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## MAGPIES: Math & Girls + Inspiration = Success: Creating and Implementing a Virtual Math Circle for Girls

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# MAGPIES: Math & Girls + Inspiration = Success: Creating and Implementing a Virtual Math Circle for Girls

### **Cover Page Footnote**

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## MAGPIES: Math & Girls + Inspiration = Success: Creating and Implementing a Virtual Math Circle for Girls

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#### Abstract

During the academic year 2020-2021, we started a virtual math outreach program for upper elementary and middle school girls, MAGPIES: Math & Girls + Inspiration = Success. The online community was created to be an inclusive and collaborative environment for the attending girls, and the lessons were designed to provide a learning experience for all levels of participants. In this article, we provide details about how MAGPIES began, the layout for each event, coordination and training of volunteers, and the impacts on community members. Our hope is to inspire others to start their own (virtual) math circle for girls.

## 1 Introduction

MAGPIES is a virtual math circle for 4th-9th grade girls. Originally designed to provide online outreach opportunities for undergraduate students during the COVID-19 pandemic, it has become a thriving virtual community of upper elementary and middle school girls, Bard College undergraduates, Bard College alumnae, Simon's Rock undergraduates, and volunteer math educators from all over the world. The program is designed to have mentorship at many levels, as each individual member compassionately supports one another's personal and intellectual growth, within and between all levels of the mentorship hierarchy. MAGPIES is a program that exists based on the passion and dedication of each person involved.

### 1.1 How MAGPIES Began

In the Fall of 2019, Professor Lauren Rose taught a course, Math 116: *Mathematics of Puzzles and Games*, to non-math majors at Bard College. The main goal was to get them excited to learn math through the use of puzzles and games. As an "Engaged Liberal Arts and Science" class, the course had a required community engagement component. The students helped to develop outreach activities and brought puzzles and games to nearby schools, libraries and local STEM events for K-8 students. Bard students would sit at a table and guide K-8 students through activities involving Sudoku, SET, Nim, and the Rubik's cube. "It was rather sneaky of me to do this, because the Bard students, most of whom did not have a love for math, were supposed to instill a love for math in the K-8 students. In the process, many of them came out of the course with a new appreciation for, and a raised confidence level in, math," shares Lauren. Several of the students went on to take additional math and computer science courses.

In Fall 2020, during the COVID-19 pandemic, the Puzzles and Games class was offered again, but this time as a virtual, synchronous course. The two primary challenges for the course design were (1) to engage students with games and puzzles in a similar manner to the prior year, and (2) to have students participate in outreach activities during the pandemic. Fortunately, the Julia Robinson Math Festival (JRMF), an organization that started online webinars for K-12 students and teachers in the early days of the pandemic, had already developed a format that had found relative success. Lauren, who had been volunteering with JRMF throughout the pandemic, decided to start an online math circle, using the JRMF webinars as a model, with Bard student facilitators and local participants.

With a clearer idea for how to design the outreach component of her

Fall course, Lauren and her course assistant, Jazmin Zamora Flores, a senior in mathematics at Bard College, spent several Zoom sessions brainstorming about what this could look like for their particular course. Bard alumna Joy Sebesta, currently a community college instructor, joined the planning sessions. They wanted to make sure to have a critical mass of female participants since many math circle programs have an overwhelming majority of male participants. They decided that the best way to do this was to have a girls-only program among participants, but allow facilitators of any gender. While deciding on a name for the virtual math circle, they looked for words with the letters M and G for "Math" and "Girls". They came upon the word "magpies" and discovered that they are among the most intelligent animals, sometimes annoying other birds with their intense curiosity. That was perfect for a math circle for girls! MAGPIES became an acronym for "Math And Girls Plus Inspiration Equals Success" or "Math & Girls + Inspiration = Success".

### **1.2** The Importance of a Girls-Only Math Circle

Data demonstrates that girls and women still face obstacles in STEM despite their average to high performances in these courses. In the 2018 OECD's Programme for International Student Assessments (PISA), data collected from more than 60 countries indicate that few countries have significant differences in average math test scores in favor of boys [5]. In most countries, however, there is either no significant difference in average math test scores, or the difference is in favor of girls. According to the National Girls Collaborative Project, as of 2018, although women make up about 50% of the college educated workforce in the U.S., only 28% of the science and engineering workforce identify as female [4]. This small percentage continues to shrink post-graduation. In "Women in Science, Technology, Engineering, and Mathematics (STEM): Quick Take," an online article published by Catalyst, a global non-profit that helps to build workplaces for women, data is shown that in 2016, women accounted for fewer than one-third of those employed in scientific research and development throughout the world [1]. The 2018 PISA insights report states that the majority male workforce in STEM lies in the young women's lack of *desire* to pursue science, technology, engineering, or mathematics. This absence of aspiration and support for these jobs results in a smaller percentage of young women who go into STEM and STEM-related fields in college. Moreover, women leave STEM-related fields at a higher rate than their male counterparts. It is significant to note here that an even smaller percentage of women in science and engineering are also Latina, Black, or Indigenous. Thus, even more inclusive programming is necessary for these subgroups of young women.

One proven way to increase girls' desire to seek STEM-related fields and create multiple pathways of entry is to have the programming occur in an allgirl space. The *Higher Education Research Institute* (HERI) at the University of California, Los Angeles demonstrates in their 2018 study, "Fostering Academic and Social Engagement: An Investigation into the Effects of All-Girls Education in the Transition to University," that girls who graduate from all girls schools have higher levels of confidence in their scientific knowledge and abilities than girls from co-ed schools [6].

Yet these all-girl spaces need to go beyond simply admitting only femaleidentified students. The *National Coalition of Girls' Schools* state that girls learn best (a) with female role models and strong mentoring, (b) with experiential learning activities, (c) in a collaborative environment, and (d) with frequent feedback [3].

MAGPIES is a program that attempts to change the status quo for women and girls in STEM. At MAGPIES events, all of the young students and the vast majority of facilitators are female-identified. Our non-female facilitators are allies and fully committed to supporting the passion and growth of women and girls. Thus, the girls have many role models, mentors and supporters at each event. MAGPIES events revolve around inquiry-based activities in which the girls are expected to collaborate with one another in order to fully participate in the designed activity, as facilitators provide constant feedback in the form of open-ended guiding questions.

Jazmin Zamora Flores, one of MAGPIES lead undergraduate coordinators, says this about the design of the program:

Our intent for MAGPIES was not to show the coolest math out there or to change the education system. Our goal was simple: to show that math was fun, and something girls could do. As a program, we amplified girls' voices and provided role models and mentors that represent all kinds of backgrounds. We want girls to come into a session and know that they can become anyone they set their minds to.

### 1.2.1 Civic Engagement for Female Undergraduate Retention in Mathematics

MAGPIES, as a math circle, is not only important for fostering interest and increasing retention for the young girls who attend our program. In [2], the authors highlight significant evidence that civic engagement opportunities in the STEM fields also increase confidence and mathematical interest in female undergraduates. This addresses the issue of retention of female students in STEM as described earlier. It is significant to highlight this fact because MAGPIES is unique for empowering women and girls at a variety of stages.

## 2 Structure of the Program

In this section, we outline the goals of the program, a description, and rationale for each goal, and introduce the various roles of the people involved. MAG-PIES is a multifaceted program, designed with several broad goals in mind. *First*, MAGPIES aims to provide virtual opportunities for middle school girls to engage in mathematical activities in a supportive environment, meet other girls who love math, and develop problem-solving skills and mathematical habits of mind. *Second*, MAGPIES aims to use the JRMF Math Circle model of using breakout rooms with a small number of participants guided by experienced facilitators. *Third*, the program intends to provide opportunities for undergraduates to engage in math outreach, develop leadership and facilitation skills, and experience math outside of the classroom setting. *Fourth*, MAGPIES is a laboratory for developing collaborative lesson plans. A *fifth* goal is to build a community of math students and math educators working together. And, *lastly*, a MAGPIES goal is to invite guest presenters who teach participants and facilitators alike new activities and new outreach tools.

This is an ambitious program, made possible by the deep commitment and experience level of the faculty mentors and math educators. There are a variety of ways one could develop a simpler program, depending on the interests and goals of the math circle leaders. We address some of those modifications. The first goal listed is likely to be the cornerstone of any math circle to support girls. The JRMF model requires the coordination of many volunteers as well as running regular training sessions for volunteers. This could be streamlined by limiting the number of participants, having fewer breakout rooms and less formal guidance, or running the program in the main Zoom room without breakout rooms. The parallel goal of mentoring undergraduates makes the MAGPIES program unique, but it does require a good deal of work on the part of the faculty mentors. This is both the most challenging and the most rewarding feature of MAGPIES. Moreover, the majority of the undergraduates are women, and they report that being involved with MAG-PIES has had a profound effect on their view of themselves as mathematicians, as detailed in section 8. Developing lesson plans can be viewed as optional, as there are many ready-made math circle activities that can be freely used and adapted. For example, since the pandemic started, the Julia Robinson Mathematics Festival (JRMF) has created dozens of freely available virtual activities, together with PowerPoint slides, facilitator notes, and easy-to-use apps. We chose to develop our own lesson plans in order to provide activities that would be appealing to most girls, to provide tactile materials in addition to online tools, and to promote active discussions and collaborative problemsolving in the breakout rooms. In addition, we included undergraduates in lesson planning and brainstorming sessions, which enhanced their education as STEM majors. As with most math circle activities, our lesson plans are designed around "low floor, high ceiling" activities, problems that encourage the development of analytical thinking and problem-solving techniques, and are connected to (and often arise from) topics in advanced mathematics. Facilitators of the activities are trained to guide the participants to come up with their own ideas, and participants are discouraged from blurting out answers. Our goal of creating a vibrant community of math educators (and math students) was based on the faculty mentors' extensive experience as volunteers with the JRMF online webinars, training sessions, and problem incubators. The volunteer math educators (many of whom are regular JRMF volunteers or Bard math alumnae) help mentor the undergraduates to run math circle activities, with special attention paid to the needs of young girls, and act as an advisory committee to the faculty organizers, helping to keep the program on track, fresh, and relevant. We relied on guest presenters for our more skillbased activities, such as coding and the Rubik's cube. For most math circle activities, guest presenters are not necessary, although in our case it became a way to expand our community of educators and give participants and undergraduates a chance to meet a variety of STEM professionals. It also gave the faculty organizers a break from having to develop detailed lesson plans for each session.

Most of the organizing was done by Lauren and four undergraduate coordinators, all math majors at Bard College. The Bard Center for Civic Engagement provided a grant to pay the coordinators a small stipend, and the group met weekly to plan events and work on organizational tasks. Starting in Spring 2021, Professor Amanda Landi, a regular volunteer at the fall events, joined Lauren as a faculty organizer and oversaw undergraduate volunteers from Bard College at Simon's Rock. A description of MAGPIES roles (Table 1) gives an idea about how the tasks were divided.

Role	Responsibilities		
Faculty Coordinators	Oversee the program; Invite presenters and		
	math educators; Develop lesson plans; Re-		
	cruit, mentor, and train undergraduates		
Undergraduate Coor-	Logistics; Organizational tasks; Advertising;		
dinators	Communications to participants and volun-		
	teers; Develop and maintain the website; As-		
	sist with lesson plans and training sessions;		
	Co-facilitate breakout rooms and mentor the		
	participants		
Volunteer Math Edu-	Co-facilitate breakout rooms and mentor the		
cators	undergraduates; Brainstorm with the faculty		
	mentors		
Guest Presenters	Develop and present the lesson plan; Coordi-		
	nate with faculty mentors and undergraduate		
	coordinators; Run training sessions		

 Table 1: MAGPIES roles and role descriptions

## 2.1 Layout of a MAGPIES Event

MAGPIES events are held virtually, once per month on a Sunday afternoon. In the fall our sessions were 90 minutes long, but when we discovered that some of the activities seemed rushed, we extended the sessions to two hours in the spring. While specific layouts depended on a particular activity, there was a general template that most sessions followed:

• The event starts with an icebreaker, followed by a 15-30 minute interactive presentation, usually by a math educator, to introduce the topic and provide scaffolding for the breakout room activities.

- The girls are placed into breakout rooms based on grade level, with two facilitators in each breakout room. Usually, there are four to six girls in each breakout room, in order to give each participant individual attention and a chance to speak out in a safe and supportive space.
- Inside the rooms, facilitators do a round of introductions and a brief icebreaker activity, and then guide the girls' thinking to get them started on the activity given by the presenter. The girls explore the problems together, with friendly guidance from the facilitators. They are invited to share observations, ideas, and conjectures which are then discussed as a group.
- After working on the activity for 30 to 60 minutes, students and facilitators come back together in the larger room, where the presenter inquires into the findings discovered by each group.
- Students and facilitators share their results and their experiences, the presenter may provide additional insights, and either an online poll or a jamboard is presented to get feedback from participants.
- Participants say goodbye, but facilitators remain, discussing how the breakout rooms went, sharing their strategies, and suggesting improvements for the next event.

Technologies commonly used during events include Google Jamboard, Zoom's whiteboard with annotation, Google Sheets, and Zoom's chat and polling features. These (free) tools allow for real-time interaction and collaboration, and help to create a dynamic virtual environment for all.

## 3 Fall Lesson Plans

For the first semester programming, we wanted to stick closely to the literature describing how girls learn best (see Introduction). Thus, we selected topics that would more readily allow for a tactile experience, despite the virtual setting. Since many girls learn best with experiential activities, we chose topics where the girls could use printouts, supplies, their creativity, and their hands to participate. For less hands-on activities, we relied on collaborative, community-based learning. Moreover, we designed our activities with math circle core values in mind. In each session, the girls explored meaningful mathematical tasks and questions, our facilitators modeled and fostered problem-solving habit of mind, all the while cultivating a supportive and inclusive community.

September: Domino Necklaces with Prof. Lauren Rose
October: Math and Voting with Prof. Jeff Suzuki
November: Word Cycles with Usha Kotelawala, PhD
December: Ready, SET<sup>®</sup>, Go with Prof. Lauren Rose

### 3.1 Domino Necklaces and Bracelets

The inaugural activity, *Domino Necklaces and Bracelets*, was an adaptation of a lesson plan called 'Domino Circles' which has since been published in the Journal of Math Circles [7]. We wanted the girls to have a tactile experience, so we emailed them domino templates to print out in case they did not own a domino set. We also used an app for those who did not have access to a printer. Several of us collaboratively made the slides for the Domino Activity (Figure 1). The girls who came were dedicated, curious and excited, making the first session a great success.

The activities in this lesson build on each other, and as patterns are discovered, students have many "aha" moments. For a multi-session event, or an event with older students, participants can be guided towards a graph theory interpretation of the activities, leading to Euler Circuits and Paths and cycle decompositions of complete graphs. Here is a brief description of the session.

- 1. Students had in front of them a set of Double-6 Dominoes, dominoes cut from the print-out, or an app.
- 2. The presenter described the usual domino connecting rules and gave examples of domino necklaces and bracelets.
- 3. Breakout Room Activities



(a) We used the Zoom Annotation tool to get girls involved, even during the presentation.



(b) Some presenters used a camera to work hands-on alongside the girls.

Figure 1: Screen shots from the Zoom domino activity

- (a) Activity 1: Using the usual Domino Rules, try to make a domino necklace using all of the dominoes.
- (b) Activity 2: Take out the dominoes with a 6 on them. Now try to make a domino necklace using all of the dominoes.
- (c) Activity 3: Take out the 5's, then the 4's, 3's, and 2's. In each case, record whether you can make a domino necklace using all of the dominoes.

- (d) Discussion: At this point students will likely see a pattern and will discuss why some of the sets work and others do not. The facilitators may ask what would happen if you have larger sets of dominoes, for example a Double-9 set.
- (e) After this, the facilitators could choose from several additional activities, depending on the background and interests of the participants. We describe two of these below.
- (f) Activity 4: Make as many domino bracelets that you can, using a complete Double-6 set of dominoes. Share your findings with your group.
- (g) Activity 5: Play the domino bracelet game: Players take turns placing a domino onto an existing line of dominoes (using an app and screen sharing). If a player completes a domino bracelet, they take it and the next player starts a new bracelet. When there are no moves left, the player with the most bracelets wins. Can you figure out a winning strategy?
- 4. Students return to the main Zoom room and share their findings with the larger group.

The informal feedback we received from the girls was that they had fun exploring math through the activity, and many of the participants stated they were likely to attend future sessions.

### 3.2 Math and Voting

Our second event was *Math and Voting*. The presenter, Jeff Suzuki, is a former Bard Professor, currently at Brooklyn College. The slides were created collaboratively by faculty mentors and coordinators.

For the first activity, students were asked to choose a field trip for their class from a list of 4 choices: Musical Theater performance, Horseback riding, Science museum, or Ice Skating. Here, a poll was taken, the results to be analyzed later in the session. The poll was followed by a Breakout Room Activity. In the breakout rooms, the girls were asked to choose a preferred field trip twice, first based on their own preferences, and second based on what they thought was best for the group. This was followed by a group



Figure 2: A slide defining voting systems

discussion to decide (a) what system should be used to determine the field trip based on their preferences, and (b) should they make their choices based on their personal preferences or with others in mind. When these discussions completed, the girls were brought back to the main room, where the presenter introduced three types of voting systems — Plurality , Borda Count, and Instant Run-off. See Slide 2 in Figure 2. A real-world example followed:

The Pittsburgh Zoo is running a naming contest for five cheetah cubs born in 2019. Voters will choose a theme for the names: Pittsburgh Pride, Girl Power, and Africa.

Each voter cast a vote by ranking their choices 1-3. Using the ballot in Figure 3, we analyzed and compared the different voting systems. Using plurality, "Africa" won. Using Borda Count and Instant Run-off, "Girl Power" theme won. Back in the breakout rooms, the girls applied the voting systems to the field trip data, putting their findings in a shared Google spreadsheet. Then they had to decide which Voting System gave the "fairest" results. We encouraged the participants to consider the questions "Why would we want to choose one voting method over another?", "Is there a better system?", and "Is there a fairest system?"

Here is feedback from one of the undergraduate facilitators who was enrolled in MATH 116. This quote does a good job of highlighting the in-depth and thoughtful discussions the participants have in MAGPIES events.

## The Votes are in!

Using this new method, suppose 60 people voted with ballots below

Ranks	15 ballots	10 ballots	14 ballots	1 ballot	20 ballots
1st choice	А	А	Р	G	G
2nd choice	Р	G	G	А	Р
3rd choice	G	Р	А	Р	А

Number of ballots with a given Ranking

A= Africa P= Pittsburg Pride G= Girl Power

Figure 3: Voting ballot for activity

[The girls] had detailed discussions about which method [of counting votes] they thought was the most fair, and they showed real emotional maturity when it came to choosing which option they felt would be the best for a larger group of students. It was really nice to see younger girls thinking about their own moral code and how it applied to the problem of vote counting. I thought it was especially nice to see how seriously they took making sure the voting solutions were fair for everyone, not just representative of what they personally wanted. Girls would admit that while one solution gave them the outcome they wanted, it wasn't the most fair and therefore not the best one to use.

— MATH 116 Student.

### 3.3 Math Word Cycles

Our third session was *Math Word Cycles*. Usha Kotelawala, PhD, presented this session, introducing us to Google Jamboards, which allow for real-time collaboration. In keeping with our goal to give students a tactile experience, the girls had a choice of using the Jamboard or printable graph paper that we emailed to them. The goal of this session was to create visual representations of words, first by coding a word with a sequence of numbers and then by creating a graph based on the numerical sequence. In Figure 4, the word "marie" is coded to get "41995".



(a) Coding the word "marie" to a series of numbers

### Marie



# Closure occurs when we complete a number segment on the starting point.

(b) The representation of "marie" into an image

Figure 4: Pictures from the Word Cycles Activity

The graphical representation of the sequence is determined by drawing line segments of length n, where n is the current number in the sequence, first in a downward direction, then right, then left, then up, repeating the sequence until a pattern materializes. The code of the word "marie" to "41995" can be drawn by randomly selecting an initial spot on the Google Jamboard, going 4 units down, 1 unit to the right, 9 units up, 9 units to the left, and 5 units down. The word is then repeated, keeping the pattern down-right-up-left. In Figure 4, the word cycle of "marie" is completed after repeating the numerical sequence several times.

In the breakout rooms, the girls began by coding and drawing their names, followed by other words of their choosing. Facilitators invited the girls to think about the following questions and to find examples to illustrate answers to the questions.

- 1. How many cycles of your name did you do to complete your image?
- 2. Why do you think some words don't complete a cycle?
- 3. What kind of words might give us a certain shape, like a square?
- 4. What happens if a word is longer? Shorter?



Figure 5: A Google Jamboard was used to demonstrate how to create a math word cycle. Two examples are in this figure.

This was the most popular event of the fall semester. The basic activity was easy to do, and each of the girls were able to have a success. Some went further and addressed the questions, while others used the names of others in their family and/or enjoyed creating artistically pleasing cycles. Many of the girls shared their creations proudly in the main breakout room by holding up their drawings in their Zoom box.

### 3.4 Ready, SET<sup>®</sup>, Go

Our final session of the fall was a mathematical exploration of the popular card game SET<sup>®</sup>. We used a new technology called "Flippity," where items

can be moved around on a webpage, and each girl could go online and work on their board. Although we shared a screen to play SET together, the girls were encouraged to use their own SET cards or the printables we emailed to them. We made sure that all printables were in black and white since most students do not have color printers at home.

We introduced the game using only 9 cards with different numbers and shapes. We invited them to guess which groups of 3 cards would make a SET, guiding them toward the rules of the general 81 card game. See Figure 6 to see the initial 9-card layout. We then moved to 3 colors and 27 cards and invited them to figure out what SETs looked like in this case.



Figure 6: A picture of the opening explanation to SET

In breakout rooms, the task at hand was to find all possible sets among the initial 9-card layout and also to try to explain why their collection was complete. Students were encouraged to draw SETs on a piece of paper or annotate on the Flippity-9 on a shared screen. Facilitators then posed the following questions:

- 1. How many SETs are all different in both number and shape?
- 2. How many are alike in one and different in the other?
- 3. Do you notice any patterns?

Following this breakout room activity, everyone went back to the main room, where the presenter introduced the full deck of SET cards, with four attributes and 81 cards total. In a second breakout session, students worked to find and answer questions about SET using Flippity-27 and Flippity-81 (see Figure 7).



Figure 7: Flippity-27 and Flippity-81

This activity worked well even though some of the girls already knew how to play SET. None of them had been exposed to the combinatorial questions posed, and those who were new to the game learned quickly.

## 4 Spring Lesson Plans

In Spring 2021, the topics selected for MAGPIES events were highly influenced by input from the girls in attendance during the last event of the Fall. Girls had responded well to suggestions of Math and Art, Rubik's cube, and computer programming. Thus, in the Spring semester, our event calendar was

February: Girls Who Cube with Sydney Weaver

March: Girls Who Code with Fred Gluck

April: Graph Theory Games presented by Bard College students

May: Math and Art presented by Bard and Simon's Rock students

All of these events were rooted in community-based learning and hands-on experience, as with the Fall events. While the first two events were led by guest educators, the final two events were run by undergraduate math students from Bard College and Simon's Rock. The April event was designed and presented by Bard College students enrolled in Math 317: Graph Theory, and the May event was designed by the MAGPIES undergraduate coordinators, including Cathy Zhang from Simon's Rock.

### 4.1 Girls Who Cube

The rationale for holding a Rubik's Cube event was threefold. First, the girls expressed excitement when we mentioned it during the fall sessions. Second, the world of speedcubing is overwhelmingly male, and we wanted to encourage more girls to learn this skill. Third, the algorithmic and problem solving skills developed in learning to solve a cube translate well to the computational facility and analytical reasoning skills needed to learn and understand mathematics.

Girls Who Cube was led by Sydney Weaver, a nine-time gold medalist professional speedcuber and Rubik's cube educator. Each girl needed to bring a Rubik's cube to participate. Sydney ran several training sessions to teach facilitators both how to solve and how to teach the cube. Together, we developed an instruction plan to teach girls at each stage of solving (the 10 steps of solving are listed below). During the event, Sydney gave an overview of how to solve the cube, an introduction to the mathematics behind the cube, and concluded her presentation with several demos of her solving a cube and other twisty puzzles. Following this, girls were put into breakout rooms where the facilitators helped the girls learn to solve a Rubik's cube layer by layer.



Figure 8: The Steps of the Cube

The goal of this session was to introduce the girls to the complexities of the Rubik's cube, including the algebraic notation used in the algorithms, and to have each girl complete at least one of steps toward solving a cube. The steps are summarized below and illustrated in Figure 8.

- 1. Get to know the cube, understand the difference between corners, edges and centers
- 2. Learn the algebraic notation used in the algorithms.
- 3. Make a daisy (a white cross with a yellow center).
- 4. Make a white cross with matching centers.
- 5. Solve the first layer, all white on top, matching colors on the sides.
- 6. Solve the second layer.
- 7. Solve the yellow cross.
- 8. Solve the yellow face.
- 9. Solve corners on the 3rd layer.
- 10. Solve the edges on the 3rd layer.

After the breakout rooms, we invited feedback from the girls, asking them to put in the chat how far they got, and then Sydney did a few more demos (solving a cube one-handed, blindfolded, and with her feet!). All of the girls left having solved the daisy (Step 3) and many students were able to complete the first layer and beyond. We encouraged the girls to go online and continue learning to solve the cube with the help of their friends, parents or siblings.

### 4.2 Girls Who Code

For *Girls Who Code*, Fred Gluck, a retired engineer, and Math and Computer Science Educator, presented on a Game Design activity using AgentCubes (https://agentsheets.com/). AgentCubes allows students to make their own 2D and 3D games. Fred began with the logic behind programming a video game using the AgentCubes platform. The goal of the session was for the girls



(a) Sydney showing the girls some math related to the cube



(b) The poster for the event

Figure 9: The Rubik's cube activity

to understand coding concepts such as variables and conditionals and to create a simple video game using the AgentCubes platform.

In the breakout rooms, each girl worked on creating a video game using the same "agents": a floor agent, a traveler agent, a chaser agent, a goal agent, and a wall agent. The floor agent was used to create the world of the game. The objective of the game was for the traveler to obtain the goal agent. If the traveler and chaser agent were to interact, the game would be over. The traveler agent won if it obtained the goal agent. The wall agent was used to create obstacles for the traveler agent. The girls learned about coding by thinking about how their different agents were interacting with each other. Figure 10 shows a demo game with all five agents.

Following the breakout rooms, the girls did a show-and-tell of the worlds they created! Even though everyone learned the same concepts, all the games were vastly different from one another.



Figure 10: Demo game on AgentCubes

### 4.3 Graph Games

Our *Graph Games* session was led by undergraduates in Lauren's Graph Theory course. The undergrads developed activities that incorporated graph theory concepts, and the students stayed in one breakout room as facilitators for each activity visited each breakout room. The four activities they designed were River Crossings, the Art Gallery Problem, Graph Coloring, and Graph as Mazes. We describe two of these activities below.

The Graph Coloring activity was based on the 4 Color Theorem. The goal was to color maps with as few colors as possible, with no colors sharing a border, except for corners. Figure 11 shows a graph coloring of Australia using three colors. The girls went through several activities of increasing difficultly, stopping after each one to discuss observations, ideas, and conjectures. A Jamboard was used so that the girls could work together on the different tasks.



Figure 11: A graph coloring of Australia

The Graphs as Mazes activity was based on Hamiltonian paths and cycles. Upon entering a maze, the girls had to obtain as many keys as possible (see Figure 12) while avoiding a monster. Girls shared how many keys they were able to collect and how many hallways they passed through. Discussions and additional activities followed.



Figure 12: Monster Maze for the Graphs as Mazes activity

### 4.4 Math and Art

Our last event of MAGPIES 2020-2021 was *Math and Art*. Participants attended two breakout room sessions, learning about (1) Tesselations and (2) Frieze Patterns. Polypad and Jamboard were used to conduct the tesselation and Frieze Pattern activity, respectively.

In the Tessellation activity, the girls were shown examples of tessellations using a new online geometry program called Polypad (https://mathigon.org/polypad). They considered and discussed the following questions:

- 1. What shapes can we repeat over and over (in the plane) without leaving gaps?
- 2. What is it about these shapes that makes this work?
- 3. What kind of shapes don't work and why?

The participants were tasked with creating tesselations using one, two, and three shapes. The girls discovered which regular polygons can tesselate the plane, and explored the questions of allowing more than one type of regular polygon. For more than one shape, using regular polygons, the girls explored the question of how many there are to cover a plane. Figure 13 shows three different tesselations created on Polypad.



Figure 13: Tesselations on Polypad

In the Frieze Pattern activity, the girls were introduced to the different one-dimensional symmetries such as reflections, translations, and glide reflections. They were then asked to identify the different symmetries in the letters of the alphabet, shapes, and the seven different Frieze patterns. A Jamboard was used so that the girls could collaborate on the various activities. Figure 14 shows the girls' annotations of the English alphabet and its symmetries.

While some Frieze images were simulated, we also included images of St. Louis Cathedral Basilica, Pavement from the Greek Island of Rhodes, and a Lakota (Sioux) blanket band. We wanted girls to recognize the significance of Frieze patterns and their use throughout different cultures and history.



Figure 14: Jamboard of the alphabet and its symmetries

## 5 Recruiting Participants and Volunteers

In order to advertise the MAGPIES program to participants, we first created a mission statement and a logo (see Table 2 for both).

Once an email blurb was created, a beginning list of participants was needed. Lauren has co-directed Bard Math Circle since 2007 and had a substantial mailing list of families in the Mid-Hudson Valley region of New York State. This would be the first list to send the MAGPIES advertisement to. Another list came from the Mid-Hudson Math Teachers Circle that some of us ran from 2013-2018. Finally, in addition to Jazmin's connections in New York City, the Bard Center for Civic Engagement sent a notice to their community partners. The first MAGPIES event yielded 60 registrations. Although 60 girls had signed up, only about half of the registrants actually attended the session (unsurprisingly, as it was a free event).

Since the initial advertisement, MAGPIES has built its own participant list. Additionally, the MAGPIES media (see section 6) has helped to garner more attention. While most of the participants come from the greater New York area, we have had a number of participants from other parts of the country. We hope to gain more global participation as we begin to work with other institutions in the Open Society University Network (OSUN).

In Fall 2019, the facilitators were primarily MATH 116 students, as part

Mission	To provide an opportunity for girls in 4th through
Statement	9th grade to develop and enhance their problem
	solving skills and become independent and confi-
	dent thinkers. Activities are both accessible and
	open-ended, providing engaging challenges for all
	ages.
Advertising	
Blurb	MAGPIES: Math & Girls $+$ Inspiration $=$ Suc-
	cess is a new monthly online program for girls in
	4th-9th grade. Hands-on Math Activities based
	on puzzles and games will help them to develop
	problem solving skills and become independent
	and confident thinkers. Each session is standalone.
	Fall 2020 Dates: Sept 20, Oct 18th, Nov 15th,
	Dec 13th. For more information or to register (link
	to register)
	π
	λλ
	MAGPIES is a program of the Bard Math Circle
	and the Bard Center for Civic Engagement.

Table 2: The original MAGPIES mission statement and the Fall 2020 advertisement

of their course requirement, Bard alumnae, and volunteer math educators from the JRMF community. In Spring 2020, Amanda, from Bard College at Simon's Rock, advertised the program to Simon's Rock students studying in STEM fields. At least 20 students signed up to volunteer, and 5 of those students regularly attend MAGPIES events.

We used a Google Group to keep track of emails for participants as well as volunteers. To become part of either email list, advertisements now include a

link to the MAGPIES website where people can click the "Subscribe" button on the home page.

## 6 MAGPIES Media

MAGPIES has an official website at bardmagpies.org, created by undergraduate coordinator Julia Crager (see Figure 15), and is currently maintained by undergraduate coordinators from both Bard and Simon's Rock. Julia also designed the logo we use to this day (see Figure 16).



Figure 15: Screen shots of the MAGPIES website

In Spring 2020, MAGPIES was invited to submit a video for a 24 hour Online Math Buffet, sponsored by JRMF. Undergraduate coordinator Shea Roccaforte created a short MAGPIES promotional video, using screenshots and clips from MAGPIES events and Zoom facilitator training sessions. (View on YouTube https://www.youtube.com/watch?v=DL05b5HReLk). Shea interviewed several other undergraduate student coordinators. The video was one of the first pieces where we were able to show the program from the point of view of the coordinators.

MAGPIES has also attempted to break into social media with a Twitter

account, handle @magpiesFTW. The account follows many associated programs, and retweets important messages. However, social media is definitely an area of future improvement for the program.



Figure 16: The MAGPIES logo went through many variations before the final logo was selected

## 7 Logistics, Challenges, and Lessons Learned

The running of the MAGPIES program took a good deal of work on the part of the faculty and student coordinators, although organizational tasks became more streamlined in the spring semester. Initially, we had to learn how to manage mailing lists, do Zoom registrations, use breakout rooms, train facilitators, design a website, design email flyers, use new technologies, and design appropriate lesson plans. This was made possible for several reasons. First, Lauren had been running the Bard Math Circle and the Mid-Hudson Math teacher's circle for many years, and had experience mentoring students to do outreach and designing lesson plans for math enrichment. For example, three of the four activities in Fall 2020 were based on lesson plans she had previously designed. Second, at the start of MAGPIES, Lauren had been volunteering for JRMF for 5 months, running breakout sessions at their weekly webinars, so she had become experienced in online modalities for outreach. Third, we invited Bard alumnae and math educators (most of whom were also JRMF volunteers) to help train and mentor the Bard students, and to be lead facilitators in the breakout sessions. Their participation was immeasurable, and provided opportunities for undergraduates to meet and connect with a variety of math educators.

The planning for each MAGPIES event followed the rough format below.

1. Coordinators meet to go over logistics: one person is assigned to coordinate with the main presenter, the website is updated, an email blurb is updated and sent out to participants, an email blurb is updated and sent out to volunteers, google forms and spreadsheets are used to collect and organize data about volunteers and participants, and training session(s) are planned.

- 2. Lesson plans are created, including a guide for facilitators.
- 3. A facilitator training is held
- 4. The main event!

### 7.1 Lesson Plan Logistics

In the fall, the lesson plans were primarily created by math professionals, but student coordinators helped out by making Google slides and helping out at the volunteer training sessions. In the spring, the coordinators took on more of a role, working directly with the math educators in the first two sessions, and designing their own activities in the later sessions.

Since the Bard students didn't have any expertise in designing lesson plans, Lauren made an assignment for students in Math 317: Graph Theory for students to work in small groups and come up with activities to illustrate concepts they were learning in the course. She worked with each group to come up with a plan, and the students practiced their activities on their classmates. Below is a quote from a graph theory student after presenting at MAGPIES. This quote does a very good job of demonstrating that undergraduate students learned a lot about teaching through participating in MAGPIES. It also illustrates how thoughtful the undergraduate facilitators are while designing and implementing their activities.

[The participants] very quickly made the intuitive leap to solve the simple version of the river crossing problem, and immediately started making informal conjectures about when it was solvable and when not. Thus, I also learned that our activity was probably not complicated enough for all the groups. I think I liked the foundation of our lesson plan, introducing a simple version of the problem and then showing how it can be solved with some mathematical tool and then asking the kids to apply it to a more complicated version, but I think there were better puzzles we could have picked. I also learned that it seems usually a bad idea to wait a long time for people to find an answer to something or to wait for someone to speak up when the answer is extremely obvious. If people are feeling too shy to talk, waiting creates a really tense atmosphere. The better approach seems to be to make sure you have some momentum, so that you're always having something for the students to pay attention to, be that calling on specific students and directly engaging with them, or keeping them engaged with humor or personal anecdotes.

— MATH 317 student.

### 7.2 Training Coordinators and Facilitators

MAGPIES is unique because of its inclusion of undergraduates in all facets of the program. Undergraduates are mentored by the faculty directors and the volunteer math educators, most of whom are regular facilitators with JRMF. The nature of this mentoring includes guidance about working with young students (in particular young girls), online facilitating, creating a friendly and inclusive atmosphere in breakout rooms, working with technology, and developing leadership skills. The great thing about this program is that once the coordinators feel comfortable in their role, they are able to train and mentor other undergraduate volunteers. Initially, we paired each volunteer undergraduate with a seasoned math educator as the lead facilitator, and as time went on the Bard student coordinators became lead facilitators and helped both to run the training sessions and develop lesson plans.

Prior to each event we run training sessions for all volunteers. In these sessions, we not only teach the activity and introduce online tools, but we give facilitators extensive guidance on how to create an inclusive, collaborative and supportive environment for the girls to explore math in their own way and at their own pace. Here is an example of the detailed notes we provided to facilitators: https://tinyurl.com/mathandvoting.

For specialized events, we run several training sessions. For example, for the *Girls Who Cube* and *Girls Who Code* events, we held three training sessions for each event so that the facilitators could (1) learn how to solve the Rubik's cube and/or program with AgentCubes, and (2) learn how to teach

the Rubik's cube (online!) and/or guide the girls in making a video game with AgentCubes. The multiple training sessions helped to create a community among the facilitators, and made this a learning experience for everyone, not just the girls. We provide feedback from two undergraduate facilitators; it demonstrates how useful the training and mentorship was for undergraduate coordinators and facilitators.

Running a MAGPIES training session gave me such a confidence boost as both a leader in STEM and as a peer to my fellow facilitators. It was intimidating to present our lesson plans to the group — these were seasoned math educators and professors! What did I have to offer as a senior mathematics major? As it turns out, I had a lot to bring to the table, and I am so grateful for the opportunity. Not only did we get to create lesson plans for these events, but we got to explain our concepts to students and teachers of all ages. I could see how everyone approached math, and we got to apply our strategies to both children and adults. I enjoyed the training sessions just as much as the events themselves; the trainings allowed us to workshop strategies for getting our points across to the girls. We often discussed the importance of representation for the girls, in both female and male presenters, who were supportive and committed to enhancing the girls' mathematical journeys. At the end of every session, we had a debrief meeting to discuss how the event went. These procedures were invaluable for the constant improvement to the program.

## — Shea Roccaforte, Undergraduate Coordinator, Bard College.

The training sessions helped me not only by providing me with the math-related knowledge needed for an upcoming event, but, more importantly, by teaching me how to communicate with and guide the girls in an interactive, interesting, and inspiring way. Everyone participating in the training sessions was encouraged to give feedback, comments, and edits on both the mathematical content and materials to be presented at the events. We always met after the events as well to self reflect on our performances and find ways to improve. In this way, we were each other's mentors and could learn and reflect together as a larger community. — Cathy Zhang, Undergraduate Facilitator, Bard College at Simon's Rock.

### 7.3 Challenges and Lessons Learned

There were several aspects of running the program that were very challenging since MAGPIES had a small staff of full-time undergraduate students and full-time math professors. Building a program from scratch and without administrative support proved to be daunting and time consuming. We spent a good deal of time on the following tasks:

- Learning all of the technologies required to for a smooth operation
- Developing lesson plans and making slides and/or jamboards
- Advertising to recruit participants
- Advertising to recruit volunteer facilitators
- Planning and running training sessions for facilitators

As with all free events, more participants signed up than actually attended. This added other complications to planning, such as unknown number of facilitators needed, and an inability to create breakout rooms prior to the events. Another challenge with respect to participants is the large age range and skill levels. While we created lessons with this diversity in mind, not knowing who would show up made it difficult to plan the skill level of the activities. Although we faced these challenges, we believe it is important to maintain the intended design of the program since we our goal was to support as many girls as possible.

Volunteer-based programs often result in some volunteers backing out at the last minute, and because the program was virtual, not having a stable internet connection was an issue at times. The comfort and skill levels of facilitators also varied. This was only an initial challenge, though, because students who facilitated repeated sessions became much more comfortable over time. We have learned to adapt to many of these challenges. For example, at this point, we have become skilled in many of the technologies required, so our operations roll out more smoothly. Moreover, many of our facilitators are regular volunteers, so training is not as time-intensive as in the past. Because we can count on a regular set of math educator volunteers, we are not scrambling to find new undergraduate volunteers for each session.

Finally, because undergraduate students graduate or move on, each year we have to train a new crop of coordinators. This is probably the most important concern we have regarding the sustainability of the program. Without institutional buy-in, it may be that MAGPIES serves as a model for those wanting to create a math circle for girls, rather than a long term project at Bard College.

### 8 Impacts

While we did survey participants at the end of each event, as well as send out a cumulative feedback form at the end of the academic year, we did not receive a substantial enough response to include here. At the end of each event, Zoom polls indicated that many of the responses enjoyed the activity and planned to come back! At the end of last Fall, we asked participants to suggest Spring topics, and students responded positively to "math and art" and "Rubik's cube," so we designed our Spring offerings based on their feedback. Many participants came back for several MAGPIES sessions over the academic year, indicating they enjoyed the environment we cultivated for them to learn math and interact with each other. We plan to do better about collecting response data in the future.

In a short amount of time, MAGPIES was transformed into more than just another math circle. It became a community of practice and collaboration. We all came together for the common goal of creating a space for girls to explore their interest in mathematics, but we all learned so much from each other, the girls themselves, and the program we built. The program was sustained through the efforts of many people who strongly believed in the MAGPIES mission. We share some of their stories, and we do so through their unique voices. We hope these stories demonstrate the multifaceted ways in which MAGPIES has transformed volunteers' lives. Impacts from peers:

I really appreciate the [coordinator] students that develop Magpies events. My team was amazing and I am lucky to have worked with them. We chose to do the art gallery problem, which I was unfamiliar with prior to this. Jazmin was a great group leader and she took charge for the most part. I learned a lot from her and hope to do it myself some day.

— MATH 317 Student.

Impacts through mentorship:

The program exposed me to more math majors and enthusiasts who encourage me to continue on my math journey. It has been both inspiring and illuminating, as an undergraduate, to be connected to people who have a career involving math.

— Julia Crager, Undergraduate Coordinator, Bard College.

Impacts from leading the program:

I learned a lot about what it takes to run programs like this — from lesson planning to Zoom technicalities, those skills are important to me. For a program that was completely online, I met many interesting people this past year, educators and students alike. I'm grateful to have worked with educators who have tons of knowledge to share and who are just as dedicated in uplifting girls.

— Jazmin Zamora Flores, Undergraduate Coordinator, Bard College.

Impacts on the adult facilitators:

I was never the girly-girl type that my parents might have preferred, even as I was raised in a relatively progressive household; it's understandable that Black parents have additional concerns about anything they may perceive as a challenge to their children's well being. In my time spent as a facilitator for MAGPIES, I've seen girls and women focused on problem solving through serious play. I've seen sisterhood bonds shaped by shared experience. I've seen curiosity and confidence rise with girls who are taken seriously as learners, collaborators, and contributors. I've seen women grow as math educators and become strong leaders. Girliness here is being a girl with the freedom to triumph and fail – spectacularly and unapologetically. All forms of "girl" are supported and welcomed, and we're doing it all online. Being an extreme introvert and a gender non-conforming person, I am used to being on the outside looking in, which is actually quite comforting for me. Being different and finding relative contentment with striving to be an accomplished outsider in a world that is often dominated by hegemonic masculinity, I had not really understood the dynamic of "girl power" until my recent experiences with MAGPIES.

— A. Gwinn Royal, Math Professor at Ivy Tech Community College.

## 9 Girl's Math JAM Festival



Figure 17: The MAGPIES Math JAM logo

After volunteering ourseves at JRMF webinars for a year, we invited JRMF volunteers to partner with MAGPIES to create a virtual Julia Robinson Math Festival for Girls, as a way to expand our reach. Skona Brittain, from JRMF co-organized the event. The Girls Math JAM Festival was occured in late June 2021 of (JAM refers to JRMF And MAGPIES). We were joined by Cat Lennon

from the Girls Adventures in Mathematics (GAIM), and Leanne Holder, a math professor at Rose-Hulman Institute of Technology. Julia combined the JRMF & MAGPIES logos to produce a new logo for the event — a magpie perched on the graph in the JRMF logo (Figure 17).

We recruited using mailing lists from all four of the organizations present and female-identified facilitators from both JRMF and MAGPIES. We ran two training sessions for 21 volunteer facilitators. Six different activities from the JRMF archives were prepared and each student chose two of them. Almost 100 girls signed up, and over half of them showed up. There were eight breakout rooms and a community-building activity in-between the two sessions. Lauren and Skona visited every breakout room, and, without exception, they found girls and facilitators happily engaged in exploring math together.

We hope to continue such collaborations in the future in addition to encouraging other math circles to either find ways to recruit more girls into their programs or start a math circle for girls. We are not proprietary about the MAGPIES name and would be happy for there to be MAGPIES programs throughout the US and beyond!

### 10 Future Plans

In the first year of this program, we intended to provide a safe, virtual space for girls to explore mathematics, but in fact MAGPIES quickly became a safe, virtual space for all involved. We plan to continue to cultivate this community online in the next academic year.

In summary, we built a large vibrant community of undergraduates and math educators and would like to maintain this community aspect of MAG-PIES. Our facilitators included 30 Bard students, 6 Bard College at Simon's Rock students, 7 Bard Alumnae, 3 professors from the Open Society University Network (Bard, Simon's Rock, and American University of Central Asia), and over 25 volunteer math educators from the US and beyond.

We plan to further broaden our community to include participants in more parts of the US as well as international students. Furthermore, we plan to have more MAGPIES events as part of larger events like the Math JAM, partnerships with other institutions and organizations, and we encourage others to start their own MAGPIES programs.

**Contact Information:** For anyone looking to start a MAGPIES of their own, we invite you to reach out to the authors directly — Lauren Rose (rose@bard.edu), Amanda Landi (alandi@simons-rock.edu). We are happy to share lesson plans, train leaders, and help others to adapt our model to a different setting. To connect with MAGPIES, our email is magpies@bard.edu.

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