UNIVERSIDADE CATÓLICA PORTUGUESA

Frequency-domain information for active multi-asset portfolio management

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Católica Porto Business School
April 2022

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Final Assignment in the modality of Dissertation presented to Universidade Católica Portuguesa to obtain the degree of Master in Finance

by

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Católica Porto Business School

April 2022

Acknowledgements

I would like to thank my whole family and friends for their unconditional support through life and during the writing of this dissertation.

To my supervisors, Professor Gonçalo Faria and Professor Fabio Verona, for the encouragement, patience, availability, and well-being. It truly was an honor to get to deal with you and follow your work.

Resumo

Nesta dissertação avaliamos as vantagens da utilização de informação no domínio da frequência no contexto de uma carteira de ações, obrigações e *commodities* gerida ativamente. O estudo da informação no domínio da frequência na gestão ativa de ativos é muito incipiente na literatura: há apenas um artigo para um conjunto de investimentos de ações e obrigações (Faria e Verona, 2020). Baseando-nos nesta descoberta e, aplicando a mesma metodologia, estendemos o estudo aos mercados de *commodities* e avaliamos os possíveis ganhos económicos da utilização de informação no domínio da frequência no contexto de uma carteira de ações, obrigações e *commodities* gerida ativamente.

Concluímos que o uso de informação no domínio da frequência na gestão ativa de uma carteira de ações, obrigações e *commodities* é economicamente benéfica, e que *commodities* têm um elevado poder de diversificação e de proporcionar retornos equivalentes a ações.

Palavras-chave: *equity risk premium, bond risk premium, commodity returns*, previsibilidade, análise multi-resolução, carteiras de vários ativos, gestão ativa de carteiras

Abstract

In this thesis we evaluate the advantages of using frequency-domain information in the context of an actively managed portfolio exposed to equity, bonds, and commodities. Studying frequency-domain information in active asset management is very incipient in the literature: there is only one paper for an investment set of equity and bonds (Faria and Verona, 2020). Based on this finding, and applying the same methodology, we extend the work to commodity markets and study if there are eventual economic gains from the use of frequency-domain information in the context of an actively managed portfolio exposed to equity, bonds, and commodities.

We conclude that using frequency-domain information in active multi-asset portfolio management is beneficial and that commodities have a high diversification power.

Keywords: equity risk premium, bond risk premium, commodity returns, predictability, multiresolution analysis, multi-asset portfolios, active portfolio management

Word count excluding abstract, references, tables and figures and appendixes: 7285
Word count total: 10642

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Introduction

The main research question to be addressed is this study is: can frequency-domain information be used to improve the performance of an actively managed portfolio exposed to equity, bonds and commodities?

Active portfolio managers are forecasters. An active portfolio management success is deeply rooted on good forecasts for the assets in case. Over the last two decades, the explosion of index tracking industry mainly reflects the difficulty of active portfolio management consistently outperforming a given benchmark index. Therefore, it is fundamental for a manager to be able to identify reliable predictors and the best forecasting methods.

During decades, economic researchers made efforts to study the predictability of assets, mostly equity and fixed income returns. Recently, academicians began to raise awareness about the predictability of commodity prices and the behavior of commodity futures markets. Historically, investing in commodity futures appears to be as rewarding as investing in equities (e.g., Erb and Harvey (2006)). Commodity futures are used by investors to hedge against inflation (Bodie (1983); Edwards and Park (1996); Bjornson and Carter (1997)). Also, commodity future investments have had low correlations with equity and fixed income. Since commodity prices are among the direct drivers of inflation, commodities are often considered one of the key real assets that can protect against rising inflation. While equity and fixed income returns are negatively correlated with inflation, commodity returns have a considerable positive correlation with both expected and unexpected inflation (Kang, 2012).

These reasons make that commodity futures are seen as good financial instruments for strategic asset allocation (Bauer, Molenaar, Steenkamp and Vrugt (2004); Wang and Yu (2004); Erb and Harvey (2006) and (X. Gao and Nardari (2018)).

Using out-of-sample exercises are of a major importance for real time traders. A good active asset management relies on good forecasts for the asset classes of interest. Therefore, identifying predictors and the best forecasting methods is essential. Literature on the out-of-sample predictability is vast and is dominated by time-series analysis (e.g., Beveridge and Nelson (1981)). However, frequency-domain techniques, like Fourier transforms, are comparatively new procedures that are of a major relevance. This frequency-domain technique extracts information that is hidden in a time-series analysis that is crucial - the best frequencies to be used as predictors by testing the OOS predictive capacity of these new predictors. In recent studies, Ian Dew-Becker and Stefano Giglio (2016) stated that "frequency domain is the natural setting in which to analyze dynamics".

In the context of forecasting equity and bond returns, Faria and Verona (2018, 2020a, 2020b) and Bandi et al. (2019) introduce models where the predictors for equity and bond returns are frequency-aggregated components and predictability is frequency specific.

In this thesis, we follow Faria and Verona's (2020b) procedure and use wavelet filtering methods to extract cycles in the initial dataset of Goyal and Welch's (2008) predictors, to decompose them into more predictors of the bond risk premium (BRP), commodity returns (CR) and equity risk premium (ERP).

The expected contributions of this project are twofold. First is to extend the study of frequency-domain forecasting to commodity markets. Second, is to evaluate the economic significance of eventual forecasting gains in different commodity returns in the context of an actively managed multi-asset (equity, bonds, commodities) portfolio. The actively managed portfolio's objective is to beat a given benchmark. Up until today, the literature is focused on portfolios of equity and bonds. We adopt the perspective of a power utility investor, where the BRP, CR and ERP forecasts are seen as the investor's active views on the bond, commodity, and stock markets. We use a mean-variance portfolio optimization framework and an allocation of 48% to stocks, 32% to bonds and 20% to commodities as a benchmark. Our main finding is that using frequency-domain information leads to better portfolio performance when compared with the original time series of the predictors. This finding is robust towards alternative benchmark portfolios and different portfolio constraints.

The thesis is organized as follows. We review the existing literature on the main topics in Chapter 1. Chapter 2 presents the data and methodology used to construct the predictive models. In Chapter 3 we report the out-of-sample forecasting results and the performance of the active management strategy. Robustness tests results are presented in Chapter 4. At last, we conclude and describe the main findings in Chapter 5.

Chapter 1

1. Literature Review

In finance, predicting stock returns has a long tradition. In fact, the stock market risk premium is the most important tool for capturing predictable variation of the stock portfolios, while premiums associated with interest rate risks capture predictability of bond returns. However, predicting commodity future returns was mainly ignored up until early 2000s.

This thesis builds on three strands of literature, which are reviewed in this section. In specific, in section 1.1 we review literature on forecasting equity, fixed income and commodity returns, in section 1.2 we review the OOS forecast, in section 1.3 we review literature on frequency-domain and Wavelet analysis and in section 1.4 we review literature on Active Portfolio Management.

1.1 Forecasting equity, fixed income and commodity returns

Forecasting the equity risk premium (ERP) is of major importance when comes to asset allocation decisions. For that reason, this topic justifies the immense attention in finance research.

In 1977, Fama and Schwert (1977) studied the quality of a diversity of assets as hedges against the expected and unexpected components of the inflation rate, highlighting that private residential real estate was a total hedge against both expected and unexpected inflation and found that the common stocks were negatively related to expected component of the interest rate.

In 1984, Rozeff (1984) investigated the level of significancy of dividend yields in the prediction of equity risk premiums. The conclusion was that today's dividend yield gives a clue to future return predictability.

Following the dividend yield, Fama and French (1989) investigated if there was a variable that could predict simultaneously the equity risk premium and the corporate bonds' risk premium. In fact, they both moved together and could be forecasted using the dividend yield. In addition, they also noticed that long-term business cycles were associated with big movements in the dividend yield.

Ferson and Harvey (1991) claim that most of the predictability is explained by an asset pricing model that focuses on risk. The two most acknowledged ideas up until this date stated that market inefficiencies and changes in the required return were the source of predictability. In their paper, Ferson and Harvey found that at a portfolio level, the time variation in the expected risk premiums is the main source of predictability.

The efforts to develop and test new predictors continued. As it has been a target for financial economics, Møller and Rangvid (2015) researched for connection between macroeconomics and financial markets. They showed that the influence of macroeconomic growth on expected returns of risky financial assets depend on the time of the year. A pattern was discovered across many different asset classes: macroeconomic growth in the fourth quarter of the year strongly influences expected returns on risk financial assets, whereas during the rest of the year does not. Investors are more likely to make decisions regarding their investments at the end of the year (Jagannathan and Wang (2007)). Therefore, Møller and Rangvid used the relation between economic growth, surplus consumption ratio and expected returns as evidence for the infrequent portfolio adjustment hypothesis.

In contrast, academic researchers paid little attention to the forecasting ability of technical indicators. Rapach and Zhou (2013) used technical indicators as complements of macroeconomic variables to provide additional information over a business-cycle. They found that "technical indicators better detect the typical decline in the equity risk premium near business-cycle peaks, while macroeconomic variables more readily pick up the typical rise in the equity risk premium near cyclical troughs". Combining these two notably improve equity risk premium forecasts.

Since bonds play an important role in investor's portfolios, understanding the risk-return dynamics is of major importance. The in-sample predictability of treasury bonds excess returns was shown by Fama and Bliss (1987) with forward spread variable; by Campbell and Shiller (1991) with yield spread; and by Cochrane and Piazzesi (2005) with a linear combination of macroeconomic variables.

Commodities were left aside by the investor community for a long time, but recently emerged to potentially enhance portfolio diversification, offer protection against inflation, and provide equity-like returns.

Gorton and Rouwenhorst (2006) pointed some characteristics that may suggest that commodities may improve diversification of a traditional portfolio. They discovered that commodities tend to perform differently than other asset classes: in recession period, equity returns dropped significantly while commodities performed well.

Commodity returns can be equity-like if the risk premium is large enough. Keynes (1930) developed the theory of normal backwardation - the futures price should be less than the expected future spot price. If today's futures price is below the future spot price, then as the futures price converges towards the spot price at maturity, excess returns should be positive. As so, investors that go long commodity futures should receive a positive excess rate of return. This normal backwardation theory argues that commodity futures offer companies the ability to hedge their commodity price exposure. Over the period 1986 to 1994, De Roon, Nijman and Veld (2000) analyze twenty futures markets and find that hedging pressure plays an important role in explaining futures returns.

The in-sample predictability of commodity futures has been addressed in the literature. Jensen, Johnson and Mercer (2002) stated that a measure of the U.S. monetary policy predicts the performance and role of commodity futures in mean-variance efficient portfolios. Acharya, Lochstoer and Ramadorai (2009) proved that the default risk of commodity producers is a determinant of their hedging demand in futures markets and risk premium.

1.2 Out-of-sample forecast

Several studies discuss the in-sample predictability of stock returns using predictors such as the treasury bill rate, dividend yield, dividend–price ratio, term spread, equity market volatility or the consumption–wealth ratio (see e.g. Ferson and Harvey 1991; Lettau and Ludvigson, 2001; Cochrane, 2008).

However, for real time trading, the use of out-of-sample exercises to test predictive models is essential. Hence, over the past decades, empirical researchers made efforts to find the best forecasting exercise. Goyal and Welch (2003) showed that the dividend ratios could not outperform the unconditional mean out-of-sample. They then concluded that "good in-sample performance is not a guarantee of out-of-sample performance in the equity premium prediction context". Also, Goyal and Welch (2008) symbolize the turning point in forecasting literature. They tested the out-of-sample quality of the literature's best in-sample predictors. They discovered that: 1) the academic predicting models have failed both in-sample and out-of-sample; 2) the models are unstable: the out-of-sample performance was surprisingly mediocre; and most important, 3) "most models not only cannot beat the unconditional benchmark, but also outright underperform it". Campbell and Thompson (2008) argue that imposing restrictions to the out-of-sample predicting models would make them perform better out-of-sample than the historical average return forecast. Although the models' predicting power could be small, is economically meaningful.

Since the out-of-sample performance is poor, researchers explored two different paths to improve the forecastability. Firstly, focused their work on developing and testing new predictors. Regarding predictability of stock returns, Bollerslev et al. (2009) checked the use of the variance risk premium, Cooper and Priestley (2009, 2013) use the output gap and the world business cycle, Rapach et al. (2013) assessed the influence of lagged US market returns for the out-of-sample predictability of stock returns of other industrialized countries, Li et al. (2013) study the aggregate implied cost of capital, Neely et al. (2014) consider the significance of technical indicators to serve as complementary predictors to the traditional set of variables, and recently, Faria and Verona (2019) test the use of frequency-decomposed variables as new predictors of equity returns. Regarding bonds, Thornton and Valente (2012) found that forward spread predictors do not lead to higher out-of-sample Sharpe ratios or higher economic utility compared with expectation hypothesis (EH) approaches. Later, Timmermann, Gargano and Pettenuzzo (2017) showed that by comprising the forward spread, the combination of forward rates and macro factors generates gains in out-of-sample forecasting accuracy when compared with EH models. The gains translated into higher risk-adjusted portfolio returns. Regarding the commodity market, the literature on OOS forecasting is recent and not as vast as the literature on stock returns. Furthermore, most of the literature on forecasting commodities is on spot prices and not future prices. For instance, Wang, Liu and Wu (2019) used technical indicators as predictors for commodity prices since their performance is not affected by data mining problems or time changes. Gargano and Timmermann (2012) demonstrate that inflation rate has no predicting power at the monthly horizon, whereas growth in industrial production, money supply growth, and the change in the unemployment rate have predictive power over returns at the annual horizon. Regarding commodity future prices, Cotter, Eyiah-Donkor and Potì (2020) showed that, when studying the relation between commodity futures excess returns and the individual predictors, combination forecasts perform both statistically and economically better when comparing to the historical average forecast. Most recently, Guidolin and Pedio (2021) demonstrate that including commodity-specific factors like basis, hedging pressure and momentum can generate economic value. However, statistically, commodity-specific factors carry limited predictive power for commodity futures returns.

The second path aims to improve existing forecasting methods. Regarding new methodologies to forecast the equity risk premium, Ludvigson and Ng (2007) and Kelly and Pruitt (2013) proposed dynamic factor analysis. Ferreira and Santa-Clara (2011) decomposed the stock market return into three components (sum-of-parts (SOP) method) and extracted each component's time series characteristics by forecasting each one individually. Recently, Faria and Verona (2018a and 2018b) suggested applying wavelet decomposition techniques: 1) to the SOP method; 2) to extract cycles in the term spread to analyze their role for predicting the equity risk premium. In contrast, the literature on improving the out-of-sample predictability of futures commodity prices through new methodologies is scarce. Kwas and Rubaszek (2021), on forecasting nominal commodity prices, found that futures-based forecasts are a better method when compared to the random walk (benchmark). Lutzenberger (2014), following the research from Rapach et al. (2010), used combinations of individual models to OOS forecast returns on commodity futures and concluded that there are economically significant gains.

1.3 Frequency-domain and wavelet analysis

Time series information is the most frequent method for making predictive regressions, and historically, the literature on asset return forecast is heavily rooted on it.

Although time-series gives a perception of the signal over time, it gives no clue about the frequencies that these signal changes at any moment in time. In contrast, frequency domain is an analysis of signals, in reference to frequency, instead of time. Consequently, when examining a time-series we have a clear idea in the time domain, yet a lot of uncertainty in the frequency domain.

Dew-Becker and Giglio (2016) stated that a frequency domain analysis technique, like Fourier Transforms, are new procedures that can be used as a complementary tool in time-series analysis. However, the Fourier Transforms are not ideal for non-stationary signals, i.e., that have frequency variation throughout. Then, a new technique was developed, the Wavelet Transform. This technique can divide a non-stationary signal into its stationary parts, using Fourier Transforms to capture all frequencies exposed.

The pioneer work of Ramsey and Lampart (1998a,b) draws on wavelets to study the relationship between various macroeconomic variables. Crowley

(2007), in its Guide to Wavelets for Economists[1], provide a comprehensive review of what wavelets could offer to finance and economics. However, “Wavelet analysis, although used extensively in disciplines such as signal processing, engineering, medical sciences, physics and astronomy, has not fully entered the economics discipline yet”. Also, Ramsey (2002) stated that “wavelets are treated as a “lens” that enables the researcher to explore relationships that previously were unobservable”, i.e, introducing this new statistical procedure can improve the results of the economic and finance research.

Ramsey and Lampart (1998) used wavelets to study the relationship between macroeconomic variables: income versus consumption and money supply versus income. Kim and In (2005) stated a positive relationship between nominal stock returns and inflation. Gençay et al. (2005) proposed a new approach based on wavelets to estimate systematic risk.

Gallegati (2007) studies the relationship between stock market returns and economic activity over different time scales using signal decomposition techniques based on wavelet analysis and found that stock market returns tend to lead the level of economic activity but only on scales corresponding to periods of 16 months and longer (lowest frequencies). Aguiar-Conraria et al. (2012) used wavelet analysis to study business cycle synchronization across European countries.

Adding to this, the studies of Rua (2011) and Rua (2017) suggest a wavelet-based multiscale principal component analysis to predict GDP growth and inflation. The result was that are improvements in short-run predictions when this procedure is complemented with factor-augmented models.

[1] See Crowley (2007) for more details.

Later, Faria and Verona (2020), using wavelets, isolated the best frequencies of different predictors and used them in the context of active portfolio management.

1.4 Active Portfolio Management

The literature on active portfolio management is little.

The studies of Almadi, Rapach and Suri (2014) on tracking the best timing for portfolio rebalancing found that out-of-sample forecasts can be helpful. If the investors can rebalance the portfolio on right timing, returns can be higher. However, De Miguel et. al. (2019) state that obtaining higher returns from optimal portfolios are hardly reached out-of-sample. In fact, historically, active portfolio management struggles to consistently beat a given benchmark.

Da Silva et. al. (2009) applied the Black-Litterman framework in active investment management and found that it leads to a risker portfolio and the risk-adjusted performance is lower.[2]

Recently, Faria and Verona (2020b) demonstrate that using information from different frequencies of different predictors improves the forecasts of bond and equity returns; and when used in the context of active portfolio management, these forecasts lead to a superior performance of an actively managed equity-bond portfolio.

[2] See Black, F., and R. Litterman (1992): Global Portfolio Optimization for details.

Chapter 2

2. Data and Methodology

The expected contribution of this dissertation is to extend the study of frequency-domain forecasting to commodity markets and evaluate the economic significance of eventual forecasting gains in different commodity returns in the context of an actively managed multi-asset (equity, bonds, commodities) portfolio.

2.1 Data Description

Our sample period is from January 1972 to June 2010[3], with a monthly sample frequency, where all prices and returns are in U.S. dollars. Bond risk premium (BRP) and equity risk premium (ERP) of month t are determined as the difference between the return on the 10-year US Treasury bond and the return on the S&P500 index in month t, respectively, and the one-month T-bill known at the beginning of month t (lagged-risk free rate).

The equally weighted portfolio of 27 commodity futures is constructed by Asness et al. (2013) and the data is obtained from Tobias J. Moskowitz's website. The portfolio covers aluminium, copper, nickel, zinc, lead, tin, brent crude oil, gas oil, live cattle, feeder cattle, lean hogs, corn, soybeans, soy meal, soy oil,

[3] The sample period is only until 2010 due to data availability.

wheat, WTI crude, RBOB gasoline, heating oil, natural gas, gold, silver, cotton, coffee, cocoa, sugar, and platinum.[4] The futures returns are calculated, as Asness et al. (2013) explained, by computing the “daily excess return of the most liquid futures contract every day, which is typically, the nearest-or next nearest-to-delivery contract”. Next, the daily returns are compounded to a total return index, and the monthly returns are computed from this index. Following the literature, and to smooth the series, we use the log of these monthly returns as Commodity Returns (CR).

Generally, commodities have low or negative correlation with traditional asset classes over the long-term and can act as a portfolio diversifier. However, during periods of global economic downturn such as in the early 1980s, early 1990s and late 2000s, commodities’ correlation with other asset classes–especially equities–tended to sharply increase before reverting to relatively low levels, as seen in Graph 1.

[4] Data on Aluminum, Copper, Nickel, Zinc, Lead, and Tin are from the London Metal Exchange (LME). Brent Crude and Gas Oil are from the Intercontinental Exchange (ICE). Live Cattle, Feeder Cattle, and Lean Hogs are from the Chicago Mercantile Exchange (CME). Corn, Soybeans, Soy Meal, Soy Oil, and Wheat are from the Chicago Board of Trade (CBOT). WTI Crude, RBOB Gasoline, Heating Oil, and Natural Gas are from the New York Mercantile Exchange (NYMEX). Gold and Silver are from the New York Commodities Exchange (COMEX). Cotton, Coffee, Cocoa, and Sugar are from New York Board of Trade (NYBOT), and Platinum data are from the Tokyo Commodity Exchange (TOCOM).

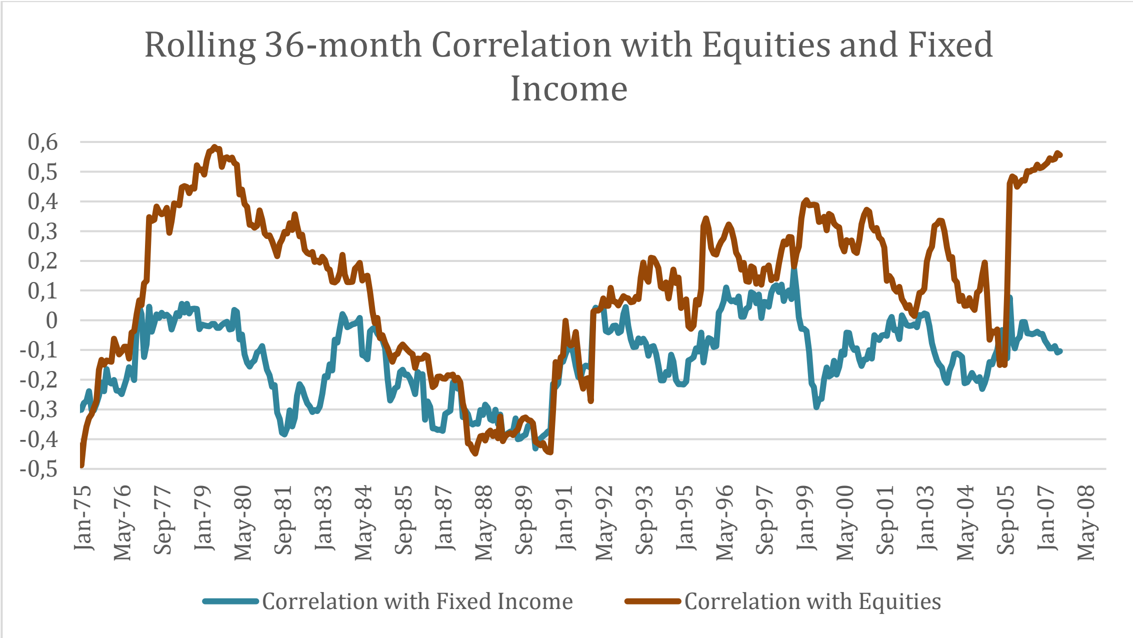

Figure 1 - Rolling 36-month correlation with equities and fixed income

The rolling correlation is computed using the correlation of two time series on a rolling window (36 months). Hence, we compute the correlation between CR and ERP, and CR and BRP. This type of correlation allows us to visualize the correlation between two time series over time.

In fact, the commodity correlation with equities and fixed income is very volatile. Therefore, the forecasting exercise is challenging. Then, the better we capture these changes in correlation dynamics, the better will be the dynamic asset allocation. This relationship between the asset classes is not new, however, the most important and focal point of this thesis is to use a method (frequency-domain forecasting) to assess the possible economic gains of introducing commodities in an actively managed portfolio (market-timing adaptation).

Regarding predictors, we use twelve variables taken from Goyal and Welch (2008: log dividend-price ratio (DP), log dividend yield (DY), log earnings-price ratio (EP), excess stock return volatility (RVOL), book-to-market ratio (BM), net equity expansion (NTIS), long-term bond yield (LTY), long-term bond return

(LTR), term spread (TMS), default yield spread (DFY), default return spread (DFR), and lagged inflation rate (INFL). These predictors are briefly described in Appendix 1. Table 1 reports the summary statistics for BRP, ERP, CR, and the predictors. Figure 1 provides their time series.

2.2 Methodology

Regarding our methodology, there are three key subjects: the frequency domain analysis and wavelet multiresolution analysis (sub-section 2.2.1), the out-of-sample (OOS) procedure (sub-section 2.2.2), and the asset allocation framework (sub-section 2.2.3).

2.2.1 Frequency-domain analysis and Wavelet Transform

The term "wavelets" mean small waves, as they have finite length (compactly supported) and oscillatory behavior. In economics and finance, its main ability is to deal with both stationary and non-stationary data.

The Wavelet Transform analyzes the signal at different frequencies with different resolutions using a multiresolution analysis (MRA). The multiresolution analysis approach, by using short windows at high frequencies and long windows at low frequencies, can overcome the resolution problem as it adaptively divides the time-frequency plane. This way, both time and frequency resolutions can vary in the time-frequency spectrum and the original characteristics of the time series is kept.

There are two types of Wavelet Transforms, the Continuous Wavelet Transform (CWT,) and the Discrete Wavelet Transform (DWT), differing on how the wavelets are scaled and shifted, and on the signal: CWT assumes continuous signal and DWT assumes a signal consisting of observations sampled at evenly spaced points in time (Crowley, 2008). We use DWT in this dissertation.

In Appendix 2, we describe the wavelet transform. Equation (**Erro! A origem da referência não foi encontrada.**) shows that the original series y_t, exclusively defined in the time domain, can be decomposed in different time series components, each defined in the time domain and capturing the fluctuation of the original time series in a specific frequency band. For small j, the j wavelet detail components represent the higher frequency characteristics of the time series (i.e., its short-term dynamics). As j increases, the j wavelet detail components represent lower frequencies movements of the series. Lastly, the wavelet smooth component (${y_t}^{S_J}$) captures the lowest frequency dynamics (i.e., its long-term behavior or trend).

The classic DWT has limitations, for example, is non-shift variant and is restricted to sample size (dyadic length requirements). Then, we use the Haar wavelet filter [5] and the maximal-overlap discrete Wavelet Transform Multiresolution Analysis (MODWT MRA)[6] to do the wavelet decomposition analysis. In our analysis, given the size of the sample, we apply a J = 6 levels MRA[7] for each of the original predictors. Hence, the wavelet decomposition provides seven time-frequency series: six wavelet details (${y_t}^{D_1}$ to ${y_t}^{D_6}$) and a wavelet smooth ${y_t}^{S_6}$.

Since we are dealing with monthly data, the first wavelet detail component ${y_t}^{D_1}$ captures oscillations between 2 and 4 months, while the other ${y_t}^{D_2}$, ${y_t}^{D_3}$,

[5] "The Haar filter makes a neat connection to temporal aggregation as the wavelet coefficients are simply differences of moving averages" (see Faria and Verona (2020) and Ortu et. al (2016))

[6] As Percival and Walden (2000) note, the MODWT is also commonly referred to by various names in the wavelet literature). More details on the MODWT MRA can be found in Percival and Walden (2020) and Crowley (2008)

[7] Regarding the choice of J, the number of observations dictates the maximum number of frequency bands that can be used. In our case, t_0= 204 is the number of observations in the in-sample period, so J is such that $J \leq log_2\, t_0 \simeq 7$.

$y_t{}^{D_4}$, $y_t{}^{D_5}$, $y_t{}^{D_6}$ capture oscillations with periods of 4-8, 8-16, 16-32, 32-64 and 64-128 months, respectively. Ultimately, the smooth component $y_t{}^{S_6}$ captures oscillations with a period longer than 128 months.

As an example, figure 2 plots the time series of the term spread (one of the predictors used) and its MODWT MRA decomposition (seven time-frequency series components). We can observe that then lower the frequency, the smoother the resulting filtered time-series. However, the time-series dynamics vary a lot depending on the components and thus, only some are good ERP, BRP and CR predictors. For example, regarding ERP, Faria and Verona (2019) showed that the low-frequency component of the term spread (TMS^{D_7}) is an excellent OOS predictor, whereas the other frequency components are not.[8]

2.2.2 Out-of-Sample Forecasts

The out-of-sample exercise is done using the information available at moment t in time, to predict the value of a certain variable at $t+1$. The OOS forecasts of the BRP, CR and ERP are generated using a sequence of expanding windows. We use an initial in-sample (IS) period from 1972:01 (January 1972) to 1989:12. Then, the sample is increased by one observation and a new one-step-ahead OOS forecast is generated. We repeat this process until the end of the sample, getting a sequence of 246 one-step-ahead OOS forecasts. The full OOS period is from 1990:01 to 2010:06.

As the MODWT MRA is a two-sided filter, we use the original predictors and compute their frequency components at each iteration of the OOS forecasting process using current and past data from the sample. The procedure

[8] See Faria and Verona (2019) for more details.

is important to avoid look-ahead bias. In terms of boundary treatment, we use the reflection rule, i.e., at the boundary, the original time series is expanded symmetrically (the time series doubles in size) before computing the MODWT MRA.

The CR predictive model is

$$CR_{t+1} = \alpha + \beta \boldsymbol{X}_t + \varepsilon_{t+1} \tag{1}$$

where $\boldsymbol{X}$ is a vector of predictors and the one-step-ahead OOS forecast of the commodity return, $\widehat{CR}_{t+1}$, is given by:

$$\widehat{CR}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t \boldsymbol{X}_t \tag{2}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the OLS estimates of parameter α and vector of parameters β, respectively.

The same predictive regression model is used to forecast ERP and BRP.

We consider two types of predictive regressions when running model (1)-(2):

- $\boldsymbol{X}$ including all original predictors, we run multi-variate regressions using various original predictors. This model is designated *multi_ts;*
- $\boldsymbol{X}$ including the decomposed frequencies of the original predictors (obtained with MODWT MRA), i.e., we run multi-variate regressions using different frequencies of different original predictors. This model is designated *multi_wav.*

The benefits of using data from the frequency decomposition of the original predictors can be evaluated when we compare the *ts* and *wav* models.

To evaluate the OOS forecasting performance of the predictive models, we use the Campbell and Thompson (2008) R^2_{OS} statistic. As it is the standard in the literature, we consider the historical mean (HM) forecast $\bar{r}_t$, which is the average ERP, CR and BRP up to time t, as the benchmark model. The R^2_{OS} statistic computes the proportional decrease in the mean squared forecast error for the predictive model ($MSFE_{PRED}$) relative to the historical mean ($MSFE_{HM}$), i.e., comparing the predictive capability of the predictor and the historical sample mean. The R^2_{OS} statistic can be written as

$$R^2_{OS} = 100\left(1 - \frac{MSFE_{PRED}}{MSFE_{HM}}\right) = 100\left[1 - \frac{\sum_{t=t_0}^{T-1}(r_{t+1} - \hat{r}_{t+1})^2}{\sum_{t=t_0}^{T-1}(r_{t+1} - \bar{r}_t)^2}\right]$$

where $\hat{r}_{t+1}$ represents the CR (ERP; BRP) forecast from the predictive model for period *t+1*, and r_{t+1} is the realized CR (ERP; BRP) from *t* to *t+1.* A positive value of R^2_{OS} means that the predictive model outperforms the historical mean (HM) in terms of mean squared forecast error (MSFE).

Like in Faria and Verona (2019), the statistical significance of the results is evaluated using the Clark and West (2007) statistic.[9] This statistic tests the null hypothesis that the $MSFE_{HM}$ is less than or equal to the $MSFE_{PRED}$ against an alternative hypothesis that the $MSFE_{HM}$ is greater than the $MSFE_{PRED}$, i.e.:

$$H_0 : R^2_{OS} \leq 0$$
$$H_A : R^2_{OS} > 0$$

Then, we compare the resulting t-statistic from the forecasts with the critical values: 1,282 for a 10% level, 1,645 for a 5% level and 2,326 for a 1% level of significance. If the t-statistic is higher than the critical values, the null hypothesis can be rejected. If the null-hypothesis is rejected, $MSFE_{PRED}$ outperforms the $MSFE_{HM}$ with a given statistical significance level.

[9] Advantage of this test is that it corrects small-sample forecast bias. ***, ** and * denote significance levels of 1%, 5% and 10%, respectively. See Clarke and West (2007) for more details.

2.3 Asset allocation framework

The asset allocation framework allows us to analyze the significance of frequency domain information for active portfolio management (APM). The goal is to beat a benchmark. Hence, the actively managed portfolio is compared to the given benchmark.

We adopt the standard in the literature, i.e., the perspective of a mean-variance investor, who invests in bonds, equities, and commodities to maximize returns that are risk-adjusted. The weight of bonds in the portfolio is given by ϖ_b, the weight of equities is given by ϖ_e and the weight of commodities is given by ϖ_c, represented by a vector $\boldsymbol{\varpi} = (\varpi_b, \varpi_e, \varpi_c)$.

The initial wealth of the investor who follows this APM strategy is set to 1 and the rebalancing of the portfolio is done monthly, using the one-step-ahead monthly return forecasts. The goal is to optimize the trade-off between risk and return. As so, the optimization problem can be written as

$$\min_{\varpi} \left[\gamma \theta_p(\varpi) - \varpi' \hat{R}\right] \tag{3}$$

where γ is the relative risk aversion coefficient (which, following the literature, we assume to be equal to 2), $\hat{R} = \left(\hat{R}_{b,t+1}, \hat{R}_{e,t+1}, \hat{R}_{c,t+1}\right)$ is the vector of one-step ahead return forecast of bonds ($\hat{R}_{b,t+1}$), equities ($\hat{R}_{e,t+1}$) and commodities ($\hat{R}_{c,t+1}$), and $\theta_p(\varpi)$ is the portfolio risk function. The one step-ahead commodity forecast ($\hat{R}_{c,t+1}$) is the result of the one-step ahead forecast of the commodity returns ($\widehat{CR}_{t+1}$) minus the risk-free rate that is known at the beginning of each period. The same procedure was applied to the one step-ahead equity return forecast ($\hat{R}_{e,t+1}$) and bond return forecast ($\hat{R}_{b,t+1}$). The portfolio risk function $\theta_p(\varpi)$ is set as $\theta_p(\varpi) = \sqrt{\varpi' \hat{\Sigma} \varpi}$, where $\hat{\Sigma}$ is the estimated monthly returns covariance matrix. The covariance matrix is estimated using the exponentially weighted moving average approach, with the decay parameter set to 0.97[10], standard for monthly data.

We introduced some restraints to the vector ϖ to make sure that the APM portfolio leveraging is realistic and to make our analysis more reliable. The first constraint is to set a lower bound l to the weight of each asset $(\varpi_b, \varpi_e, \varpi_c)$. We intend to exclude short selling, i.e., the weight of each asset must be above or equal to 0. So, $l = 0$. The second constraint sets an upper bound h to the total of the portfolio weights, $\varpi' \boldsymbol{I}_3 = h$, where $\boldsymbol{I}_3$ is a 3-vector of ones and h denotes the maximum leverage. So, we set h = 1,5, meaning that an investor cannot borrow more that 50% of the value of his investment.

The active strategy portfolio return at $t+1$, $R_{p,t+1}$ is given by:

$$R_{p,t+1} = \widehat{\varpi}'_t R_{t+1} + (1 - \widehat{\varpi}'_t \boldsymbol{I}_3) r_f \quad ,$$

where R is the vector of expected returns of bonds R_b , equities R_e and commodities R_c and r_f is the one-month risk-free rate.

For constructing the benchmark portfolio, we follow two standards in the literature. Anson (1999) states that an investor with high risk aversion should invest about 20% in commodities. Jensen, Johnson and Mercer (2000) defend that depending upon risk tolerance, commodities should represent 5-36% of the investor's portfolio. We assume the weight of commodities represents 20% of the benchmark portfolio. Regarding stocks and bonds, we follow the standard 60-40 for the remaining 80% of the benchmark portfolio. As so, our benchmark portfolio considers an allocation of 48% of the investment in stocks, 32% in bonds and 20% in commodities, and six performance measures are used:

[10] The choice of the decay factor was based on Reuter's Risk Metrics-Technical Document. The standard for daily data is 0.94.

Sharpe ratio, composite annual growth rate of returns (CAGR), tracking error, information ratio, maximum drawdown, and certainty equivalent return (CER) gain. The Sharpe ratio, that is commonly used as a metric to assess the portfolio performance, is calculated with the one-year moving average of the annualized SR of the portfolio. To compute the tracking error, we use the annualized standard deviation of the active strategy's monthly excess return relative to the benchmark. Then, the information ratio is computed as the annualized active strategy's monthly excess return relative to the benchmark divided by the tracking error. Both the IR and the TE are important metrics to evaluate the performance for actively managed portfolio because they show the eventual economic advantages of deviating from the benchmark. The maximum drawdown gives the maximum potential loss (percentage reduction in the portfolio's cumulative returns) of following the active strategy.

The power utility is measured as $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$, where x = 1 + R_p and R_p is the portfolio return. We define the average utility an investor would get by following the active portfolio management strategy (APM) as $\overline{U}_{APM}$ and the average utility an investor would get by following the benchmark portfolio as $\overline{U}_{benchmark}$. The CER is given by $CER_i \; = \; [(1-\gamma)\overline{U}_i]^{1/(1-\gamma)}$ – 1, where i = *APM*, *benchmark*. We can compute the annualized utility gain and interpret it as the premium an investor is willing to pay to have access to the APM portfolio instead of the benchmark.

Chapter 3

3. Results

3.1 Out-of-sample forecasting statistical performance

As it was previously described in section 2.2.2, we run two predictive models: regressions using multiple original predictors (*multi_ts)* and regressions using different decomposed frequencies from different original predictors (*multi_wav).* We based our regression models' performance analysis on the Campbell and Thompson (2008) out-of-sample R-square (R^2_{OS}) statistic. For simplification purposes, we only report the model specification that maximizes the R^2_{OS} statistic, i.e., the predictor (or combination of predictors) that have the best OOS predictability. Table 2[11] summarizes the two regression models' best R^2_{OS}.

On a first note, the predictability of the bond risk premium (BRP) is higher than the commodity returns (CR) and the equity risk premium (ERP), no matter what forecasting model we consider. Secondly, there are noticeable forecast improvements from using different frequencies from multiple original predictors at a time (*multi_wav)* against using more than one original predictor at a time (*multi_ts*). Using combinations of different frequencies of the original predictors

[11] For computational reasons, we consider at most three frequencies from all possible predictors in the models.

($multi\_wav$) translated in an increase in the maximum R^2_{OS}. The best R^2_{OS} for the BRP forecast improves from 2,42% to 7,78%; from 1,06% to 6,53% for CR; and from -0,28% to 5,02% for ERP. However, for CR and ERP there is no statistically significant forecasting when using time series.

These results indicate that using frequency-domain information strengthen the forecastability of bond risk premium, commodity returns and equity risk premium. In the next section, we analyze if these gains directly translate to superior portfolio performances.

3.2 Active portfolio management performance

The outputs of the BRP, CR and ERP forecasting are used to feed the vector of active views $\hat{R} = (\hat{R}_{b,t+1}, \hat{R}_{e,t+1}, \hat{R}_{c,t+1})$. This vector then serves as an input for the APM strategies. The BRP, CR and ERP forecasts obtained through the *multi_wav* models will feed the *APM_WAV* strategy. For comparison reasons, we also report other APM strategy: *APM_TS*. The BRP, CR and ERP forecasts obtained using the original time-series of predictors (*multi_ts* models) will feed this *APM_TS* strategy. As mentioned before, our benchmark portfolio considers an allocation of 48% of the investment in stocks, 32% in bonds and 20% in commodities, and is denoted $Benchmark_{48-32-20}$.

In panel A of Table 3, we report the performance measurements of the two active strategies and the $Benchmark_{48-32-20}$. Clearly, the APM strategies outperform the $Benchmark_{48-32-20}$ and the *APM_WAV* outperforms the *APM_TS*. The two APM strategies enhance the average annual return whilst decreasing the maximum drawdown. This leads to higher Sharpe ratios (annualized). Actively deviating from the benchmark benefit the active investor, as reflected in the annualized information ratios of 0,47 (*APM_WAV*) and 0,29 (APM_TS). The APM_WAV strategy outperforms the APM_TS strategy. This suggests that there are economic gains from using frequency-domain information in active portfolio management.

In Figure 3, we highlight the cumulative wealth of an investor who invests 1$ in January 1990 and reinvests monthly along the OOS period adopting the APM_WAV strategy (orange line), the APM_TS strategy (blue line) and the $Benchmark_{48-32-20}$ strategy (yellow line). The three strategies have common trends, being the downtrend in world financial crisis period (late 2000's) the most noticeable. Following a cumulative return perspective, the APM_WAV strategy outperforms the APM_TS and $Benchmark_{48-32-20}$ strategies. By the end of the OOS period (June 2010), the investor gained 12,74$ with the APM_WAV strategy, versus 9,46$ with the APM_TS strategy or 6,14$ with the $Benchmark_{48-32-20}$. Despite the APM_WAV strategy dominating the APM_TS strategy across the whole period, it is not the best strategy through the whole OOS period. In late 90's and early 2000's, the benchmark outperforms both active strategies. However, considering the cumulative perspective, the APM_WAV strategy clearly outperforms the other ones.

The previous analysis demonstrates that using frequency-domain information (namely adding commodities to the Faria and Verona's (2020) work) is beneficial for active portfolio management. We test the robustness of our findings by considering alternative benchmark portfolios and introducing changes to the constraints used.

Chapter 4

4. Robustness

4.1 Alternative benchmarks

To test the robustness of our findings, we constructed different benchmark portfolios. First, we consider a portfolio based on equity and bonds solely (60% equity, 40% bonds and 0% commodities). Then, we consider the classic diversification rule 1/N (33% equity, 33% bonds and 33% commodities). After that, we consider different portfolios with commodity weights varying from 5% to 36% (Jensen, Johnson and Mercer (2000)) of the investor's total portfolio. The remaining percentage is then divided between equity and bonds following the 60/40 standard. The results are reported in Panel B of table 3. We highlight two main conclusions. The first one is that no matter the weights of the Benchmark portfolio, both APM strategies will tend to converge into the "ideal" portfolio weights at the end of the OOS period. Given that, all APM strategies reported have the same value of average return, CAGR, Sharpe ratio and maximum drawdown. The second conclusion is that the tracking error, information ratio and CER gains of the *APM_WAV* strategy are higher than those of the *APM_TS* strategy. These results confirm that our findings are robust towards alternative benchmarks.

As a final note, we can conclude that increasing the weight of commodities, and keeping all constraints unchanged (risk aversion coefficient = 2 and maximum leverage = 1,5), lead to higher CER gains.

4.2 Alternative set of portfolio constraints and investor risk aversion

As an additional robustness test, considering the same $Benchmark_{48-32-20}$ and the same level of risk aversion of the representative investor, we test APM strategies for different leverage and short-selling constraints. First, we allow short-selling but not leverage (h = 1 and l = -0,5); secondly, we don't allow short-selling nor leverage (h = 1 and l = 0); and finally, we allow both leverage and short-selling (h = 1,5 and l = -0,5). The results indicate that for higher levels of leverage and short-selling, the level of outperformance of *APM_WAV* strategy versus the *APM_TS* strategy is higher. A final robustness test is considered: lower level of risk aversion while keeping other constraints constant. We conclude that the lower the level of risk aversion, the higher is the outperformance of the *APM_WAV* strategy when compared with the *APM_TS* strategy.[12]

[12] Results are available upon request.

Conclusion

In this thesis we tried to extend the work of Varia and Verona (2020) by introducing commodities in the investment opportunity set alongside equity and fixed income. Hence, our aim is to test the forecastablity and diversification power of commodities in the context of an actively managed multi-asset (equity, bonds, commodities) portfolio using frequency-domain information.

Firstly, we test whether the use of information from different frequencies of the variables used improves the out-of-sample forecasting performance on bond, commodity and equity returns. We conclude that using frequency domain information leads to improved forecasts of the three asset classes returns versus when using exclusively original time series.

Secondly, and the focal point of this work, is to test the economic significance of the frequency domain information. Hence, we assess if the use of frequency domain information would lead to superior portfolio performances in the context of an actively managed multi-asset portfolio.

The results are clear. It is unequivocal that the use of frequency domain information in an actively multi-asset (equity, bonds, commodities) portfolio is beneficial.

We identify a secondary finding: the higher the commodity weight in the benchmark portfolio, the higher the CER gains of the active strategy.

The main limitation of this thesis is the commodity futures and the dataset length. The initial goal was to obtain data until October 2021, but we were not able to have access to more updated data than June 2010.

As a final note, we would like to present one suggestion for future research. The OOS forecasting horizon could be enlarged beyond one month. As Timmerman (2014) shows, the strength of the commodity predictability is as higher as the forecasting horizon. This can be particularly useful for long-term driven investors with lower turnover ratios of their portfolios, requiring fewer rebalancing trades.

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Tables & figures

	Mean	Median	Min	Max	Std. dev.
BRP (%)	-0,76	-0,44	-12,52	10,30	4,51
ERP (%)	0,42	0,79	-11,33	11,47	4,50
CR (%)	0,44	0,49	-11,23	11,99	4,33
DP	-3,58	-3,52	-4,47	-2,83	0,45
DY	-3,58	-3,52	-4,47	-2,83	0,45
EP	-2,81	-2,85	-4,66	-1,97	0,51
RVOL (ann.)	0,15	0,14	0,06	0,31	0,05
BM	0,51	0,41	0,13	1,15	0,30
NTIS	0,01	0,01	-0,05	0,04	0,02
LTY (%, ann.)	7,61	7,47	3,95	14,00	2,45
LTR (%)	0,74	0,82	-7,05	9,58	3,11
TMS (%, ann.)	2,00	2,11	-2,38	4,37	1,53
DFY (%, ann.)	1,11	0,96	0,56	2,91	0,48
DFR (%)	0,01	0,05	-3,95	3,95	1,41
INFL (%)	0,36	0,32	-0,54	1,29	0,38

Table 1: Summary Statistics

This table reports summary statistics for the bond risk premium (BRP), equity risk premium (ERP), commodity returns (CR) and the set of predictors. BRP and ERP are measured as the difference between the return on the 10-year US Treasury bond and the return on the S&P500 index, respectively, and the return on a one-month T-bill. BRP, ERP, LTR, DFR, and INFL (LTY, TMS, and DFY) are measured in percent (annual percent). CR is the log of portfolio's monthly returns. The set of predictors is described in Appendix 1. The sample period runs from 1972:01 to 2010:06.

	multi_ts		*multi_wav*	
	R^2_{OS}	Predictors	R^2_{OS}	Predictors (frequency)
BRP	2,42*	DP, DY	7,78***	BM (D_2), NTIS (D_1), LTY (D_3)
CR	1,06	DFR	6,53**	DP (D_2), BM (D_2), LTR (D_5)
ERP	-0,28	LTR	5,02***	EP (D_3), RVOL (D_5), TMS (D_7)

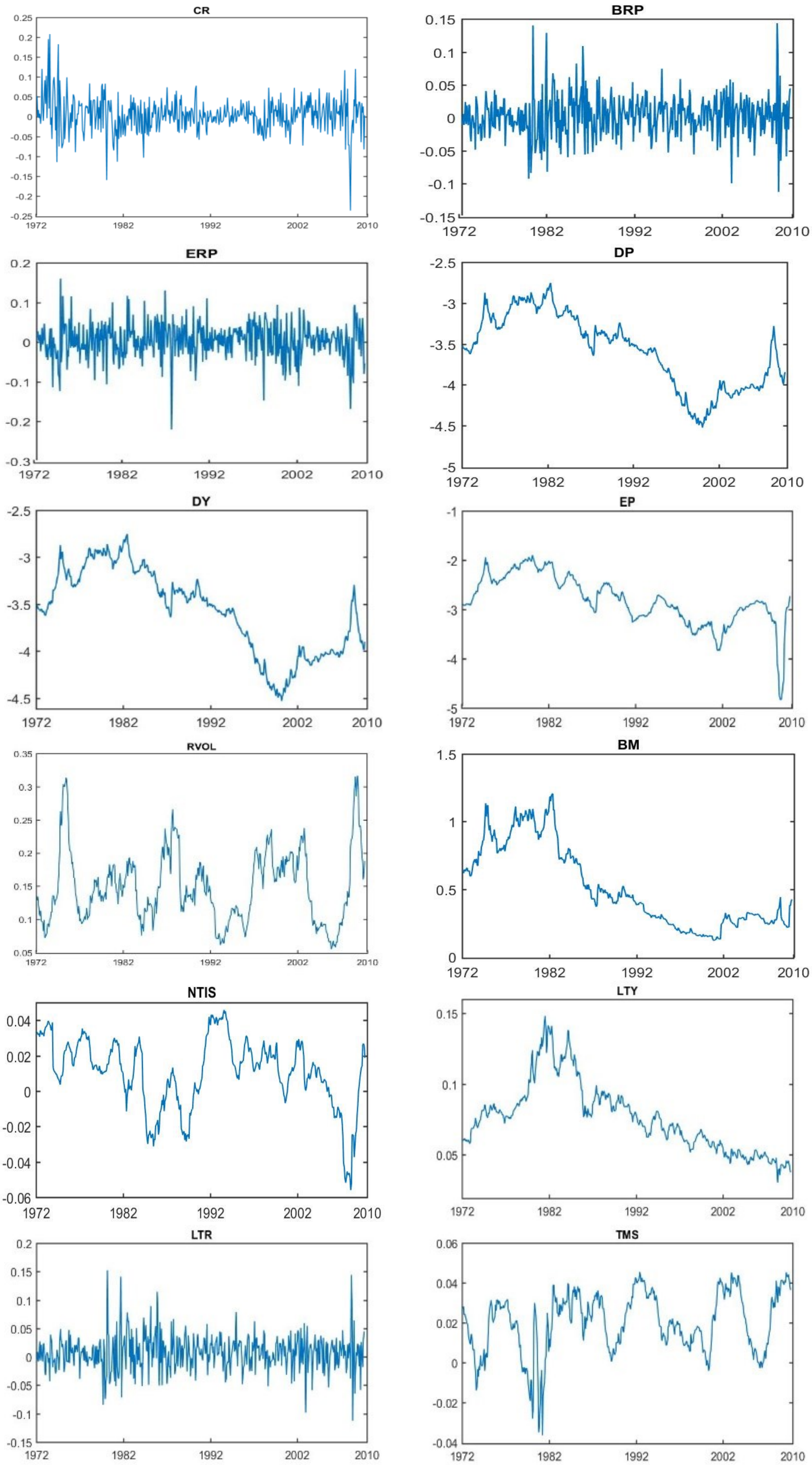
Table 2: Out-of-sample R-squares (R^2_{OS})

This table reports the maximum R^2_{OS} (out-of-sample R-squares) for the bond risk premium (BRP), commodity returns (CR) and equity risk premium (ERP) forecasts of the two predictive models: using more than one predictor at a time (*multi_ts*) and using different frequencies from multiple original predictors at a time (*multi_wav*). The R^2_{OS} measures the reduction in the mean squared forecasting error from the use of the predictive model against the forecast based on the historical mean. We use an initial in-sample (IS) period from 1972:01 (January 1972) to 1989:12, with a monthly sample frequency, and the full OOS period is from 1990:01 to 2010:06. The asterisks represent the significance of Clark and West (2007)'s statistic. ***, ** and * denote significance levels of 1%, 5% and 10%, respectively.

	Average return	CAGR	Sharpe ratio	Maximum drawdown	Tracking error	Information ratio	CER gain
Panel A: baseline							
APM_WAV	13,20%	11,02%	0,80	19,90%	7,63%	0,47	3,23%
APM_TS	11,63%	11,02%	0,68	21,19%	7,47%	0,29	1,85%
$\text{Benchmark}_{48-32-20}$	9,22%	8,81%	0,57	30,79%	-	-	-
Panel B: different benchmark portfolios							
APM_WAV	13,20%	11,02%	0,80	19,90%	6,48%	0,62	3,51%
APM_TS	11,63%	11,02%	0,68	21,19%	6,28%	0,42	2,13%
$\text{Benchmark}_{33-33-33}$	8,76%	8,42%	0,58	30,41%	-	-	-
APM_WAV	13,20%	11,02%	0,80	19,90%	8,95%	0,34	2,85%
APM_TS	11,63%	11,02%	0,68	21,19%	8,82%	0,18	1,46%
$\text{Benchmark}_{60-40-0}$	9,88%	9,34%	0,56	29,09%	-	-	-
APM_WAV	13,20%	11,02%	0,80	19,90%	8,54%	0,37	2,93%
APM_TS	11,63%	11,02%	0,68	21,19%	8,40%	0,21	1,54%
$\text{Benchmark}_{57-38-5}$	9,71%	9,22%	0,57	29,49%	-	-	-
APM_WAV	13,20%	11,02%	0,80	19,90%	7,88%	0,44	3,12%
APM_TS	11,63%	11,02%	0,68	21,19%	7,72%	0,26	1,73%
$\text{Benchmark}_{51-34-15}$	9,39%	8,95%	0,57	30,34%	-	-	-
APM_WAV	13,20%	11,02%	0,80	19,90%	7,45%	0,50	3,35%
APM_TS	11,63%	11,02%	0,68	21,19%	7,28%	0,32	1,97%
$\text{Benchmark}_{45-30-25}$	9,06%	8,66%	0,56	31,60%	-	-	-
APM_WAV	13,20%	11,02%	0,80	19,90%	7,25%	0,56	3,66%
APM_TS	11,63%	11,02%	0,68	21,19%	7,07%	0,38	2,27%
$\text{Benchmark}_{38-26-36}$	8,70%	8,31%	0,53	33,88%	-	-	-

Table 3: Portfolio performance statistics

This table reports the performance statistics of different portfolio strategies. The performance statistics are: average return, which is the annualized first moment of returns time series; CAGR (composite annual growth rate of returns time series; Sharpe ratio, which is measured as the 1-year moving average of portfolio's annualized Sharpe ratio; maximum drawdown, that is measured as the maximum percentage decrease in the portfolio's cumulative return; tracking error, measured as the annualized standard deviation of the APM monthly excess return towards the benchmark; the information ratio, measured as the annualized average APM monthly excess return towards the benchmark divided by the tracking error; CER gain, measured as the annualized gain in certainty equivalent return that a power-utility maximizing investor with relative risk aversion γ=2 would have by owning the APM portfolio instead of the benchmark one. The initial benchmark portfolio (Panel A) is constructed considering an allocation of 48% of the investment in stocks, 32% in bonds and 20% in commodities. Panel B portfolios are alternative benchmark portfolios constructed for robustness testing reasons. APM_WAV is the active portfolio management strategy that uses the forecasts obtained from the *multi_wav* models and the APM_TS is the active portfolio management strategy that uses the forecasts obtained from the *multi_ts* models. The sample period runs from 1972:01 to 2010:06. The initial in-sample (IS) period is from 1972:01 to 1989:12, with a monthly sample frequency, and the full OOS period refers to the period from 1990:01 to 2010:06



CR
BRP
ERP
DP
DY
EP
RVOL
BM
NTIS
LTY
LTR
TMS
1972
1982
1992
2002
2010

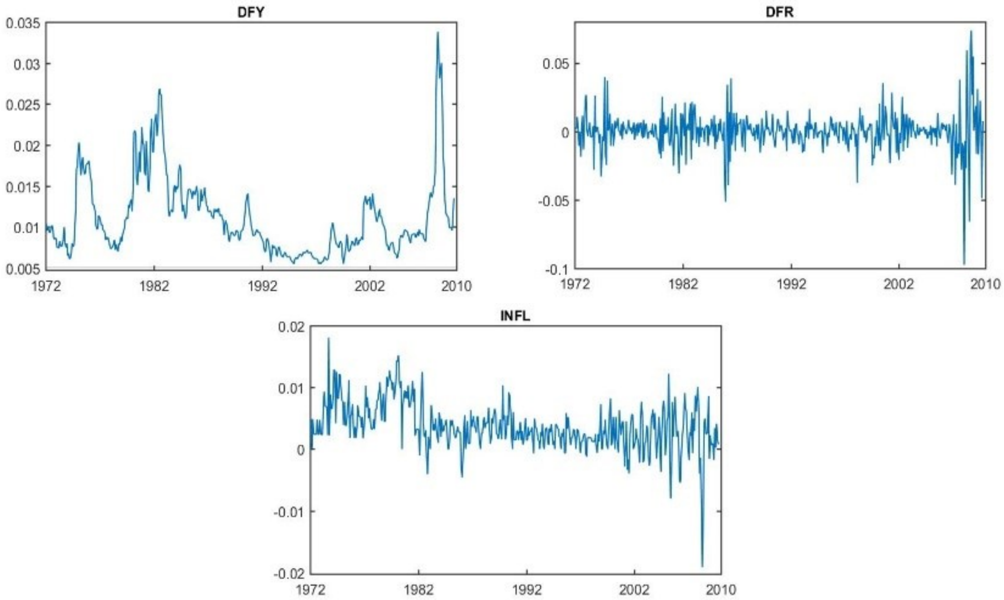


Figure 2: Monthly time-series of the BRP, CR, ERP, and their predictors

This figure plots the time series of the bond risk premium (BRP), the commodity returns (CR) and the equity risk premium (ERP) and of the twelve predictors used. Bond risk premium (BRP) and equity risk premium (ERP) of month t are determined as the difference between the return on the 10-year US Treasury bond and the return on the S&P500 index in month t, respectively, and the one-month T-bill known at the beginning of month t (lagged-risk free rate). Commodity returns (CR) are computed as the log of the commodity portfolio monthly returns. The set of predictors is described in Appendix 1. The sample period runs from 1972:01 to 2010:06.

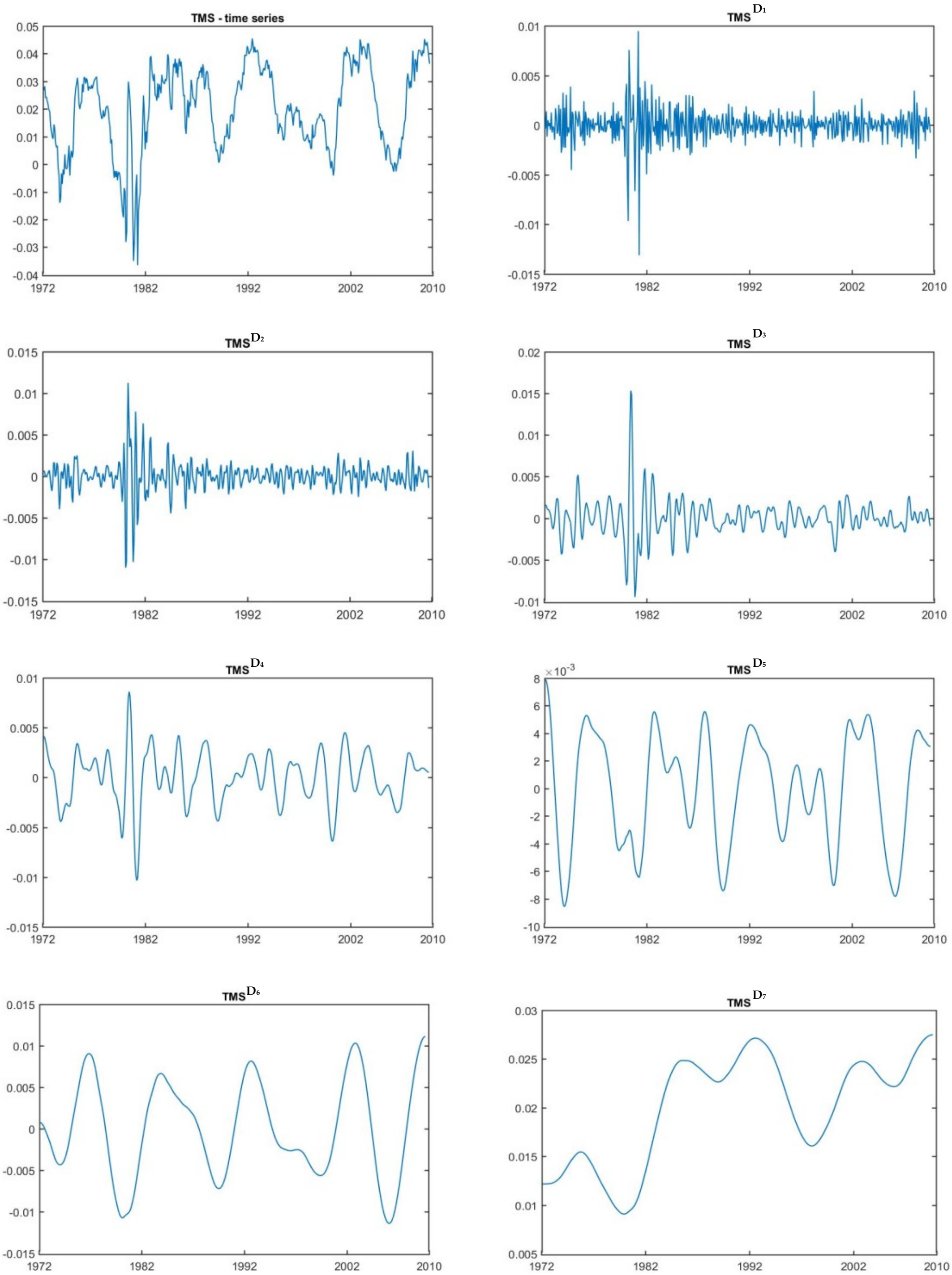


Figure 3: Term spread time-series and wavelet decomposition

This figure plots the time series of the term spread (TMS) and the seven frequency components into which the time series is decomposed. It is applied a $J = 6$ level wavelet decomposition, which produces six wavelet details *(D_1, D_2, ..., D_6)*, each representing higher-frequency characteristics of the series, as well as a wavelet smooth *(D_7)*, which captures the low-frequency dynamics of the series. The sample period runs from 1972:01 to 2010:06 (monthly frequency).

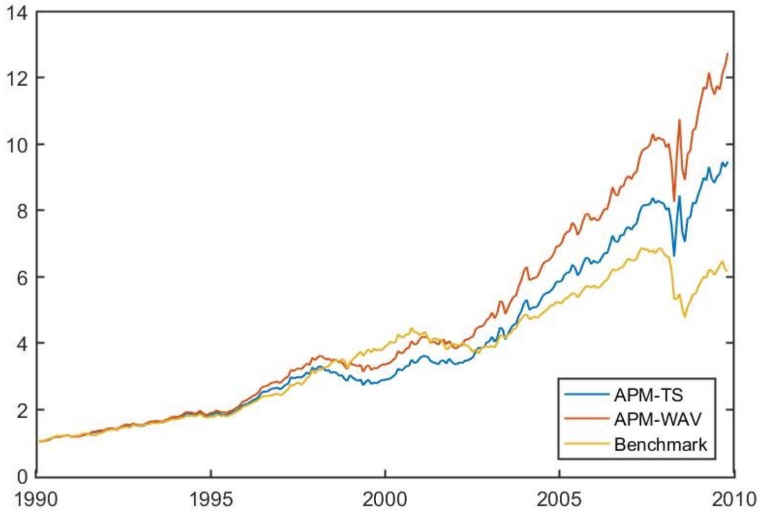


Figure 4: Cumulative wealth for APM_WAV, APM_TS and Benchmark$_{48-32-20}$ investors

This figure represents the cumulative wealth of an investor who begins with \$1 and reinvests all proceeds monthly, adopting an APM_WAV, APM_TS, and $Benchmark_{48-32-20}$ strategy (orange, blue and yellow lines, respectively). The APM_WAV and APM_TS active portfolio management strategies are based on asset return forecasts from *multi_wav* and *multi_ts* methodologies, respectively. The $Benchmark_{48-32-20}$ portfolio considers an allocation of 48% of the investment in stocks, 32% in bonds and 20% in commodities. The sample period extends from 1972:01 to 2010:06. The out-of-sample forecasting period runs from 1990:01 to 2010:06 (monthly frequency)

Appendix 1. Predictors of risk premiums

- Log dividend-price ratio (DP): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of prices (S&P 500 index).
- Log dividend yield (DY): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of lagged prices (S&P 500 index).
- Log earnings-price ratio (EP): difference between the log of earnings (12-month moving sums of earnings on S&P 500) and the log of prices (S&P 500 index price).
- Excess stock return volatility (RVOL): calculated using a 12-month moving standard deviation estimator.
- Book-to-market ratio (BM): ratio of book value to market value for the DJIA.
- Net equity expansion (NTIS): ratio of 12-month moving sums of net equity issues by NYSE-listed stocks to the total end-of-year NYSE market capitalization.
- Long-term yield (LTY): long-term government bond yield.
- Long-term return (LTR): long-term government bond return.
- Term spread (TMS): difference between the long-term government bond yield and the T-bill.
- Default yield spread (DFY): difference between Moody's BAA-and AAA-rated corporate bond yields.
- Default return spread (DFR): difference between long-term corporate bond and long-term government bond returns.
- Inflation rate (INFL): calculated from the Consumer Price Index (CPI) for all urban consumers.

Appendix 2. Maximal overlap discrete wavelet transform

Wavelets have genders: there are father wavelets (ϕ) that represent the smooth and trend (low frequency) of the series; and mother wavelets (ψ) that represent the detailed (high frequency) part. The father wavelet integrates to 1 and the mother wavelet integrates to 0, where $\int \phi(t)dt = 1$ and $\int \psi(t)dt = 0$.

The discrete wavelet transform (DWT) multiresolution analysis (MRA) allows the decomposition of a time series into its constituent multiresolution (frequency) components.[13] Given a time series y_t, its wavelet multiresolution representation can be written as

$$y_t = \sum s_{J,k} \phi_{J,k}(t) + \sum d_{J,k} \psi_{J,k}(t) + \sum d_{J-1,k} \psi_{J-1,k}(t) + \cdots + \sum d_{1,k} \psi_{1,k}(t) \tag{4}$$

where J is the number of multiresolution levels (or scales), i.e., the number of frequencies used, k defines the length of the filter, $\phi_{J,k}(t)$ and $\psi_{J,k}(t)$ are the wavelet functions, and $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}$, … , $d_{1,k}$ are the wavelets coefficients.

The mother and father wavelets generate the wavelet functions through scaling and translation, as follows

$$\phi_{j,k}(t) = 2^{-J/2}\phi\left(\frac{t - 2^j k}{2^j}\right) \tag{5}$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t - 2^j k}{2^j}\right) \tag{6}$$

[13] Detailed analysis of wavelet methods can be found in Crowley (2008) and Percival and Walden (2000)

where the base scale in DWT is set to 2 and the different scales are obtained by raising it to integer numbers (2^j).

Hence, the Wavelet Transform coefficients are given by

$$s_{J,k} = \int y_t \, \phi_{J,k}(t) \, dt \tag{7}$$

$$d_{j,k} = \int y_t \, \psi_{j,k}(t) \, dt \tag{8}$$

where $s_{J,k}$ are the smooth coefficients generate from the father wavelet at a maximal scale 2^J and $d_{j,k}$ are the detail coefficients generated from the mother wavelet at all scales from 1 to J, with $j = 1, 2, \ldots, J$.

Given that,

$$S_J = \sum_k s_{J,k} \, \phi_{J,k}(t) \tag{9}$$

$$D_J = \sum_k d_{j,k} \, \psi_{j,k}(t) \tag{10}$$

the function y_t can be rewritten as:

$$y_t = \sum_{j=1}^{J} {y_t}^{D_j} + {y_t}^{S_J} \tag{11}$$

where ${y_t}^{D_j}$ are the J wavelet detail components and ${y_t}^{S_J}$ is the wavelet smooth component.

In the mother wavelet equation (**Erro! A origem da referência não foi encontrada.**), $2^j k$ determines the time position of the wavelet and 2^j is the size (scale) of the wavelet, making the window smaller or larger. Therefore, if 2^j

increases, the levels in the wavelet spectrogram go down (time window is stretched), and $2^j k$ slides that window across the signal. This means that low scales capture rapidly changing details (high frequencies), whereas higher scales capture slowly changing features (low frequencies).