Optimization of Operation of a System of Flood Control Reservoirs

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ABSTRACT

Optimization of operation of a system of flood-control reservoirs is established by the application of mathematical programming. The mathematical procedure is applied to two different types of systems, reservoirs in parallel and reservoirs in tandem.

The operational matrix to be optimized is made up of the objective function and the constraining equations. The objective function that is to be maximized is made up of the time sequence of releases from the reservoirs. The physical, structural and hydrological limitations are described by the constraint equations. All equations in the operational matrix are linear.

Inflows to the reservoirs of the system and the initial conditions are assumed to be known, as are the reservoir capacities and downstream-channel maximum and minimum capacities. The objective of the operational matrix is to maximize the sum of releases thus minimizing the storage occupied by flood water.

Set up of the operational matrix is carried out using a digital computer program and the optimization is carried out by applying the Linear-Programming algorithm of MPS/360. Results of the procedures are shown for a

three reservoir system in the Kansas river basis (U.S.A.) using actual data.

keywords - Flood Control, Probability, Linear Programming

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CHAPTER I

INTRODUCTION

1.1 INTRODUCTION TO THE PROBLEM AREA

1.1.1 General. A flood is an overflow onto land which although adjacent to water is not normally covered by it. Flooding simply can be considered as an inverse pollution, water "contaminating" the land.

Flood-plain land, alongside rivers and streams has always attracted man, because of the potential advantages it offers to him. Flood-plain fertility encourages agriculture, flatness encourages urban development, railroad and highway construction. Many other advantages are offered by flood-plains that overshadow the disadvantages, by which nature exacts a price for the advantages it offers. The way that nature has found of exacting a price for the advantages it offers is by infrequent flooding.

The flood control program in the United States developed as the country grew. At first, individuals struggled with the flood control problem, but as the population increased and flood damages became greater local governments and ultimately the Federal Government became involved. Today the flood control program is a major responsibility of the Corps of Engineers of the U.S. Army.

Floods are essentially caused by large volumes of water in a short period of time, which arrive and occupy the stream channel and its flood-plain, thus causing damages to economic activities. Generally the essential problem of flood control is;

a) To contain the flow within a designated damage free area (Improve channel flow conditions).

b) To store the water until it can be safely released to the channel (Store the water in a reservoir and operate the reservoir to get safe releases).

c) Do both (a and b).

1.1.2 Flood Control Measures. From probability theory it is known that all rivers are subject to a non-zero probability that any particular flow will be equalled or exceeded. This means that for every flood-plain there is a probability that the river or stream shall leave its banks and flood the adjacent plains. Since each level of flow can be exceeded it may be stated that "absolute flood control" is rarely feasible, either physically or economically.

Therefore, the purpose of a flood control measure is to "reduce flood damages to the greatest extent possible". The widely accepted flood control measures are classified into two categories.

a) Structural measures. These measures make the use of structures (physical) which may,

1. Contain the flow within a designated area

free of damages (channel improvement.)

2. Store the flood water (behind dams) or do both.

Common structural measures are the following:

1. Flood Control Reservoirs. The function of a flood control reservoir is to store a portion of the flood water to minimize the flood peak downstream. Of great importance in operation of flood control reservoir is the size of releases that must be made given the size of the inflows.

2. Levees. Construction of levees is actually a way of increasing the carrying capacity of the channel. By this method the flood is confined within predetermined areas free of damage.

3. Channel Improvement Works. Achieve higher velocities and lower stages for the same rate of flow.

4. Diversion Structures. Divert the flood water through a bypasser of floodways.

b) Non-structural Measures. This type of measures do not control or reduce the flood but reduce or avoid flood damages. Some measures of this type are;

1. Temporary or permanent evacuation evacuation of the flood-plain.

2. Flood proofing of specific properties.

Measures of type a and b are effective in cases of emergencies such as when floods can be partly controlled by structural measures until temporary evacuation of the flood-plain is carried out.

1.1.3 Flood Control by Reservoirs & Reservoir Operation.

In a large river basin the primary line of defense against flood damages is a system of flood control reservoirs. A flood control reservoir provides control for the area downstream. Its effectiveness is reduced with increasing distance downstream, due to the lack of control over the local inflow between the dam and the area to be protected. It is obvious that if the local area is big enough it may be capable of producing a flood over which the reservoir would have little or no control. From the above it can be concluded that the relative positions of flood control reservoir and the area to be protected are major factors in the effectiveness of the reservoir to reduce the flood peak.

Another factor determining the reservoir effectiveness on flood reduction is the amount of storage allocated for flood control. The potential of a reservoir in reducing peak flows by reservoir operation increases as flood control storage increases, because a greater portion of the incoming flood water can be stored. However, economic and topographic limitations control the maximum feasible size of the reservoir.

Since the flood control storage is limited, by economic and other physical factors, this volume has to be used wisely to yield maximum effectiveness. For a one reservoir system the operation is simple as is shown in section 1.2.1 but for more than one reservoir the problem becomes more involved. This report deals with the multiple reservoir problem in detail.

The reservoir operation shall be optimized by mathematical optimization procedures. Such procedures have been applied on many aspects of water resources for several years. They have not, however, been applied to flood control operations.

1.2 STATEMENT OF THE PROBLEM

1.2.1 General. The main objective of reservoir regulation, for flood control, is to reduce downstream flood damages to the greatest extent possible with available facilities. Reduction of flood damages requires keeping the flows below flood stage at all times, while excess flood waters are stored in the flood control reservoir.

Consider a single reservoir located immediately upstream the area to be protected. The inflow hydrograph, the reservoir capacity and initial condition, as well as the channel safe capacities, are known. Given the above determine the releases from the reservoir such that;

a) The reservoir is not spilling nor has a negative volume in storage.

b) The channel safe capacities are not exceeded, nor is the flow less than the minimum acceptable.

The release schedule from the reservoir should be the one shown on Fig. 1. Pass all incoming inflow until the outflow reaches the zero damage capacity of the channel downstream. (Point A on Fig. 1) All flow above this safe rate is stored until the inflow drops below the zero damage capacity of the channel (Point C on Fig. 1) and the stored water is released to recover storage for the next flood. This schedule of release is possible provided that the volume of water A, B, C (Fig. 1) does not exceed the flood storage available by the reservoir. If that is not the case then the release schedule shall be changed, according to the reservoir initial conditions.

For a system of reservoirs more than one, the optimum schedule of releases can be determined by trial and error. For a system of reservoirs with different initial conditions, different inflow hydrographs, different capacities, more than one flow to consider, and different channels to consider, the problem becomes very involved. A trial and error method would be possible but the time needed for such a solution would render the solution trivial. Therefore a systematic solution to this problem

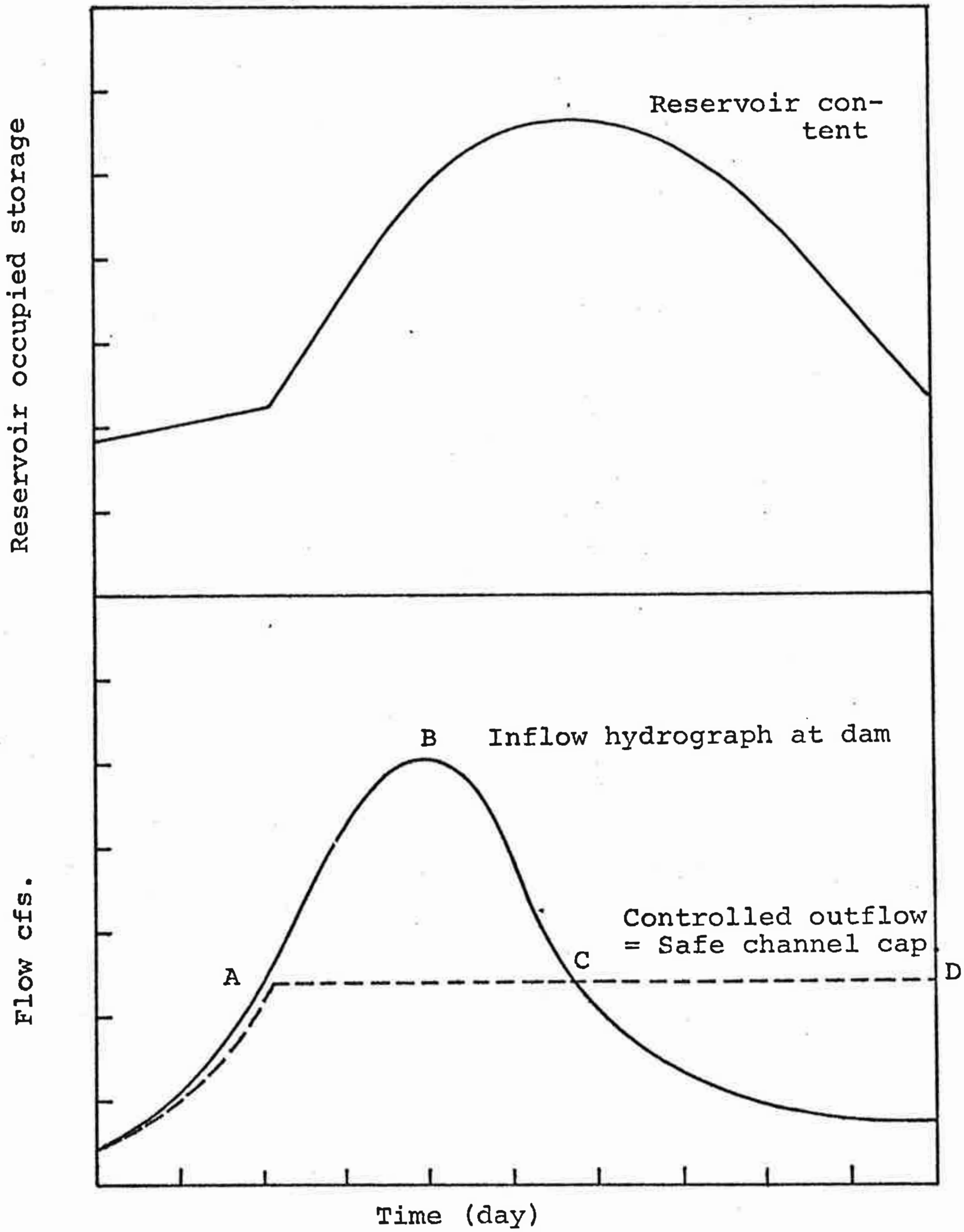


Fig. 1 Idealized Operation of a Flood Control Reservoir

is to be sought in this study by using mathematical optimization procedures.

1.2.2 Objectives of the Project. The aim of this study is to establish a procedure, using Linear Programming, for the optimization of flood control operation of a system of several reservoirs, operating in parallel, in tandem or of mixed type systems.

The procedure is to employ a digital computer to produce a program applicable to any river basin in which the floods are controlled by a system of reservoirs, and shall be applied to an actual river basin with actual data.

1.2.3 Problem Statement. In a system of flood control reservoirs, either multipurpose or single purpose, the portion of storage allocated for flood control operations is limited, whereas the channel capacities are bounded above and below by the maximum and minimum allowable flows. The problem in final analysis reduces to the one of limited resource allocation (flood storage allocation) among different demands (flood waters). This is to be stated as follows.

Maximize the total sum of releases from a system of reservoirs, thus maximizing the free flood control storage volume, (minimizing the volume occupied by flood water) provided always that the physical and hydrological limitations of the system are satisfied.

The objective function to be maximized shall be a performance function and the optimal solution shall be the one that maximizes the releases provided that it satisfies the following limitations.

a) The reservoirs shall not have any uncontrolled spilling. This means that the water level in every reservoir in the system must be kept below crest level.

b) The volume in storage cannot take any negative values. This limitation expresses the hydrology of the system. The outflow cannot be more than the inflow plus the water in storage.

c) The flows in the tertiary, secondary and main channels must always be kept within the range of safe capacities.

The objective function and the constraint equations shall be linear or linearized by approximation to fit Linear Programming requirements.

The Linear Programming algorithm of the Mathematical Programming System/360 shall be applied to carry out the optimization.

1.2.4 Input Data. To formulate and process the mathematical model, to be developed, the following data must be known;

a) Reservoir capacities. Expressed in units of volume the reservoir capacity or flood control storage for every reservoir in the system.

b) Initial conditions of reservoirs. The amount of water in every reservoir at the beginning of the operations.

c) Maximum and minimum level of water in every reservoir (preferred, for any reason) must be decided in advance, expressed in units of volume.

d) Inflow to every reservoir for a certain time to a certain accuracy must be known in advance, before operation begins, expressed in units of volume per time interval.

e) Acceptable channel flows. The minimum acceptable flows in every channel in the system must be known. Such flows are determined generally on the basis of:

1. Water quality criteria
2. Navigation, and
3. Other reasons.

f) Acceptable, maximum channel flows. Engineering economic techniques can be employed to derive the most optimum stage of flow in a certain channel. However, such methods become very involved and in many cases are not the best to use.

Other methods, more empirical, as the one outlined in paragraphs 4-2 and 4-6 [8], have been used to define the upper limits of channel flows. This method being developed and in use by the U.S. Army Corps of Engineers for the regulation of flood control reservoirs, relates individual reservoir storage space in use to normal seasonal zones of demand. Each zone or "phase" is numbered phase I through phase IV in the increasing order of the storage demand severity.

Operation of the reservoir can be carried out in any of the four different phases according to the seasonal evacuation demand.

Phase I:	2/3 channel capacity
Phase II:	Channel capacity
Phase III:	Flow is such that it causes appreciable damage
Phase IV:	Flows are under no control.

A typical graph showing the guidelines is shown on Fig. 2. For the purpose of this study it will be assumed that such guideline graphs are available for defining the maximum acceptable flows in the channel downstream the dam.

1.3 DEFINITIONS AND BASIC ASSUMPTIONS

1.3.1 Definitions. In what follows some of the many terms to be used in this paper are to be defined.

a) Reservoirs in Parallel. A system of reservoirs is defined as reservoirs in parallel when inflow to

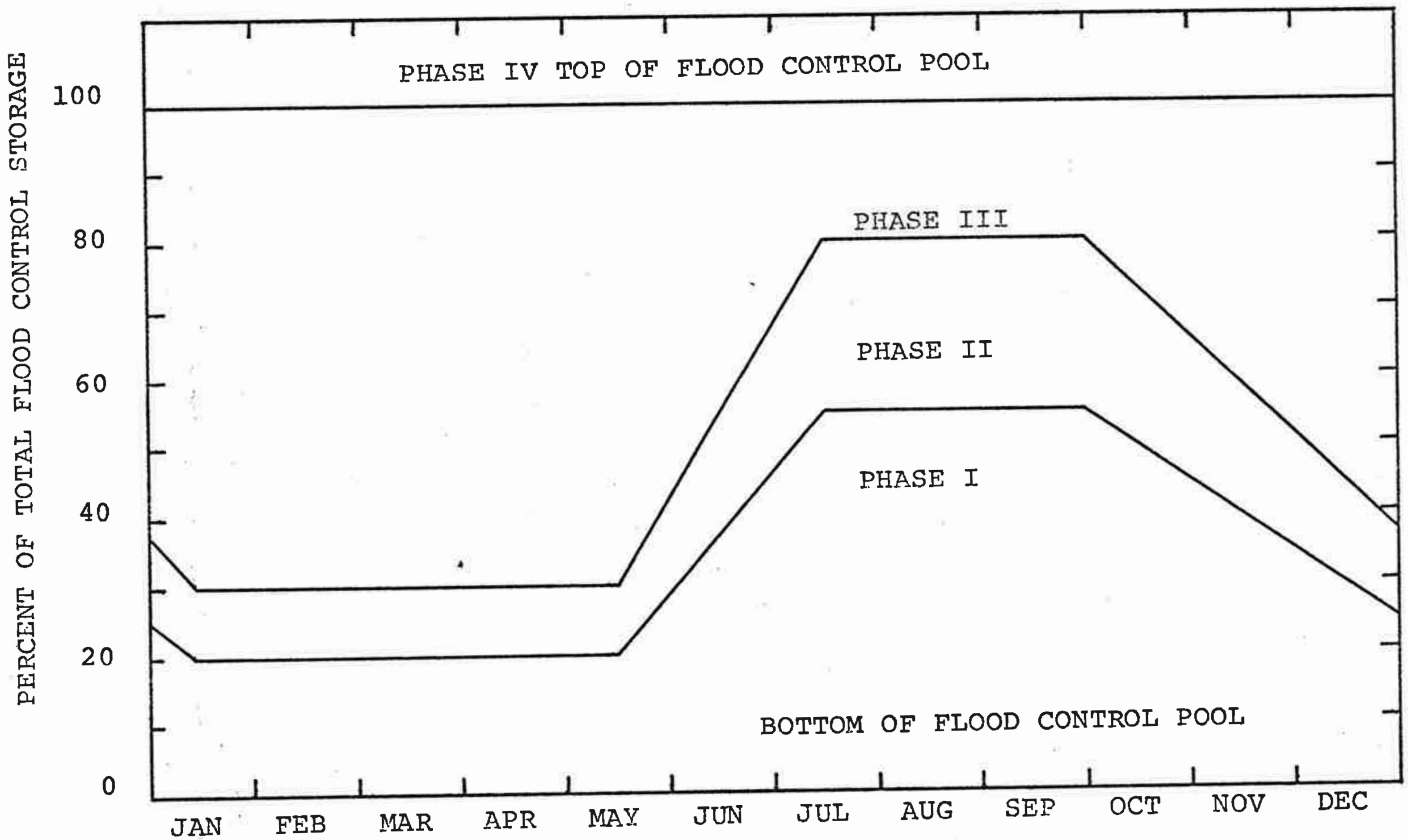


Fig. 2 Typical Guidelines for Flood Control Operation

every reservoir in the system is independent of the outflows from other reservoirs in the system (See Fig. 3).

b) Reservoirs in Tandem. A system of reservoirs is said to be in Tandem when the inflow to at least one reservoir is a function of the outflows from the rest of the reservoirs in the system. In Figure 4 inflow to reservoir no. 4 is a function of outflows from reservoirs no. 1, 2, and 3.

c) Reservoirs in Series. A system of reservoirs is said to be operating in series when the inflow to every reservoir is a function of the outflow of one reservoir situated upstream. In Fig. 5 reservoir no. 2 is in series with reservoir no. 1 and reservoir no. 3 is in series with reservoir no. 2.

d) Mixed type reservoir system. A mixed type reservoir system is made up of subsystems of reservoirs in series, in parallel, and in Tandem. In Fig. 6 reservoir no. 4 is in series with reservoir no. 2. Subsystem of reservoirs no. 3, 6, and 5 is in tandem and subsystem of reservoirs no. 1, 4, 6, and 7 is in parallel.

Any mixed type system can be broken down into a number of the basic subsystems, ie. into reservoirs in tandem, in parallel, and in series.

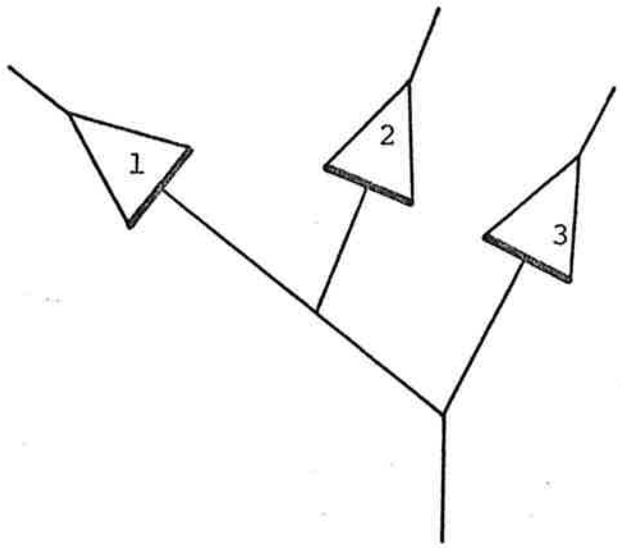


Fig. 3 . Reservoirs in Parallel

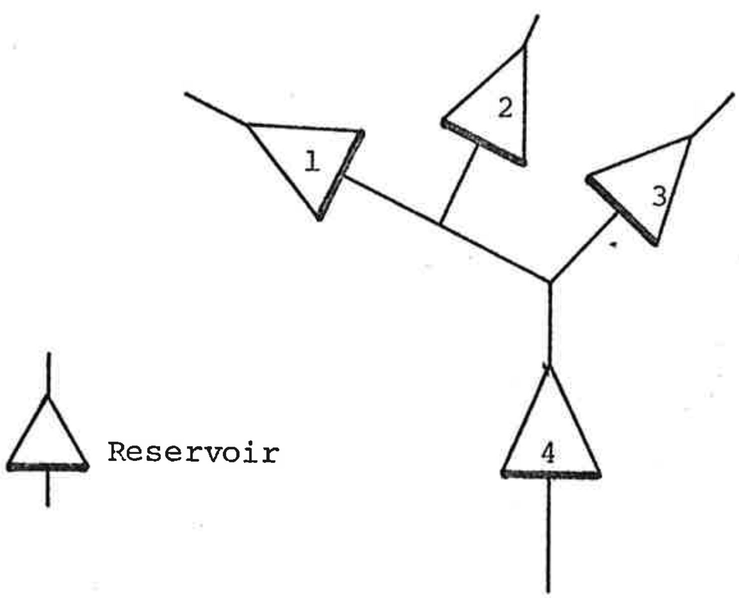


Fig. 4 Reservoirs in Tandem

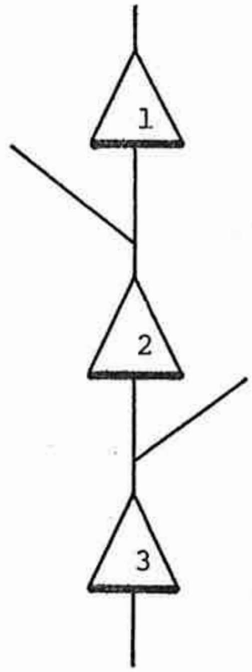


Fig. 5 Reservoirs in Series

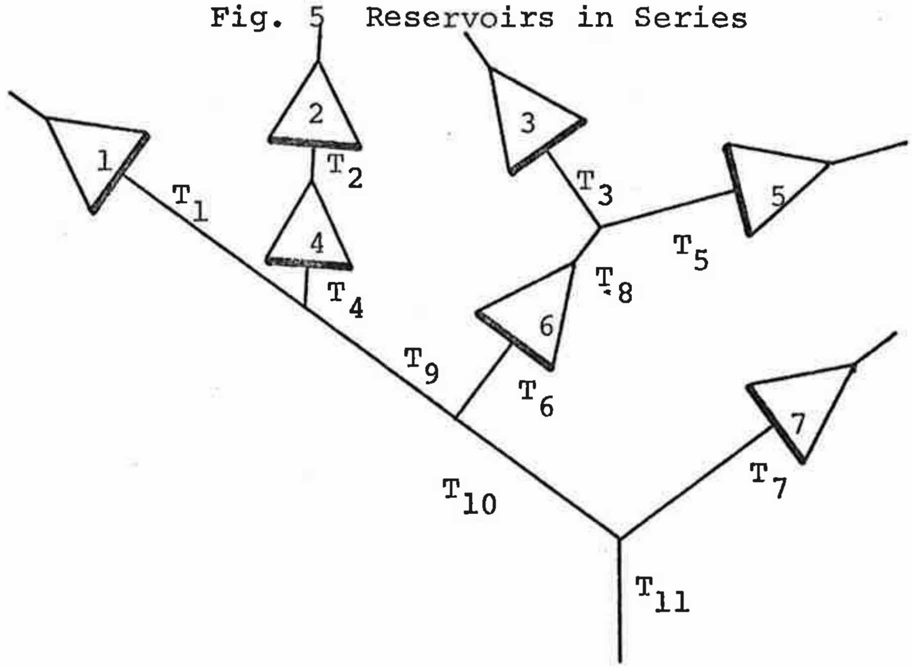


Fig. 6 Mixed Type Reservoir System

e) Channel Operating Phases. A tertiary, secondary or main channel is to be allowed to operate at any moment with a maximum flow defined by the guidelines for flood control operations. The U.S. Army Corps of Engineers breaks the flood control operations into four phases as defined previously. So when it is stated that the channel is operated under phase II it means that the maximum allowable flow in that channel is the one defined by phase II according to the U.S. Army Corps of Engineers. The channel operating phase defines the upper limit of flow.

f) Channel Minimum Flows. The minimum flow in a channel is established to a certain extent by requirements of pollution control, municipal and industrial water supply, navigation and other uses. This quantity may vary with the season of the year and the quality of water.

g) Flood Storage Reservoirs. Reservoirs that are equipped with outlet works which can be used to regulate the outgoing flow, are referred to as flood storage reservoirs.

h) Tertiary Channel. Tertiary channel is defined as the channel associated directly with the outflow from only one reservoir. With reference to Fig. 6, channels T_1, T_2, T_3, T_4, T_5, T_6, and T_7 are tertiary channels.

i) Secondary Channel. A secondary channel is defined as the channel that receives flows from more than one tertiary channel (or reservoir) but not from all the reservoirs in the system. In Fig. 6 channels T_8, T_9, and T_{10} are secondary channels.

j) Main Channel. Main channel is defined as the channel that receives flows from all of the reservoirs. There is only one main channel in a system of reservoirs. In Fig. 6 channel T_{11} is the main channel.

1.3.2 Assumptions. In the formulation of the mathematical model for determining the optimal sequence of releases from a system of reservoirs, the following assumptions were made.

a) The inflow hydrograph to every reservoir in the system is known in advance for a reasonable period of time.

b) The initial conditions for every reservoir are known.

c) The operating guidelines for allowable flows in every channel in the system are known.

d) Minimum and maximum allowable water levels of every reservoir at the end of each time interval during the operation period, under consideration, are known.

e) Minimum acceptable flows for every channel in the system are known.

f) Local inflows to the river downstream the flood control reservoir and upstream the damage center is neglected or assumed to be considered in selecting the maximum safe capacity of the particular channel.

g) The outlet works of any dam are designed to allow any discharge within the range of minimum to maximum safe capacities. By this it is assumed that the outlet works do not restrain in any way the releases. (Release is independent of the water level in the reservoir.)

h) The water travel time is not a function of the discharge. This assumes that the water travel time from any reservoir to the damage center is constant, whatever the flow in the channel. (This is not absolutely true.)

1.4 RESUME OF RELATED RESEARCH

The advantage and need of coordinated regulation of a system of reservoirs in a river basin has been recognized for years. Regulation experience of many years has shown the importance of maintaining the available storage space in the various reservoirs of the system in hydrological and seasonal balance at all times in anticipation of floods. Further, releases that could augment downstream flood damages should be avoided.

In 1958 the U.S. Army Corps of Engineers released a report [1] on "Reservoir Regulation". This report, prepared for use by district engineers and their technical personnel, contains the results of many years of experience.

In the report it is indicated that the method by which a reservoir is regulated, to minimize flood damages, is very important on the effectiveness of the reservoir.

A total of three methods are described which have been successfully applied in many flood control projects. These methods are:

a) Maximum beneficial use of available storage during each flood event. The success of this method depends on accurate forecast of the inflow.

b) Regulation is based on control of project design flood, and

c) Combination of methods A and B. The general procedure for the derivation of releases for a system of reservoirs by any method is as follows:

1. Develop first general schedules for the tributary reservoirs operating as separate units.

2. Adjust the individual regulation schedule for coordinated operation of the various tributary and main river project, considering the analysis of the basin project plan and design flood.

Leo Beard [2] gave a comprehensive description of the problems involved in flood control operation and some of the experiences in operating a system of flood control reservoirs in California. The practice used in California, in multipurpose reservoirs, was to release water whenever necessary at the highest practical rates, so that a minimum space should be needed for flood control.

The operation of reservoirs in California is based on flood control diagrams prepared for each reservoir in the system. The flood control diagrams based on observations and practical experience of operation of each reservoir gives the flood storage requirement as a function of time and antecedent precipitation.

Although mathematical optimization procedures have been used in general planning and operation of water resources for several years, problems of flood control have been studied only recently. A notable contribution was made by William C. Hughes using methods [3] from calculus. Hughes [3] optimizes the releases from a system of reservoirs using procedures based upon the minimization of a cost function that includes the future flood risk cost, resulting from storage and direct downstream damage cost associated with the magnitude of reservoir releases.

The mathematical model which consists of a cost function, relating outflow magnitude with both direct flood damage cost and the cost of foregone flood control capability due to storage, is applied over discrete time periods. The schedule which minimizes the cost function is defined as the optimum one.

The solution involves the application of the Lagrange technique and involves the solution of a polynomial. In case of n reservoirs in parallel the problem involves the solution of n polynomials with n unknowns.

CHAPTER II

MATHEMATICAL MODELS

2.1 INTRODUCTION

The general problem of water resources either water conservation or flood control, is to transpose a matrix describing the natural occurence of water and its properties, to another matrix, related to the objectives of the project. [4]

The natural occurence of water can be described by a matrix F consisting of three vectors: L (location), T (quantitative distribution in time), and Q (quality) thus,

$$F = \begin{bmatrix} L \\ T \\ Q \end{bmatrix} \qquad (2.1.1)$$

The location vector L has two kinds of components, x and y, which determine the spatial extent of the waters. In the flooding case x and y define the locations at which flooding occurs and the locations of the various reservoirs in the system. Thus,

$$L = (x_1, x_2, \ldots x_n; y_1, y_2, \ldots y_n) \qquad (2.1.2)$$

The time vector T consists of parameters of the quantitative occurence of flood waters in time, ie. the flood flow hydrographs at the reservoir sites and at the damage center. The quantities are expressed as averages over some time period. In this case the period is expressed in hours. Then,

$$T=(\mu_{1,1},\mu_{1,2}\cdots\mu_{1,n};\mu_{2,1}\cdots\mu_{2,n};\cdots\mu_{n,n}) \quad (2.1.3)$$

The quality vector Q, although of no great consideration in this study, is considered to be made up of a number of elements, such as biological quality, the mineral quality, the caloric quality and so on. Then,

$$Q= (qb,qm, \ldots qn) \quad (2.1.4)$$

The construction of flood control reservoirs and the mode of operation, amounts to the transformation of the original matrix F, into another matrix F* in which the elements of the vectors L,T, and Q assume desired values. Thus,

$$F^*=\begin{bmatrix} L^* \\ T^* \\ Q^* \end{bmatrix} \quad (2.1.5)$$

The vector L* indicates the locations at which the flood water must be controlled, by structural or non structural measures. T* pertains to the time distribution of the quantities of flood water; ie., the flows are to be kept below flood damage stages. Q* represents the quality standards at which the flowing water has to be maintained.

The transformation of matrix F to F* is to be achieved by,

$$F^*= (\theta_1 \; \theta_2) \; F \quad (2.1.6)$$

where θ_1 is a submatrix which is composed of design parameters of the physical components of the system ("hardware"), and θ_2 is a submatrix which contains the operational aspects of the system ("software").

The analysis and solution of equation 2.1.6 is the realm of water resources engineering. However, for flood control operations where the system already exists ("hardware") and the submatrix θ_1 is known, it remains to find the operational aspects of the system (θ_2) and to try to optimize this matrix.

The objective of the problem at hand is to optimize the operational matrix θ_2 given the matrices F, θ_1, and the desired values of the matrix F*.

The determination of matrix F involves investigation in natural sciences; goelogy, hydrology, meteorology, and others. On the other hand the matrix F* requires, for its evaluation, investigations in economics, sociology and other sciences.

In this study it is assumed that:

1. The matrix F is known in detail.
2. The submatrix θ_1 has been established.
3. That matrix F* is given values within the desired range.

2.2 LINEAR PROGRAMMING

The objective function and the constraint equations shall be expressed in linear equations or inequalities and the Linear Programming algorithm is to be applied to carry out the optimization.

The mathematical statement of a generalized linear programming problem (or model) [5] is the following. Find the values of $x_1, x_2, x_3, \ldots x_n$ called the decision variables, which maximize or minimize the objective function;

$$Z = c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots\ldots c_n x_n \tag{2.2.1}$$

subject to the following relationships (called constraints)

$$\begin{aligned} a_{1,1}\, x_1 + a_{1,2}\, x_2 + \ldots\; a_{1,n}\, x_n \;\{\geq = \leq\}\; b_1 \\ a_{2,1}\, x_1 + a_{2,2}\, x_2 + \ldots\; a_{2,n}\, x_n \;\{\geq = \leq\}\; b_2 \\ a_{m,1}\, X_1 + a_{m,2}\, X_2 + \ldots\; a_{m,n}\, X_n \;\{\geq = \leq\}\, b_m \end{aligned} \tag{2.2.2}$$

and $x_j \geq 0$.

The decision variables, $x_1, x_2, x_3 \ldots x_n$ represent the levels of n competing activities, where z is the overall measure of effectiveness. In the flood control operational matrix, the decision variables represent the magnitude of the releases to be made during each discrete time interval and z expresses the total release to be made from all reservoirs during the total period of operation.

c_j is the increase in the objective function that would result by an increase of one unit of x_j.

The first m linear inequalities correspond to a restriction to the availability of one of the resources, where in this case resources are considered to be the amount of flood water available at the various reservoirs. b_i is the amount of resource i available to the n activities, where $a_{i,j}$ is the amount of resource i consumed by each unit of activity j. The last restriction $x_j \geq 0$ rules out the possibility of negative activity levels.

A problem formulated by a set of linear inequalities and made up of only two decision variables can be easily solved, graphically, provided that there are feasible solutions. For small problems with more than two variables the simplex method can be employed, where for problems involving a large number of decision variables and many constraint equations, optimization is most easily carried out by applying an electronic data processing Linear Programming algorithm such as MPS/360. In this study optimization of the operational matrix is carried out by the application of the Linear Programming algorithm of MPS/360.

2.3 LINEAR MODEL OF OPERATIONAL MATRIX

In order to translate the physical system and its operational limitations into a mathematical model the following notations are used. $X_{i,j}$; $I_{i,j}$; R_i; C_i; $Z_{i,j}$; $Y_{i,j}$;

$b_{i,j}$; $B_{i,j}$; Bl_j; $c_{i,j}$; $d_{i,j}$; $D_{i,j}$; Dl_j. The definitions of these notations are given in Appendix I.

The mathematical model of the operation of a system of reservoirs shall be made up of the objective function, which is to be maximized, and the constraints, as follows.

1. Objective function: Maximize the releases from the system of N reservoirs, for n time intervals,

$$\sum_{j=1}^{n} \sum_{i=1}^{N} c_{i,j} X_{i,j} = Z \qquad (2.3.1)$$

subject to the following physical and hydrological limitations of the facilities of the system.

2. Constraints: Each set of constraints is to be made up of n constraints, each constraint equation expressing the limitations for one time interval only.

a) Minimum cumulative release constraints. Mathematically this constraint is given by,

$$\sum_{j=1}^{k} X_{i,j} \geq \sum_{j=1}^{k} I_{i,j}+R_i-C_i+\dot{Z}_{i,j} \qquad (2.3.2)$$

where i= the reservoir call number and j= time interval j=1,2,3...k...n. For each reservoir in the system there will be one such set of constraints made up of n inequalities.

Inequality 2.3.2 states that the cumulative release, from the time operation of reservoir i has started (j=1) to any time interval j, after, (j≤n), cannot be less than the sum of inflows during that period of time, plus the initial storage plus a certain volume (z) minus the capacity of the reservoir. Simply this equation avoids the uncontrolled spilling from the reservoir.

b) Maximum cululative release constraints. Hydrological limitations of the basin and initial conditions of the reservoir are also to be considered in the mathematical model. It is true that it is not possible to get more water from the reservoir than was initially put in. So this set of constraints states that the maximum cumulative release from reservoir i, shall be equal or less, than the cumulative inflow plus the initial storage minus a value Y. Mathematically,

$$\sum_{j=1}^{k} X_{i,j} \leq \sum_{j=1}^{k} I_{i,j} + R_i - Y_{i,j} \qquad (2.3.3)$$

where i= the reservoir call number and j= the time interval, j= 1,2, ... k ...n. There will be one set of such constraints for each reservoir, made up of n time sequence inequalities.

c) Tertiary channel flow constraints. This set of "Bounds" (according to section 2.10.4) states that the flow in the tertiary channels cannot be less than a minimum acceptable value neither can it be more than a maximum acceptable value. The minimum and maximum acceptable values for each channel are known. Mathematically this set of Bounds is given by,

$$d_{i,j} \leq X_{i,j} \leq b_{i,j} \tag{2.3.4}$$

Each reservoir having a release to a tertiary channel will be associated with one set of Bounds each set to be made up of n inequalities.

d) Secondary channel flow constraints; Maximum. By definition a secondary channel is the one whose flow is made up of releases from two or more reservoirs. Mathematically this set of constraint equations is generally expressed by,

$$x_{1,j} + x_{2,k} + \ldots x_{N-1,m} \leq B_{i,j} \tag{2.3.5}$$

where: j,k, and m represent the time interval the releases are to be made from reservoirs 1,2,...N-1. The releases are not made at the same time interval but reach the secondary channel at the same time. To avoid complications the starting time for each reservoir in the system shall be adjusted so

$$x_{1,j} + x_{2,j} + \ldots x_{N-1,j} \leq B_{i,j} \tag{2.3.6}$$

is possible. This is to be explained at a later section titled, "Commencement and Termination of Reservoir Operations".

e) Secondary channel flow constraints; Minimum. Flows in the secondary channels must be kept above a certain minimum. Mathematically,

$$x_{1,j} + x_{2,j} + x_{3,j} + \ldots x_{N-1,j} \geq D_{i,j} \qquad (2.3.7)$$

In the formulation of 2.3.7 it has been assumed that each reservoir has its own starting time, as is explained in section 2.9.3.

Each constraint equation expresses the total release from certain reservoirs to the secondary channel during the time interval j for each reservoir. Time interval j for reservoir i might occur at a different time than the time interval j of reservoir k.

f) Main channel flow constraints; Maximum. By definition there is only one main channel (or none) in a system of reservoirs. The damage center to be protected is usually located somewhere along this channel. Therefore the flow should be kept below flood stages as long as possible to reduce flood damages.

Mathematically,

$$\sum_{i=1}^{N} X_{i,j} \leq BIj \qquad (2.3.8)$$

where i represents the reservoir call number (1,2,...N) and j is the time interval under consideration or time interval during which the release x is made. Reference time for the different reservoirs in the system is assumed different. In a system of N reservoirs there will be only one (or none) set of constraint equations made up of n inequalities of the form 2.3.8.

g) Main channel flow constraint; Minimum. For water quality control and other criteria the flow in the main channel cannot be less than the minimum acceptable.

This set of constraints is to be made up of n inequalities expressed as follows,

$$\sum_{i=1}^{N} X_{i,j} \geq DIj \tag{2.3.9}$$

where the notations are the same as explained in Appendix I.

2.4 NOTATIONS OF SYSTEM FACILITIES

In the mathematical model given in a general way in section 2.3 the releases and the input data were defined by referring to a certain structure or facility (reservoir or channel) and to a certain time interval. The reservoir or channel number has been given by i, where i= 1,2,3, ...N and the time interval by j where j= 1,2,3, ...n.

In this section a procedure is to be given for assigning call numbers to the different reservoirs and channels of the system.

a) Reservoir notation. Arrange the reservoirs in order, in terms of the size of their water travel time to the damage center, assigning call number 1 the one with the longest water travel time and reservoir no. N to the one with the shortest water travel time, where N is the number of reservoirs in the system. This method of assigning call numbers indicates the order of water travel time from each reservoir and the order of commencement of operations.

b) Channel notation. Tertiary channel: Each tertiary channel is assigned the same call number as the reservoir upstream. Secondary and main channel: Any suitable notation will do, provided that it does not cause confusion with the reservoir and tertiary channel notation.

2.5 TIME INTERVALS

In the mathematical model, the decision variables and the inflows are expressed in units of volume per time interval. Also, the model describes the operation over a number of discrete time intervals. The sum of all discrete time intervals equals the period for which operation of reservoirs is planned.

The number of time intervals in a finite period can range from 1 to an unlimited number. For practical and other reasons it is proposed that a time interval of 6-48 hours be used depending on;

a) The size of the problem. Small size intervals may generate a large size problem.

b) The period of time for which operation is planned.

c) The efficiency desired.

d) The shape of the Inflow Hydrograph. A fair size time interval would be the one that allows the approximation of a smooth curve hydrograph, by a histogram without distorting the actual graph considerably.

2.6 WATER TRAVEL TIME

In section 2.4 it was stated that the reservoirs are assigned call numbers according to the order of their water travel time. Further, as will be shown later, the value of the water travel time for each reservoir must be known for the determination of the commencement and termination of operations of each reservoir.

From hydraulics the water travel time is dependent on:

a) The distance of travel.

b) The flow velocity which velocity depends on the stage of flow.

It is realized that for a specific channel the water travel time is not constant but varies with the quantity of flow in the river. Schematic illustration of variation in travel time, depending on the flow stage is given in reference [8], plate no. 2 .

In this study, to avoid additional complications, it will be assumed that the water travel time is constant. For the formulation of the mathematical model the water travel time is to be subdivided into a number of equal time intervals.

2.7 FORECAST OF INFLOW

The optimization of the operational matrix requires the prior knowledge of the inflow to every reservoir in the system. The inflow should be forecast with a certain acceptable accuracy for the flood period.

Forecasting of inflows is to be derived from the forecasted precipitation, in the basin, by hydrological methods. In the United States forecasting of flood flows is sponsored cooperatively by the Weather Bureau and the Corps of Engineers. The forecasts are issued daily for a period of 48-72 hours, which breaks down into individual periods of 6, 12, or 24 hours, as required.

2.8 UNITS OF MEASURE OF PARAMETERS

For homogenuity of units in the objective function and the constraint equations, the parameters appearing in the mathematical model shall be expressed in the following units.

a) Decision variables. These are to be expressed in units of volume per time interval.

b) Reservoir storage and initial conditions. Reservoir volumes shall be expressed in units of volume.

c) Inflows, upper and lower channel flows. It is obvious that the units must be the same as those of the decision variables. Since the inflows and the other flow parameters are usually expressed in units of volume per second (c.f.s. cum./sec.) the following equation can be used to make the transformation to units of volume per time interval,

$$Q = 3600 \times DIP \times Q_0 \qquad (2.8.1)$$

where Q= the flow rate parameter expressed in units of volume per time interval. Q_0 = the flow rate parameter expressed in units of volume per second. DIP= the time interval expressed in hours.

2.9 TIME RELATIONSHIPS

2.9.1 General. In section 2.3d it was mentioned that in order for controlled releases to be additive, in a secondary channel, they should be made at different time intervals (not simultaneously) from the reservoirs upstream. For example consider the system of two reservoirs on Fig. 7.

A release made at time period 1 from reservoir No.1 is scheduled to reach point A after 5 time intervals, whereas a release from reservoir No.2 needs only two time intervals. From section 2.3b equation 2.3.5, the set of constraints for the secondary channel shall be;

$$x_{1,1} + x_{2,4} \le B_{1,1}$$

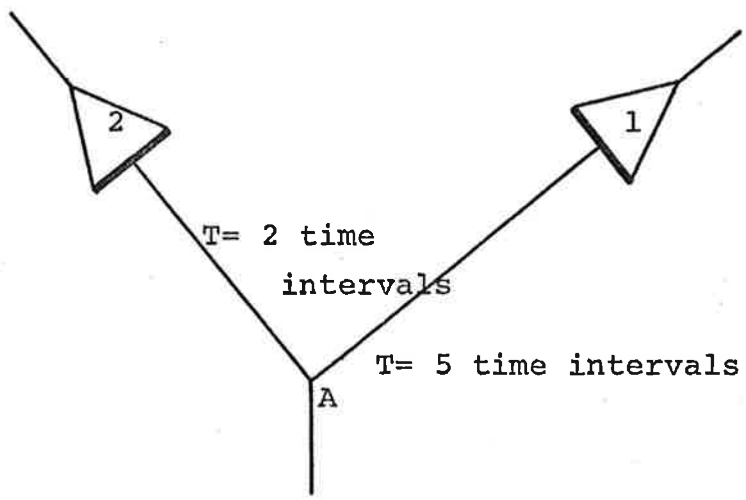


Fig. 7 System of Reservoirs

which means that the release made at time intervals No.1 and No.4 are additives. All releases, from reservoir No. 2, made earlier are not considered.

This simple example shows that the geometry of the system complicates the model when no adjustments are made to avoid the time factor being directly involved in the mathematical model.

The rest of this section discusses the time relationships connected with the geometry of the system and equations are developed, the results of which will bring the necessary adjustments to the model. The equations to be developed deal with discrete time intervals, and for this purpose it is assumed that:

a) The water travel time from the different reservoirs is expressed in discrete time intervals.

b) The forecasting period is also expressed in discrete time intervals.

2.9.2 Number of Time Intervals for Which Operation is Determined. n is defined as the number of discrete time intervals over which the mathematical equations shall be applied, to optimize the releases from a system of reservoirs.

The number "n" is a function of:

a) The number of time intervals for which inflow is forecasted and,

b) The water travel time difference (number of time intervals) between reservoir No.1 and reservoir No.N for reservoirs in parallel, and reservoir No.1 and reservoir No. (N-1) for reservoirs in tandem. Mathematically for a system of N reservoirs the number of time intervals for which operation can be formulated into a mathematical model is given by;

For reservoirs in parallel,

$$n_p = TF + T_N - T_1 \qquad (2.9.1)$$

and for reservoirs in tandem

$$n_t = TF + T_{(N-1)} - T_1 \qquad (2.9.2)$$

where n_p = the number of time intervals for reservoirs in parallel; n_t = the number of time intervals for reservoirs in tandem; TF = the forecast period expressed by a number of discrete time intervals; T_1 = the water travel time of reservoir No.1 expressed in discrete time intervals; T_N = the water travel time of reservoir No. N (last reservoir) expressed in discrete time intervals. T_{N-1}= the water travel time of reservoir No. (N-1) (last but one reservoir) expressed in discrete time intervals.

Equations 2.9.1 and 2.9.2 require the reservoirs be given call numbers according to the procedure outlined in section 2.4. Further, equations 2.9.1 and 2.9.2 should give positive results, for the model to be applied, ie.,

$$n_p \geq 1$$

$$n_t \geq 1$$

or

$$[TF - (T_1 - T_N)\ ] \geq 1 \qquad (2.9.3)$$

$$[TF - (T_1 - T_{N-1})\ ] \geq 1 \qquad (2.9.4)$$

If n_p or n_t are found to be negative then the system must be solved by the methods of decomposition. (Break the system into a number of subsystems).

2.9.3 Commencement & Termination of Operations. The time at which operations of the different reservoirs commences and terminates is another time relationship arising from the general geometry of the system. Commencement of operation of reservoirs, with different water travel times, occurs at different times.

Assuming that operation of reservoir No.1 commences at time interval one, then the time interval at which operation of the rest of the reservoirs will commence is given by,

$$CT_i = T_1 - T_i + 1 \qquad (2.9.5)$$

where termination occurs at,

$$TT_i = CT_i + n - 1 \qquad (2.9.6)$$

where CT_i = the time interval at the beginning of which operation of reservoir i shall commence. TT_i = the time interval at the end of which operation of reservoir i shall terminate. Other symbols are explained in Appendix I.

2.9.4 Inflow Data. (Forecast Hydrograph). The hydrographs calculated from the precipitation data give the inflow to the reservoirs in the system for a fixed time period (or for a number of time intervals). However, due to the geometry of the system, as was shown in section 2.9.2, the number of time intervals (time period) over which the mathematical model can be formulated might be less. (As given by equations 2.9.1 and 2.9.2). The next thing related directly to the system geometry is the time of commencement and termination of operations of the reservoirs given by equations 2.9.5 and 2.9.6. It is therefore necessary to know during which time intervals each reservoir is operative so it can be decided which parts of the given inflow hydrographs can be used as input data.

Given the water travel time of each reservoir in the system, in time intervals, the time intervals during which each reservoir is operative are given by,

$$j(i,k) = T_1 - T_i + k \qquad (2.9.7)$$

$$\begin{array}{l} k=1 \\ k=2 \\ k=3 \\ \vdots \\ k=n \end{array}$$

where j(i,k) = the time interval at which the reservoir i is operated. Range of j(i,k) is 1 to n.

Using the results of 2.9.7 the input data hydrographs can be derived and the necessary time corrections be made by,

$$Ii,k = I'i,\ j(i,k) \qquad (2.9.8)$$

where I' = the given hydrograph, I= the derived hydrograph, and k= the time interval index (time interval). Applications of equations 2.9.1 to 2.9.8 are to be shown in section 2.11.

2.10 PROBLEM SIZE

2.10.1 General. The problem size which MPS/360 solves, depends on the amount of core storage available for data.

Given a computer, the total amount of core storage to be used by MPS/360 is divided into two parts,

a) Program storage whose size is approximately 29800 bytes, and

b) Data storage, the amount of which is available, depends on many factors.

IBM Application Program GH20-0156-1 shows the maximum and design norm problem size which may be solved with the indicated number of data bytes available.

The purpose of the following discussion is to establish the size of the problem and indicate the computer size necessary to solve the problem. The discussion shall be concentrated on reservoirs in parallel but equations for reservoirs in tandem are also given.

2.10.2 Decision Variables. The objective function is made up of the decision variables. For any system of reservoirs the number of decision variables is given by,

$$N.D.V. = N \times n \qquad (2.10.1)$$

where N= the number of reservoirs and n= the number of time intervals over which the model is formulated.

2.10.3 Constraint Equations. Constraint equations are defined as those equations that express a relationship between two or more decision variables. The number of constraint equations depends on:

a) The number of reservoirs in the system.

b) The general geometry of the system.

c) The number of time intervals over which the mathematical model is to be formulated.

Assuming that geometry is not considered and that no more than two channels meet at any confluent point in the system, the number of constraint equations can be expressed by the number of time intervals and the number of reservoirs in the system. Thus, for reservoirs in parallel;

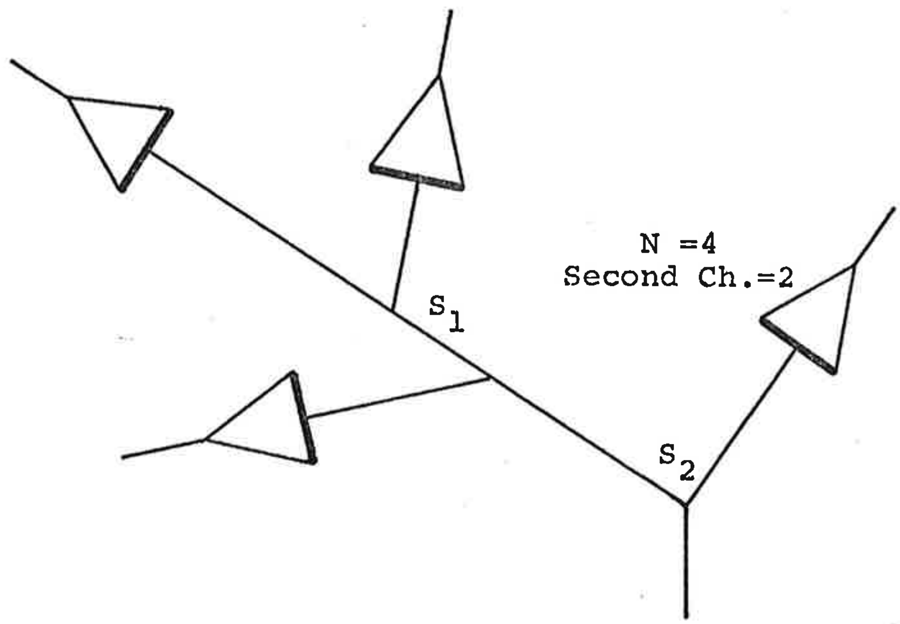
$$NP = (4N-2)n \qquad (2.10.2)$$

For reservoirs in tandem;

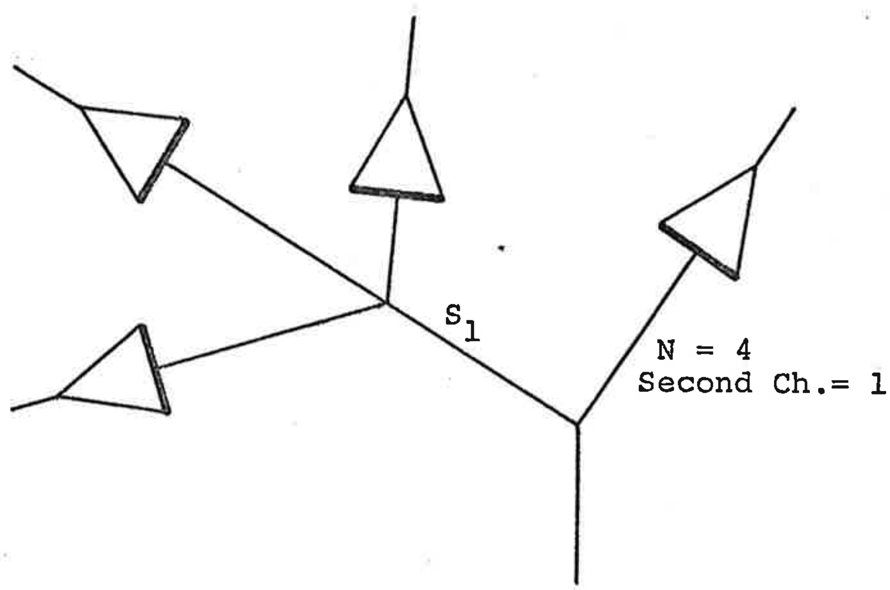
$$NT = (4N-4)n \qquad (2.10.3)$$

where NP = the number of constraint equations for reservoirs in parallel. NT= the number of constraint equations for reservoirs in tandem. N= the number of reservoirs in the system. n= the number of time intervals.

It must be realized that the constraint equations describe the operational limitations of the reservoirs, secondary and main channels. So any variation of the number of secondary channels in a system due to the geometrical set up will change the number of constraint equations. For example consider the two systems of reservoirs shown on Fig. 8. Although the number of reservoirs in both systems is the same and the reservoirs are in parallel, the number of secondary channels is different, and as a result the number of constraint equations is different.



System A



System B

Fig. 8 Systems of Reservoirs

In system A there are 2 secondary channels, S_1 and S_2 where in system B there is only one secondary channel, S_1. In such cases equation 2.10.2 and 2.10.3 can be corrected by subtracting a quantity 2n from the above equations for each node where more than two channels meet.

2.10.4 Bounds. Bounds constitute a part of the constraints and actually they put limits on the values of the decision variables. In this case, there will be only one set of bounds which define upper and lower values for each reservoir. Therefore the number of bounds for a problem of N reservoirs and n time intervals there will be a total of N x n bounds.

2.10.5 Problem Size. The problem size is defined by the number of rows in the problem, where the number of rows consists of;

a) The objective function plus the constraint equations and the bounds.

2.11 EXAMPLES OF MATHEMATICAL MODELS

2.11.1 General. The procedures developed in sections 2.3-2.10 shall now be applied to formulate the mathematical models of two different systems of reservoirs. The problems shall be presented in such a way, with all data given as in actual cases.

Following are given the mathematical models of reservoirs in parallel, section 2.11.2 and reservoirs in tandem, section 2.11.3.

2.11.2 Mathematical Model of Reservoirs in Parallel. By definition a system of reservoirs is operating in parallel when the inflow to each reservoir in the system, is independent of the outflows from any other reservoir in the system.

Formulation of the mathematical model of a system of reservoirs operating in parallel is most clear if done by example. General presentation of the matrices involved is given in Appendix II.

For our purpose consider that a system of reservoirs, as shown in Fig. 9, is to be operated for flood control. Given the data shown on Tables 2.1 and 2.2 as well as the inflow hydrographs presented on Fig. 10, optimize the operation of the system.

The approach to the above problem shall be as follows.

a) Decide the size of the time interval.

b) Calculate the number of time intervals for which operation can be planned.

c) Calculate the commencement and termination time of operations of the reservoirs.

d) Derive from the data given and the inflow hydrographs I', the new hydrographs I to be used as input data.

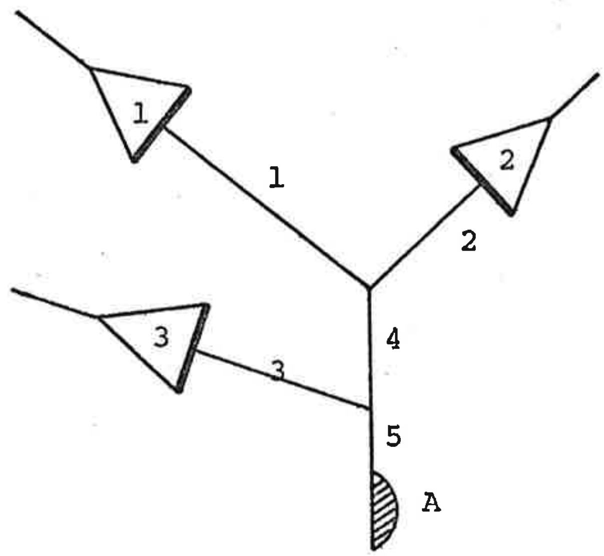


Figure 9 Reservoir System

e) Calculate the problem size.

Table 2.1 Reservoir Data.

Reservoir No.	Capacity	Initial Condition	Water Travel Time	
			Days	Time Int.
1	C_1	R_1	2.5	5
2	C_2	R_2	2.0	4
3	C_3	R_3	1.0	2

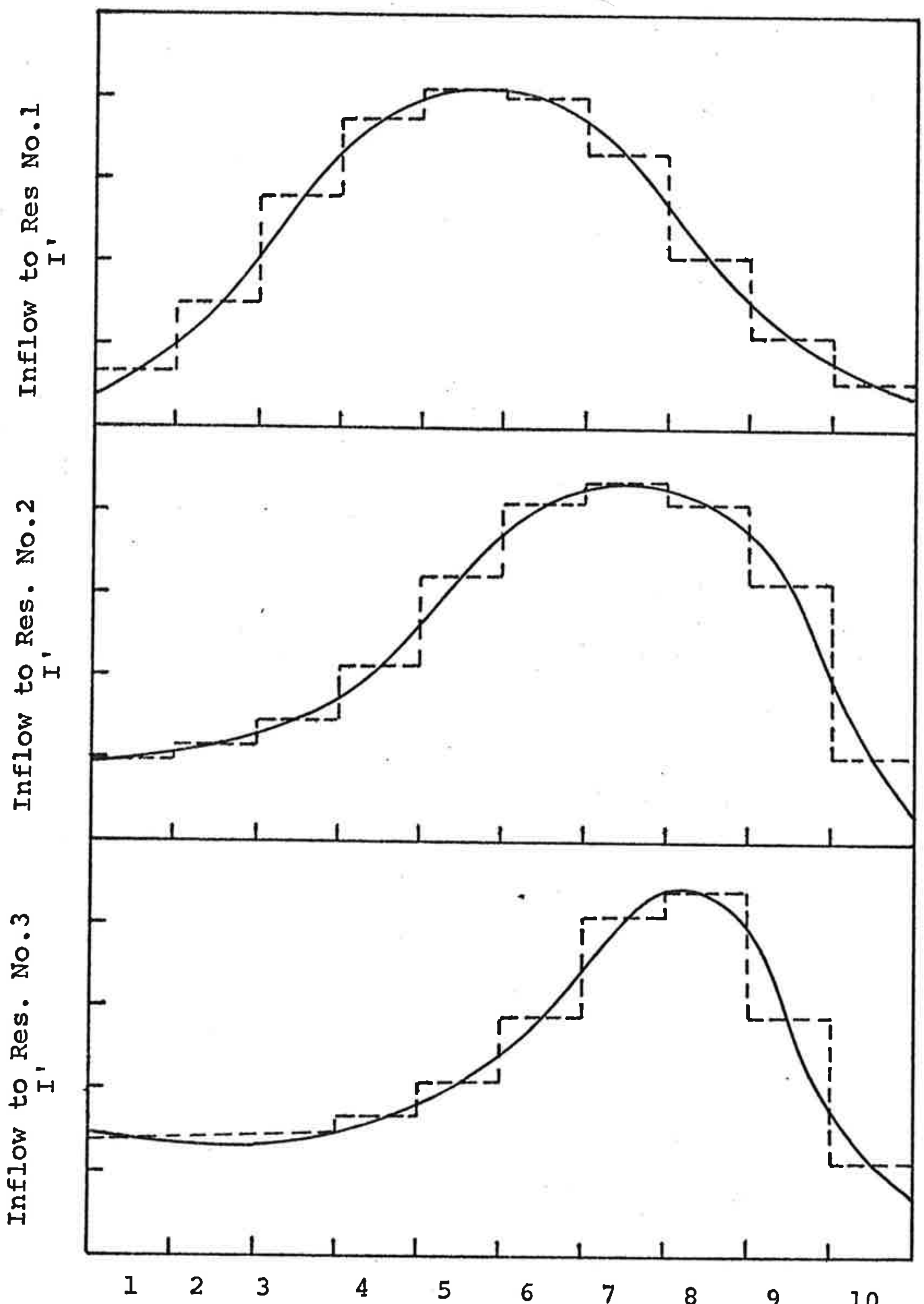


Fig. 10 Inflow Hydrographs I'

Table 2.2 Channel Data.

Channel No.	Minimum Flow*	Maximum Flow	Remarks
1	$d_{1,j}$	$b_{1,j}$	Tertiary Ch.
2	$d_{2,j}$	$b_{2,j}$	Tertiary Ch.
3	$d_{3,j}$	$b_{3,j}$	Tertiary Ch.
4	D_j	B_j	Secondary Ch.
5	DI_j	BI_j	Main Channel

a) Size of Time Interval. Considering the inflow hydrograph shape and the forecast period, it is decided to select a time interval of 12 hours. Using this time interval the histogram hydrographs are derived from the smooth curve hydrographs as shown on Figure 10. Note that each of the areas under the smooth curve hydrographs is closely equal to that under each of the histograms.

b) Number of Time Intervals n. Inflow to the reservoir is forecasted for a period of 5 days or 10 time intervals. Applying the relationship 2.9.1 the number of time intervals for which operation can be programmed is equal to:

---

*Minimum and maximum flows are expressed in units of volume per time interval.

$n = TF + T_N - T_1$ where $TF = 10$

$n = 10 + 2 - 5$ $T_N = T_3 = 2$

$n = 7$ $T_1 = 5$

c) <u>Commencement & Termination of Operations</u>. Commencement time of operation of reservoirs is given by equation 2.9.5 where the termination time is given by equation 2.9.6. Substituting the known values into the above equations, the following results are given;

Commencement	Termination
$CT_1 = 1$	$TT_1 = 7$
$CT_2 = 2$	$TT_2 = 8$
$CT_3 = 4$	$TT_3 = 10$

The figures given above represent the time intervals. Commencement occurs at the very beginning of the time interval, where termination occurs at the very end of the time interval. A chart on Figure 11 shows graphically the commencement and termination times of operations.

It is clear that, if the model is to be applied repeatedly, the next time period commencement of operation of the reservoir occurs at the time it terminated before. For the above example for the next application of the procedure, the commencement of operation of reservoirs No. 1, 2, and 3 will occur at the very beginning of the time intervals

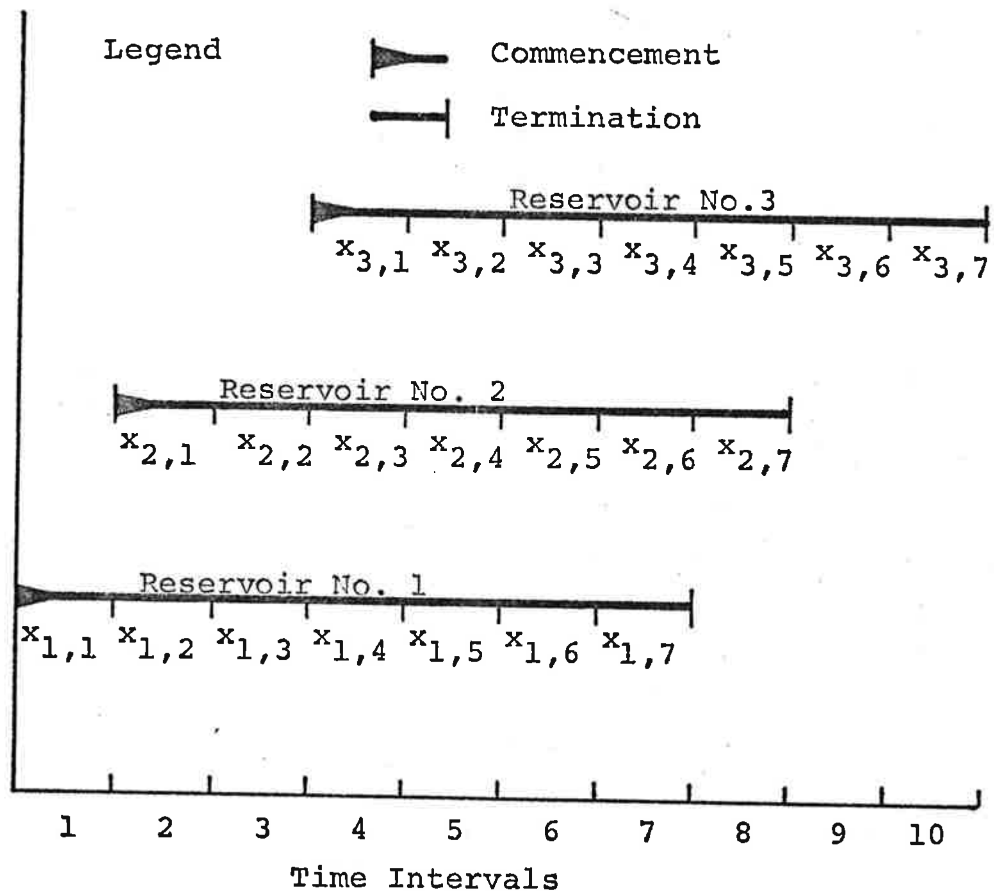


Fig. 11 Commencement & Termination of Operation of Reservoirs

8, 9, and 11, respectively.

d) <u>Inflow Hydrographs for Input Data</u>. It has been shown in paragraph b that the inflow hydrographs are known for 10 time intervals and that due to the geometry of the system the operational matrix shall be derived for only 7 time intervals. This paragraph answers the question which inflows are to be considered as input data and at what time interval.

Direct substitution into equations 2.9.7 and 2.9.8

give the results shown in Table 2.3.

Table 2.3 Inflow Hydrographs I

Reservoir No.	$I_{i,1}$	$I_{i,2}$	$I_{i,3}$	$I_{i,4}$	$I_{i,5}$	$I_{i,6}$	$I_{i,7}$
1	$I'_{1,1}$	$I'_{1,2}$	$I'_{1,3}$	$I'_{1,4}$	$I'_{1,5}$	$I'_{1,6}$	$I_{1,7}$
2	$I'_{2,2}$	$I'_{2,3}$	$I'_{2,4}$	$I'_{2,5}$	$I'_{2,6}$	$I'_{2,7}$	$I_{2,8}$
3	$I'_{3,4}$	$I'_{3,5}$	$I'_{3,6}$	$I'_{3,7}$	$I'_{3,8}$	$I'_{3,9}$	$I'_{3,10}$

$I_{i,k}$ represents the inflow to reservoir i during time interval k to be used in the mathematical programming as input data, where $I'_{i,j}$ represents the forecasted inflow to reservoir i during time interval j. The table shows the derivation of inflow hydrographs I from hydrographs I'.

The hydrographs I for each of the three reservoirs in the system are shown on Figure 12.

e) Problem Size. By inspection of the physical system of Fig. 9, it is seen that the result of equation 2.10.2 can be used to define the problem size. Therefore by applying equations

$$NP = (4N-2)n$$

$$N.D.V. = n \times N$$

$$\text{Bounds} = N \times n$$

by direct substitution, it is calculated that the problem is made up of:

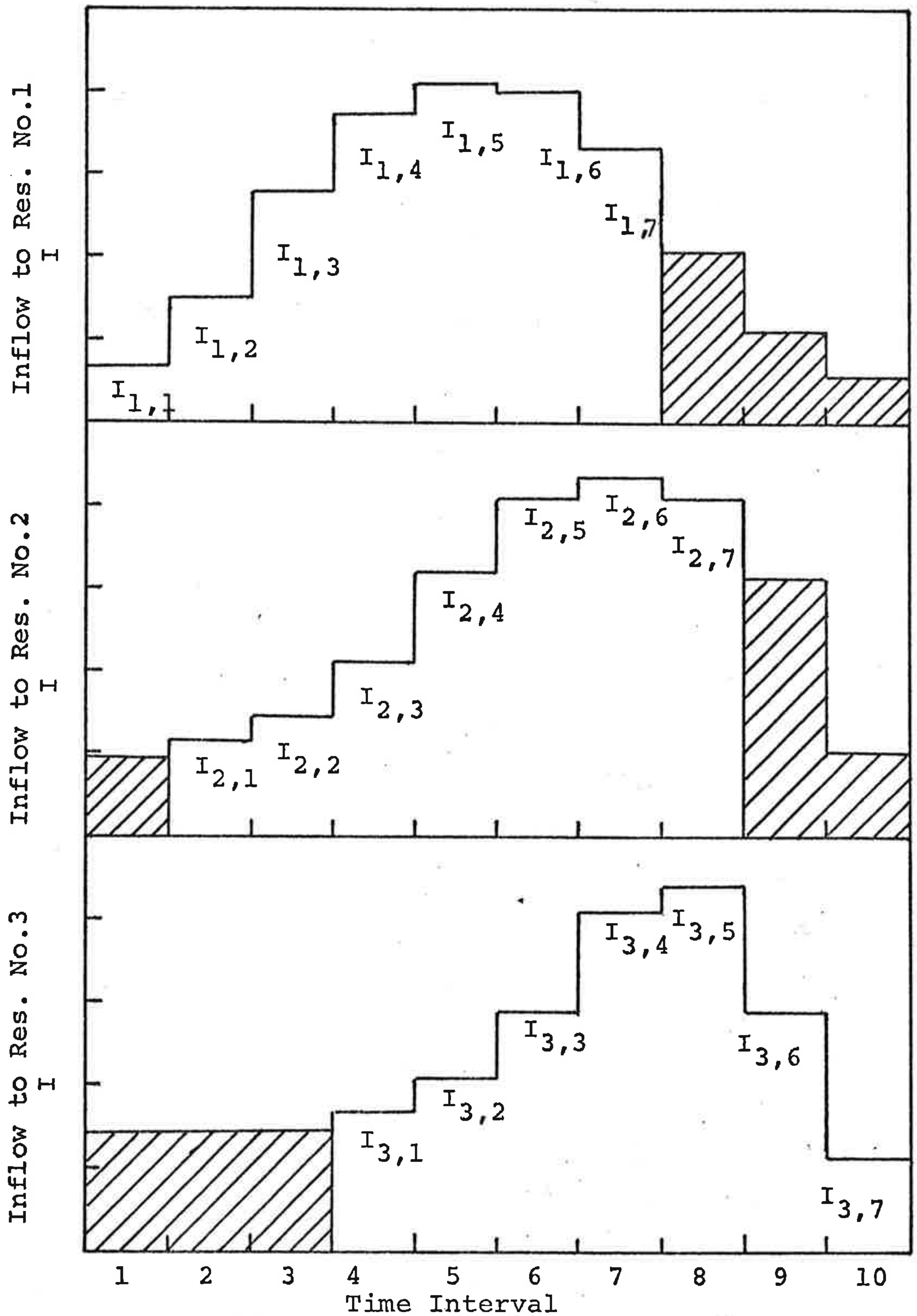


Fig. 12 Derived Inflow Hydrographs I

1. One objective function.
2. 70 constraint equations.
3. 21 bounds.

a total of 92 rows and 21 decision variables.

Now that all necessary parameters have been calculated, the operation of the physical system shall be formulated to a mathematical model.

The mathematical model shall be expressed as follows. Maximize the function

$$\sum_{j=1}^{7} \sum_{i=1}^{3} C_{i,j} X_{i,j} = Z \qquad (2.11.1)$$

subject to the constraints.

a) Minimum cumulative release,

Reservoir No.1

$$X_{1,1} \geq I_{1,1}+R_1-C_1 + Z_{1,1} \qquad (2.11.2)$$

$$\sum_{j=1}^{2} X_{1,j} \geq \sum_{j=1}^{7} I_{1,j} + R_1-C_1 + Z_{1,2} \qquad (2.11.3)$$

$$\vdots$$

$$\sum_{j=1}^{7} X_{1,j} \geq \sum_{j=1}^{7} I_{1,j} + R_1-C_1 + Z_{1,7} \qquad (2.11.8)$$

Reservoir No.2

$$X_{2,1} \geq I_{2,1} + R_2-C_2 + Z_{2,1} \qquad (2.11.9)$$

$$\sum_{j=1}^{2} X_{2,j} \geq \sum_{j=1}^{2} I_{2,j} + R_2 - C_2 + Z_{2,2} \qquad (2.11.10)$$

$$\vdots$$

$$\sum_{j=1}^{7} X_{2,j} \geq \sum_{j=1}^{7} I_{2,j} + R_2 - C_2 + Z_{2,7} \qquad (2.11.15)$$

Reservoir No. 3

$$X_{3,1} \geq I_{3,1} + R_3 - C_3 + Z_{3,1} \qquad (2.11.16)$$

$$\sum_{j=1}^{2} X_{3,j} \geq \sum_{j=1}^{2} I_{3,j} + R_3 - C_3 + Z_{3,2} \qquad (2.11.17)$$

$$\vdots$$

$$\sum_{j=1}^{7} X_{3,j} \geq \sum_{j=1}^{7} I_{3,j} + R_3 - C_3 + Z_{3,7} \qquad (2.11.22)$$

b) Maximum cumulative release

Reservoir No. 1

$$X_{1,1} \leq I_{1,1} + R_1 - Y_{1,1} \qquad (2.11.23)$$

$$\sum_{j=1}^{2} X_{1,j} \leq \sum_{j=1}^{2} I_{1,j} + R_1 - Y_{1,2} \qquad (2.11.24)$$

$$\vdots$$

$$\sum_{j=1}^{7} X_{1,j} \leq \sum_{j=1}^{7} I_{1,j} + R_1 - Y_{1,7} \qquad (2.11.29)$$

Reservoir No. 2

$$X_{2,1} \leq I_{2,1} + R_2 - Y_{2,1} \tag{2.11.30}$$

$$\sum_{j=1}^{2} X_{2,1} \leq \sum_{j=1}^{2} I_{2,j} + R_2 - Y_{2,2} \tag{2.11.31}$$

$$\vdots$$

$$\sum_{j=1}^{7} X_{2,j} \leq \sum_{j=1}^{7} I_{2,j} + R_2 - Y_{2,7} \tag{2.11.36}$$

Reservoir No. 3

$$X_{3,1} \leq I_{3,1} + R_3 - Y_{3,1} \tag{2.11.37}$$

$$\sum_{j=1}^{2} X_{3,j} \leq \sum_{j=1}^{2} I_{1,j} + R_3 - Y_{3,2} \tag{2.11.38}$$

$$\vdots$$

$$\sum_{j=1}^{7} X_{3,j} \leq \sum_{j=1}^{7} I_{3,j} + R_3 - Y_{3,7} \tag{2.11.43}$$

Values of Y and Z may range between 0 and C_i.

c) Secondary channel flow. In the system of reservoirs on Figure 9, there is only one secondary channel, channel No.4 to which channels No. 1 and No. 2 flow. Therefore, for maximum flows constraints,

$$X_{1,1} + X_{2,1} \leq B_{4,1} \tag{2.11.44}$$

$$X_{1,2} + X_{2,2} \leq B_{4,2} \tag{2.11.45}$$

$$X_{1,3} + X_{2,3} \leq B_{4,3} \tag{2.11.46}$$

$$X_{1,4} + X_{2,4} \leq B_{4,4} \quad (2.11.47)$$

$$X_{1,5} + X_{2,5} \leq B_{4,5} \quad (2.11.48)$$

$$X_{1,6} + X_{2,6} \leq B_{4,6} \quad (2.11.49)$$

$$X_{1,7} + X_{2,7} \leq B_{4,7} \quad (2.11.50)$$

Minimum Flows cons.

$$X_{1,1} + X_{2,1} \geq D_{4,1} \quad (2.11.51)$$

$$X_{1,2} + X_{2,2} \geq D_{4,2} \quad (2.11.52)$$

$$X_{1,3} + X_{2,3} \geq D_{4,3} \quad (2.11.53)$$

$$X_{1,4} + X_{2,4} \geq D_{4,4} \quad (2.11.54)$$

$$X_{1,5} + X_{2,5} \geq D_{4,5} \quad (2.11.55)$$

$$X_{1,6} + X_{2,6} \geq D_{4,6} \quad (2.11.56)$$

$$X_{1,7} + X_{2,7} \geq D_{4,7} \quad (2.11.57)$$

d) Main channel flows. In a system of reservoirs by definition there is only one main channel. In Fig. 9 the main channel is channel No. 5, and channel No. 1, No. 2, and No. 3 discharge their flows into this channel.

For maximum flows,

$$X_{1,1} + X_{2,1} + X_{3,1} \leq Bl_1 \quad (2.11.58)$$

$$X_{1,2} + X_{2,2} + X_{3,2} \leq Bl_2 \quad (2.11.59)$$

$$X_{1,3} + X_{2,3} + X_{3,3} \leq Bl_3 \quad (2.11.60)$$

$$X_{1,4} + X_{2,4} + X_{3,4} \leq Bl_4 \quad (2.11.61)$$

$$X_{1,5} + X_{2,5} + X_{3,5} \leq Bl_5 \quad (2.11.62)$$

$$X_{1,6} + X_{2,6} + X_{3,6} \leq Bl_6 \quad (2.11.63)$$

$$X_{1,7} + X_{2,7} + X_{3,7} \leq Bl_7 \quad (2.11.64)$$

For minimum flows,

$$X_{1,1} + X_{2,1} + X_{3,1} \geq Dl_1 \quad (2.11.65)$$

$$\sum_{i=1}^{3} X_{i,2} \geq Dl_2 \quad (2.11.66)$$

$$\sum_{i=1}^{3} X_{i,3} \geq Dl_3 \quad (2.11.67)$$

$$\sum_{i=1}^{3} X_{i,4} \geq Dl_4 \quad (2.11.68)$$

$$\sum_{i=1}^{3} X_{i,5} \geq Dl_5 \quad (2.11.69)$$

$$\sum_{i=1}^{3} X_{i,6} \geq Dl_6 \quad (2.11.70)$$

$$\sum_{i=1}^{3} X_{i,7} \geq Dl_7 \quad (2.11.71)$$

e) Tertiary channel flow bounds. The flow in the tertiary channels, like that in the secondary and main channels, is bounded above and below. Due to the fact that the expression contains only one decision variable, this part of the input data can be expressed as follows. Therefore for,

Reservoir No. 1

$$d_{1,1} \leq x_{1,1} \leq b_{1,1} \quad (2.11.72)$$

$$d_{1,2} \leq x_{1,2} \leq b_{1,2} \quad (2.11.73)$$

$$d_{1,3} \leq x_{1,3} \leq b_{1,3} \quad (2.11.74)$$

$$d_{1,4} \leq x_{1,4} \leq b_{1,4} \quad (2.11.75)$$

$$d_{1,5} \leq x_{1,5} \leq b_{1,5} \quad (2.11.76)$$

$$d_{1,6} \leq x_{1,6} \leq b_{1,6} \quad (2.11.77)$$

$$d_{1,7} \leq x_{1,7} \leq b_{1,7} \quad (2.11.78)$$

Reservoir No. 2

$$d_{2,1} \leq x_{2,1} \leq b_{2,1} \quad (2.11.79)$$

$$d_{2,2} \leq x_{2,2} \leq b_{2,2} \quad (2.11.80)$$

$$d_{2,3} \leq x_{2,3} \leq b_{2,3} \quad (2.11.81)$$

$$d_{2,4} \leq x_{2,4} \leq b_{2,4} \quad (2.11.82)$$

$$d_{2,5} \leq x_{2,5} \leq b_{2,5} \quad (2.11.83)$$

$$d_{2,6} \leq x_{2,6} \leq b_{2,6} \quad (2.11.84)$$

$$d_{2,7} \leq x_{2,7} \leq b_{2,7} \quad (2.11.85)$$

Reservoir No. 3

$$d_{3,1} \leq x_{3,1} \leq b_{3,1} \quad (2.11.86)$$

$$d_{3,2} \leq x_{3,2} \leq b_{3,2} \quad (2.11.87)$$

$$d_{3,3} \leq x_{3,3} \leq b_{3,3} \qquad (2.11.88)$$

$$d_{3,4} \leq x_{3,4} \leq b_{3,4} \qquad (2.11.89)$$

$$d_{3,5} \leq x_{3,5} \leq b_{3,5} \qquad (2.11.90)$$

$$d_{3,6} \leq x_{3,6} \leq b_{3,6} \qquad (2.11.91)$$

$$d_{3,7} \leq x_{3,7} \leq b_{3,7} \qquad (2.11.92)$$

In this problem is was assumed that the inflow from the local areas, downstream between the dam and the damage center, was neglected. The capacities of the outlet works of any reservoir in the system do not offer any limitation. All notations are given in Appendix I.

2.11.3 Mathematical Model of Reservoirs in Tandem. By definition a system of reservoirs is operating in tandem when inflow to one of the reservoirs is a function of the releases from all other reservoirs.

Section 2.11.2 has dealt with the mathematical model of reservoirs in parallel. To see the differences in the mathematical models of the two systems, this section deals with reservoirs in Tandem.

Suppose that the system of reservoirs shown on Figure 13 is to be operated for flood control purposes. The reservoir and channel data are given on Tables 2.4 and 2.5, where the inflow hydrographs given on Figure 14 are known for a period of 5 days.

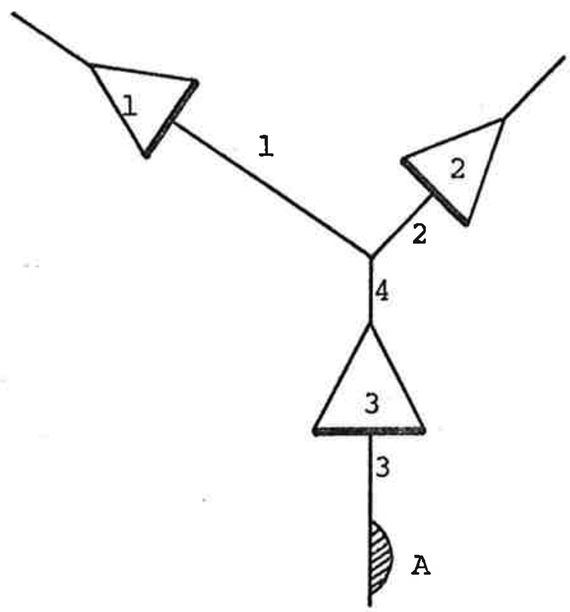


Fig. 13 Reservoirs in Tandem

Table 2.4 Reservoir Data

Reserv. No.	Capacity	Initial Cond.	Water Travel Time	
			Days	12 Hr. Interv.
1	C_1	R_1	3.5	7
2	C_2	R_2	3.0	6
3	C_3	R_3	1.0	2

Formulate the physical system as a mathematical model (in linear programming) so that the operation of the reservoir can be optimized.

Table 2.5 Channel Data

Channel No.	Minimum Flow*	Maximum Flow*	Remarks
1	$d_{1,j}$	$b_{1,j}$	Tertiary Ch.
2	$d_{2,j}$	$b_{2,j}$	Tertiary Ch.
3	$D_{3,j}$	$B_{3,j}$	Secondary Ch.
4	D_j	B_j	Tertiary Ch.

To be able to derive the mathematical model of the system the following additional information must be known.

a) The size of the time intervals.

b) The number of time intervals over which the operational matrix can be applied.

c) Commencement and termination time of operations of reservoirs.

d) The inflow hydrographs to be used as input data.

e) The problem size.

The above data shall be calculated by using the equations developed in sections 2.9 and 2.10.

*Minimum and maximum flows are expressed in units of volume per time interval.

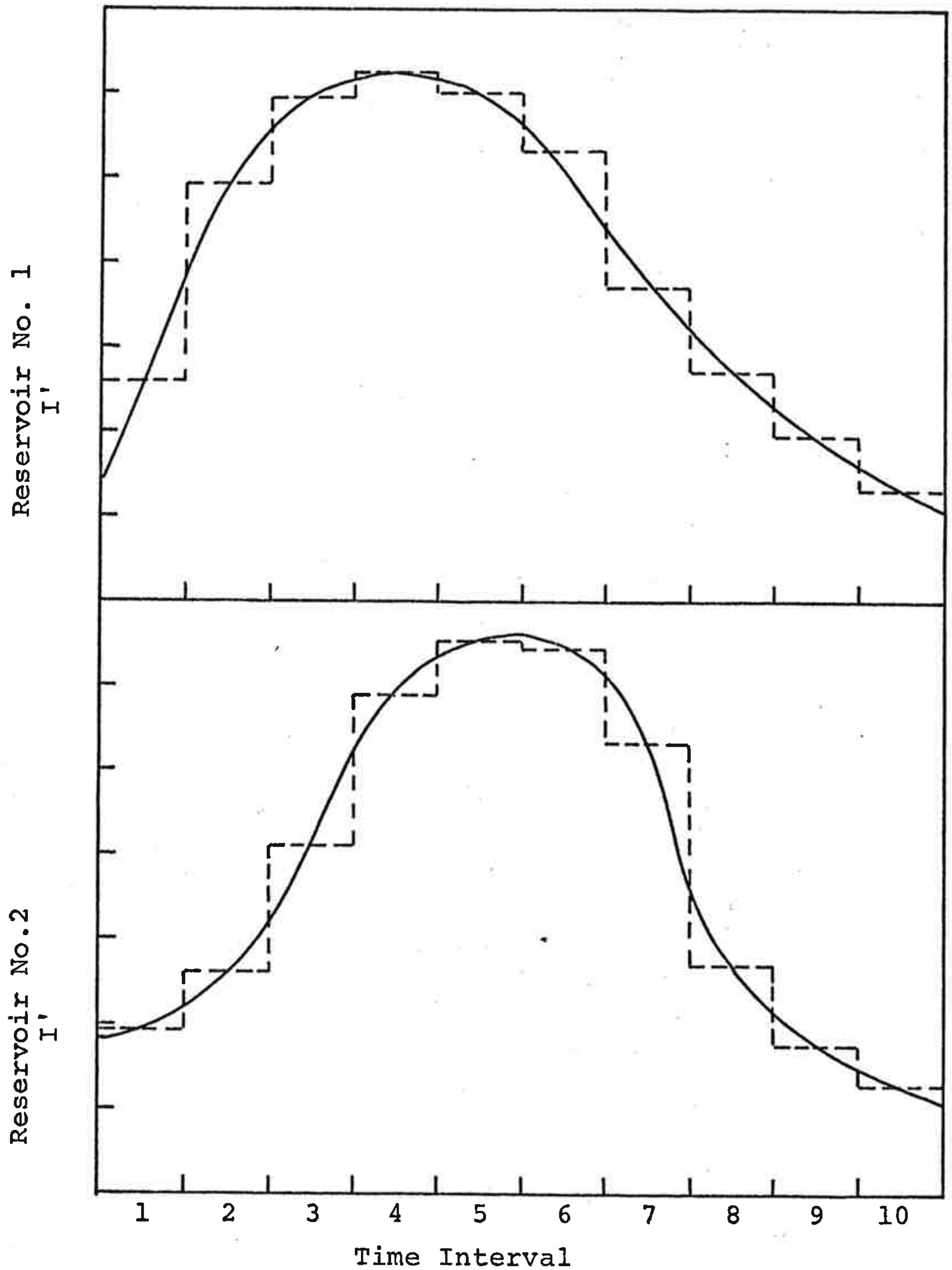


Fig. 14 Inflow Hydrographs I'

a) Size of Time Interval. By considering the inflow hydrographs and the period for which the inflows are given it is decided to select a time interval of 12 hours. Based on this, the water travel time of reservoirs No.1, No.2, and No.3 are calculated to be equal to 7,6, and 2 time intervals.

b) n. Number of time intervals for which operation can be programmed.

Given that TF=10 and T_1, T_2, and T_3 shown in Table 2.4, by substitution into equation 2.9.2

$$n = TF + T_2 - T_1$$

$$n = 10 + 6 - 7$$

$$n = 9$$

The model shall be applied over nine time intervals.

c) Commencement and Termination of Reservoir Operations. By direct substitution of known data into equations 2.9.5 and 2.9.6 the following results are given:

Commencement	Termination
$CT_1=1$	$TT_1=9$
$CT_2=2$	$TT_2=10$
$CT_3=6$	$TT_3=14$

Note that operation of reservoir No. 3 extends four time intervals beyond the forecast period. Figure 15 shows the operation chart for the reservoirs in the system.

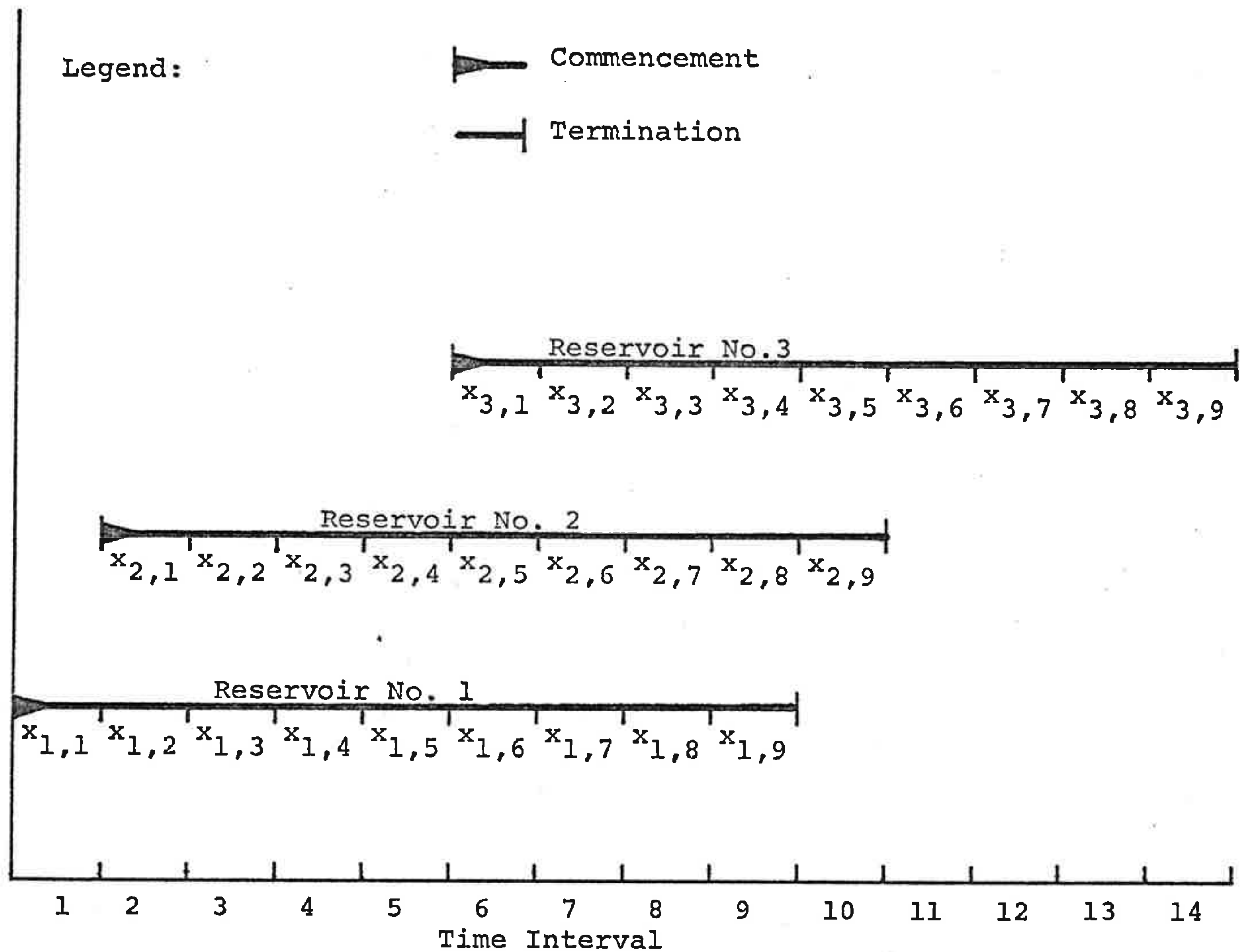


Fig. 15 Commencement and Termination of Operations Chart

It must be realized that commencement of operation occurs at the very beginning of the time interval calculated and termination occurs at the very end of the time interval calculated.

d) Derivation of Inflow Hydrographs for Data Input. Given on Figure 14 are the inflow hydrographs, in continuous and histogram form, for reservoirs No. 1 and No. 2, for 10 time intervals. According to (b) the number of time intervals over which the mathematical model shall be formulated is nine (9). The question asked is, which 9 out of the 10 flows given in Figure 14 shall be used as input data.

By direct application of equations 2.9.7 and 2.9.8 on reservoir No. 1 data, the following results are given:

$$j_{(1,1)} = 7-7+1 = 1$$

$$j_{(1,2)} = 2$$

$$j_{(1,3)} = 3 \qquad \text{Reservoir No.1}$$

$$\vdots$$

$$j_{(1,9)} = 9$$

and

$$I_{1,1} = I'_{1,1} \qquad \text{Reservoir No.1}$$

$$I_{1,2} = I'_{1,2}$$

$$\vdots \qquad \vdots$$

$$I_{1,9} = I'_{1,9}$$

Direct application of the equations for both reservoirs in the system, the results shown on Table 2.6 are taken.

Table 2.6 Derived Inflow Hydrographs I

Reserv. No.	$I_{i,1}$	$I_{1,2}$	$I_{i,3}$	$I_{i,4}$	$I_{i,5}$	$I_{i,6}$	$I_{i,7}$	$I_{i,8}$	$I_{i,9}$
1	$I'_{1,1}$	$I'_{1,2}$	$I'_{1,3}$	$I'_{1,4}$	$I'_{1,5}$	$I'_{1,6}$	$I'_{1,7}$	$I'_{1,8}$	$I'_{1,9}$
2	$I'_{2,2}$	$I'_{2,3}$	$I'_{2,4}$	$I'_{2,5}$	$I'_{2,6}$	$I'_{2,7}$	$I'_{2,8}$	$I'_{2,9}$	$I'_{2,10}$

$I'_{i,j}$ represents the inflow to reservoir i, during time interval j as given by hydrograph on Figure 14. $I_{i,k}$ represents the derived hydrograph to be used as input data. The results of Table 2.6 are plotted on Figure 16.

e) Problem Size. By inspection of the physical model, equation 2.10.3 applies for the determination of the number of the constraint equations,

$$NT= (4N-4)\ n$$

$$NT= (4x3-4)\ 9$$

$$NT= 72$$

The bounds equal to nxN = 27.

In summary the problem is made up of:

72 Constraint equations

1 Objective function

27 Bounds and 27 decision variables

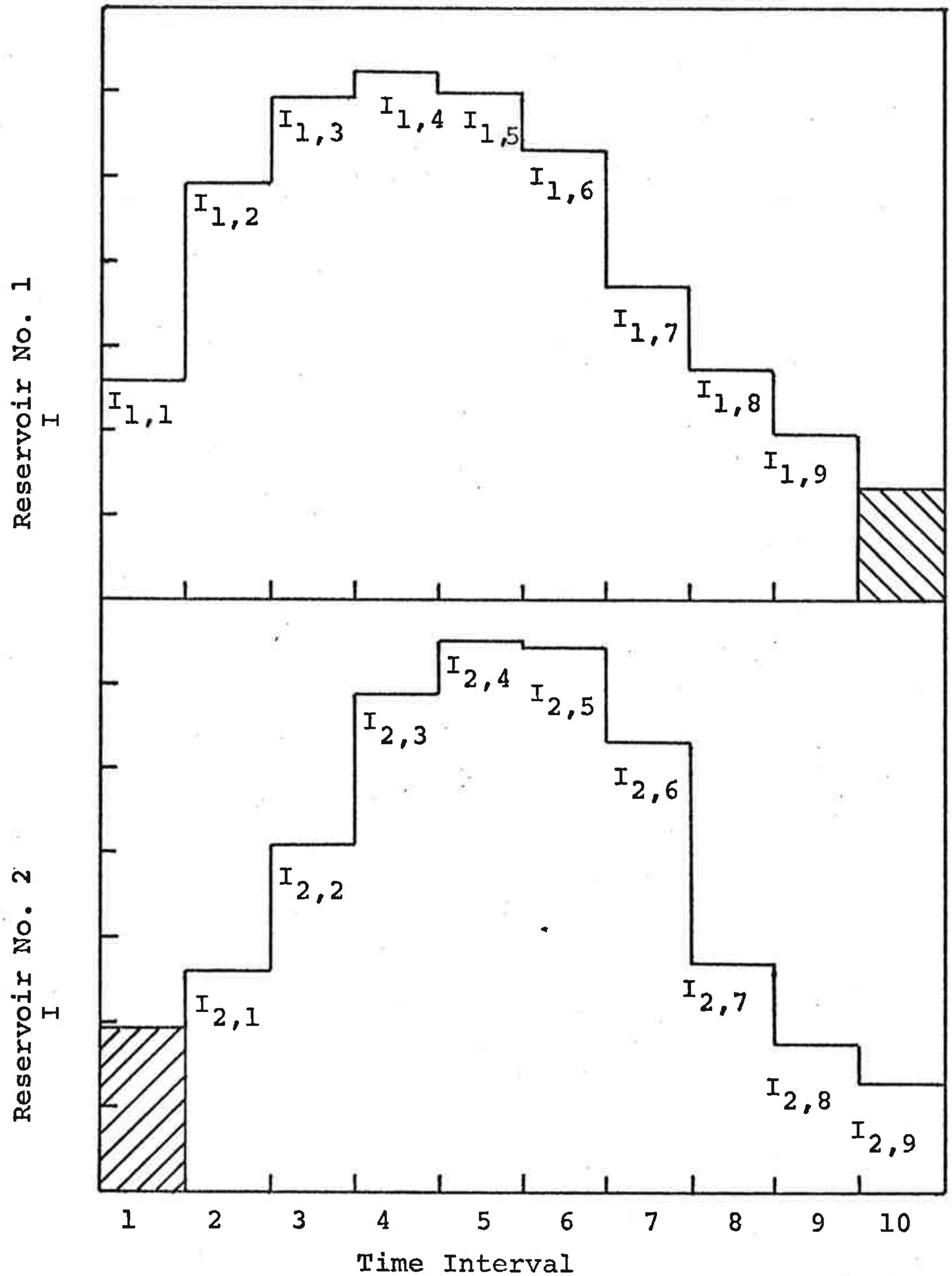


Fig. 16 Derived Inflow Hydrographs I

Now that the necessary parameters are known we can proceed with the mathematical formulation of the problem.

Generally the problem shall be stated as follows.

Maximize the function:

$$\sum_{j=1}^{9} \sum_{i=1}^{3} C_{i,j} X_{i,j} = Z \qquad (2.11.93)$$

Subject to the following constraints,

a) Minimum cumulative release;

Reservoir No.1

$$X_{1,1} \geq I_{1,1} + R_1 - C_1 + Z_{1,1} \qquad (2.11.94)$$

$$\sum_{j=1}^{2} X_{1,j} \geq \sum_{j=1}^{2} I_{1,j} + R_1 - C_1 + Z_{1,2} \qquad (2.11.95)$$

$$\vdots$$

$$\sum_{j=1}^{9} X_{1,j} \geq \sum_{j=1}^{9} I_{1,j} + R_1 - C_1 + Z_{1,2} \qquad (2.11.102)$$

Reservoir No.2

$$X_{2,1} \geq I_{2,1} + R_2 - C_2 + Z_{2,1} \qquad (2.11.103)$$

$$\sum_{j=1}^{2} X_{2,j} \geq \sum_{j=1}^{2} I_{2,j} + R_2 - C_2 + Z_{2,2} \qquad (2.11.104)$$

$$\vdots$$

$$\sum_{j=1}^{9} X_{2,j} \geq \sum_{j=1}^{9} I_{2,j} + R_2 - C_2 + Z_{2,9} \qquad (2.11.111)$$

Reservoir No. 3 Inflow to this reservoir is not known until the problem is solved, since the inflow to reservoir No. 3 is the sum of the outflows from reservoirs No.1 and No.2. The general form of this set of constraint equations is as follows:

$$- X_{1,1} - X_{2,1} + X_{3,1} \geq R_3 - C_3 + Z_{3,1} \qquad (2.11.112)$$

$$-\sum_{j=1}^{2} X_{1,j} -\sum_{j=1}^{2} X_{2,1} + \sum_{j=1}^{2} X_{3,j} \geq R_3 - C_3 + Z_{3,1} \qquad (2.11.113)$$

$$\vdots$$

$$-\sum_{j=1}^{9} X_{1,j} -\sum_{j=1}^{9} X_{2,j} + \sum_{j=1}^{9} X_{3,j} \geq R_3 - C_3 + Z_{3,9} \qquad (2.11.120)$$

b) Maximum cumulative release,

Reservoir No.1

$$X_{1,1} \leq I_{1,1} + R_1 - Y_1 \qquad (2.11.121)$$

$$\sum_{j=1}^{2} X_{i,j} \leq \sum_{j=1}^{2} I_{1,j} + R_1 - Y_{1,2} \qquad (2.11.122)$$

$$\vdots$$

$$\sum_{j=1}^{9} X_{1,j} \leq \sum_{j=1}^{9} I_{1,9} + R_1 - Y_{1,10} \qquad (2.11.129)$$

Reservoir No.2

$$X_{2,1} \leq I_{2,1} + R_2 - Y_{2,1} \qquad (2.11.130)$$

$$\sum_{j=1}^{2} X_{2,j} \leq \sum_{j=1}^{2} I_{2,j} + R_2 - Y_{2,2} \qquad (2.11.131)$$

$$\vdots$$

$$\sum_{j=1}^{9} X_{2,j} \leq \sum_{j=1}^{9} I_{2,j} + R_2 - Y_{2,9} \qquad (2.11.138)$$

Reservoir No.3: Inflow to this reservoir is the outflow from reservoirs No.1 and No.2. Therefore, the constraint equations shall be:

$$-X_{1,1} - X_{2,1} + X_{3,1} \leq R_3 - Y_{3,1} \qquad (2.11.139)$$

$$-\sum_{j=1}^{2} X_{1,j} - \sum_{j=1}^{2} X_{2,j} + \sum_{j=1}^{2} X_{3,j} \leq R_3 - Y_{3,2} \qquad (2.11.140)$$

$$\vdots$$

$$-\sum_{j=1}^{9} X_{1,j} - \sum_{j=1}^{9} X_{2,j} + \sum_{j=1}^{9} X_{3,j} \leq R_3 - Y_{3,9} \qquad (2.11.147)$$

c) Secondary channel flow. Figure 13 shows that there is only one secondary channel (channel No.4) which receives flows from reservoirs No.1 and No.2. Therefore, for maximum flows constraints,

$$X_{1,1} + X_{2,1} \leq B_{4,1} \qquad (2.11.148)$$

$$X_{1,2} + X_{2,2} \leq B_{4,2} \tag{2.11.149}$$

$$X_{1,3} + X_{2,3} \leq B_{4,3} \tag{2.11.150}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$X_{1,9} + X_{2,9} \leq B_{4,9} \tag{2.11.156}$$

where for minimum flow constraints

$$X_{1,1} + X_{2,1} \geq D_{4,1} \tag{2.11.157}$$

$$X_{1,2} + X_{2,2} \geq D_{4,2} \tag{2.11.158}$$

$$X_{1,3} + X_{2,3} \geq D_{4,3} \tag{2.11.159}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$X_{1,9} + X_{2,9} \geq D_{4,9} \tag{2.11.165}$$

d) Main channel Constraints: Considering the definition of the main channel it is concluded that the behavior of channel No.3 is actually not that of a main channel. The release to channel No.3 is controlled by one reservoir only. Therefore, channel No.3 shall be treated as a tertiary channel.

e) Tertiary channel flow bounds: The bounds section of the input data, of the MPS/360 algorithm, expresses the upper and lower limits of the values that a decision variable can obtain. Since the flow in tertiary channels is expressed by only one decision variable, then this set of constraints must come under the bounds section, and the equations are expressed as follows:

Reservoir No.1,

$$d_{1,1} \leq X_{1,1} \leq b_{1,1} \tag{2.11.166}$$

$$d_{1,2} \leq X_{1,2} \leq b_{1,2} \tag{2.11.167}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$d_{1,9} \leq X_{1,9} \leq b_{1,9} \tag{2.11.174}$$

Reservoir No.2,

$$d_{2,1} \leq X_{2,1} \leq b_{2,1} \tag{2.11.175}$$

$$d_{2,2} \leq X_{2,2} \leq b_{2,2} \tag{2.11.176}$$

$$d_{2,3} \leq X_{2,3} \leq b_{2,3} \tag{2.11.177}$$

$$\vdots \qquad \vdots$$

$$d_{2,9} \leq X_{2,9} \leq b_{2,9} \tag{2.11.184}$$

Reservoir No.3,

$$d_{3,1} \leq X_{3,1} \leq b_{3,1} \tag{2.11.185}$$

$$d_{3,2} \leq X_{3,2} \leq b_{3,2} \tag{2.11.186}$$

$$d_{3,3} \leq X_{3,3} \leq b_{3,3} \tag{2.11.187}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$d_{3,9} \leq X_{3,9} \leq b_{3,9} \tag{2.11.194}$$

CHAPTER III

COMPUTER PROGRAMMING

3.1 INTRODUCTION

Optimization of the operational matrix (mathematical model) is to be carried out by the Linear Programming algorithm of MPS/360.

The mathematical model discussed in Chapter II involves linear inequalities which can be expressed generally in a matrix form as shown on Appendix 2. The L.H.S.[1] matrices include invariants (or data) which do not change for a specific system of reservoirs, provided that the operation is always planned for the same number of time intervals. The R.H.S.[2] matrices in contrast to the L.H.S.'s contain invariants (or data) that change each time the program is used. (New time period)

Before discussing the individual program in detail, it is necessary to define several terms used throughout the discussion. The term "job" constitutes a submission of the entire program and associated input data to the computer. Every time the program and data decks are deposited on the computer console, another job is initiated. Each

---

[1]Left Hand Side Matrices See Appendix II

[2]Right Hand Side Matrices

job includes certain invariants, as discussed above which might change or not, according to the specific job.

Each job in turn consists of one or more studies, where each study is dealing with a particular part of the mathematical model. Each study may be carried out separately and have its results stored on a magnetic tape to be used by another study at a later stage of the job.

3.2 GENERAL FLOW CHART OF THE PROGRAM

The general flow chart of the program for the set up and optimization of the operational matrix is shown on Figure 17.

The left hand side matrix data (transition matrix data associated with the size and geometry of the system as well as the number of time intervals for which operation is to be programmed) are read at the start of the job. In the next step the program proceeds with the generation of the L.H.S. transition submatrices to form one big matrix as shown on Figure 18. Once the L.H.S. transition matrix has been generated it is stored by column order on a sequential file, in a format complying with the MPS/360 input data format. Before the columns are stored, the rows section generated by the same program is stored on the same sequential file. A sample of the stored data is shown on Figure 19. For more details regarding the format of the MPS/360 input data refer to "MPS/360 Version 2, Linear and Separable Programming User's Manual" IBM Application Program GH20-0476-2. It is clear that to study a different system of reservoirs, a different program must be prepared to generate the L.H.S. transition matrix. However, if it is desired to investigate the same system many times, each time for the same number of time intervals, the generated matrix can be used repeatedly, which means that the program need not start from the very beginning. Therefore, program A-B (see Fig. 17) can be put to run once, generate the transition matrix (L.H.S.) and then start the rest of the jobs from step B. Avoiding repetition of this step shall save computing time. The data stored on a magnetic tape can be used repeatedly.

The next step calls for the invariants associated with the R.H.S. matrices (inflow to reservoirs, reservoir capacity, reservoir initial conditions, channel maximum and minimum acceptable flows, and other general data) to be read to the program. The program proceeds with the processing of the input data and the generation of the R.H.S. matrices. The resulting matrices are then stored on a direct access file for later use by the Linear Programming of MPS/360. A sample of the R.H.S. matrix data, stored on a disk is shown on Figure 20.

Since the input data to this study program is always different, this program must be included in every job.

After the input data to MPS/360 is stored, the Linear Programming algorithm of MPS/360 is called to read the data and proceed with the optimization. If there is a feasible optimum solution the program proceeds to give a printout of the results through the Readcom subroutine. If however, there are infeasibilities the program is diverted to print out a table (trace) which includes all vectors that are infeasible. The "trace" helps the programmer to identify the infeasible vectors and any other constraints involved in the infeasibilities, which, by relaxation might give a feasible solution.

3.3 INPUT DATA

Three classes of input data are required for the execution of each job; (1) Hydrologic data, (2) System parameters data, and (3) Miscellaneous data.

The hydrologic data constitutes the derived inflow hydrographs, I, from the forecasted inflow hydrographs I'. The inflows are expressed in units of volume per time interval. The system parameter data comprise the sizes and capacities of the structures, (reservoirs and channels) and other data expressing the geometry of the system (water travel time). The miscellaneous data comprise the time interval size, the number of time intervals for which the analysis shall last, and other general data necessary to process the hydrologic and system parameter data.

All data is to be provided to the programs on punched cards. The hydrologic data will be different for different jobs, whereas the system parameters stay the same for the same system. Miscellaneous data is generally varied according to the conditions imposed on the system by each specific job.

3.4 EXTENSIONS & LIMITATIONS OF COMPUTER PROGRAMS

The general flow chart given in section 3.2 can be translated into a computer program for the processing of the data given and optimization of the operation of any system of reservoirs. Each program has to be written to fit a specific system. Generalized programs cannot be developed because of the geometrical dissimilarities between systems. For this study two programs were written for two specific systems of flood control reservoirs. One of the programs is dealing with reservoirs in parallel and the other with reservoirs in tandem.

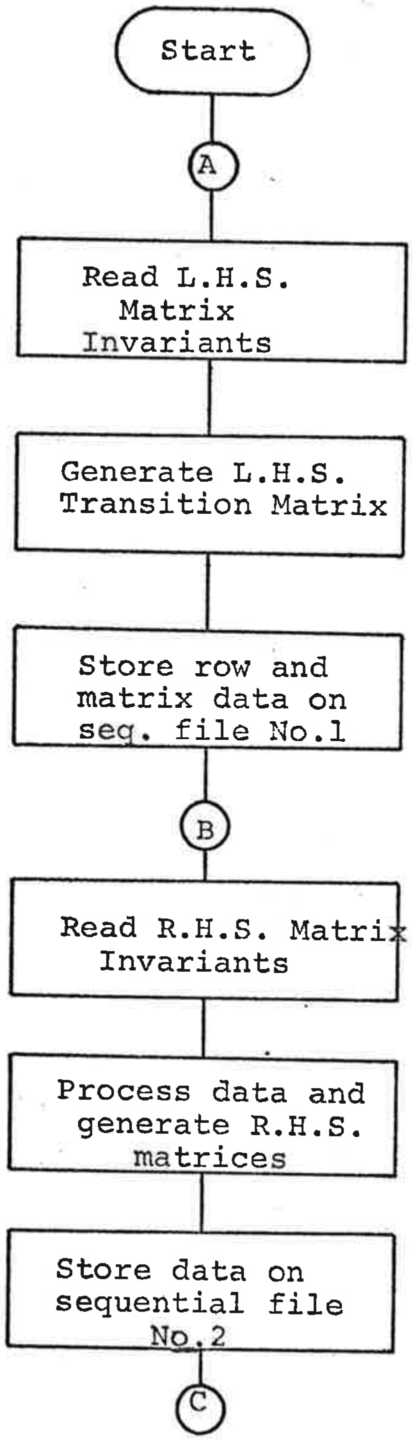


Figure 17 General Flow Chart of the Program

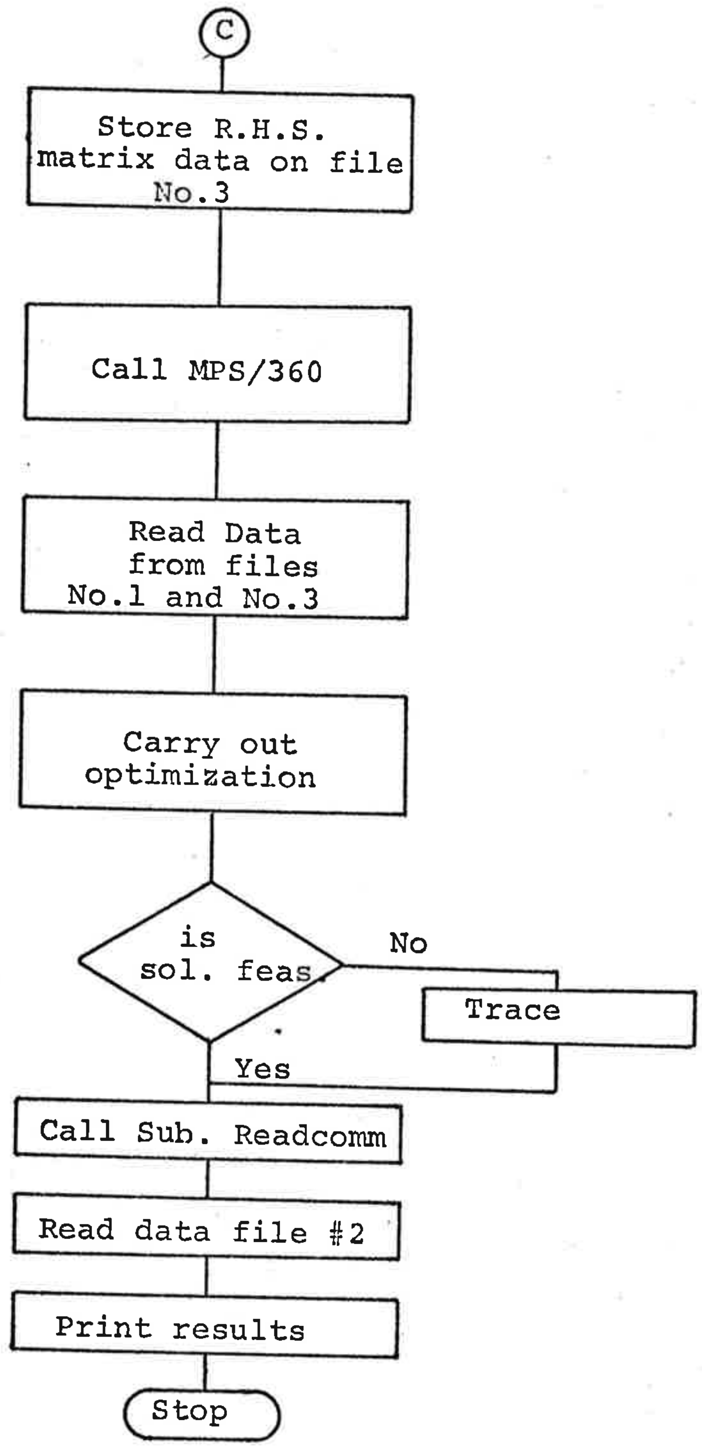


Fig. 17 Continued

TRANSITION MATRIX (RESERVOIRS IN TANDEM)

0.9	1.1	0.9	1.1	0.9	1.1	0.9	1.1	0.9	1.1	0.9	1.1
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0
-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
-1.0	-1.0	0.0	0.0	-1.0	-1.0	0.0	0.0	1.0	1.0	0.0	0.0
-1.0	-1.0	-1.0	0.0	-1.0	-1.0	-1.0	0.0	1.0	1.0	1.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0
-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
-1.0	-1.0	0.0	0.0	-1.0	-1.0	0.0	0.0	1.0	1.0	0.0	0.0
-1.0	-1.0	-1.0	0.0	-1.0	-1.0	-1.0	0.0	1.0	1.0	1.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0

Figure 18 Transition Matrix

```
NAME          PROB2
ROWS
 G CON101
 G CON102
 G CON103
 G CON104
 G CON105
 G CON106
 G CON107
 G CON108
 G CON109
 G CON110
 G CON111
 G CON112
 L CON113
 L CON114
 L CON115
 L CON116
 L CON117
 L CON118
 L CON119
 L CON120
 L CON121
 L CON122
 L CON123
 L CON124
 L CON125
 L CON126
 L CON127
 L CON128
 G CON129
 G CON130
 G CON131
 G CON132
COLUMNS
    X101      FX         0.90
    X101      CON101      1.00         CON102     1.00
    X101      CON103      1.00         CON104     1.00
    X101      CON105      0.0          CON106     0.0
    X101      CON107      0.0          CON108     0.0
    X101      CON109     -1.00         CON110    -1.00
    X101      CON111     -1.00         CON112    -1.00
    X101      CON113      1.00         CON114     1.00
    X101      CON115      1.00         CON116     1.00
    X101      CON117      0.0          CON118     0.0
    X101      CON119      0.0          CON120     0.0
    X101      CON121     -1.00         CON122    -1.00
    X101      CON123     -1.00         CON124    -1.00
    X101      CON125      1.00         CON126     0.0
    X101      CON127      0.0          CON128     0.0
    X101      CON129      1.00         CON130     0.0
    X101      CON131      0.0          CON132     0.0
```

Figure 19 Rows & Columns Input To MPS/360

```
RHS
    RHS1        CON101          0.0
    RHS1        CON102          0.0
    RHS1        CON103          0.0
    RHS1        CON104          0.0
    RHS1        CON105          0.0
    RHS1        CON106          0.0
    RHS1        CON107          0.0
    RHS1        CON108          0.0
    RHS1        CON109          0.0
    RHS1        CON110          0.0
    RHS1        CON111          0.0
    RHS1        CON112          0.0
    RHS1        CON113          0.0
    RHS1        CON114          0.0
    RHS1        CON115          0.0
    RHS1        CON116          0.0
    RHS1        CON117     -146.75343323
    RHS1        CON118     -146.75343323
    RHS1        CON119     -146.75343323
    RHS1        CON120     -146.75343323
    RHS1        CON121     -146.75343323
    RHS1        CON122     -146.75343323
    RHS1        CON123     -146.75343323
    RHS1        CON124     -146.75343323
    RHS1        CON125       54.87836647
    RHS1        CON126       66.88795471
    RHS1        CON127       70.08474731
    RHS1        CON128       71.54490662
    RHS1        CON129       72.49530029
    RHS1        CON130       73.92089844
    RHS1        CON131       77.98168945
    RHS1        CON132       80.20696106
    RHS1        CON133       59.71099663
    RHS1        CON134       66.88219643
    RHS1        CON135       70.04443359
    RHS1        CON136       71.96250916
    RHS1        CON137       72.63642883
    RHS1        CON138       73.10211182
    RHS1        CON139       73.43473816
    RHS1        CON140       73.80451965
    RHS1        CON141      174.23997498
    RHS1        CON142      174.23997498
    RHS1        CON143      174.23997498
    RHS1        CON144      174.23997498
    RHS1        CON145      174.23997498
    RHS1        CON146      174.23997498
    RHS1        CON147      174.23997499
    RHS1        CON148      174.23997498
BOUNDS
UP  P123        X101       1.81439972
UP  P123        X102       1.81439972
UP  P123        X103       1.81439972
UP  P123        X104       1.81439972
UP  P123        X105       1.81439972
UP  P123        X106       1.81439972
UP  P123        X107       1.81439972
UP  P123        X108       1.81439972
UP  P123        X109       1.64159966
UP  P123        X110       1.64159966
```

Figure 20 RHS & Bounds Input to MPS/360

CHAPTER IV

APPLICATIONS
(Mathematical Model Validation)

4.1 INTRODUCTION

Having devised the mathematical models in the appropriate form and having settled upon the strategy for optimizing the operation of a system, (chapters 2 and 3), the validity of both the mathematical model and computer procedure can be explored. They can be checked by applying the procedure to the optimization of operation of actual systems, already operating, in the Kansas river basin.

Since two models have been developed (reservoirs in Parallel and reservoirs in Tandem) both shall be applied to two distinct systems. Reservoir and channel data are taken from reference [8] and hydrologic data from reference [9]. Local inflows to channels downstream the reservoir were neglected.

4.2 APPLICATION: RESERVOIRS IN PARALLEL

The first application was made to a system of reservoirs whose basic configuration is shown on Fig. 21. The representation shows that the system is made up of three reservoirs, three tertiary channels, one secondary channel, and one main channel. The reservoirs and tertiary channels are numbered as shown on Table 4.1. The difference in

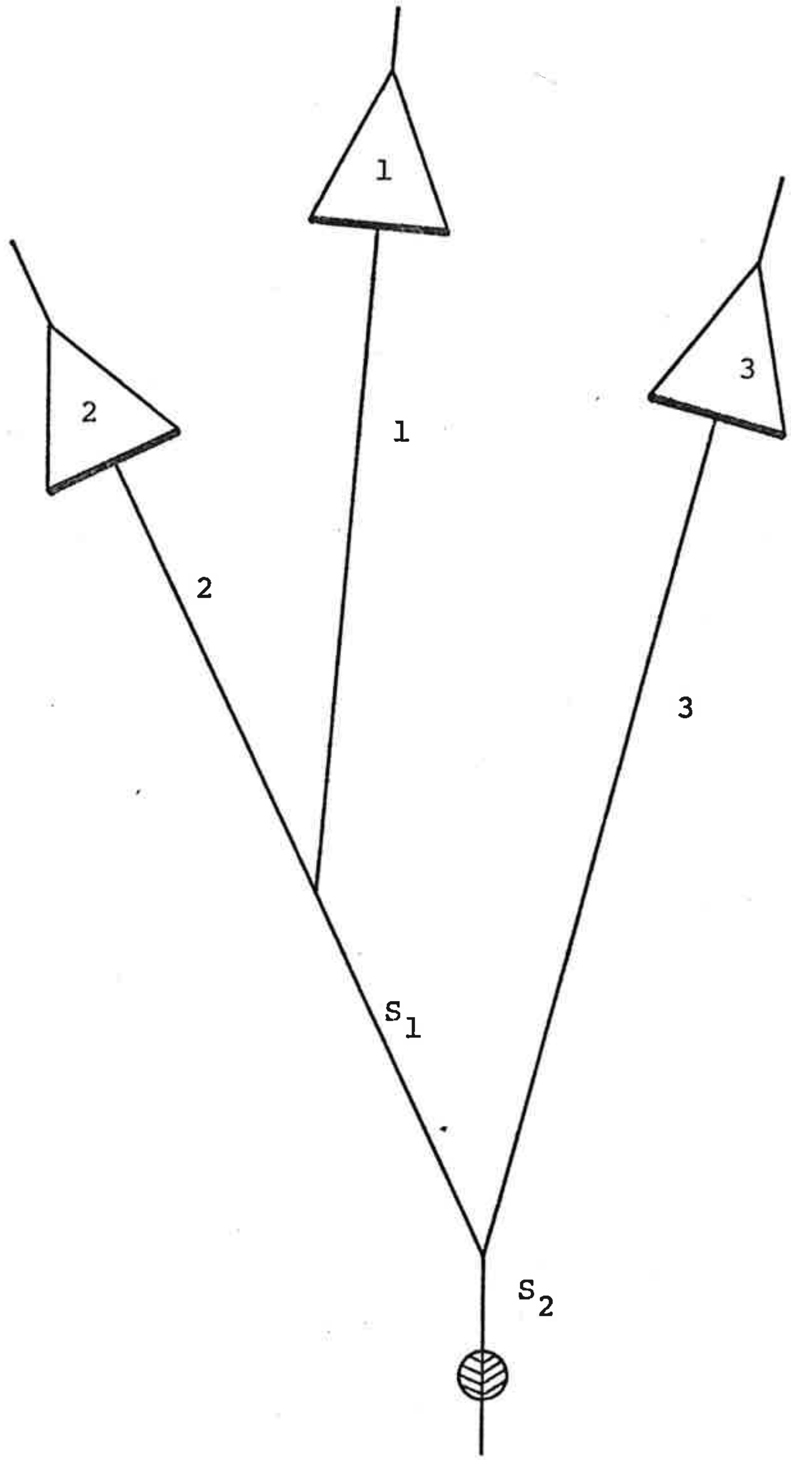


Figure 21 Schematic of Reservoirs in Parallel

water travel time between reservoir No.1 and the others is shown on the same Table.

Table 4.1

Reserv. Name	Reserv. No.	Tertiary Chan. No.	DT_i* Days
Wilson	1	1	0
Kanopolis	2	2	1
Glen Elder	3	3	1

DT_i shows the time at which operation of each reservoir shall commence with relation to reservoir No. 1. Secondary and main channels shall be referred to with their names.

1. Input data: The system's parameters and hydrologic data are shown on Figure 22. The data includes:

a) Reservoir capacities. (Actual flood control capacities)

b) Reservoir initial conditions. The values indicated have been assumed on the basis of inflows and permissible releases.

c) Allowable flows in tertiary channels. This data has been taken from reference [8].

d) Allowable flows in secondary and main channels. This data has been taken from reference [8].

---

* See Appendix I for notations.

RESERVOIR AND CHANNEL DATA

A RESERVOIR CAPACITIES

RESERVOIR NO	CAPACITY IN ACRE-FEET	CAPACITY IN UNITS	CAPACITY IN CFT
1	531000.	231.304	23130357760.
2	385600.	167.967	16796733440.
3	736900.	320.993	32099360768.

B RESERVOIR INITIAL CONDITIONS
FLOOD STORAGE INITIALLY OCCUPIED

RESERVOIR NO	VOLUME IN ACRE-FEET	VOLUME IN UNITS	VOLUME IN CFT
1	318000.	138.521	13852078080.
2	232000.	101.059	10105917440.
3	444000.	193.406	19340636160.

C ALLOWABLE FLOWS IN TERTIARY CHANNELS

CHANNEL NO	MAXIMUM FLOW IN TERT.CHAN.IN CFS PHASE 1	PHASE 2	PHASE 3	MINIMUM FLOWS IN CFS
1	2400.	4000.	5500.	100.
2	4800.	8000.	15000.	100.
3	3300.	5500.	8000.	100.

D ALLOWABLE FLOWS IN SECONDARY & MAIN CHANNELS

CHANNEL NO	MAXIMUM FLOW IN SEC & MAIN IN CFS PHASE 1	PHASE 2	PHASE 3	MINIMUM FLOW IN CFS
S1	4900.	9000.	11500.	100.
S2	10000.	16700.	38000.	100.

E INFLOWS TO THE RESERVOIRS IN CFS RECORDED EVERY 24. HOURS

PERIOD	RESERVOIR NO 1	2	3
1	4070.	6980.	10400.
2	7680.	11800.	25800.
3	15900.	19200.	38200.
4	13200.	12500.	27900.
5	7380.	7060.	14200.
6	3250.	4180.	9140.
7	4800.	4190.	7030.
8	5380.	7230.	7110.

Figure 22 Input Data - Reservoirs in Parallel

PERMISSIBLE FLOWS IN TERTIARY CHANNELS
CHANNEL NO

PERIOD	1	2	3
1	2400.	4800.	3300.
2	2400.	4800.	3300.
3	2400.	4800.	3300.
4	2400.	4800.	3300.
5	2400.	4800.	3300.
6	2400.	4800.	3300.
7	2400.	4800.	3300.
8	2400.	4800.	3300.

PERMISSIBLE FLOWS IN SEC.& MAIN CHAN.
CHANNEL NO

PERIOD	S1	S2
1	4900.	10000.
2	4900.	10000.
3	4900.	10000.
4	4900.	10000.
5	4900.	10000.
6	4900.	10000.
7	4900.	10000.
8	4900.	10000.

Figure 23 Input Data - Reservoirs in Parallel

TRANSITION MATRIX (RESERVOIRS IN PARALLEL)

0.9	1.1	0.9	1.1	0.9	1.1	0.9	1.1	1.1	0.9	1.1	0.9	1.1	0.9	1.1	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

NUMBER OF RESERVOIRS 3

NUMBER OF PROGRAMMING PERIODS 8

Figure 24 Transition Matrix- Reservoirs in Parallel

TABLE 1 PLANNED RELEASES & RESERVOIR STATUS

RESERVOIR NO 1

PERIOD	INFLOW IN CFS	OUTFLOW IN CFS	INIT. STATE ACRE-FEET	FIN.STATE ACRE-FEET	INFLOW IN ACRE-FEET	OUTFLOW IN ACRE-FEET	RESERVOIR STATUS
1	4070.	100.	318000.	325874.	8073.	198.	RISING
2	7680.	2400.	325874.	336347.	15233.	4760.	RISING
3	15903.	100.	336347.	367686.	31537.	198.	RISING
4	13200.	2400.	367686.	389107.	26182.	4760.	RISING
5	7380.	100.	389107.	403547.	14638.	198.	RISING
6	3250.	2400.	403547.	405233.	6446.	4760.	RISING
7	4800.	100.	405233.	414555.	9521.	198.	RISING
8	5380.	2400.	414555.	420465.	10671.	4760.	RISING

RESERVOIR FLOOD CONTROL CAPACITY=531000.ACRE-FT

RESERVOIR FINAL WATER CONTENT=420465.ACRE-FEET

FLOOD STORAGE VOLUME NOT OCCUPIED=110535.ACRE-FT

Figure 25 Output Data - Reservoirs in Parallel

TABLE 2 PLANNED RELEASES & RESFRVOIR STATUS

RESERVOIR NO 2

PERIOD	INFLOW IN CFS	OUTFLOW IN CFS	INIT. STATE ACRE-FEET	FIN.STATE ACRE-FEET	INFLOW IN ACRE-FEET	OUTFLOW IN ACRE-FEET	RESERVOIR STATUS
1	6980.	4800.	232000.	236324.	13845.	9521.	RISING
2	11800.	2500.	236324.	254770.	23405.	4959.	RISING
3	19200.	4800.	254770.	283332.	38083.	9521.	RISING
4	12500.	2500.	283332.	303167.	24793.	4959.	RISING
5	7060.	4800.	303167.	307649.	14003.	9521.	RISING
6	4180.	2500.	307649.	310981.	8291.	4959.	RISING
7	4190.	4800.	310981.	309771.	8311.	9521.	DROPPING
8	7230.	2500.	309771.	319153.	14340.	4959.	RISING

RESERVOIR FLOOD CONTROL CAPACITY=385600.ACRE-FT

RESERVOIR FINAL WATER CONTENT=319153.ACRE-FEET

FLOOD STORAGE VOLUME NOT OCCUPIED= 66447.ACRE-FT

Figure 26 Output Data - Reservoirs in Parallel

TABLE 3 PLANNED RELEASES & RESERVOIR STATUS

RESERVOIR NO 3

PERIOD	INFLOW IN CFS	OUTFLOW IN CFS	INIT. STATE ACRE-FEET	FIN.STATE ACRE-FEET	INFLOW IN ACRE-FEET	OUTFLOW IN ACRE-FEET	RESERVOIR STATUS
1	10400.	3300.	444000.	458082.	20628.	6545.	RISING
2	25800.	3300.	458082.	502710.	51174.	6545.	RISING
3	38200.	3300.	502710.	571934.	75769.	6545.	RISING
4	27900.	3300.	571934.	620727.	55339.	6545.	RISING
5	14200.	3300.	620727.	642347.	28165.	6545.	RISING
6	9140.	3300.	642347.	653930.	18129.	6545.	RISING
7	7030.	3300.	653930.	661328.	13944.	6545.	RISING
8	7110.	3300.	661328.	668885.	14102.	6545.	RISING

RESERVOIR FLOOD CONTROL CAPACITY=736900.ACRE-FT

RESERVOIR FINAL WATER CONTENT=668885.ACRE-FEET

FLOOD STORAGE VOLUME NOT OCCUPIED= 68015.ACRE-FT

Figure 27 Output Data - Reservoirs in Parallel

FLOW IN MAIN & SECONDARY CHANNELS

PERIOD	SEC.CHANNEL CFS	MAIN CHANNEL CFS
1	4900.	8200.
2	4900.	8200.
3	4900.	8200.
4	4900.	8200.
5	4900.	8200.
6	4900.	8200.
7	4900.	8200.
8	4900.	8200.

Figure 28 Output Data - Reservoirs in Parallel

e) Hydrologic data. Inflow to reservoirs in c.f.s. recorded every 24 hours. The length of time interval is 24 hours and a total of 8 time intervals are considered for optimization.

Figure 23 shows the permissible maximum flows in tertiary, secondary, and main channels. These values are derived from the operational guideline graphs, similar to the one shown on Figure 2.

2. Output:

a) The first output from the program is the transition matrix shown on Figure 24. The transition matrix for the system under consideration for a period of 8 time intervals is made up of 81 rows and 24 columns.

b) Releases and reservoir status. The final output from the computer program gives the optimum releases and reservoir status for each reservoir in the system. The results are shown on Figures 25, 26, and 27. The last output, Figure 28, indicates the flows to be observed in the secondary and main channels when the schedule of releases is followed.

4.3 APPLICATIONS: RESERVOIRS IN TANDEM

This section deals with the application of the mathematical model and computer program for the optimization of operation of a system whose basic configuration is shown on Figure 29. The representation shows that the system is made up of three reservoirs and three tertiary channels. Secondary and main channels do not exist (by definition) in the system.

The reservoirs and channels are numbered as shown on columns 1, 2, and 3 of Table 4.2 where the difference in water travel time, between reservoir No. 1 and the others, is shown on column 4 of the same table.

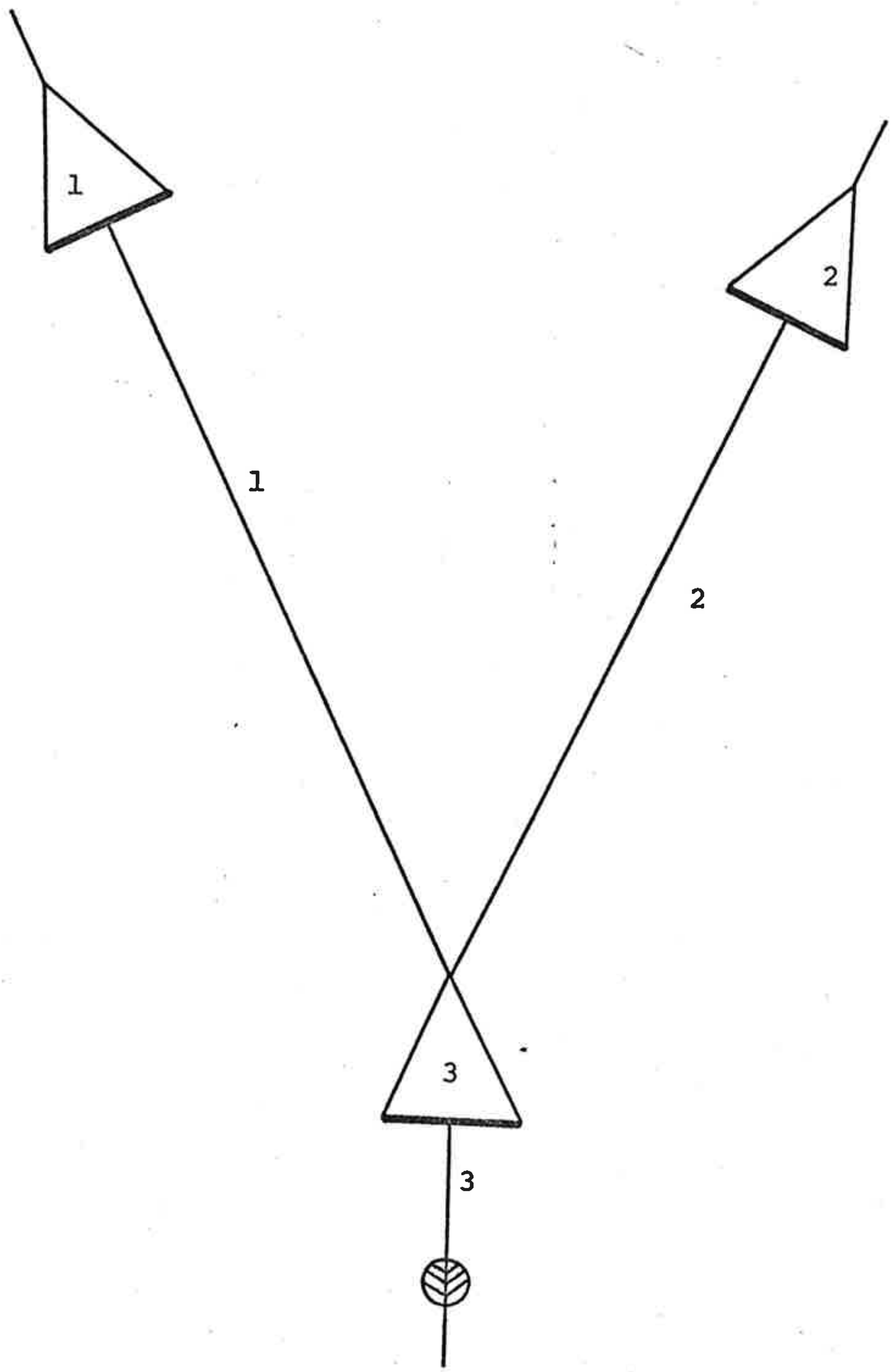


Figure 29 Schematic of Reservoirs in Tandem

RESERVOIR AND CHANNEL DATA

A RESERVOIR CAPACITIES

RESERVOIR NO	CAPACITY IN ACRE-FEET	CAPACITY IN UNITS	CAPACITY IN CFT
1	193600.	84.332	8433213440.
2	219400.	95.571	9557061632.
3	736900.	320.993	32099360768.

B RESERVOIR INITIAL CONDITIONS
FLOOD STORAGE INITIALLY OCCUPIED

RESERVOIR NO	VOLUME IN ACRE-FEET	VOLUME IN UNITS	VOLUME IN CFT
1	100000.	43.560	4355997696.
2	120000.	52.272	5227196416.
3	400000.	174.240	17423998976.

C ALLOWABLE FLOWS IN TERTIARY CHANNELS

CHANNEL NO	MAXIMUM FLOW IN TERT.CHAN.IN CFS PHASE 1	PHASE 2	PHASE 3	MINIMUM FLOWS IN CFS
1	2100.	3500.	5000.	100.
2	1800.	3000.	8000.	100.
3	3300.	5500.	8000.	100.

E INFLOWS TO THE RESERVOIRS IN CFS RECORDED EVERY 24. HOURS

PERIOD	RESERVOIR NO 1	2
1	13100.	8610.
2	13900.	8300.
3	3700.	3660.
4	1690.	2220.
5	1100.	780.
6	1650.	539.
7	4700.	385.
8	3270.	428.

Figure 30 Input Data- Reservoirs in Tandem

PERMISSIBLE FLOWS IN TERTIARY CHANNELS

	CHANNEL NO		
PERIOD	1	2	3
1	2100.	1900.	3300.
2	2100.	1900.	3300.
3	2100.	1900.	3300.
4	2100.	1900.	3300.
5	2100.	1900.	3300.
6	2100.	1900.	3300.
7	2100.	1900.	3300.
8	2100.	1900.	3300.

Figure 31 Input Data- Reservoirs in Tandem

TRANSITIUN MATRIX (RESERVUIRS IN TANDEM)

1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Figure 32 Transition Matrix

TABLE 1 PLANNED RELEASES & RESERVOIR STATUS

RESERVOIR NO 1

PERIOD	INFLOW IN CFS	OUTFLOW IN CFS	INIT. STATE ACRE-FEET	FIN.STATE ACRE-FEET	INFLOW IN ACRE-FEET	OUTFLOW IN ACRE-FEET	RESERVOIR STATUS
1	13100.	2100.	100000.	121818.	25983.	4165.	RISING
2	13900.	2100.	121818.	145223.	27570.	4165.	RISING
3	3700.	2100.	145223.	148397.	7339.	4165.	RISING
4	1690.	2100.	148397.	147583.	3352.	4165.	DROPPING
5	1100.	2100.	147583.	145600.	2182.	4165.	DROPPING
6	1650.	2100.	145600.	144707.	3273.	4165.	DROPPING
7	4700.	2100.	144707.	149864.	9322.	4165.	RISING
8	3270.	2100.	149864.	152185.	6486.	4165.	RISING

RESERVOIR FLOOD CONTROL CAPACITY=193600.ACRE-FT

RESERVOIR FINAL WATER CONTENT=152185.ACRE-FEET

FLOOD STORAGE VOLUME NOT OCCUPIED= 41415.ACRE-FT

Figure 33 Output Data - Reservoirs in Tandem

TABLE 2 PLANNED RELEASES & RESERVOIR STATUS

RESERVOIR NO 2

PERIOD	INFLOW IN CFS	OUTFLOW IN CFS	INIT. STATE ACRE-FEET	FIN.STATE ACRE-FEET	INFLOW IN ACRE-FEET	OUTFLOW IN ACRE-FEET	RESERVOIR STATUS
1	8610.	1900.	120000.	133309.	17078.	3769.	RISING
2	8300.	1900.	133309.	146003.	16463.	3769.	RISING
3	3660.	1900.	146003.	149494.	7260.	3769.	RISING
4	2220.	1900.	149494.	150128.	4403.	3769.	RISING
5	780.	1900.	150128.	147907.	1547.	3769.	DROPPING
6	539.	1900.	147907.	145207.	1069.	3769.	DROPPING
7	385.	1900.	145207.	142202.	764.	3769.	DROPPING
8	428.	1900.	142202.	139282.	849.	3769.	DROPPING

RESERVOIR FLOOD CONTROL CAPACITY=219400.ACRE-FT

RESERVOIR FINAL WATER CONTENT=139282.ACRE-FEE

FLOOD STORAGE VOLUME NOT OCCUPIED= 80118.ACRE-FT

Figure 34 Output Data - Reservoirs in Tandem

TABLE 3 PLANNED RELEASES & RESERVOIR STATUS

RESERVOIR NO 3

PERIOD	INFLOW IN CFS	OUTFLOW IN CFS	INIT. STATE ACRE-FEET	FIN.STATE ACRE-FEET	INFLOW IN ACRE-FEET	OUTFLOW IN ACRE-FEET	RESERVOIR STATUS
1	4000.	3300.	400000.	401388.	7934.	6545.	RISING
2	4000.	3300.	401388.	402777.	7934.	6545.	RISING
3	4000.	3300.	402777.	404165.	7934.	6545.	RISING
4	4000.	3300.	404165.	405553.	7934.	6545.	RISING
5	4000.	3300.	405553.	406941.	7934.	6545.	RISING
6	4000.	3300.	406941.	408330.	7934.	6545.	RISING
7	4000.	3300.	408330.	409718.	7934.	6545.	RISING
8	4000.	3300.	409718.	411106.	7934.	6545.	RISING

RESERVOIR FLOOD CONTROL CAPACITY=736900.ACRE-FT

RESERVOIR FINAL WATER CONTENT=411106.ACRE-FEET

FLOOD STORAGE VOLUME NOT OCCUPIED=325794.ACRE-FT

Figure 35 Output Data - Reservoirs in Tandem

Table 4.2

Reserv. Name	Reserv. No.	Tertiary Ch. No.	DT_i* Days
Webster	1	1	0
Kirwin	2	2	0.5
Glen Elder	3	3	1.5

1. Input data: The input data to the program is shown on Figure 30. The differences between Figure 22 and 30 are that inflow to reservoir No. 3 in the later case is not given. Figure 31 lists the permissible maximum flows to be permitted in the tertiary channels at any time during the 8 time intervals. (One time interval is equal to 24 hours.)

2. Output:

a) Transition matrix. The transition matrix (Figure 32) for this system is smaller (49x24) than the one for the system in parallel. This is due to the fact that no secondary or main channels exist in the system.

b) Releases and reservoir status. The output shown on Figure 33, 34, and 35 shows the optimal releases and the reservoir status before and after each re-

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* See Appendix I

lease. On the same figure the inflows to the reservoirs are shown, expressed in c.f.s. and acre-feet. Note that inflow to Reservoir No. 3 is the sum of the outflows from Reservoirs No. 1 and No. 2.

4.4 DISCUSSION OF RESULTS

The results shown on Figures 25, 26, 27, 28, 33, 34, and 35 indicate that the mathematical and computer procedures followed are valid. This conclusion is reached after a careful consideration of the results and the input data.

For example, it can be found that it is true that each of the releases computed is within the preassigned range, a well the reservoir's content at any time during the operational period, under study, are also within the preassigned range.

Studying the releases to be made during each time interval, it is noted that they vary according to the coefficient of the respective decision variable. For example, in Figure 25 the releases are shown to alternate between 100 c.f.s. and 2400 c.f.s. From Figure 24 it is seen that the coefficients of the decision variables vary alternatively as the releases computed. This indicates that priorities of releases to be made from different reservoirs in the system can be established by assigning the proper values to the decision variable coefficients. In some cases it is necessary to assign special values to the coefficients of the decision variables to avoid multiple solution results. In such cases the algorithm picks up only one solution which might be in favor of one of the reservoirs. (Larger release)

4.5 LIMITATIONS & EXTENSIONS OF PROCEDURES

The procedures developed in Chapter 2 and Chapter 3 have been applied and found to be valid. However, the procedure has certain limitations which have to be studied carefully before it is applied to accomplish its objective. The most important limitations are:

1) The procedure cannot be applied in big systems with considerable water travel time differences between reservoirs. In mathematical terms, when the results of equations 2.9.1 and 2.9.2 are not positive the model cannot be applied to the total system. Such systems must be studied by the decomposition method.

2) The measure of effectiveness of each release is proportional to the level of the release, conducted individually. Thus, by considering the problem as a linear one it is assumed that the marginal measure of effective-

ness and the marginal usage of each release are constant over the entire range of the permissible releases. This is a disadvantage because releases at different stages and from different levels from the reservoir have different marginal values.

3) The model does not optimize the releases from each reservoir from the economic point of view directly. What it gives is the releases to be made from the different reservoirs of the system, provided that the optimum range of flows in the channels downstream is known.

The mathematical model has been applied successfully to two discrete systems, though with small modifications can be applied to more complicated systems. Extensions of the models discussed can be made to consider the following:

1) Mixed type systems: The mathematical model of a mixed type system is constructed after studying its geometrical setup. For example, consider the system on Figure 36. The system is made up of two subsystems such as: Subsystem 1 is made up of reservoir No. 1, No. 3, and No. 5 operating in tandem and Subsystem 2 is made up of reservoirs No. 2 and No. 4 operating in parallel.

In modifying the model to consider this system these subsystems have to be considered, but in the final analysis

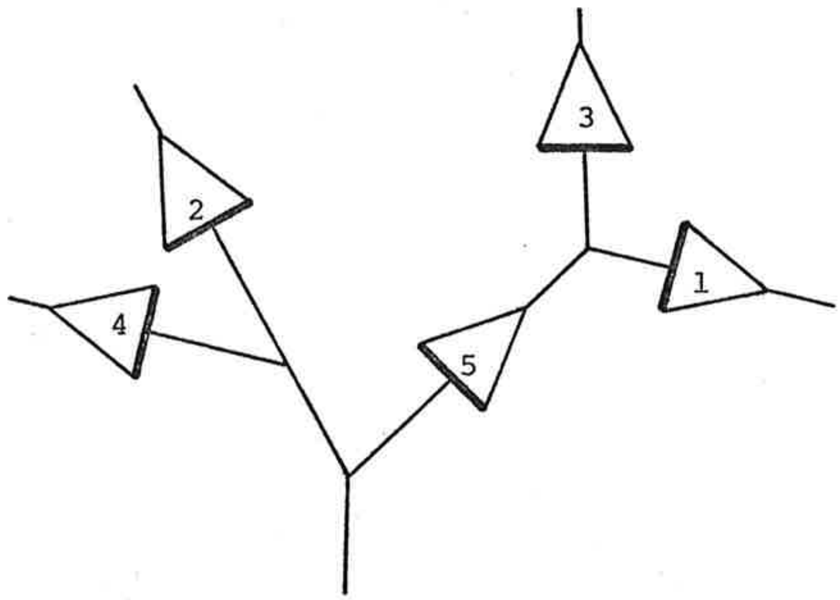

Figure 36 Mixed Type System

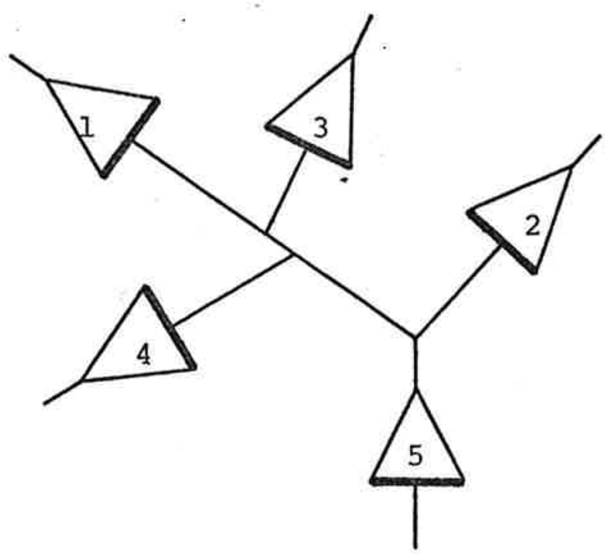

Figure 37 Mixed Type System

the system shall be treated as reservoirs in parallel.

A second example is the system shown on Figure 37 (quite different) which, in the final analysis shall be treated as reservoirs in tandem.

2) Local inflows to channels: This concerns the local inflows to channels downstream the flood control reservoirs. The model can be extended to include these flows by introducing them as another input to the channel constraints or considering the flows when deciding the maximum permissible flows in the channels of the system.

3) Local inflows to reservoir: In cases of reservoirs in tandem, it is assumed that the last reservoir in the system (Figure 29 reservoirs) has an inflow which is equal to the sum of releases from the reservoirs upstream. However, this is not absolutely true. In most cases local inflow does enter the reservoir. This can be included in the mathematical model by considering the inflow in the mathematical model as another input.

4) Multiple purposes reservoirs: The model can be extended to deal with multiple purpose reservoirs, taking into consideration the demand of reservoir storage for other uses. The parameters Y and Z in the constraint equation can be given values that include the demand of storage for other uses.

4.6 CONCLUSIONS & RECOMMENDATIONS

A mathematical model for the optimization of operation of a system of flood control reservoirs has been presented. All functions of the model have been expressed in linear form, suitable for application of a Linear Programming Algorithm.

The inflow to the reservoir has been assumed deterministic and the range of flows in the system channels known.

Application of the model on two different systems of reservoirs (reservoirs in Parallel and reservoirs in Tandem) by utilizing the Linear Programming Algorithm of MPS/360 has been successful. The result obtained indicate that operation of a system of flood control reservoirs can be optimized by the application of Linear Programming procedures.

Practical application of the model needs reasonable knowledge of the hydrologic conditions of the system in general and of every specific part of the system. Forecast of inflows to reservoirs as well as good knowledge of the capacities of the channels and reservoirs, must be known.

The procedure by itself does not provide the optimum (from the economic point of view) release from a reservoir. More studies must be carried out in establishing the optimum range of release from reservoirs as a function of inflow, reservoir level, and downstream stage. A study incorporating economic values should help to define guidelines for establishing the maximum allowable flows in the channel.

Further studies have to be carried on large scale systems where this model is difficult to apply. Methods of decomposition or by part optimization can be developed to deal with such systems.

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APPENDIX I

NOTATIONS

The following symbols and notations were used in this report.

N = Number of reservoirs in the system.

n = Number of time intervals for which operation of the system is to be optimized.

C_i = Flood storage capacity of reservoir i in units of volume; i=1, 2, ...N.

R_i = Flood storage capacity of reservoir i initially occupied in units of volume.

$I_{i,j}$ = Forecasted inflow to reservoir i during time interval j in units of volume.

$X_{i,j}$ = Decision variable (release) from reservoir i during time interval j in units of volume.

$d_{i,j}$ = Minimum acceptable flow in tertiary channel i during time interval j in units of volume.

$D_{i,j}$ = Minimum acceptable flow in secondary channel i during time interval j in units of volume.

DI_j = Minimum acceptable flow in main channel during time interval j in units of volume.

$b_{i,j}$ = Maximum acceptable flow in tertiary channel i during time interval j in units of volume.

$B_{i,j}$ = Maximum acceptable flow in secondary channel i during time interval j in units of volume.

BI_j = Maximum acceptable flow in main channel during time interval j in units of volume.

T_i = Water travel time (expressed in time intervals) from reservoir i to damage center. (Time interval may be equal to 6, 12, 18, or 24 hours.)

DT_i = Difference of water travel time between reservoir No. 1 and reservoir i in the system.

TF = Forecast period expressed in time intervals.

$Y_{i,j}$ = Minimum predetermined level the reservoir i shall be allowed to drop or rise at the end of period j in units of volume.

$Z_{i,j}$ = Minimum storage volume that must be empty in reservoir i by the end of time interval j.

APPENDIX II

MATRIX REPRESENTATION OF THE MODELS

II.1 GENERAL

The objective function and each set of time sequence constraint equations shall be presented in their matrix form, of the general form AX $\{\leq = \geq\}$H. For each set of constraints one and only one of the signs $\leq = \geq$ holds and the sign shall vary from one set of constraints to another.

Matrices of type A shall present the operating characteristics of the object or the system and shall be referred to as Transition Matrices.

Matrices of type X represent the decision variables, one column matrix, where matrices of type H, a column vector of constants express the lower or upper bounds of the left hand side (LHS) matrix (product of AX).

Matrices of type A are of the order nxL, matrices of type X are of the order Lx1 and matrices of type H are of the order nx1. (L=Nxn)

Following are general rules of how to develop the transition submatrices of a transition matrix of a system.

II.2 RESERVOIRS IN PARALLEL

The following symbols and notations are to be used in this section.

$A1_i$ = Transition submatrix of the order nxL representing the reservoir i operation requirements. (Minimum or maximum cumulative flow constraints).

$A2_i$ = Transition submatrix of the order nxL representing the secondary channel i operational requirements. (Minimum or maximum flows).

A3 = Transition submatrix of the order nxL representing the main channel operational requirements (minimum or maximum releases).

C = Transition submatrix of the order 1xL representing the unit merit of the decision variables.

H1 = Column vector matrix of constants expresses the minimum cumulative release constraint.

H2 = Column vector matrix of constants express the maximum cumulative release constraints.

D = Column vector matrix of constants, expresses the minimum flow constraint for the secondary channels.

D1 = Column vector matrix of constants, expresses the minimum flow constraint for the main channels.

B = Column vector matrix of constant, expresses the maximum flow constraints for the secondary channels.

B1 = Column vector matrix of constant, expresses the maximum flow constraint for the main channels.

Submatrices A1, A2, A3, and C make up the transition matrix (left hand side: LHS input data) where column matrices H1, H2, D, D1, B, and B1 give the limiting values upper or lower (Right Hand Side: RHS values input data). Since we can reverse inequality signs by multiplying the equation by -1, we can assume that this has been done where necessary to get the $\leq$ sign. Therefore, the general form of the operational matrix of a system of reservoirs is as shown by Eq. 1.

$$\begin{bmatrix} -A1_1 \\ -A1_2 \\ \cdot \\ \cdot \\ -A1_N \\ A1_1 \\ A1_2 \\ \cdot \\ \cdot \\ A1_N \\ -A2_1 \\ -A2_2 \\ \cdot \\ \cdot \\ -A2_M \\ A2_1 \\ \cdot \\ A2_M \\ -A3 \\ A3 \end{bmatrix} \times \begin{bmatrix} X \end{bmatrix} \leq \begin{bmatrix} -H1_1 \\ -H1_2 \\ \cdot \\ \cdot \\ -H1_N \\ H2_1 \\ H2_2 \\ \cdot \\ \cdot \\ H2_N \\ -B_1 \\ -B_2 \\ \cdot \\ \cdot \\ -B_M \\ D_1 \\ \cdot \\ D_M \\ -B1 \\ D1 \end{bmatrix} \qquad 1)$$

where N = Number of Reservoirs

M = Number of Secondary channels

The general form of the submatrices is given as follows:

Submatrix type A1: This is an nxL matrix whose general form is given by equation 2.

Submatrix type A2: This submatrix expresses the operational requirements of the secondary channel. Suppose that the secondary channel K is fed by channels from reservoir i and j. The general form of the transition submatrix shall be as shown by equation 3 $(i<j)$.

Submatrix type A3: According to the definition this submatrix describes the operational requirement of the main channel. In general this submatrix is a special form of submatrix A2 where all reservoirs in the system discharge in the channel. Since there is only one main channel in the system (reservoirs in parallel) this submatrix shall appear in the Transition matrix twice, one for upper bounds and one for lower bounds. The general form of the type A3 submatrix is shown by equation 4.

$$
Al_i = \begin{array}{cc}
\begin{array}{cccccccccccccc} 1 & & (i-1)n & (i-1)n+1 & & & & & & 1xn & ixn+1 & ixn+2 & & Nxn=L \end{array} & \\
\begin{bmatrix}
0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & & & & & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & & \\
0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 & \cdots & 0
\end{bmatrix}
&
\begin{matrix} 1 \\ 2 \\ 3 \\ \\ n-2 \\ n-1 \\ n \end{matrix}
\end{array}
\qquad 2)
$$

Where: i refers to the reservoir. i=1, 2, 3, ... N

n=number of time intervals for which operation is planned.

$$A2_k = \begin{array}{c} \begin{array}{cccccccccccccc} 1 \quad (i-1)xn & (i-1)xn+1 & & & & ixn & (j-1)xn+1 & & & & & & jxn & Nxn \end{array} \\ \begin{bmatrix} 0 \ldots 0 & 1 & 0 \ldots\ldots & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots\ldots 0 \\ 0 \ldots 0 & 0 & 1 \ldots\ldots & 0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 0 & 0 & \ldots\ldots 0 \\ 0 \ldots 0 & 0 & 0 \ldots\ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots\ldots 0 \\ \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \\ \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \\ \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \\ 0 \ldots 0 & 0 & 0 \ldots\ldots & 1 & 0 & 0 & 0 & 0 & \ldots & 1 & 0 & 0 & 0 & \ldots\ldots 0 \\ 0 \ldots 0 & 0 & 0 \ldots\ldots & 0 & 1 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots\ldots 0 \\ 0 \ldots 0 & 0 & 0 \ldots\ldots & 0 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & \ldots\ldots 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \\ \\ \\ \\ n \\ n \end{array} \qquad 3)$$

$$A3 = (I_1 \ I_2 \ I_3 \ I_4 \ \ldots \ I_N) \qquad 4)$$

where I_i = unit matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & & \ddots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \qquad 4a)$$

It is seen that the submatrix type A3 is made up of N, identity matrices.

C type matrix: This is a one row matrix which contains the coefficients of the decision variables. This can be included in the general transition as shown on Figure 23. The submatrix type C is the first row on the Transition Matrix on figure 23 (i time intervals, three reservoirs) where the rest gives the other submatrices as described above. The R.H.S. submatrices shall be discussed in Section II-4.

II.3 RESERVOIRS IN TANDEM

The general form of the operational matrix of a system of reservoirs in tandem is as shown by equation 5.

$$\begin{bmatrix} -A1_1 \\ \vdots \\ -A1_{N-1} \\ -A4 \\ A1_1 \\ \vdots \\ A1_{N-1} \\ A4 \\ -A2_1 \\ \cdot \\ \cdot \\ -A2_M \\ A2_1 \\ \cdot \\ A2_M \end{bmatrix} \times \begin{bmatrix} X \end{bmatrix} \leq \begin{bmatrix} -H1_1 \\ \vdots \\ -H1_{N-1} \\ -H_3 \\ H2_1 \\ \vdots \\ H2_{N-1} \\ H_4 \\ -B_1 \\ \cdot \\ \cdot \\ -B_M \\ D1 \\ \cdot \\ D_M \end{bmatrix} \qquad 5)$$

Submatrix type A3 does not appear in this transition matrix because by definition there is not a main channel in the system. Submatrices of type A1 and A2 have already been described by equations 2 and 3.

Submatrix type A4: This submatrix expresses the operational requirements of the tandem reservoir to which inflows are given by the outflows from the rest of the reservoirs upstream. The general mathematical form of this submatrix can be expressed as follows:

$$A_4 = [-E_1 -E_2 - \ -E_3 \ \ldots \ -E_{N-1} \ E_N] \tag{6}$$

where $E_i =$

$$\begin{array}{c c}
\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & \cdots & n-2 & n-1 & n \end{array} & \\
\left[\begin{array}{ccccccccc}
1 & 0 & 0 & 0 & & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot \\
1 & 1 & 1 & 1 & & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & & 1 & 1 & 1
\end{array}\right] &
\begin{array}{c} 1 \\ 2 \\ 3 \\ \\ \\ n-2 \\ n-1 \\ n \end{array}
\end{array} \tag{7}$$

i = Reservoir Number.

II.4 R.H.S. SUBMATRICES

The right hand side matrices listed in equations 1 and 5 are fed into the computer as column vectors of constant elements. This should be the final form of all R.H.S. submatrices after carrying out the operations as given in section 2.

For example, submatrix type H1 is given by the equation 8,

$$H1 = [B] \times [I] + [R] - [C] + [Z] \tag{8}$$

where B is a lower triangular matrix, I is a column matrix with constant elements, R is a column matrix with constant elements, and Z is a column matrix with constant elements.

$$B = \begin{array}{cc} \begin{array}{cccccc} 1 & 2 & 3 & \cdots & n-1 & n \end{array} & \\ \begin{bmatrix} 1 & 0 & 0 & & 0 & 0 \\ 1 & 1 & 0 & & 0 & 0 \\ 1 & 1 & 1 & & 0 & 0 \\ \cdot & & & \ddots & & \\ \cdot & & & & \ddots & \\ \cdot & & & & & \\ 1 & 1 & 1 & & 1 & 0 \\ 1 & 1 & 1 & & 1 & 1 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ \\ \\ \\ n-1 \\ n \end{array} \end{array} \qquad 9)$$

$$I = [I_1, I_2, I_3, I_4 \ldots\ldots I_n] \qquad 10)$$

where $[I_1 \ldots\ldots I_n]$ represents column matrix, and $(I_1 \ldots\ldots I_n)$ represents row matrix.

$$R = [R_1, R_2, R_3 \ldots\ldots R_n] \qquad 11)$$

$$C = [c^1 \; c^2 \; c \ldots\ldots c] \qquad 12)$$

$$Z = [Z_1, Z_2 \ldots\ldots Z_n] \qquad 13)$$

Since all elements of the matrices 9, 10, 11, 12, and 13 are known by carrying out the indicated operations, the equation results to one column matrix of constant elements.

Submatrix H_1 is related with the minimum cumulative release constraints.

Submatrix type H_2 is given by:

$$H2 = [b] \times [I] + [R] - [Y] \qquad 14)$$

where B, I, and R have the same form as before.

$$Y = [Y_1, Y_2 .. Y_n] \qquad 15)$$

Submatrix type H2 is related with the maximum cumulative release constraint.

Submatrix H3 is related to the minimum cumulative release constraint for the reservoir in tandem and is given by equation 16.

$$H_3 = [R] - [C] + [Z] \qquad 16)$$

Submatrix type H_4 given by equation 17

$$H_4 = [R] - [C] - [Y] \qquad 17)$$

is related to the maximum cumulative release constraint of reservoir No. N for the reservoir in tandem.

The rest of the R.H.S. submatrices B, B1, D, D1 are also column vectors with constant elements.