Resources Related to Mathematics, Art and Nature

Sabbatical Project
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"What I did not draw, I did not see." - Goethe

Table of Contents

I. INTRODUCTION

The purpose of my sabbatical work was to learn about resources related to mathematics and art, nature, music and architecture. These subjects appeal to a wide audience, yet their mathematical connections are often on the periphery of a mathematics student's studies. In this way I planned to increase my knowledge of mathematics but at the same time broaden my understanding of cross-disciplinary subjects.

I spent the semester gathering books, articles, examples of artwork and natural objects, and websites. These were then reviewed for content and value. From this, I organized a collection for my own library and then prepared an overview that would be a useful resource to others who might be interested in these topics, or to myself for further study. I also collected information on related organizations, and on influential people. I selected a number of the works for more in-depth study, and attended a five-day international conference. I also prepared an extensive display on the mathematics of tile patterns, which is currently installed in the exhibit case outside of the Mathematics Department office, M120, at Parkland College. During the semester following the sabbatical (Spring 2004), I gave a short presentation to the Mathematics Faculty during Professional Development Day, mostly showing a few of the best websites that I found.

In the last twenty years or so there has been a great increase in the number of books, articles, and such that is related to mathematics and art, and to the natural world and other non-technical disciplines. Colleges and universities are offering more cross-disciplinary and general education courses in mathematics than ever before, such as Dartmouth's *Mathematics of Art and Architecture*, developed by Dartmouth mathematician Paul Calter. There are also many new cross-disciplinary organizations, such as the International Society of the Arts, Mathematics and Architecture.

I think that this blending of mathematics and the arts – which is not new, as evidenced by the tile patterns of the Alhambra of the 13th century, and M.C. Escher's work beginning early in the 20th - has increased considerably over the last ten years especially because of technological advances. For example, the computer has encouraged many artists to create their own Escher-like patterns, and allowed sculptor Helaman Ferguson to create engraved bronzes with a precision that would not be possible otherwise. On the other hand, the use of computer graphics has greatly improved the ability for scientists and mathematicians to visualize their ideas. Without the computer, fractal patterns would have never even been visible, and mathematical research in that direction would not have progressed as it has. The advance of computer graphics itself made possible what several people are calling a new field: *mathematics visualization.* More than just interesting pictures, it has provided insight to the solution of mathematical problems and led to new discoveries.

As the printing press revolutionized communication and the dissemination of ideas, the internet has brought together more people with common interests across traditionally different professions. Also, the ability that we have today to quickly download still images and animations of remarkable quality has given us immediate access to a wealth of materials right at our own desks. Just the improvements in the last few years have given us excellent images of all sorts of mathematical art, movies of rotating graphs in four dimensions, interactive keyboards for learning about musical scales, mini-texts on the mathematics of leaf arrangement, including text-book quality pictures and illustrations; and colorful explanations of the 17 symmetry groups of abstract algebra that gave rise to Islamic tile patterns. That all of this is so easily accessible and of such high quality has greatly increased collaboration between the disciplines.

To begin the sabbatical project, in July of 2003 I attended a five-day joint meeting of The International Society of the Arts, Mathematics and Architecture and a related group called Bridges, whose mission is to explore and promote the connections between mathematics, music, art, and science. This remarkable conference was held in Granada, Spain, at the University of Granada. The speakers and attendees were an interesting mix of people – many had talents in two fields, such as engineering and art or mathematics and music. The sessions dealt with topics that are not the focus of the usual mathematics conferences that I attend. Some examples of the topics are: spiral patterns in nature, how the human eye creates gaze patterns, and the geometry used by Salvador Dali. I commend the organizing committee for their efficient scheduling – it was easy to attend five or six sessions in a day without being overwhelmed with information. The last day we gathered for a tour of the Alhambra, but it was not the usual sort of guided tour. We were accompanied by a mathematics graduate student from the University of Granada who provided us with on-site explanations of the mathematics of the 17 different tile patterns used to cover the floors and walls.

At the end of this summary is the overview of the resources that proved to be the most worthwhile. In deciding what to include, part of the decision was made for me. As it turned out, there is a flood of recent and interesting materials related to mathematics and the arts and nature - enough for a lifetime of study if one wished - and so this is where I concentrated my efforts, and decided not to pursue the topic of music. (Architecture is somewhat addressed in the materials on the Golden Ratio.) In gathering and reviewing potential materials, I found them to be scattered, numerous, and varied in quality and accessibility. However, I believed that it was important to arrive at a list of materials that would not only be appropriate to the subject at hand, but would also beautifully capture the connections between mathematics and the applications in ways that would give justice to both disciplines and inspire students of either. I also wanted to include a significant amount of material that could be appreciated without requiring too much mathematical background, in the belief that everyone should have the opportunity to see mathematics as inherent in the natural order. Most of all, I wanted to learn something new.

II. THE GOLDEN RATIO AND FIBONACCI NUMBERS

The Golden Ratio phi ($\phi \approx 1.618034$) and the related Fibonacci sequence of numbers (1, 1, 2, 3, 5, 8, 13, ...) are naturally occurring in the patterns and growth of nature, in a way similar to how the more familiar constant pi ($\pi \approx 3.14159$) is inherent in everything circular. In addition, the Fibonacci numbers and phi have some remarkable properties, many relating them to each other. Almost as interesting is how phi and/or the Fibonacci sequence have been dubiously "found" in architectural design going back to the pyramids. Discussions about mathematics in art, design, or nature will often involve these numbers.

A note about terminology: The Golden Ratio ϕ (phi) is an irrational number approximated by 1.618034. Related to this number is $1 \div \phi$, or 0.618034. These can be distinguished from one another by using Phi for the larger and phi for the smaller; however I find here and there in the literature the smaller number labeled as ϕ. This is generally not a problem, due to the way the ratio that defines phi is constructed and its properties.

Website. **www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci**
There are other websites dealing with the Fibonacci sequence and the Golden Ratio, but this can easily be claimed as the best, and maybe the only one you need. There are literally hundreds of webpages here, covering just about any facet of the subject you might want, but none of it requiring advanced mathematical knowledge. All of it is very nicely presented and organized. They start with how Fibonacci originally got his idea, rather than assume you already know. You will find lots of natural examples of where the numbers turn up, with drawings and explanations. There are also puzzles and activities. A great resource.

Livio, Mario. ***The Golden Ratio: The Story of Phi, the World's Most Astonishing Number**, Broadway Books, New York,* 2003.
The "Golden Number" or "Golden Ratio" has been known since ancient times, and Livio gives a very enjoyable and accessible account of its history and its occurrences. The presence of phi in natural patterns and growth is emphasized, and there are lots of wonderful examples. It is nice that special attention is given many inappropriate claims about phi in architecture and art, most of which rely on sloppy measurements or research. The mathematics used in *The Golden Ratio* is just adequate and not beyond the use of the quadratic formula, so is accessible by a wide audience. There are lots of good references for further reading.

Roger Herz-Fischler. ***A Mathematical History of the Golden Number***, Dover Publications, Feb 1998.
Note: This author is sometimes listed as Roger Fischler. This book is an unabridged reprinting of *A Mathematical History of Division in Mean and Extreme Ratio,* Wilfrid Laurier University Press, Ontario, 1987. The Golden Number does arise from the division of a length into these ratios, but the new title sounds less technical. Livio regards this book as "excellent", and says it does a good job of debunking sightings (my word) of the golden ratio in ancient architecture and in a wide range of paintings, especially pre-1850. The new title is not listed in the Lincoln Trail Library System.

A scholarly work that is not for casual reading, Fischler has put together what is probably the most comprehensive reference to the written history of ϕ. Each topic is dealt with in detail, including a good deal of his own commentary together with numerous and lengthy references to other sources. The bibliography is excellent, and includes two additional features not usually found: indexing to the text and library codes.

George Markowsky. "Misconceptions About the Golden Ratio", *College Mathematics Journal*, 23 (1992): 2-19. This article in MAA's journal provides reasons for doubting the presence or use of the Golden Ratio in such things as ancient building design (the Pyramids, the Parthenon), the human body, and so on. These misconceptions pervade much of the literature on the subject, even from otherwise reliable authors, probably just because they were accepted too readily. UIUC Math Library, Reserves.

III. THE ARTS - WEBSITES

Because of the large number of websites for the arts, I have separated them from the list of books and periodicals.

M. C. Escher. **www.mcescher.com**

This is the official M.C. Escher website. The art gallery is absolutely wonderful; it doesn't say if it is complete, but you will find every print you have ever seen, plus a whole lot more. There is a shop where you can buy posters, books, neckties, etc. Be sure to check out the Virtual Ride under Downloads, where they take you on sort of a roller coaster trip through one of his architectural impossibilities. Unfortunately, the graphics are not very good and the clip is fast and short (maybe the user's fault), but the idea of these is to point out that the only reason these buildings seem to work on paper is because the angle of sight for the observer has been carefully chosen – your position is frozen so that you cannot walk around the object to see if reality matches up with what you think you are seeing. The ride that best illustrates this, to me, anyway, seems to be Waterfall.

M. C. Escher. **www.Escher.info**

This is another good resource for studying Escher's art.

Thomas F. Banchoff. www.**math.union.edu/~dpvc/TFB/Art**

Brown University geometer Thomas F. Banchoff has a couple of websites, but a nice place to start is his art website, given above. There are some very nice animations of four-dimensional objects. He is also known as an expert on Edwin A. Abbott, author of *Flatland.*

Dick Termes. **www.termespheres.com**

Painter Dick Termes creates fascinating scenes on spheres using a technique called six-point perspective. It's somewhat like being inside a fishbowl and looking out, and is reminiscent of Escher's *Hand With Globe.* There are lots of things to look at on this website, including an article on six-point perspective and a long list of articles featuring Termes. He will bring his spheres and give a talk for a fee.

John Sullivan. **www.torus.math.uiuc.edu/jms/**

Geometer John Sullivan has some interesting artwork of soap films, and an animated work called The Optiverse. This film "shows an optimal way to turn a sphere inside out" and is a joint work with George Francis, Stuart Levy and Camille Goudeseune. The film is accessible from the website, but make sure you also get the sound that accompanies it and explains what you are seeing. Also included on the website are some articles written by John. He points out that at smaller scales, the physics of the *surfaces* are often more important than those of the volume underneath; recall a bug that skates across the pond. This is one reason people study minimal surfaces and soap films. He has attended many conferences related to mathematics and visualization.

MAA. **http://mathforum.org/mam**
This is MAA's website for Mathematical Awareness Month, April 2003.
MAA (Mathematical Association of America) offers a mathematical discussion of the poster art selected for Mathematics Awareness Month, April 2003. This poster celebrates the connections between mathematics and art with a colorful computer graphics rendering of Escher's *Circle Limit III.* This work is an application of hyperbolic geometry (one of the many non-Euclidean geometries) in which the whole flat plane is contained within a circle. The receding effect of the figures near the boundary is similar to that of ships receding on the horizon.

Geometry in Art and Architecture, Dartmouth. **www.Math.Dartmouth.edu**
Dartmouth offers some wonderful online interdisciplinary courses as part of their MATC series (Math Across the Curriculum), which is a program funded by the National Science Foundation. One such course is *Geometry in Art and Architecture*, developed by Paul Calter. Calter's background is interesting: he holds B.S. and M.S in mechanical engineering, and a M.F.A. in sculpture. He has been a university professor in mathematics since 1968. (I had the pleasure of attending a session given by Calter on his course at an AMATYC conference a number of years ago, which was the seed for this sabbatical project.) His book resulting from the course will be coming out in August, 2004.

Helaman Ferguson. **www.helasculpt.com**
This is sculptor Helaman Ferguson's website. Ferguson's bronze, *Umbilic Torus,* will be recognized by mathematics instructors as the cover art for a recent calculus text. He includes photos of his works, and some insight to how he used mathematics to create them.

LEONARDO, The International Society for the Arts, Sciences and Technology (ISAST)
www.mitpress2.mit.edu/e-journals/Leonardo.
Founded in 1968 by kinetic artist and scientist Frank J. Malina, this group has some nice artwork in their online gallery They serve first the artist, and states their focus as being "... on writings by artists who work with science- and technology-based art media." Their journal, *Leonardo*, published by MIT Press, is also available at various libraries at UIUC. More details about this group are given under Organizations.

Bridges, Mathematical Connections in Art, Music, and Science
www.sckans.edu/~bridges
This is a very nice website, containing a virtual art museum, related links, and resources for those interested in mathematics and the arts. The Bridges and ISAMA organizations are among the foremost in promoting collaboration across these disciplines, and often hold joint meetings, such as the one in Granada, Spain, in 2003. Information about the Bridges meeting in August, 2005 is given.

III. THE ARTS – BOOKS AND PERIODICALS

Peterson, Ivars. ***Fragments of Infinity: A Kaleidoscope of Math and Art***, John Wiley & Sons, 2001. Wonderful reading for a general audience, and includes color photos and sketches. Ivars Peterson is also the mathematics writer for Science News Online.

Ernst, Bruno. ***The Magic Mirror of M.C. Escher,*** Taschen America, Inc. 1978, 1994. A great place to begin to learn about the development, inspiration and methods of Escher's works. This book includes lots of examples of his work – some not so familiar - with accompanying explanations. The writing is non-technical, but very informative.

Banchoff, Thomas. ***The Hypercube: Projections and Slices***, 1978. Animated film. Banchoff is a geometer at Brown. This film is mentioned in the literature here and there and so seemed to be worth a look. The U of I Math Library has this film in their catalog listing but they couldn't find it.

H.S.M. Coxeter, M. Emmer, R. Penrose, M.L. Teuber, editors, ***M.C. Escher: Art and Science, Proceedings of the International Congress on M.C. Escher,*** **Rome, Italy, 26-28 March, 1985** , Elsevier Science Publishers, 1986.
Not the layman's treatment of Escher. The introduction gives the flavor of what is to come and is worth quoting here: "The participants include mathematicians, physicists, crystallographers, chemists, biologists, psychologists, psychiatrists, art historians and experts in computer graphics and visual communications. The lively discussions that followed the presentation of each paper confirm than Escher's works are not only *good examples of visualization of scientific problems, but also stimulate real scientific research."* (My emphasis.) The two articles Mathematical Challenges in Escher's Geometry by Branko Grunbaum of the University of Washington, and Doris Schattschneider's M. C. Escher's Classification System for His Colored Periodic Drawings address Escher's remarkable abilities despite his lack of formal mathematical training. He struck out on his own path, yet his work foreshadowed results that would be established years later by others' research in mathematics and crystallography.
UIUC Math Library.

Note: After investigation, I determined that this meeting was not an activity of International Congress of Mathematicians, a group that holds meetings every four years (and by the way, awards the Fields Medal). There was no ICM meeting in 1985. It appears that the Escher Congress in Rome in 1985 was simply organized by interested faculty at the University of Rome. No mention of any mathematical organization is made in the Acknowledgements.

Emmer, Michele, Ed., ***The Visual Mind: Art and Mathematics,*** MIT Press, Cambridge, 1993.
This is a compilation of a number of articles from other sources but primarily from various issues of *Leonardo,* the journal of ISAST, The International Society for the Arts, Sciences and Technology, edited by Frank Malina. The list of authors is a who's who list of the frontrunners in the new field of visual mathematics. Names familiar to those in mathematics are: mathematician Michele Emmer, physicist Roger Penrose,

mathematician and sculptor Heleman Ferguson, geometer H.S.M. Coxeter (who taught M.C. Escher a thing or two about hyperbolic geometry), and mathematician Benoit Mandelbrot, the father of fractals.

The volume is divided into four parts: Geometry and Visualization; Computer Graphics, Geometry and Art; Symmetry; and Perspective, Mathematics and Art. It is a very interesting book that provides a good look at the visualization of mathematics, how it can be done, and why it is important. I liked this book so much that I wrote a more detailed review for my own files.

Elaine Strosberg. ***Art and Science,*** Abbeville Press Publishers, NY, 2001.
Architecture and Art Library, UIUC.
This is a collection of easy-reading articles about connections between the arts and science. Her comment about ancient city planning is worthwhile because it points to the practical application of mathematical design. She says: "The Egyptians used the grid as an aid in architectural design, but the Greeks extended orthogonal (right angle) planning to the layout of entire cities." (p. 47) There is a piece of artwork or illustration on every page, and an extensive bibliography.

Abbott, Edwin A. ***Flatland, A Romance of Many Dimensions*** *by A. Square* (HarperCollins, 1983)
In 1884, Edwin Abbott wrote a story about the inhabitants of a world that had only two dimensions, like the surface of your coffee table. They could see right and left, ahead and behind, but not up nor down. Their interactions with people from Spaceland, which is just our ordinary three-dimensional world, provide an interesting adventure that can give insight into higher dimensions - such as the fourth. Abbott also used this as a study of Victorian social attitudes.

Companion books to *Flatland* include:

> **Dionys Burger,** ***Sphereland, A Fantasy About Curves Spaces and an Expanding Universe by A. Hexagon*** (HarperPerennial, 1965)
> This can also be found bound together with *Flatland*, in a 1994 HarperCollins edition, one upside down to the other. In this world, the inhabitants live on the surface of a sphere. I suppose that *Sphereland* could be called an Einsteinian response to *Flatland.*
>
> **Stewart, Ian.** ***Flatterland*** (Perseus Publishing, 2001)
> Stewart uses the writing style of *Flatland* to investigate other geometrical spaces, such as hyperbolic discs, places that have fractional dimension, and so on. The main character is Victoria Line, a Flatlander and descendant of A. Square, the main character of *Flatland.* The writing style echoes that of *Flatland*, being somewhat whimsical and fairy-tale like.

Abbott, Edwin A., Ian Stewart, ***The Annotated Flatland, A Romance of Many Dimensions*** (Perseus Publishing, 2002)
Ian Stewart provides the lengthy notes that accompany the original *Flatland,* written by Edwin A. Abbott in 1884. Arranged in two columns per page- one for each of the authors - it is interesting, entertaining, readable, and contains a wealth of sources for further study. A single word by Abbott can elicit 100 from Stewart of explanation and elaboration. The introduction is very good, too, and contains quite a bit of biographical information about Abbott (whose middle name, by the way, was also Abbott). In a book of another format it would have been a chapter in itself. A very nice volume; if you are shopping for *Flatland*, consider *The Annotated Flatland* instead.

Dewdney, A. K. ***The Planiverse: Computer Contact with a Two-Dimensional World*** (John Wiley & Sons) Not reviewed; however, this is another companion to *Flatland.*

V. THE NATURAL WORLD

Matila Ghyka, *The Geometry of Art and Life*, Dover, 1977. UIUC, Architecture and Art Library.
This was first published in 1946; the 1977 edition is given as a "slightly corrected republication". This is a nice little book, providing more than an overview of the geometry found in nature, especially the golden mean and the logarithmic spiral.
The first few chapters expound on the mathematics involved, as many books do, but the author starts out a little differently by giving a nice explanation of the generalization of proportions and the reason *why* one would want to divide a length according to the extreme and mean ratio, which results in the famous Golden Section.

There is a nice overview of the five Platonic Solids and the Archimedean semi-regular polyhedra and their role in tiling the plane or partitioning space. On p. 85 in the discussion of configurations of crystallized matter, she notes that Curie's law says: "a body tends to take a shape presenting the minimum surface energy made possible by the directing forces. In many cases…this minimum… corresponds to the solution for which a given volume produces the smallest surface..." This result, or perhaps its two-dimensional counterpart, is familiar to any student of calculus.

In her discussion of botany, she says that Church was the first to discover that the Golden Ratio (and its related angles) corresponded to the best distribution of leaves, and that "the mathematical confirmation was given by Wiesner in 1875." (I was interested in finding out more about Wiesner, but Ghyka's book has no bibliography. Theodore Cook, in *The Curves of Life,* referred to him as J. Wiesner, and further research turned up Jacob Wiesner, a mathematician, and Julius Wiesner, botanist, both living and working at a time contemporary with Ghyka's reference. There are some books on botany at the Library authored by Julius Wiesner, but they are all in German.)

Ghyka also includes standard material on the use of the Golden Mean in the construction of ancient architecture and in medieval artwork. There is disagreement or at least doubtfulness that this actually occurred found in more recent writings, especially Livio and Herz-Fischler.

Overall, sketches are provided as needed, historical tidbits are included, and there are plenty of examples to accompany each discussion.

Mandelbrot, Benoit, *The Fractal Geometry of Nature.*
This is Mandelbrot's 1982 classic on the structure of natural forms as fractals. The subject of fractals was new at the time; if you had stumbled upon this book without any prior exposure to the subject, you might have thought that the man was a nut. His terminology is certainly unusual: curds and whey, dust, rivers and streams, dragons, squigs, fudgeflakes, sponges, and gaskets are some of the terms he uses to describe fractal sets. But then, one must remember that this identification of heretofore oddball

mathematical "monsters" with all sorts of common structures of nature was largely a new thing. It begs for its own vocabulary, and Mandelbrot provides it wonderfully.

Mandelbrot's credentials are top-notch. He has degrees in Aeronautics and Mathematics, and has lectured at numerous prestigious schools around the world. This book is a collection of essays that grew out of a series of lectures, and in them he discusses fractal geometry and dimension, and gives a great many examples of how fractals occur naturally. In places he is generous with speculation and admits so, but the spirit of the book seems to be one of sharing his ideas and thinking out loud. Being a group of essays, the presentation is not rigorous. The varied examples of fractals are interesting; besides the familiar examples of coastlines and fluid turbulence, he discusses the human lungs, brain folds, vascular geometry, high-energy particle collision, Brownian motion, watersheds, isothermal surfaces, and even economics. There is even a fractal sort of calculus, where the power rule is generalized to reducing (or increasing) the power by a *non-integer.*

The mathematical level overall is advanced, although he is kind to actually mark the paragraphs (he calls them digressions) that one may skip without losing the general idea under discussion. Proofs and such are reserved for the appendix, and the Biographical Sketches and Historical Sketches at the end are a nice addition. Even so, it is still not a volume for the general reader. It is especially helpful to have an understanding of the Cantor set and Sierpinski triangle sorts of constructions, which can provide a framework for appreciating the rest. Regardless of the depth at which one understands what Mandelbrot is saying, one comes away with amazement at his observations and imagination.

Theodore A. Cook, *The Curves of Life,* Dover Publications, 1979. UIUC Math Library. (This is an unabridged republication of the original 1914 edition.)
This is a classic. The text is a detailed, scholarly treatment of the spiral in nature, art, and design. Mathematical support is included where appropriate but most of it is in the Appendix. The author takes care to distinguish the logarithmic spiral as the one with self-similarity (p. 61). The cover's claim that the volume is "Profusely Illustrated" is justified by 415 illustrations and eleven plates. Examples include a wide variety of shells and plants, feathers, bones, animal horns and spiral nebulae. Spiral growth and its mechanics are discussed at length. Examples from art and design include a number of Da Vinci's drawings, spiral columns, the Bois spiral staircase, and examples of spiral patterns used as decorations since Paleolithic times. The discussion of the origin of symbols deriving from the chambered nautilus is interesting. Note the photo of the prehistoric altar from Central America, which is exactly the Yin-Yang symbol (p. 451). Cook also authored *Spirals in Nature and Life,* which is part of the UIUC General Collection. No copies of that book were available at this writing.

Goethe, Johann Wolfgang Von. *The Spiral Forms in Nature* (1831)
This is listed simply to make note of Goethe's contribution to this subject, and to point out that the study of patterns in nature is not a new thing. Quoting from Strasburg, "The Romantic writer Goethe was also a biologist. He introduced the notion of *morphology*

which would be fundamental to the development of the theory of evolution of species." D'Arcy Thompson agrees with this, but unfortunately the only other places Goethe is mentioned in Thompson's book are simply to reprint quotes from him in the original German.

Douady, Stephane, and Coulder, Yves. "Phyllotaxis as a Physical Self-Organized Growth Process", *Physical Review Letters,* 68 (1992): 2098-2101
The patterns of plants' leaves have been known for quite some time, but Douady and Coulder are the French physicists who in 1992 mathematically nailed down the process of phyllotaxis. They have several very technical articles giving the details of this. See *Physical Review Letters,* Engineering Library, UIUC; and the *Journal of Theoretical Biology*, Biology Library UIUC. (Abstracts can be found online, but reprints of the articles were for purchase and expensive.)

Stewart, Ian. *Life's Other Secret: The New Mathematics of the Living World* (John Wiley & Sons, 1998)
Stewart's basic premise is that DNA is only one secret of life, and mathematics is the other. Strong arguments are made that the development of living organisms depends not just on DNA instructions but also on physical and chemical laws applied on a larger scale. A very nice example of this is stomach formation in frogs and its comparison to the buckling of a sphere (pp. 41-43.)

Stewart, Ian. *Nature's Numbers, The Unreal Reality of Nature's Numbers* (Basic Books, HarperCollins, 1995) Popular science writer and mathematician Ian Stewart discusses the mathematical order of nature, including shapes (morphology), rhythyms, growth patterns, and chaos and order. His writing is fresh, discussing new ideas and discoveries, and offering conjectures to make one think.

Thompson, D'Arcy Wentworth. *On Growth and Form*, *The Complete Revised Edition* (Dover reprint, 1992)
It seems that everyone who writes on mathematics and nature lists Thompson's book as a reference, but it is no wonder. This book is the classic work on the mathematics of morphology, which is the study of organic form. It would be a welcome addition to the library of anyone interested in the subject.

Thompson, a zoologist, first published this book in 1917, and revised it in 1942. Those interested in books in their own right will be pleased to find a copy of the first edition in the General Collection of the UIUC Library. It is a lovely volume: fat and compact, with the original red buckram binding now faded to rose; frayed at the corners, and worn soft to the touch.

The list of topics is lengthy. Included are factors affecting growth (temperature, sunlight, weight, etc.); phyllotaxy, which is the system of leaf arrangement; physical structures and their properties (from single cells to skeletons), symmetry, properties of surfaces, the logarithmic spiral, and the theory of mathematical transformations as applied to related organic forms. Large numbers of examples are included, accompanied with charts, sketches or photographs. A few representatives are pollen grains, crystals, radiolarians, seashells, animal horns, seed heads, ferns, and zebra stripes. The logarithmic spiral is treated at length.

The writing style shows its age, but is still quite readable, even for a scientific work. Be warned, however; there are over 1000 pages. Thompson revised the work in 1942, and this newer edition contains the bulk of the original material, updated where needed. (It is still about the same length.) Dover Publications offers reprints, and the UIUC Library lists perhaps a dozen copies at various libraries on campus (all but two checked out at the time of this writing.) There is also an abridged version of the book in publication, with a different ISBN, reducing the content to about 300 pages. I was curious to learn what they left out, but was not able to obtain this edition for examination.

Website: **www.smith.edu/~phyllo/About/index.html**
This is a great website on phyllotaxis, the study of the arrangement of leaves. The graphics are good, and the writing covers all the main ideas with thoughtful explanations, yet it remains accessible to the non-scientist.

Istvan Hargittai, Clifford Pickover, Ed., *Spiral Symmetry* (World Scientific, 1992). This is a collection of essays on the spiral, with emphasis on the "role that spirals play in science". Note in particular the articles on phyllotaxis: Roger Jean's article on branching, "Determination of Spiral Symmetry in Plants and Polymers", and Robert Dixon's "Green Spirals". Dixon writes at a level that could be appreciated by a calculus student. Ian Stewart also contributes with "Broken Symmetry and the Formation of Spiral Patterns in Fluids." See also Michael Cortie's lovely computer simulated seashells, pp. 378-379.

Jean, Roger V. *Mathematical Approach to Pattern and Form in Plant Growth*, 1984. Botanist Roger Jean has authored a number of articles and books on the mathematics of plant growth, of which this is one.

Kruhl, J.H., Ed. *Fractals and Dynamic Systems in Geoscience* (Springer-Verlag, 1994) UIUC Geology Library.
This is a collection of essays dealing with the application of fractal geometry to the geosciences, and was an offshoot of an international conference in 1993 by the same name. Yes, this is somewhat dated, and it is definitely stuff for the geologists - in some places just the vocabulary will stop you - but for a look at what they are doing with fractals, this is an interesting volume to peruse. Tired of fractal dragon pictures? Want to know how fractals have been applied to the study of earthquakes? Here's a place to look.

Doczi, György. *The Power of Limits: Proportional Harmonies in Nature, Art and Architecture* (Shambhala Publications, Boulder, 1981) UIUC Art and Architecture Library.
This title is included in part as an example of a good idea gone awry. Doczi's book is loaded with examples of mathematical patterns either found in nature or used intentionally. There are lots of interesting pictures and photos, and he does provide detailed diagrams clearly showing all the measurements made of the objects. Some of his

claims are well-established (and already well-known), such as the sine waves found in the rhythm of breathing or an ocean's waves, the logarithmic spiral found in shells, and certain patterns in leaves. However, he also claims that the golden ratio (phi) was used in the Pyramids, the Parthenon and other such places. This is a claim that has been repeated in the literature for years, even by otherwise respected writers, but has more recently been discredited due to lack of real evidence (sloppy measuring and acceptance of numbers anywhere near 0.6 usually produces the results; see Herz-Fischler or Markowsky.)

Where Doczi's book is different is how he connects everything to music. He starts out early in the book with a discussion of musical harmony, and how the Greeks developed the musical scale. This is good, but he connects this to the golden ratio 0.618 simply because the musical fifth has the proportion 2:3, which is about 0.666. He also does this with 3:5, or 0.6, which allows the Pythagorean right triangle to be "related" to the golden ratio. He then, without much difficulty, finds phi in the architecture and pottery of ancient Greece and Crete; the Tower of Babel, the Triumphal Arch of Constantine in Rome, the Colosseum, statues of Buddha, the Yakushiji Pagoda, and in the natural forms of fish, insects, and the human body.

Perhaps we might forgive architect Doczi for some of this, until we see some of the *other* places he has found phi: Stonehenge, Native American hats, dinosaurs, and the Boeing 747. There's lots more, and unfortunately this is what most of the book is really about. Check out the drawing of the frog skeleton and the accompanying musical scale on p. 71.

V. GENERAL/COMBINED/MISCELLANEOUS

Javier Barrallo, Nathaniel Friedman et al, *Meeting Alhambra, ISAMA-Bridges 2003 Conference Proceedings*, (University of Granada, Faculty of Sciences, 2003).
These are the complete conference proceedings for the 2003 meeting of the International Society of the Arts, Mathematics and Architecture, which I attended. This is a wonderful resource for mathematics related especially to the visual arts and nature, but there are also materials on music and architecture. It contains abstracts of all the presentations at the conference, and for most of them there are also several pages of notes, often including graphics, which at other conferences might be given as handouts at the sessions. The lists of references are a wealth of materials that could give one years of further study. The best thing I ever took home from a conference. 588 pages.

Mathematical Association of America, *Math Horizons*. UIUC Math Library, Reserves.
A quarterly journal published by MAA to encourage undergraduate students toward careers in mathematics. Included are articles on contemporary mathematics and recent items in the popular press (for example, the feature film, A Beautiful Mind), advice on job-hunting and graduate programs; cartoons and puzzles, and a column by Martin Gardner. Even the covers are inviting. The level of mathematics varies but does not exceed that of the undergraduate student. The issue of September, 2001 features artist Dick Termes' fascinating "termespheres"on the front and back covers, with a lengthy article inside.

Devlin, Keith. *Mathematics: The Science of Patterns* (Scientific American Library, 1994). Champaign Public Library.
This book traces the development of mathematics as a study of patterns through history, beginning with a pile of sticks used to "count". Written for those with a general interest in mathematics, it is neither a text nor a book on math history; a better description would be that it is a collection of articles that explain the best ideas of mathematics. The mathematical level of the writing varies considerably, increasing as the book goes on. Nevertheless, quite a bit of it would be accessible to the average high school graduate. There is a colorful piece of artwork, photo, graph, or historical note on almost every page. The major topics are counting, logic and abstraction, motion and change, geometries of all sorts (including illustrations of the use of perspective and representations of the four-dimensional hypercube), symmetry (tiling, packing, art and crystals), and position or topology (Möbius strips, knots, networks).
There is no bibliography, but the short but sweet list of Further Readings given at the back is welcome. A listing of the sources for all of the illustrations is given as well.
A very nice book overall.

Herman Weyl. *Symmetry* (Princeton University Press, 1952). UIUC Math Library.
Although this book is over fifty years old, symmetry is a fundamental notion in mathematics that plays a big role in artistic patterns and natural phenomena, and these are topics that are enjoying increased interest and new discovery today.

This isn't your algebra book's treatment of symmetry. Symmetry across a line or through the origin is just one example of a more general notion, and Weyl's purpose in the book is to explain what it is and illustrate its occurrence in the arts and nature.

The book is derived from four lectures he had given at Princeton (it is not clear to what audience), and is not mathematically rigorous in the whole, but there are places where his language is quite abstract – he talks of groups, fields and linear transformations - and this will be lost on the untrained reader. Nevertheless, I think Weyl has a lot to offer in this short book, which contains only 145 pages plus appendices. His intent is to convey the basic ideas without getting into too much detail or technicality, and most of the time he pulls it off. It is not awash with graphics and artwork as today's books are, but he does include several that are useful.

Richard K. Guy and Robert E. Woodrow, Ed. ***The Lighter Side of Mathematics - Proceedings of the Eugene Strens Memorial Conference on Recreational Mathematics and its History***, Mathematical Association of America ,1994. UIUC Math Library.
This is part of MAA's Spectrum Series, which contains books on mathematical topics that will appeal to a wide range of readers, including students, teachers, amateurs, and researchers. The book is a collection of articles, the most of which concern mathematical puzzles or games. However, the first group, Tiling and Coloring, contains an article by Doris Schattschneider. Schattschneider is an expert on M.C. Escher, and has written extensively on Escher's works. Her article, *Escher: A Mathematician in Spite of Himself,* explains that Escher's use of mathematical principles in his drawings went beyond his formal training, and it was interesting to note that she says his accomplishments probably benefited from *not* being "hampered by advice and criticism from experts." Other familiar names from the art/science pantheon that authored articles are: Ivars Peterson, H.S.M. Coxeter, Stan Wagon, and Sol Golomb. At the back of the book there is contact information for a long list of participants from several countries.

Jamie H. Eves. ***Introduction to the History of Mathematics with Cultural Connections***
6th Ed., UIUC Math Library.
There were probably half a dozen copies of *Howard* Eves' history of mathematics sitting on the shelves, so either it is popular or has been required reading. Perhaps Jamie is related to Howard. A very brief description of Jamie's book (found after some web search) suggests that it is a joining of Howard's *History* with the added *Cultural Connections* written by Jamie. This seems to be a good reason to leave the older volume on the shelf. Unfortunately, there was only one copy of Jamie's book listed in the UIUC catalog and it was checked out at this writing, and not due anytime soon.

Website. The Pythagoreans. **http://about Scotland.com/harmony/prop**
This presents the Pythagoreans and the development of the musical scale. There is a lot to see here, but what is special is a feature that lets you "pluck" the strings and listen to the notes.

VI. ORGANIZATIONS

LEONARDO, The International Society for the Arts, Sciences and Technology (ISAST)

Founded in 1968 by kinetic artist and scientist Frank J. Malina, this group has some nice artwork in their online gallery (**www.mitpress2.mit.edu/e-journals/Leonardo**). They serve first the artist, and states their focus as being "... on writings by artists who work with science- and technology-based art media." Their journal, *Leonardo*, is available at various libraries at UIUC.

> *The Leonardo Almanac - International Resources in Art, Science and Technology*, Harris, Craig, Ed., MIT Press, 1993.
> This is a resource book which can help one navigate through the fields where art, science, and technology converge. Contains profiles of dozens of organizations, however I noted there was no listing for Bridges or ISAMA (The International Society of the Arts, Mathematics and Architecture), which might suggest that Leonardo is less mathematically oriented. The resource book is clearly heavy on the arts, especially computer generated art, and music.
>
> Other publications include the *Leonardo Music Journal*, the *Leonardo Electronic News*, and the Leonardo Book Series, which includes *The Visual Mind,* M. Emmer, Editor. This last book is wonderful, and is described elsewhere in this report.

International Society for the Arts, Mathematics and Architecture (ISAMA)

This group holds an interdisciplinary conference each summer. Founded in 1998 by Nat Friedman, mathematician and sculptor at University of Albany-SUNY, ISAMA promotes collaboration and communication between the arts and science, and provides a needed forum for those whose interests connect art and science. It arose from a series of Arts and Mathematics conferences, AM92 – AM98, first held at SUNY, and then later at Berkeley, which attracted the attention of other scientist/artists in the U.S. as well as internationally. This group consists of multitalented people, who are often professionally accomplished in two completely different fields.

Bridges, Mathematical Connections in Art, Music and Science

This organization was begun at Southwestern College in Kansas in 1998 by Reza Sarhangi, as a response to the Arts and Mathematics (AM) conferences mentioned above. They share interests and members with ISAMA, although there is a different mix of topics and they don't claim to be an international group. In 2003 Bridges held a joint meeting with ISAMA in Granada, Spain.

The Fibonacci Association

There is a lot more to the Fibonacci sequence than what one can read about in the popular literature. This group deals with all sorts of deeper mathematics related to the famous sequence. Their journal is the ***The Fibonacci Quarterly,*** whose content is heavy with esoteric theorems and their proofs. Occasional applications to other areas, e.g. Markov chains can be found. Their ninth international conference was held in 2000 in Luxembourg. This is a journal of interest primarily to number theorists. UIUC Math Library.

Joint Policy Board

Responsible for Math Awareness Month each April, this is a collaborative effort of the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. Of particular interest was April 2003, which celebrated Mathematics and Art. (See Arts for a listing of the website.)