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Cover Page Footnote

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Synopsis

The mathematician Georg Cantor, the writer Jorge Luis Borges, and the protagonist of Borges' short story *The Aleph*, Carlos Argentino Daneri, are seen here as three pieces of a single puzzle. We put these pieces together and we look at the surprising figure that we obtain.

Keywords: Cantor, Borges, aleph, infinity

1. Georg Cantor

Georg Ferdinand Ludwig Philipp Cantor (1845–1918) was born in Saint Petersburg, Russia, but he spent most of his life in Germany, where he moved with his family at the age of eleven. He is considered one of the greatest mathematicians of all time, and he had a terrific way of looking at mathematics. He believed that *the essence of mathematics is its freedom*, and he suggested that *pure mathematics* should rather be called *free mathematics* [13]. He gave birth to *set theory*, described by David Hilbert as:

> *The paradise that Cantor has created for us and from which no one shall expel us.* [16]

Cantor believed that there could not be one single infinity, but there had to be infinities greater than others. In order to distinguish them, within the setting of set theory in which numbers are seen as sets, he defined the *transfinite numbers*. He also talked about the *Absolute Infinite* (denoted Ω), that

he identified with God and which, as he said, could not be formalized mathematically. Cantor named the "small" infinite numbers (small with respect to God) Aleph-zero, Aleph-one, Aleph-two (denoted $\aleph_0, \aleph_1, \aleph_2$), and so on. Moreover, he defined operations between them.

Despite having shaped a paradise, though, Cantor lived with his mind and body in hell, as he struggled with mental illness and he spent his last years in a psychiatric hospital, in Halle. Also, in 1901 his paradise seemed to collapse, because Bertrand Russell showed that his set theory contained a paradox within itself. In fact, Cantor's theory did not exclude the existence of a set containing all the sets that do not belong to themselves,

$$R = \{S : S \text{ is a set and } S \text{ is not element of } S\}.$$

Does this set belong to itself? Any answer leads to a contradiction, and the discovery of this paradox led the mathematical community to doubt the robustness of Cantor's theory. Set theory was saved later on; the mathematicians Ernst Zermelo, Adolf Abraham Halevi Fraenkel and Albert Thoralf Skolem managed to free Cantor's paradise from its paradoxes, thanks to a new axiomatization of the theory. However, Cantor's life has never been saved, and today, a crater on the dark face of the Moon is named after Georg Cantor, in accordance with his dark destiny.

2. The mathematics of Borges

Jorge Francisco Isidoro Luis Borges Acevedo (1899–1986) was born in Buenos Aires, Argentinia. At his death in Geneva, he left an immortal literary legacy, made of marvelous works full of creativity, imagination, dreams and mathematics. When Herbert Simon, laureate of both the Nobel Prize in Economics and the Turing Award in computing, interviewed him in 1971, Borges said:

> *It is true that many of my ideas come from reading books on logic and mathematics. But having said that, each time I set about reading these books I get the feeling that they defeat me, that I haven't been able to understand them to the full.* [24]

As a mathematician who has read many of Borges' works, I believe that he was actually able to understand to the full many mathematical concepts that fascinated him. He exploited this capability and used it to create magic. Borges did something similar to what Lewis Carroll, the mathematician author of *Alice's Adventures in Wonderland* [11], did. Namely, he demolished a wall that was built by men which separates mathematics from literature.

There is a vast literature about Borges' use of mathematics in his writing (see for instance [1, 2, 10, 12, 14, 15, 17, 18, 19, 20, 21, 22, 25]). Two mathematical concepts that are very often present in his work are *paradoxes* and *infinity*. On the infinity of time, he says that

> *Eternity is a splendid artifice that liberates us, even if for just a few moments, from the unbearable oppression of the passage of time.* [4]

He discusses the paradox of Achilles and the tortoise, both from a philosophical and from a mathematical point of view, in *Kafka and His Precursors* [9], *The Perpetual Race of Achilles and the Tortoise* [3] and *Avatars of the Tortoise* [5]. In *Death and the Compass* [7], he makes use of this paradox without explicitly mentioning it:

> *Scharlach, when, in some other incarnation you hunt me, feign to commit (or do commit) a crime at A, then a second crime at B, eight kilometers from A, then a third crime at C, four kilometers from A and B, halfway enroute between the two. Wait for me later at D, two kilometers from A and C, halfway, once again, between both.*

During his interview with Herbert Simon [24], Borges also said that he took inspiration from Russell's mathematical work. It is therefore not surprising that Russell's paradox appears often and in various forms in Borges' works. For instance, Borges mentions several times [22] a map that Josiah Royce, in his book *The World and the Individual* [23], describes as a map of England which is perfectly traced on a portion of the soil of England. The map is so precise that it contains a map of the map which contains a map of the map of the map, and so on. This self-referring map reminds one of Russell's paradox; we can find a similar idea in Borges' *The Library of Babel* [6]:

> *Like all men of the Library, I have traveled in my youth; I have wandered in search of a book, perhaps the catalogue of catalogues;*

as well as in *The Aleph* [8]. This is the story we will now analyze further.[1]

3. Carlos Argentino Daneri

There are only two active characters in Borges' short story *The Aleph* [8]: Borges himself, and Carlos Argentino Daneri — a poet who is described by Borges as an *extravangant* and a *madman*. When Carlos was a child, he discovered, in his cellar, an *Aleph*:

> *One of the points in space that contains all other points. [...] The only place on earth where all places are — seen from every angle, each standing clear, without any confusion or blending.*

And since its discovery, Carlos has been obsessed with it. As he could see, from the Aleph, every angle of the universe, he wanted to write verses about every angle of the earth. But Borges did not appreciate Carlos' poems and personality. He could not even fully understand them, until the day he himself saw the Aleph in Carlos' cellar:

> *I saw the Aleph from every point and angle, and in the Aleph I saw the earth and in the earth the Aleph and in the Aleph the earth; I saw my own face and my own bowels; I saw your face; and I felt dizzy and wept, for my eyes had seen that secret and conjectured object whose name is common to all men but which no man has looked upon — the unimaginable universe.*
>
> *I felt infinite wonder, infinite pity.*

[1] The story is included in *The Aleph and Other Stories 1933–1969*, by Jorge Luis Borges and Norman Thomas Di Giovanni. One might also find and read it online; see for example https://web.mit.edu/allanmc/www/borgesaleph.pdf, last accessed on July 29, 2022.

4. The Aleph of Borges and the Paradise of Cantor

Although the Aleph of Borges is conceptually very different from Cantor's Aleph numbers, it is clear that Borges wanted to pay a tribute to Cantor's set theory. It is clear not only by the choice of the name, by the reference to Russell's paradox (*in the Aleph I saw the earth and in the earth the Aleph and in the Aleph the earth*) and by the knowledge of the topic that we know Borges had. Borges also mentions Cantor's work directly, at the end of the short story, when talking about the name of that point that contains all other points. He says that he heard the name *Aleph* from Carlos, but he doesn't know where Carlos took it from. He says that ℵ is the first letter of the Hebrew alphabet, as well as the letter used in *Mengenlehre* (in German, *set theory*) for indicating the transifinite numbers. Interestingly, Cantor's name does not appear in the original Spanish version of the short story, but it does appear in the English version, translated by Norman Thomas Di Giovanni in collaboration with Borges.

I also ask myself: What if Borges wanted Carlos Argentino Daneri to somehow interpret Georg Cantor? Carlos is the one who discovers the Aleph (*my Aleph,* he says) and he is so obsessed with it that he becomes a *madman.* Cantor is the father of Aleph numbers, and he struggled with mental health issues. But Cantor's life did not have a happy ending, while Carlos does recover from his issues in Borges' story (I will not tell you how). I like to think that Borges wanted to give a happy ending to Cantor's life, too. I like to think that, at the end of these pages of Borges, Cantor found again the freedom that had always characterized his work.

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