USE OF A MICROWAVE REMOTE SENSOR FOR DETERMINATION OF WATER IN SUBSOILS

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1. OBJECTIVES AND STATEMENT OF PROBLEM

The development of instruments for rapidly measuring the water content of subsoils is important in many areas of water resources research. For example, knowing the excess or lack of subsoil water can improve crops. The marked dielectric contrast, which exists between water and other common soil constituents (Fig. 1 and Table I) at microwave frequencies, provides a valuable means of sensing water content in soil.

In order to investigate the feasibility of using a microwave remote sensor to determine the amount of water in subsoils, the following studies are to be considered:

1. Calculation of the complex dielectric constant corresponding to each measured value of the complex propagation constant in order to provide a comparison of the measured data with previously measured values of the complex dielectric constant of various topsoils. The calculated values of dielectric constant will also be used to obtain the reflection coefficients at layers.
2. Calculation of the two-way (radar) attenuation[1] versus depth associated with each frequency, water content, and each locale for which measurements are made.
3. Calculation of the brightness (radiometer) temperature[2] associated with each frequency, water

content, and locale for which measurements are made.

4. Evaluation of the possibilities for using passive and active microwave remote sensing devices for sensing water content at the various locales for which measured and calculated data are available.

TABLE I. DIELECTRIC CONSTANT OF SOIL CONSTITUENTS.

Kaolinite	4.85	Augite	6.90
Halite	5.70	Biotite	6.19
Aragonite	6.46	Muscovite	6.19
Calcite	7.80	Feldspar	4.5
Dolomite	6.11	Quartz	4.19
Barite	6.99	Wood	4.0
Gypsum	5.00	Air	1.0003

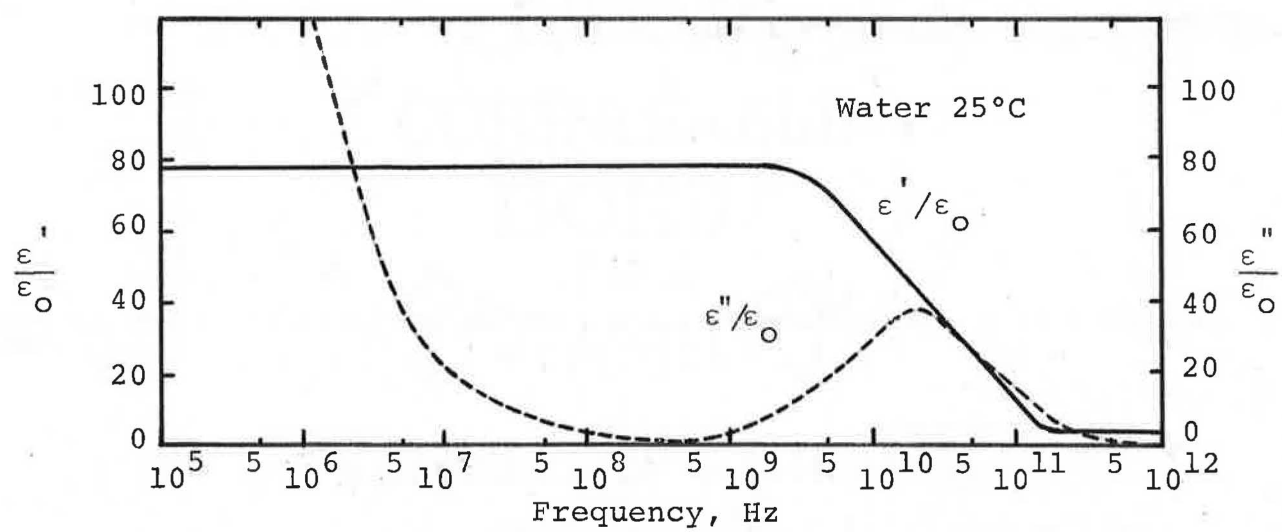

FIGURE 1. THE DIELECTRIC CONSTANT $\varepsilon'/\varepsilon_o$ AND DIELECTRIC LOSS $\varepsilon''/\varepsilon_o$ OF WATER AT 25°C AS A FUNCTION OF FREQUENCY.

Moisture content was determined by weighing the moist soil in a container of known weight immediately on completion of the microwave measurements. The soil was again weighed after it was slowly heated for six to eight hours or until subsequent heating produced no further change in weight.

Test results are given in the next section.

2. INTRODUCTION

The complex dielectric constant of soils at a function of moisture content was measured at 10.525 GHz. The dielectric constant is computed from measured values of the input impedance of a known length of waveguide filled with soil when the soil sample is terminated in a short circuit. Because the phase of the reflection coefficient of the shorted sample is known only as modulo 2π, a second measurement is required to resolve this ambiguity; therefore, the impedance was also measured when the sample was terminated in an open circuit. The basic method is discussed in detail by vonHippel (Reference 1), but a brief synopsis is given here in Appendix A. A further complication is imposed, however, because the soil cannot support itself in the transmission system; hence additional media must be introduced to contain the sample. A satisfactory extension of the procedure given by vonHippel is derived in Appendix B along with a suitable technique for reducing data. The soil was contained by a dielectric window at the input terminal plane and by either a fixed short or a second window of the output. This equipment and the procedure for using it are described in Section 3.

Input impedance was measured by slotted line techniques, which are described in detail in Section 4, with the supporting theory given in Appendices B and C.

3. RESULTS AND CONCLUSIONS

Three soil types were presented for testing; two bore the labels Jefferson City Sand and Roubideux, while the third was not labeled. The Jefferson City Sand was largely what the label implies--a grayish-white sand reminiscent of the type currently sold for concrete aggregate. The Roubideux soil and the unlabeled specimen appeared to be basically clay, intermingled with tiny pebbles and organic matter.

As of this writing, testing of the Jefferson City and Roubideux specie has been completed. The resulting data are given in Tables II and III. In each case, the moisture content ranges from less than 2% (by weight of the moist sample) to saturation. A discussion of the tests results follows Table III.

TABLE II. TEST RESULTS FOR JEFFERSON CITY SAND

Water Content (% by weight)	Complex Dielectric Constant $\varepsilon_c/\varepsilon_o$	Weight of Moist Sample -Grams-	Propagation Constant, α/β_o From Short Circuit Test	Short Circuit Input Impedance
0	2.95-j.080	2.53	$2.05\underline{/89.2°}$	$.629\underline{/-85.4°}$
2.4	3.99-j.59	2.46	$2.45\underline{/85.4°}$	$.219\underline{/-56.2°}$
4.4	2.51-j.37	2.47	$1.88\underline{/85.0°}$	$.980\underline{/-60.8°}$
8.4	1.42-j.34	2.59	$1.33\underline{/81.0°}$	$.413\underline{/37.6°}$
8.6	1.42-j.28	2.31	$1.32\underline{/82.3°}$	$.370\underline{/41.5°}$
11.6	1.49-j.40	2.52	$1.39\underline{/80.0°}$	$.448\underline{/24.1°}$
13.2	4.33-j1.56	2.53	$2.64\underline{/79.2°}$	$.220\underline{/-13.2°}$
13.9	5.63-j2.34	2.83	$3.07\underline{/78.0°}$	$.207\underline{/1.7°}$

TABLE III. TEST RESULTS FOR ROUBIDEUX SOIL

Frequency = 10.525 GHz

Water Content % by Wt.	Complex Dielectric Constant $\varepsilon_c/\varepsilon_o$	Wt. of Moist Sample -Grams-	Propagation Constant, α/β_o		Short Circuit Input Impedance Z_{sc}/Z_o	Open Circuit Input Impedance Z_{oc}/Z_o
			From Short Circuit Test Only	From Combined Open and Short Circuit Tests		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.6	2.45-j.62	1.71	1.88/86.6°	2.04/84.3°	.969/-69.9°	.248/81.3°
5.4	3.88-j.69	1.89	2.42/84.4°	--	.247/-52.3°	--
7.6	4.15-j.98	1.86	2.53/82.7°	--	.208/-35.1°	--
8.8	5.14-j1.50	1.87	2.86/81.2°	--	.157/4.6°	--
11.0	8.10-j2.36	2.42	3.64/81.5°	3.20/82.1°	.289/42.4°	.337/-26.6°
12.1*	11.47-j.31	2.55	4.27/89.2°	3.75/89.4°	.367/17.4°	.194/-16.2°
13.1*	11.70-j.24	3.03	4.31/89.4°	4.64/87.0°	.267/-10.8°	.174/16.8°
13.7	11.64-j.12	2.69	4.30/89.7°	3.98/84.5°	.327/4.4°	.193/6.6°
13.7	11.64-j.31	2.85	4.30/89.2°	--	.352/2.0°	--
15.7	11.64-j.20	2.75	4.30/89.5°	--	.246/5.9°	--
20.1	17.50-j7.84	2.65	5.56/77.7°	5.25/77.7°	.162/6.7°	.225/17.9°
23.5	21.2-j12.0	2.46	6.29/75.0°	5.73/78.3°	.151/16.9°	.202/6.4°

In Tables II and III, dielectric constant is relative to free space, while the propagation constant γ of a soil-filled section of waveguide (see Section 4) is normalized to the phase constant β_o in empty X band waveguide. The tabulated impedances are normalized to the wave impedance Z_o of the TE_{10} mode in empty X band waveguide.

3.1. JEFFERSON CITY SAND

All test data for Jefferson City Sand are derived from short-circuit tests. Only enough open circuit tests were run to resolve the ambiguity mentioned in Section 2. This material was tested during the developmental phase of the measurement scheme, and considerable difficulty was experienced at that time in obtaining satisfactory agreement between the open and short circuit values of γ/β_o, particularly in the real part of γ (the attenuation constant). Temporarily, it was considered sufficient if the imaginary part of γ could be resolved well enough that ambiguities were eliminated; for this reason, open circuit test data are not reported for Jefferson City Sand. Subsequent improvements in both equipment and procedures, however, led to greater accuracy so that these data are given for later measurements. More will be said of this in Section 3.2.

Other factors also cast more suspicion on the data of Table II than on subsequent measurements. For one thing, the precision attenuator was not available until testing of Roubideux soil commenced; therefore, it was necessary to accept the reading on the scale of the VSWR indicator--a less accurate procedure, usually. Second, a Varian V-54 reflex klystron was used as the power source at that time. This tube is not so easily tuned as the X-13 which replaced it in later measurements, so that small frequency drifts had to be corrected by changes in reflector voltage. This procedure can increase the incidental frequency modulation (always present to some degree in modulated sources) and thus alter the structure of the standing wave pattern at a deep minimum. A further, though smaller source of error, was in the calculation of VSWR, whereby the corrections provided in the table of Appendix C were not introduced until refinements in equipment justified the added labor.

A quantitative assessment of error in the data of Table II is difficult to obtain, although the table is believed to provide a reasonable indication of the behavior of γ/β_o versus percent moisture. If the need for more accurate data justifies the additional expenditure, it is recommended that Jefferson City Sand be re-tested with the refinements used in the preparation of Table III, which are further described in Section 4.

3.2. ROUBIDEUX SOIL

A high level of confidence is ascribed to the data of Table III for reasons to be enumerated here.

First, γ/β_o was computed solely from the short circuit measurement with the open circuit data used only to determine the proper branch of γ as a multivalued function of the short circuit impedance Z_{sc} (see Appendix A). These values are given in column 4 of Table III. A second method of calculating γ/β_o was used, whereby the influence of the open circuit impedance (derived from an independent measurement) is injected. These values of γ/β_o are given in column 5 of Table III. The calculations are illustrated in Section 5[1]. If one accepts the disparity between these two calculations as a figure of merit, then the following analysis can be given.

Based on values obtained from short circuit tests, one sees that the worst errors in $|\gamma/\beta_o|$ occur at 12.1% and 11.0% moisture where the errors are 12.2% and 12.1%, respectively. All other errors in $|\gamma/\beta_o|$ range from 8.9% downward. The worst error in argument (γ/β_o) is 5.8% and this occurs at 13.7% moisture.

The lines marked * in Table III are especially significant, for they were obtained without use of the windows

[1] A completely independent calculation based entirely on the open circuit impedance would be still better, but this required more extensive tables of the complex function coth $\gamma d/\gamma d$ (see Appendix A) than were available.

described in Section 4. It was discovered that Roubideux soils with moisture content in the range of 11 to 14% could support themselves during measurement. Because the error in these two measurements is no worse than in those which required the windows, it is concluded that the windows and the series of measurements required to evaluate them (described in Section 4) have not added to experimental error. It is further concluded that the dominant error is caused by: (a) nature's unwillingness to provide soils which are uniform dielectrics, and (b) difficulty in wetting and packing soils uniformly into the sample holder. The latter conclusion is substantiated by the following observations:

1. Small pebbles and roots of millimeter cross sections were present in samples which showed large error.
2. Color striations were observed in the 12.1% sample as it was packed into the sample holder; this sample shows 12.2% error in $|\gamma/\beta_o|$.
3. Roubideux soils of 11 to 14% moisture tended to be non-plastic and were hard to work into corners of the sample holder; this is the range of moisture content which shows most error.

Several samples were rejected after testing due to excessive error attributed to condition 3, and, to a lesser extent, to condition 1. Table III is believed to be reliable, however, in those circumstances where nature is benevolent enough to provide soils of uniform electrical properties.

4. EXPERIMENTAL PROCEDURES

Several pieces of hardware had to be prepared before data could be collected. This hardware, instrumentation for microwave measurements, and methods by which data were obtained are described in this section.

4.1. HARDWARE

Special hardware needed for the microwave measurements consisted of a sample holder, two dielectric windows self jigged to the sample holder, and a fixed short mated to the sample holder.

The sample holder used to mount the specimen on the waveguide is shown in Figure 4-1.

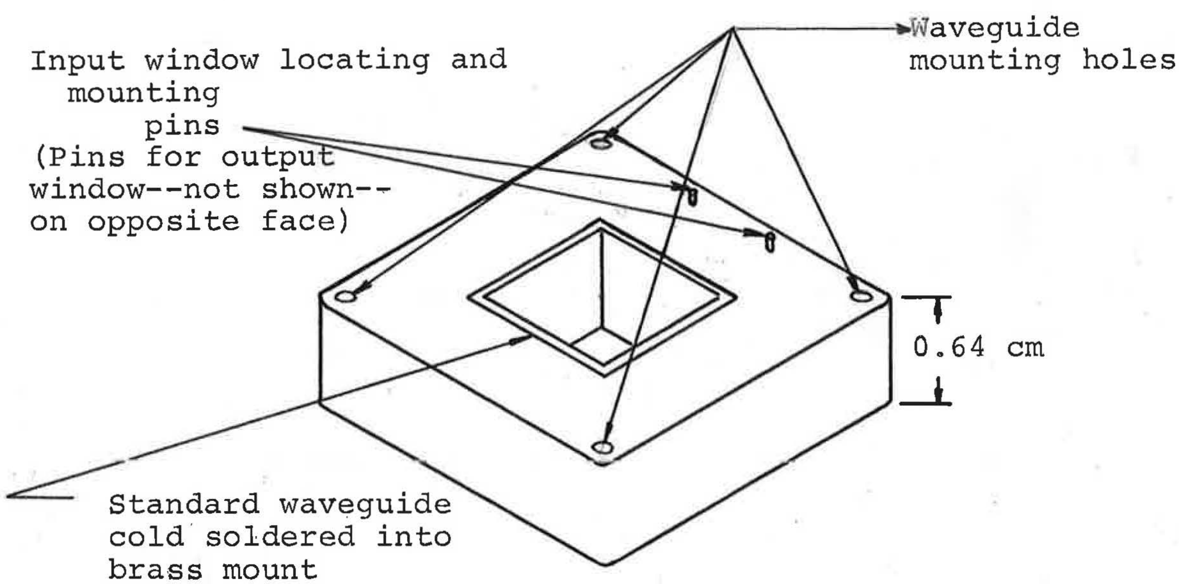

FIGURE 4-1. SAMPLE HOLDER

The inner opening and the mounting holes in Figure 4-1 were dimensioned to mate with X band waveguide, EIA size WR90 (inner dimensions 0.900x0.400 inches nominal). An input window shown in Figure 4-2 was constructed to mate with the sample holder.

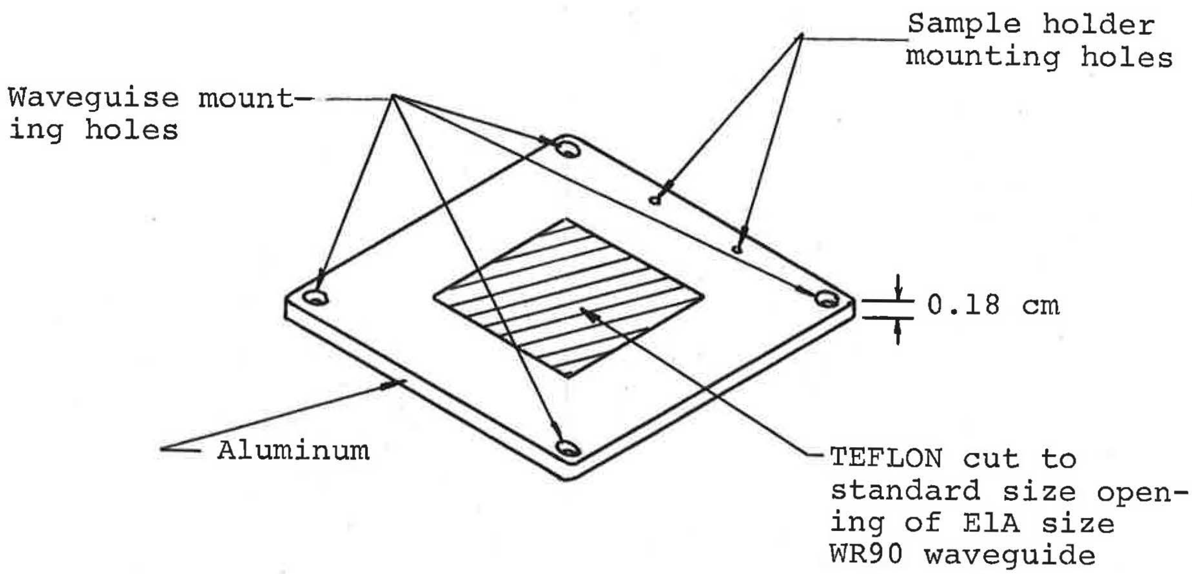

FIGURE 4-2. INPUT WINDOW

To ensure a standard size opening in the sample holder without extensive machining, an oversize opening was cut in the brass and a short piece of standard waveguide was cold soldered into the brass mount. Because the window was thin, no difficulty was experienced in cutting the proper opening there.

An output window was similarly constructed of brass and teflon. Because of experimental procedures (see Appendix C), its dimensions are not critical (except that it must mate with the sample holder) and thus are not shown.

Teflon for the windows was cut to a snug fit so that it supports itself in place but can be removed with sufficient pressure. After the windows were completed, the teflon was removed, and the windows were aligned in place and clamped to the sample holder. Sample holder mounting holes were drilled through the window frames and into the sample holder. Locator pins were driven into place. Again a snug fit was provided so that when they were attached to the locator pins, the windows were held rigidly, but could readily be pried free of the sample holder for ease in changing soil specimens.

Finally, a brass shorting plate was constructed with holes drilled to clear the window locator and mounting pins on the sample holder. This shorting plate replaced the output window during short-circuit impedance measurements.

4.2. EXPERIMENTAL EVALUATION OF HARDWARE

Evaluation of hardware included measurement of the physical length of the sample holder and the electrical and physical lengths of the input window.

The length of the sample holder was measured by placing the shorted sample holder on the slotted line and recording the position of the minimum in the standing wave pattern. The sample holder was next replaced by a fixed short and the distance the minimum shifted, 0.64 cm, is the length of the sample holder. This distance was measured before locator pins were installed on the sample holder. An identical measurement was made on the input window with the teflon removed; its physical length was found to be 0.18 cm.

The electrical length of the input window was found by measurement of the open circuit impedance. Relative to a short circuit, the input window (with teflon in place) produced .505 cm shift of the minimum along the slotted line toward the load; that is a shift of +0.138 λ_g which, according to the Smith chart, corresponds to a normalized impedance of -j1.18 if the VSWR is infinite. Putting this value into equation (A.3) with $\gamma = j\beta_1$[1] yields:

$$\frac{1}{\beta_o S} \frac{Z_{oc}}{Z_o} = -j \frac{\cot \beta_1 S}{\beta_1 S} \tag{4.1}$$

where β_1 = phase constant in teflon filled waveguide

S = physical length of the teflon filled section

Z_{oc} = open circuit input impedance of the window

Z_o = wave impedance in empty waveguide

β_o = phase constant in empty waveguide.

At 10.525 GHz, the guide wavelength λ_g measured 3.66 cm and S measured .18 cm; then since $\beta_o = 2\pi/\lambda_g$, equation (4.1) becomes:

$$\frac{\cot \beta_1 S}{\beta_1 S} = 3.82 \tag{4.2}$$

for which the lowest root is $\beta_1 S = .490$. Because $\beta_o S = 0.309$, one finds:

$$\frac{\beta_1}{\beta_o} = 1.59. \tag{4.3}$$

To check the validity of taking the lowest root of (4.3), the dielectric constant of teflon was computed by substituting (4.3) into (A.6); the result is that the measured dielectric constant for teflon is:

$$\frac{\varepsilon_c}{\varepsilon_o} = 1.99. \qquad (4.4)$$

The value in (4.4) compares to a value 2.08 with a loss tangent of less than 6×10^{-4} given for DuPont teflon (see p. 332, Reference 1). Although the specific variety of teflon used in the window is not known, equation (4.4) is thought to be more accurate than the tabulated value of a specific manufacturer. Use of the least root of (4.2) and the approximation of a lossless window appear to be justified, however, by comparison of (4.4) to the tabulated value.

4.3. INSTRUMENTATION.

Instrumentation for the microwave tests is standard for slotted line impedance measurements; nevertheless, for completeness, it is shown in Figure 4-3 and will be discussed briefly.

The klystron power supply provides for either CW or modulated operation via 60 Hz sine wave or 1000 Hz square wave applied to the klystron reflector. The directional coupler with crystal detector (coupled to the incident wave) allows the klystron mode diagram to be displayed continuously on the oscilloscope. This facilitates rapid adjustment and

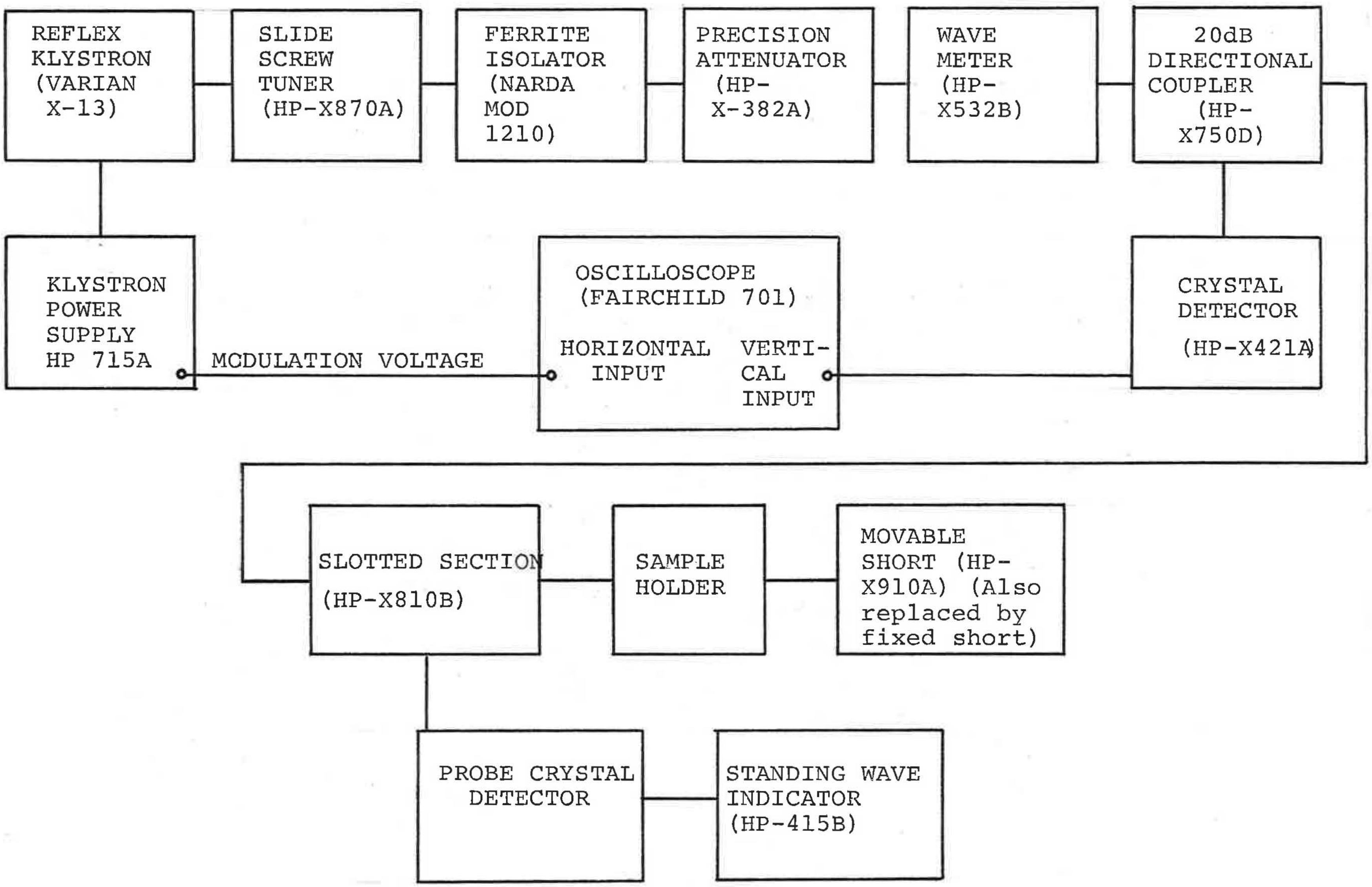

Figure 4-3. Microwave Equipment.

checking of equipment and is a most convenient method of making numerous frequency checks.

Waveguide components were interconnected by ElA size WR90 waveguide rated from 8.2 to 12.4 GHz.

Forced air cooling must be provided for the Varian X-13 klystron. This klystron is tuned by a micrometer screw from a frequency of 8.5-12.4 GHz and can supply as much as 700 mw power, depending on frequency, when it is operated into the optimum load. Load optimization is the purpose of the slide screw tuner. Because all measurements are to be made in the vicinity of a voltage minimum, and because large VSWR may be anticipated with some samples, it was thought desirable to provide for optimization of the load should maximum power be required. A ferrite isolator following the slide screw tuner still provides a well-matched generator as seen from the input to the slotted line.

The precision attenuator is used so that probe depth can be adjusted at will without incurring the tedium of numerous crystal calibrations. Insertion of 3 dB attenuation during VSWR measurements allows the standing wave indicator to be used only as a fixed level indicator in locating the points of double minimum power.

4.4. MEASUREMENT OF IMPEDANCE

The sample holder backed by the shorting plate was packed with moistened soil whose dielectric constant was to be measured. Packing was facilitated by use of a piece of stycast machined to waveguide dimensions. Special care had to be exercised to maintain soil surfaces flush with the faces of the sample holder. The input window was then attached to the sample holder and the sample was positioned on the slotted section as shown in Figure 4-3.

Input impedance was obtained by measurement of the VSWR and the shift in position of the voltage minimum produced by the sample relative to the position of the minimum when the slotted line was terminated in a fixed short. The VSWR was determined by the double minimum method, described in some detail in Reference 2, and briefly reviewed in Appendix C. This method affords superior accuracy when moderately large standing waves are encountered and adequate power is available because the probe is restricted to the proximity of the minimum. Probe interference effects, with attendant errors, are thereby minimized.

Guide wavelength was given by twice the separation of adjacent minima when the line was shorted.

The foregoing information was recorded on a Smith chart in the usual fashion and the impedance looking into the input window (normalized to the wave impedance of the empty guide, Z_o) was read from the chart. According to the equations of Appendix A, it is the impedance looking into the soil sample normalized to Z_o, however, which is required for determination of the dielectric constant. The influence of the input window must, therefore, be removed from the data. That this influence is appreciable is shown by the measurement of Section 4.2 and the calculations of Section 4. Appendix B details a method whereby the required impedance can be retrieved through further calculation; moreover, the entire calculation is illustrated by a sample data reduction presented in Section 5.

Open-circuit impedance data were similarly measured and reduced except that the fixed short had to be replaced by an open circuit terminating the sample. To this end, the sample with attached input window was removed from the line and the fixed shorting termination replaced by the output window. The output window was then terminated by a movable short which had been previously adjusted to place an open circuit coincident with the output end of the soil sample. That such an adjustment is possible (subject to neglect of dielectric losses in the output window and wall losses in the moving short) is shown in Appendix B. Appendix B further shows how the adjustment is to be made.

The assumption that losses could be neglected in the movable short and in both windows was tested as follows. First, the movable short, and later, each window backed by the movable short, was attached to the slotted line, and the movable short was varied through its full range. In each case, the minimum of the standing wave was below noise level as evident by random motion of the needle of the standing wave indicator. In each case, also, the average incident power (as read by a Hewlett-Packard model 432A power meter connected to the 20 dB coupler) was in excess of 100 mw and the standing wave indicator was at maximum gain. The test suggests that wall losses in the slotted line, too, can be neglected, as is hereafter assumed.

The neglect of the above mentioned losses is further supported by the fact that for certain samples (e.g., Roubideux soils with 12 to 15% moisture), complex propagation constants with arguments in excess of 89° were measured.

4.5. ADJUSTMENT AND MEASUREMENT OF MOISTURE CONTENT

Attempts to adjust soil samples to a prescribed moisture content met with limited success; nevertheless, a great deal of time was expended on that rather essential task so that some discussion of it seems in order.

Several grams of dry soil were placed in a plastic cup and weighed. The weight of water needed to yield a given percent moisture was computed, total weight of cup, soil, and water were added, and the cup of soil was placed on one pan of a laboratory balance. The desired weight of cup, plus wet soil, was placed on the other pan and water was added from an eye dropper until the mixture came to balance. During this procedure, it was necessary to stir the soil vigorously after the addition of just a few water droplets. Small globules of soil a few millimeters in diameter tended to form. These globules consisted of wet soil surrounding a dry core, so that they had to be broken up often to obtain a uniform sample. Frequent stirring produced a rapid evaporation rate, so that once a sample was prepared, it was just a guess how much moisture remained during the microwave tests. Several hours were required to dry a portion of the wet sample for an accurate determination of water content, and if this was done before the impedance measurement, it was impossible to keep the moisture content of the original sample from changing while the water content was being measured. Because of the extreme difficulty in obtaining the impedance of samples with a pre-determined water content, the following method was finally adopted.

A batch of 20-30 grams of soil was moistened slightly and a sample removed on which short circuit measurements were quickly made. Immediately after the microwave tests, the soil was removed from the sample holder, placed in a beaker or crucible, and weighed. The sample with its container was then placed on a hot plate for drying. To the original batch, a few drops of water were added, the batch was stirred, and another sample removed and tested. In this fashion, moisture content could be increased by 2 to 4% for each succeeding specimen so that if the procedure continued until the original batch was exhausted, a good range of data could be obtained with acceptable increments in moisture content. Open circuit tests were eliminated from this series of measurements since they consumed much more time than short circuit tests alone; thus the change in moisture content of the original batch changed less while microwave tests were being run. Ideally, this sequence of tests should be carried from dry samples to saturated soil with no long interruptions. This required long working hours of the experimenter--perhaps 10 to 14 hours or more.

The ambiguity due to use of short circuit tests alone must be resolved by a separate series of tests. Open circuit tests are not required for each specimen; usually 4 or 5 open circuit tests suitably dispersed throughout the range from 0% moisture to saturation are adequate. With fewer data points required, one can rely on more-or-less hit-or-miss methods of moisture control for open circuit measurements. For consistent data, however, it is necessary to repeat short circuit tests on each sample at the same time the open circuit data are taken.

Samples were weighed on a Fisher model 100 precision balance. A Thermolyne model HP-A 1915B hot plate was used for drying the soil during measurement of moisture content. The hot plate was adjusted to a dial setting of 200 to 250 which kept the sample uncomfortably warm to touch but presumably not so warm as to cause appreciable chemical decomposition of soils. Drying usually took 3 to 6 hours.

5. REDUCTION OF MICROWAVE DATA

As an illustration of the procedures by which dielectric constant is obtained from the microwave data, the data for Roubideux soil of 11.0% moisture content will be reduced in this section.

Raw data for the open-circuit test are as follows:

1. Distance between twice minimum power points = Δx = 0.35 cm.
2. Guide wavelength = λ_g = 3.66 cm.
3. Position of minimum with short on slotted line = 12.84 cm.
4. Position of minimum with sample on slotted line = 12.78 cm.
5. Frequency = 10.525 GHz.

The input impedance looking into the soil sample is computed by the formulas given in Appendices B and C. With the data from (1) and (2) above, one finds:

$$\frac{\pi\Delta x}{\lambda_g} = \frac{.35\pi}{3.66} = 0.300. \qquad (5.1)$$

The correction term is found from the table of Appendix C to be 0.016 so that:

$$\frac{1}{S} = 0.300 - 0.016 = 0.284. \qquad (5.2)$$

According to data items 3 and 4, the shift of the minimum (relative to a short) is:

$$d = 12.78 - 12.84 = -0.06 \text{ cm}$$

or

$$\frac{d}{\lambda_g} = -\frac{.06}{3.66}$$

$$= -0.0164 \qquad (5.3)$$

where the negative sign indicates that the minimum shifted toward the generator. If the data of (5.2) and (5.3) are plotted on a Smith chart, the result is the point marked (1) on the next page. The corresponding impedance is:

$$\frac{Z_{i1}}{Z_o} = .29 + j.10. \qquad (5.4)$$

This must be re-normalized to Z_1, the characteristic impedance inside the input window. It was found from the measurements described in Section 3 that $Z_o/Z_1 = 1.59$; thus let:

$$\frac{Z_{i2}}{Z_o} = 1.59 \frac{Z_{i1}}{Z_o} = 1.59(.29 + j.10)$$

$$= .461 + j.159. \qquad (5.5)$$

The point represented by (5.5) is plotted on the Smith chart (see the previous page) and is marked (2) .

The phase of the reflection coefficient corresponding to point 2 is 157.7°. From the measurements of Section 4, it was found that the teflon window advances the reflection coefficient by 56.3° toward the load without change in

SMITH CHART FORM 5301-7560-N	GENERAL RADIO COMPANY, WEST CONCORD, MASSACHUSETTS	DWG. NO. 27
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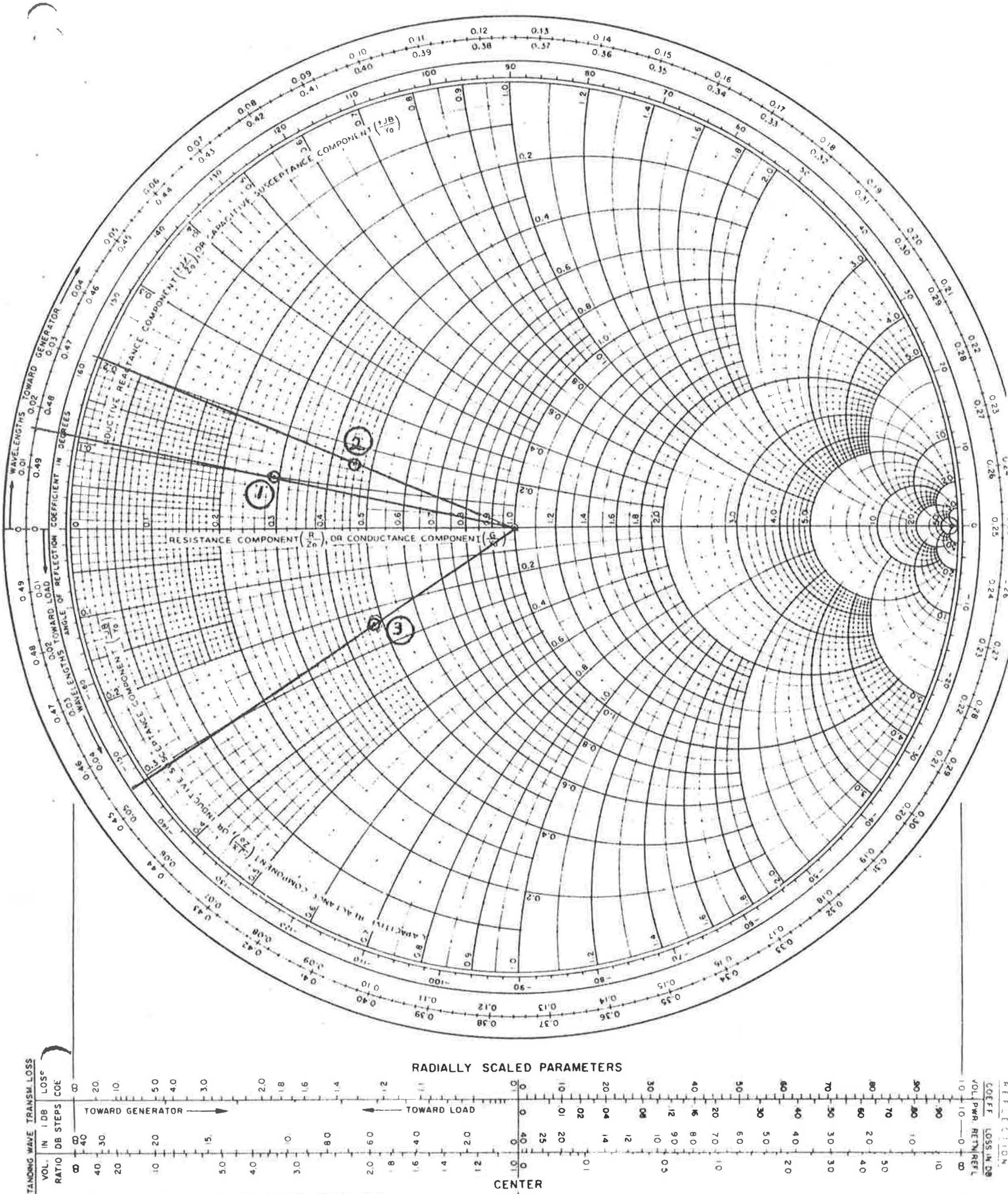
IMPEDANCE OR ADMITTANCE COORDINATES



magnitude; point 3 on the Smith chart is the result of this transformation, and the corresponding impedance is:

$$\frac{Z_{i3}}{Z_1} = 0.48 - j0.24$$

$$= 0.536\underline{/-26.6°} \ .$$

This is the open-circuit input impedance to the soil sample, but it must be renormalized to the wave impedance of empty guide, i.e.,

$$\frac{Z_{oc}}{Z_o} = \frac{Z_{i3}}{Z_1}\left(\frac{Z_1}{Z_o}\right) = \frac{1}{1.59}\ (.536\underline{/-26.6°}$$

$$= 0.337\underline{/-26.6°} \ . \qquad (5.6)$$

A similar procedure for the short circuit measurement produced:

$$\frac{Z_{sc}}{Z_o} = 0.289\underline{/42.4°} \ . \qquad (5.7)$$

With γ the propagation constant in the soil and β_o the phase constant in empty waveguide, one has from Appendix A:

$$\frac{\gamma}{\beta_o} = j\,\frac{1}{\sqrt{(Z_{sc}/Z_o)\ (Z_{oc}/Z_o)}} = j\,\frac{1}{\sqrt{(.289\underline{/42.4°})(.337\underline{/-26.6°}}}$$

$$= 3.20\underline{/81.2°} \qquad (5.8)$$

which is the value determined from both open and short circuit data.

For the calculation from short circuit data only, Appendix A gives:

$$\frac{\tanh \gamma d}{\gamma d} = -j \frac{1}{\beta_o d} \left(\frac{Z_{sc}}{Z_o}\right) .$$

By the measurements in Section 3, $\beta_o d = 1.097$ and with 5.7, one obtains:

$$\frac{\tanh \gamma d}{\gamma d} = -j \frac{1}{1.097} (.289\underline{/42.4^\circ}$$

$$= 0.263\underline{/-47.6^\circ} . \qquad (5.9)$$

As discussed in Appendix A, reference 1 gives charts of the multivalued function tanh $\gamma d/\gamma d$. The result of 5.8 is needed to select the proper chart--in this case, Chart III-A, page 93 of reference 1--which yields:

$$\gamma d = 4.0\underline{/81.5^\circ}$$

and since $\gamma/\beta_o = \gamma d/\beta_o d$, one finds:

$$\frac{\gamma}{\beta_o} = \frac{4.0\underline{/81.5^\circ}}{1.097}$$

$$= 3.64\underline{/81.5^\circ} \qquad (5.10)$$

from short circuit data only.

Finally, according to Appendix A,

$$\frac{\varepsilon_c}{\varepsilon_o} = \frac{\left(\frac{\lambda_g}{\lambda_c}\right)^2 - \left(\frac{\gamma}{\beta_o}\right)^2}{\left(\frac{\lambda_g}{\lambda_c}\right)^2 + 1}$$

where $\lambda_c = 4.57$ cm for the waveguide used, and $\lambda_g = 3.66$ cm. If γ/β_o from 5.10 is used for the calculation, one has:

$$\frac{\varepsilon_c}{\varepsilon_o} = \frac{(.801)^2 - (3.64\underline{/81.5^\circ})^2}{(.801)^2 + 1}$$

$$= 8.10 - j2.36. \qquad (5.11)$$

If the impedance Z_{sc} (normalized to the wave impedance Z_o of the TE_{10} waveguide mode) is measured, equation (A.2) can be solved for γd and the dielectric constant follows from an expression to be given subsequently. vonHippel gives charts (pp. 86-101) on which level curves of magnitude and phase of γd are plotted with the magnitude and phase of the right side of (A.2) as abscissa and ordinate, respectively. The main problem in using these curves is that γd is multiplied valued, so that one never knows a priori which branch of the function to use. This difficulty can be circumvented by a second impedance measurement in which the short of Figure A.1 is replaced by an open circuit; the input impedance Z_{oc} is then

$$\frac{Z_{oc}}{Z_o} = j \, \frac{\beta_o}{\gamma} \coth \gamma d. \qquad (A.3)$$

If (A.1) is now multiplied by (A.3), an expression for γ in terms of measured quantities follows:

$$\frac{\gamma}{\beta_o} = \pm \frac{1}{\sqrt{\left(\frac{Z_{sc}}{Z_o}\right)\left(\frac{Z_{oc}}{Z_o}\right)}} \; . \qquad (A.4)$$

The sign on the right of (A.4) is chosen such that γ lies in the first quadrant. Of course, if one wishes to go to the trouble of measuring both open and short circuit impedance for each data point, he can eliminate need for the charts altogether. The open circuit measurement, however, is

APPENDIX A.

INPUT IMPEDANCE METHOD OF MEASURING DIELECTRIC CONSTANT

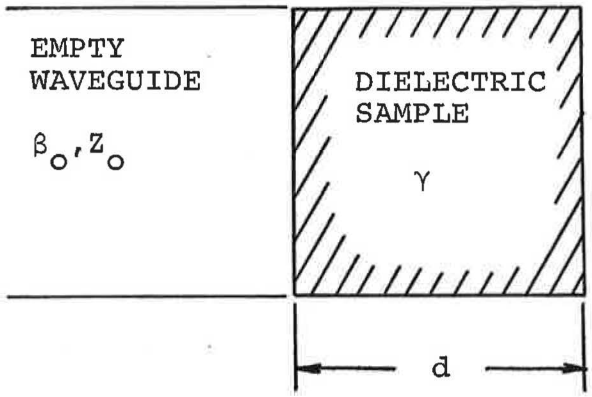


Figure A-1.

When the sample is terminated in a short circuit, the input impedance, Z_{sc}, at the air-dielectric interface is given by:

$$\frac{Z_{sc}}{Z_o} = j \frac{\beta_o}{\gamma} \tanh \alpha d \tag{A.1}$$

or

$$\frac{\tanh \gamma d}{\gamma d} = -j \frac{1}{\beta_o d} \frac{Z_{sc}}{Z_o} \tag{A.2}$$

where β_o = phase constant in empty waveguide, γ = propagation constant inside guide filled with sample whose dielectric constant is to be measured, and Z_o = wave impedance of TE mode of empty guide.

considerably more tedious than the short circuit measurement, and it is preferable to measure the open circuit impedance only as often as necessary to resolve ambiguity in the short circuit data. In this way, both effort and error buildup due to added measurements are reduced.

The complex dielectric constant may be derived in terms of the measured propagation constant as follows. From the wave equation for rectangular waveguide, it is known that:

$$\omega^2 \mu_o \varepsilon_c = k_c^2 - \gamma^2 \tag{A.5}$$

where ε_c = complex dielectric constant of material filling the guide, k_c = cutoff wave number of the guide, and the material inside the guide is taken to have the free space permeability μ_o. In empty guide, ε_c is to be replaced by the free space permitivity ε_o and $\gamma = j\beta_1$ so that

$$\omega^2 \mu_o \varepsilon_o = k_c^2 + \beta_o^2 \; . \tag{A.6}$$

If (A.5) is divided by (A.6), the result can be put into the form:

$$\frac{\varepsilon_c}{\varepsilon_o} = \frac{\left(\frac{k_c}{\beta_o}\right)^2 - \left(\frac{\gamma}{\beta_o}\right)^2}{\left(\frac{k_c}{\beta_o}\right)^2 + 1}$$

or since $k_c/\beta_o = \lambda_g/\lambda_c$, where λ_g = guide wavelength of the TE_{10} mode in empty guide and λ_c = cutoff wavelength, in the guide,

$$\frac{\varepsilon_c}{\varepsilon_o} = \frac{\left(\frac{\lambda_g}{\lambda_c}\right)^2 - \left(\frac{\gamma}{\beta_o}\right)^2}{\left(\frac{\lambda_g}{\lambda_c}\right)^2 + 1} \tag{A.7}$$

APPENDIX B.

ADAPTATION OF IMPEDANCE METHOD FOR SOIL MEASUREMENTS

Need to confine the soil obviates use of the simple two-layer medium depicted in Figure A-1. Confinement may be accomplished by interpassing a dielectric window of thickness S and phase constant β_1 between the soil sample and the empty waveguide. The fiscal geometry for the short circuit test may be then represented by the three-layer medium shown in Figure B-1.

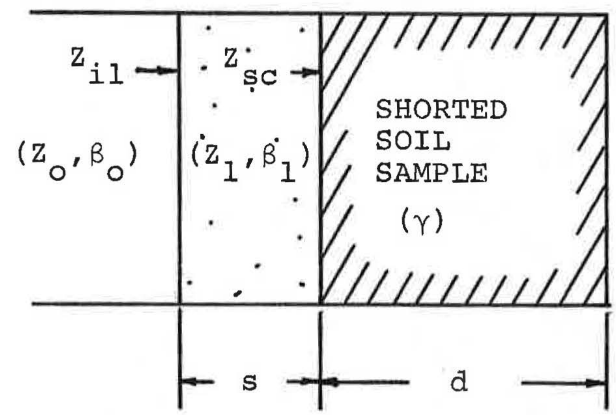


Figure B-1.

Impedance measurement by slotted line techniques now yields the input impedance normalized to the wave impedance of the empty guide at the air-dielectric window interface. Data reduction procedures can, however, be extended simply (if through increased tedium) to yield the impedance of the soil sample at the soil-window interface normalized to the characteristic impedance of <u>empty guide</u> as needed for the equations of Appendix A. This procedure is, briefly, as follows:

1. If Z_{i1} is the input impedance at the air-dielectric interface (see Figure B-1), then Z_{i1}/Z_o is known by measurement, and is plotted on a Smith chart.
2. The impedance Z_{i1}/Z_o is re-normalized to the characteristic impedance inside the dielectric window and it, too, is plotted on a Smith chart.
3. The reflection coefficient just inside the dielectric (at the air-dielectric boundary) is known from step 2. This reflection coefficient is advanced by $2\beta_1 S$ in phase without change in magnitude and the corresponding impedance is read from the Smith chart.
4. The impedance which results from step 3 is the input impedance to the shorted soil sample normalized to the wave impedance inside the dielectric window. This impedance is now re-normalized to the empty waveguide characteristic impedance.
5. The normalized impedance from step 4 is the quantity Z_{sc}/Z_o to be used in equation (A.2) or (A.4).

Implicit in step 3 is the assumption that losses in the window can be neglected; thus a thin low loss dielectric should be used for the window. The equation (see vonHippel, equation (2.15 on any reference on waveguides):

$$\frac{Z_o}{Z_1} = \frac{\beta_1}{\beta_o} \tag{B.1}$$

is needed in steps 2 and 4; thus in step 2,

$$\frac{Z_{il}}{Z_1} = \left(\frac{Z_{il}}{Z_o}\right) \left(\frac{\beta_1}{\beta_o}\right) \tag{B.2}$$

and in step 4,

$$\frac{Z_{sc}}{Z_o} = \left(\frac{Z_{sc}}{Z_1}\right) \left(\frac{\beta_o}{\beta_1}\right) . \tag{B.3}$$

The open-circuit test actually involves a five-layer medium which gives rise to a rather formidable task in data reduction. Fortunately, however, judicius adjustment of equipment allows the problem to be reduced to that of three layers. Here, in addition to the input window, the sample is backed by an output window followed by an air filled movable short. The arrangement is represented in Figure B-2.

From either transmission line theory or impedance transformations on the Smith chart, it follows that with a lossless output window, the short can be adjusted to place an open circuit at the sample-output window interface. This is readily accomplished as follows. With a fixed short on the output end of a slotted line, the position of the minimum in the standing wave is noted. The fixed short is then replaced by the output window backed by the moveable short, and the moveable short is adjusted to place the minimum at the same position as with the fixed short. When output window and moveable short are re-attached to the sample, an open circuit appears at the plane A-A' of Figure B-2. Input impedance may now be measured, and the steps 1-5 repeated as the short-circuit test to obtain Z_{oc}/Z_o. All quantities needed for the computations described in Appendix A are now available.

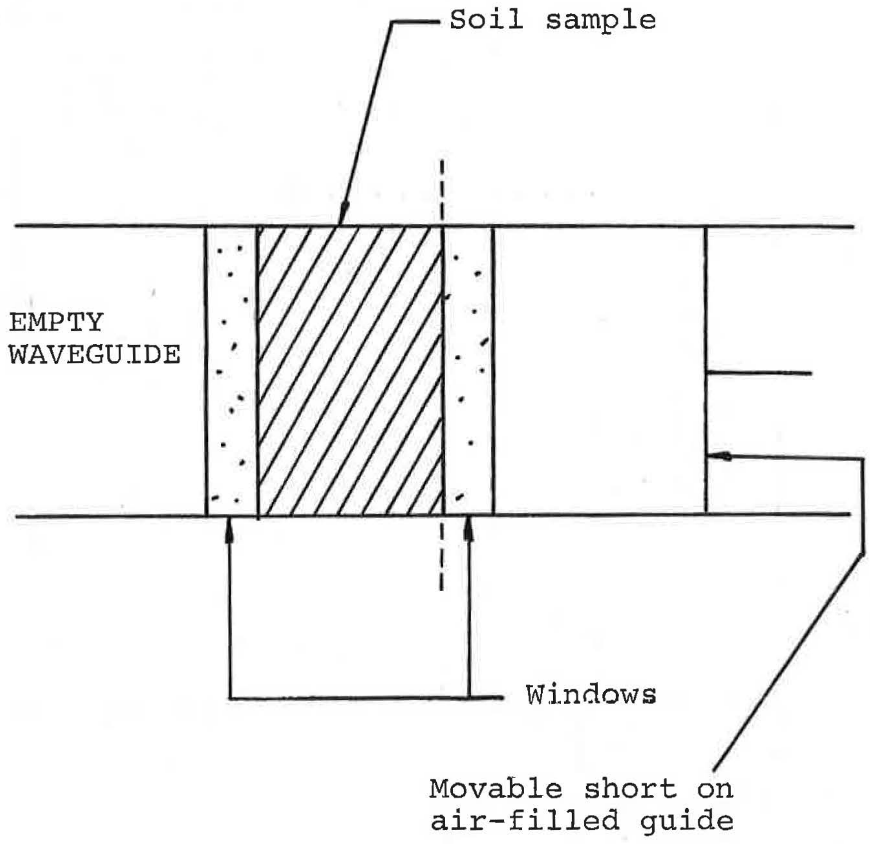


Figure B-2.

APPENDIX C.

THE DOUBLE MINIMUM METHOD

The double-minimum method is a standard method of measuring moderate to high VSWR and is described in most references on microwave measuremtns. That description is repeated here for the convenience of the reader. The double minimum method has the advantage that the sampling probe is restricted to use in regions of low power (except for small VSWR) thus minimizing probe interference effects. This is very important if accurate measurements are required.

In the double minimum method, a small amount of power coupled from the line by a probe and a crystal detector is measured at the minimum of the standing wave pattern. The probe is then moved to each side of the minimum to a point at which the coupled power is 3 dB above that at the minimum, and the distance Δx between the 3 dB points is measured. If S denotes the voltage standing wave rating then S is given by:

$$\frac{1}{S} = \frac{\sin \theta}{\sqrt{1+\sin^2 \theta}} \qquad \text{(C.1)}$$

where

$$\theta = \pi \frac{\Delta x}{\lambda_g} \quad . \qquad \text{(C.2)}$$

When S is large, θ is small and to very good approximation, equation (C.1) reduces:

$$\frac{1}{S} = \pi \frac{\Delta x}{\lambda_g} \quad . \tag{C.3}$$

The full accuracy afforded by equation (C.1) may be retained along with the convenience of equation (C.3) through use of a table given by vonHippel (see vonHipple, p. 86). The table, repeated below for convenience, gives correction terms which are to be subtracted from values calculated from C.3, and may be used for storying wave ratios ranging from 2 to 10.

TABLE C-1. SUBTRACTION TERM TO OBTAIN CORRECT E_{min}/E_{max}

$\frac{\pi \Delta x}{\lambda}$	Subtraction	$\frac{\pi \Delta x}{\lambda}$	Subtraction
0.075	0.0000	0.375	0.0308
0.100	0.0002	0.400	0.0367
0.125	0.0010	0.425	0.0436
0.150	0.0019	0.450	0.0506
0.175	0.0032	0.475	0.0585
0.200	0.0050	0.500	0.0670
0.225	0.0072	0.525	0.0762
0.250	0.0098	0.550	0.0862
0.275	0.0130	0.575	0.0969
0.300	0.0164	0.600	0.1082
0.325	0.0207	0.625	0.1200
0.350	0.0255		

REFERENCES

1. vonHippel, A. (Editor), Dielectric Materials and Applications, Wiley, New York, 1954.

2. Ginzton, Edward L., Microwave Measurements, McGraw-Hill, New York, 1957.