices. Fiske goes on to say that compared with colleagues in other nations, mathematics teachers in the United States already face a demanding and draining workload.

However, Fiske quotes McKnight et al. (1987) that despite the time-consuming nature of their job, more than half of all eighth grade mathematics teachers think that mathematics is easy to teach (Fiske, 1990).

Marilyn Burns, in her article titled "What to Do and Why," states that not all teachers understand the difference between teaching procedures and teaching reasoning in arithmetic. She goes on to say that the test of whether mathematics instruction is effective is if students can be successful in a lesson by merely performing mechanically, or must they think and reason? If students should know more than just rote process, children must be taught to apply higher-order thinking skills in order to make sense out of what they are being asked to learn, and to understand what they are being asked to do (Burns, 1986).

Lynn Steen (1989) says that there is no better way to learn mathematics than by working in groups, by arguing about strategies, and by expressing arguments carefully in written form. Mathematics teachers must employ classroom strategies that make students active participants in their own learning rather than passive receivers of knowledge. He concludes by saying that curriculum, teaching, and testing must change together to improve mathematics education. Unless all improve in concert, nothing will change.

Cooperative Learning.

Student discovery of mathematical concepts can also be enhanced by pair and group problem-solving (cooperative learning). Students working together on a problem that is not routine for them (for which they do not possess a solution strategy at the outset) are faced with the tasks of generating possible solution strategies, selecting one to try first, deciding how long to stay with that strategy, and determining what strategy to try next.

Through this process, students are led into a knowledge about their own cognitive processes and their ability to use them. Having a storehouse of knowledge and strategies is useless without the ability to select appropriately from that storehouse (Simon, 1986).

According to Slavin (1995), cooperative learning has typically referred to a variety of teaching methods in which students work in small groups to help one another learn academic content. In cooperative classrooms, students are expected to help each other, to discuss and argue with each other, to assess each other's current knowledge and fill in gaps in each other's understanding.

Students in lecture-oriented classrooms may be thinking, but the interactions that occur in cooperative learning classrooms are more productive. Students discuss, observe, compare, imagine, describe, create, listen, evaluate, and solve problems (Slavin, 1995).

There are four basic principles central to the structural approach of cooperative learning: simultaneous interaction, equal participation, positive interdependence, and individual accountability. Simultaneous interaction occurs when there is more than one active participant at a time in a classroom.

Simultaneous interaction gets students actively involved. Students cannot be the outsiders, they must have equal participation in cooperative groups. Only when they take part in discussion can they really learn.

According to Johnson and Johnson (1991), the first requirement for an effectively structured cooperative lesson is that students believe that they sink or swim together (p. 16). In cooperative learning situations, students have two responsibilities: to learn the assigned material and to ensure that all members of the group learn the assigned material. The technical term for this dual responsibility is "positive interdependence." Positive interdependence exists when students perceive they are linked with other members in the group in a way that they cannot succeed unless the other members do.

Individual accountability is another basic principle of cooperative learning. According to Kagan and Kagan (1994), individual accountability is making each member accountable to her own learning or contribution (p. 129). Johnson and Johnson (1991) also state that individual accountability is the key to ensuring that all group members are strengthened by cooperative learning.

In a cooperative learning classroom, the teacher should give students directions, quiet signals, and classroom rules. The desired student role should be both active and interactive. They want to "do" and to talk. The teacher has to tolerate their noise, so the management is very important. According to Kagan and Kagan (1994), cooperative classroom management differs radically from that of the traditional classroom. In the traditional classroom, student behavior means instituting a system to keep students from talking or interacting. In contrast, in the cooperative classroom, student-student interaction is encouraged. The teacher establishes a quiet signal that at any given moment, he or she can quickly focus all attention away from peer interaction and toward the teacher (p. 132).

Bessarear and Davidson (1992) indicated that frequent use of cooperative learning methods in mathematics can foster the following benefits for students:

- Opportunities to discuss and clarify concepts, to exchange ideas freely, ask questions, give and receive help, explore situations, look for patterns and relationships in sets of data, and formulate and test conjectures
- Learning varied approaches for solving the same problem
- Support for problem-solving, logical reasoning, and making mathematics connections
- Learning to communicate the language of mathematics
- The chance to learn from "mistakes" in a nonthreatening environment
- Decreasing math anxiety and increasing math confidence
- Accommodating diverse learning styles
- Making friends with group members across boundaries of race, class, and gender
- Improving the ability to cooperate with others and to develop social skills
- Providing a lively, engaging, and enjoyable mathematics class (p. 249)

A growing number of educators share the view that cooperative group learning can be particularly useful in advancing the development of critical thought among adolescents (Charles, et al., 1990). Ferguson (1955) stated that one advantage favoring the employment of cooperative learning as a vehicle for advancing critical thinking is its increasing popularity among middle and high school classroom teachers (p. 55).

According to Owens (1995), in secondary mathematics, as in many other fields, cooperative learning has been proposed as a solution to a range of problems, both academic and social (p. 154). Davidson (1990) focused on cooperative learning as a way of organizing mathematics classrooms for maximum student participation and peer support, thereby reducing student anxiety and belying the view of mathematics as "isolated, individualistic, or competitive" (p. 24).

Robertson, Davidson, and Dees (1994) have explained how the study of mathematics has often been viewed as an isolated, individualistic, or competitive subject matter. Students typically work alone and struggle to understand the material or solve the assigned problems (p. 246). Perhaps it is not surprising that many students and adults are afraid of mathematics and develop math avoidance or math anxiety. They sometimes believe that only a few talented individuals can be successful in the mathematical realm. These authorities have concluded, however, that small group cooperative learning can solve these problems in several ways, as follows:

- Small groups provide a social support mechanism for the learning of mathematics. "Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism, and summarize their discoveries in writing" (National Council of Teachers of Mathematics, 1989, p. 79).
- Students learn by talking, listening, explaining, and thinking with others.

- Small-group cooperative learning offers opportunities for success for all students in mathematics.
- Mathematics problems are ideally suited for cooperative group discussion because they have solutions that can be shown objectively.
- The field of mathematics is filled with exciting and challenging ideas that invite discussion.

School Reform and Assessment in Mathematics

One of the many issues in school reform is redefining content standards and benchmarks--what should be taught and when students should be expected to have learned it (Northwest Regional Educational Report, 1994).

Assessment is the means by which we decide what students know and can do. It tells teachers, students, parents, and policy makers something about what students have learned--the mathematical terms they recognize and can use, the procedures they can carry out, the kind of mathematical thinking they can do, the concepts they understand, and the problems they can formulate and save. It provides information that can be used to award grades, to evaluate a curriculum, or to decide whether to review fractions. Authentic assessment can help convince the public and educators that change is needed and that efforts to change mathematics education are worthwhile and overdue. Conversely, ineffective assessment can thwart attempts at change.

Curriculum and Evaluation Standards for School Mathematics (1989), presented by the National Council of Teachers of Mathematics notes that standards tell what students should learn. Teaching standards tell how students learn and how teachers should teach. Delivery standards tell what is necessary of schools so that students can learn and teachers can teach. Assessment standards tell what students should know and can do and how evaluators can judge levels of performance.

Assessment that is out of synchronization with curriculum and instruction gives the wrong signals to all those concerned with education (Mathematical Science Education Board, 1993, p. 29). In addition to providing an over-arching philosophy for mathematics instruction and suggesting change in content emphasis in mathematics, the Curriculum and Evaluations Standards for School Mathematics (1989) advocates changes in assessment strategies used in the classroom as well, since assessment is viewed as an integral part of mathematics instruction. The document called for a shift from traditional testing programs aimed at the lowest level of skill development and designed to determine what students do not know, to alternative assessments that encourage students to demonstrate and communicate their thinking and understanding of the underlying concepts of mathematics.

The Curriculum and Evaluation Standards for School Mathematics (1989) contended standardized testing has major shortcomings when we rely on these tests as our primary (if not sole) source of information regarding student performance and program evaluation. We must recognize the difficulties this practice causes, particularly with respect to fulfilling the vision of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). Conventional standardized tests are generally inconsistent with promoting effective mathematics teaching practices in terms of what is assessed, how it is assessed, and how the results are used (NCTM, 1989).

New assessment strategies and practices need to be developed that will enable teachers and others to assess students' performance in a way that reflects the NCTM's reform vision for school mathematics. As stated in the NCTM's vision (1989), the mathematics we expect students to know and use, the way they have learned it, and how their progress is to be assessed. For school assessment practices to inform educators as they progress toward this vision, it is essential that we move away from the "rank order of achievement" approach in assessment toward an approach that is philosophically consistent with NCTM's vision of school mathematics and classroom instruction.

Many standardized tests emphasize isolated low-level procedural skills at the expense of conceptual understanding and problem solving. A recent study of the six commonly used commercial achievement tests (Romberg, et al., 1989) found that at grade eight, on average, only 1% of the items were problem solving while 77% were computation or estimate. These instruments also offer little assessment of important attitudes or behaviors, such as persistence, flexibility, or creativity.

According to the NCTM Mathematics Assessment Handbook (1991), the commonly used multiple-choice format may yield important and useful data. However, it can have a negative impact on how students are taught and evaluated at the local school level. Factors that do not contribute to good classroom practices include the following:

- Student scores are generated solely from right or wrong answers with no consideration (or credit) given to the student's strategies and processes.

- Routine timing measures how quickly students can respond but not necessarily how well they can think.
- Mathematics tools such as calculators, manipulatives, and measurement devices are not allowed.

Results of tests of this nature tend to be used for four purposes: (a) ranking or sorting students for placement in specific grades or special programs; (b) reporting on the effectiveness of a program or curriculum; (c) comparing teachers, schools, or districts; and (d) making policy decisions such as the allocation of resources (NCTM, 1991).

Recognizing that public demands for accountability and consequently for standardized tests are not likely to disappear in the future, some states are customizing the development of standardized tests that more closely align themselves with both the content and the pedagogy implicit in the NCTM curriculum and evaluation standards. These states have recognized that accountability testing does not in fact drive instruction, and because that is the case, the tests must reflect, as far as possible, what would be the appropriate educational practice.

The National Council of Teachers of Mathematics (1998) comments that in order for external standardized testing instruments to be a more appropriate component of a school's evaluation program, the following issues need to be addressed:

- Scores on standardized tests are only one piece of information and should be viewed in context with other information.

- Students' attitudes, behaviors, and oral and written skills must be assessed to provide a complete picture. Important data is obtained from student questionnaires or interviews.
- Students' strategies and thought processes must be looked at with their answers. Currently, some states' assessments include open-response or open-ended items that can yield such information.
- Test items need a balance of content and cognitive process that is consistent with the ideas in the curriculum and evaluation standards, not over weighing on computations.
- Students must be permitted to use calculators; as the use of these "fast pencils" becomes routine in classrooms, evaluation must keep pace.
- Manipulative usage should be part of the formal evaluation process.

Summary

The research and literature summarized in Chapter 2 supported the following themes:

1. In today's complex and global economy, mathematics plays an important role by preparing individuals to absorb new ideas, adopt to constant change, cope with ambiguity, perceive patterns and solve unconventional problems.
2. Knowing and applying core concepts and principles of mathematics; and thinking analytically, logically, and creatively to form reasoned judgments and solve problems have been identified by the National

Council of Teachers of Mathematics and the Washington state Legislature as an essential learning skill for all students.

3. Current research on instructional options such as Multiple Intelligences Theory, problem-solving, and cooperative learning can be incorporated into a teaching and learning of mathematics and help students to become more engaged with mathematical processes.
4. Assessment is a guidance system of mathematical reform that can be used as a tool to help teachers and students keep track of their progress toward higher standards in mathematics.

CHAPTER 3

PROCEDURES OF THE PROJECT

The purpose of the project was to design and develop a model secondary level mathematical curriculum, in alignment with Washington State Essential Academic Learning Requirements, for the Easton School District in Washington. To accomplish this purpose, a review of current research and literature regarding Washington State Essential Academic Learning Requirements related to secondary mathematics was conducted. In addition, related information from selected sources was obtained and analyzed. Chapter 3 contains background information describing the following:

1. Need for Project
2. Development of Support for the Project
3. Procedures
4. Planned Implementation and Assessment of the Project

Need for the Project

The need for this project was influenced by the following considerations:

1. The writer (Seyed Victor Nourani), a secondary mathematics teacher in the Easton School District since 1996 has been engaged in an ongoing search for instructional methods, techniques, and strategies

to provide a better mathematics curriculum and learning atmosphere for middle and high school students.

2. The need to prepare students at Easton School to meet new higher standards in mathematics enacted by Washington State Legislature in 1993 to improve teaching and learning in Washington State (RCW 28A.630.885).
3. The need to align mathematics curriculum and related activities in the ESD with Washington State Essential Academic Learning Requirements for secondary mathematics (RCW 28A.630.885).
4. The development of this project coincided with the writer's graduate studies in Educational Administration at Central Washington University.

Development of Support for the Project

Beginning with the 1997-98 school year, events occurring at Easton School provided ongoing support for the present project. A Mathematics Learning Improvement Committee was formed and the writer was invited to chair the committee. The committee's mission was to align the K-1 Easton School mathematics curriculum with the Washington State Essential Academic Learning Requirements for mathematics. The committee, which consisted of four elementary teachers and two secondary mathematics teachers, met monthly during the 1997-98 school year and accomplished the task of curricular alignment. During the 1998-99 school year, the committee determined the need for grades K-12 to develop a mathematics continuum consistent with the mathematics curriculum developed by the committee in the 1997-98 school year (see Appendix C).

Mr. Darell Cain, Easton School Superintendent, invited the writer, Seyed Victor Nourani, to begin planning activities related to his forthcoming principal internship. This planning focused on the need for the writer to assume a leadership role in the self-study project.

In addition, Central Washington University's graduate program has allowed the writer to pursue an in-depth study of the research topic that was the subject of this project.

Procedures

An Educational Resources Information Center (ERIC) computer search was conducted to review and obtain background information for developing a model, secondary level, mathematics curriculum aligned with Washington State Essential Academic Learning Requirements. The writer participated in Educational Service District 105-sponsored workshops and specialized training that focused on aligning the Easton School District mathematics curriculum with Washington State Essential Academic Learning Requirements for mathematics. Finally, a hand research of various other resources concerned with the project topic was also undertaken.

Planned Implementation and Assessment of the Project

The model, secondary level, mathematics curriculum aligned with Washington State's ELAR's and instructional strategies developed for this project have been incorporated into lesson plans presented in Chapter 4 of this project. The model curriculum was field-tested and used in the classroom by the writer during the 1998-99 school year. In the coming school year, the writer will use performance-based assessment procedures needed to determine program success. Examples of assessment include classroom observations, student interviews, testing and evaluation of student level of mathematical skill development, and student ability to apply mathematics in day-to-day situations. Samples of Washington State Essential Academic Learning Requirements will be given to students as well. Assessment data obtained will be used to modify the curriculum.

CHAPTER 4

THE PROJECT

The secondary level mathematics curriculum for use by students in the Easton School District, has been designed to coincide two model instructional units for grades 8, 9, and 10, and has been presented in Chapter 4 as follows:

Unit One: Displays . . . . Grade 8

Unit Two: Real Numbers and Area . . . . Grade 8

Unit Three: Exponents and Powers . . . . Grade 9

Unit Four: Slopes and Lines . . . . Grade 9

Unit Five: Surface Area and Volume . . . . Grade 10

Unit Six: Polygons and Symmetry . . . . Grade 10

The six instructional units cited above have been developed in alignment with the Washington State Essential Academic Learning Requirements and related benchmarks (Appendix A).

A MODEL, SECONDARY LEVEL, MATHEMATICS CURRICULUM

DEVELOPED IN ALIGNMENT WITH

WASHINGTON STATE ESSENTIAL ACADEMIC LEARNING REQUIREMENTS

EASTON SCHOOL DISTRICT

Easton School District

Secondary Mathematics

Seyed Victor Nourani, Instructor

1999

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UNIT ONE

DISPLAYS

GRADE 8

UNIT ONE

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DISPLAY

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DISPLAYS

Unit Overview

The display unit introduces the student to the different types of displaying mathematical information. In this unit the students will see, work with, and examine the stem-and-leaf displays, bar graphs, and coordinate graphs. Stem-and-leaf displays are used by some statisticians to explore data. Bar graphs and coordinate graphs appear in daily newspapers and magazines. In mathematical terms, graphs of solutions to equations are fundamental in understanding relationships and underlie much of advanced mathematics.

Student Learning Objectives

Student will be able to . . .

- Identify reasons for having graphs
- Use numerical data in a stem-and-leaf display to solve problems
- Interpret display information in bar graphs
- Discuss the rationale, reading, and construction of bar graph
- Plot and name points on a coordinate graph

Student Learning Activities

Activities will be consistent with unit student learning objectives. Activities include:

- Taking daily quizzes
- Participating in lesson introduction warm-ups
- Participating in class reading
- Completing writing assignments
- Completing mathematics assignments
- Completing a unit project
- Completing a unit test
- Developing a unit portfolio
- Keeping a daily journal

Teaching Strategies

Strategies include:

- Cooperative learning groups
- Problem solving
- Using manipulatives (computers, calculators, games, tools . . .)
- Student-centered instruction (peer tutoring, presentations, oral reports, etc.)

Assessment

Assessment will be consistent with Washington state Essential Academic Learning Requirements and unit student learning objectives.

Assessment includes:

- Daily quizzes
- Classroom assignments
- Communication proficiencies (reading, writing, listening, and oral)
- Unit project
- Unit test
- Student portfolio

Instructional Materials

Instructional materials include:

- Textbook, Transition Mathematics
  Scott Foresman Integrated Mathematics, 2nd Edition
- Algebra Thinking First Experiences, Creative Publications
- Computer and computer software
- Calculator, ruler, paper, overhead projector
- Algebra manipulatives

Unit One: DISPLAY

Introduction Activity for Lesson: ***Graphs and Other Displays***

Material needed: News paper, Paper, Pencil

Work with a partner and write examples of Display graphs you find in the newspaper.

Grade 8

Unit One: DISPLAY
Work with a partner

Introduction Activity for Lesson : ***Bar Graphs***

Unit One: DISPLAY

Introduction Activity for Lesson: **Coordinate Graphs**
Work with a partner

Unit One: DISPLAY
Work with a partner

Unit Project

Grade 8

Unit One : DISPLAY

Assessment Sample

UNIT TWO

REAL NUMBERS AND AREA

GRADE 8

UNIT TWO

REAL NUMBERS AND AREA

TABLE OF CONTENTS

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REAL NUMBERS AND AREA

Unit Overview

The Real Numbers and Area unit introduces the student to the real numbers and, in particular, two applications of infinite decimals. Repeating decimals are shown to be equal to simple fractions in Lesson 1. In Lesson 2, square roots are introduced. A square root of an integer is either an integer or an infinite non-repeating decimal. Square roots are necessary with applications of the Pythagorean theorem in Lesson 3.

Student Learning Objectives

Student will be able to . . .

- Find a simple fraction equal to any terminating or repeating decimal.
- Estimate the square roots of a number without a calculator.
- Know how square roots and geometric squares are related.
- Use the Pythagorean theorem to find lengths of third sides in right triangles.

Student Learning Activities

Activities will be consistent with unit student learning objectives. Activities include:

- Taking daily quizzes
- Participating in lesson introduction warm-ups
- Participating in class reading
- Completing writing assignments
- Completing mathematics assignments
- Completing a unit project
- Completing a unit test
- Developing a unit portfolio
- Keeping a daily journal

Teaching Strategies

Strategies include:

- Cooperative learning groups
- Problem solving
- Using manipulatives (computers, calculators, games, tools . . .)
- Student-centered instruction (peer tutoring, presentations, oral reports, etc.)

Assessment

Assessment will be consistent with Washington State Essential Academic Learning Requirements and unit student learning objectives.

Assessment includes:

- Daily quizzes
- Classroom assignments
- Communication proficiencies (reading, writing, listening, and oral)
- Unit project
- Unit test
- Student portfolio

Instructional Materials

Instructional materials include:

- Textbook, Transition Mathematics
  Scott Foresman Integrated Mathematics, 2nd Edition
- Algebra Thinking First Experiences, Creative Publications
- Computer and computer software
- Calculator, ruler, paper, overhead projector
- Algebra manipulatives

Unit Two: REAL NUMBERS AND AREA

Introduction Activity for Lesson: **Square Roots**

Work with a partner

Without using a calculator or paper and pencil, write the squares of as many integers as you can.

Grade 8

Unit Two: REAL NUMBERS AND AREA

Introduction Activity for Lesson : ***Converting Decimals to Fractions***
Work with a partner

Please note: Text in this unit has been redacted due to copyright concerns.

Grade 8

Unit Two : REAL NUMBERS AND AREA

Unit Two: REAL NUMBERS AND AREA

Work with a partner

Grade 8

Unit Two: REAL NUMBERS AND AREA

Introduction Activity for Lesson: ***The Pythagorean Theorem***

Work with a partner

UNIT THREE

EXPONENTS AND POWERS

GRADE 9

UNIT THREE

EXPONENTS AND POWERS

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EXPONENTS AND POWERS

Unit Overview

The Exponents and Powers unit introduces the student to one of the most used mathematical concepts in modern business, finance, biology, physics, and sociology. In Lesson 1, students will learn about compound interest and how to calculate related problems and discover the difference between annual rate and annual yield. Lessons 2 and 3 stress the uses and representations that arise from exponential growth and contrast them with situations of constant increase or decrease. This work builds on the strong ties between powers and repeated multiplication.

Student Learning Objectives

Student will be able to . . .

- Evaluate integer powers of real numbers
- Calculate compound interest
- Evaluate zero and positive integer powers of real numbers
- Identify and apply the zero exponent property
- Solve problems involving exponential growth and decay
- Graph situations involving exponential growth
- Graph exponential relationships
- Make connections between this unit and real life application of the concepts
- Read, write, and communicate clearly the concepts and information discussed in this unit

Student Learning Activities

Activities will be consistent with unit student learning objectives. Activities include:

- Taking daily quizzes
- Participating in lesson introduction warm-ups

- Participating in class readings
- Completing writing assignments
- Completing mathematics assignments
- Completing a unit project
- Completing a unit test
- Developing a unit portfolio
- Keeping a daily journal

Teaching Strategies

Strategies include:

- Cooperative learning groups
- Problem solving
- Using manipulatives (computers, calculators, games, and tools . . .)
- Student-centered instruction (peer tutoring, presentation, oral reports, etc.)

Assessment

Assessment will be consistent with Washington state Essential Academic Learning Requirements and unit student learning objectives.

Assessment includes:

- Daily quizzes
- Accuracy of classroom assignments (80% or better)
- Communication proficiencies (reading, writing, listening, and oral)
- Unit project
- Unit test
- Student portfolio

Instructional Materials

Instructional materials include:

- Textbook, Algebra
  Scott Foresman Integrated Mathematics, 2nd edition
- Computer and computer software
- Calculator, ruler, paper, overhead projector
- Algebra manipulatives

Grade 9

Unit Three: EXPONENTS AND POWERS

Introduction Activity for Lesson: ***Compound Interest***

Work with a partner

Please note: Text in this unit has been redacted due to copyright concerns.

Grade 9

Unit Three: EXPONENTS AND POWERS

Introduction Activity for Lesson: ***Exponential Growth***

Work with a partner

Grade 9

Unit Three: EXPONENTS AND POWERS

Introduction Activity for Lesson : Comparing Constant Increase and Exponential Growth

Work with a partner

Grade 9

Unit Three: EXPONENTS AND POWERS
Work with a partner

Unit Project

Grade 9

Unit Three : EXPONENTS AND POWERS Assessment Sample

UNIT FOUR

SLOPES AND LINES

GRADE 9

UNIT FOUR

SLOPES AND LINES

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SLOPES AND LINES

Unit Overview

In this unit, students will have a good understanding of the relationship between linear equations and their graphs. The emphasis in this unit is on finding and analyzing equations of lines, and relating the equations to the applications that give rise to them and with the geometry of the plane. The unit begins with rates and students will interpret the slope of a line as a rate. In Lesson 2, a line emerges as a graph of a situation involving a constant rate of change, and the slope is defined in the traditional fashion.

Student Learning Objectives

Student will be able to . . .

- Calculate rates of change from real data
- Find the slope of the line through two given points
- Use the definition of slope
- Use the definition and the properties of slope
- Graph a straight line given a point and the slope

Student Learning Activities

Activities will be consistent with unit student learning objectives.

Activities include:

- Taking daily quizzes
- Participating in lesson introduction warm-ups
- Participating in class readings
- Completing writing assignments
- Completing mathematics assignments
- Completing a unit project
- Completing a unit test
- Developing a unit portfolio
- Keeping a daily journal

- Participating in a field trip to Civil Engineering office

Teaching Strategies

Strategies include:

- Cooperative learning groups
- Problem solving
- Using manipulative computers, calculators, games, and tools . . .
- Student-centered instruction (peer tutoring, presentation, oral reports, etc.)

Assessment

Assessment will be consistent with Washington state Essential Academic Learning Requirements and unit student learning objectives.

Assessment includes:

- Daily quizzes
- Accuracy of classroom assignments (80% or better)
- Communication proficiencies (reading, writing, listening, and oral)
- Unit project
- Unit test
- Student portfolio

Instructional Materials

Instructional materials include:

- Textbook, Algebra
  Scott Foresman Integrated Mathematics, 2nd edition
- Computer and computer software
- Calculator, ruler, paper, overhead projector
- Algebra manipulatives

Grade 9

Unit Four : SLOPES AND LINES

Introduction Activity for Lesson : ***Rate of Change***
Work with a partner

Please note: Text in this unit has been redacted due to copyright concerns.

Grade 9

Unit Four : SLOPES AND LINES

Introduction Activity for Lesson : ***The Slope of a Line***
Work with a partner

Grade 9

Unit Four : SLOPES AND LINES

Introduction Activity for Lesson : ***Properties of Slopes***

Grade 9

Unit Four: SLOPES AND LINES
Work with a partner

Unit Project

Grade 9

Unit Four : SLOPE AND LINES

Assessment Sample

UNIT FIVE

SURFACE AREA AND VOLUME

GRADE 10

UNIT FIVE

SURFACE AREA AND VOLUME

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SURFACE AREAS AND VOLUMES

Unit Overview

This unit develops and discusses the formulas for the surface area and the volume of common three-dimensional shapes: prisms, cylinders, pyramids, cones, and spheres. In this unit, students will learn how to apply these formulas in both theoretical and real situations. Lessons 1 and 2 look at surface area using the concept of a net. Lesson 3 introduces the basic properties of volume.

Student Learning Objectives

Student will be able to . . .

- Calculate lateral areas and surface areas of cylinders and prisms from appropriate length and vice versa
- Apply formulas for lateral and surface areas of prisms and cylinders to real situations
- Calculate lateral areas and surface areas of pyramids and cones from appropriate length, and vice versa
- Apply formulas for lateral and surface area of pyramids and cones to real situations
- Calculate volumes of rectangular prisms from appropriate length and vice versa
- Calculate cube roots
- Apply formulas for volumes of rectangular prisms to real situations

Student Learning Activities

Activities include:

- Taking daily quizzes
- Participating in lesson introductory warm-ups
- Participating in class readings
- Completing writing assignments
- Completing mathematics assignments
- Completing a unit project
- Developing a unit portfolio
- Keeping a daily journal

Teaching Strategies

Strategies include:

- Cooperative learning groups
- Problem solving
- Using manipulatives (computers, calculators, games, tools . . .)
- Student-centered instruction (peer tutoring, presentation, oral report)

Assessment

Assessment will be consistent with Washington state Essential Academic Learning Requirements and unit student learning objectives.

Assessment includes:

- Daily quizzes

- Accuracy of classroom assignments (80% or better)
- Communication proficiencies (reading, writing, listening, and oral)
- Unit project
- Unit test
- Student portfolio

Instructional Materials

Instructional materials include:

- Textbook, Geometry
  Scott Foresman Integrated Mathematics, 2nd edition
- Computer and computer software
- Drawing tools
- Geometry manipulatives

Grade 10

Unit 5 : SURFACE AREA AND VOLUME

Introduction Activity for Lesson : Surface Areas of Prisms and Cylinders

Work with a partner

Please note: Text in this unit has been redacted due to copyright concerns.

Grade 10

Unit 5 : SURFACE AREA AND VOLUME

Introduction Activity for Lesson : *Surface Areas of Pyramids and Cones*

Work with a partner

Grade 10

Unit 5 : SURFACE AREA AND VOLUME

Introduction Activity for Lesson : ***Fundamental Properties of Volume***

Grade 10

Unit Five: SURFACE AREA AND VOLUME

Work with a partner

Grade 10

Unit Five : SURFACE AREA AND VOLUME

Assessment Sample

UNIT SIX

POLYGONS AND SYMMETRY

GRADE 10

UNIT SIX

POLYGONS AND SYMMETRY

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POLYGONS AND SYMMETRY

Unit Overview

This unit introduces students to the definition of reflection-symmetric figures. The definition is then used to deduce that segment, angles, and circles are reflection-symmetric. Lesson 2 establishes the symmetry of the isosceles triangle and uses it to deduce that the base angles are congruent; this theorem is then used to prove theorems involving equilateral angles.

Student Learning Objectives

Student will be able to . . .

- Locate symmetry lines and centers of symmetry of geometric figures
- Apply properties of symmetry to assert and justify conclusions about symmetric figures
- Locate and draw symmetry lines in real world designs
- Locate symmetry lines and centers of symmetry of geometric figures
- Draw polygons satisfying various conditions
- Apply theorems about isosceles triangles to find angle measures and segment length
- Know the properties of various types of triangles and regular polygons
- Write proofs using properties of triangles
- Draw polygons satisfying various conditions
- Apply theorems about quadrilaterals to find angle measures and segment length
- Know the properties of the seven special types of quadrilaterals
- Write proofs using properties of quadrilaterals
- Draw and apply hierarchies of polygons

Student Learning Activities

Activities include:

- Taking daily quizzes
- Participating in lesson introduction warm-ups
- Participating in class readings
- Completing writing assignments
- completing mathematics assignments
- Completing a unit project
- Developing a unit portfolio
- Keeping a daily journal
- Drafting with computer-aided design (CAD)

Teaching Strategies

Strategies include:

- Cooperative learning groups
- Problem solving
- Using manipulatives (computers, calculators, games, tools . . .)
- Student-centered instruction (peer tutoring, presentation, oral report)

Assessment

Assessment will be consistent with Washington state Essential Academic Learning Requirements and unit student learning objectives.

Assessment includes:

- Daily quizzes
- Accuracy of classroom assignments (80% or better)
- Communication proficiencies (reading, writing, listening, and oral)
- Unit project
- Unit test
- Student portfolio

Instructional Materials

Instructional materials include:

- Textbook, Geometry
  Scott Foresman Integrated Mathematics, 2nd edition
- Computer and computer software
- Drawing tools
- Geometry manipulatives

Grade 10

Unit 6 : POLYGONS AND SYMMETRY

Introduction Activity for Lesson : *Reflection Symmetric Figures*

Please note: Text in this unit has been redacted due to copyright concerns.

Grade 10

Unit 6 : POLYGONS AND SYMMETRY

Introduction Activity for Lesson : Isosceles Triangles

Grade 10

Unit 6 : POLYGONS AND SYMMETRY

Introduction Activity for Lesson : Types of Quadrilaterals

Grade 10

Unit Six: POLYGONS AND SYMMETRY
Work with a partner

Unit Project

Grade 10

Unit Six : POLYGONS AND SYMMETRY

Assessment Sample

CHAPTER 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this project was to design and develop a model secondary level mathematics curriculum developed in alignment with Washington state Essential Academic Learning Requirements for Easton School District #28, Easton, Washington. To accomplish this purpose, a review of current research and literature regarding curriculum and instruction related to secondary mathematics and Washington State standards for the Essential Academic Learning Requirements was conducted. Additionally, related information from selected sources was obtained and analyzed.

Conclusions

Conclusions reached as a result of this project were:

1. Preparing our young people for living, learning, and working successfully in the 21st century will call for their ability to apply mathematical sciences in practical and new ways.
2. A need to update and elevate the standards of academic achievement in mathematics to bring school district curricula into alignment with Washington State mandated Essential Academic Learning Requirements to insure compliance with the law.

3. Educational reform demands that educators find ways to make curriculums relevant to students, to involve students in setting their own goals, to vary the ways of learning, to use approaches that employ all of the senses, and to be sure that there are opportunities for relating the knowledge to experience or actually using it.

Recommendations

As a result of this project, the following recommendations have been suggested:

1. To help students succeed in today's complex world, mathematics curricula should reflect on practical applications and new ways of problem solving.
2. To align and carry out the Washington State Essential Academic Learning Requirements into the educational content area.
3. To use approaches that employ all of the senses, and to be sure that there are opportunities for relating the knowledge to experience or actually using it.
4. Other educators seeking to design, develop, and align a secondary mathematics curriculum with Washington state Essential Academic Learning Requirements may wish to adapt and/or use the model curriculum development for this project or undertake further research on this subject to meet their unique needs.

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APPENDIX A

THE ESSENTIAL ACADEMIC LEARNING REQUIREMENTS IN MATHEMATICS

The Essential Academic Learning Requirements in

MATHEMATICS

1. **The student understands and applies the concepts and procedures of mathematics.**

To meet this standard, the student will:

1.1 understand and apply concepts and procedures from number sense
number and numeration, computation, and estimation

1.2 understand and apply concepts and procedures from measurement
attributes and dimensions, approximation and precision, and systems and tools

1.3 understand and apply concepts and procedures from geometric sense
shape and dimension, and relationships and transformations

1.4 understand and apply concepts and procedures from probability and statistics
probability, statistics, and prediction and inference

1.5 understand and apply concepts and procedures from algebraic sense
relations and representations, and operations

2. The student uses mathematics to define and solve problems.

To meet this standard, the student will:

2.1 investigate situations
by searching for patterns and exploring a variety of approaches

2.2 formulate questions and define the problem

2.3 construct solutions
by choosing the necessary information and using the appropriate mathematical tools

3. The student uses mathematical reasoning.

To meet this standard, the student will:

3.1 analyze information
from a variety of sources; use models, known facts, patterns and relationships to validate thinking

3.2 predict results and make inferences
and make conjectures based on analysis of problem situations

3.3 draw conclusions and verify results
support mathematical arguments, justify results, and check for reasonableness of solutions

4. The student communicates knowledge and understanding in both everyday and mathematical language.

To meet this standard, the student will:

4.1 gather information
read, listen, and observe to access and extract mathematical information

4.2 organize and interpret information

4.3 represent and share information
share, explain, and defend mathematical ideas using terms, language, charts, and graphs that can be clearly understood by a variety of audiences

5. **The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.**

To meet this standard, the student will:

5.1 relate concepts and procedures within mathematics
recognize relationships among mathematical ideas and topics

5.2 relate mathematical concepts and procedures to other disciplines
identify and apply mathematical thinking and notation in other subject areas

5.3 relate mathematical concepts and procedures to real-life situations
understand the connections between mathematics and problem solving skills used every day at work and at home

1. The student understands and applies the concepts and procedures of mathematics.

To meet this standard, the student will:

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
1.1 understand and apply concepts and procedures from number sense		
number and numeration		
use objects, pictures, or symbols to demonstrate understanding of whole and fractional numbers, place value in whole numbers, and properties of the whole number system	use pictures and symbols to demonstrate understanding of fractions, decimals, percents, place value in non-negative decimals, and properties of the rational number system	understand and use properties and symbolic representations of real numbers
identify, compare, and order whole numbers and simple fractions	compare and order whole numbers, fractions, and decimals	explain the magnitude of numbers by comparing and ordering real numbers
	understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules	understand concepts of and use processes involving prime and composite numbers, factors and multiples, and divisibility
	understand the concepts of ratio and direct proportion	understand and apply the concepts of ratio and both direct and indirect proportion
computation		
show understanding of whole number operations (+, -, ×, ÷) using blocks, sticks, beans, etc.	understand operations on rational numbers	understand operations on real numbers
add, subtract, multiply, and divide whole numbers	add, subtract, multiply, and divide non-negative fractions and decimals using rules for order of operation	compute with real numbers, powers, and roots
use mental arithmetic, pencil and paper, or calculator as appropriate to the task involving whole numbers	use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving rational numbers	use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real numbers
estimation		
identify situations involving whole numbers in which estimation is useful	identify situations involving rational numbers in which estimation is sufficient and computation is not required	identify situations involving real numbers in which estimation is sufficient and computation is not required
use estimation to predict computation results and to determine the reasonableness of answers, *for example, estimating a grocery bill*	use estimation to predict computation results and to determine the reasonableness of answers involving rational numbers, *for example, estimating a tip*	use estimation to predict computation results and to determine the reasonableness of answers involving real numbers, *for example, estimating the interest on a loan*

Approved February 26, 1997
Reformatted June 1998

Mathematics - Essential Academic Learning Requirement 1 (Continued)

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
1.2 understand and apply concepts and procedures from measurement		
attributes and dimensions		
understand concepts of perimeter, area, and volume	understand the relationship among perimeter, area, and volume	understand how changes in dimension affect perimeter, area, and volume
use directly measurable attributes *such as length, perimeter, area, volume/capacity, angle, weight/mass, money, and temperature* to describe and compare objects	measure objects and events directly or using indirect methods *such as finding the area of a rectangle given its length and width*	measure objects and events directly or use indirect methods *such as finding the volume of a cone given its height and diameter*
	understand the concept of rate and how to calculate rates and determine units	calculate rate and other derived and indirect measurements
approximation and precision		
understand that measurement is approximate	understand that precision is related to the unit of measurement used and the calibration of the measurement tool	understand that the precision and accuracy of measurement is affected by the measurement tools and calculating procedures
estimate to predict and determine when measurements are reasonable, *for example, estimating the length of the playground by pacing it off*	use estimation to obtain reasonable approximations, *for example, estimating the length and width of the playground to approximate its area*	use estimation to obtain reasonable approximations, *for example, estimating how much paint is needed to paint the walls of a classroom*
systems and tools		
understand the benefits of using standard units of measurement for measuring length, area, and volume	understand the benefits of standard units of measurement for both direct and indirect measurement	understand the benefits of standard units of measurement and the advantages of the metric system
understand appropriate units of measure for time, money, length, area, volume, mass, and temperature	understand the relationship among units within both the U.S. and metric systems	compare, contrast, and use both the U.S. and metric systems
use appropriate tools for measuring time, money, length, area, volume, mass, and temperature	select and use tools that will provide an appropriate degree of precision, *for example, using meters vs. kilometers*	select and use tools that will provide an appropriate degree of precision, *for example, using kilometers vs. light years*

Mathematics - Essential Academic Learning Requirement 1 (Continued)

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
1.3 understand and apply concepts and procedures from geometric sense		
shape and dimension		
use shape and size to identify, name, and sort geometric shapes	use multiple attributes to describe geometric shapes	compare, describe, and classify 2- and 3-dimensional geometric figures
recognize geometric shapes in the surrounding environment, *for example, identify rectangles within windows*	identify and describe objects in the surrounding environment in geometric terms, *for example, describe the triangles that make up a bridge structure*	construct geometric models and scale drawings using tools as appropriate, *for example, designing a house plan or building a model of a bridge*
relationships and transformations		
describe the location of objects relative to each other on grids or maps	describe location of objects on coordinate grids	understand and use coordinate grids
understand concepts of parallel and perpendicular	understand and identify properties and relationships of plane geometry including ray; angle; isosceles; equilateral; and degrees in a circle, triangle, or quadrilateral	identify simple differences between geometric properties of a plane and a sphere
understand concepts of symmetry, congruence, and similarity	construct symmetric, congruent, and similar figures	understand and use properties of symmetry, similarity, and congruence
understand and construct simple geometric transformations using slides, flips, or turns	understand and construct simple geometric transformations using combinations of slides, flips, or turns	understand and construct multiple geometric transformations using combinations of translation, reflection, or rotation
construct simple shapes using appropriate tools *such as a straightedge or a ruler*	use a compass and straightedge, and/or computer software to perform geometric constructions	use a variety of tools and technologies to perform geometric constructions

Mathematics - Essential Academic Learning Requirement 1 (Continued)

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
1.4 understand and apply concepts and procedures from probability and statistics		
probability		
understand the difference between certain and uncertain events	know how to calculate numerical measures of uncertainty for simple events	understand the properties of dependent and independent events
know how to list all possible outcomes of simple experiments	understand procedures for counting outcomes to determine probabilities	understand and use appropriate counting procedures to determine probabilities
understand and use experiments to investigate uncertain events	know how to conduct experiments and simulations and to compare results with mathematical expectations	use both experimental and theoretical methods to determine probabilities
statistics		
know that data can be represented in different forms *such as tabulations of events, objects, or occurrences*	identify how statistics can be used to support different points of view	use statistics to support different points of view, *for example, in a debate or a position paper*
collect data in an organized way	collect a random sample of data that represents a described population	collect data using appropriate methods and technology
organize and display data in numerical and graphical forms *such as tables, charts, pictographs, and bar graphs*	organize and display data in appropriate forms *such as frequency tables, circle graphs, and stem-and-leaf graphs*	organize and display data in appropriate forms *such as tables, graphs, scatter plots, and box plots*
use different measures of central tendency *such as "most often" and "middle"* in describing a set of data	calculate and use mean, median, and mode as appropriate in describing a set of data	calculate and use the different measures of central tendency, variability, and range as appropriate in describing sets of data
prediction and inference		
predict outcomes of simple activities and compare predictions to experimental results	predict outcomes of experiments and simulations and compare the predictions to experimental results	design and conduct experiments to verify or disprove predictions
understand and make inferences based on experimental results using coins, number cubes, spinners, etc.	understand and make inferences based on experimental results	understand and make inferences based on the analysis of experimental results

Mathematics - Essential Academic Learning Requirement 1 (Continued)

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
1.5 understand and apply concepts and procedures from algebraic sense		
relations and representations		
recognize, create, and extend patterns of objects and numbers using a variety of materials *such as beans, toothpicks, pattern blocks, calculator, cubes, or colored tiles*	recognize, create, and extend patterns and sequences	recognize, create, extend, and generalize patterns, sequences, and series
understand and use guess and check in the search for patterns	represent number patterns with tables, graphs, and rules	understand, develop, and express rules describing patterns
represent number patterns symbolically, *for example, using tiles, boxes, or numbers*	represent equalities and inequalities symbolically using =, ≠, >, <, ≤, ≥,	translate among tabular, symbolic, and graphical representations of relations, *for example, displaying information from a table as a graph*
use standard notation in reading and writing open sentences, *for example,* $3 \times \square = 18$	understand and use variables in simple equations, inequalities, and formulas, *for example,* $3x > 18$	represent situations that involve variable quantities with expressions, formulas and equations, and inequalities
operations		
evaluate simple expressions using blocks, sticks, beans, pictures, etc.	evaluate simple expressions	evaluate and simplify expressions
solve simple equations using blocks, sticks, beans, pictures, etc.	set up and solve single-variable equations	create and solve equations and inequalities

2. The student uses mathematics to define and solve problems.

To meet this standard, the student will:

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
2.1 investigate situations		
search for patterns in simple situations use a variety of strategies and approaches	search systematically for patterns in simple situations develop and use a variety of strategies and approaches	search systematically for patterns in complex situations analyze and use multiple strategies
recognize when information is missing or extraneous recognize when an approach is unproductive and try a new approach	identify missing or extraneous information recognize the need to modify or abandon an unproductive approach	identify what information is missing or extraneous and compensate for it analyze an unproductive approach and attempt to modify it or try a new approach
2.2 formulate questions and define the problem		
identify questions to be answered in familiar situations define problems in familiar situations identify the unknowns in familiar situations	identify questions to be answered in new situations define problems in new situations identify the unknowns in new situations	identify questions to be answered in complex situations define problems in complex situations identify the unknowns in complex situations
2.3 construct solutions		
organize relevant information select and use appropriate mathematical tools	organize relevant information from multiple sources ➡ select and use appropriate mathematical tools	organize and synthesize information from multiple sources ➡ select and use appropriate mathematical tools
apply appropriate methods, operations, and processes to construct a solution	➡ apply appropriate methods, operations, and processes to construct a solution	➡ apply appropriate methods, operations, and processes to construct a solution

3. The student uses mathematical reasoning.

To meet this standard, the student will:

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
3.1 analyze information		
interpret and compare information in familiar situations	interpret, compare, and contrast information from a variety of sources	interpret and integrate information from multiple sources
validate thinking using models, known facts, patterns, and relationships	validate thinking and mathematical ideas using models, known facts, patterns, relationships, and counter-examples	validate thinking and mathematical ideas using models, known facts, patterns, relationships, counter-examples, and proportional reasoning
3.2 predict results and make inferences		
make conjectures and inferences based on analysis of familiar problem situations	make conjectures and inferences based on analysis of new problem situations	make and explain conjectures and inferences based on analysis of problem situations
3.3 draw conclusions and verify results		
test conjectures by finding examples to support or contradict them	test conjectures and inferences and explain why they are true or false	test conjectures and inferences by formulating a proof or by constructing a counterexample
support arguments and justify results based on own experiences	support arguments and justify results using inductive reasoning	support arguments and justify results using inductive and deductive reasoning
check for reasonableness of results	➡ check for reasonableness of results	➡ check for reasonableness of results
reflect on and evaluate procedures and results in familiar situations	reflect and evaluate on procedures and results in new problem situations	reflect on and evaluate procedures and results and make necessary revisions

4. The student communicates knowledge and understanding in both everyday and mathematical language.

To meet this standard, the student will:

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
4.1 gather information		
follow a plan for collecting information	develop a plan for collecting information	develop or select an efficient system for collecting information
use reading, listening, and observation skills to access and extract mathematical information from a variety of sources *such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations*	use reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as pictures, diagrams, physical models, oral narratives, and symbolic representations*	use reading, listening, and observation skills to access and extract mathematical information from multiple, self-selected sources *such as pictures, diagrams, physical models, oral narratives, and symbolic representations*
use available technology to browse and retrieve mathematical information from a variety of sources	choose appropriate available technology to browse, select, and retrieve relevant mathematical information from a variety of sources	integrate the use of a variety of available technologies to browse, select, and retrieve mathematical information from multiple sources
4.2 organize and interpret information		
organize and clarify mathematical information in at least one way - reflecting, verbalizing, discussing, or writing	organize and clarify mathematical information by reflecting, verbalizing, discussing, or writing	organize, clarify, and refine mathematical information in multiple ways - reflecting, verbalizing, discussing, or writing
4.3 represent and share information		
express ideas using mathematical language and notation *such as physical or pictorial models, tables, charts, graphs, or symbols*	clearly and effectively express or present ideas and situations using both everyday and mathematical language *such as models, tables, charts, graphs, written reflection, or algebraic notation*	express complex ideas and situations using mathematical language and notation in appropriate and efficient forms
express mathematical ideas to familiar people using everyday language	express mathematical ideas with clarity using both everyday and mathematical language appropriate to audience	express or present mathematical ideas clearly and effectively using both everyday and mathematical language appropriate to audience

5. **The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.**

To meet this standard, the student will:

BENCHMARK 1 - GRADE 4	BENCHMARK 2 - GRADE 7	BENCHMARK 3 - GRADE 10
5.1 relate concepts and procedures within mathematics		
connect conceptual and procedural understandings among familiar mathematical content areas	connect conceptual and procedural understandings among different mathematical content areas	relate and use conceptual and procedural understandings among multiple mathematical content areas
recognize equivalent mathematical models and representations in familiar situations	relate and use different mathematical models and representations for the same situation	relate and use multiple equivalent mathematical models and representations
5.2 relate mathematical concepts and procedures to other disciplines		
recognize mathematical patterns and ideas in familiar situations in other disciplines	identify mathematical patterns and ideas in other disciplines	extend mathematical patterns and ideas to other disciplines
use mathematical thinking and modeling in familiar situations in other disciplines	use mathematical thinking and modeling in other disciplines	apply mathematical thinking and modeling in other disciplines
describe examples of contributions to the development of mathematics *such as the contributions of women, men, and different cultures*	➡ describe examples of contributions to the development of mathematics *such as the contributions of women, men, and different cultures*	➡ describe examples of contributions to the development of mathematics *such as the contributions of women, men, and different cultures*
5.3 relate mathematical concepts and procedures to real-life situations		
give examples of how mathematics is used in everyday life	recognize the extensive use of mathematics outside the classroom, *for example, in banking or sports statistics*	identify situations in which mathematics can be used to solve problems with local, national, or international implications *such as calculating resources necessary for interstate highway maintenance*
identify how mathematics is used in career settings	investigate the use of mathematics within several occupational/career areas of interest	investigate the mathematical knowledge and training requirements for occupational/career areas of interest

APPENDIX B

EASTON SCHOOL

SECONDARY SCHOOL

MATHEMATICS CURRICULUM

Easton School
MATHEMATICS

ESSENTIAL LEARNING 1.2: The student **understands and applies the concepts and procedures of mathematics: measurement**.

COMPONENTS	**Benchmark 2**	**Transition Mathematics**	**Related Materials / Resources** (Math Teacher's Book of Lists *)
Attributes and Dimensions	a) understands and can use different ways in which various objects and events can be measured directly or indirectly b) understands the concepts of and the relationships among linear dimensions, area, and volume c) understands how dimensional changes affect perimeter, area, and/or volume d) understands the concept of rate and the procedures for calculating rates and determining units	Chapter 3 pages 117 Chapter 3 pages 117 Chapter /lesson 3-8, 3-9 pages 158 Chapter/lesson 9-5 pages 493,521,597,	⇒ Math Teacher's Book of Lists: Section 2, pages 65-97 science and technology reinforces concepts. using computer as an aid using graph paper drawing and designing shapes and figures
Approximation and Precision	e) understands that precision is related to the unit of measurement used and the calibration of the measurement tool f) knows how and when to use estimation to obtain reasonable approximations	Chapter 1 pages 16-55	class activities industrial technology discuss distances and times
Systems and Tools	g) understands the benefits of using standard units of measurement h) knows relationships among units within each system, U.S. and metric i) knows how to select and use tools that will provide an appropriate degree of precision	Chapter 3 pages 119-131 Chapter 3 pages 118-136 Chapter 3	science handouts industrial technology

- *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 1.3: The student **understands and applies the concepts and procedures of mathematics: <u>spatial sense</u>**.

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Shape and Dimension	a) knows and can use attributes of shape and size to describe geometric shapes b) understands that objects in nature, architecture, and other items in the surrounding environment can be identified and described in geometric terms	Chapter 5, 11, & 12 Chapter 5	⇒ Math Teacher's Book of Lists: pages 112, 122, and 134 computer aids, geo board discussion of shapes
Relationships and Transformations	c) knows how to describe location of objects on coordinate grids d) understands that properties of plane geometry apply to a curved surface (e.g., the earth) only as an approximation for small areas e) understands and constructs symmetric, congruent, and similar figures f) understands and constructs simple geometric transformations g) knows how to use compass and straightedge and/or computer software to perform geometric constructions	Chapter/lesson 8-3 pages 422 Chapter/lesson 8-5 pages 436 Chapter/lesson 8-6 reflections Chapter/lesson 8-7 reflections Chapter/lesson 8-8 tessellation's	⇒ Math Teacher's Book of Lists: pages 181, 182 (graph paper) science - discussion of latitude and longitude ⇒ Math Teacher's Book of Lists: pages 127, 150 graph paper ⇒ Math Teacher's Book of Lists: page 152,153 hands on - drawing tools possibly using computer aided drawing

• *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 1.4: The student **understands and applies the concepts and procedures** of **mathematics: probability and statistics**.

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Chance	a) knows how to calculate numerical measures of uncertainty for simple events b) knows how to conduct experiments and simulations and to compare results with mathematical expectations c) understands procedures for counting outcomes to determine probabilities	Chapter/lesson 4-9 page 219 (Open Sentences) used throughout the text in problem solving. Chapter /lesson 4-8 pages 214	using ratios to find unknowns by measuring the known lengths than calculating the unknowns. science ⇒ Math Teacher's Book of Lists: pages 255, 257, 258, 260, 261, & 262 (dice and cards)
Data Analysis	d) understands that there can be different interpretations of the same set of data and how statistics can be used and misused to support different points of view or arguments e) knows how to describe a population and to collect a random sample of data that represents it f) knows how to organize and display data in tables, charts, and graphs as appropriate g) understands, calculates and uses mean, median, and mode as appropriate in describing a set of data	Chapter 8 pages 408 Chapter 8 pages 408 Chapter 8 pages 408	⇒ Math Teacher's Book of Lists: Finding Statistics In baseball - page 295 story problems ⇒ Math Teacher's Book of Lists: page 254 science, class activities class activities
Prediction and Inference	h) knows how to predict outcomes of experiments and simulations and to compare the predictions to experimental results i) understands and makes inferences based on experimental results	Chapter/lesson 4-8 pages 216	class activities, outside work class activities, outside work

• *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 1.5: The student **understands and applies the concepts and procedures** of **mathematics: <u>functions and relationships</u>.**

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Relations and Representations	a) knows how to recognize, create, and extend sequences b) knows how to represent number patterns with tables, graphs, and rules c) knows how to symbolically represent equalities and inequalities d) understands and uses variables in simple equations, inequalities, and formulas	Chapter/lesson 6-5 pages 320	class activities ⇒ Math Teacher's Book of Lists: pages 168, 169 ⇒ Math Teacher's Book of Lists: pages 175, 176, class activities
Operations	e) knows how to set up and solve single-variable equations	Chapter 4 pages 176-233	⇒ Math Teacher's Book of Lists: pages class activity

- *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 2: The student **solves problems using mathematics.**

*** Problem solving is found throughout this book. Selected appearances are identified on page T83 in the Index.**

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Investigates Situations	a) conducts systematic and open-ended explorations b) searches systematically for patterns c) develops and uses a variety of strategies d) identifies what information is missing or extraneous e) recognizes when an attempted approach is unproductive and tries to modify it or tries a new approach	Every lesson in this text has an **<u>Exploration question or questions.</u>** Many of these are open-end. Example: page T82 Index Throughout the text different strategies identifying information and using new approaches are discussed and used. Example: page 342 objective A or Example: page T83 Index	**1-12 are covered and discussed throughout the year, students working together to investigate and solve problems.**
Formulates Questions	f) identifies questions to be answered in new situations g) defines problems in new situations h) identifies the unknowns in new situations	Example: page T82 Index Example: page T82 - T83 Index	students develop questions
Constructs Solutions	i) organizes relevant information collected from a variety of sources j) selects and uses tools appropriate for situation and degree of required precision k) uses appropriate strategies l) applies appropriate concepts and procedures from Essential Learning 1, Benchmark 2.	**Covering the Reading and Applying the Mathematics** are found in every lesson of this book. The strategies used to construct solutions are used throughout.	⇒ Math Teacher's Book of Lists: Math in Other Areas - section 5 pages 253 - 303 Outside math activities magazines, newspapers metric / English measurement

• *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 3: The student **uses mathematical reasoning.**

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Analyzes Situations	a) interprets information from a variety of sources b) compares and contrasts information accessed from different sources c) finds examples that contradict a mathematical argument	Many problems in this text interprets information from a variety of sources.	Using mathematical reasoning is discussed throughout the year. Discussion using supplemental materials. <u>magazines, newspapers, etc</u>. Example: negative number times a negative number equals a positive number.
	d) validates thinking and mathematical ideas using models, known facts, patterns, relationships, and proportional reasoning in relatively familiar situations	This book does an excellent job using models or representations that students identify with. Examples are found on every page. Model, relationships, and proportional examples on page T81 in Index.	Other disciplines: science, history, art, and industrial technology.
Predicts Results	e) makes conjectures and inferences based on analysis of problem situations		
Draws Conclusions and Verifies Results	f) tests conjectures and inferences and discusses why they are true or not true g) supports arguments and justifies results using inductive reasoning	Exploration Questions- examples found on T80 in the Index.	
	h) checks for reasonableness of results using concepts and procedures in Essential Learning 1, Benchmark 2.	Assessment Source book, Progress Self-Test, and Chapter Review are all ways to check concepts and procedures.	discussion
	i) reflects on and evaluates procedures and results		discussion

• *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 4: The student **communicates knowledge and understanding in mathematical and everyday language**.

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Gathers Information	a) develops a system for collecting information b) uses reading, listening, and observation skills to access and extract mathematical information from multiple sources such as pictures, diagrams, physical models, oral narratives, and symbolic representations c) chooses appropriate technology to browse, select, and retrieve mathematical information from a variety of sources	One of the basic goals designed in this book is to concentrate on reading and comprehend what is read. The first questions asked after reading each lesson is **Questions Covering the Reading.** Example: Notes on Reading page 7	class activities / science outside sources: science . magazines / newspapers
Interprets Information	d) organizes mathematical information within given parameters e) clarifies mathematical understandings in multiple ways - reflecting, verbalizing, and discussing	Read and discuss every lesson In this text.	class discussion student journal
Represents and Shares Information	f) expresses ideas using mathematical language and notation in multiple forms such as physical or pictorial models, tables, charts, graphs, and algebraic notation g) expresses mathematical ideas with clarity in ways appropriate to audience	Students explain in writing questions pertaining to diagrams pictures and charts. Class discussion	

• *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

Easton School
MATHEMATICS

ESSENTIAL LEARNING 5: The student **makes mathematical connections.**

COMPONENTS	Benchmark 2	Transition Mathematics	Related Materials / Resources
Relates Within Mathematics	a) links conceptual and procedural understandings among a variety of mathematical content areas b) uses equivalent mathematical models and representations c) sees relationships among parts of mathematics	This text is designed with built in extensions covering many areas of study. Examples: Multicultural Connection , Technology Connection, and Science Connections	⇒ Math Teacher's Book of Lists: Math in Other Areas - section 6 Potpourri - section 7
Relates Among Disciplines	d) identifies mathematical patterns and ideas in other disciplines e) uses mathematical thinking and modeling in other disciplines f) describes examples of the contributions of various cultures to the historical development of number systems, measurement, and patterns g) describes examples of the contributions of both women and men to the development of number systems, measurement, and patterns	Throughout this text are examples. Example: Chapter/lesson 1-1 Readings discuss men and women that have made contributions in math. Example: Chapter/lesson 1-1 and 1-2	Uses math concepts in science, art, industrial technology, history, etc. ⇒ Math Teacher's Book of Lists: pages 251-304 ⇒ Math Teacher's Book of Lists: History of Math - pages 307-308 ⇒ Math Teacher's Book of Lists: Famous Mathematicians - page115 Read / report about important contributions people made in math & science:

ESSENTIAL LEARNING 5 (cont.): The student **makes mathematical connections.**

Relates in Real-life Situations	h) recognizes the widespread use of mathematics in daily life i) recognizes the extensive use of mathematics in situations that relate to society j) understands that mathematically related careers are open to all students k) understands that access to many diverse career opportunities is dependent upon mastery of the Essential Learnings in Mathematics	Is discussed throughout the text.	⇒ Math Teacher's Book of Lists: ⇒ Math Teacher's Book of Lists: Mathematical Idioms - page 333 ⇒ Math Teacher's Book of Lists: Careers in Mathematics - page 263 ⇒ Math Teacher's Book of Lists: Books About Careers That Require Mathematics - page 317 - 318

• *The benchmarks describe the necessary knowledge and essential skills students would be expected to achieve at approximately grade 7.*

APPENDIX C

EASTON SCHOOL

SECONDARY MATHEMATICS

CONTINUUM

Easton Math Continuum
7th Grade

Name ____________________
Year __________

Fluent (6th grade)	*7th Grade Continuum*
____Compares and orders fractions and decimals ____Multiplies and divides fractions ____Adds and subtracts decimals	_____ Understands fractions, decimals, percents, place value in non-negative decimals, and properties of the rational number system
____Finds product of two decimals ____Divides decimals by decimals ____Reads and writes numbers having exponents	_____ Understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules
____Identifies prime and composite numbers ____Calculate percents	_____ Understand the concepts of ratio and direct proportion
____Understands concept of ratio and proportion ____Identifies and use metric and standard measurement systems	_____ Add, subtract, multiply, and divide non-negative fractions and decimals using rules for order of operation
____Calculates and use mean, median, and mode to describe a set of data ____Investigates situation, formulate questions, define problem and use appropriate method to solve it	_____ Uses estimation to predict computation results _____ Understands the relationship among perimeter, area, and volume.
____Makes conjectures and inferences based on analysis of new problem situations ____Draws conclusions and verify results	_____ Measures objects and events directly or using indirect methods *such as finding the area of a rectangle given its length and width.*
____Browses and retrieves information from a variety of sources	_____ Understands the concept of rate and how to calculate rates and determine units.
____Represents ideas or situations using models, graphs, tables, charts, written reflection or algebraic notation	_____ Understands that precision is related to the unit of measurement and what tool used.
____Uses different mathematical models and representation of same situation (e.g., bar and circle graph) ____Recognizes use of mathematics outside of classroom and within different occupations	_____ Understands the relationship among units within both the U.S. and metric systems.
	_____ Uses multiple attributes to describe and identify geometric shapes and constructs symmetric, congruent, and similar figures
	_____ Describes location of objects on coordinate grids.
	_____ Understands and identify properties and relationships of plane geometry including ray; angle; isosceles; equilateral; and degrees in a circle, triangle, or quadrilateral
	_____ Understands procedures for counting outcomes to determine probabilities.
	_____ Organizes and displays data in appropriate forms *such as frequency tables, circle graphs, and stem-and-leaf graphs.*
	_____ Understands, setups, evaluates and use variables in simple equations, inequalities, and formulas, *for example, $3x > 18$.*

	_____ Makes conjectures and inferences based on analysis of new problem situations. (Size of car to amount of fuel consumed.)
	_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as texts, pictures, diagrams, physical models, oral narratives, and symbolic representations.*
	_____ Clearly and effectively express or present ideas and situations using both everyday and mathematical language *such as models, tables, charts, graphs, written reflection, or algebraic notation.*
	_____ Uses mathematical thinking and modeling in other disciplines.

Easton Math Continuum
7th & 8th Grade

Name ____________________
Year __________

7th Grade Continuum	*8th Grade Continuum*
_____ Understands fractions, decimals, percents, place value in non-negative decimals, and properties of the rational number system	_______ Uses the concept of square roots and Pythagorean Theorem to solve problems.
_____ Understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules	_____ Uses putting together with overlap model. (Venn diagram.)
_____ Understand the concepts of ratio and direct proportion	____ Uses various methods of operations for solving simple algebraic equations.
_____ Add, subtract, multiply, and divide non-negative fractions and decimals using rules for order of operation	_____ Understands models for multiplication and division of negative numbers and zero.
_____ Uses estimation to predict computation results _____ Understands the relationship among perimeter, area, and volume.	_____ Understands various forms and uses of display graphs and charts.
_____ Measures objects and events directly or using indirect methods *such as finding the area of a rectangle given its length and width.*	_____ Understands the patterns leading to multiplication such as rectangular arrays and uses the model to calculate surface area and volume of rectangular solids.
_____ Understands the concept of rate and how to calculate rates and determine units.	_____ Uses formulas to calculate of perimeters, areas, and volumes of different geometric shapes.
_____ Understands that precision is related to the unit of measurement and what tool used.	_____ Uses tools to measure and compare angles.
_____ Understands the relationship among units within both the U.S. and metric systems.	_____ Understands the concept of size changes. (Contractions and expansions.)
_____ Uses multiple attributes to describe and identify geometric shapes and constructs symmetric, congruent, and similar figures	_____ Uses methods of slide, reflection, and tessellation to transfer simple geometric shapes on the coordinate graph.
_____ Describes location of objects on coordinate grids.	_____ Uses coordinate axis on graphs and plots points and lines.
_____ Understands and identify properties and relationships of plane geometry including ray; angle; isosceles; equilateral; and degrees in a circle, triangle, or quadrilateral	_____ Understands and compares the relationship and identifies various angles and lines.
_____ Understands procedures for counting outcomes to determine probabilities.	_____ Understands multiplying probabilities of events.
_____ Organizes and displays data in appropriate forms *such as frequency tables, circle graphs, and stem-and-leaf graphs.*	_____ Understands various forms and uses of display graphs and charts.
_____ Understands, setups, evaluates and use variables in simple equations, inequalities, and formulas, *for example,* $3x > 18$.	_____ Understands models for algebraic addition and subtraction as well as solving single variable equations.
_____ Makes conjectures and inferences based on analysis of new problem situations. (Size of car to amount of fuel consumed.)	_____ Understands the concept of proportion defined as means and extremes property to solve unknowns.

_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as texts, pictures, diagrams, physical models, oral narratives, and symbolic representations.*	_____ Understands and uses proportional thinking to get or estimate and answer to a proportion without solving an equation.
_____ Express or present ideas and situations using both everyday and mathematical language *such as models, tables, charts, graphs, written reflection, or algebraic notation.*	_____ Uses proportions property to compare similar figures.
_____ Uses mathematical thinking and modeling in other disciplines.	_____ Uses mathematical thinking and modeling in other disciplines.
	_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as texts, pictures, diagrams, physical models, oral narratives, and symbolic representations.*
	_____ Clearly and effectively express or present ideas and situations using both everyday and mathematical language *such as models, tables, charts, graphs, written reflection, or algebraic notation.*

Easton Math Continuum
8^{th} & 9^{th} Grade

Name ____________________
Year ____________________

8^{th} Grade Continuum	*9^{th} Grade Continuum*
_______ Uses the concept of square roots and Pythagorean Theorem to solve problems.	_____ Applies Pythagorean Theorem to solve advanced problems.
_____ Uses putting together with overlap model. (Venn diagram.)	_____ Understands the difference between intersection and union sets. Uses sets and domain to organize whole numbers, integers and real numbers
____ Uses various methods of operations for solving simple algebraic equations.	_____ Uses variables to create patterns.
_____ Understands models for multiplication and division of negative numbers and zero.	_____ Uses Multiplying Fractions Property to solve algebraic equations.
_____ Understands various forms and uses of display graphs and charts.	____ Uses products and powers with negative numbers to solve algebraic problems.
_____ Understands the patterns leading to multiplication such as rectangular arrays and uses the model to calculate surface area and volume of rectangular solids.	_____ Understands and uses Multiplication Property of Inequality to solve algebraic problems.
_____ Uses formulas to calculate of perimeters, areas, and volumes of different geometric shapes.	_____ Understands and uses the Multiplication of Inequality to solve algebraic problems.
_____ Uses tools to measure and compare angles.	_____ Understands and uses multiplication counting principles to solve problems.
_____ Understands the concept of size changes. (Contractions and expansions.)	_____ Understands and uses factorial and permutations to solve problems.
_____ Uses methods of slide, reflection, and tessellation to transfer simple geometric shapes on the coordinate graph.	_____ Understands and uses models and properties of addition to solve problems.
_____ Uses coordinate axis on graphs and plots points and lines.	_____ Understands and uses the distributive property and adding like terms.
_____ Understands and compares the relationship and identifies various angles and lines.	_____ Understands and uses linear expressions to solve problems.
_____ Understands multiplying probabilities of events.	_____ Uses simple equations to graph linear patterns and to demonstrate algebraic problems.
_____ Understands various forms and uses of display graphs and charts.	_____ Uses graphs to compare linear expressions.
_____ Understands models for algebraic addition and subtraction as well as solving single variable equations.	_____ Understands sums and differences in geometry and uses the triangle inequality to solve problems.
_____ Understands the concept of proportion defined as means and extremes property to solve unknowns.	_____ Understands the algebraic definitions of division, including rates, ratio, relative frequency, probability percents proportions, size change and similar figures.
_____ Uses mathematical thinking and modeling in other disciplines.	_____ Understands and uses rate of change to solve problems.
	_____ Understand the slope of a line to graph algebraic equations and problems.

_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as texts, pictures, diagrams, physical models, oral narratives, and symbolic representations.*	_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as texts, pictures, diagrams, physical models, oral narratives, and symbolic representations.*
_____ Clearly and effectively express or present ideas and situations using both everyday and mathematical language *such as models, tables, charts, graphs, written reflection, or algebraic notation.*	_____ Recognizes the extensive use of mathematics outside the classroom, *for example, in banking or sports statistics.*

Easton Math Continuum
9th & 10th Grade

Name ____________________
Year _____________________

9th Grade Continuum	*10th Grade Continuum*
_____ Applies Pythagorean Theorem to solve advanced problems.	_____ Understands and uses properties and symbolic representations of real numbers
_____ Understands the difference between intersection and union sets. Uses sets and domain to organize whole numbers, integers and real numbers	_____ Uses mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real numbers
_____ Uses variables to create patterns.	_____ Calculates rate and other derived indirect measurements. Measures objects and events directly or use indirect methods *such as finding the volume of a cone given its height and diameter.*
_____ Uses Multiplying Fractions Property to solve algebraic equations.	_____ Uses estimation to obtain reasonable approximations, *for example, estimating how much paint is needed to paint the walls of a classroom.*
____ Uses products and powers with negative numbers to solve algebraic problems.	_____ Understands and computes with real numbers, powers, and roots.
_____ Understands and uses Multiplication Property of Inequality to solve algebraic problems.	_____ Understands the benefits of standard units of measurement and the advantages of the metric system. Compares, contrast, and use both the U.S. and metric systems.
_____ Understands and uses the Multiplication of Inequality to solve algebraic problems.	_____ Constructs geometric models and scale drawings using tools as appropriate, *for example, designing a house plan or building a model of a bridge*
_____ Understands and uses multiplication counting principles to solve problems.	_____ Identifies simple differences between geometric properties of a plane and a sphere.
_____ Understands and uses factorial and permutations to solve problems.	_____ Understands and uses properties of symmetry, similarity, and congruence
_____ Understands and uses models and properties of addition to solve problems.	_____ Understands and constructs multiple geometric transformations using combinations of translation, reflection, or rotation.
_____ Understands and uses the distributive property and adding like terms.	_____ Understands the properties of dependent and independent events.
_____ Understands and uses linear expressions to solve problems.	_____ Uses statistics to support different points of view, organize and display data in appropriate forms such as table graphs, scatter plots, and box plots. Calculate and use the different measures of central tendency, variability, and range as appropriate as describing sets of data.
_____ Uses simple equations to graph linear patterns and to demonstrate algebraic problems.	_____ Recognizes, creates, extends, and generalizes patterns, sequences, and series.
_____ Uses graphs to compare linear expressions.	_____ Evaluates, creates, simplifies and solves expressions, equations and inequalities.
_____ Understands sums and differences in geometry and uses the triangle inequality to solve problems.	_____ Analyzes and uses multiple strategies to solve problems.
_____ Understands the algebraic definitions of division, including rates, ratio, relative frequency, probability percents proportions, size change and similar figures.	_____ Interprets and integrates information from multiple sources.

_____ Understands and uses rate of change to solve problems.	_____ Makes and explains conjectures and inferences based on analysis of problem situations.
_____ Understand the slope of a line to graph algebraic equations and problems.	_____ Organizes clarifies and refines mathematical information in multiple ways - reflecting, verbalizing, discussing, or writing.
_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple sources *such as texts, pictures, diagrams, physical models, oral narratives, and symbolic representations.*	_____ Uses reading, listening, and observation skills to access and extract mathematical information from multiple, self-selected sources *such as pictures, diagrams, physical models, oral narratives, and symbolic representations.*
_____ Recognizes the extensive use of mathematics outside the classroom, *for example, in banking or sports statistics.*	_____ Identifies situations in which mathematics can be used to solve problems with local, national, or international implications *such as calculating resources necessary for interstate highway maintenance.*