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A Critical Review of Current Literature on Modern Arithmetic

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A CRITICAL REVIEW OF CURRENT LITERATURE ON MODERN ARITHMETIC

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INTRODUCTION

"MATHEMATICIANS HAVE NEVER BEEN IN FULL AGREEMENT ON THEIR SCIENCE, THOUGH IT IS SAID TO BE THE SCIENCE OF SELF-EVIDENT VERITIES --- ABSOLUTE, INDISPUTABLE AND DEFINITE. THEY HAVE ALWAYS BEEN IN CONTROVERSY OVER THE DEVELOPING ASPECTS OF MATHEMATICS, AND THEY HAVE ALWAYS CONSIDERED THEIR OWN AGE TO BE A PERIOD OF CRISIS."

Henri Lebesque

Current literature and other communications media have made most of our citizens aware that great changes have been taking place in our world of science and technology. These changes are seriously affecting the lives and future of every person in every country in our now small world. Most significant is the role that mathematics has played and is playing in effecting these changes. The explosive expansion of both the theory and application of mathematics has been the key that has unlocked the mass of learning and discovery that has made all this possible, and scientists claim that the surface has hardly been scratched.

In addition to discovering new areas and uses for mathematics, and probably most important, mathematicians have gained a new insight into the nature and structure of mathematics. G. Bailey Price calls this the "Golden Age of Mathematics". He stated, "more

mathematics, and more profound mathematics, has been created in this period than in all the rest of history." (23:1)

> The ever increasing importance of the role of mathematics in our biological and physical science, in all areas of technology, business and industry has made mathematics vital to our national interests and needs. In order to appreciate its influence on our culture, to react and interact with it, laymen and educators alike must learn to comprehend mathematics with a greater insight than has been true up to the present time. (34:3)

I. THE PROBLEM

Statement of the problem. It was the purpose of this study (1) to review current literature regarding new trends and programs in elementary mathematics; (2) to investigate the need for the adoption of a new mathematics program in the elementary schools; (3) and investigate the need for the retraining of elementary teachers in the new mathematics.

During the past few years millions of dollars have been made available, by the government and private foundations, to groups of mathematicians and educators to study the problem of teaching mathematics in our schools and to prepare new mathematics programs for our schools. At the present time several such groups are busy perfecting programs already in use and extending them to either higher or lower grade levels.

School administrators and teachers desks are being flooded with information and propaganda about specific programs and the new

movement in mathematics teaching. The press, radio, television, professional and popular magazines have given much emphasis to this new mathematics and the general public is developing an awareness to this movement.

> This interest in the "new mathematics" is not the result primarily of the readily recognizable inherent worth of the new but the great desire of leaders to improve mathematics instruction. Much of the interest in this new movement stems from the need for mathematically trained personnel which became evident during World War II, and became much more evident as the great technological, scientific, and industrial achievements of the post war period began to unfold. (31:2-3)

At the same time that this demand for change is being felt throughout the country, many administrators, mathematicians and teachers stand staunchly behind the traditional program of mathematics being taught in our schools --- stating that the present program is adequate and that, if there is a weakness, it is in the teaching of the subject and not in the program itself.

Importance of the study. The commitment of our nation to a full scale scientific and technological development --- to the extensive use of nuclear fusion, digital computers, and automation have placed a heavy burden on our schools. The demand for highly trained scientists, mathematicians, technologists and others in academic pursuits dependent upon mathematical skills and abilities indicates a need for change in the training of our youth. Price states that:

> The present dependency on scientific and technological development have called for an ever increasing number of people,

> highly skilled in the areas of science, mathematics and mathematics teaching and it is highly improbable that there will ever be an adequate supply. (23:1-14)

"Mathematics has now invaded almost every field of study and is becoming more influential in them with each passing day." (23:15-16)

It is of utmost importance that the general populace have an adequate background and understanding of present day mathematics and its language in order to cope with the problems of everyday living. The problem is probably most aptly expressed by the oft quoted "Rockefeller Report on Education" which comments on the highly critical situation in science and mathematics education as follows:

> Though we cannot discuss in detail each of the fields of study, it is worthwhile to say a few words about education in science and mathematics. The public reaction to this subject have been so intense and so diverse that it has not been easy for the informed citizen to appraise the issues. The simplest way to avoid confusion is to keep a few basic ideas firmly in mind.
>
> First, the crisis in our science education is not an invention of the newspapers, or scientists, or the Pentagon. It is a real crisis.
>
> Second, the USSR is not the "cause" of the crisis. The cause of the crisis is our breath-taking movement into a new technological era. The USSR has served as a rude stimulus to awaken us to that reality.
>
> The heart of the matter is that we are moving with headlong speed into a new phase of man's long struggle to control his environment, a phase beside which the industrial revolution may appear a modest alteration of human affairs.
>
> The fateful question is not whether we have done well, or whether we are doing better than we have done in the past, but whether we are meeting the stern demands and unparalleled opportunities of the times. And the answer is that we are not. (24:346)

The race into space with Russia and the launching of

Spudnik I, although they are not directly related to the problem, did turn the attention of the nation toward the schools; due to irresponsible reporting by the press, uncomplementary statements by eager politicians wanting to climb on the bandwagon and by others whose sole aim was to discredit the schools, They all, either directly or indirectly, blamed the schools for the shortage of capable scientists, mathematicians and technologists, who, they claimed, would have kept us in the lead in the race into space.

Limitations of the study. The materials used in this study were limited to a review of current literature on the subject. These included professional and popular periodicals, newspapers, reports of various mathematic study groups, and pamphlets and books on or about mathematics. No attempt was made to survey or evaluate any of the "new" mathematics programs as that would involve an entirely separate study.

II. DEFINITION OF THE TERMS USED

C U P M This is an abbreviation for "The Committee on the Undergraduate Program in Mathematics", which is a committee of the Mathematical Association of America. The purpose of the committee is to develop a broad program of improvement in the undergraduate mathematics curriculum of the nation's colleges and universities. It is in part financed by the National Science Foundation. (16-421)

Modern Mathematics. The term "modern mathematics" when used to refer to suggested changes in the school curriculum refers to an approach to the teaching of mathematics which emphasizes the importance of concepts, patterns and mathematical structures, as well as the development of mathematical skills, as opposed to the traditional approach which places principal stress on computational skill alone. (15:4-5)

The character of the curriculum changes being proposed, in a broad sense, fall into two classifications. These are:

1. Placing the teaching and learning of arithmetic on a sound mathematical basis. Such practice may be referred to as the mathematizing of arithmetic.
2. Introducing into the elementary school curriculum certain topics that have been labeled "modern". Some of these topics formerly were reserved for more advanced mathematics. (32:1-7)

Retraining. In service or college programs specifically designed to give those elementary teachers with limited mathematical background a broad basic understanding and appreciation for mathematics structure based on concepts and principles and their relationships. This program should also include a course in methodology and materials based on teaching concepts, patterns and mathematical structure and discovery, rather than the more common explain, practice and perform method of the traditional mathematics.

Revolution. Refers to the degree of change in the "new mathematics" programs. Many writers on the subject claim that the change in the programs are so great, so far reaching, that it can best be described as a revolution.

S M S G School Mathematics Study Group. A group of

mathematicians, teachers and administrators who, under the sponsorship of the National Science Foundation, have as their main objective the improvement of the teaching of mathematics in our schools, are developing a mathematics program for grades and high school that has the same name as the group.

Traditional Mathematics. The type of mathematics program that has been offered in the public schools since the turn of the twentieth century to the present time and is based primarily on the development of computational skills.

CHAPTER II

REVIEW OF THE LITERATURE

Our present position: Why change? Hundreds of teachers in many schools are deeply involved in working experimentally with many thousands of students in one or another of the new mathematics programs. Reports of progress being made range from a modest "satisfactory" to glowing reports of astounding progress, interest and enthusiasm on the part of the students participating. Yet the total involved in experimental programs is only a fraction of the student body of our nation. Many of the administrators and teachers responsible for the education of the latter group are asking, "Why make a change? What evidence do you have that the new mathematics programs are better than the traditional ones?" Others are sitting back patiently waiting to see what happens.

Many teachers who have taught mathematics for years and have seen their students achieve success in the study of mathematics and science in college, seriously question the need for change or the superiority of the new programs.

Some educators and mathematicians contend that the general direction taken by elementary school mathematics during the past twenty-five years has been good.

Weaver quoting from Brownell's recommendations of 1935 on

the Teaching and Learning of Arithmetic (38:270-271), and Spitzer on the recommendations of Buckingham and himself, in 1947 and 1948 respectively, (30:11) contend that these recommendations are quite in line with many things being stressed in the "new" programs.

Neither author implies that all is well with the present programs but suggest that perhaps it would be better to use the best of the "old" as a basis for the introduction of the "new".

J. Fred Weaver expresses his views thusly:

> The need for improvement in the elementary school mathematics program is clearly recognized. However, any attempt on improvement must be based on a valid appraisal of present status: that a careful consideration of "things as they are" will reveal strengths as well as weaknesses: and it would be a grave error to play up the weaknesses and disregard the strength. (38:269-273)

He continues the theme in another article by stating:

> We must recognize that there is much good in the present elementary school mathematics curriculum. We must start with this, build on it, and not lose strengths we have already gained. (45:95-96)

Spitzer states:

> The improvement in elementary school arithmetic programs, as I see it, will result from the modification of the present programs and not a complete discarding of what we now have and an adoption of anyone of the half-dozen or so new programs that are now being offered. (30:11)

Professor Phillip Jones has voiced a caution regarding attempts to change the mathematics program:

> --- as I read and listen to the expositions of the denunciators and innovators, I feel, happily, that with thought, discussion, and experimental teaching we are progressing toward a substantially improved mathematics program which will, however, be less changed and less radically modified than some of the

> loudest of the early outcries seem to imply --- I think someone should speak out for the "old" mathematics --- and for the conscientious teachers whose valuable experience in teaching the "non-modern mathematics" should not be lost. (18:65-72)

Sage states:

> It is evident that children in the first grade can learn to multiply, those in the second grade can learn much geometry --- but what child needs to know geometry at the second grade level, or how to multiply at the first grade level. The depressing of upper level fields of mathematical specialization further into the elementary curriculum must be stopped. (27:181-189)

Many mathematics teachers claim that the "good" teacher has always done what the new programs are advocating and that they see no reason for the change of content and grade placement that is being recommended. Others, although not adverse to change, are sounding a note of caution. Frances Keppel writes:

> The eager desire for results - fast, dramatic, tangible results --- has led to the temptation to engage in a variety of loose practices under the rubric of "research", and to advance premature claims for the results. There is a haunting danger that too hasty action may result in short-changing the next generation. Just as serious is the danger that irresponsible or misleading claims may lead to public disillusionment and resistance to further change and experimentation. (12:354-357)

W. D. Wall continues with this note of caution:

> --- we should try a pilot run of these new programs on a limited scale and under carefully controlled conditions before we attempt to persuade our colleagues in the schools to adopt new ideas on a large scale. We have no right to ask society and the teachers to accept the new simply because it has proved moderately successful. (32:101)

Read observes: "Select what seems feasible --- for your school. Do not be afraid to experiment; but do not make changes merely to be different, and do not discard the old merely because

it is old." (25:163-174)

REASONS FOR CHANGE

The general contention, of those demanding change in our mathematics curriculum, is that the majority of children never succeed in understanding the real meanings of mathematical concepts. Many students become deft technicians in manipulating complicated sets of symbols, while others are completely baffled by the impossible position in which present day situations place them.

To make matters worse, some proponents of change contend that, there is a general practice in our schools of segregating those students who are mathematically capable into special classes under the leadership of the most able teachers. The balance of the students are told to drop out of mathematics classes or are scheduled into general mathematics classes under more or less mathematically untrained teachers where they can never hope to reach a satisfactory standard of achievement or understanding in mathematics. Dienes concluded a discussion of the above practices by stating that, "the mathematically fit survive by natural selection; the rest get gradually relegated to the mathematical lumber-yard as second-class citizens unfit for initiation into the mysteries of true mathematics." (8:23-24)

Lucienne Felix contends that there is a scandal in the

present day teaching of mathematics and that it is just a repetition of other scandals throughout history in the teaching of the subjects.

> Now that a general conception of the universe has evolved, all educated men, whether or not they are specialists in science, have the right to an explanation which, however elementary, will give them an insight into the new aspects of scientific thought. So the scandal really exists, but it is on the level of teaching, and it is being reduced little by little as necessary adaptations are made. (11:12)

Marshall Stone, in an address to the group developing S M S G remarked:

> The commitment of our nation to an intensive scientific and technological development puts heavy pressure on our schools to produce an increased number of graduates well trained to pursue careers dependent on mathematical skills and abilities. While this pressure is felt especially in high schools and colleges, its effects are reaching the grade schools as well. From the stream of students flowing through our educational systems we must obtain a greater portion willing to pursue mathematical studies to more or less advanced levels, and we must in particular guarantee that among the students leaving grade schools an increased number will have a taste and inclination for mathematics sufficiently strong for them to study further mathematics in high school. This necessitates an improvement and enrichment of the mathematical elements in grade school education. (32:177-179)

The able English mathematician, Z. P. Dienes states the problem thusly:

> At the present time there can hardly be a single member of the teaching profession concerned with the teaching of mathematics at any stage, from infants upward, who can honestly say to himself that all is well with the teaching of mathematics. There are far too many children who dislike it. (8:13)

The principal criticisms or concerns of those who champion the new mathematics seems to fall into the following categories:

> That the majority of students do not like the subject. (13:8)

> That altogether too many students fail in the subject. (7:87)

Too many students are dropped, or drop out of the mathematics program as soon as they are allowed to do so. (8:24)

That there is a lack of training, understanding and appreciation of the subject on the part of teachers. (8:24)

> That, in most instances, the subject is poorly taught. (38:270)

That the traditional arithmetic is the rote learning of rules, the mechanical operation of the four fundamental processes; all of which lead to the acquisition of inert skills. (34:)(13:18)

That the schools are failing to interest students in mathematics or develop the solid basis of competence that is needed for high school and college programs in mathematics. (38:269-270)

Many persons take an extremely dim view of the present day situation. Their appraisal of the present status is expressed by Professor Van Engen:

> In spirit, the present program is a complete stranger to mathematics; in content it lacks modernity; in its exclusive attention to computational aspects of the subject, arithmetic as practiced in the schools is not mathematics. We all recognize that the present elementary mathematics program leaves much to be desired. We would be in a dangerous state of complacency if we felt otherwise. (35:3-6)

When mathematics is taught, wrote George Boehm:

> It is presented mainly as a collection of slightly related techniques and manipulations. The profound, yet simple, concepts get little attention. If art appreciation were taught the same way, it would consist mostly of learning how to chip stones and mix paint. (22:11)

To continue in the same vein, E. P. Northrop remarked: "In saying that there is room for change, I do not mean in the hackneyed, there is always room for improvement sense. I mean urgent, critical need for change." (21:386-393)

THE ROLE OF THE TEACHERS

Many of the critics of the traditional program of mathematics are, in reality, not critical of the program itself, but of the role of the teacher in the classroom and the methodology applied. It is charged by this group, that a great proportion of teachers are poorly trained in this area, have little or no understanding of the underlying concepts of the subject, are not mathematics conscious, are afraid of the subject or have little or no interest or appreciation of the program. They claim that the unpopularity of mathematics and the large number of failures in the field have been the results of the teaching of the subject in the early years, or to be more specific, in the elementary grades.

C. L. Davis in summarizing the 1961 Summer Conference of the National Association of Teachers of Mathematics wrote:

> The speakers were in complete agreement on the need for, and the general direction of, change in the area of elementary curriculum and of teacher training programs --- the negative attitude of many teachers toward mathematics must be altered. Teachers must understand and enjoy mathematics if they are to inspire pupils to do so. Mere computational skill is not enough. Teachers must know the rational. (6:14-18)

> Studies conducted by Glennon, Weaver and Phillips indicate many prospective elementary school teachers in the United States have neither the facility in the computational processes which they are expected to teach nor the firm grasp of the mathematical concepts which underlie these processes. (21:147-151)

> A large number of elementary teacher education programs in both Canada and the United States devote less time to the study of mathematics than any other subject commonly taught in the elementary school. If the quality of mathematics instruction and hence the level of pupil achievement depend, at least, in part, upon the mathematical competence of the teacher, then those who are concerned about the relative mathematical attainment of students might find the solution to this problem lies in improving preparation programs in mathematics for prospective elementary teachers. (21:147-151)

A recommendation of the 1961 Summer Conference in Arithemetic and Mathematics for College Teachers of Arithmetic Curriculum and Methods and Supervisors of Elementary Arithmetic programs was that:

> The key to the improvement of the elementary arithmetic curriculum and to the greater effectiveness of the elementary teacher lies in the inclusion of increased mathematical content in the curriculum of teacher education both at the pre-service and in-service levels. This should lead to the alteration of the arithmetic program in the elementary schools in the direction of greater mathematical understanding developed through a discovery approach so that it may be more universally applicable both to life situations and to further mathematical study. More effective presentation of mathematics to children is predicated on the assumption that teachers are well versed in the most efficient instructional procedures. (6:14-18)

Commenting on the problems current in the elementary school mathematics programs Stone remarked: "If we wish to improve the teaching of grade school mathematics it will be necessary for us to give much better training to the future teachers of elementary mathematics." (32:179)

Beagle on the same subject writes: "There is a woefully inadequate number or competent teachers of elementary mathematics." (22:1-5)

Professor Wilson states:

> Today a high percentage of pupils show no enthusiasm for arithmetic. The attitude of pupils toward mathematics in high school can be changed and it has been changed in school systems where good teaching accomplishes the legitimate aims of arithmetic in grade schools. (46:168-171)

Nuclear physicist, Dr. Edward Teller blames unimaginative instruction for a large part of the "loss" of understanding students to other less demanding fields. "Science and mathematics courses are too frequently taught as dull exercises in fundamentals rather than as intellectual adventures and so fall short of the spirit of the subjects." (39:119)

Dienes surveying the present position of mathematics in the elementary school wrote:

> The limited number of highly skilled mathematics teachers are assigned to the classes for the most capable, in spite of the fact that these pupils are much more able to work for themselves, and the classes for the less competent students and those with little or no ability or understanding are assigned to teachers whose only qualifications in the field is the ability to do computation. (8:24)

Many teachers have had little or no training in mathematics other than that required for a high school diploma or as an entrance requirement to college.

> This is particularly true in the elementary schools where these mathematically untrained teachers, teach the subject as they were taught, however poorly, and that the fear, apprehension, distaste and lack of understanding is quickly transferred

from the teachers to their students. (28:291-294)

Dutton sums up the problem as follows:

> Mathematics is the language of science, and arithmetic is the corner stone of mathematics. Moreover, never before in history have so many people used so much arithmetic in their daily lives. It becomes, then, one of the major responsibilities of the elementary school to supply children of our times with a proper foundation in arithmetic. To do this, it is essential that teachers acquire a background of understanding and skills in facts, processes, application and appreciation of arithmetic. (9:1)

CHAPTER III

NEW MATHEMATICS

Arithmetic, in the past, has been the term used to identify the area of mathematics taught in the elementary school. It, primarily, was classified as a "tool subject" which tended to emphasize the computational phase of the four fundamental processes. Its aim was "social utility", to provide the individual with the equipment necessary to cope with the computational problems of everyday living. The emphasis was only on one phase, "the machinery of mathematics".

The new approach, while certainly not de-emphasizing the importance of computation or the mechanical portion of the subject area --- tends to be more concerned with the science of numbers --- the awareness of structures and relationships - of discovery and understanding. According to Professor Sueltz the greatest single trend in the elementary school is one of attitude toward mathematics.

> The older pattern of explain - practice - perform is being replaced with a new spirit or attitude, a spirit of adventure, of speculation, thinking, discovery leading to understanding and self-projected learning. It is the same spirit of adventure that has established a favorable climate for experimentation. (33:274-280)

Dr. Herbert Spitzer, in discussing this same spirit or attitude relating to creativity and discovery by students of mathematics states:

The effective teacher of mathematics encourages creativity

> by helping pupils discover basic laws or principles of mathematics; he aims at understanding ahead of skills of operation; and he seeks to give students stimulation that comes from accepting and realizing worthwhile goals. (31:5)

Although creativity and discovery are as applicable to the "old" as well as the "new" programs, it has been the "new" that has advocated and stressed this approach, Spitzer continues:

> It is important to note that most mathematics teaching; including the teaching of arithmetic has not given discovery much place. The fact that the "new" mathematics has boosted and is boosting discovery procedures is, then, a boon to all who are interested in improving mathematics instruction. (31:10)

Another seemingly important change of the "old" to the "new" mathematics is the introduction of algebra and geometry very early in the students mathematical experiences. In this manner they are treated quite naturally as a normal part of the field of mathematics and not as separate subjects "bordering on magic and surrounded by mysticism" (11:35); as was often the case when introduced as separate subjects in the secondary school years.

Dienes observed that:

> There is no meaning nowadays in any attempt to separate different branches of mathematics, such as arithmetic and algebra; there is so much connection between them that is impossible to speak about one without introducing some of the other. (8:75)

Fehr made a similar observation when he stated:

> The number and variety of mathematical disciplines have greatly increased in the last sixty years. New braches of knowledge based on mathematical methods have been created. These new conceptions have broken up the traditional compartments that housed arithmetic, algebra and geometry; and the classical treatment of mathematics in schools has therefore become in considerable part obsolete. (11:VII-IX)

Professor Beagle, Director of the S M S G project stated:

> New mathematics is an easy but not particularly accurate description. It isn't the mathematics that is new, but rather the approach, the teaching technique, the rate and order in which the elements are taught, and the emphasis given them. (22:1-5)

There appears to be some misuse or misunderstanding of the tern "new" as it applies to the present day mathematics program as it apparently has distinct and different meanings. First, there is the "new mathematics" that is definitely new to the field as much new mathematics has been discovered in recent years. Secondly, there is the "new" that is only new to the school program. This mathematics new to the program may be mathematics that has been in general use in schools but now moved to a new level in the program or that which has previously been used only by "true mathematicians".

This brings us to the third use of "new" and that is the development of a "language of mathematics". Dr. Fehr stated that the language we use must be clear, clean, concise and coherent instead of ambiguous, unclear, verbose and incorrect, as it was frequently in the past. With the increased importance of mathematics, the invasion of the language of mathematics into most other areas of study, it is highly important that one be literate in mathematics, if one is going to be able to understand the literature of other areas of study.

According to many writers in the field, the amount of new mathematics will be rather limited but will include such things as sets and operation of sets, mappings or matching, number as distinct

from numeral, one-to-one correspondence, order, quantity (less than, more than), operation, base numbers, factors, exponents, number lines, ordered pairs, inequalities, intuitive geometry, modular arithmetic and non-metric geometry, commutative,associative and distributive laws - to name a few.

Several distinguishing characteristics of the "new" mathematics as listed by the National Education Association as quoted by Spitzer include:

1. Explanations are given of the why as well as the how.
2. Extensive use is made of deductive reasoning and proof.
3. The structure of mathematics is emphasized. Mathematics is developed as an organized body of knowledge, founded on a surprisingly small number of assumptions.
4. The discovery method of teaching is utilized. Questions and illustrations, examples often lead the student to make and test conjectures of his own ---
5. Greater emphasis is placed on the precise use of language. Definitions are stated carefully. The ability to read intensively for meaning is essential for success.
6. The new courses are built on unifying ideas (structure, operation and their inverses, logical deductions, valid generalizations, etc.)
(30:1-13)

FUNDAMENTAL ISSUES

Today, mathematical literacy is imperative to a degree that has been unprecedented in the history of the world. On the one hand, it cannot be ascertained just what mathematical knowledge will be required of future citizens when they assume leadership roles in directing society. There is general agreement on one point, however, and that is: young people must be exposed to the kind of mathematics content and instruction that will be basic and adaptable to whatever scientific, technological and social innovations may occur to mankind through chance and necessity. (26:369-372)

Just how this is to be accomplished is a question that is perplexing too many educators who are interested in bringing a better program to their students.

Agreeing that we can no longer predict what the student of today will need to know, it is necessary that the student today learn to deal with problems for which he has no previous instruction. He must learn how to learn. He must learn basic concepts that he will be capable of applying to new situations to reach a logical solution. In other words --- he must learn how to think. This is a basic demand of present day society and the best way to assure thinking, currently and in the future, is to provide practice throughout the students school experience from kindergarten on.

Thinking is based on the understanding of basic concepts and principles. The new mathematics indicate that while some things are best learned directly or authoratively (mermorization), that the spirit and method of discovery may be more lasting and the more valuable part of the learning process.

Experience in discovery in a wide range of situations, is the critical factor in the development of genuine mathematical thinking.

The summary statement of the Twenty-first Yearbook of the National Council of the Teachers of Mathematics predicts that:

> Instruction in mathematics for general education will come to rely more and more on the following:
> Learning is thinking.
> Successful thinking is possible at any grade or achieve-

ment level.
Successful thinking is heavily dependent on concepts and relationships.
The satisfactions following successful thinking provide enduring enrichment for the learner. (5:388-394)

Brownell stated that: "The basic tenet in the proposed instructional reorganization is to make arithmetic less a challenge to the pupils memory and more a challenge to his intelligence." (4:42-45)

CHAPTER IV

TEACHER TRAINING

Howard Fehr stated:

> No one can deny that the new programs appearing in American schools are more challenging to the intellect, more in harmony with contemporary mathematical thought, and far more meaningful to students than those that existed ten or twenty years ago. (10:402-411)

Mathematics has always held a major position in our elementary school program. Educators have the same responsibility or obligation to eliminate the "deadwood", the "outdated", and the unimportant parts of the arithmetic program as they have had in updating science, reading or social studies programs to keep atuned to our rapidly changing times.

However, the creation of new and better programs in mathematics is not the "end" or complete solution to the problem. According to many writers on the subject, it is in fact, only the beginning --- for although mathematics is receiving far greater attention than ever before in history, and has assumed a far greater role of importance in all segments of society --- even though the "new" programs have incorporated in them all the new learning on how students learn --- it still remains true that the classroom teacher is the dominant factor.

> It is the teacher who establishes the atmosphere of the classroom, who selects the appropriate subject matter and adjusts

her methods to meet the needs of the group, it is the teacher who provides for individual differences, who sets the scene that will create the desire to learn and who will establish the standards and goals for the group. (33:277)

Hannon comments:

Certainly as final consideration the quality of instruction obtained depends to a large extent upon the qualifications of the teacher. This aspect of the present day curriculum problem is not an easy one to solve. If the new curriculum is to be properly implemented, the teacher must be adequately trained to carry forward the program. (16:171-177)

Glennon adds:

Many educators are of the opinion that modernizing a school mathematics program is as much a problem of changing the methods by which we teach as it is of changing the content. (12:354-358)

Many persons recognize the problem of teacher education to be the most critical problem we face in our effort to improve the elementary school mathematics program.

According to the Twenty-fifth Yearbook of the National Council of Teachers of Mathematics:

The careful preparation of teachers in mathematics subject matter is the pre-requisite to an improved arithmetic program in elementary schools --- that for too long we have tried to do the impossible - in training teachers for the elementary school we have tried to expect them to provide high-level instruction in virtually all subject matter areas. As a consequence we have developed entirely too many elementary teachers who are simply Jacks-of-all-trades and masters of none. (38:269-273)

Stone sums up the thinking of many when he stated:

If we wish to improve the teaching of grade school mathematics, it will be necessary to give much better training to teachers of elementary mathematics. It is only too certain that today's mathematically ill-prepared teachers, many of whom are ill disposed toward the subject of mathematics, are infecting too large a number of our boys and girls with an enduring fear and hatred of mathematics, which can rarely be overcome

> later on. The new mathematics training which may be developed for teachers should give them an appreciation and understanding of the subjects they may be expected to teach and as a result should inspire them with a certain degree of respect and admiration for mathematics, if not a real liking for it. (32:177-179)

The Mathematical Association of America's Committee on Undergraduate Program in Mathematics recently conducted a study of requirements and offering of mathematics in the pre-service education programs for teachers in the elementary schools. (16:89-93)

Their study reported that programs in elementary education are offered in 906 colleges and universities. Of this group 762 submitted usable responses to the Committee's inquiries. The results indicate that 22.4 percent of the respondents required no mathematics of prospective elementary school teachers, and 68.9 percent require the equivalent of four or fewer semester hours of mathematics. Especially significant is the fact that 55.6 percent offer no mathematics courses specifically designed for prospective elementary school teachers. This report also indicated that elementary school teachers are, on the whole, less well prepared to teach the mathematics for the elementary school than any other subject.

The need for specialized training in mathematics for elementary teachers was generally recognized. However, all the conferees agreed that the nature of the content of required courses for elementary school teachers is just as important as the number of hours of training required. The spirit of the course and the manner in which it is presented is of the utmost importance. The use of the

"discovery method" by the student is essential, and the development of proper attitude toward mathematics is imperative.

The C U P M, as part of a mandate from the parent organization, established a panel instructed to prepare a set of recommendations of minimum standards for the training of teachers of mathematics on all levels. This report included for each classification a recommendation as to the type and minimum amount of mathematics which should be taken by students preparing for teaching careers. The courses recommended are specifically designed for prospective teachers and it is recommended that they be taught by persons who are masters of their subject matter and who have, in addition, a knowledge of the problems which teachers face.

The Committee (C U P M) specifically recommended that:

> As a pre-requisite for college training of elementary school teachers, at least two years of college preparatory mathematics. This is to consist of one year of elementary algebra and one year of geometry, or the same material in an integrated course. For their college training a minimum of twelve semester hours was recommended. This would include, (a) a two course sequence devoted to the structure of the real number system and subsystem, (b) a course devoted to the basic concepts of algebra; (c) a course in informal geometry. In addition, recognizing that special problems may be connected with the teaching of primary children, the Committee suggests that a special program might be adviseable for teachers in this area. (40:421-425)

> The Cooperative Committee on the Teaching of Science and Mathematics (a Committee of the American Association for the Advancement of Science and Mathematics) made a similar study previous to the C U P M report. In addition, similar sequences have been recommended by the Commission on Mathematics, the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, and others. (40:421-425)

CHAPTER V

SUMMARY AND IMPLICATIONS

The literature reviewed for this paper indicates that there has been an awareness developing among educators and the general public of the importance of the role that mathematics is now playing in our present day world. Along with this awareness of the importance of mathematics there seems to be agreement among the writers on this subject of the need for far greater insight and understanding on the part of students of the basic concepts and applications of mathematics than has been true up to the present. The reasons advanced for these needs are many-fold. Among them are, (1) the need for the general populace to better understand the developments taking place in the world in which we live and to be better prepared to cope with present and future problems, (2) the need for ever increasing numbers of highly trained mathematicians, scientists and technologists.

Most writers reviewed seem to agree that to accomplish these aims noted above, there is need for a much improved program of mathematics in our elementary schools. There appears to be, however, some diversity of opinion as to how this is to be accomplished. Their opinions generally fall into the following categories:

1. Those who contend that the traditional mathematics of the past twenty-five years is adequate.

2. Those who advocate the discarding the present program

in favor of one of the "new" programs such as S M S G or G C M P, etc.

3. Those who advocate the retention of much of the present program with some modifications.

Some of the writers such as Spitzer and Brownell are confident that the end result will be a combination of the best of the old or traditional program with what proves valuable in the "new". However, much of the criticism levied at the present day program in our elementary schools appears to be directed toward the competency of the teacher and the methodology used rather than at the actual mathematics content of the programs themselves. Many opinions are expressed and some evidence listed that too many elementary teachers do not have adequate training in mathematics, have little or no understanding or appreciation for the subject and that both their attitudes and methodology are poor. They further contend that the unpopularity of the subject, fear, distaste, and outright dislike of mathematics by the students are directly attributed to the teacher and her presentation of the subject.

Studies made by mathematicians and educators seem to bear out the assumption that many elementary teachers have had little or no training in mathematics above the secondary level. Further more, these same studies show that teacher training institutions, in general, require little or no study on the college level of either the subject area itself or the methodology used in teaching the subject.

Some further implications appear to be:

1. That mathematics is assuming a far more important role in elementary education than has been true in the past.

2. That there is a definite need for improvement and enrichment of the mathematical elements in elementary school education.

3. That some of the content of the mathematics program will be new and some content will be introduced at a different (lower) level.

4. That in the lower grade levels arithmetic, algebra and geometry will be introduced as a unified subject.

5. That some of the language of mathematics will be new and that the language of mathematics will assume a role of major importance.

6. That the major emphasis will be on developing a greater understanding of the science of numbers, on structure and relationships ahead of computational skills. That creativity through discovery of the basic laws will be stressed.

7. That a greater number of students need to pursue mathematical studies to more advanced levels.

8. That there will be increased emphasis on teacher training.

9. That there is a need for increased content both in mathematics and methodology in the curriculum of teacher training institutions.

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