Volatility scaling applied to investment-grade bond portfolios

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Abstract

Scaling excess returns in investment-grade bond portfolios by their past volatility does not increase risk-adjusted returns nor Sharpe Ratios, even considering longer or shorter periods with different degrees of volatility. This is observed for the United States bond market in US dollars. I would expect that volatility scaling could increase alphas for the lowest rated bond portfolios of my sample, that theoretically incorporate a higher degree of equity features, BAA bond portfolios, but that was not the case. When I isolate the credit or default risk from the expected returns, I also verify the inexistence of volatility management risk-adjusted returns. Major institutional holdings, buy and hold strategies typical of bondholders, mean reversion of returns for long-term investments, liquidity constraints, regulatory procedures, transaction costs, all can be reasons why the volatility scaling strategies are not worthwhile.

Keywords: bond portfolios, return volatility, scaling

JEL Classification: G11, G12, G24

Resumo

Aplicar uma estratégia ativa de gestão de carteiras de obrigações de grau de investimento com um nível alvo de volatilidade constante não aparenta ser eficaz na medida em que não permite aumentar o grau de rendibilidade face ao risco dessas carteiras (habitualmente designado na literatura por "*alfa*"), nem o Rácio Sharpe. Esta conclusão aplica-se mesmo considerando períodos de observação distintos, de maior ou menor longevidade, onde se notam diferentes níveis de volatilidade histórica. Observo estas conclusões para o mercado de obrigações nos Estados Unidos da América em dólares norte-americanos. Seria expectável que essa estratégia pudesse funcionar para as carteiras de obrigações com notação de rating BAA, que teoricamente incorporam mais características de instrumentos de capital como ações, mas tal não se verifica. Quando isolo a componente de risco de crédito ou de incumprimento dessas obrigações na rendibilidade esperada, continuo a verificar a inexistência de aumentos no grau de rendibilidade face ao risco dessas carteiras de obrigações com a aplicação de uma estratégia ativa. O facto de a maioria das emissões de obrigações serem detidas por investidores institucionais, com perspetivas de aquisição e manutenção das posições até à maturidade, da liquidez reduzida observada no mercado de obrigações, imposições regulatórias, a tendência de reversão da rendibilidade para o seu valor médio no longo prazo, observável em investimentos desta natureza, bem como, custos de transação elevados, podem ser motivos pelos quais esta estratégia ativa de gestão de carteiras com base em volatilidade constante não se traduz em rendimentos acrescidos.

Palavras-Chave: carteiras de obrigações, volatilidade da rendibilidade esperada, estratégias de gestão de carteiras com base na volatilidade

Classificação *JEL*: G11, G12, G24

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1. Introduction

Debt instruments represent an important funding source for corporations, governments, municipalities, states, provinces, agencies, and other economic agents. The amount of Treasury securities outstanding in the United States (Treasury bills, other Treasury notes, bonds and TIPS - Treasury inflation-protected securities) at the end of 2021 was of USD 22.55 trillion, while the amount outstanding of bonds issued by nonfinancial corporate businesses was of USD 6.65 trillion for the same period[1]. There are several debt instruments available for investors, from the simplest to debt instruments with embedded complex derivative structures, making it widely used by portfolio managers worldwide. Each debt instrument has implicit a probability of default or the degree of risk that the debtholder face about receiving the capital invested. Unlike equity assets in which gains are unlimited, the return of the capital invested in common debt instruments is capped by its face value plus some interest or other sort of return, if any.

According with Merton (1973), a corporation faces two classes of claims, a homogeneous class of debt and a residual claim designated equity. These two types of claims may be explained by using a combination of derivative instruments; the equity value of the firm is equivalent to a long position in a call option on the firm's assets in which the exercise price is the debt's face value. Likewise, the market value of debt issued by the firm is equivalent to a short position in a put option on the firm's assets plus a long position in a risk-free asset (partially) bought with the proceeds from selling the put. The put's exercise price is also the debt's face value at the moment the put option can be exercised. The relationship between the instantaneous standard deviation of the return on the bond and the instantaneous standard deviation of the return of the firm, is a measure of the relative riskiness of the bond in terms of the riskiness of the firm at a defined moment. Consequently, the debt of the firm can never be riskier than the firm as a whole and the equity of a levered firm must always be at least as risky as the firm itself. With the increase in the ratio of the present value of the promised payment of the debt to the current value of the firm, the probability of debt default becomes larger, the market value of the debt approaches that of the firm and the risk characteristics of the debt approaches that of equity. Similarly, as the probability of debt default approaches zero, its risk characteristics become the same as riskless debt. Between these two extremes, the debt will behave like a combination of riskless debt and equity.

[1] Tables L.210 and L.213 of Federal Reserve Statistical Release, Financial Accounts of the United States, March 10, 2022

As the volatility of the stock returns affects the value of the call and the put issued on the firm's assets, indirectly it has an impact on the value of the firm's outstanding debt and thus on its expected returns.

In the present analysis I do not study the volatility of equity returns but instead focus on the volatility of the bond's or bond portfolios' excess returns and on the eventual benefits of the volatility management strategies on different bond portfolios with diverse classes of credit risk, albeit being all investment-grade[2].

These benefits of scaling returns can be assessed using the Sharpe Ratio formula given by:

$$(1)\ SR = {}^{R_p}/_{\sigma_p}$$

In which, R_p is the portfolio excess return and σ_p the portfolio volatility of returns.

Scaling a portfolio aims to obtain a constant level of volatility over time, instead of a constant amount in the long and short position of the asset with varying volatility of the portfolio over time. In volatility scaled strategies, the amount invested in a certain asset at a specific time may vary and is proportional to the inverse of its past volatility and calibrated by a pre-defined constant level of volatility.

If the strategy works it should not be difficult to implement, save for the existence of a liquid market for the asset that allows the investors to trade and short it at any moment. The definition of the optimal constant level of volatility can be done ex-ante, Barroso and Santa-Clara (2015), or it can be constant and set so that the scaled and unscaled past excess returns present the same average volatility, Moreira and Muir (2017).

Since the weights are defined as a function of the inverse of past volatility, investors may decrease their long positions when volatility starts to increase and only come back to the market

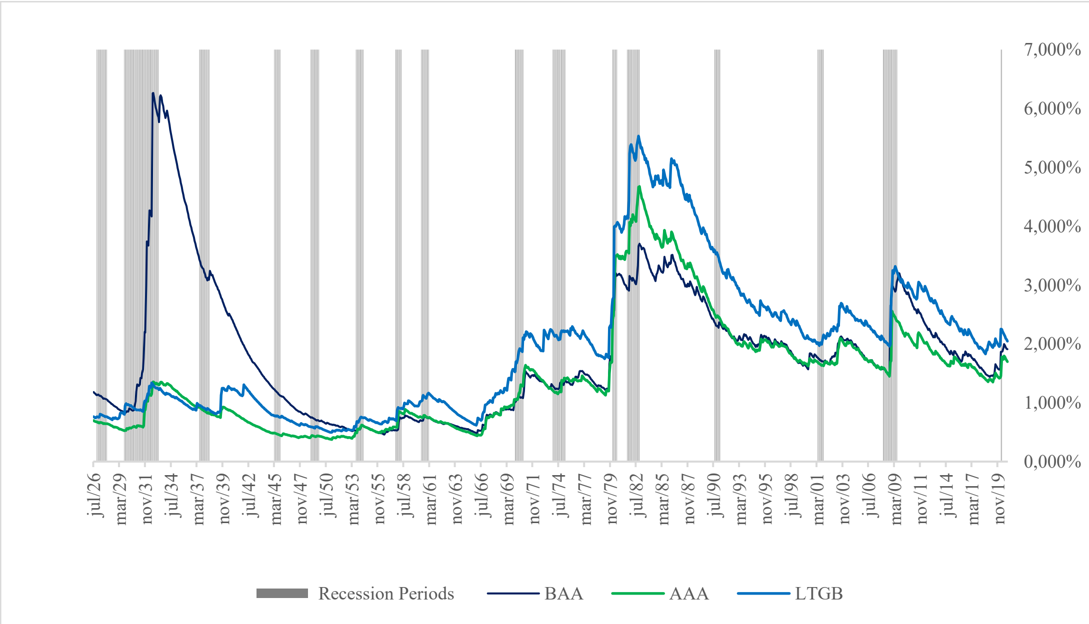
[2] Investment-grade is a definition used by Rating Agencies in which the best known are Standard & Poor's and Moody's, that defines the quality of a company's credit having implicit a default probability. To be considered an investment-grade issue, the company must be rated at 'BBB' or higher by Standard and Poor's or Baa by Moody's. Anything below this 'BBB' or 'Baa' rating is considered non-investment grade.

when volatility levels start decreasing again. In that sense they may avoid tumultuous times and market crashes, periods in which typically volatility is high. Therefore, that suggests that increasing investment in an asset or portfolio of assets when its past recent volatility has been low might lead to an improvement in performance measured by higher alphas (risk-adjusted returns) and Sharpe Ratios than unscaled (raw) returns. This strategy is called volatility timing or volatility scaling.

Figures 1 and 2 present historical volatility levels observed in the United States ("US") market for the analyzed samples, namely considering three classes of credit risk bond portfolios (BAA and AAA corporate bonds and Long-Term Government Bond "LTGB")[3] and two datasets, a longer dataset with monthly observations for the period from July 1926 until November 2020 (Figure 1) and a shorter dataset with daily observations and the corresponding monthly values, for the period from January 1986 until November 2020 (Figure 2), and also the recession periods in US as per National Bureau of Economic Research for the same periods.

Figure 1: Volatility of returns for BAA, AAA and Long-Term Government Bond portfolios in the US, July 1926: November 2020

The figure below presents the monthly volatility levels of returns with volatility calculated using the EWMA- Exponentially Weighted Moving Average formula and using monthly observations from July 1926 until November 2020 for the BAA, AAA and LTGB portfolios. Shaded areas represent the National Bureau of Economic Research recession periods.



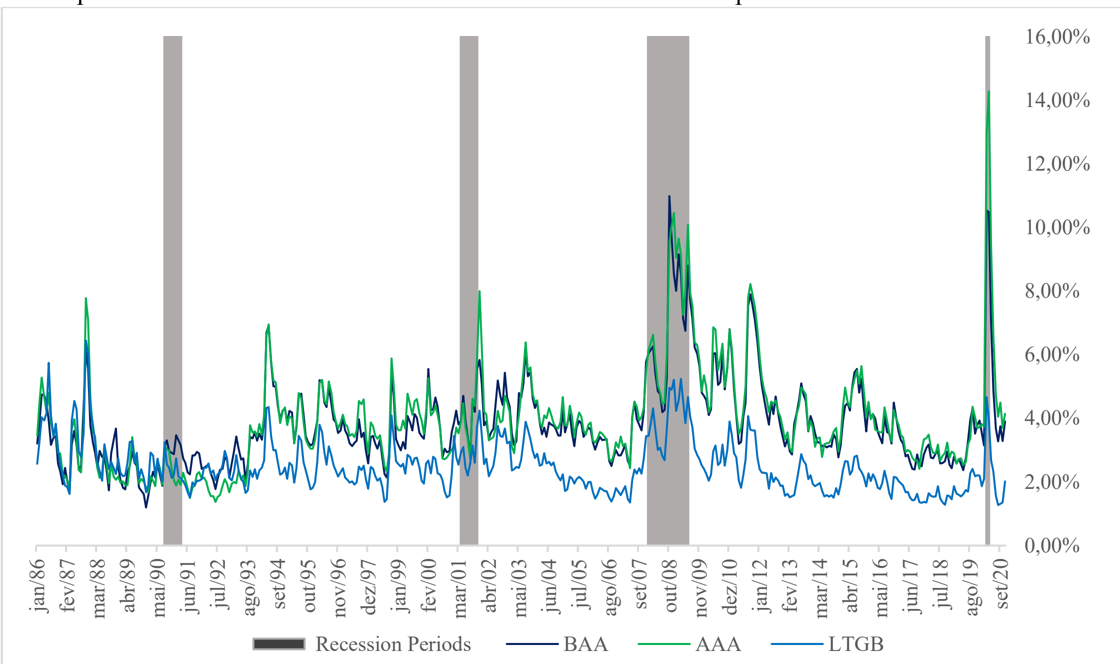
[3] BAA and AAA are credit risk notations which correspond to Baa and Aaa rating notations from Moody's Investors Services and BBB and AAA from Standard & Poor's and each correspond to a certain probability of default for the security. AAA is the best credit risk notation and corresponds to the lowest probability of default for that instrument.

From Figure 1 and Figure 2 we may note that there are several critical periods with significant volatility, the 1929 crisis, the early eighties and early nineties recessions, the dot.com bubble and the 2008 financial crisis. Also, we can conclude that for the longer dataset the volatility levels are lower when compared with the shorter dataset, maximum annualized volatility of returns of 21.70%, 16.20% and 19.16% respectively for the BAA, AAA and LTGB portfolios, while for the shorter dataset the maximum annualized volatility of returns reached 38.06%, 49.47% and 22.34% respectively for the BAA, AAA and LTGB portfolios.

Any reference to BAA and AAA corporate bonds and LTGB means the bond portfolios whose majority of assets are composed with corporate or Treasury bonds of the same class of credit risk within the portfolio as explained in detail in the Data caption.

Figure 2: Volatility of returns for BAA, AAA and Long-Term Government Bond portfolios in the US, January 1986: November 2020

The figure below presents the monthly volatility levels of returns with volatility calculated using the EWMA- Exponentially Weighted Moving Average formula and using daily observations to compute monthly values from January 1986 until November 2020 for the BAA, AAA and LTGB portfolios. Shaded areas represent the National Bureau of Economic Research recession periods.



This analysis is based on some existing literature of the eventual benefits of volatility scaling, particularly the paper presented by Wang, Yan and Zheng (2020), in which they try to demonstrate that timing volatility through scaling returns in a set of mutual funds composed mainly with equity instruments, improves risk-adjusted returns (alphas) and the Sharpe Ratios extending their analysis from long-short Fama-French factors. My analysis is different in the sense that I use investment-grade bond portfolios instead of mutual funds. I use for this assessment a time series of returns of different credit risk bond portfolios in the United States market (BAA and AAA corporate bonds and LTGB) and the purpose is to verify if the volatility scaling applied to those bond portfolios outperform the unscaled or passive strategies of buy and hold and conclude that, at least for the samples studied, there seems to be no benefits in using a volatility scaling strategy, as the risk-adjusted returns obtained with scaling the returns are null or even negative including the case of portfolios with higher credit risk and with more embedded equity features (BAA portfolio).

For performance evaluation I regressed the unscaled and scaled excess returns against the Market Factor and the Fama-French 5 Factor Model (besides the Market Factor, assess if the bond portfolio excess returns are explained by SMB, HML, RMW and CMA equity market factors[4]) and compared the alphas in both strategies. My test consists in verifying the existence of positive differences between the scaled alphas and the unscaled ones and to check whether volatility management may result in larger risk-adjusted returns in what regards to these factor models. If I find no difference between scaled alphas and the unscaled ones or that difference is negative, then the active strategy based on constant volatility seems not to work. Also, I test the bond portfolios for the existence of a risk-return trade-off relationship and evaluate the relationship of the portfolios' yield to maturity for each degree of default risk and the previous period volatility of returns. Lastly, I performed a regression of the scaled excess returns (dependent variable) against unscaled excess returns (independent variable) to test if the estimated alpha coefficient is zero. Intuitively, these zero alphas imply that the mean-variance frontier is not expanded, and that Sharpe-Ratios do not increase with volatility timing. If that situation occurs, it might be another confirmation that scaling excess returns is not worthwhile for these samples.

Furthermore, as previously mentioned, I consider two types of data sets, a monthly (longer) dataset for the period July 1926 until November 2020 in which the volatility of returns is determined using the EWMA - exponentially weighted moving average formula and a daily (shorter) dataset through which I obtain monthly values comprising data from January 1986 until November 2020. I calculate the volatility of returns, in this last case, using the EWMA

[4] SMB (Small Minus Big), HML (High Minus Low), RMW (Robust Minus Weak), CMA (Conservative Minus Aggressive) are Fama-French equity factors available at Kenneth French website Kenneth R. French - Description of Fama/French Factors (dartmouth.edu)

formula and the Realized Volatility formula (the variance of the returns corresponds to the square of daily observed return values of the previous month). Also, I isolate the credit risk for each dataset (monthly and daily), through studying only the default spread or the difference in excess returns obtained for the BAA and AAA corporate bond portfolios and the Long-term Government Bond portfolio.

Fama and French (1993), considered two factors to explain bond returns, a Term factor which is the difference between the monthly long-term Government bond return and the one-month Treasury-bill, and a Default factor which is the difference between the return on a market portfolio of long-term corporate bonds and the long-term government bond return. In the same field of explaining bond returns, Bai, Bali, and Wen (2019), studied the common factors that can explain corporate bond returns and concluded that besides the commonly used stock market factors and macroeconomic variables, the downside risk factor, or the possibility of losing all the investment, the credit risk factor and the liquidity risk factor have a significant power to explain the cross-section of expected corporate bond returns. Their results also indicate that the abnormal returns on corporate bond portfolios generated by the existing factor models, are compensated by these other factors, downside, credit and liquidity risk factors. In my analysis, besides studying bond portfolio's excess returns I will only focus on the default or credit risk factor.

This empirical work demonstrates that scaling returns for the bond portfolios hereby studied apparently does not increase risk adjusted returns (alphas) compared with the buy and hold strategy. It is confirmed also by default spread regressions where the alphas are very close to zero and not statistically significant[5] and from the estimated alpha coefficients obtained from scaled (dependent variable) against unscaled (independent variable) regressions. I note that for all the data samples analyzed the regressed monthly alpha estimated coefficients are close to zero or negative and not statistically significant, which confirms that I cannot reject the hypotheses that are all zero.

Also, I notice a positive risk-return trade-off in all the bond portfolios, that typically exist when timing volatility is not beneficial. Moreira and Muir (2017), state that if investors face a positive and strong risk-return trade-off and time volatility, i.e, leave the market and sell their positions

[5] We have considered in all our analysis a two-sided *t-stat* with a significance level of 5%

when volatility starts to increase, they may not benefit from the eventual excess returns of the periods with high volatility, meaning that the risk-adjusted returns or alphas tend to zero.

Furthermore, I study the relationship of the Fama-French 5 factors and check if those factor excess returns can explain the bond excess returns for both datasets. I obtain results in line with the work presented by Bektic, Wenzler, Wegener, Schiereck, and Spielmann (2019), in which they conclude that Fama-French equity factors are not fully translated to fixed income markets particularly in what regards investment-grade bonds.

I conclude that for the sample studied the eventual benefits of volatility scaling are almost inexistent, which might be due to the lower levels of liquidity and short sales restrictions in corporate bond markets or because institutional investors dominate the corporate bond market and perceive risk differently compared to individual investors. Edwards, Harris, and Piwowar (2007), studied the impact of increased transparency in transaction costs and concluded that average transaction costs in US corporate bond market are inversely related with trade size (the higher the trade size the lower transaction costs) and are much lower for institutional sized transactions. Cao, Goyal, Xiao, and Zhan (2020), in their study about the impact of option implied volatility in corporate bond returns, concluded that non-investment grade bonds have higher transaction costs and have less institutional holdings. Chung, Wang, and Wu (2019), referred that regulation requirements may prevent some types of investors to diversify their portfolios, as an example, insurance companies and pension funds often only invest in certain types of bonds due to regulatory constraints. I observe from Table L.213 of the Federal Reserve Statistical Release, Financial Accounts of the United States, March 10, 2022, that 45.36% of the amount of corporate bonds outstanding at the end of 2021 (excluding asset-backed securities) belong to life insurance companies, mutual funds, pension funds and exchange-traded funds, which confirms the dominance of the corporate bond market by institutional investors in the United States.

The remainder of this analysis is divided as follows: Section 2 presents a brief literature review, Section 3 presents the Data, Section 4 the methodology and the main results and Section 5 concludes.

2. Literature review

This analysis is based on the paper presented by Wang, Yan and Zheng (2020) in which they conclude that there is an increase in risk-adjusted returns and Sharpe Ratios with volatility scaling considering a set of mutual funds in the US market, composed mainly with equities. My contribution to the literature is to verify the existence of eventual benefits of volatility scaling in a diferent kind of financial intrument, investment-grade bond portfolios also in the US market, and conclude that for the samples studied those benefits are not present. This work may be important as although there are a multitude of research regarding volatility management strategies those are mainly related with equity factors and anomalies. In this case I only focus on the assessment if volatility management strategies based on constant levels of volatility may imply higher risk-adjusted returns when dealing with investment-grade bond portfolios and with its single credit risk component (Default spread).

Besides the work of Wang, Yan and Zheng (2020), there is vast literature that studies the existence of benefits on applying constant levels of volatility in equity portfolio management strategies that result in an increase in risk-adjusted returns (alphas) and Sharpe Ratios of the portfolios.

Fleming, Kirby, and Ostdiek (2003), evaluated the benefits of a Realized Volatility approach in the context of investment decisions for a mean-variance investor, considering a portfolio composed of four asset classes: stocks, bonds, gold and cash, for an observed period from January 1984 until November 2000 and concluded that volatility timing computed at a daily level leads to performance gains over longer horizons and that investors were available to pay around 50 to 200 basis points per year to capture the incremental gains from volatility timing.

Also, Verbeek and Marquering (2004), studied the economic value of predicting stock index returns and volatility, using stock indexes monthly data from 1954 until 2001 and observed that the success of volatility timing varies significantly over the sample period, also it appears easier to forecast returns at times when volatility is high. They observe that for a mean-variance investor there are gains with a volatility timing strategy even when are not allowed short sales and transaction costs are high.

More recently, Barroso and Santa-Clara (2015) studied the impact and eventual benefits of volatility management in the performance of momentum portfolios through setting an ex-ante level of constant volatility and concluded that controlling for volatility of these portfolios may eliminate crash risk and almost double the Sharpe Ratio comparing with the buy and hold strategy.

Moreira and Muir (2017), analysed the benefits of volatility timing, using an ex-post level of constant volatility, from the viewpoint of a mean-variance investor, that take less risk when volatility is high and conclude that volatility timing produces large alphas and Sharpe Ratios for market, value, momentum, profitability, return on equity, investment and betting-against-beta (a self-financing strategy that is long the low-beta portfolio and short the high-beta portfolio) equity factors.

Volatility management strategies have been widely discussed in more recent literature, besides Moreira and Muir (2017) and Barroso and Santa-Clara (2015), also Eisdorfer and Misirli (2017) studied a company's financial distress or probability of default level as a new factor to explain bond excess bond returns. This new factor called healthy-minus-distressed (HMD) trading strategy, is a no investment and self-financing strategy, the same strategy I apply in this analysis, which aims to produce high average returns to investors. They notice that this does not hold in market downturns, and that the profitable long-short strategy is subject to sudden crashes during market recoveries. They decompose the equity beta into two components: equity elasticity, which is primarily affected by financial leverage, and asset beta, which is determined by operating leverage, default risk, and the fraction of growth options in firm value. Also, they suggest a risk-management method that scales HMD by market volatility, putting less weight on the HMD portfolio in high volatility periods thereby decreasing the vulnerability of this zero-cost strategy to sudden crashes. I follow this methodology of scaling by volatility in my analysis, although consider that the probability of default is directly related with the rating notation of the bond portfolios and not indirectly by the company's equity beta. Moreover, Grobys and Äijö (2018) followed the work from Moreira and Muir (2017), that suggested that volatility-managed equity portfolios take less risk when volatility is high or time the volatility, produce large alphas, increase Sharpe ratios, and produce large utility gains for mean-variance investors but extended the analysis to verify the profitability of volatility-managing the Fama and French local risk factors in international equity markets, concluding that volatility-managing adds value for local risk factors in Europe and Asia. This is different from my study as I only base the empirical analysis on bond portfolios and not equity ones and limit it to the US market.

Moreover, Barroso, Detzel, and Maio (2021), studied the weak relationship of risk and return or the low-risk anomaly in which assets with low-risk earn positive abnormal returns or alphas and the opposite applies for high-risk assets, through creating a new factor or anomaly, BAR – Betting Against Risk, which is also a self-financing strategy and consists in buying low-risk stocks and selling high-risk stocks. They conclude that volatility timing used with BAR factors results in significant performance improvements, particularly when ex-ante volatility is low. They argue that the results may be due to leverage constraints in the sense that relatively risk-tolerant investors have the chance of earning higher returns without borrowing through buying high-beta stocks which may appear overvalued. Other reasons might be limits to arbitrage, missing risk factors or the fact that investors may prefer lottery-like payoffs of high-risk stocks. Although in my analysis I study the impact of volatility scaling to verify if it can imply higher risk-adjusted returns in the investment-grade bond portfolios, I do not study the cross-sectional relationship nor test factors or anomalies that could explain those returns.

3. Data

3.1 Monthly data

All the data refers to the United States bond market in USD (United States dollars) and refers to a basket of bonds with the same credit risk and similar maturity. I obtained monthly data from July 1926 until November 2020 (1,133 monthly observations) for BAA, AAA and Long-Term Government Bonds yield to maturity from the work of Goyal and Welch (2008), updated data (up to 2020), which by its way was retrieved from Ibbotson and Harrington (2020), Stocks, Bonds, Bills and Inflation®. The basket is composed of bonds with the same rating notation and for a term of 20 years. Since 1986 the U.S. Treasury stopped issuing 20-year Government Bonds and from October 2001 until January 2006 did not issue 30-year Government Bonds, making the 10-year bond the longest-term Government Bond issued over the October 2001–January 2006 period.

To get the monthly returns from the yield to maturity, I consider that the monthly change in yields is divided by capital and interest gains and therefore calculate the price at the beginning

and at the end of the month. The change in price is the total return obtained for the bond for the monthly period. For portfolio excess returns deducted the risk-free rate.
Risk-free rates (Rf), equity market premium (Rm-Rf), SMB (Small Minus Big), HML (High Minus Low), RMW (Robust Minus Weak), CMA (Conservative Minus Aggressive) equity risk factors, were all obtained from Kenneth French's website Kenneth R. French - Data Library (dartmouth.edu). SMB, HML, RMW and CMA data sample refers to the period from July 1963 until November 2020.

According with Fama & French, 2015, and for the determination of the 5-factor model they do double sorts (2x3), meaning that they create pairs of double factors generating 6 major portfolio combinations: size (SMB) and equity market premium or value-weight (Rm-Rf) against book-to-market (HML), operating profitability (RMW) and investment (CMA).

Furthermore, for SMB, and regarding each pair (SMB and HML, SMB and RMW, SMB and CMA), they divide each pair in terciles (small, neutral and big) and calculate for each one the difference in the average return of the three big, neutral and small stock portfolios in order to obtain nine outputs (3x3). Small and big are the firm's size measured by its market capitalization (market cap) and the breakpoint is the NYSE (New York Stock Exchange) median market cap.

The same principle applies for the remaining factors. For HML (the pair is value-weight vs HML), the sample is split by small (percentile 30^{th}) and big (percentile 70^{th}) of the stocks listed on the NYSE. The factor is computed as the average return of the two value portfolios (small value and big value) minus the average return on the two growth portfolios (small growth and big growth). Value is each stock's B/M (book value to market value ratios). RMW and CMA are calculated exactly the same way as HML, being each firm's operating profitability measured by revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book value of equity and investment, the change in each firm's total assets from one fiscal year compared with the preceding and divided by the total assets observed in the previous year. The breakpoints are the 30^{th} and 70^{th} percentiles of the stocks listed on NYSE. The sample includes all NYSE, AMEX, and NASDAQ stocks on both

CRSP (Center for Research in Security Prices) and Compustat with share codes 10 or 11[6] and for the breakpoint determination, Fama & French, 2015, only considered the NYSE stock universe.

3.2 Daily data

Daily data refers to the United States bond market in USD and comprises a set of daily observations of yield to maturities (8,788 observations that represent 418 monthly observations, considering months of 21 business days) for corporate bond baskets of different classes of risk, BAA and AAA bonds.

BAA and AAA yield to maturity is the Moody's Seasoned Baa or Aaa Corporate Bond Yield, Percent, daily, not seasonally adjusted for bonds with maturity of 20 years and above, downloaded from FRED – Federal Reserve Bank of St. Louis database (https://fred.stlouisfed.org) and corresponds to the period from 02/01/1986 until 01/12/2020.

Long-Term Government Bond yield refers to Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity [DGS10], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DGS10, also for the period from 02/01/1986 until 01/12/2020.

Daily total returns were obtained from yields to maturity and include a capital appreciation plus an interest return component.

Risk-free rates, and the remaining five Fama-French factors were also retrieved from Kenneth French's website.

4. Methodology and results

I study five types of bond portfolios with the same degree of credit risk within each portfolio, namely BAA, AAA, Long-Term Government Bond portfolios and isolate the credit risk by considering a third and fourth portfolios that consist of the Default spread or the difference between excess returns of BAA and AAA bond portfolios and the Long-Term Government Bond portfolio excess return, Fama and French (1993). This last portfolio composed of US Treasuries excess returns may be considered as a proxy to a long-term risk-free rate if we disregard the Term spread or the difference between the yield to maturity of long-term Treasury notes and the risk-free short term interest rate.

[6] The first digit 1 means the security type and the second digit, 0 or 1 in this case, gives more detailed information about the type of security traded. The first digit 1 means ordinary common shares and the second digits 0 means securities which have not been further defined and 1 means securities which need to be further defined.

To compare both strategies, unscaled and scaled, I use first the monthly data for the period from July 1926 until November 2020, the longer dataset, and then consider the same type of bond portfolios but use daily observations to determine monthly data for the shorter period, January 1986 until November 2020, the shorter dataset.

4.1 Summary statistics

Table 1 and Table 3 present summary statistics for unscaled and scaled monthly excess returns for different classes of credit risk of corporate bond monthly observations in the US market from July 1926 until November 2020 and daily observations from January 1986 until November 2020. BAA and AAA represent different classes of corporate bond default risk, being AAA the bond portfolio with the lowest default risk and BAA the first degree of investment-grade, but with a credit risk higher when compared with the AAA bond portfolio. Each rating classification has implicit a certain probability of default. $\bar{X}$ and σ are the annualized mean and standard deviation, respectively for the excess return of each type of corporate bond portfolio. The excess return is calculated deducting the risk-free rate from the monthly observed return.

Table 1: Summary statistics for unscaled excess returns
This table presents descriptive statistics for unscaled (raw) excess returns for July 1926: November 2020 and January 1986: November 2020. The statistics are $\bar{X}$ (annualized average excess return), σ (annualized standard deviation of excess returns), Max (maximum monthly excess return), Min (minimum monthly excess return), Skew (Skewness), Excess Kurtosis and Sharpe Ratio.

	$\bar{X}$ (%)	**σ (%)**	**Max (%)**	**Min (%)**	**Skew**	**Excess Kurtosis**	**Sharpe Ratio**
Panel A - July 1926: Nov 2020							
BAA	3.88	7.32	27.26	-13.26	1.74	30.85	0.53
AAA	2.71	5.81	11.09	-10.83	0.51	8.64	0.47
LTGB	2.21	7.79	15.52	-9.74	0.58	6.96	0.28
Default Spread BAA	1.67	8.65	27.12	-16.72	0.49	17.83	0.19
Default Spread AAA	0.50	6.60	9.69	-9.87	-0.30	4.85	0.08
Panel B - Jan 1986: Nov 2020							
BAA	5.51	15.45	15.92	-22.26	-0.27	2.94	0.36
AAA	4.26	14.67	25.84	-12.22	0.70	3.52	0.29
LTGB	1.86	9.73	14.82	-8.43	0.43	2.02	0.19
Default Spread BAA	3.76	11.20	12.89	-21.81	-1.03	12.32	0.34
Default Spread AAA	2.37	8.07	10.02	-9.96	0.48	2.30	0.29

The variance of monthly returns using monthly data is obtained through EWMA – Exponentially Weighted Moving Average formula:

$$(2)\ \sigma_t^2 = r_t^2$$

Equation (2) for the first observation, in which r is the observed monthly return and σ is the volatility at time t.

$$(3)\ \sigma_t^2 = [(1-\lambda) * (r_{t-1}^2)] + (\lambda * \sigma_{t-1}^2)$$

And equation (3) for the remaining observations.

The decay factor λ, should vary between 0 and 1. I use the same λ values of 0.97 and 0.94 for monthly and daily data respectively as used by Mina and Xiao (2001).

The variance of monthly returns using daily data is obtained through the Realized Variance formula.

$$(4)\ RV_t^2 = \sum_{j=1}^{21} r_j^2,$$

Where r_j^2 is the square of the observed daily returns and the monthly Realized Variance ("RV^2") is the sum of the squared daily returns within the month. Realized Volatility ("RV") is the square root of Realized Variance.

Scaling corresponds to having a weight in the long and short legs that is different from one and varies over time. I assume that this strategy is a zero investment and self-financing strategy and did not include constraints in the leverage levels, being the weights simply the inverse of observed previous month volatility times the constant *c*. The constant is defined so that scaled and unscaled excess returns present the same volatility and apply the formula in (5) to calibrate excess returns.

$$(5)\ r_{\sigma,t} = \frac{c}{\sigma_{t-1}} * r_t$$

The monthly fund excess return is given by r_t and σ_{t-1} is the past month's volatility of returns, estimated from equations (3) and (4).

Table 2: Constant c used for excess return calibration

This table presents the values of the constant *c* used to calibrate excess returns for both data samples, July 1926: Nov 2020 and Jan 1986: Nov 2020, set so that the unscaled and scaled excess returns present the same volatility. σ is the non-annualized monthly volatility of excess returns.

	c (%)	**σ (%)**
Panel A - July 1926: Nov 2020		
BAA	1.97	2.11
AAA	1.60	1.68
LTGB	2.13	2.25
Default Spread BAA	2.23	2.50
Default Spread AAA	1.74	1.91
Panel B.1 - Jan 1986: Nov 2020 (EWMA)		
BAA	3.41	4.46
AAA	3.48	4.23
LTGB	2.36	2.81
Default Spread BAA	2.36	3.23
Default Spread AAA	2.32	2.33
Panel B.2 - Jan 1986: Nov 2020 (RV)		
BAA	3.22	4.46
AAA	3.34	4.23
LTGB	2.31	2.81
Default Spread BAA	2.30	3.23
Default Spread AAA	2.21	2.33

Comparing Table 1 and Table 3, I notice that scaling decreases the excess kurtosis for the different bond portfolios for the longer dataset, being that decrease impressive for BAA bond portfolio (excess kurtosis decreases from 30.85 for raw data compared to 9.72 for scaled data), suggesting that, considering only this excess kurtosis reduction effect, an active strategy based on scaling may be advantageous at least to decrease the crash risk for this class of bonds. The same is not true for the SR, it decreases slightly for scaled excess returns for every bond portfolio but decreases most for the BAA bond portfolio in which the monthly reduction is of 0.17 p.p.. Considering the shorter dataset, the decrease in SR is smaller (0.08 p.p. for BAA portfolio with Realized Volatility) but is consistent, i.e., decreases with scaled excess returns, for all the bond portfolios independently of the credit risk. The only exception in which the SR slightly increases (0.03 p.p.) is in the case of BAA Default Spread estimated using with EWMA volatility.

Cederburg, O'Doherty, Wang, & Yan, May 2020, test 103 equity trading strategies and find no statistical or economic evidence that volatility-managed portfolios systematically generate higher Sharpe Ratios than unmanaged ones, although observe systematically positive alphas for those volatility managed portfolios.

Table 3: Summary statistics for scaled excess returns
This table presents descriptive statistics for scaled excess returns for July 1926: November 2020 and January 1986: November 2020. The statistics are $\overline{X}$ (annualized average excess return), σ (annualized standard deviation of excess returns), Max (maximum monthly excess return), Min (minimum monthly excess return), Skew (Skewness), Excess Kurtosis and annualized Sharpe Ratio.

	$\overline{X}$ (%)	σ (%)	Max (%)	Min (%)	Skew	Excess Kurtosis	Sharpe Ratio
Panel A.1 - July 1926: Nov 2020 (EWMA)							
BAA	2.62	7.32	12.76	-17.44	-1.07	9.72	0.36
AAA	2.59	5.81	10.44	-10.33	-0.57	5.93	0.45
LTGB	2.08	7.79	11.95	-11.18	-0.10	3.20	0.27
Default Spread BAA	1.08	8.65	14.58	-17.69	-1.04	8.26	0.12
Default Spread AAA	0.63	6.60	7.90	-9.94	-0.51	3.40	0.10
Panel B.1 - Jan 1986: Nov 2020 (EWMA)							
BAA	4.72	15.45	26.88	-26.87	-0.33	7.26	0.31
AAA	3.47	14.67	14.76	-19.00	-0.10	2.06	0.24
LTGB	1.70	9.73	10.60	-8.97	0.28	0.96	0.18
Default Spread BAA	4.12	11.20	15.09	-25.18	-1.49	14.64	0.37
Default Spread AAA	2.04	8.07	12.08	-9.79	0.42	2.85	0.25
Panel B.2 - Jan 1986: Nov 2020 (RV)							
BAA	4.28	15.45	25.31	-23.56	-0.26	5.54	0.28
AAA	4.01	14.67	15.54	-22.65	-0.08	2.84	0.27
LTGB	1.75	9.73	12.47	-11.14	0.23	1.63	0.18
Default Spread BAA	3.14	11.20	13.14	-27.64	-1.84	16.37	0.28
Default Spread AAA	1.88	8.07	10.48	-9.67	0.29	2.32	0.23

4.2 Performance evaluation

For performance evaluation I run a set of OLS – Ordinary Least Squares regressions to each bond portfolio, starting with equation (6), to verify the existence of positive and higher alphas when comparing scaled excess returns with unscaled ones. I did that for five different types of credit risk bond portfolios, including the BAA and AAA default spreads. Also, for the period from January 1986 until November 2020 (shorter dataset), volatility was determined using two methods, EWMA and Realized Volatility. Considering this I obtain two series of scaled returns for this shorter dataset. The first regression is to verify if CAPM – Capital Asset Pricing Model, or the Fama-French stock market factor can explain the bond portfolios' excess returns.

$$(6)\ r_{i,t} = \alpha_i + \beta_i MKT_t + \varepsilon_{i,t}$$

In which, $r_{i,t}$ is the bond portfolio excess return (unscaled or volatility-scaled) and MKT is the Fama-French market factor (R_m-R_f).

Table 4: CAPM- Capital Asset Pricing Model

This table presents estimated alpha coefficients (α or the regression constant) and β, the estimated coefficient of Fama-French market factor (Rm-Rf). t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for α and in decimals for β, t-stat and adjusted R2.

	α	**β(Rm-Rf)**	**Adjusted R^2**
Panel A - July 1926: Nov 2020 (unscaled)			
BAA	0.22	0.15	0.14
	(3.78)	**(13.79)**	
AAA	0.20	0.04	0.01
	(4.03)	**(3.97)**	
LTGB	0.21	-0.03	0.01
	(3.09)	**(-2.74)**	
Default Spread BAA	0.01	0.18	0.16
	(0.21)	**(14.45)**	
Default Spread AAA	-0.01	0.07	0.04
	(-0.11)	**(6.82)**	
Panel A.1 - July 1926: Nov 2020 (scaled EWMA)			
BAA	0.14	0.12	0.09
	(2.26)	**(10.85)**	
AAA	0,18	0.05	0.03
	(3.61)	**(5.92)**	
LTGB	0.20	-0.03	0.01
	(2.92)	**(-2.70)**	
Default Spread BAA	-0.02	0.16	0.11
	(-0.24)	**(12.11)**	
Default Spread AAA	0.00	0.08	0.05
	(-0.03)	**(7.70)**	
Panel B - Jan 1986: Nov 2020 (unscaled)			
BAA	0.28	0.24	0.06
	(1.32)	**(5.21)**	
AAA	0.32	0.05	0.00
	(1.53)	(1.06)	
LTGB	0.21	-0.07	0.01
	(1.5)	**(-2.40)**	
Default Spread BAA	0.09	0.32	0.19
	(0.60)	**(10.05)**	
Default Spread AAA	0.11	0.12	0.05
	(0.96)	**(4.89)**	
Panel B.1 - Jan 1986: Nov 2020 (EWMA)			
BAA	0.18	0.25	0.06
	(0.84)	**(5.26)**	
AAA	0.27	0.08	0.01
	(1.31)	(1.82)	
LTGB	0.19	-0.06	0.01
	(1.39)	**(-2.14)**	
Default Spread BAA	0.04	0.30	0.18
	(0.30)	**(9.55)**	
Default Spread AAA	0.06	0.13	0.06
	(0.57)	**(5.13)**	

	α	β(Rm-Rf)	Adjusted R^2
Panel B.2 - Jan 1986: Nov 2020 (RV)			
BAA	0.22	0.24	0.06
	(1.03)	**(5.08)**	
AAA	0.22	0.09	0.01
	(1.06)	**(2.05)**	
LTGB	0.19	-0.07	0.01
	(1.39)	**(-2.32)**	
Default Spread BAA	0.14	0.28	0.15
	(0.96)	**(8.67)**	
Default Spread AAA	0.08	0.12	0.05
	(0.74)	**(4.73)**	

Comparing the results of the unscaled excess returns with the scaled ones for the different portfolios and both datasets, as in
Table 4, I detect that the difference in alphas is in general negative for the scaled sample compared with the unscaled results and that difference is higher for the lower rated portfolio (the only exception is the BAA default spread scaled with RV in which the alpha increases 0.06 p.p. for the scaled strategy). The highest decrease in monthly alpha is detected for BAA bonds (0.10 p.p.). That difference decreases to 0.01 p.p. for the Long-Term Government Bond.

As expected, considering that the higher the probability of debt default the more similar are debt instruments to equity ones, the excess return over the market risk premium explains a great proportion of the monthly BAA bond unscaled and scaled excess returns (0.15 and 0.12 - longer dataset and 0.24 and 0.25 – shorter dataset respectively) when compared with monthly AAA bond unscaled and scaled excess returns which ranges from a minimum of 0.04 for the unscaled longer dataset and a maximum of 0.09 for the shorter dataset scaled with Realized volatility.

Observing the default spread regressions (BAA and AAA) thus leaving only the pricing of the credit risk in the bond portfolio excess returns, I conclude that alphas are inexistent as by observing *t-stat* values do not reject the hypothesis that the estimated parameters are zero and therefore there is no alpha or risk-adjusted returns for these default spreads.

Even though the decrease in alphas for the scaled versions, an investor who buys only the unscaled types of the bond portfolios can benefit from risk-adjusted returns as alphas are positive for the analyzed samples, those alphas decrease and tend to zero with scaling.

Continuing the analysis, I tested if the Fama-French five factor model can explain the bond portfolio excess returns, using equation (7).

$$(7)\; r_{i,t} = \alpha_i + \beta_i\, MKT_t + s_i\, SMB_t + h_i\, HML_t + w_i\, RMW_t + c_i\, CMA_t + \varepsilon_{i,t}$$

Where, $r_{i,t}$ is the bond portfolio excess return (unscaled or volatility-scaled) and MKT, SMB, HML, RMW, CMA are, respectively, market, size, value, profitability and investment Fama-French factors.

I notice from Table 5 that the Fama-French five factor model presents a poor performance when explaining the bond portfolio excess returns, only the market factor presents in general statistically significant values, and I cannot reject the hypothesis that the remaining factor estimated parameters are null. This explanatory power increases significantly when the credit risk is isolated. Monthly estimated *Beta* coefficient of BAA Default Spread is of 0.207 and 0.211 (longer dataset, unscaled and scaled respectively) and 0.321, 0.302 and 0.280 (shorter dataset, unscaled, scaled with EWMA and RV volatility, respectively) all presenting statistically significant *t-stat* values. Not surprisingly, this estimated parameter is negative or null when I observe Long-Term Government Bond results, which may be due to the safe-haven function of this type of debt instrument and since its returns might be better explained by factors different from the equity market factors.

Table 5: Fama-French 5 factor model

This table presents estimated alpha coefficients (α or the regression constant), β, the estimated coefficient of Fama-French market factor (Rm-Rf), *s*, the estimated coefficient of Fama-French SMB factor, *h*, the estimated coefficient of Fama-French HML factor, *w*, the estimated coefficient of Fama-French RMW factor and *c*, the estimated coefficient of Fama-French CMA factor. t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for α and in decimals for β, *s*, *h*, *w* and *c* estimated coefficients, t-stat values and adjusted R^2.

	α	Rm-Rf	SMB	HML	RMW	CMA	Adjusted R^2
Panel A - July 1926: Nov 2020 (unscaled)							
BAA	0.25	0.13	-0.06	0.04	0.00	0.03	0.06
	(3.22)	**(6.69)**	**(-2.29)**	(1.03)	(0.00)	(0.50)	
AAA	0.23	0.06	-0.10	-0.01	0.01	0.03	0.02
	(2.75)	**(3.24)**	**(-3.44)**	(-0.18)	(0.29)	(0.59)	
LTGB	0.29	-0.08	-0.04	-0.06	-0.00	0.02	0.02
	(2.61)	**(-2.98)**	(-1.03)	(-1.27)	(-0.08)	(0.26)	
Default Spread BAA	-0.04	0.21	-0.02	0.10	0.00	0.01	0.11
	(-0.39)	**(8.59)**	(-0.66)	**(2.23)**	(0.10)	(0.10)	
Default Spread AAA	-0.06	0.15	-0.06	0.06	0.02	0.01	0.06
	(-0.69)	**(6.57)**	(-1.86)	(1.39)	(0.36)	(0.22)	

	α	Rm-Rf	SMB	HML	RMW	CMA	Adjusted R^2
Panel A.1 - July 1926: Nov 2020 (scaled EWMA)							
BAA	0.08	0.15	-0.05	0.04	-0.00	0.05	0.07
	(0.93)	**(7.29)**	(-1.62)	(0.97)	(-0.05)	(0.89)	
AAA	0.07	0.07	-0.07	-0.03	0.01	0.09	0.03
	(1.04)	**(4.14)**	**(-3.06)**	(-1.03)	(0.33)	(1.83)	
LTGB	0.18	-0.08	-0.03	-0.09	0.01	0.05	0.03
	(1.99)	**(-3.68)**	(-0.83)	**(-2.18)**	(0.12)	(0.78)	
Default Spread BAA	-0.11	0.21	-0.03	0.10	0.01	0.01	0.12
	(-1.20)	**(9.15)**	(-0.84)	**(2.40)**	(0.12)	(0.16)	
Default Spread AAA	-0.08	0.13	-0.05	0.05	0.01	0.03	0.07
	(-1.06)	**(7.05)**	(-1.83)	(1.46)	(0.31)	(0.57)	
Panel B - Jan 1986: Nov 2020 (unscaled)							
BAA	0.22	0.28	-0.05	0.09	0.09	0.01	0.06
	(1.00)	**(5.20)**	(-0.66)	(0.90)	(0.84)	(0.09)	
AAA	0.25	0.09	-0.13	-0.04	0.11	0.06	0.01
	(1.14)	(1.78)	(-1.70)	(-0.43)	(1.12)	(0.39)	
LTGB	0.18	-0.04	-0.16	-0.05	0.05	0.00	0.04
	(1.28)	(-1.31)	**(-3.08)**	(-0.88)	(0.82)	(0.03)	
Default Spread BAA	0.05	0.32	0.10	0.13	0.03	0.01	0.21
	(0.35)	**(9.06)**	(1.93)	**(2.08)**	(0.47)	(0.14)	
Default Spread AAA	0.06	0.14	0.02	0.02	0.06	0.05	0.05
	(0.55)	**(4.84)**	(0.53)	(0.31)	(1.02)	(0.64)	
Panel B.1 - Jan 1986: Nov 2020 (EWMA)							
BAA	0.14	0.27	-0.07	0.12	0.09	-0.06	0.06
	(0.63)	**(5.14)**	(-0.92)	(1.24)	(0.83)	(-0.43)	
AAA	0.22	0.12	-0.13	-0.01	0.10	0.00	0.01
	(1.02)	**(2.31)**	(-1.74)	(-0.11)	(0.98)	(0.02)	
LTGB	0.16	-0.04	-0.15	-0.09	0.07	0.01	0.04
	(1.14)	(-1.09)	**(-3.02)**	(-1.47)	(1.04)	(0.13)	
Default Spread BAA	0.04	0.30	0.07	0.21	-0.02	-0.04	0.20
	(0.27)	**(8.49)**	(1.28)	**(3.20)**	(-0.32)	(-0.42)	
Default Spread AAA	0.04	0.14	0.00	0.03	0.01	0.05	0.05
	(0.31)	**(4.95)**	(0.01)	(0.58)	(0.24)	(0.64)	
Panel B.2 - Jan 1986: Nov 2020 (RV)							
BAA	0.18	0.27	-0.07	0.11	0.10	-0.07	0.06
	(0.81)	**(4.98)**	(-0.93)	(1.10)	(0.98)	(-0.48)	
AAA	0.17	0.13	-0.14	0.00	0.10	0.00	0.01
	(0.78)	**(2.52)**	(-1.76)	(0.00)	(1.00)	(-0.03)	
LTGB	0.16	-0.04	-0.15	-0.08	0.06	0.03	0.04
	(1.13)	(-1.20)	**(-3.05)**	(-1.30)	(0.85)	(0.30)	
Default Spread BAA	0.14	0.28	0.06	0.18	-0.02	-0.02	0.17
	(0.90)	**(7.72)**	(1.19)	**(2.74)**	(-0.34)	(-0.20)	
Default Spread AAA	0.06	0.13	0.00	0.03	0.00	0.05	0.05
	(0.51)	**(4.56)**	(0.02)	(0.69)	(0.08)	(0.62)	

Volatility scaling in these samples presents a negative effect in the regressions as monthly alphas decrease for all types of bond portfolios and for both datasets (major decrease for BAA portfolio of 0.17 p.p. for the longer dataset). The only exception, in which I note a slight increase in alpha with volatility scaling, is for the BAA Default Spread scaled using RV for the shorter dataset (monthly alpha of 0.09 p.p.). Also, observing regression estimated parameters, I conclude that besides the market factor, the remaining Fama-French factors do not present statistically significant values and thus may not reject the hypotheses that they are null and have no explanatory power to explain these bond portfolio excess returns.

These results are consistent with the work presented by Bektic, Wenzler, Wegener, Schiereck, and Spielmann (2019), in which they tested if Fama-French equity market factors can explain corporate bond return dynamics. They conclude that those equity factors are not fully translated to fixed income markets, and one reason could be market segmentation as equity and debt present a different position in a company's capital structure. Also, this market segmentation can arise as corporate bond markets are dominated by institutional investors with different risk perception compared to individual investors. Structural credit risk models that study the relationship between the company's credit risk and its capital structure, claim that as equity and debt belong to the same company and in order to avoid arbitrage, the risk premium in equity and bond markets shall be related. In this case the authors notice that cross sectional return premiums in US corporate bonds are not equal to those in the equity markets, implying market segmentation. They defend that Fama-French factors present a poor explanatory capacity to explain bond portfolios excess returns.

This may also be related with the type of credit risk embedded in each bond portfolio. Either BAA, AAA and Long-Term Government Bond are investment-grade bond portfolios, considering that a corporate bond is the sum of a risk-free component and an equity component, in the case of investment-grade bond portfolios the risk-free component is higher (probability of default lower) when comparing with non-investment grade portfolios and in this last case more similar to equity instruments.

Therefore, with debt instruments with higher equity related features and riskier in terms of credit quality, like the non-investment-grade bond portfolios, and since Fama-French factors show a significant explanatory power to explain equity market returns, it is possible that the Fama-French model is more adequate in explaining excess returns than in my analysis.

Continuing the work and for performance evaluation, I realize another test in order to verify if there are positive alphas for a regression in which the independent variables are the market factor and the Long-Term Government Bond excess returns as per equation (8).

$$(8)\ r_{i,t} = \alpha_i + \beta_i\, MKT_t + l_i\, LTGB_t + \varepsilon_{i,t}$$

In which, $r_{i,t}$ is the bond portfolio excess return (unscaled or volatility-scaled), MKT is Fama-French market factor and LTGB is the Long-Term Government Bond excess return.

Table 6: Excess returns against Market factor and LTGB excess returns
This table presents estimated alpha coefficients (α or the regression constant), β, the estimated coefficient of Fama-French market factor (Rm-Rf) and, *l,* the estimated coefficient of LTGB excess returns. t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for α and in decimals for β and *l* estimated coefficients, t-stat values and adjusted R^2. n.a. means not applicable.

	α	**Rm-Rf**	**LTGB**	**Adjusted R^2**
Panel A - July 1926: Nov 2020 (unscaled)				
BAA	0.15	0.16	0.36	0.29
	(2.75)	**(16.28)**	**(15.06)**	
AAA	0.11	0.05	0.43	0.34
	(2.75)	**(6.76)**	**(23.72)**	
LTGB	n.a.	n.a.	n.a.	
Default Spread BAA	0.15	0.16	-0.64	0.49
	(2.75)	**(16.28)**	**(-27.11)**	
Default Spread AAA	0.11	0.05	-0.57	0.49
	(2.75)	**(6.76)**	**(-31.63)**	
Panel A.1 - July 1926: Nov 2020 (scaled EWMA)				
BAA	0.07	0.13	0.32	0.21
	(1.25)	**(12.57)**	**(12.64)**	
AAA	0.11	0.07	0.33	0.22
	(2.48)	**(7.98)**	**(16.88)**	
LTGB	n.a.	n.a.	n.a.	
Default Spread BAA	0.10	0.14	-0.59	0.39
	(1.79)	**(12.72)**	**(-22.79)**	
Default Spread AAA	0.10	0.06	-0.48	0.36
	(2.13)	**(7.46)**	**(-23.65)**	
Panel B - Jan 1986: Nov 2020 (unscaled)				
BAA	0.04	0.33	1.15	0.58
	(0.30)	**(10.42)**	**(22.76)**	
AAA	0.05	0.14	1.31	0.75
	(0.43)	**(6.16)**	**(35.05)**	
LTGB	n.a.	n.a.	n.a.	
Default Spread BAA	0.05	0.33	0.15	0.21
	(0.38)	**(10.42)**	**(2.99)**	
Default Spread AAA	0.04	0.14	0.30	0.18
	(0.43)	**(6.18)**	**(8.16)**	

	α	Rm-Rf	LTGB	Adjusted R^2
Panel B.1 - Jan 1986: Nov 2020 (EWMA)				
BAA	-0.06	0.33	1.14	0.57
	(-0.40)	**(10.33)**	**(22.21)**	
AAA	0.01	0.17	1.26	0.70
	(0.08)	**(6.86)**	**(30.90)**	
LTGB	n.a.	n.a.	n.a.	
Default Spread BAA	0.00	0.32	0.19	0.20
	(0.02)	**(10.08)**	**(3.81)**	
Default Spread AAA	0.01	0.15	0.28	0.16
	(0.05)	**(6.28)**	**(7.40)**	
Panel B.2 - Jan 1986: Nov 2020 (RV)				
BAA	0.00	0.32	1.13	0.55
	(-0.09)	**(9.87)**	**(21.59)**	
AAA	-0.04	0.18	1.25	0.68
	(-0.34)	**(7.08)**	**(29.90)**	
LTGB	n.a.	n.a.	n.a.	
Default Spread BAA	0.10	0.29	0.21	0.18
	(0.68)	**(9.25)**	**(4.03)**	
Default Spread AAA	0.03	0.14	0.28	0.16
	(0.24)	**(5.85)**	**(7.38)**	

I detect that the estimated alphas or risk-adjusted returns seem to be not statistically significant for the shorter dataset, BAA and AAA bond portfolio excess returns, and the corresponding Default Spreads, meaning that these portfolios' excess returns can be essentially explained by the market factor excess return and the LTGB excess returns. This assessment is not the same for the longer dataset. I notice there the existence of positive alphas or risk-adjusted returns for all the bond portfolios, BAA, AAA and Default Spreads, considering raw or scaled excess returns. From these results, it might be worthwhile for an investor to hold corporate bond portfolios that present these positive alphas and not simply a combination of instruments related with stock market premiums and Treasury securities.

I verify from Table 6 that for the longer dataset, alphas decrease for scaled returns for all the bond portfolios being that decrease larger for the BAA bond portfolio (-0.08 p.p. monthly). Regarding the shorter dataset, I observe that all the alpha *t-stat* values are within the rejection area and therefore alphas may be zero, either for unscaled or scaled bond returns, confirming that volatility scaling does not increase risk adjusted returns for the analyzed sample.

Furthermore, there are significant Adjusted R-Squared (R^2) values, maximum of 0.68 for the AAA unscaled bond portfolio for the longer dataset and thus I may conclude that market factor combined with LTGB excess returns presents a significant explanatory power to explain the AAA corporate bond excess returns.

Considering Merton's (1973) findings that corporate debt can be considered as a combination of risk-free instruments and equity, I may conclude that a significant part of the corporate bond portfolios (BAA and AAA) excess returns is composed by the risk-free rate, which is true for investment grade bond portfolios as this is the case hereby studied. I assume for simplicity that the LTGB excess returns are equivalent to the risk-free rate, ignoring the Term factor in this analysis.

The risk-free component or the monthly estimated LTGB parameter increases for AAA bond portfolio in all datasets, scaled or unscaled, and the same applies for the default spread portfolios, in which the risk-free component is less for the BAA default spread portfolio compared with the AAA one. This is consistent with the Merton Model (1973) and the findings of Bektic, Wenzler, Wegener, Schiereck, and Spielmann (2019).

I regressed the excess returns of volatility managed bond portfolios (scaled) against the original (unscaled) ones to verify the existence of positive alphas as per equation (9). The existence of positive alphas or positive intercepts in this regression means that the volatility managed portfolios may increase Sharpe Ratios and expand the mean-variance frontier relative to unscaled strategies, Moreira and Muir (2017).

This method of verifying the existence of positive and statistically significant alphas in the scaled against unscaled regressions was also used in the most important studies of volatility-management literature. Cederburg, O'Doherty, Wang, and Yan (2020), also performed this analysis in their study regarding equity trading strategies and the evidence of positive alphas in volatility-managed portfolios. Barroso and Detzel (2021) also used this methodology of checking the existence on non-zero alphas to measure abnormal returns by running regressions on a set of equity factors and Sharpe-Ratios, of excess returns over unmanaged ones.

$$(9)\ Sr_{i,t} = \alpha_i + \beta_i\, Ur_{i,t} + \varepsilon_{i,t}$$

Where, $Sr_{i,t}$ is the bond portfolio scaled excess return and $Ur_{i,t}$ is the bond portfolio unscaled excess return.

I notice from Table 7 that all portfolios for both datasets present alpha values that vary from a minimum of -0.07 p.p. monthly (*t-stat*=-0.76) for the BAA bond portfolio considering the shorter dataset with volatility determined using EWMA and a maximum of 0.06 p.p. monthly (*t-stat*=0.89) for the BAA Default spread portfolio also considering the shorter dataset with

Realized Volatility. Looking at the *t-stat* values for all the portfolios I may conclude that cannot reject the hypothesis that all the alpha coefficients are zero (whether the long or short dataset and independently of the volatility estimation formula) and therefore that volatility timing strategies do not increase Sharpe-Ratios, nor expand the mean-variance frontier for the portfolios herein studied.

Table 7: Scaled excess returns against unscaled excess returns
This table presents estimated alpha coefficients (α or the regression constant), β, the estimated coefficient of unscaled excess returns. t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for α and in decimals for β estimated coefficient, t-stat values and adjusted R^2.

	α	**Unscaled excess returns**	**Adjusted R^2**
Panel A - July 1926: Nov 2020			
BAA	-0.06	0.86	0.73
	(-1.76)	**(55.47)**	
AAA	0.03	0.84	0.70
	(0.95)	**(51.63)**	
LTGB	0.02	0.85	0.72
	(0.49)	**(53.40)**	
Default Spread BAA	-0.03	0.88	0.77
	(-0.91)	**(61.94)**	
Default Spread AAA	0.02	0.87	0.75
	(0.57)	**(58.29)**	
Panel B.1 - Jan 1986: Nov 2020 (EWMA)			
BAA	-0.07	0.92	0.84
	(-0.76)	**(47.67)**	
AAA	0.01	0.92	0.84
	(0.11)	**(46.32)**	
LTGB	0.00	0.95	0.90
	(-0.05)	**(62.46)**	
Default Spread BAA	-0.03	0.92	0.85
	(-0.45)	**(48.77)**	
Default Spread AAA	-0.03	0.93	0.87
	(-0.67)	**(53.09)**	
Panel B.2 - Jan 1986: Nov 2020 (RV)			
BAA	-0.02	0.90	0.80
	(-0.19)	**(41.29)**	
AAA	-0.03	0.90	0.81
	(-0.34)	**(42.56)**	
LTGB	0.00	0.94	0.89
	(-0.09)	**(57.32)**	
Default Spread BAA	0.06	0.91	0.83
	(0.89)	**(44.52)**	
Default Spread AAA	-0.01	0.92	0.85
	(-0.26)	**(48.26)**	

4.3 Robustness testing

Furthermore, I perform a set of tests that lead me to confirm that volatility scaling strategies do not present benefits in the analyzed bond portfolios.

First, regress the unscaled excess returns of the five bond portfolios, considering the longer and shorter datasets, with the volatility observed for the previous period as per equation (10).

$$(10)\ Ur_{i,t} = c_i + B_i\, RV_{i,t-1} + \varepsilon_{i,t}$$

In which, $Ur_{i,t}$ is the bond portfolio unscaled excess return and $RV_{i,t-1}$ is the volatility of returns of the previous period, either EWMA estimated volatility or historic Realized Volatility.

This is a measure of the risk-return relationship or risk-return trade-off. Risk neutral investors usually require the same level of excess return for incurring an additional unit of risk or the same level of risk implicit in that rate of return, while risk-averse investors require an excess return higher than the implicit level of risk in the corresponding level of return.

The measurement of the risk-return trade-off relation to justify the existence of benefits in the volatility management strategies seems to be standard in the literature. Cederburg, O'Doherty, Wang, and Yan (2020) state that volatility management is likely to be successful if volatility is persistent and risk-return trade-off is flat. Positive risk-return trade-off makes volatility management less effective. Barroso and Maio (2021), also studied the risk-return trade-off relation among equity factors through using Realized Volatility. Wang and Yan (2021), on their study of the performance of volatility-managed portfolios scaling by downside volatility exhibit also use this risk-return trade-off relation as a measure of strategy performance.

I verify from Table 8 that there is a strong risk and positive risk-return relationship for the BAA portfolio considering the longer dataset, namely it presents an estimated monthly *Beta* coefficient of 0.31 and a *t-stat* value of 5.67. Considering this sample, the only exception is concerning the AAA Default spread in which I do not perceive the same results, the monthly *Beta* estimated coefficient is 0.02 and the *t-stat* value of 0.25 lead me not to reject the hypothesis that the estimated coefficient could be zero thus making that risk-return trade-off relationship inexistent for this bond portfolio.

Table 8: Relationship of unscaled excess returns with previous month volatility

This table presents estimated coefficients (*c* or the regression constant) and *B*, the estimated coefficient of previous period volatility. t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for *c* and in decimals for *B* estimated coefficient, t-stat values, adjusted R^2 and the correlation coefficient (ρ).

	c	Vol_{t-1}	Adjusted R^2	ρ (correlation coefficient)
Panel A - July 1926: Nov 2020				
BAA	-0.24	0.31	0.03	0.17
	(-2.06)	**(5.67)**		
AAA	-0.02	0.17	0.01	0.10
	(-0.25)	**(3.29)**		
LTGB	-0.05	0.12	0.00	0.06
	(-0.37)	**(2.18)**		
Default Spread BAA	-0.24	0.18	0.01	0.09
	(-1.60)	**(2.88)**		
Default Spread AAA	0.02	0.02	0.00	0.01
	(0.14)	(0.25)		
Panel B.1 - Jan 1986: Nov 2020 (EWMA)				
BAA	-1.31	0.46	0.02	0.15
	(-2.10)	**(3.02)**		
AAA	-0.33	0.17	0.00	0.07
	(-0.62)	(1.39)		
LTGB	-0.07	0.09	0.00	0.03
	(-0.16)	(0.54)		
Default Spread BAA	-1.08	0.60	0.03	0.19
	(-2.82)	**(3.98)**		
Default Spread AAA	-0.43	0.25	0.01	0.13
	(-1.65)	**(2.65)**		
Panel B.2 - Jan 1986: Nov 2020 (RV)				
BAA	-0.7	0.31	0.01	0.11
	(-1.24)	**(2.22)**		
AAA	-0.66	0.26	0.01	0.11
	(-1.38)	**(2.35)**		
LTGB	-0.16	0.13	0.00	0.04
	(-0.42)	(0.88)		
Default Spread BAA	-0.36	0.30	0.01	0.10
	(-1.03)	**(2.15)**		
Default Spread AAA	-0.47	0.27	0.02	0.15
	(-1.98)	**(3.19)**		

Considering the shorter dataset, I do not perceive the risk-return trade-off relationship for the LTGB portfolio, considering volatility estimation trough EWMA and Realized Volatility, monthly *Beta* coefficients are respectively 0.09 and 0.13, while *t-stat* values are 0.54 and 0.88 respectively. The strongest risk-return relationship, as in the longer dataset, is observed for the BAA bond portfolio, the lowest credit risk bond portfolio with monthly estimated *Beta* coefficient of 0.46 (*t-stat* of 3.02) in the case of volatility estimation using EWMA and 0.31 (*t-stat* of 2.22) in the case of Realized Volatility, although the adjusted R^2 values are not significant, 0.02 and 0.01 respectively.

The existence of a risk-return trade-off positive relation that confirms that volatility scaling strategies do not increase risk-adjusted returns and Sharpe Ratios in our bond portfolios, is consistent with the intuition formulated in the work of Moreira and Muir (2017), in which is stated that investors should be more risk-averse when volatility increases and risk is high.

Yet, unlike equities where risk-return trade-off relations are empirically weak, for bonds they are relevant. If investors face a positive and strong risk-return trade-off and time volatility, leaving the market and selling their positions when volatility is high, they will not benefit from the excess returns of the periods with high volatility, meaning that the risk-adjusted returns or alpha tend to zero. That is the case of my analysis.

Observing the effects of volatility in explaining the cross-section of corporate bond returns, Chung, Wang, and Wu (2019) stated that bonds with lower ratings tend to have a higher exposure to aggregate volatility risk and have a larger volatility risk premium, stating that aggregate volatility risk is priced in the cross-section of corporate bonds within relatively homogeneous groups bearing the same credit rating. Also, they found positive cross-sectional relation between idiosyncratic (specific) bond volatility and expected bond returns and that relation is stronger for lower-grade bonds. Chordia, Goyal, Nozama, Subrahmanyam and Tong (2017) also studied the effects of idiosyncratic volatility in returns and concluded that although corporate bond market's volatility is non-negligent, for investment-grade bonds the volatility of returns is about 1/5 of the volatility detected for the equity market, idiosyncratic volatility is not a good predictor of bond returns. Firms with greater levels of real investments have lower required returns and small firms and those with low or negative profits are considered riskier by bond market investors and thus require higher rates of return.

Moreover, I perform another robustness test to confirm the results of this analysis, namely an AR(1) regression analysis of Realized Volatility for the shorter dataset. As by definition the volatility estimated using EWMA depends on its previous values, I exclude volatility values estimated using that methodology and concentrate only on the shorter dataset. Equation (11) and Table 9, present the analysis performed and the results obtained.

$$(11)\ RV_{i,t} = c_i + B_i\, RV_{i,t-1} + \varepsilon_{i,t}$$

Where, $RV_{i,t}$ is the Realized Volatility of bond portfolio returns at time t and $RV_{i,t-1}$ is the Realized Volatility of bond portfolio returns at time t-1.

I notice a strong relation between current and previous monthly Realized Volatility values of returns, indicated by the estimated *Beta* coefficients that vary between a maximum of 0.60 for the BAA bond portfolio (*t-stat*=15.34 and adjusted R^2=0.36) and a minimum estimated *Beta* coefficient of 0.48 for the AAA Default spread portfolio (*t-stat*=11.22 and adjusted R^2=0.23).

Also, I detect that volatility of returns is persistent over time as autoregressive coefficients are strongly significant and above 50, Barroso and Maio (2021). The highest AR(1) coefficient detected is for the BAA bond portfolio meaning that Realized Volatility presents a significant predictive power to estimate future returns.

Table 9: Relationship of RVt and RVt-1

This table presents estimated coefficients (*c* or the regression constant) and *B*, the estimated coefficient of previous period volatility. t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for *c* and in decimals for *B* estimated coefficient, t-stat values and adjusted R^2.

	c	**Vol $_{t-1}$**	**Adjusted R^2**
Panel B - Jan 1986: Nov 2020			
BAA	1.51	0.60	0.36
	(9.41)	**(15.34)**	
AAA	1.68	0.57	0.32
	(9.69)	**(14.01)**	
LTGB	1.10	0.55	0.30
	(10.33)	**(13.31)**	
Default Spread BAA	1.07	0.53	0.28
	(10.10)	**(12.78)**	
Default Spread AAA	1.26	0.48	0.23
	(10.61)	**(11.22)**	

Finally, I execute a test to study the relationship between bond portfolios' yield to maturity (*YTM*) and previous month volatility, as per equation (12).

$$(12)\ YTM_{i,t} = c_i + B_i\, RV_{i,t-1} + \varepsilon_{i,t}$$

In which, $YTM_{i,t}$ is the bond portfolio yield to maturity at time t and $RV_{i,t-1}$ is the observed volatility of returns at time t-1 (EWMA volatility and Realized Volatility).

I determine from Table 10 that for the longer dataset there is a positive and strong relationship between yield to maturity, or the expected return an investor would face if maintained its bondholding until the maturity and reinvested the returns received at the same yield to maturity, and previous period volatility. I would expect that like the risk-return trade-off and being the yield to maturity a measure of return, that higher risk or high volatility would imply a higher required yield to maturity by the investor. This is noticed for the longer dataset, in which this relationship is strong with monthly *Beta* coefficients ranging from a minimum of 0.09 (*t-stat*=6.33) in the case of AAA default spread portfolio and 2.32 (*t-stat*=44.19) for the AAA bond portfolio.

The conclusions for the longer dataset are not the same as for the shorter one. I perceive a negative relationship, still strong but less strong that in the longer dataset, between yield to maturity and previous period volatility for BAA and AAA bond portfolios, both considering volatility estimated through EWMA and Realized Volatility. The strongest negative results are observed for the AAA bond portfolio (monthly *Beta* coefficient of -0.40 and *t-stat* of -7.12 for the portfolio with volatility estimated using EWMA and *Beta* coefficient of -0.31 and *t-stat* of -6.00 for the portfolio with volatility estimated using Realized Volatility formula).

Longstaff and Schwartz (1993) in their study of Interest Rate Volatility and Bond Prices, provide a possible explanation for this negative relationship as they refer that the uncertainty of future returns means that investors are willing to pay more for securities that allow them to lock in a guaranteed rate of return when uncertainty increases, thus increasing the prices paid for those debt assets and therefore reducing the yield to maturity, justifying somehow the negative detected relation for the BAA and AAA bond portfolios presented in my analysis.

Table 10: Relationship of YTM (yield to maturity) and previous month volatility

This table presents estimated coefficients (*c* or the regression constant) and *B*, the estimated coefficient of previous period volatility. t-stat values are presented in brackets, values in bold are statistically significant for a significance level of 5%, two-sided test. Values in percentage points for *c* and in decimals for *B* estimated coefficient, t-stat values, adjusted R^2 and the correlation coefficient (ρ).

	c	Vol $_{t-1}$	Adjusted R^2 (%)	ρ (correlation coefficient)
Panel A - July 1926: Nov 2020				
BAA	4.86	1.08	0.18	0.42
	(32.40)	**(15.59)**		
AAA	2.36	2.32	0.63	0.80
	(25.86)	**(44.19)**		
LTGB	1.60	1.77	0.59	0.77
	(16.10)	**(40.69)**		
Default Spread BAA	0.83	0.46	0.35	0.59
	(17.82)	**(24.69)**		
Default Spread AAA	0.56	0.09	0.03	0.19
	(22.25)	**(6.33)**		
Panel B.1 - Jan 1986: Nov 2020 (EWMA)				
BAA	8.14	-0.25	0.03	-0.17
	(28.97)	**(-3.61)**		
AAA	7.79	-0.40	0.11	-0.33
	(32.40)	**(-7.12)**		
LTGB	3.44	0.57	0.04	0.20
	(9.82)	**(4.24)**		
Default Spread BAA	1.34	0.42	0.37	0.61
	(19.66)	**(15.86)**		
Default Spread AAA	0.96	0.16	0.17	0.41
	(20.10)	**(9.16)**		
Panel B.2 - Jan 1986: Nov 2020 (RV)				
BAA	7.90	-0.19	0.02	-0.15
	(31.06)	**(-3.02)**		
AAA	7.40	-0.31	0.08	-0.28
	(33.65)	**(-6.00)**		
LTGB	3.82	0.42	0.03	0.17
	(12.35)	**(3.57)**		
Default Spread BAA	1.59	0.33	0.27	0.52
	(23.77)	**(12.44)**		
Default Spread AAA	1.04	0.13	0.13	0.37
	(23.79)	**(8.09)**		

5. Conclusions

Unlike for equity instruments, scaling returns of the investment-grade bond portfolios herein analyzed does not increase risk-adjusted returns or alphas, nor the Sharpe Ratios. The scaling strategy seems to work even worse for the BAA bond portfolio, the one with the least proportion of the risk-free component from my sample.

This may be due to diverse reasons like the mean-reversion of returns in the long-run; many bondholders are long-term investors who often follow a buy and hold strategy so that increases in volatility can be considered as short-term movements that do not affect their long-term view of returns, bearing in mind that in the long-run those expected returns revert to its mean. The long-term view is consistent with the work of Moreira and Muir (2019) who concluded that due to the mean reversion existent in stock returns, investors with long time horizons should not view increases in volatility as increases in risk and therefore should not adopt a short-term approach selling their positions when volatiliy starts to increase, as suggested with timing volatility strategies. This long-term approach of bond investors is also according with the fact that US bond market is dominated by institutional investors, namely pension funds and life insurance companies, who typically face a long-term investment horizon.

Transaction costs related with trade sizes can be another reason to explain my findings, as very often is noticed that the higher the trade size the lower the associated transaction costs, thus keeping aside from the corporate bond market the individual investors that usually trade small tickets and confirming the institutional investors strong presence in the bond markets.

Low liquidity levels could be another reason to explain these findings. US bond market faces a considerable number of over-the-counter transactions compared with the stock market, Bai, Bali, and Wen (2019) suggested that as bondholders are long-term investors who often follow a buy-and-hold strategy and the market is dominated by those kinds of investors, liquidity in the corporate bond market is lower compared with the stock market in which active trading is partially attributable to the existence of individual investors.

Regulatory constraints may distort bond market dynamics as incomplete diversification or high idiosyncratic bond volatility of returns may arise from regulations. The case of insurance companies and pension funds, which often only invest in certain types of bonds due to regulatory constraints as stated in the work of Chung, Wang, and Wu (2019) is an example.

The bond's probability of default implicit in the portfolios can be another reason for the results achieved. The bond portfolios that I analyzed are all investment-grade, meaning that the implicit credit risk and probability of default are lower contrasted with below investment-grade portfolios. These non-investment grade riskier portfolios present more equity like features and a lower component of risk-free in their expected returns compared with investment-grade bond portfolios. It might be the case, save for the remaining reasons mentioned, that volatility scaling actually works for these riskier bond portfolios vs the buy-and-hold strategies.

I believe in the existence of some limitations in my findings; besides the mentioned rating notation of the bond portfolios and its embedded probability of default, volatility-managed strategy might work well in high-yield markets due to the quasi-equity features of the non-investment grade bonds. Also, controlling for transaction costs and using different methods to determine volatility (GARCH[7] or other stochastic volatility models) or even performing the same analysis but in other international markets with different bondholding characteristics, might lead to different conclusions from the ones hereby presented.

[7] GARCH or Generalized AutoRegressive Conditional Heteroskedasticity models assume that the variance of the error term follows an autoregressive moving average process.

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