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Equity Index Returns Predictability and Fama-French factors: a frequency domain analysis

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Equity Index Returns Predictability and Fama-French factors: a frequency domain analysis

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by

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Resumo

Extendemos o trabalho de Faria e Verona (2020) para o modelo de 5 fatores Fama French para prever o prémio de risco de mercado do índice S&P500.

O modelo de 5 factores Fama French é decomposto através do método *maximal overlap discrete wavelet transform* para que seja possível estudar o poder de previsão das várias frequências de cada fator.

A principal conclusão deste trabalho é que os fatores por si não preveem o prémio de risco de mercado *out-of-sample*, mas a frequência de 16 a 128 meses dos fatores HML (*high minus low*) e RMW (*robust minus weak*) não só conseguem prever, como têm uma performance superior à média histórica no período a seguir à grande crise financeira de 2008.

Os nossos resultados corroboram com os resultados da literatura mais recente, no sentido em que as frequências de médio prazo de variáveis financeiras são bons preditores do prémio de risco do mercado. Como tal, estes resultados são de elevada relevância para académicos e investidores.

Palavras-chave: Fatores Fama French; Retornos de Índices; Previsibilidade de Retornos de Índices; Prémio de Risco de Mercado; Domínio de Frequência;

Número de palavras excluindo sumário executivo, bibliografia, anexos e apêndices: 6996
Número de palavras total: 8926

Abstract

We extend the analysis of Faria and Verona (2020) to the Fama French 5 factor model to predict the Equity Risk Premium in the S&P500 index.

The Fama French 5 factor model is decomposed using the maximal overlap discrete wavelet transform so that we can study the forecasting performance of each factor's different frequencies and test them out-of-sample.

The main findings of this study are that the factors themselves are not good predictors of the equity risk premium out-of-sample, but the 16-128 month frequency of the HML (high minus low) and, especially the RMW (robust minus weak), are good predictors of the Equity Risk Premium especially in post-2008 crisis.

Our results support recent findings in the asset pricing literature that the business-cycle frequency components of financial variables play a crucial role in forecasting the equity premium. Thus, for both investors and academics, these findings are of great relevance.

Keywords: Fama French Factors; Index Returns; Index Returns Forecasts; Equity Risk Premium; Frequency Domain;

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Chapter 1

1. Introduction

Efforts to successfully predict the equity risk premium (ERP) have a long tradition in finance, dating back to the beginning of the XX century, (e.g., Dow (1920)). There is an extensive literature on the predictability of equity returns, but the prediction methods vary across studies. As pointed by Goyal and Welch (2008), "The literature is difficult to absorb. Different articles use different techniques, variables, and time periods. Results from articles that were written years ago may change when more recent data is used, and some articles contradict the findings of others." Several studies over the years have suggested whether the equity return premium can be predicted by numerous variables such as dividend yield, interest rate spread and GDP among others financial and macroeconomic variables. These predictors have been tested and have been good predictors in-sample, but their out-of-sample (OOS) performance has not been robust (Goyal and Welch, 2008).

At the same time, the literature is dominated by time series analysis for making predictive regressions, with frequency domain techniques being comparatively new methods in the financial world. The frequency domain techniques we use in this thesis – wavelet filtering methods - have the advantages of working with both stationary and non-stationary signals and capturing both time and frequency information, whereas time series analysis captures only time information. Even though the original variables are decomposed into new variables fluctuating at different frequencies, the information in the original set of data is preserved. This decomposition allows us to isolate the frequencies with the highest predictive power.

We extend the analysis of Faria and Verona (2020) to predict the equity risk premium in the S&P500 index using a maximal overlap discrete wavelet transform to decompose the 5 Fama-French factors. The Fama-French 5 factor model (Fama and French, 2015) represents a set of factors widely employed in asset pricing literature in the context of stock returns predictability. Fama and French earlier created a model in 1993 aiming at capturing the association between average return and company size and the relation between average return and pricing ratios such as book to market. Their research offers evidence that those three components comprise an imperfect model for expected returns since they ignore most of the variation in average returns due to profitability and investment. The authors then developed a five-factor model including the three factors of the previous model as well as two additional factors that account for firm investment and profitability.

Although the Fama-French factor model has been extensively used among researchers, so far the literature has not been analyzing it in frequency domain and especially with wavelet transform techniques.

Overall, we find that the factors themselves are not good predictors of the equity risk premium out-of-sample, but the 16-128 month frequency of the HML (high minus low) and RMW (robust minus weak), are good predictors and actually overperform the historical mean benchmark during recession periods.

The rest of this paper is structured as follows: Section 2 presents a brief theoretical background on the Fama-French factor models and the use of frequency domain techniques in Finance. The data and methodology are described in section 3. Section 4 presents the results and subsequent discussion. Finally, section 5 concludes.

Chapter 2

2. Literature Review

2.1. Fama-French factor models

The forecast of stock returns is one of the most prominent areas studied in finance and, although a great deal of research has been done on this topic so far, the topic is still relevant as new studies are done on a regular basis.

The Capital Asset Pricing Model (CAPM) of Sharpe and Sharpet (1964) and Lintner (1965) is perhaps the most studied asset pricing model and marks "the birth of asset pricing theory" (Fama and French, 2004). The CAPM models the expected return as the sum of the return on the risk-free asset plus a risk premium, with the risk premium determined by the asset's correlation (β) with the market return. A security's expected return in CAPM is given by:

$$ER_i = Rf + \beta_i(ER_M - RF) + \varepsilon_i, \tag{1}$$

where *ER* is the expected return of security i, β is the so called beta (asset's correlation with the market return), *RF* is the return on a risk-free asset, *M* is the market, and *i* is one specific security.

Even though the CAPM is commonly used, through the years authors have found problems in the model and suggested other factors that could help predict securities returns. If the CAPM were to be right model, over a long period of time securities with higher (lower) β should on average, overperform (underperform) securities with lower (higher) β. However, Black et al., (1972) found that portfolios composed of securities with higher risk, this is, a higher β, on average, earned less than what the CAPM would predict. Basu (1977) found that portfolios composed of stock with a lower price to earnings ratio (P/E) yielded larger risk adjusted returns between 1957 and 1971 than high P/E portfolios. This was challenged by Reinganum (1981) who found that that effect should be more related to firm's size than the P/E ratio as the P/E effect disappeared when controlling for size, but the size effect remains significant when controlling for P/E. Likewise, Banz (1981) and Grinblatt and Titman (1989) showed that smaller firms outperform larger firms by a significant amount. Rosenberg et al. (1985) found that stocks with a high book-to-market value (B/M) generated higher returns than firms with a low B/M. Later Chab et al. (1991) studied the returns of Japanese stocks and found the book to market ratio and cash flow yield to be great predictors of stock returns. Finally, Bhandari (1988) used the debt/equity ratio (D/E) as a proxy for the risk of equity in a firm, and found that even when beta estimates are computed similar to Black et al. (1972), and the size effect is taken into account, as in Banz (1981), D/E still played a role in explaining a stock's expected return.

These findings made clear that additional factors were missing from the CAPM model, supporting the work of Black et al. (1972), and motivating Fama and French (1992) to use additional factors such as size, P/E, D/E, and B/M to study their effects on expected stock returns. They found that the relation between the market beta and average returns disappears during the 1963-1990 period and revealed that the impact of leverage and earnings-to-price ratio could be represented by the factors size and book-to-market equity as these factors do a good job in explaining the average returns during that period. Additionally, they claim that beta doesn't explain expected stock returns, as it has no explanatory power when used alone in the tests.

Fama and French (1993) extended the findings of Fama and French (1992) with the creation of six types of portfolios with combinations of size and book-to-market values to explain the returns in both stocks and bonds, which allowed them to calculate the difference in returns between small and big firms (SMB), between high and low book-to-market companies (HML) and the excess market return (MKT). The results showed that the three-factor model works well in predicting returns in portfolios compromised with only stocks[1] leading to the creation of the 3 factor model:

$$ER_i = \alpha_i + \beta 1_i(MRP) + \beta 2_i(SMB) + \beta 3_{iB}(HML) + \varepsilon_i \quad (2)$$

Since then, the Fama-French 3 factor model has been widely used as an asset pricing model. Notwithstanding it's good performance, throughout the years some authors have suggested additional factors that could predict stock returns more precisely.

For instance, Jegadeesh and Titman (1993) found a momentum factor, this is, when stocks outperform (underperform) the market over a period of three to twelve months they tend to continue to do well (poorly) for the next few months. This effect was different from the value effect captured by book-to-market equity and other price ratios up until that point.

[1] Fama and French (1993) tested whether their model would be appropriate to explain average returns in both stocks and bonds, with two additional factors related to the term structure and default risk to better evaluate the latter.

The relationship between the 3 factors and other variables was studied by Lakonishok et al. (1994) and Fama and French (1995), who found a negative relation between B/M and profitability and investment, meaning that firms with a lower B/M tend to be more profitable and invest more. Following this, Sloan (1996) showed that firms with higher accruals tend to have both low profitability and lower returns, suggesting a possible relationship between profitability and stock returns. This idea was confirmed by Haugen and Baker (1996), who showed that profitable firms do have higher expected returns. Similarly Novy-Marx (2013) found that profitable firms generate significantly higher stock returns than unprofitable firms, even when accounting for lower book-to-markets and higher market capitalizations, further suggesting profitability as an additional factor.

A negative relationship between investments and stock returns was found by Richardson and Sloan (2005), arguing that over-investment is the driving force behind this relation. Similar results are obtained by Fairfield et al. (2003) and Titman et al. (2004), with lower stock returns for a five year period being achieved by firms that increase their level of investment. Likewise, Aharoni et al. (2013) found not only a positive relation between expected profitability and returns, but also a negative relation between expected investment and returns.

Finally, Fama and French (2006), studied the impact of investment and profitability on stock returns and found similar results: profitability lead to higher stock returns and a higher rate of investment lead to lower expected rate of returns, controlling for B/M and investment/profitability in both cases. Nonetheless, Fama and French (2015) proposed a 5 factor model, which did not include the momentum factor found by Jegadeesh and Titman (1993), since Fama and French (2004) found that the impact of the momentum factor in portfolio performance was not significant and its effect was short-lived, and therefore its estimates for abnormal returns should be irrelevant. The five-factor model focus instead on two additional factors to represent the profitability and investment to explain the variation in average returns that the three-factor model failed to. To take the new factors into account, Fama and French (2015) introduced the Robust Minus Weak factor (RMW) as the difference between the returns of stocks with high and low profitability, and the Conservative Minus Agressive (CMA) as the returns of stocks with low and high investment profiles. The Fama-French 5 factor model can be written such as:

$$ER_i = \alpha_i + \beta 1_i(MRP) + \beta 2_i(SMB) + \beta 3_i(HML) + \beta 4_i(RMW) + \beta 5_i(CMA) + \varepsilon_i \quad (3)$$

They found that the five-factor model fits better the data than the three-factor model with the only exception of a portfolio composed by small size stocks with high investment profile and low profitability. The same results were also found later in Fama and French (2017), where both models were applied and compared on a global and regional level across North America, Europe, Asia Pacific and Japan.

2.2. Out-of-sample (OOS) forecasting

The use of out-of-sample (OOS) exercises to forecast the equity risk premium has been growing in the recent decades, due to their real-time usefulness and empirical evidence in the literature the variables employed in-sample to predict the equity risk premium did not perform as well OOS.

In fact, in a seminal paper, Goyal and Welch (2008) demonstrated that most of the models that aim to forecast equity premium up until that time, despite having good in-sample performance, did not guarantee similarly good OOS gains. They showed that the OOS equity premium predictability using several economic and financial variables is not statistically robust nor economically relevant. They also found that variables proposed in the literature, such as dividend price ratios and dividend yield, would not have helped an investor to outperform the historical mean OOS in a significant manner. They further stated: “Our profession has yet to find a variable that has had meaningful robust empirical equity premium forecasting power, at least from the perspective of real-world investor.”

An OOS exercise gives the correct information to the investor as it shows how a model would have performed over the months in the sample period if only the information available to an investor had been used. It also eliminates various econometric issues such as in-sample over-fitting, small sample size distortions, and look-ahead bias (Goyal and Welch 2008). As a result, it is widely understood that forecasting models need OOS validation, as Campbell and Thompson (2008) stated: "The ultimate test of any predictive model is its out-of-sample performance."

As such, in recent years there has been a focus of the literature in the equity premium forecast out-of-sample, in terms of both new predictors and new methodologies. Regarding new predictors, Rapach et al., (2013) studied the impact of lagged US equities returns on the OOS predictability of stock returns in other industrialized countries. Yin, (2019) found the stock market variance to be a good predictor of equity return premium OOS. Faria and Verona (2018, 2020) tested frequency-decomposed variables as new predictors of equity returns OOS. As for new forecasting methods Ludvigson and Ng (2007) and Neely et al. (2014) used dynamic factor analysis. Pettenuzzo et al. (2014) studied stock excess returns OOS under economic restrictions. Finally, Ferreira and Santa-Clara (2011) developed the SOP (sum-of-the-parts) methodology, that was later expanded by

Faria and Verona (2018) and suggested the addition of wavelet decomposition techniques to the SOP method.

Despite this efforts, Goyal et al. (2022) reexamined whether the variables suggested by numerous studies published after Goyal and Welch (2008) in predicting the equity premium in-sample and out-of-sample maintained their predictive ability after extending their sample period. They stated that "Overall, the predictive performance remains disappointing. (…) Because our paper reuses the data that the authors themselves had originally used to discover and validate their variables and theories, all that the predictors had to do in the few added years was not to "screw up" badly. The original results should still hold. Yet, we find that most variables have already lost their predictive ability"

2.3. Frequency Domain

Most of the financial forecasts done in the literature are based on time series methods, with time domain analysis being the most popular approach. Time domain analysis allows the analysis of signals displayed by time series with respect to time. Although it allows the study of temporal properties of a given economic variable – i.e. stock returns – it's not able to analyze all the information contained in the series when the fluctuations of a variable occur heterogeneously across different frequencies. Frequency domain, in contrast to time domain, allows to study these fluctuations.

Although popular in other study fields, the use of frequency domain is still recent in Finance but is seeing a rapid growth with wavelet transform techniques being a promising tool to use.

Wavelet techniques present two main advantages over other popular Frequency Domain methods such as the Fourier transform. First, its ability to work with non-stationary data, which is especially useful when studying

Financial variables. Second, the possibility to separate the dynamics of a time series maintaining both frequency and time domain information, whereas in the Fourier analyses the time domain information is lost. As Ranta (2010) states “Wavelet techniques possess an inherent ability to decompose this kind of time series into several sub-series which may be associated with a particular time scale. Processes at these different time-scales, which otherwise could not be distinguished, can be separated using wavelet methods and then subsequently analyzed with ordinary time series methods”

Capobianco (2004) studied the Nikkei stock index data with wavelet methods, Crowley and Lee (2005) applied wavelet multiresolution analysis to analyze the different frequency components of European business cycles, Harris and Yilmaz (2009) decomposed the spot exchange rate into its regular and irregular components and made forecasts of the spot exchange rate using the low frequency trends of the short-term momentum. In et al. (2010) showed that the risk factors are more relevant at the lower frequencies than at the higher frequencies in the CAPM. (Rua, 2011) proposed a wavelet approach for factor-augmented forecasting to test the forecasting ability of GDP growth and found that wavelet multiresolution analysis can improve forecast accuracy. Chaudhuri and Lo (2016) studied stock-return dynamics through spectral analysis. Zhang et al. (2017) focused on improving stock return forecasts through wavelet methods. Bandi et al. (2019) and Faria and Verona (2018) focused on improving the equity risk premium (ERP) predictability by employing a model that aggregated the frequency components dependence between the ERP and its predictors. Finally, Faria and Verona (2020) extracted cycles from the term spread using wavelet filtering methods to predict the equity premium.

In the context of the Fama-French model, this literature is scarcer. Trimech et al. (2009) used wavelet methods to decompose the Fama-French 3 factor model in the French stock market and found that the explanatory power of the factors In-Sample increases as the wavelet scale increases. To the best of our knowledge there are no studies that decompose the Fama-French factor model to predict equity returns out-of-sample. We fill this gap and extend the analysis of Faria and Verona (2020) to the Fama-French 5 factor model.

Chapter 3

3. Data and Methodology

3.1. Data description

For this research project, the S&P500 index is used as the equity index of study, with data retrieved from Thomson Datastream. The Fama French 5 factors were downloaded from Kenneth French's library[2]. The 3 month U.S treasury bills extracted from Thomson Datastream were used as a risk free interest rate.

All the data was used with a monthly frequency for the period ranging from January 1971 to December 2020, which comprises periods of expansions as well as recessions, including the dot com bubble and the Great Financial crisis in 2008-2009.

In what follows, we briefly describe the variables used:

Market Return Premium (MKT) – Excess returns on the market, computed as the value-weight return of all CRSP firms incorporated in the U.S and listed on the NYSE, AMEX, or NASDAQ.

Small Minus Big (SMB) – Excess return obtained by companies with a small market capitalization versus larger companies' returns, computed as follows:

$$
\begin{aligned}
SMB_{(B/M)} &= 1/3\,(Small\ Value + Small\ Neutral + Small\ Growth) \\
&\quad - 1/3\,(Big\ Value + Big\ Neutral + Big\ Growth) \\
SMB_{(OP)} &= 1/3\,(Small\ Robust + Small\ Neutral + Small\ Weak) \\
&\quad - 1/3\,(Big\ Robust + Big\ Neutral + Big\ Weak)
\end{aligned} \tag{4}
$$

[2] See https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5_factors_2x3.html

$$SMB_{(INV)} = 1/3\,(Small\ Conservative + Small\ Neutral + Small\ Aggressive) - 1/3\,(Big\ Conservative + Big\ Neutral + Big\ Aggressive)$$

$$SMB = 1/3\,(SMB_{(B/M)} + SMB_{(OP)} + SMB_{(INV)})$$

High Minus Low (HML) – Difference between the returns on diversified portfolios of high book-to-market stocks (value stocks) and low book-to-market stocks (growth stocks), computed as:

$$HML = 1/2\,(Small\ Value + Big\ Value) - 1/2\,(Small\ Growth + Big\ Growth) \quad (5)$$

Robust Minus Weak (RMW) - Difference between the average return of the two robust operating profitability portfolios and the average return on the two weak operating profitability portfolios, computed as:

$$RMW = 1/2\,(Small\ Robust + Big\ Robust) - 1/2\,(Small\ Weak + Big\ Weak) \quad (6)$$

Conservative Minus Aggressive (CMA) – Difference between the average return on the two conservative investment portfolios and the average return on the two aggressive investment portfolios, computed as:

$$CMA = 1/2\,(Small\ Conservative + Big\ Cosnervative) - 1/2\,(Small\ Aggressive + Big\ Aggressive) \quad (7)$$

S&P500 index return (SPR) – Computed as:

$$SPR = \left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) - 1, \quad (8)$$

where S&P500 corresponds to the dividend adjusted of the S&P500 index in period *t*. The adjusted prices were extracted from Thomson Datastream.

Risk free interest rate (rf) – It's the one-month risk free interest rate. The interest rate available from Thomson Datastream is the three-month US treasury bond, that was converted to a one-month rate as follows:

$$rf_t = [(1 + rf3_t)^{1/12} - 1], \tag{9}$$

where $rf3_t$ represents the three-months rate.

Equity Risk Premium (ERP) – Calculated as the log return on the S&P500 index minus the log return on a one-month US treasury bond:

$$ERP_t = \log(1 + SPR_t) - \log(1 + rf_t) \tag{10}$$

3.2. Methodology

This thesis extends the work of Faria and Verona (2020) to the Fama French 5 factors. Faria and Verona (2020) studied the role of the decomposed frequencies of the Term Spread as equity premium predictors. We follow a similar methodology and evaluate the forecasting power of the frequency decomposition of the Fama French factors. Each factor was decomposed into 3 different frequencies:

- HF captures the high frequencies of the series, i.e. oscillations smaller than 16 months.
- BCF captures business cycle fluctuations of the series, i.e. oscillations between 16 months and 128 months.
- LF captures the low frequencies of the series, i.e. oscillations greater than 128 months.

3.2.1 Wavelets

Wavelets are a signal processing method defined across a finite time span that decomposes an original signal into several sub-series, each one occurring at a different frequency. They enable the extraction of both time-varying and frequency-varying characteristics simultaneously simply by changing the time span, maintaining the original characteristics of the time series. With a short time span, we can see the time series' high frequency components. If we use the same signal, but a large time span, we can see its low frequency properties. This represents an advantage over the Fourier analysis which require the time series to be stationary, and do not have the ability to identify the moment in time when a given frequency exists. Wavelets are thus particularly beneficial when time series feature structural breaks or leaps, as well as when time series are non-stationary, which is the case for financial variables.

There are two types of Wavelet Transforms, the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The CWT assumes that the signal is continuous, quantifying the variation in a signal at a given frequency and at a particular point in time. Whereas the DWT decomposes a signal composed of observations sampled at evenly spaced points in time using a limited number of combinations of the mother wavelet to decompose.

There are some limitations to the DWT as suggested by Masset (2008) such as peaks in the original time-series may not be appropriately aligned with similar events in the multiresolution analysis and the DWT not being shift variant, if the series is shifted one period to the right, the multiresolution coefficients will not be equal.

To overcome such limitations a maximal overlap discrete wavelet transform multiresolution analysis (MODWT MRA) is used as our prevailing method. As Faria and Verona (2018) suggested, this methodology has the advantages of not being restricted to a particular sample size; is translation-invariant, which means it is not affected by the starting point for the time series being studied; and does not cause phase changes in wavelet coefficients.

The MODWT MRA uses two types of wavelets: father wavelets (ϕ), capturing the smooth and low-frequency part of the series, and mother wavelets (Ψ), capturing the high-frequency components of the series, where $\int \phi (t)\, dt = 1$ and $\int \Psi (t)\, dt = 0$.

Using a wavelet filter, a time series y_t can be decomposed as follows:

$$y_t = \sum_{J=1}^{J} y_t^{D_J} + y_t^{S_J} \tag{11}$$

where $y_t^{DJ}, j = 1,2, \dots, J,$ are the J wavelet detail components and $y_t^{S_J}$is the smooth component of the wavelet.

Equation (11) illustrates the decomposition of the original series y_t across several components, each capturing the original time series' fluctuation within a specific frequency. A smaller wavelet detail components J means the wavelet is better suited to study the short-term dynamics of the series. On the other hand, a bigger J indicates that the wavelet is well suited to study the long-term dynamics of the series. The series trend is captured by the smooth component $y_t^{S_J}$ (lowest frequency component).

In this study, we use a J=6 level MODWT MRA with a Haar wavelet filter. There are six wavelet detail components D_1-D_6 and one wavelet smooth component S_6. As the data used is monthly, the first wavelet detail component D_1 captures oscillations between 2 and 4 months, while the other wavelet detail components D_2 , D_3, D_4, D_5, D_6, capture oscillations with periods between 4-8, 8-16, 16-32, 32-64 and 64-128 months, respectively. The wavelet smooth component S^6captures oscillations with a period longer than 128 months.

3.2.2 Forecasts

We start by testing the predictability of the frequency decomposed Fama-French factors In Sample. The equity premium is given by r_t for month *t*. For each predictor x_t, the predictive regression is given by:

$$r_{t+1} = \alpha + \beta x_t + \varepsilon_t \quad \forall t = 1, \dots, T, \tag{12}$$

We estimate Eq (12) by OLS In Sample to test the significance of estimated beta coefficients. We use a heteroscedasticity and autocorrelation-robust t-statistic and compute a wild bootstrapped p-value to test the null hypothesis that β = 0 against an alternative hypothesis that β > 0. To enhance comparisons across predictors, predictors are standardized to have a standard deviation of 1 before estimating. The In Sample period runs between January 1971 and December 2020 with 588 observations.

We then use an initial sample between January 1971 and December 1994 to make the first OOS forecast. The sample is then increased by one observation and a new OOS forecast is produced. The procedure follows this pattern until the end of the sample. The full OOS period ranges from January 1995 to December 2020, with 300 monthly observations. For robustness reasons we first consider an OOS period from 1995 to 2020, and then split that into two different periods from 1995 to 2007 and from 2008 onwards. The equation representing the one month-ahead forecast of the equity risk premium is given by:

$$\hat{r}_{t+1} = \hat{\alpha} + \hat{\beta} x_t, \tag{13}$$

where $\hat{\alpha}$ and $\hat{\beta}$ the OLS estimators of α and β, respectively.

We evaluate the forecasting performance of the predictive models using Campbell and Thompson (2008) OOS R-square (R^2_{OS}) statistic, as follows:

$$R^2_{OS} = 100\left(1 - \frac{MSFE_{PR}}{MSFE_{HM}}\right) = 100\,[1 - \frac{\sum_{t=t0}^{T-1}(r_{t+1} - \hat{r}_{t+1)}{}^2}{\sum_{t=t0}^{T-1}(r_{t+1} - \hat{r}_{t)}{}^2}], \tag{14}$$

where $\hat{r}_t$ is the ERP forecast from the predictive model and r_t is the actual value of the ERP in that period. We use the historical mean of the ERP until time t as the benchmark model, as is common in the literature. The R^2_{OS} statistic measures the proportional reduction in the mean squared forecast error for the predictive model ($MSFE_{PR}$) relative to the mean squared forecast error for the historical mean model ($MSFE_{HM}$). A value bigger than 0 means that the predictive model outperforms the historical mean in terms of MSFE.

As is standard in the literature (Rapach et al. (2016) and Faria and Verona, (2020)), to evaluate the statistical significance of the results, we use the Clark and West (2007) statistic. We test the null hypothesis that the R^2_{OS} statistic is smaller or equal to zero, meaning that the $MSFE_{HM}$ is smaller or equal than the $MSFE_{PR}$. The alternative hypothesis is that the $MSFE_{HM}$ is bigger than the $MSFE_{PR}$.

To test for the hypothesis, we compare the t-statistic from the forecasts with the critical values of 1.282, 1.645 and 2,326 for a 10%, 5% and 1% level of significance respectively. The null hypothesis can be rejected if the t-statistic is greater than the critical values, indicating that the $MSFE_{PR}$ outperforms the $MSFE_{HM}$.

To understand the results obtained from a financial point of view we evaluate the forecasts from an asset allocation framework. The perspective is that of a mean-variance investor that actively manages a portfolio composed of equities and risk-free bills. Let EW_t be the percentage of the portfolio allocated to equities:

$$EW_t = \frac{1}{\gamma}\frac{\hat{R}_{t+1}}{\hat{\sigma}^2_t}, \tag{15}$$

wγ is the investor's relative risk of aversion, $\hat{R}_{t+1}$ is the OOS forecast of the equity returns at time t and $\hat{\sigma}^2_t$ is the variance of the equity returns. Following Rapach et al. (2016) and Faria and Verona (2020), we assume a relative risk aversion coefficient of three and use a ten-year moving window of past excess returns to estimate the variance of the excess return. Two different analyses were done, one constraining the weights EW_t to −0.5 and 1.5 range, so that short-selling and leverage are allowed, and another that constrain the weights EW_t to a range between 0 and 1 so that short-selling and leverage are prohibited. The investor uses the model prediction of excess returns over the next month and equation (15) gives us the equity weight for the next month.

The certainty equivalent return (CER) is used to quantify the gains from an economic perspective and is computed as CER = $\overline{RP} - 0{,}5\gamma\sigma_{RP}^2$, where $\overline{RP}$ and σ_{RP}^2 are the sample mean and the sample variance of the portfolio return, respectively. We further compute CER gains as the difference between the CER of an investor that uses the predictive model in Eq (13) to forecast excess returns and the CER of an investor who uses the historical mean benchmark.

Chapter 4

4. Results

4.1. In Sample Results

Predictors	$\hat{\beta}$	R^2
MKT	0.16	0.13
MKT_{HF}	-0.39	0.79
MKT_{BCF}	1.66	14.22***
MKT_{LF}	0.71	2.61***
SMB	0.22	0.24
SMB_{HF}	0.21	0.22
SMB_{BCF}	0.29	0.44*
SMB_{LF}	-0.31	0.50
HML	-0.22	0.26
HML_{HF}	-0.04	0.01
HML_{BCF}	-0.40	0.83
HML_{LF}	-0.38	0.73
RMW	-0.30	0.47
RMW_{HF}	-0.12	0.07
RMW_{BCF}	-0.62	1.99
RMW_{LF}	-0.01	0.00
CMA	-0.27	0.37
CMA_{HF}	-0.02	0.00
CMA_{BCF}	-0.64	2.10
CMA_{LF}	-0.41	0.86

This table reports the β estimation by OLS of the predictive model (12) and the corresponding R^2 statistic in percentage, for the various predictors. The predictors are the original time series of the Fama French 5 factor model and the three frequency components HF, BCF and LF obtained through wavelets decomposition using a haar filter capturing oscillations of the factors less than 16 months, between 16 and 128 months, and greater than 128 months, respectively. Each predictor variable is standardized to have a standard deviation of one. ∗∗∗, ∗∗, and ∗ denote significance at the 1%, 5%, and 10% levels, respectively, accordingly to wild bootstrapped p-values. The sample period runs from 1971:01 to 2020:12, monthly frequency.

Table 1 - In Sample predictive regression results

Table 1 shows the OLS regression of the Fama French factors and their frequency decomposed predictors and its in sample performance. None of the 5 factors themselves are statistically significant, whereas the low frequency component of the MKT factor is at the 1% level, as well as both the business cycle factor of both the MKT and SMB factor at the 1% and 10% levels respectively. The MRP_{BCF} and MRP_{LF} depict a rather big R^2_{OS} of 14.22% and 2.61% respectively. Meanwhile the SMB_{BCF} has a smaller R^2 of only 0.44% indicating that In Sample the MRP_{BCF} and MRP_{LF} are better predictors of the equity return premium than the SMB_{BCF}.

4.2. Out-of-sample (OOS) Results

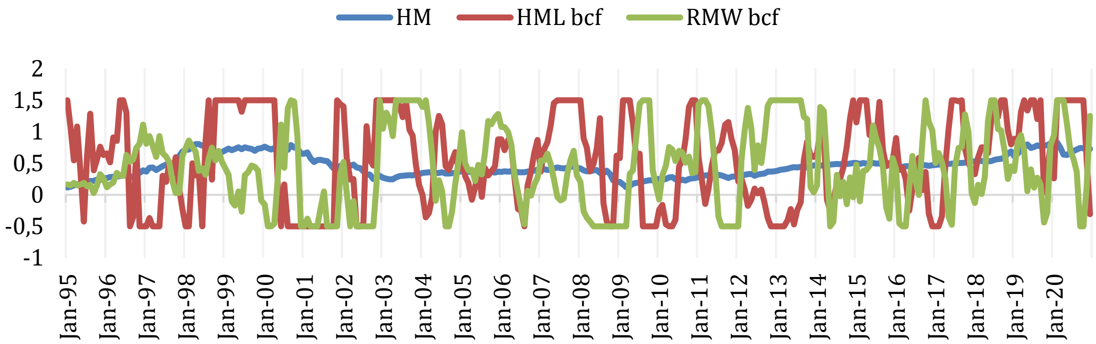
Predictor	R^2_{OS}	CER gains	CER gains (no short selling)
MKT	-0.17	-0.24	0.26
MKT_{HF}	-1.86	-3.20	-0.81
MKT_{BCF}	-23.19	1.66	2.24
MKT_{LF}	-3.91	1.27	2.69
SMB	-0.63	-1.36	-0.17
SMB_{HF}	0.46	0.36	0.53
SMB_{BCF}	-3.73	-4.42	-2.17
SMB_{LF}	0.31	0.39	0.79
HML	-0.13	-0.56	0.23
HML_{HF}	-0.78	-1.69	-1.16
HML_{BCF}	0.19*	0.72	1.17
HML_{LF}	-1.05	-1.41	0.48
RMW	-0.15	0.92	1.15
RMW_{HF}	-0.31	-0.40	0.02
RMW_{BCF}	1.26**	4.23	2.84
RMW_{LF}	0.35	0.02	0.65
CMA	-0.98	-0.69	0.36
CMA_{HF}	-0.59	-1.33	-0.77
CMA_{BCF}	-2.13	0.11	1.08
CMA_{LF}	-2.00	0.25	1.82

This table reports the OOS R-Squares R^2_{OS} (in percentage) for the excess returns as given by Eq (13) and the CER gains (in percentage). The predictors are the original time series of the Fama French 5 factor model, and the three frequency components HF, BCF and LF obtained through wavelets decomposition using a haar filter capturing oscillations of the factors less than 16 months, between 16 and 128 months, and greater than 128 months, respectively. ∗∗∗, ∗∗, and ∗ denote significance at the 1%, 5%, and 10% levels, respectively, based on the Clark and West (2007) MSFE-adjusted statistic. The Out-of-Sample period runs from 1995:01 to 2020:12, monthly frequency.

Table 2 – OOS predictive regression results

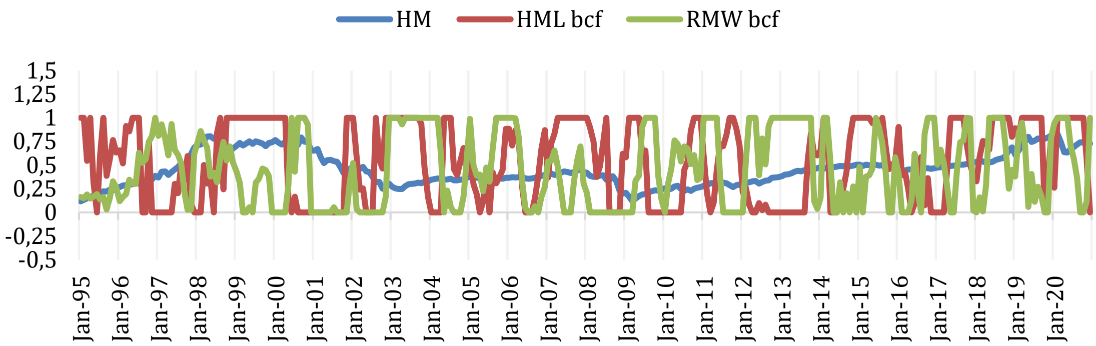
Table 2 shows the OOS performance of the Fama-French factors and it's OOS performance and their frequency decomposed predictors. Unlike the IS regression the MRP_{BCF}, MRP_{LF} and SMB_{BCF} do not hold any predictive power. The variables that have predictive power are the HML_{BCF} with a statistical significance at the 10% level and a rather small R^2 of 0.19%, and the RMW_{BCF} with a statistical significance at the 5% level and R^2 of 1.25%. Our results show that the performance of the RMW_{BCF} is strong from an economical perspective, with CER gains of 423 bps when allowing for short selling (equity weights limited to the -0.5 to 1.5 range) and 283 bps when short selling is not possible (equity weights limited to the 0 to 1 range). The performance of the HML_{BCF} has smaller CER gains of 72 bps and 117 bps when short selling is allowed and prohibited, respectively.

Figures 1 and 2 display the evolution of the optimal equity weights obtained using the historical mean, HML_{BCF} and RMW_{BCF} one-month ahead forecasts, when the equity weights are constrained to the −0.5 to 1.5 range (allowing short selling) and 0 to 1 range (not allowing short selling), respectively. In both analysis the weights allocated using the RMW_{BCF} and HML_{BCF} forecasts exhibit more variance and require bigger changes in the portfolio over time than the historical mean forecasts. Furthermore, figure 3 depicts the log cumulative wealth of an investor who invested 1$ at the beginning of our sample period and reinvested all proceeds throughout the sample. The greater returns of the RMW_{BCF} are clear.



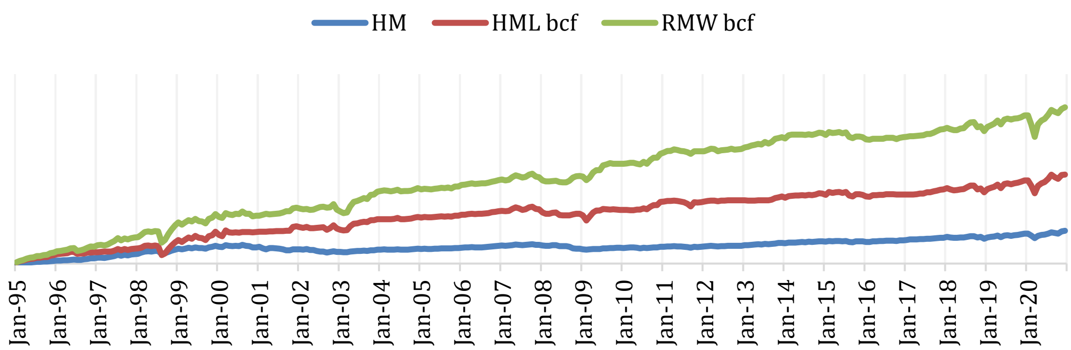
Equity weight in the portfolio of a mean-variance investor who allocates monthly his wealth between equities and risk-free bills according to Eq (15), using stock return forecasts based on the historical mean benchmark (blue line), the RMW_{BCF} (green line) and HML_{BCF} (red line). The equity weight is constrained to a range between −0.5 and 1.5. Sample period runs from January 1995 to December 2020, monthly frequency.

Figure 1 - Equity weight allocation using the HM, HML-bcf and RMW-bcf forecasts with limits between -0.5 and 1.5.



Equity weight in the portfolio of a mean-variance investor who allocates monthly his wealth between equities and risk-free bills according to Eq (15), using stock return forecasts based on the historical mean benchmark (blue line), the RMW_{BCF} (green line) and HML_{BCF} (red line). The equity weight is constrained to a range between 0 and 1. Sample period runs from January 1995 to December 2020, monthly frequency.

Figure 2 - Equity weight allocation using the HM, HML-bcf and RMW-bcf forecasts with limits between 0 and 1



Log cumulative wealth of an investor who invested 1$ using at the beginning of our sample period and reinvested all proceeds throughout the sample, using forecasts based on the historical mean benchmark (blue line), the RMW_{BCF} (green line) and HML_{BCF} (red line). The equity weight is constrained to a range between 0 and 1. Sample period runs from January 1995 to December 2020, monthly frequency.

Figure 3 - Log cumulative returns using the HM, HML-bcf and RMW-bcf forecasts

Not surprisingly, there is a mismatch between the variables that that are good predictors of the ERP In Sample and OOS. These results confirms the findings in Goyal and Welch (2008) and subsequent literature in the sense that predictors that are good predictors In Sample are not necessarily good predictors OOS.

Similar to the In Sample regression results, these findings support the notion that wavelets are a useful method and reinforces the validity of frequency domain techniques to forecast the ERP, given that as the RMW and HML factors do not predict the ERP but their BCF do.

In fact, our main finding is that the Fama-French factors are not predictors of the equity risk premium OOS, but the 16-128 month frequency of the HML and RMW factors are. To the best of our knowledge, this is the first time that the 16-128 month frequencies of this specific factors have been suggested as good ERP predictor OOS. The RMW and SMB factors themselves have been shown to be significant predictors OOS in the UK stock market as shown by Foye (2017). The RMW factor has also been argued to capture mispricing away from value, caused by noise trading (Ülkü, 2017). The relevancy of the 16-128 month frequency, however, is not new. Stein (2020) found that all the predictive power of the equity premium comes from periods between 16 to 64 months for technical indicators such as the moving average and trading volume, without any evidence of predictability outside of this frequency band. And, relevant to the Fama-French factor model, Trimech et al. (2009) found that in the French stock market the wavelet decomposed Fama-French 3 factor model achieved a higher R^2 in frequencies bigger than 12 months, when studied In-Sample.

4.3. Robustness

4.3.1. Different sample periods

As a robustness exercise, we evaluate the OOS results in two subsamples: the first from January 1995 to December 2007, just before the emergence of the 2008 Financial Crisis, and the second from January 2008 until December 2020.

Predictors	1995 - 2007		2008 - 2020	
	R^2_{OS}	CER gains	R^2_{OS}	CER gains
MKT	-0.33	-0.71	-0.09	0.15
MKT_{HF}	-1.68	-2.40	-1.96	-3.88
MKT_{BCF}	-26.84	1.27	-21.31	1.98
MKT_{LF}	-5.02	0.93	-3.97	1.53
SMB	0.01	-0.83	-0.97	-1.82
SMB_{HF}	1.74	1.93	-0.49	-0.96
SMB_{BCF}	-5.55	-6.34	-1.53	-2.80
SMB_{LF}	0.08	0.44	0.43	0.34
HML	0.44	0.75	-0.50	-1.67
HML_{HF}	-1.12	-1.86	-0.59	-1.54
HML_{BCF}	-0.91	2.99	1.14	-1.22
HML_{LF}	-1.88	-1.16	-0.69	-1.64
RMW	-1.12	0.44	0.65	1.33
RMW_{HF}	-0.76	-0.85	0.03	-0.02
RMW_{BCF}	-0.66	3.93	2.69**	4.48
RMW_{LF}	1.03	1.36	-0.36	-1.10
CMA	-0.57	1.14	-1.16	-2.24
CMA_{HF}	-0.63	-1.62	-0.39	-1.09
CMA_{BCF}	-4.72	0.54	-0.70	-0.28
CMA_{LF}	-3.39	3.42	-1.67	-2.45

This table reports the OOS R-Squares R^2_{OS} (in percentage) for the excess returns as given by Eq (13) and the CER gains (in percentage). The predictors are the original time series of the Fama French 5 factor model, and the three frequency components HF, BCF and LF obtained through wavelets decomposition using a haar filter capturing oscillations of the factors less than 16 months, between 16 and 128 months, and greater than 128 months, respectively. ∗∗∗, ∗∗, and ∗ denote significance at the 1%, 5%, and 10% levels, respectively, based on the Clark and West (2007) MSFE-adjusted statistic. Two different sample periods are used, one from 1995:01 to 2007:12, and another from 2008:01 to 2020:12, monthly frequency.

Table 3 - OOS predictive regression results (sample split)

In table 3 both the RMW_{BCF} and HML_{BCF} lose their predictive power between 1995 and 2007, but from 2008 to 2020 the RMW_{BCF} has a higher R^2_{OS} (of 2.69%) when compared to the full OOS period, and a statistical significance at the 5% level. From an economic perspective, the RMW_{BCF} also has a superior performance in the sample period between 2008 and 2020, with CER gains of 448 bps.

The sample period between 2008 and 2020 coincides with the aftermath of the Great Financial Crisis, which makes the RMW_{BCF} a good predictor in post recessions period. Interestingly, Jareño et al. (2020) found that the RMW factor itself was found a relevant variable in post-crisis periods when forecasting European financial institutions returns. Additionally, the notion that the historical mean is outperformed during recessions and in its aftermath is a well-established fact in the literature (Stein 2020; Henkel et al. 2011; Rapach and Zhou, 2013).

4.3.2. Different filtering methods

As a robustness exercise we extract the three different frequencies from the Fama-French 5 factor model using additional filtering methods.

Like the Haar transform, the Daubechies wavelet transform is implemented as a succession of decompositions with the main difference being in the filter length that bigger than two making it more localized and smoother (Sharif and Khare, 2014). In particular, we use a Daubechies filter length of 4.

Similar to Faria and Verona (2020) we also use the Christiano and Fitzgerald (2003) asymmetric band-pass filter (with a unit root with drift) to extract the frequency components. In particular, the frequency bands of the filter are chosen to extract the same frequency components as in our analysis with wavelets: the high-frequency (BP−HF), the business cycle-frequency (BP−BCF), and low-frequency (BP−LF) components.

Predictors	Daubechies wavelet		Band Pass	
	R^2_{OS}	CER gains	R^2_{OS}	CER gains
MKT_{HF}	-2.06	-3.14	-2.32	-3.60
MKT_{BCF}	-20.16	2.67	-5.06	2.88
MKT_{LF}	-3.34	1.58	-2.06	-1.11
SMB_{HF}	0.57	0.63	0.10	-0.09
SMB_{BCF}	-3.85	-5.40	-1.60	-3.06
SMB_{LF}	0.09	-0.02	0.33	0.05
HML_{HF}	-0.76	-1.45	-0.95	-1.67
HML_{BCF}	0.63**	1.62	1.76**	2.26
HML_{LF}	-1.53	-2.24	-0.86	-1.14
RMW_{HF}	-0.36	-0.45	-0.15	0.14
RMW_{BCF}	1.20**	3.88	0.79**	2.52
RMW_{LF}	0.42	0.55	-0.39	-0.75
CMA_{HF}	-0.57	-1.35	-0.62	-1.53
CMA_{BCF}	-1.42*	0.79	0.42*	1.67
CMA_{LF}	-1.88	-0.15	-0.24	0.16

This table reports the OOS R-Squares R^2_{OS} (in percentage) for the excess returns as given by Eq (13) and the CER gains (in percentage). The predictors are the original time series of the Fama French 5 factor model, and the three frequency components HF, BCF and LF obtained through wavelets decomposition using a Daubechies wavelet transform filter (columns 2 and 3) and band-pass filter (columns 3 and 4) capturing oscillations of the factors less than 16 months, between 16 and 128 months, and greater than 128 months, respectively. ∗∗∗, ∗∗, and ∗ denote significance at the 1%, 5%, and 10% levels, respectively, based on the Clark and West (2007) MSFE-adjusted statistic. The Out-of-Sample period runs from 1995:01 to 2020:12, monthly frequency.

Table 4 - OOS predictive regression using alternative filtering methods

Tables 4 depicts the results obtained using the alternative filtering methods. Both the RMW_{BCF} and HML_{BCF} predictors remain statistically significant at the 5% level using the Daubechies and Band Pass filters. Both the R^2_{OS} and CER gains are superior for the HML_{BCF} predictor in both filters. On the other hand, the RMW_{BCF} exhibits smaller R^2_{OS} and CER gains in both filters. Overall, the results are similar across theses different filtering methods tested, thus confirming the results.

Chapter 5

5. Conclusion

In this study we extend the work of Faria and Verona (2020) to the predict the Equity Risk Premium (ERP) of the S&P 500 index with the Fama-French 5 factor model.

The major novelty of this study is that we extend the frequency domain methodology to the Fama-French factor models, something that is still scarce in the literature and we foresee as highly relevant for both academics and investors.

Our In-Sample regression analysis showed that the MKT_{BCF}, MKT_{LF} and SML_{BCF} are statistically significant predictors. We then run predictive regressions OOS and find the HML_{BCF} and RMW_{BCF} to be the relevant ones. Finally, we split the sample period in two and find an significant increase in the predictability power and economic gains of the RMW_{BCF}, in the aftermath of the 2008 Financial crisis.

Interestingly, in all the regressions made the Fama-French factors themselves were never shown to be good predictors of the ERP.

We have two main findings. First, there are forecasting gains from using frequency-domain information from the Fama-French factor model. Second, those forecasting gains are stronger in post crisis periods.

Going forward, it would be interesting to extend this methodology to other equity indexes across the world and in longer forecasting horizons.

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