An investigation of the use of mobile technologies in the teaching and learning of congruence in mathematics

By

Elamo Blessing Chibaya

(213573538)

A full dissertation for the Degree of

Doctor of Education

(Mathematics Education)

In the School of Education

(Edgewood Campus)

University of KwaZulu-Natal

Supervisor: Prof Vimolan Mudaly

Declaration

I, **Elamo Blessing Chibaya, student number 2013573538,** hereby declare that:

1. The research reported in this dissertation, except where otherwise indicated, is my original work.

2. This dissertation has not been submitted for any degree or examination at any other university.

3. This dissertation does not contain other persons' data, pictures, graphs, or other information unless specifically acknowledged as being sourced from other persons, the details of which are to be found as in-text citations and in the References.

4. This dissertation does not contain other persons' writing unless specifically acknowledged as being sourced from such researchers.
   Where other written sources have been quoted then:

   4.1 their words have been re-written, but the general information attributed to them has been referenced.

   4.2 where their exact words have been used, their writing has been placed inside quotation marks and referenced.

Student: _[redacted]______________ Date: 21/02/2022

Abstract

The study explored the feasibility of using mobile technology in teaching and learning the congruence of triangles, with a particular focus on conceptual understanding. A mixed-method case study methodology was used, in which 25 grade 9 learners who were purposefully selected from a school in Durban took part. Two questionnaires and semi-structured interviews were administered. The validity and reliability of both questionnaires were assessed and confirmed. The triangulation of the results from the first questionnaire, the second questionnaire, and the semi-structured interviews revealed that learners were willing to use mobile technology in learning mathematics. The results also showed that it was practical to use curriculum-tailored GeoGebra applets on mobile devices to learn the congruence of triangles. The study could directly benefit the grade 9 educators and learners in South Africa. Educators can use the applets and the worksheet designed in this study when teaching the congruence of triangles. The study responds to the call made in the Action plan to 2024, to align technology integration to the improvement of learning outcomes. To the board of knowledge of social sciences, the research contributes a modified FRAME or model that emerged in this study, which I named the FRAME_applet model; that emphasizes the use of curriculum tailored applets on mobile devices in teaching and learning, to enhance conceptual understanding. This study is relevant in the current context of the COVID-19 pandemic, where the time for face-to-face learning and teaching has been reduced. Learners can self-learn new content through the use of curriculum-tailored GeoGebra applets.

Acknowledgments

Many people walked this research journey with me. Colleagues from work added their academic voices to the study through their professional critique, while brothers and sisters in Christ spiritually supported me. Family and social friends supplemented my moral and physical upkeep through informal engagements. I thank God for His intervention, for life, wisdom, and for being in control all the way.

My supervisor, Professor Vimolan Mudaly was also a pillar to lean on. I would want to express my heartfelt gratitude for his patience, expertise in research, and his unconditional availability whenever I needed guidance. I salute you for the prompt feedback and the critically engaging comments.

I would like to thank the University of KwaZulu-Natal for allowing me to study. I also want to thank the Higher Degrees Research staff for all the assistance with the registration and funding administration processes.

I would also want to express my sincere gratitude to the school governing board of the selected school, for allowing me to research at the school, the grade 9 teachers for giving me time to talk to their learners during registration, and the Head of the Mathematics Department, Mr. Mansingh, for allowing me to use his classroom. A special thank you to the participants who sacrificed their time to take part in the research. It would not have been possible without them. I would also want to thank all my colleagues for their support and encouragement.

I would also thank my family, particularly my husband, Dr. Colin, who tirelessly assisted when I called for help. Special mention also goes to my sons, Onil, Victor, and Shaun, who also directly or indirectly proofread this thesis, took over the household chores every time I could not fulfill them, as well as encouraged me now and again when I felt down. A huge thank you to you all. Special thanks to Shaun for consistently instructing me not to sleep until I submit this thesis.

I am grateful to Dr. J. Sibanda for editing this thesis.

Acronyms

NSC	National Senior Certificate
MST	Mathematics, Science and Technology
ICT	Information and Communications Technology
USAO	Universal Service and Access Obligation
DBE	Department of Basic Education
EPubs	Electronic publications
NEEDU	National Education Evaluation and Development Unit
TED	Teacher Education and Development
LoLT	Language of Learning and Teaching
ISPFTEDSA	Integrated Strategic Planning Framework for Teacher Education and Development in South Africa
DGS	Dynamic geometry software
FRAME	Framework for the Rational Analysis of Mobile Education

List of the labels for the Department of Basic Education Reports

It was difficult for me to track the Department of Basic Education reports that were cited at any instance, because they shared the same author and, in some cases, they had the same year as well. Therefore, I decided to label the reports as indicated in the list below.

Department of Basic Education_a:	Action plan
Department of Basic Education_d:	Diagnostic report
Department of Basic Education_cf:	Curriculum Assessment Policy Statement for foundation phase, grades 1-3
Department of Basic Education_ci:	Curriculum Assessment Policy Statement for intermediate phase, grades 4-6
Department of Basic Education_cs:	Curriculum Assessment Policy Statement for senior phase, grades 7-9
Department of Basic Education_cfet:	Curriculum Assessment Policy Statement for Further Education and Training phase, grades 10-12
Department of Basic Education_l:	The Status of The Language of Learning and Teaching in South African Public Schools.
Department of Basic Education_ar:	Annual report
Department of Basic Education_m:	Moodle teacher training for teachers and subject specialists on using MOODLE. moodle teacher training
Department of Basic Education_g:	Mathematics, Science, and Technology Conditional Grant registers significant progress in provinces.
Department of Basic Education_wp:	Whitepaper
Department of Basic Education_f:	Integrated Strategic Planning Framework for Teacher Education and Development in South Africa
Department of Basic Education_pr:	National Senior Certificate Performance Report
Department of Basic Education_ict:	Information Communication and Technology in Schools
Department of Basic Education_w	Winter School Programme

Table of Contents

List of Figures

List of Tables

Chapter 1 Introduction

"People can generally remember 10% of what they read, 20% of what they hear, 30% of what they see, 50% of what they see and hear, 70% of what they say and write, 90% of what they say and do." (Polásek & Sedlácek, 2015).

Mathematics is perceived as a challenging discipline to most learners, both in South Africa and abroad (Pfeiffer, 2017; Aric & Aslan-Tutak, 2015; Ünlü & Ertekin, 2017; Naidoo, 2011). Achievement in mathematics is a prerequisite for entry into the fields of science, technology, mathematics, and engineering; in both vocational and other post-school academic institutions (NEEDU, 2018; Department of Basic Education_c, 2015; Beilock,2015). Low performance in mathematics impacts negatively on the number of learners who enroll in the science faculties (Department of Education_a, 2015; Soobrayan, 2012; Beilock, 2015). Consequently, the current shortage of qualified manpower in the science sector persists (Marginson, et al., 2013; Kennedy & Kennedy, 2014; Soobrayan, 2012; Beilock, 2015). Although efforts are being made to attract learners into science and technology in faculties at higher institutions (Kennedy & Kennedy, 2014), the desired outcome of getting a large enrollment into these faculties have not been achieved (Department of Basic Education_w, 2018). One of the contributing factors for this low input in these faculties is the low performance in mathematics (NEEDU, 2018). According to the National Senior Certificate (NSC) diagnostic reports, the percentage pass rate of learners with 30% and above in mathematics has been ranging between 49,1% and 58,9% since 2014 (Department of Basic Education_d, 2020). Although 30% is considered as a pass, the chances of one entering a university science faculty with less than 40% are very slim (University of Kwa-Zulu-Natal, 2019; Durban University of Technology; 2019). This reduces the intake into science faculties even more. Since 2014, the percentage of learners with 40% or more ranges between 31,9% and 37,1% (Department of Basic Education_d, 2020). The highest pass rates of 58,9% for learners with 30% and above, and 37,1% for learners with 40% and above, were both achieved in the year 2018 (Department of Basic Education_d, 2018). However, there was a slight decrease in both 2019 and 2020. Due to disruptions in school face-to-face learning caused by the COVID-19 pandemic, it was not surprising to see a decrease in the pass rate of 2020 matriculants, not only in mathematics but also in all subjects. The performance of the students in 2020 increased by 0,6% at the 40% level and decreased by 0,8% at the 30% level.

According to the subject specialists, learners display confidence in answering less challenging, routine questions (Department of Basic Education_d, 2020). However, learners still struggle to answer questions that require deep conceptual understanding (Department of Basic Education_d, 2018; Department of Basic Education_d, 2020). Subject specialists identified the following sections as needing immediate attention and support; financial mathematics, Euclidean geometry, coordinate geometry, statistics, and probability (Department of Basic Education_d, 2017; Department of Basic Education_d, 2020). Several motives lie behind the selection of the section of congruence of triangles. Firstly, it is part of the grade 9 curriculum. Secondly, it forms the basis of some proofs in Euclidean Geometry at the Further Education and Training level (from grade 10 to grade 12). For example, congruence of triangles is applied when proving properties of some quadrilaterals in grade 10 and some circle theorems in grade 11. The use of these theorems extends to grade 12. I hope that using GeoGebra applets on mobile devices may enhance the understanding of congruence.

Subject specialists also recommended a focus on diagram analysis during content dissemination (Department of Basic Education_d, 2017; Department of Basic Education_d, 2018). These sections pull learners' grades in mathematics down. Are there creative interventions in place with which to improve learners' understanding of the congruence of triangles?

1.1 Factors contributing to low performance in mathematics

Local and international researchers have highlighted several factors believed to contribute towards learners' low performance in mathematics (Howie, 2003; Mji & Makgato, 2006; Spaull, 2013; Pournara et al., 2015; Beilock, 2015; Han & Capraro, 2014). In the succeeding paragraphs, some of these factors are discussed, coupled with the government's response to these concerns as indicated in the Action Plan to 2019 (Department of Basic Education_a, 2015). Special attention is put on the goals found in this action plan (Department of Basic Education_a, 2015). Over and above the list of factors affecting low performance in Mathematics, is language proficiency-related challenges (Howie, 2003; Mji & Makgato, 2006; Dempster & Vijay, 2007; Spaull, 2013. The transition from using the African language for teaching and learning in the foundation phase to English in the intermediate phase presents several difficulties (Soobrayan, 2012; Pillay, 2013). It is observed that access to high-quality material in both English and the language from which learners are making the transition, may remedy the problem (Pillay, 2013; Department of Basic Education_l, 2010). The applets used in this research uses familiar signs and symbols drawn from learners' contexts (Asiimwe et al., 2017). I hope this technology-related visual approach could enhance learners' understanding of congruence regardless of the language challenges (Pfeiffer, 2017; Asiimwe et al., 2017; Radović et al., 2018).

Another commonly inferred factor affecting learner performance in mathematics is inadequate subject knowledge of teachers (Naidoo, 2011; Spaull, 2013; Mji & Makgato, 2006; Howie, 2003; Visser et al., 2015). Most mathematics teachers, especially in the South African context, are under-qualified (Padayachee, 2010; Venkat & Spaull, 2007) and lack sufficient content knowledge (Venkat & Spaull, 2007; Pournara et al., 2015) required for the challenge. To make matters worse, the introduction of the new Curriculum Assessment Policy Statements (CAPS) brought new topics into the Further Education and Training phase (FET) (Department of Basic Education_fet, 2011). This meant that some educators were required to teach content areas they were not familiar with (Department of Basic Education_f, 2011). In my view, the educator's confusion during lessons also disrupts the lesson flow and extends the time spent on different sections. It is thus, essential for an educator to plan lessons thoroughly and on time, to master the content (Naidoo, 2011; Mji & Makgato, 2006; Spaull, 2013).

The government is working tirelessly to address the issue of underqualified teachers (Soobrayan, 2012). In trying to increase the number of qualified educators, the government has put measures in place to attract young, motivated, and appropriately trained students to the teaching profession (Department of Basic Education_f, 2011). Under this goal, the Department of Basic Education is calling for a more holistic approach that goes beyond the Funza Lushaka bursary, which gives higher priority to students intending to study mathematics and science teaching (Department of Basic Education_f, 2011).

Inadequate time to complete the syllabus is also a prevalent contributing factor towards low performance in mathematics (Mji & Makgato, 2006; Bush, Joubert, Kiggundu, & Van Rooyen, 2009). The absenteeism of teachers and late coming for classes affect curriculum coverage (Dempster & Vijay, 2007; Howie, 2003; Magwa & Ngara, 2014). Some researchers have pinpointed the lack of prior knowledge by learners as contributing to the extended time for syllabus coverage (Magwa & Ngara, 2014). This is affirmed in the national senior certificate diagnostic reports that most candidates lack basic mathematical competencies, which should have been acquired in lower grades (Department of Basic Education_d, 2020; Department of Basic Education_d, 2019). In trying to close the content gap, teachers use the time meant for syllabus coverage to teach basic concepts needed for the new sections (Magwa & Ngara, 2014). Teachers may, therefore, not find sufficient time to use teaching strategies that promote conceptual understanding because of the pressure to complete the syllabus (Magwa & Ngara, 2014). Goal 18 in the Action Plan to 2019 focuses on curriculum coverage in schools (Department of Basic Education_a, 2015). As a remedy to time-related factors, the Department of Education prescribes annual teaching plans (Department of Basic Education_a, 2015) that give guidance on the time needed for each section and the dates by which respective sections are expected to be completed. This is meant to ensure that learners cover all the topics and skills within their respective school year (Department of Basic Education_a, 2015). However, these interventions still pose risks related to teachers complying with the given time frames but compromising depth and the actual principles of learning (Department of Basic Education_a, 2015). More support and guidance for teachers are still crucial (Department of Basic Education_a, 2015). The use of mobile technology promotes teaching strategies that can enhance conceptual understanding without impacting syllabus coverage (Radović et al., 2018; Pfeiffer, 2017). A detailed review of mobile technologies and the statement of the problem follow in sections 1.3 and 1.5 respectively.

Some researchers pointed to the lack of instructional materials in schools as a contributing factor to learners' poor performances in mathematics (Howie, 2003; Mji & Makgato, 2006; Magwa & Ngara, 2014). In response to this, the government and non-governmental organizations have many projects underway to address this challenge (Department of Basic Education_a, 2015; Department of Basic Education_ar, 2018). The Mathematics, Science, and Technology Conditional Grant is set to improve the performance of learners in mathematics (Department of Basic Education_g, 2017). Through the Operation Phakisa Information and Communications Technology Programme, ICT equipment is now available at various teaching centers (Department of Basic Education_ar, 2018. This gives teachers access to the internet and offline resources (Department of Basic Education_ar, 2018). The Universal Service and Access Obligation Project was also launched (Department of

Basic Education_ar, 2018). Through this project, many schools are connected (Department of Basic Education_ar, 2018). More so, computers loaded with Department of Basic Education content were distributed to schools (Department of Basic Education_ar, 2018). This was meant to augment efforts invested in converting some textbooks into electronic publications for use on mobile devices (Department of Basic Education_ar, 2018). Furthermore, offline digital content packs were made available to all nine provinces (Department of Basic Education_ar, 2018). The National Education Collaboration Trust (Department of Basic Education_ar, 2018) supports these interventions with digital content provision and connectivity (Department of Basic Education_ar, 2018). Even Siyafunda digital libraries assist in this regard (Department of Education_ar, 2018). The Multi-Grade Toolkit was also supplied to schools (Department of Basic Education_ar, 2018). These myriad interventions provide teachers with the much-needed support in the mediation of the curriculum at the classroom level (Department of Basic Education_ar, 2018).

We are further inspired by the department of education's enthusiasm towards improving learner performances in mathematics. Even exemplars and common examinations for further education and training phases are implemented in low achieving schools, as an intervention (Department of Basic Education_ict, 2016). More so, the department of education has gone to the extent of collaborating with external ICT businesses, both locally and internationally. These include Vodacom, Microsoft, and the United Nations International Children's Emergency Fund, to enhance professional development for educators (Department of Basic Education_ict, 2016). Similarly, Vodacom and MTN provide access to Learning and Teaching Support Materials (Grade 10 to 12) through the department of basic education cloud where there are zero-rated sites with digital resources (Department of Basic Education_ar, 2018). Moreover, the Post Office, in partnership with Microsoft, UNICEF, Vodacom, and MIST SETA; is expanding coverage to enhance ICT in schools (Department of Basic Education_ict, 2016). These interventions are all in line with the proposed integration of mobile technology (see section 1.5).

To further support these interventions, classrooms are also being upgraded and refurbished to ensure ICT readiness (Department of Basic Education_ar, 2018; Department of Basic Education_a, 2015). In line with these plans, a large portion of the Department of Education's budget is diverted towards the procurement of technology resources (laptops, tablets, smartboards), both for learners and educators (Department of Basic Education_ar, 2018; Department of Basic Education_a, 2015). The fruits of these interventions are becoming visible, through a significant turnaround to township education (Department of Basic Education_wp, 2019). The hope is that this investment will, eventually, do away with paper-based textbooks and other traditional teaching and learning material (Mabona, 2018). Even more focused interventions have been instituted. Worth mentioning is when curriculum specialists were trained to use online Moodle to transform and integrate ICT into teaching and learning (Department of Basic Education_m, 2018). Also inspiring is the work reported on the Thutong Portal, where the Department of Basic Education further aims to lead a drive to improve learning using appropriate technology (Department of Basic Education_ar, 2018; Department of Basic Education_a, 2015). The proposed integration of mobile technology (see the statement of the problem in section 1.5) comes at the right time to augment and further support the Department of Education's efforts.

More works in the literature have broadened the scope of the possible causes and potential barriers to learners' performances in mathematics. Frequently, difficulties experienced by teachers when they manage activities in the classroom have surfaced (Mji & Makgato, 2006; Magwa & Ngara, 2014). Teachers battle with handling heavy teaching workloads (Mji & Makgato, 2006; Magwa & Ngara, 2014) and struggle to manage overcrowded classrooms (Magwa & Ngara, 2014). Unfortunately, the challenges of excessively large classes remain serious (Department of Basic Education_a, 2015; Department of Basic Education_a, 2020). Besides the overcrowded classrooms, other contributing factors of learner indiscipline in classrooms include lack of attention and concentration, disinterest, laziness, not understanding the curricular content, and attention-seeking (Silva et al., 2017; Mji & Makgato, 2006). Bringing mobile technology into teaching and learning, even under these harsh conditions, could exploit learners' digital nativity and proficiency towards fostered engagement.

In their study, the National Education Evaluation and Development Unit (NEEDU) established that the use of assistive devices allows learners to visualize concepts, boosts motivation and concentration, and enhances understanding (NEEDU, 2018). Although John et al. (2012) indicated that learners' motivation for mathematics declines as they progress from lower to higher grades, technology-driven deep learning could boost their motivation. Our intervention comes at the right time to support all these efforts.

The government is investing a lot of money in education to improve the integration of technology in schools. Currently, much has been done to provide digital learning materials, but there is still a call to fully utilize the new technologies in improving learning outcomes. The proposed integration of mobile technology could promote learning with understanding. In the next section, I present the contextual understanding of technology integration.

1.2 Technology Integration

It is apparent, from the goals set in the Action plan to 2019 (Department of Basic Education_a, 2015), that the Department of Basic Education is embarking on integrating technology in teaching and learning. This plan was affirmed by the President of South Africa in his 2019 State of the Nation Address (The Presidency, 2019), where promises were made that every learner will have a tablet in the next six years (The Presidency, 2019). However, in the Action plan to 2024, it was reported that there was slow progress in the integration of technology in the understanding of the subject content (Department of Basic Education_a, 2020). The need for online learning during the COVID- 19 pandemic exposed the unreadiness of many schools to integrate technology, and the gaps with regards to digital content for learners and teachers (Department of Basic Education_a, 2020). Moreover, emphasis was put on online content rather than the incorporation of technology to enhance the achievement of learning outcomes (Department of Basic Education_a, 2020).

In the literature, the meaning of the term technology is wide. There are different understandings of the meaning of this term. According to Ramey (2013, p.1), technology is a body of knowledge devoted to creating tools, processing actions, and extracting materials. It is used to simplify tasks, accomplish tasks, and extend people's abilities to handle tasks in their daily lives (Ramey, 2013). The Collins dictionary defines technology as a collection of methods, systems, and devices emanating from scientific knowledge being used for practical purposes (Collins, 2018). Harcourt (2010, p.1523) defines technology as a system by which a society provides its members with those things needed or desired. In this research, we contextually understand technology as a collection of all tools and components that can be used to facilitate teaching and learning. Technology integration is, therefore, the use of tools and components of technology to support and engage learners in a meaningful learning context.

As we move into a technology-based society, the role of technology in education is becoming a globally important concern (Saylan et al., 2018). A great amount of investment and research in the field of educational technologies has been made throughout the world (Saylan et al., 2018). South Africa seeks to have all schools and learners ready for technology integration. The pressure is mounting for teachers and schools to use and implement technology in their classrooms (NEEDU, 2018). Consequently, researchers, educators, and policy-makers have been exploring the best ways of integrating technology in classrooms, to enhance teaching and learning (Department of Basic Education_ar, 2018).

Implementation of technology in teaching is affected by many dynamic variables such as students, teachers, administrators, and the availability of hardware and software (Blackwell et al., 2014). Undoubtedly, the teacher is the most important factor in using and integrating technology (Blackwell et al., 2014). Effective use of technology in education can only be realized through well-equipped teachers (Blackwell et al., 2014). Without such teachers, learners' exposure to educational technology remains limited and inequitable.

It is important to then realize that technology integration is not much about the availability of technological tools, but more about the teachers' effective use of such technology. Concerns around the use of technology in ways that do not support student-centered learning were raised (Blackwell et al., 2014). Teacher input makes a difference in reforming the classroom (NEEDU, 2018; Saylan et al., 2018). It is, therefore, important to integrate technology in a way that supports teachers' capabilities, and that is easy to learn and implement, without posing a steep learning curve (Manuguerra, 2011). It is, also, important to tailor technology to specific curriculum standards. In this research, I focus on integrating mobile technology (see subsection 1.3 for a detailed description of mobile technology) using GeoGebra applets (see subsection 1.4 for a detailed description of GeoGebra applets) tailored to understand the congruence of triangles. The next section presents the contextual understanding of mobile technology.

1.3 Mobile Technology

Successful teaching requires an understanding and appreciation of the learners’ needs, backgrounds, interests, and learning styles (Roberts et al., 2012). It is inferred in the literature that technology integration should effectively match learners’ multiple learning styles and contributes positively towards academic achievement (Rossing et al., 2011). It is, therefore, essential to understand the characteristics and interests of learners in the present generation. Some learners prefer memorizing concepts while others prefer learning with understanding. One of the indicators of learners who prefer understanding is their active engagement during the learning process (Kade et al., 2019). This is demonstrated when students ask questions, participate in discussions, and share ideas with teachers or other classmates (Kade et al., 2019). Teaching strategies that enhance conceptual understanding allow students to wrestle with, and make connections of, important mathematical ideas (Zahner et al., 2012). This can be achieved through mathematical discussions where students share answers and present explanations to peers. When students share ideas, they can identify and rectify an existing misunderstanding (Zahner et al., 2012). Sharing ideas and explanations reduces drill and memorization habits (Zahner et al., 2012).

Today's learners are digital natives (Franklin & Peng, 2008; Thompson, 2013). They are referred to as ‘thumb nails’ due to their expertise in using touch screen devices (Thompson, 2013). There is a call for teaching and learning strategies to simultaneously consider learners’ learning styles and promote conceptual understanding (Kade et al., 2019). Exploration of learners’ digital proficiency for teaching and learning purposes is, therefore, essential. Mobile technology presents some hope for addressing the challenge of bringing mathematics closer to learners’ digital contexts. In this research mobile technology refers to the use of any mobile device small enough to be carried anywhere and everywhere, such as smart cellphones, tablets, iPads, or small computers to perform desired tasks (Korenova, 2017). A smart cellphone is a mobile phone that performs many of the functions of a computer, typically having a touchscreen interface, internet access, and an operating system capable of running downloaded apps (Smartphone, 2018). An App is a short form for application, which is a software program, designed to perform a specific function directly for the user (Rouse, 2011).

Mobile technology offers fresh opportunities to re-envisage some aspects of deep, meaningful learning; that enhance learners' engagement and mathematical thinking (Larkin & Calder, 2015). Mobile technology enables learners to collaborate, create knowledge, and interact with a broader range of content (Rossing et al., 2011). It allows for direct manipulation of content and intuitive learning, as the devices are cognitively simpler to use than computers (Geist, 2012). Some researchers cited more benefits of mobile technology, including developing deeper cognitive abilities of students (Polásek & Sedlácek, 2015), helping visualize difficult to see content (Andraphanova, 2015; Polásek & Sedlácek, 2015), or enhancing understanding of mathematical concepts (Andraphanova, 2015; Akçakın, 2018; Polásek & Sedlácek, 2015; Ota & de Araujo Jr, 2016). Often, learners' attention is also captured, thus improving learners' time on task and engagement. Exploiting learners' digital proficiency and digital nativity fosters deep learning, triggers learners' interest, enhances engagement, and increases learners' time on task, and most importantly, brings mathematics closer to learners' contexts (Akçakın, 2018; Polásek & Sedlácek, 2015). Additionally, the management of activities by teachers would also be simplified (Akçakın, 2018; Larkin & Calder, 2015).

I recommend aspects and features of mobile technology related to visual communication, for curbing language proficiency-related issues, enhancing deep conceptual learning, and engaging learners actively. The benefit of mobile technology is in promoting learning anywhere and anytime, as well as allowing learners to complete tasks at their own pace, and in their convenient time and space. These aspects of mobile technology are of particular interest and are attractive to, my study, as they potentially, remedy the issue of lack of interest by learners in studying mathematics. They also allow the implementation of strategies that promote active learner engagement. Additionally, these aspects of mobile technology enhance deep conceptual understanding without negatively impacting syllabus coverage. This is because they facilitate the accomplishment of tasks that would take longer without using technology.

While some limitations of mobile technology, such as small screens, inadequate memory, and slow network speeds (Cheon et al., 2012) have been noted, there are also concerns whether the claimed benefits of mobile technology are linked to enthusiasm for the tasks or simply enthusiasm for using devices to complete tasks (Falloon, 2017). The need to bridge the digital divide during technology integration has been underscored (Hatlevik et al., 2018). In compliance with this call, the South African government promised to close the digital gap by issuing tablets to every learner in public schools (Presidency, 2019; Department of Education, 2015). In the next section, I introduce GeoGebra and applets, as tools for technology integration in the teaching and learning of mathematics.

1.4 GeoGebra Applets

I propose a case study that uses applets, created in GeoGebra and run on mobile devices, to teach the concept of congruencies of triangles. In this context, an applet is a small piece of software that serves a specific task and can be executed from within a web browser (Morphett et al., 2015). I further sought learners' perceptions (upon acquiring experience in using GeoGebra applets) in the use of mobile devices, in the teaching and learning of mathematics.

GeoGebra is an open-source mathematical dynamic software package. It was developed at the University of Cambridge Education Institute, although the initiator was a lecturer, Markus Hohenwarter, from the Johannes Kepler University of Linz in Austria (Diković, 2009; Žilinskiene & Demirbilek, 2015). It is freely available at www.GeoGebra.org (Žilinskiene & Demirbilek, 2015; Nisiyatussani et al., 2018; Morphett et al., 2015). It can be downloaded or used directly from the website (Morphett et al.,2015). GeoGebra can be used as a standalone desktop application, as a tablet app, or as a browser-based web app. Its main purpose is to aid visualization of mathematical concepts and ideas (Dimitrov & Slavov, 2018; Korenova, 2017; Akçakın, 2018; Polásek & Sedlácek, 2015). The key aspect of the software is its content on dynamic geometry, calculus, symbolic algebra, and statistics functionality (Morphett et al., 2015; Stumbles, 2018). As a case study, I focus on Euclidean geometry, particularly the concept of congruencies of triangles.

GeoGebra, particularly the geometry component, is regarded as dynamic. This means that visualized or drawn objects can be moved around and manipulated while updating measurements of attributes instantly (Diković, 2009; Žilinskiene & Demirbilek, 2015). More so, geometric objects created with a particular relationship to another, maintain the relationship no matter how either object is manipulated or changed (Stumbles, 2018). Additionally, GeoGebra has a set of standard tools that allow easy construction of points, lines, and geometric figures (Stumbles, 2018;

Žilinskiene & Demirbilek, 2015; Romero & del Mar García, 2015). Furthermore, it includes measuring tools that enable students to get accurate measurements of lengths of sides, sizes of angles, areas, and perimeters of geometric figures. It is, therefore, one of the most appropriate tools for the teaching and learning of geometry, particularly Euclidean geometry; and for solving problems related to geometric concepts (Koyuncu et al., 2015).

More benefits of using GeoGebra have been presented in the literature. Precisely, this software allows educators to create an interactive learning environment to foster experimental, problem-oriented, independent, and discovery learning for learners (Žilinskiene & Demirbilek, 2015; Romero Albaladejo & del Mar García, 2015; Getenet, 2015). It facilitates learners' development of conceptual understanding in geometry (Stumbles, 2018; Žilinskiene & Demirbilek, 2015; Romero Albaladejo & del Mar García, 2015; Koyuncu et al., 2015). In particular, visualization is one of the most powerful and widely recognized didactical components of GeoGebra (Žilinskiene & Demirbilek, 2015; Akçakın, 2018; Koyuncu et al., 2015; Polásek & Sedlácek, 2015; Dimitrov & Slavov, 2018; Korenova, 2017). The slider motion tool in GeoGebra is, in fact, a good option for dynamic manipulations (Nisiyatussani et al., 2018; Denbel D. G., 2015). This tool enables users to manually manipulate the drawn geometric objects, and to monitor interactive changes (Denbel, 2015; Nisiyatussani et al., 2018).

In addition to the wide range of tools, GeoGebra has an option of applet construction which allows the author to determine the extent of interactivity for users in design time (Morphett et al.,2015). Using applets comes with some benefits. Constructed applets are dynamic. They enhance the visual representation of concepts, helping learners to create mental images of mathematical concepts (Denbel D. G., 2015; Nisiyatussani et al., 2018; Radović et al., 2018). Additionally, applets often have a specific conceptual focus. They can be used selectively by teachers to support understanding of specific key concepts or to enhance teachers' explanations. The most striking benefit is that applets are usually easy for users to master without training (Morphett et al.,2015). That alone saves the time for teaching how to use GeoGebra. There is no need to have previous experience with the software. This greatly reduces the technological divide concerns and issues experienced by learners and staff. It reduces the effort expended by teaching staff providing technical support (Morphett et al., 2015; Pfeiffer, 2017). A significant strength of GeoGebra applets, from my perspective, is that it removes potential technological barriers from learners and teachers. Fast learners may become so proficient that they would end up showing instructors better ways of exploring the applets created.

An added benefit of using applets is that the user interface can be customized to display only those tools required in a specific applet. This reduces confusion on the selection of tools for a particular problem domain. Furthermore, most icons of the app show and explain the functions of the tools. Applets can run entirely within a web browser without requiring the installation of additional software, add-ins, or plug-ins. They are easily used on mobile devices anytime and anywhere, as long as there is an internet connection. The fact that no additional software installation is required alleviates memory issues. Access to applets typically requires the user to only follow a web link, after which the applet will load and execute automatically. The flexibility of applets allows use anywhere and at any time. Furthermore, the use of applets promotes a comfortable environment that avoids anxiety in using technology for learning mathematics (Radović et al., 2018; Taleba & Hassanzadeh, 2015; Mudaly & Uddin, 2016). Several researchers affirm that the use of appropriate technology enhances students' motivation, confidence, and interest in learning mathematics (Radović et al., 2018; Arbain & Shukor, 2015; Mudaly & Uddin, 2016). This is revealed by; the learners' commitment to the learning task, enhanced enjoyment, increased self-esteem, independence, and confidence (Radović et al., 2018). For this research, online applets were developed and used to discover cases of congruence of triangles in grade 9.

As grounds for my proposed intervention, similar studies have been reported in the literature. GeoGebra applets closely linked and aligned to Indonesia National Curriculum were developed and used in the teaching and learning of quadrilateral mathematical concepts (Nisiyatussani et al., 2018). Pfeiffer (2017) studied a GeoGebra-focused learning environment for mathematics bridging program students at Stellenbosch University, which yielded plausible outcomes; where students preferred to work with prepared GeoGebra applets for circle geometry and some activities on transformations of functions (Pfeiffer, 2017). In his study, Pfeiffer (2017) concluded that GeoGebra helped students to understand the concepts of general solutions of trigonometric equations (Pfeiffer, 2017), and increased student enjoyment, interest, and confidence in the classroom

(Pfeiffer, 2017). However, he cautioned against over-reliance on one teaching style (Pfeiffer, 2017).

I am most inspired by Pfeiffer's (2017, p.304) identification of a gap in research on teaching and learning with GeoGebra, at foundation and secondary levels in South Africa. I am further inspired by the project described by Morphett et al. (2015), where a collection of interactive applets were developed to enhance the teaching and learning of subjects at the undergraduate level. The findings of Takači et al. (2015) are also inspiring, as they emphasize the usage of the GeoGebra applet to support students in mastering the mathematical area of functions (Takači et al., 2015). These views were also echoed by Taleba & Hassanzadehb (2015) when they posited that the use of interactive learning environments in mathematics education has a positive effect on students' achievement and knowledge retention. The same views are shared by Albaladejo et al. (2015), who regarded GeoGebra as having the potential to foster active and student-centered learning, by allowing mathematical experiments, interactive explorations, and discovering concepts (Romero et al., 2015). The call for technology integration that engages learners and enhances learning with understanding is, therefore, long overdue. The next section explicitly states the statement of the problem I address in this thesis.

1.5 Statement of the problem

A statement in the 2017/2018 annual report (Department of Basic Education_ar, 2018) states that there is a steady increase in learner performance in mathematics over the years. Results from the NSC examinations also attest to this progress (Department of Basic Education_d, 2018). To further confirm this, subject specialists reported that learners displayed greater confidence in answering less challenging questions (Department of Basic Education_d, 2018; Department of Basic Education_d, 2019). While the department applauds this progress, there are still concerns that the majority of learners struggle to answer questions that demand deep conceptual understanding (Department of Basic Education_d, 2018, Department of Basic Education_d, 2020; Pfeiffer, 2017). From my teaching experience, learners do not enjoy lengthy scenarios and explanations in questions. Instead, they prefer quick straightforward, and short ways of getting to the solutions. In any case, some of these quick methods do not enhance conceptual understanding (Pfeiffer, 2017).

Quick approaches are often not applicable to the resolution of a broader spectrum of problems (Pfeiffer, 2017). Consequently, learners relying on these quick approaches struggle to answer questions that require a deep understanding of concepts.

Though a statement was issued in the NEEDU report (NEEDU, 2018) about the number of learners getting 50% and more being on the increase, the rate of increase was still very slow. It prolonged the realization of the hopes of having more students enrolling into the science, technology, engineering, and mathematics faculties in higher institutions of learning. Regardless of all the measures put in place by the Department of Basic Education, and by other relevant stakeholders, the problem of low performance in mathematics persists. More worrying is that the number of learners who wrote NSC in mathematics in 2018 dropped compared to those who wrote the NSC in mathematics in 2017 (Department of Basic Education_d, 2018).

A possible cause of slow progress in alleviating low performance in Mathematics is possibly attributed to less focus on teaching strategies that simultaneously motivate learners and enhance conceptual understanding (Crawford, 2001; Jelatu, 2018). An innovative investigation of teaching and learning strategies built on the use of mobile technology could potentially help and improve learner performance in mathematics. I see the need for formalized teaching and learning strategies that enhance deep conceptual understanding, considering the time constraints to complete the syllabus. The need for teaching strategies that motivate learners towards learning and understanding mathematics is also apparent. Teaching strategies that bring mathematics closer to learners' digital contexts are long overdue. The statement of the problem can, therefore, be summarized as an investigation of a mobile technology strategy that could help learners understand the congruence of triangles.

1.6 Research questions

This research is guided by the main question and five sub-questions, all aimed at investigating a mobile technology strategy that could help learners understand the congruence of triangles. Precisely, the thesis explores learners' mobile technological proficiency in the teaching and learning of mathematics, particularly, the congruence of triangles. The sub-questions are stated below,

- What are the learners’ learning preferences?
- Is the learner’s affection or performance in mathematics associated with technology proficiency or technology appreciation?
- Is it practical to use GeoGebra applets on mobile devices to learn mathematics, particularly the congruence of triangles?
- What are the learners’ views on the benefits of using GeoGebra applets (on mobile devices in the learning of mathematics?
- What are the challenges anticipated by learners when using GeoGebra applets for learning mathematics?

In my view, responses to the questions posed in this section would address the general statement of the problem, prescribing a strategy for improving the teaching and learning of mathematics built on mobile technology.

1.7 Motivation

Availability, portability, and affordability of mobile devices are increasing in South Africa (Daichendt, 1999; contributors, 2019; Department of Education, 2011). Although half of 50 million people in South Africa live below the poverty line (Contributors, 2019), more than 75% of people in the low-income bracket who are 15 years or older, own a mobile phone (Contributors, 2019). According to Newzoo's Global Mobile Market Report (2018, p.1), 41,6% of the population of South Africa uses smartphones (contributors, 2019). The desire to exploit the digital proficiency of the current generation (Drijvers, 2013; Nisiyatussani et al., 2018), coupled with the availability of mobile devices (Falloon, 2017; Blackwell et al., 2014), was a major drive for undertaking this research.

Parallel to the first motivating factor, the government of South Africa is putting much effort to integrate technology in teaching and learning (Department of Basic Education_ap, 2020; Department of Basic Education_f, 2018; Department of Basic Education_ap, 2015;). It is further encouraging to note that the government of South Africa aims to close the digital gap by giving each learner a tablet (Presidency, 2019; Department of Basic Education_m, 2018; Department of Basic Education_ap, 2015). This research is thus, responding to the call that was made in the Action

Plan to 2019, to come up with teaching strategies that effectively explore technology facilities made available to schools. Working on ground-breaking calls by the government is on its own, a further motivating factor for this study.

Furthermore, it is inferred in the literature that school curricula need to take advantage of this growing popularity of mobile devices (NEEDU, 2018; Drijvers, 2013). There are thousands of apps available on mobile devices that focus on education (Nisiyatussani et al., 2018; Pfeiffer, 2017). However, many such apps focus on the presentation of information rather than engaging the learner in interacting with, and/or constructing their knowledge (Nisiyatussani et al., 2018; Pfeiffer, 2017). Well-designed apps that focus on specific curricula standards are thus, needed. Researchers commonly point to the potential of digital technology (Drijvers, 2013). The need for well-designed apps that focus on specific curricular standards, and the potential of using digital technology in teaching and learning, also motivated me to undertake this study.

1.8 Envisioned contributions of the study

This study makes contributions from an academic and a practical perspective. From an academic angle, the work supports the call for more research on effective uses of mobile technology in teaching and learning (Presidency, 2019; Pfeiffer, 2017; Department of Basic Education_ap, 2015), with emphasis on implementing strategies that enhance conceptual understanding of specific curriculum standards (Nisiyatussani et al., 2018; Pfeiffer, 2017). This has direct benefits to the body of knowledge, contributing additional content to the literature. Research on effective use of mobile devices in the teaching and learning of mathematics, especially in the South African context, would thus, persist.

From the practical perspective, the work demonstrates the potential of mobile devices as effective tools in the teaching and learning of mathematics. It also brings about awareness to teachers and stakeholders regarding the values of mobile devices as tools with which to disseminate knowledge and facilitate discovery and understanding of mathematical concepts, without affecting syllabus coverage. In my view, these contributions are a worth motivation for the undertaking of most of the aspects presented in this study.

1.9 Overview of the thesis

The thesis comprises seven chapters, the bibliography, and some appendices. Chapter one provided, mainly, background information about the factors which contribute to poor performance in mathematics by learners. It gave a detailed review of the need for technology integration, the coming in of mobile technology, and the values of GeoGebra applets in the teaching and learning of a particular section of mathematics. The explicit statement of the problem was presented, along with the sub-questions, and the objectives of the study. The motivation and rationale for undertaking the study were presented before the scope and boundaries of the study were marked. Envisioned contributions of the study were given from two perspectives; the academic and practical angles.

Chapter two presents related literature informing the area of investigation. Literature on technology integration in general, mobile technology, the teaching and learning of particular sections of mathematics, GeoGebra and applets, geometry, and congruence of triangles is included.

Chapter three presents the theoretical framework which informs our reasoning and argumentation in the rest of this study. Precisely, I emphasize the Framework for the Rational Analysis of Mobile Education (FRAME), which forms the basis for most of the views shared in this research study. The relevance of this theoretical framework to the study is of importance, and is thus, clearly presented in this chapter.

Chapter four dwells mainly on the discussion of the research design, the research methodology, and procedures undertaken to conduct this study. It outlines and summarizes the tools and instruments used in the design of the study.

In chapter five, data collection procedures; including the use of questionnaires and interviews; data recording procedures; and the actual analyses, interpretation, and discussion of the data collected, are the key sections. The key deliverables sought in this chapter are the meanings and interpretations emanating from the data collected.

Chapter six closes the thesis, presenting concluding remarks informed by the findings. The particular findings of this study are aligned with the research questions. Our philosophical reflections arising from the findings are presented and justified in this chapter. Some limitations of the study are shared, and recommendations and suggestions regarding the areas for further researches are discussed in this final chapter.

1.10 Conclusion of the chapter

This chapter presents an introduction to this research study, emphasizing the background information to the factors which contribute to poor performance in mathematics by learners. Among these factors, five stand out, namely; language proficiency issues by learners, teachers' lack of content knowledge, time constraints to complete the syllabus, lack of teaching materials in schools, and the general lack of classroom management skills by teachers. The demand for technology adoption, particularly the use of mobile technology, is discussed. Sections of difficulty for mathematics learners were identified, and a proposal is made to try the use of GeoGebra applets in the teaching and learning of the topic congruencies of triangles. The chapter provides the motivation and rationale for the study, its scope, and the key contributions it made. The chapter closes with an overview of the sections and chapters of the thesis, where a brief preview of the content covered in each chapter was provided. In the next chapter, I present related works to theme areas of this study.

Chapter Two Literature Review

2.1 Introduction

This chapter aims to explore the literature that informed the study. It commences with exploring definitions of geometry in the literature, and the importance of learning geometry. This is followed by a discussion on constructivism; after which, some theories specifically to teaching and learning of geometry are discussed (Van Hiele model of geometric thinking, the theory of figural concepts, the theory of figural apprehension). Also included are some general theories applied to the specifics of geometry education (prototype theory and theory of variation). Discussion of the discovery learning model is also incorporated. The chapter progresses with a discussion of spatial ability, and a review of literature on learning and teaching geometry using GeoGebra. The elements of the research paradigm, qualitative research, and quantitative research methods were briefly discussed.

2.2 Geometry

Several definitions of geometry have surfaced in the literature. A few of those definitions are discussed in this paragraph. Güven and Kosa (2002, p.1) say geometry is the study of shape and space. Some have said that geometry explores the characteristics and relationships of angles, lines, and shapes (Ünlü & Ertekin, 2017). Comparatively, geometry is also defined as the study of properties, relationships, and transformations of spatial objects, within an interconnected network of concepts and representational systems (Crompton et al., 2018). Kösa (2016) added more terms to his previous definition and said that geometry includes point, line, plane, space, spatial figures, and the relationships among these. This research focuses on the congruent relationship between triangles.

Learning geometry comes with many benefits (Crompton et al., 2018). It provides students with opportunities to better understand the physical environments in which they live (Crompton, 2016; Abdullah & Zakariab, 2012). Additionally, geometry learning helps learners develop spatial intuitions that refer to how one views space and area in the real world (Abdullah & Zakariab, 2012; Gunhan, 2014). Geometry teaching and learning are also aimed at fostering the skill to (1) analyze characteristics and properties of two and three-dimensional geometric shapes, and develop mathematical arguments about geometric relationships (Abdullah & Zakariab, 2012), (2) specify locations and describe spatial relationships using coordinate geometry and other representational systems (Alqahtani & Powell, 2015; Abdullah & Zakariab, 2012), (3) apply transformations and use symmetry to analyze mathematical situations (Abdullah & Zakariab, 2012; Aric & Aslan-Tutak, 2015), (4) use visualization spatial reasoning, and geometric modeling to solve problems (Abdullah & Zakariab, 2012; Sariyasa, 2017; Alqahtani & Powell, 2015; Aric & Aslan-Tutak, 2015). Knowledge and skills acquired through geometry learning are useful in architecture and design, engineering, and in various aspects of construction work (Abdullah & Zakariab, 2012).

This research focuses on using mobile technology to understand congruence. In real life, congruent triangles are mostly used in construction, when there is a need to reinforce structures so that they are strong and stable, and do not bend or buckle in strong winds or under load. They are used in the construction of bridges and buildings because they are considered more stable and stronger to use (Pussy, 2013).

2.3 Geometry in the South African Curriculum

The geometry section in the Curriculum Assessment Policy Statement is referred to as space and shape in the foundation phase (grade 1 to 3), intermediate phase (grade 4 to 6), and senior phase (grade 7 to 9) (Department of Basic Education_cf, 2011). At these phases, the content focuses on the properties, relationships, orientations, positions, and transformations of two-dimensional shapes and three-dimensional objects (Department of Basic Education_cf, 2011).

At the foundation level, the main progression in space and shape is achieved by focussing on new properties and features of shapes and objects in each grade; and moving from learning the language of position, and matching different views of the same objects to reading and following directions on informal maps (Department of Basic Education_cf, 2011). The learning activities include sorting, classifying, describing, and naming two-dimensional shapes and three-dimensional objects, drawing shapes, and building with objects (Department of Basic Education_cf, 2011). Additionally, the curriculum at this phase aims to produce learners who can recognize and describe shapes and objects in their environment that resemble mathematical objects and shapes

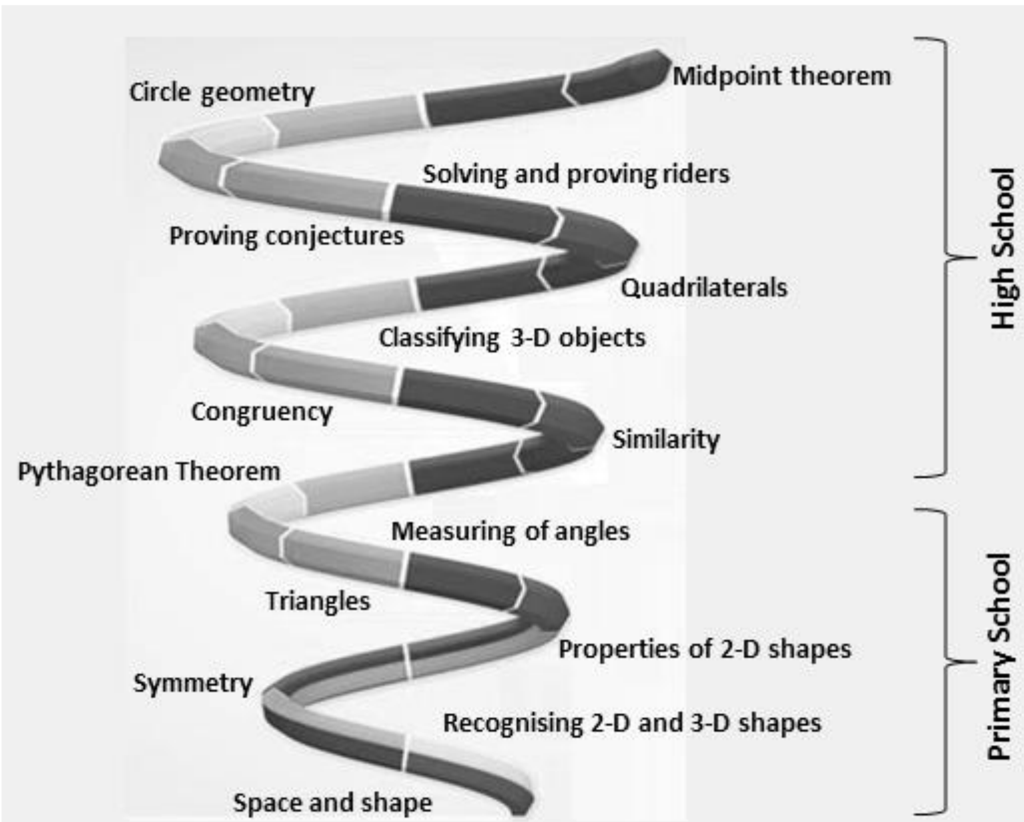
(Department of Basic Education_cf, 2011). Furthermore, learners are expected to develop skills to follow and give directions; and describe the position of objects, themselves, and others using the appropriate vocabulary (Department of Basic Education_cf, 2011).

In the intermediate phase (grade 4 to 6), the learners' experience of space and shape in this phase moves from recognition and simple description to classification and more detailed description of characteristics and properties of two-dimensional shapes and three-dimensional objects (Department of Basic Education_ip, 2011). In the senior phase (grade 7 to 9), there is a progression from informal descriptions of geometric figures to more formal definitions and classifications of shapes and objects. The formal definitions involve investigating new properties of shapes and objects and solving more complex geometric problems using known properties of geometric figures developing from inductive reasoning to deductive reasoning (Department of Basic Education_ip, 2011). In the senior phase, the section is divided into five topics; geometry of two-dimensional shapes, the geometry of three-dimensional objects, geometry of straight lines, transformation geometry, and construction of geometric figures (Department of Basic Education_ip, 2011). The geometry of two-dimensional shapes focuses on the similarity and congruence of two-dimensional shapes. In this section, learners are expected to recognize and describe similar and congruent figures by comparing shapes and sizes (Department of Basic Education_ip, 2011). As learners progress in grade 8, they are expected to identify and describe the properties of congruent shapes and similar shapes (Department of Basic Education_sp, 2011). In grade 9, learners are required to establish the minimum conditions for congruent triangles through investigation (Department of Basic Education_sp, 2011).

In this study, applets were designed to investigate the four cases of minimum conditions of congruence. This is why grade 9 learners were chosen as participants. The content covered up to this phase laid the foundation for the sections covered in the further education and training phase. At the further education and training phase, the space and shape section is split into analytical and Euclidean geometry (Department of Basic Education_cfet, 2011). The congruence of triangles is mainly applied in grade 10 (proving properties of special quadrilaterals) and grade 11 (for proving some theorems of circle geometry) (Department of Basic Education_cfet, 2011). Figure 1 illustrates the progression of geometry from primary school to high school. This progression is aligned to the van Hiele levels of geometry thinking (Naidoo & Govender, 2019).

Figure 1

Progression of Geometry in South African Curriculum



From "Exploring In-service and Pre-service Teachers' Perceptions of Integrating Technology-based Tools When Teaching Circle," by J. Naidoo and R.G. Govender, 2019, *The International Journal of Science, Mathematics and Technology Learning*, *26*(2), p. 31. Copyright 2019 by Common Ground Research Networks.

2.4 Congruence of triangles

Triangles are congruent when all corresponding sides and interior angles are equal (Department of Basic Education_sp, 2011). There are four minimum conditions for congruent triangles that are prescribed in the grade 9 curriculum: three corresponding sides equal (S, S, S); two corresponding sides and the included angle are equal (S, A, S); two corresponding angles and a corresponding side are equal (A, A, S); and right-angle, hypotenuse and one other corresponding side are equal (R, H, S) (Department of Basic Education_sp, 2011). In previous grades, learners would conclude that two triangles are congruent if all corresponding sides and angles were equal. However, in grade 9 they are required to establish congruence when given the minimum conditions listed above.

2.5 Theories underlying geometry understanding

Understanding geometry is essential in itself, as well as for understanding other areas of mathematics (Jelatu, 2018). Unfortunately, geometry and mathematics as a whole is often taught through the lecture method (Abdullah & Zakariab, 2012), which affords surface understanding, merely requiring students to memorize mathematical facts (Jelatu, 2018). Thus, learning geometry with understanding is posing a challenge in South Africa (Pfeiffer, 2017; De Villiers, 2010) and countries throughout the world (Arici & ASLAN-TUTAK, 2015; Ünlü & Ertekin, 2017). It is mentioned every year in National Senior Certificate diagnostic reports, that students fail to answer questions that need deep conceptual understanding (Department of Basic Education, 2020). Before focusing on strategies that support the understanding of geometry, it is essential to discuss theories underlying geometry understanding. Many researchers assert that constructivist approaches support critical thinking and can help students to develop a deep understanding (Kaufmann, 2004; Zulnaidi & Zakaria, 2012; Crawford, 2001). Zengin (2011, p.184) posit computer-assisted instruction, as a supplement to constructivist instruction. This is particularly the main focus of this research. I seek to use mobile technology as a supplement to constructivist instruction.

2.5.1 Constructivism

There is no universal definition of constructivism (Mvududu & Thiel-Burgess, 2012). For some, it is a theory of learning; for others, it is a theory of knowledge; and for others still, it is a pedagogical theory (Amineh & Asl, 2015; Mvududu & Thiel-Burgess, 2012). Additional views consider it a theory of science, an educational theory, or an all-encompassing worldview (Mvududu & Thiel-Burgess, 2012). In the most general sense, the contemporary view of constructivist learning is that people construct new knowledge and understandings based on what they already know and believe (Crawford, 2001). Constructivism has roots in Socrates's dialogues: 496 BC – 399 BC (Goodwin & Webb, 2014). It is acknowledged that this question-and-answer strategy between teachers and learners enhances the construction and interpretation of hidden knowledge (Amineh & Asl, 2015; Goodwin & Webb, 2014). Furthermore, it assists teachers to assess student understanding and plan learning experiences that promote independent, reflective and critical thinking (Goodwin & Webb, 2014). In line with that, as students answer the teachers' "why" questions, they get into a deeper analysis of the properties of figures, which in turn enhances conceptual understanding (Amineh & Asl, 2015; Goodwin & Webb, 2014; Swoboda & Vighi, 2016).

Some prominent philosophers in the constructivist theory are Jean Piaget, Lev Vygotsky, John Dewey, Jerome Bruner, and Seymore Papert (Pfeiffer, 2017). In their research review of constructivism and social constructivism, Amineh and Asl (2015, p.11) discuss several principles governing the constructivist view of learning. Two strands of constructivism are identified are more dominant in the classroom (Amineh & Asl, 2015). First, is cognitive or individual constructivism attributed to Jean Piaget (Amineh & Asl, 2015; Powell & Kalina, 2009). Second, is social constructivism depending on Vygotsky's theory (Powell & Kalina, 2009; Amineh & Asl, 2015).

This paragraph presents some of the principles from Piaget's theory of constructivism in the classrooms. First, it is perceived that learners construct new understandings using what they already know (Amineh & Asl, 2015; Pfeiffer, 2017; Mvududu & Thiel-Burgess, 2012). In other words, they come to learning situations with knowledge gained from previous experiences (Mvududu & Thiel-Burgess, 2012), and form meaning based upon those experiences (Amineh & Asl, 2015). Prior knowledge influences what new or modified knowledge learners will construct from the new learning experiences (Mvududu & Thiel-Burgess, 2012). Teachers should, therefore, understand and explore students' prior knowledge, and actively engage them in the construction of new knowledge (Mvududu & Thiel-Burgess, 2012; Amineh & Asl, 2015). In this research, it was assumed that learners knew that objects are congruent or identical if all corresponding attributes of the objects are equal. I, therefore, anticipated that learners extended this knowledge as they discovered that there are sufficient conditions required to affirm the congruence of triangles, instead of comparing all attributes.

Second, it is attested that discovery is the most fundamental basis of learning (Amineh & Asl, 2015). Therefore, the teacher's role is to design activities that guide students to discover knowledge rather than memorize facts (Goodwin & Webb, 2014). In line with this view, as learners negotiate, ask questions, and try hard to find the answers themselves; what they learn becomes more meaningful to them and enhances deeper conceptual understanding (Amineh & Asl, 2015). In addition, discovery augments a sense of ownership in learners, of their work; which in turn, can boost their commitment to learning (Koohang et al., 2009; Amineh & Asl, 2015). Furthermore, discovered knowledge is more likely to be retained and transferred to real-life contexts (Bada, 2015). Since mathematics is a cumulative subject, this research anticipates that as learners discover cases of congruence, they are then able to retain the aspects learned and apply them where necessary.

Third, learning is an active process in which learners confront their understanding in light of what they experience in the new learning situation (Bada, 2015; Amineh & Asl, 2015; Mvududu & Thiel-Burgess, 2012). This research focused on the use of mobile devices. If what learners encounter is inconsistent with their current understanding, their understanding can change to accommodate new experiences (Bada, 2015; Amineh & Asl, 2015; Mvududu & Thiel-Burgess, 2012).

In this paragraph, social constructivism in teaching and learning is briefly discussed. The social or realist constructivist tradition is credited to the work of Vygotsky (Amineh & Asl, 2015). The social constructivist theory acknowledges that a student can learn from a more knowledgeable adult or peer (Pfeiffer, 2017). It also encourages the learner's version of the truth that is influenced by his or her background, culture, or knowledge of the world (Amineh & Asl, 2015). This theory assumes that understanding, the significance of knowledge, and meaning are developed in collaboration with other human beings (Amineh & Asl, 2015). It suggests that knowledge is first constructed in a social context, and then internalized and used by individuals (Amineh & Asl, 2015). Other constructivist scholars agree with this and emphasize that individuals make meaning through interactions with each other, and with the environment, they live in (Bada, 2015). From the social constructivist viewpoint, it is thus, important to take into account the background and culture of the learner during the learning process (Amineh & Asl, 2015; Bada, 2015). The learner's background also helps to shape the knowledge and truth that the learner creates, discovers, and attains in the learning process (Amineh & Asl, 2015; Mvududu & Thiel-Burgess, 2012). Present-day learners live in a mobile society (Figueiredo et al. 2016) and it is, therefore, necessary for educational institutions to appropriate mobile devices in teaching and learning, to coincide with learners' mobile lifestyle (Figueiredo et al., 2016). Through the use of personal mobile devices, learners become active participants in their learning process, and creative producers of learning content (Figueiredo et al., 2016).

2.5.2 Theories specific to the teaching and learning of geometry

This section, like the preceding one, is based on a review of the research literature on geometry education by Sinclair et al. (2016), spanning from 2008 to 2015. In their review, they had two categories of theories; theories that are specifically about the teaching and learning of geometry and, general theories applied to the specifics of geometry education (Sinclair, et al., 2016).

2.5.2.1 Theories specific to the teaching and learning of Geometry

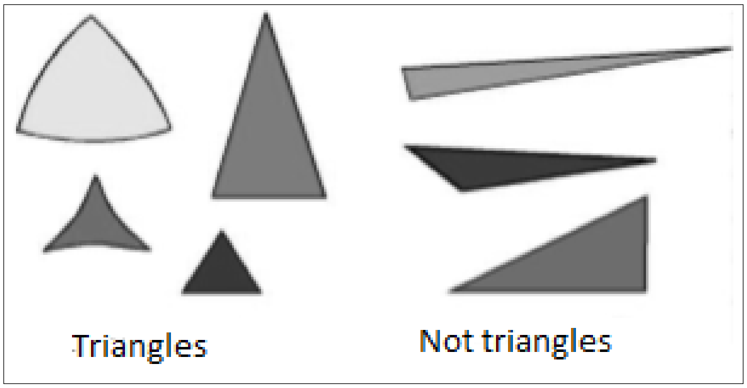
In this subsection, I discuss van Hiele's model of geometrical thinking, the theory of figural concepts, and the theory of figural apprehension and figural deconstruction. This is because they provide the grounding for educational aspects in this study.

2.5.2.1.1 Van Hiele's theory

Van Hiele's theory has been influential over decades of research on children's geometric understanding (Lai & White, 2012; Yılmaz & Koparan, 2016; Pfeiffer, 2017). Van Hiele's theory was foregrounded during both the design of the 2005 South African curriculum statement and its revision in 2012 (Alex & Mammen, 2016). Van Hiele described how children develop their geometric understanding using five levels (Lai & White, 2012; Yılmaz & Koparan, 2016; Pfeiffer, 2017). The first four levels are the most pertinent for secondary school geometry, and in the South African school curriculum (Department of Basic Education_sp, 2011; Pfeiffer, 2017).

Figure 2

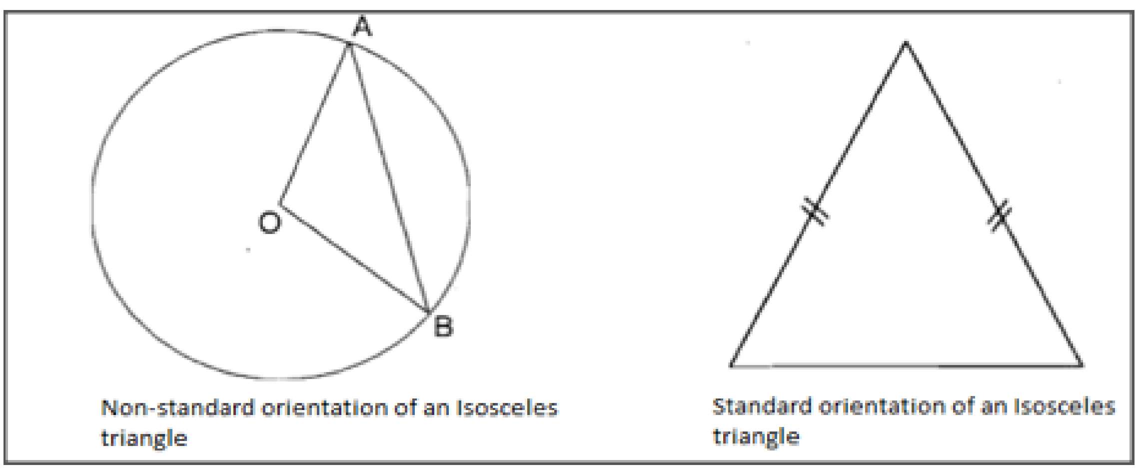
Classification of triangles by children at Van Hiele's Level One.



Note. From "The van Hiele model of geometric thinking", by I. Vojkuvkova, 2012, *WDS' 12 Proceedings of Contributed Papers,* p.72. Copyright 2012 by MATFYZ PRESS.

Figure 3

Isosceles triangles identification at level one



Note. From "A Study Of The Development Of Mathematical Knowledge In A GeoGebra Focused Learning Environment", by C. Pfeiffer, 2017, p.60. Copyright (2017) by Stellenbosch University.

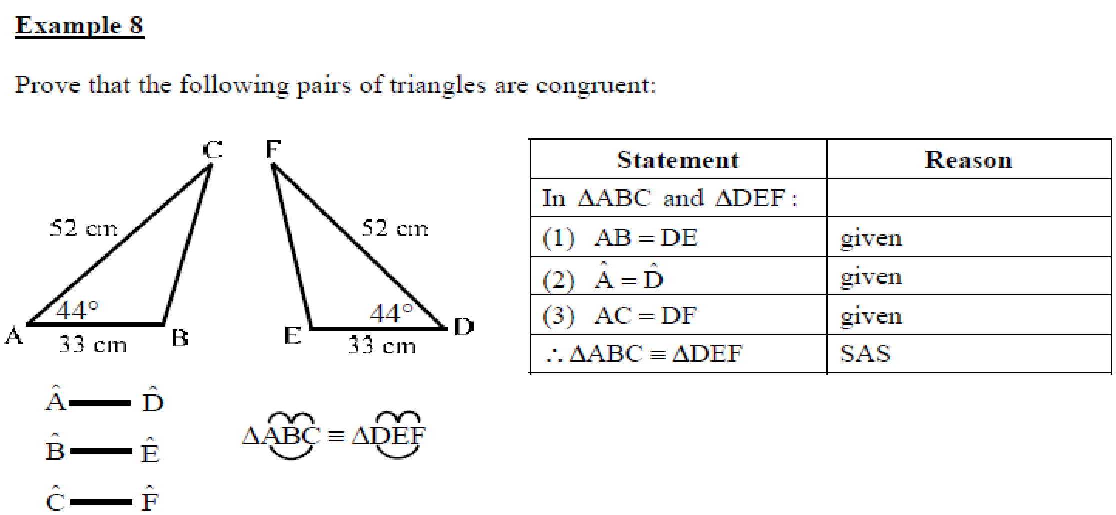
Level one is the Visualisation or Recognition level (Lai & White, 2012; Yılmaz & Koparan, 2016; Pfeiffer, 2017). This level starts with non-verbal thinking (Alex & Mammen, 2016), where students are just interested in the image of the figure (Yılmaz & Koparan, 2016), and the figures are judged by their appearance (Alex & Mammen, 2016; Pfeiffer, 2017). Students recognize the shapes of triangles, squares, parallelograms, and so forth; but they do not explicitly identify the properties of these figures (Pfeiffer, 2017). At this level, it is advised that students be exposed to non-standard orientations of concepts (Pfeiffer, 2017). Figure 2 and figure 3 give examples where students get distracted by the orientation of the shapes and fail to identify figures correctly because they are not in standard orientation (Vojkuvkova, 2012; Pfeiffer, 2017). At this level, two triangles are said to be congruent because they have the same orientation and they look the same (Pfeiffer, 2017).

Level two is the analysis level (Lai & White, 2012; Yılmaz & Koparan, 2016; Pfeiffer, 2017). At this level, students analyze figures in terms of their parts and the relationships between these parts (Pfeiffer, 2017). They begin to recognize shapes by their properties (Özerem, 2012) and establish the properties of a class of figures empirically (Alex & Mammen, 2016; Pfeiffer, 2017). Additionally, they can distinguish the features of a figure (Yılmaz & Koparan, 2016), and use properties to solve problems (Pfeiffer, 2017). Furthermore, at this stage, students have the view that all the properties are important, and they see no difference between necessary and sufficient properties (Vojkuvkova, 2012; Pfeiffer, 2017). The definitions given by students at this stage are not precise, and often include redundancies (Pfeiffer, 2017), or are not economical (Govender & De Villiers, 2004). For example, students at this level would say congruent triangles are triangles that have corresponding sides congruent and corresponding angles congruent (Pfeiffer, 2017; Govender & De Villiers, 2004). However, it is sufficient to require less than that for two triangles to be congruent. In South Africa, grade 8 students are expected to identify and write clear definitions of triangles in terms of their sides and angles; distinguishing between equilateral triangles, isosceles triangles, and right-angled triangles (Department of Basic Education_sp, 2011). However, they may not be able to classify an equilateral triangle as an isosceles triangle.

Level three is commonly known as the Informal deduction or Order Level (De Villiers, 2010; Pfeiffer, 2017; Vojkuvkova, 2012; Mudaly, 2004). Students start to see the features and relationships of shapes. They can logically order and identify which properties are implied by others (Vojkuvkova, 2012; De Villiers, 2010; Pfeiffer, 2017). In like manner, they can differentiate between necessary and sufficient conditions (Vojkuvkova, 2012; Pfeiffer, 2017). They also understand the role of definitions, theorems, axioms, and proofs (Vojkuvkova, 2012). Furthermore, at this level, an equilateral triangle is recognized as being an isosceles triangle because the definitions of the figures are precise (Pfeiffer, 2017). Over and above that, students are capable of ‘if… then’ thinking (but not formal proofs) at this level; and logical reasoning can be developed (Pfeiffer, 2017). In the South African Curriculum, this level is manifest in grade 9 (Department of Basic Education-sp, 2011). In grade 8, students identify congruence of triangles as being required to establish the minimum conditions for congruent triangles through investigation (Department of Basic Education-sp, 2011). I am inspired by the work of Govender and De Villiers (2004) where they used the dragging facility in Sketchpad to explore definitions of quadrilaterals (Govender & De Villiers, 2004). They attested that using the dragging facility, participants achieved a better understanding of necessary and sufficient conditions and were able to give economical definitions (Govender & De Villiers, 2004). I anticipate that using the slide tool in GeoGebra, students can discover the necessary and sufficient conditions to establish the four congruence axioms of triangles. Ideally, students should investigate conditions of congruence by construction (Department Of Basic Education, 2011). However, I opt for the use of applets, mainly to remove the software knowledge barrier for both teachers and students. Furthermore, construction would take time and be a constraint to completing the syllabus.

Figure 4

An example of a question on proving the congruence of two triangles



Statement	Reason
In ΔABC and ΔDEF :	
(1) AB = DE	given
(2) $\hat{A} = \hat{D}$	given
(3) AC = DF	given
∴ ΔABC ≡ ΔDEF	SAS

Note. From "Mind Action Series Mathematics Grade 9 Textbook (NCAPS)”, by M.D. Philips et. Al., 2015, p.157. Copyright (2015) by Allcopy Publishers.

Level four is the Deduction Level. At this level, students can order the relationships of figures (Yılmaz & Koparan, 2016). Additionally, they can identify necessary and enough conditions, and use them for proof or inference (Yılmaz & Koparan, 2016). Furthermore, students start developing longer sequences of statements and can use proven theorems and axioms to prove other theorems (Yılmaz & Koparan, 2016). In the South African curriculum, after the discovery of axioms of congruence of triangles, given sufficient conditions; students are required to prove the congruence of triangles. This is done by listing the necessary conditions given and a closing statement indicating the respective axiom used to justify congruence. An example of a question on proving the congruence of two triangles is given in Figure 4. It is posited that students who are not at this level, can only do proofs by memorization (Pfeiffer, 2017).

Level five is the rigor level (Alex & Mammen, 2016; Pfeiffer, 2017). Students at this level can interpret and use axioms and definitions (Alex & Mammen, 2016). They can recognize the differences, relationships, and comparisons between axiomatic systems (Yılmaz & Koparan, 2016). At this level, students understand the formal aspects of deduction, such as establishing and comparing mathematical systems (Pfeiffer, 2017). They can understand the use of indirect proof and proof by contrapositive theorem or axiom and can understand both Euclidean and non-Euclidean systems (Pfeiffer, 2017).

I move to discuss five characteristics of the van Hiele levels; hierarchical, adjacency, distinction, separation, and attainment, as elaborated by Vojkuvkova (2012). First, many researchers adopt the notion that geometric understanding is hierarchical (Vojkuvkova, 2012; Alex & Mammen, 2016; Pfeiffer, 2017). Consequently, levels are used as a means for judging the general progression in curriculum material in some 555 schools abroad, and in South Africa (Sinclair, et al., 2016; Alex & Mammen, 2016). For example, the distribution of grade-level learning expectations (GLEs) of the elementary and middle school (grades K to 8) in some states of the USA, is consistent with the general thrust of the van Hiele theory; particularly, the claim that the levels of geometric thinking are sequential (Sinclair, et al., 2016). While some researchers find some utility in the van Hiele model of levels, Papademetri-Kachrimani (2012, p.5) argues against the view that "geometrical thinking can be described through a hierarchical model formed by levels (Sinclair, et al., 2016). Instead of using the word level or lower and higher level of thinking, he/she suggests the use of the phrase "diverse modes of understanding and different ways of thinking” (Papademetri-Kachrimani, 2012). The idea that children (and adults) may think in different ways and assign different meanings to a specific geometric concept is similar to geometric understanding within the sphere of epistemological pluralism; the idea of "accepting the validity of multiple ways of knowing and thinking" (Padayachee, 2010). Papademetri-Kachrimani (2012, p.5) goes on to argue that substituting the word "levels" with the phrase "diverse modes of understanding" is consistent with the view that children may think in different ways independent of their age, and may think in different ways simultaneously.

Second, levels are adjacent. What was intrinsic in the preceding level becomes extrinsic in the current level (Vojkuvkova, 2012). Third, each level has its linguistic symbols and network of relationships connecting those symbols. It is distinct (Vojkuvkova, 2012). What may be “correct” at one level may not necessarily be correct at another level (Vojkuvkova, 2012).

Fourth, two persons at different levels cannot understand each other. They are separated (Vojkuvkova, 2012). The teacher speaks a different “language” to the student at a lower level (Vojkuvkova, 2012). Van Hiele’s thought this property was one of the main reasons for failure in geometry (Vojkuvkova, 2012; De Villiers, 2010). Thus, a teacher must establish learners’ competence level or prior knowledge before starting a new section (Killen, 2015).

Fifth, attainment, the learning process leading to complete understanding at the next level, has five phases; Information or inquiry, guided orientation, explanation, free orientation, and integration. These are not strictly sequential (Vojkuvkova, 2012). In contrast to Jean Piaget who was of the view that cognitive levels are age-dependent, van Hieles believed that cognitive progress in geometry can be accelerated by instruction (De Villiers, 2010). The progress from one level to the next one is more dependent upon instruction than on age or maturity (De Villiers, 2010; Vojkuvkova, 2012).

Some researchers focus on transferring van Hiele to other areas of mathematics, for example; Boolean Algebra, Function – Analysis – Calculus, etc (Vojkuvkova, 2012). Further studies have been done in the field of using dynamic geometry software to achieve higher van Hiele levels (Vojkuvkova, 2012; De Villiers, 2010). The van Hiele theory supplies an important explanation.

The transition from Level 1 to Level 2 particularly poses specific problems to second language learners, since it involves the acquisition of the technical terminology by which the properties of figures need to be described and explored (De Villiers, 2010). This requires sufficient time, which is not available in the presently overloaded secondary curriculum (De Villiers, 2010). De Villiers (2010) also emphasizes that the future and success of secondary school geometry depends on primary school geometry. A comparative statement was made between Japan and South Africa. In Japan, pupils start in Grade 1 with extended tangram, as well as other planar and spatial investigations; and by Grade 5, they are already dealing formally with the concepts of congruence and similarity; concepts that are formally introduced in Grades 8 and 9 in South Africa (De Villiers, 2010). De Villiers (2010) argues that, though geometry sections are introduced in the South African primary school curriculum, teachers and textbook authors do not appear to understand its relevance to the van Hiele theory. More emphasis is put on the secondary school curriculum. Challenges in completing the syllabus may lead to learners progressing to the next grade without covering all sections. Lack of content knowledge also influences educators to focus more on sections they are comfortable with, and ignore geometry sections.

Worth mentioning is the explanation given by De Villiers on how construction and measurement can assist the transition from level one to level 2. The use of applets can assist the transition from one level to the next; in particular, level one to level two transition, and level two to level three transition. He argues that students who are predominantly at van Hiele Level 2 cannot be expected to logically check their descriptions (definitions) of quadrilaterals, but should be allowed to do so by accurate construction and measurement (De Villiers, 2010). It is also emphasized that measuring cannot be used to show that some relationships hold. Instead, it helps understand relationships (Koyuncu et al., 2015; De Villiers, 2010). Similarly, in this study, measuring is used to discover necessary and sufficient conditions of congruence of triangles.

In various research studies, it has been attested that technology integration contributed positively towards the attainment of higher van Hiele models (De Villiers, 2010; Koyuncu et al., 2015). De Villiers (2010) calls for more research on how using dynamic geometry software enhances or constrains the development of geometric thinking.

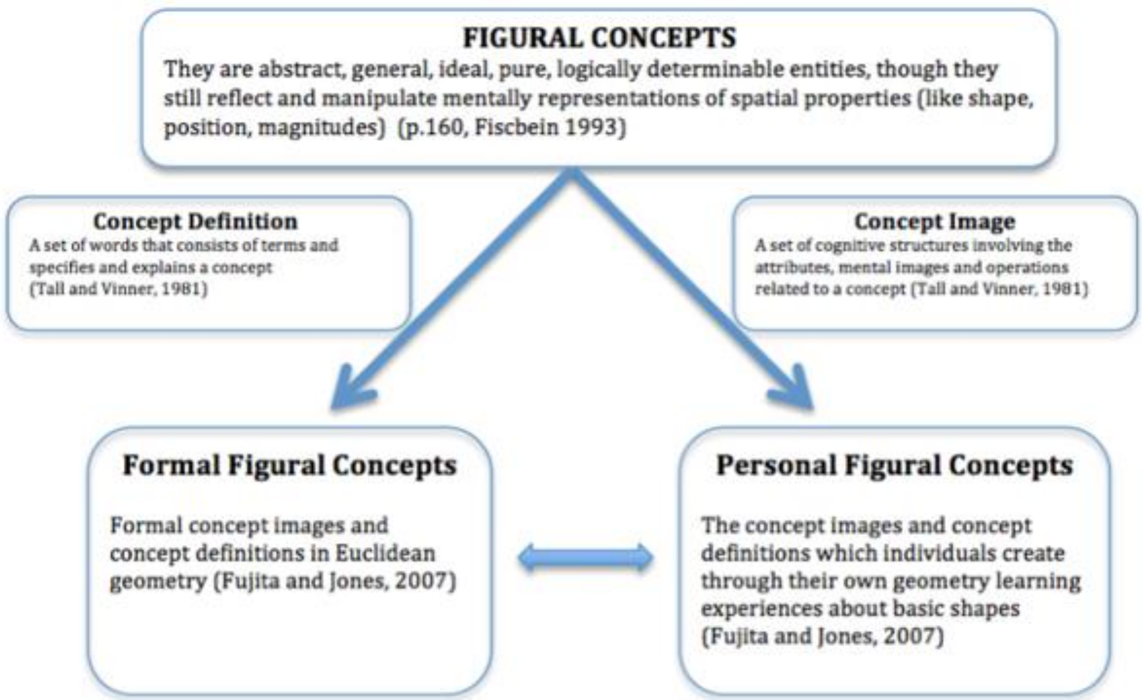
2.5.2.1.2 The theory of figural concepts

A geometrical shape should be considered as a figural concept of a dual nature, that simultaneously possesses figural properties and logical conceptual constraints (Fischbein, 1993). While the figural aspect involves spatial properties (e.g. shape, position, and magnitude), the conceptual component involves properties of an abstract and theoretical nature (e.g. ideality, abstractness, generality, and perfection) that geometrical concepts share with all other concepts (Erdogan & Dur, 2014). The figural concept theory is applied to issues of defining concepts (Sinclair, et al., 2016; Erdogan & Dur, 2014; Fujita & Jones, 2007). Definitions are particularly important in determining conceptual understanding of geometrical concepts (Erdogan & Dur, 2014). It was suggested that the figural aspect is generally more dominant than the conceptual aspect (Fischbein, 1993). Fujita and Jones (2007) claimed that while Fischbein (1993) regarded the figural concept as a process in which the coherence between the figural and conceptual aspect develops into the ideal form, he did not address the development of this process in individuals. Hence, they reinterpreted the definition of the figural concept as having two components; personal figural concept and formal figural concept (Fujita & Jones, 2007). These concepts, and the relationships among them, are illustrated in Figure 5. The personal figural concepts refer to the individual's own figural concept images and definitions, which they construct through their own experiences of learning geometry (Fujita & Jones, 2007). The formal figural concept refers to Euclidean geometry formal concept images and concept definitions found in textbooks (Fujita & Jones, 2007). For example, the personal figural concept requires evidence that all corresponding sides and angles are equal, to conclude that two triangles are congruent; whereas, the formal figural concept definition only focuses on the necessary and sufficient conditions to establish congruence. Figure 6 shows the figural concept of the SAS case of congruence of triangles. From my personal experience, I concur with the suggestion that the personal figural concept is more dominant than the formal figural concept. I also believe that failure to progress from personal figural concept to formal figural concept is likely to pose challenges when questions require the application of these formal definitions. For example, questions like, "Prove that the given parallelogram is a rectangle", may be very difficult to comprehend if one does not have the correct mental image of the parallelogram.

In their research, Erdogan and Dur (2014, p.1) investigated the relationships between pre-service mathematics teachers' figural concepts and quadrilateral hierarchical classifications. They found that the teachers' prototypical images learned at school dominated their personal figural concepts, although they possessed formal definitions of quadrilaterals (Erdogan & Dur, 2014). Their claim concurs with Fischbein's (1993) suggestion that the figural aspect is generally, more dominant than the conceptual aspect. It is observed that many mistakes made by students in geometric reasoning are caused by the gap between the two aspects of a figural concept (Erdogan & Dur, 2014). Relating this theory to van Hiele, I would say that the personal figural concept definition includes all aspects of the object, and is expected from learners in level 1 and level 2.

Figure 5

Theory of figural concept model

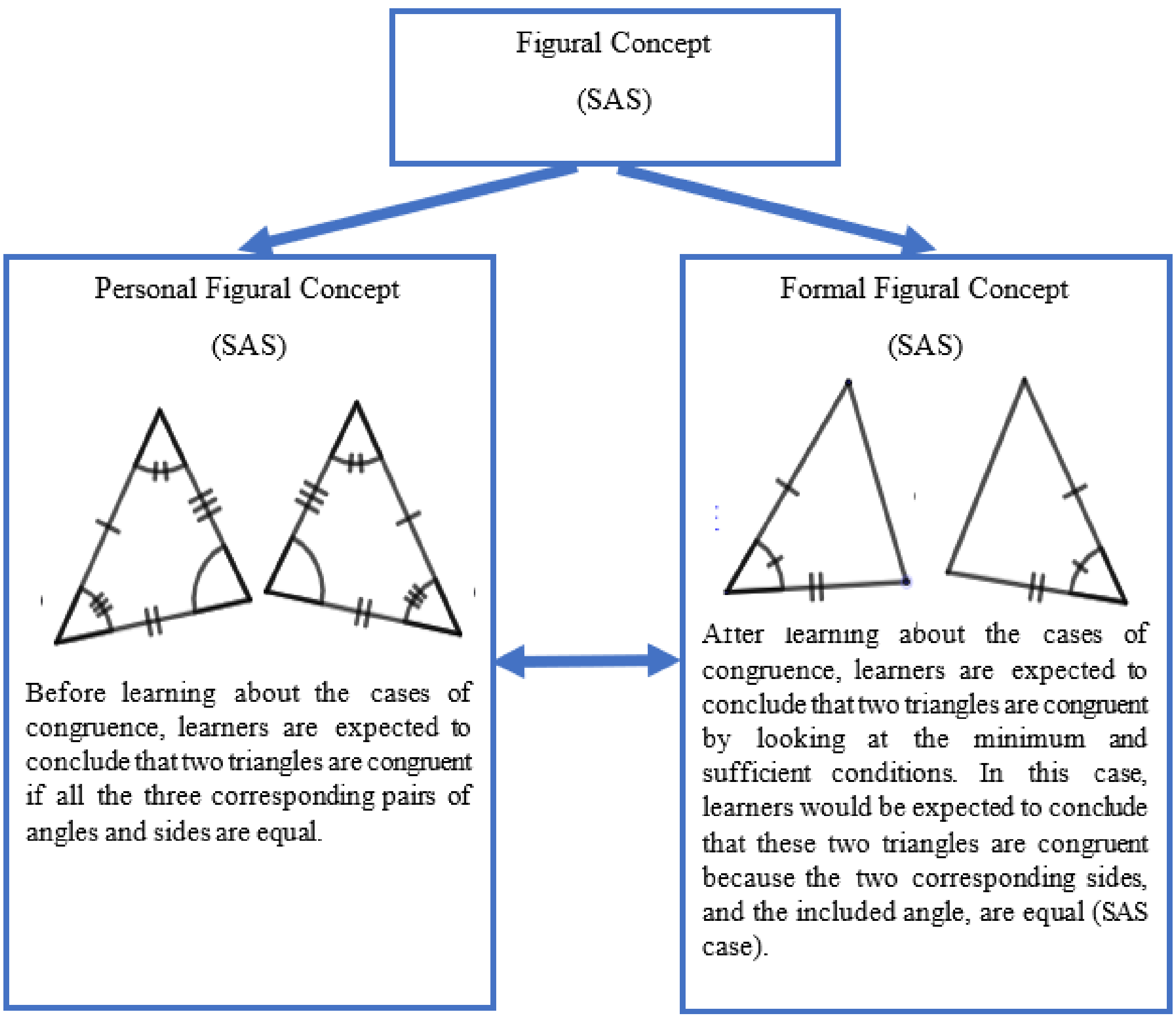


Note. Diagram to illustrate theory of figural concept model. From "Preservice Mathematics Teachers' Personal Figural Concepts and Classifications About Quadrilaterals*", Australian Journal of Teacher Education*, *39*(6), p.110, by E. O. Erdogan &, Z. Dur. Copyright 2014 by Teacher Education and Professional Development Commons.

The process of constructing formal figural concepts involves a transition from Level 2 to level 3; where students are expected to give necessary and sufficient aspects or properties of an object (Erdogan & Dur, 2014). It is connoted that the development of a personal figural concept into the ideal formal figural concept needs to be supplemented with didactic situations that keep both the figural and conceptual aspects active (Erdogan & Dur, 2014). This calls for strategies that enhance visualization of concepts and discovery of formal figural concepts; for example, the proposed use of mobile technology.

Figure 6

Figural concept development of the SAS case of congruence of triangles



2.5.2.1.3 The theories of figural apprehension and dimensional deconstruction

According to the notion of figural apprehension by Duval (as cited in Sinclair et al., 2016), there are four types of cognitive apprehension. Perceptual apprehension is the initial apprehension of a geometrical figure. This is what is recognized at first glance. I can relate this to Van Hiele's level 1 of geometric thinking. Sequential apprehension depends on the mathematical properties and is required to construct or explain the construction of figures. The discursive apprehension is the description of properties and relations of geometric figures. Operative apprehension involves modifying a figure or dividing a figure into sub-figures while maintaining its geometric properties.

Duval (as cited in Sinclair et al., 2016) suggests that work with computers may support the development of sequential apprehension and operative apprehension if the software is tailored for the specific task. Similarly, this research proposes the use of applets on mobile technology, that are tailored for specific tasks. Gal and Linchevski (2010) utilized Duval's notion of figural apprehension in their work on student difficulties in geometry (Sinclair, et al., 2016). They found that it was difficult for students to distinguish, within the configurations of a geometric diagram, the visual characteristics that are relevant from those that are not (Sinclair et al., 2016). Such difficulties, they argue, emerged because students could not progress from the visual perception to other apprehensions (Sinclair et al., 2016). Accordingly, they argue that visual perception is not only insufficient but may even hinder the use and development of geometric concepts and properties (Sinclair et al., 2016). This is because attending to the properties of a geometric figure involves dimensional deconstruction. For example, seeing the properties of the rectangles involves attending to their sides and angles. In this study, it is hoped that measuring the sides and angles would assist learners in the deconstruction, and help them discern the necessary and sufficient conditions of congruence of triangles.

In his work, Duval argues that school geometry proceeds in the wrong direction because it moves from solids to plane figures to lines. Instead, Duval proposes construction as a point of entry (Sinclair, et al., 2016; Koyuncu et al., 2015). The approach contrasts with van Hiele's proposition of construction by straight edge and compass in Level 3 (Sinclair, et al., 2016). Drawing on Duval's work, Perrin-Glorian et al. (as cited by Sinclair et al., 2016) argue that instruments such as rulers and stencils could play an important role in enabling dimensional deconstruction. In a similar vein, Koyuncu et al.'s (2015) study observed that construction indeed helped reason. Furthermore, they attested that construction using GeoGebra was more effective than construction using paper and pencil (Koyuncu et al., 2015). The approach in this study, however, did not follow Duval's suggestion because of two main reasons. First, construction would need the users to master GeoGebra, which would lead to some resistance in using the approach (Getenet, 2015). Second, construction would need more time (time to learn the software and time to implement) which would impact syllabus coverage (De Villiers, 2010). In the next subsection, I discuss the general theories applied to the specifics of geometry education.

2.5.2.2 General theories applied to the specifics of geometry education

This subsection discusses three theories that focus on specific aspects of Geometry; the prototype theory, the theory of variation (or teaching with variation, and discovery learning.

2.5.2.2.1 Prototype theory

Prototype theory is used in education research on concepts and conceptual development (Sinclair, et al., 2016). It seeks to explain why some members of a family are more central than others (Sinclair, et al., 2016). For example, Fujita (2012) investigated learners' understanding of inclusion relations of quadrilaterals. The study was based on the prototype theory to describe learners' cognitive development of such relations. The findings suggested that, in general, more than half of the above-average learners are likely to recognize quadrilaterals primarily by prototypical examples, and are able able to discern that a square is a parallelogram, even though they know the correct definition (Fujita, 2012). This concurs with the findings of Erdogan and Dur (2014), who attested that teachers' prototypical images learned at school dominated their figural concepts, even though they possessed formal definitions of quadrilaterals.

2.5.2.2.2 The theory of variation (or teaching with variation)

The theory of variation (a theory of learning and awareness) has to do with the use of counterexamples to arrive at a specific definition or concept (Sinclair, et al., 2016). This focus on differences creates a powerful way for learners to experience and discern critical features of something to be learned, and to categorize different ways of experiencing that something. Building on the theory of variation, Leung (2012) proposed a progressive discernment sequence based on different levels of contrast, to classify plane figures starting from awareness of visual intuitive features, to geometrical properties, and finally relationships between properties (Leung, 2012;

Sinclair, et al., 2016). The key idea behind this approach was shifting attention from the visual classification of shapes and objects to becoming aware of different types of critical features (e.g., geometrical properties, and relationships between properties) and their logical connection. In this study, two applets served as counterexamples. The first counterexample illustrated the importance of the order of the equal attributes in the SAS case. That is the importance of having the equal angle included between the two equal sides. Another counterexample illustrated that having three equal corresponding angles in a pair of triangles does not always imply the congruence of the two triangles.

2.5.2.2.3 Discovery Learning

A review of related literature suggests that discovery learning occurs whenever the learner is not provided with the target information or conceptual understanding and must find it independently and with only the provided materials (Alfieri et al., 2011). This method of learning is in accordance with the constructivist approach in which the students learn more effectively by constructing their knowledge (Balım, 2009; Alfieri et al., 2011). It allows active participation of the learner in discovering the unknown, thus; arousing curiosity, developing inner motivation, and boosting the learner's confidence (Balım, 2009; Mayer, 2004; Alfieri et al., 2011). Additionally, it encourages students to arrive at a conclusion based on their activities and observations (Balım, 2009). Similarly, discovery learning prioritizes reflection, thinking, experimentation, and exploration (Balım, 2009). There is a wide range of approaches within the realm of discovery learning. Some approaches focus on implicit pattern detection, others on elicit explanations, while other methods require learners to follow instructions in a given manual, and other similar strategies rely on simulations (Balım, 2009).

Concerns and limitations of discovery learning also emerged in the literature. In his research, Mayer (2004) reviewed three pieces of research that compared pure discovery and guided discovery. Based on the findings of that research, it was concluded that the constructivist view of learning is best supported by the guided discovery that is targeted towards the understanding of specific concepts. It was, however, pointed out that the challenge of teaching by guided discovery is to know how much and what kind of guidance to provide, and to know how to specify the desired outcome of learning (Mayer, 2004; Alfieri et al., 2011). It is insinuated that in some cases, direct instruction can promote the cognitive processing needed for constructivist learning, while in others, some mixture of guidance and exploration is needed (Mayer, 2004).

This research is a mixture of guided and exploration discovery approaches. The study suggests the use of applets and a worksheet with leading questions. While using applets, learners got an opportunity to explore various examples and complete a worksheet with fill-in questions, leading to aspired conclusions.

2.5.3 Designing mathematical tool-based tasks

Human intelligence is resembled by the ability to create and use tools to extend their abilities in achieving tasks that are otherwise unimaginable and time-consuming to accomplish manually (Leung & Bolite-Frant, 2015). This is the aim of this study. I hope to capitalize on students' digital proficiency and the existing technology to facilitate the process of teaching and learning mathematics. There are different perspectives used to interpret tools and representations (Leung, 2012). Tools are broadly interpreted as physical or virtual artifacts that influence cognition and enhance mathematical understanding (Joubert, 2007; Leung & Bolite-Frant, 2015). Similarly, tools can be regarded as mediators between the phenomenological world and the conceptual world (Leung & Bolite-Frant, 2015). Hence, our interaction with tools, artifacts, and cultural material is important to consider. In this current study, I propose using GeoGebra Applets on mobile devices to facilitate the teaching and learning process, while enhancing the understanding of congruence. The development of mathematical ideas and concepts has been closely associated with the development of technology, which is regarded as a cultural tool for up-to-date educational practices (Leung & Bolite-Frant, 2015). Learners of today are digitally oriented, and it is, therefore, essential to adopt teaching styles that adhere to the culture of this current generation. It is inferred that mathematics teaching that involves tools, such as technology and representations, enables teachers to guide students to (re)invent, visit and discover mathematical concepts (Leung & Bolite-Frant, 2015). It has also been confirmed that some software used in the teaching and learning of mathematics, to a greater or lesser extent, performs some mathematical processes for the user (Joubert, 2007). Likewise, in this study, the construction has been done, users are required to measure the missing attributes. A tool-based task is a thing to do or act on, designed by the teacher/researcher, aiming to activate an interactive tool-based environment where teacher, students, and resources mutually enrich each other in producing mathematical experiences (Joubert, 2007). Therefore, there is a strong relationship between tool mediation, teaching and learning, and mathematical knowledge (Joubert, 2007). This relationship is further elaborated in the Framework for the Rational Analysis of Mobile Education (FRAME) discussed in chapter 3.

The choice of a tool for pedagogical purposes can be considered as a function of how mathematical knowledge is perceived epistemologically by the teacher. Depending on their belief, a teacher can be a participationist or acquisitionist. Participationists would favor lesson designs that allow students to participate in the construction of mathematical knowledge (Leung & Bolite-Frant, 2015). However, acquisitionists opt for designs that explore established mathematical knowledge (Leung & Bolite-Frant, 2015). Teachers' conception of the tools is important (Leung & Bolite-Frant, 2015). However, teachers in high school are guided by the curriculum standards. Consequently, investigation tasks are designed to explore or discover existing axioms, theorems, or relations. The applets in this study were designed to discover the cases of congruency of triangles.

Tools can act as a mediator for mathematical communication; hence, it is important to have views on mathematical discourse in tool-based task design. The same tool can be used in two task designs that are at opposite epistemological poles. For example, using compasses and rulers may be seen in different ways. Either student can construct their geometrical models to explain a certain mathematical phenomenon that they experienced, or the students can follow a given construction procedure to check the validity of a given theorem (Leung & Bolite-Frant, 2015). A tool-based task design could shorten any distance between students' prior mathematical experiences and the intended mathematical knowledge to be learned (Leung & Bolite-Frant, 2015). In this study, triangles were constructed and the participants were required to measure missing sides and angles. Therefore, the use of GeoGebra applets shortens the distance between the students' ability to construct triangles, and the establishment of cases of congruence. Challenges that could be perceived in constructing triangles are alleviated, and the learner's focus is directed to the intended objective of the task.

The theory of instrumental genesis is important when designing tool-based tasks. This theory focuses on how tool-based tasks can be used to acquire or construct new knowledge (Leung &

Bolite-Frant, 2015). It also focuses on how a learner can develop the skill for using a particular tool while using it to solve a problem. It is hoped that mobile technology can be adopted and used as an instrument of teaching and learning.

2.5.3.1 Considerations in designing mathematical tool-based tasks

When designing tool-based tasks, epistemological and mathematical considerations play a pivotal role (Leung & Bolite-Frant, 2015). The teacher's conception of the tool influences the design and use of the tool. Participationists would favor a tool-based design with the potential for students to participate in the construction of shared mathematical experiences or discourses, whereas the acquisitionists would use tools to explore and consequently construct personal mathematical knowledge (Leung & Bolite-Frant, 2015). Dynamic digital tools, for example, GeoGebra applets on mobile devices, can be used in task design to cover both spectrums. This ranges from drawing precise geometrical figures to the exploration of new geometric theorems, and the development of argumentation discussions (Leung & Bolite-Frant, 2015).

This study followed the acquisitionist belief, and the applets were used to explore the triangles and ultimately, discover the cases of congruence; thus, constructing new knowledge. However, a challenge to tool-based task design is to determine the epistemological orientation and the type of mathematical knowledge that a tool can afford and to align it to the curriculum (Leung & Bolite-Frant, 2015; Nisiyatussani et al., 2018). The task that was used in the study was adopted from the prescribed grade 9 textbook, and the GeoGebra applets were designed specifically to complete the task. It is also recommended that applets be tailor-made for the curriculum (Nisiyatussani et al., 2018). It is essential to ensure that the tool-based task enhances the achievement of the objectives stipulated in the curriculum. Tools can be used to overcome obstacles to learning new content. There are two common obstacles: a teaching gap in classroom practices and distance between learners' prior mathematical knowledge and the intended mathematical knowledge to be learned (Leung & Bolite-Frant, 2015). The teaching gap in the classroom involves the use of inappropriate teaching strategies and practices that do not promote effective learning. These obstacles are called didactical obstacles. The lack of prior knowledge is an epistemological obstacle. Epistemological obstacles are those knowledge gaps which students need to overcome by the construction of new knowledge (Joubert, 2013). Putting epistemological obstacles into consideration while designing tool-based tasks enhances the knowledge construction process because tools can be used as a concrete bridge between students' prior mathematical experiences/knowledge and the intended mathematical knowledge to be learned (Joubert, 2013). For example, the dynamic nature of GeoGebra could help learners visualize mathematical concepts. In this study, the designed applets allowed learners to change the measurements and to visualize how the measurements changed, maintaining the relationship that existed between the original set of measurements. This could have been a challenge if measuring was done manually. For example, the challenges of measuring angles using a protractor could be an epistemological obstacle in discovering cases of congruence. Learners could not arrive at the anticipated conclusions if their measurements were incorrect.

2.5.3.2 Tool-Representational Considerations

Effective learning of mathematics involves the creation of representations of mathematical knowledge. It is, therefore, very important to design a tool that represents the intended mathematical knowledge (Leung & Bolite-Frant, 2015; Nisiyatussani et al., 2018). There are two basic tool-representational aspects to consider when designing a tool. The first one is the difference between symbolic representation in the tool and the mathematical concept. Applets should be tailored to develop mathematical concepts as stipulated by the curriculum, and to implement the language that is used in the standard mathematical concept representations (Morphett et al., 2016; Nisiyatussani et al., 2018). The second aspect to consider is whether the tool helps in constructing the knowledge it is intended to construct. This study is centered on the use of curriculum tailored applets on mobile devices in teaching and learning mathematics, hoping to enhance understanding of congruence.

2.6 Conceptual approach in teaching and learning mathematics

To cope with the demands of 21st-century skills, there is a dire need for teaching and learning strategies that promote critical thinking and problem-solving abilities (Kade et al., 2019). Learners relying on procedural knowledge face difficulties in answering complex questions (level 3 and level 4 questions) (Zahner et al., 2012).

The development of students' conceptual understanding is one central goal of mathematics learning (Zahner et al., 2012). Conceptual understanding should have serious attention due to the low performance in mathematics in South Africa (Department of Basic Education_d, 2018). This research explores the use of mobile technology as a strategy for promoting understanding of congruence in the teaching and learning of mathematics. Therefore, it is worth discussing conceptual knowledge or approach in mathematics. There exists a myriad of conceptual approach definitions in the literature. This teaching and learning approach includes translations between verbal, visual (graphical), numerical, and formal/algebraic mathematical expressions (representations); linking relationships, interpretations, and applications of concepts (for example by way of diagrams) to mathematical situations (Engelbrecht et al., 2017). It encompasses, not only what is known (knowledge of concepts), but also one way that concepts can be known deeply and with rich connections (Rittle-Johnson & Schneider, 2014). The prominent feature of conceptual knowledge is its richness in relationships (Bartell et al., 2013). Relationships infuse individual facts and propositions so that all pieces of information are linked to some network (Bartell et al., 2013; Rittle-Johnson & Schneider, 2014). During the learning process, students wrestle with the content and make connections among important ideas (Zahner et al., 2012). In doing that, they develop a deep understanding of the content.

The procedural approach is contrary to the conceptual approach. A procedure is a series of steps, or actions, done to accomplish a goal (Rittle-Johnson & Schneider, 2014). The procedures can be algorithms or a predetermined sequence of actions that lead to the correct answer when executed correctly, or possible actions that must be sequenced appropriately to solve a given problem (Rittle-Johnson & Schneider, 2014). This knowledge develops through problem-solving practice, and is thus, tied to particular problem types (Rittle-Johnson & Schneider, 2014). Learners who rely on procedural knowledge are comfortable with answering routine questions (Rittle-Johnson & Schneider, 2014). They tend to get lost once a familiar question is slightly twisted. Additionally, they are not flexible. They do not want alternative procedures to solve the same problem. Instead, they prefer short methods with no understanding of the concepts underlying the calculations. This may lead to insufficient understanding and mere completion of routine mathematical tasks (Wiest & Amankonah, 2019). Contrary to procedural knowledge, conceptual knowledge is flexible and not tied to specific problem types, and therefore, generalizable (Rittle-Johnson & Schneider, 2014;

Bartell et al., 2013). Learners relying on conceptual knowledge develop sophisticated mathematical thinking and strategies in answering simple questions (Rittle-Johnson & Schneider, 2014). Those who rely on procedural knowledge face difficulties in handling complicated conceptual structures. Although conceptual knowledge is often emphasized as being particularly important in mathematics in comparison with procedural knowledge of mathematics, many educators and researchers believe that both are important for successful mathematics learning (Wiest & Amankonah, 2019). Not only are both conceptual and procedural knowledge necessary for mathematics competence, but also the relationship between these two types of knowledge (Wiest & Amankonah, 2019). In line with this, much has emerged in the literature about the relationship between conceptual approach and procedural approach (Engelbrecht et al., 2017).

Four different theoretical viewpoints on the causal relations between conceptual and procedural knowledge emerge in the literature (Rittle-Johnson & Schneider, 2014). The concepts-first view posits that children initially acquire conceptual knowledge and then derive and build procedural knowledge from it, through repeated practice of solving problems (Rittle-Johnson & Schneider, 2014). The procedures-first view postulates that children first learn procedures and then gradually derive conceptual knowledge from them, through abstraction processes such as representational redescription and analysis of extreme cases (Rittle-Johnson & Schneider, 2014). A third possibility is the inactivation view, which conceives that conceptual and procedural knowledge develop independently (Rittle-Johnson & Schneider, 2014). A fourth possibility is the iterative view. This view postulates a bi-dimensional relationship, where an increase in conceptual knowledge leads to subsequent increases in procedural knowledge and vice versa (Rittle-Johnson & Schneider, 2014).

In the literature, the iterative view is currently the most well-accepted perspective (Rittle-Johnson & Schneider, 2014). An iterative view accommodates gradual improvements in each type of knowledge over time. In addition, the iterative view accommodates evidence in support of concepts-first and procedures-first views; as initial knowledge can be conceptual or procedural, depending upon environmental input and relevant prior knowledge of other topics. In my teaching experience, I came across scenarios where some concepts are best learned through procedural approach, or conceptual approach, or both. However, I have also noticed that it is very difficult to introduce the conceptual approach to learners who are used to the procedural approach. These learners may not have the patience to listen to lengthy explanations and do lengthy calculations or discovery activities. In line with this, I also noted that many average-performing learners prefer the procedural approach. It has also been affirmed in the National Senior Certificate diagnostic reports that learners display confidence in answering less challenging routine questions (Department of Basic Education_d, 2018).

The South Africa program of assessment is guided by four cognitive levels that were adopted from the Trends in International Mathematics and Science Study (TIMSS) (Department of Basic Education_sp, 2011, Reddy, et al., 2016). At the Further Education and Training phase, all assignments, tests, and examinations are expected to have the following approximate percentages at each respective cognitive level; knowledge questions (20%), routine procedure questions (35%), complex procedure questions (30%), and problem-solving questions (15%) (Department of Basic Education_sp, 2011). Procedural knowledge is dominant in the knowledge and routine procedure questions, whereas, conceptual knowledge is mainly visible in the complex procedure and problem-solving questions (Department of Basic Education_sp, 2011). Table 1 indicates skills that are demonstrated at each cognitive level at the Further Education and Training phase. In 2018, 48,6 % of learners who wrote the national senior certificate mathematics exam scored 40% or more. I would intuitively conclude that 51,4% were operating at the first two levels since 55% of the assessment is accounted for at these levels. This would imply challenges to conceptual knowledge. Within this program of assessment, approximately 55% of the assessment depends on procedural knowledge. Although conceptual knowledge is often emphasized as being particularly important in mathematics in comparison to procedural knowledge, many educators and researchers believe that both are important to successful mathematics learning (Wiest & Amankonah, 2019).

One way to engage students in attending to concepts and making connections between important ideas is through engaging mathematical discussions (Zahner et alo., 2012). As students participate in classroom mathematical discussions, they learn to communicate mathematically, by making conjectures, presenting explanations, and constructing arguments (Zahner et al., 2012).

Table 1

Skills that are demonstrated at each cognitive level

Cognitive level	Skills to be demonstrated at the level
Knowledge (20%)	• Straight recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary
Routine Procedures (35%)	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject). • Performance of well-known procedures. • Simple applications and calculations might involve few steps. • Derivation from given information may be involved • Identification and use (after changing the subject) of the correct formula. • Generally similar to those encountered in class.
Complex procedures (30%)	• Problems involve complex calculations and/or higher-order reasoning • There is often not an obvious route to the solution. • Problems need not be based on a real-world context. • Can involve making significant connections between different representations.

	• Require conceptual understanding.
Problem Solving (15%)	• Non-routine problems (which are not necessarily difficult). • Higher-order reasoning and processes are involved. • May require the ability to break the problem down into its constituent parts.

Note. Reprinted from the *Curriculum and Assessment Policy Statement grade 7-9* (pp 156) by the Department of Basic Education.

2.7 Geometric habit of thinking

Geometry is one of the branches of mathematics that is used in many areas of our daily lives, perhaps, without even noticing. For this reason, individuals are geometric thinkers, not only in geometry classes but also in different areas of life. It is thus, important for individuals to acquire geometric habits of mind (Erşen et al., 2018). Creative ways of thinking that promote the learning and application of geometry may be labeled as geometric habits of mind. Such thinking involves exploring, reasoning with, generalizing, and investigating relationships, thus, creating meaningful geometry (Erşen et al., 2018). The habits of the mind support students in understanding a problem to establish possible approaches to solving the problem (Grant et al., 2017). According to the researchers, individuals possessing geometric habits of mind have four main habits (Gürbüz et al., 2018). These are; reasoning with relationships, generalizing geometric ideas, investigating invariants, as well as balancing exploration and reflection. This study drew on the work of Driscoll (as cited by Gürbüz et al., 2018), which describes the four geometric habits of mind which contribute to the understanding of geometry as depicted in Table 2.

2.7.1 Reasoning with Relationships

Reasoning with relationships involves establishing relationships between geometric shapes (such as congruence, similarity, parallelism, etc.), and being able to apply these relationships in the problem-solving process (Gürbüz et al., 2018; Erşen et al., 2018). Individuals with this reasoning can easily identify shapes by their definition, can easily identify properties of geometric shapes, can modify shapes to make them similar, can define shapes using different properties, can use symmetry to reason with geometric shapes, and can work with compound shapes (Erşen et al., 2018). Additionally, they can reveal relationships with relevant justification and can use proportional reasoning. Similarly, grade 9 learners are expected to show and justify the congruence of given triangles. This habit is the most critical and basic requirement in learning geometry.

2.7.2 Generalizing Geometric Ideas

Generalizing geometric ideas involves the process of formulating generalized rules (Erşen et al., 2018). Individuals with this habit are interested in constructing new rules for a set of geometric shapes (Erşen et al., 2018). Additionally, they have the habit of checking if the discovered rule works in all cases, and also strive to reformulate the rule until it works in all cases (Erşen et al., 2018).

2.7.3 Investigating Invariants

In geometry, invariance refers to the characteristics that stay the same if the geometric shape is transformed (Erşen et al., 2018). Individuals with this characteristic can quickly identify the characteristics that either change or stay the same after a transformation (Erşen et al., 2018). Additionally, they are curious to establish and explain the reasons for such occurrences (Erşen et al., 2018).

Table 2

The four geometric habits of mind

Geometric Thinking Habits	Indicators of the GHoM Habits	Student Indicators
Reasoning with relationships	Focus on relationships among separate figures	Determines the relationship between the properties of geometric shapes
	Focus on relationships among the pieces in a single figure	Identifies/classifies the properties of shapes
	Use special reasoning skills to focus on relationships	Associates more geometric shapes with proportional reasoning (congruence-similarity)
Generalizing geometric ideas	Seek solutions from familiar cases or known solutions	Makes generalizations from the special case to explain the problem situation
	Seek a range of solutions using assumed simplifying conditions	Adapts a general situation in a problem for the special case
	Seek complete solution sets or general rules	Can think of all possible situations based on the data in the problem
Investigating invariants	Use dynamic thinking and searching	When a geometric shape is transformed in any way or enlarged/reduced in a specific rate, solves the problem by determining which features of the shape have changed and which ones remained fixed
	Check evidence of effects	Can imagine the geometrical structure as mobile so as not to disturb the conditions of the problem and can explain the emergent effect
Balancing exploration and reflection	Put exploration in the foreground	Can make additional drawings to help solve the problem / Can develop different strategies for solving the problem
	Put end goals in the foreground	Asks questions related to retro-metacognitive capacity in problem solving / Can make explanations through mathematical language for correctness of problem solving

Note: Reprinted from "Investigating Geometric Habits Of Mind By Usingpaper Folding," by M. C. Gürbüz, M. Ağsu, H. K. Güler, 2018, *Acta Didactica Napocensia, 11*(3), p. 159. Copyright 2018 by Eric.

2.7.4 Balancing Exploration and Reflection

Exploration involves applying various strategies to solve a geometric problem, while reflection is understanding the steps that one follows when solving a geometric problem (Erşen et al., 2018). Individuals with this habit can explain each step of their solution. Additionally, they are creative and can solve geometric problems in different ways (Erşen et al., 2018).

2.8 Spatial Ability

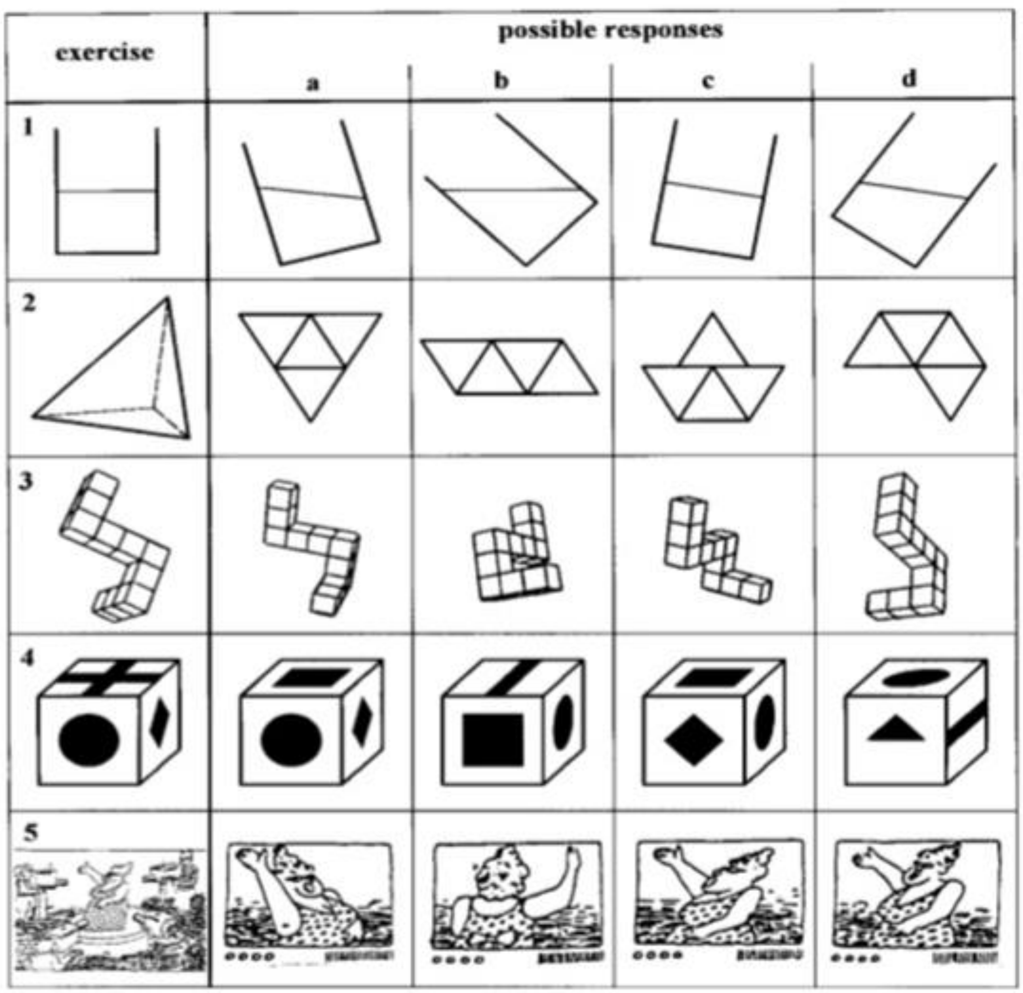
From the literature, I noted that geometry learning and spatial ability are concomitant. As I searched for the definition of spatial ability, several meanings and terms emerged. Some researchers stated that the definition of spatial ability is contentious (Maier, 1996; Kösa, 2016; Buckley et al., 2018). In some cases, different words refer to the same skill or factor; whereas in other cases, the same terms have different meanings (Maier, 1996). While studying the literature, I noted that many researchers referenced McGee's 1979 article. In that article, McGee (1979,p.891) reviewed literature that was published then, on spatial ability. One of the review points was that spatial ability consists of at least two spatial factors; visualization and orientation (McGee, 1979). Spatial visualization involves the ability to mentally rotate, manipulate, and twist two- and three-dimensional stimulus objects (McGee, 1979). Spatial orientation involves the comprehension of the arrangement of elements within a visual stimulus pattern, the aptitude to remain unconfused by the changing orientations in which a spatial configuration may be presented, and the ability to determine spatial orientation concerning one's body (McGee, 1979). Individuals who possess well-developed spatial visualization ability can imagine the visual representations of different views of an object (McGee, 1979). However, individuals who possess a high level of spatial orientation ability, are not distracted by the change of orientation of the object (McGee, 1979).

Other researchers have examined spatial ability in two sub-dimension, spatial relations and spatial visualization (Burnett & Lane, 1980; Clements & Battista, 1992). Olkun (2003, p.2) defines spatial ability as the skill of mentally manipulating objects and their parts in 2D and 3D space. Whiteley et al. (2015, p.2) focused on spatial reasoning, which is characterized by a range of processes that include locating, orienting, decomposing, or recomposing, balancing, diagramming, symmetry, navigating, comparing, scaling, and visualizing. Correspondingly, Guzel and Sener (2009, p.1765) claimed that spatial ability is associated with the comprehension of symbols, shapes, and figures. Similarly, Mulligan (2015, p.513) defined spatial ability as the ability to recognize and mentally manipulate the spatial properties of objects, and the spatial relations among objects.

I am inspired by Maier's (1996, p.70) definition of spatial ability because it covers most of the aspects included in the definitions of other researchers. Maier (1996, p.70) conceptualizes spatial ability in five factors as spatial perception, visualization, mental rotation, spatial relations, and spatial orientation. Spatial perception is the ability to locate the horizontal or the vertical, despite distracting information (Maier, 1996; Buckley et al., 2018). Exercise 1, in Table 3, gives an example to test horizontality using “water-level tasks”. Individuals who possess spatial perception can draw or identify a horizontal line into a tilted glass (Aszalos & Bako, 2004).

Table 3

Exercises of the five elements of spatial ability.



Note. Reprinted from "How can we improve spatial intelligent", by Aszalos and M. Bako, 2004, 6th International Conference on Applied Informatics, 27-31. Copyright [2004] by Researchgate.

Visualization is the ability to visualize the components that make an object/figure, in which there is movement or displacement among (internal) parts of the configuration (Maier, 1996). An example to test this factor is exercise 2 in Table 3. In this test, individuals with spatial visualization can identify the nets that belong to the tetrahedron (Aszalos & Bako, 2004). Mental rotation involves the ability to rotate a two or three-dimensional figure and identify identical objects (Maier, 1996). Spatial relations is the ability to understand the configuration of one part with another object and the relationship of one. A test of spatial relations is illustrated in exercise 4 in Table 3 by identifying drawings that could represent the same or different cubes. Spatial orientation is the ability to position oneself physically or mentally in space. An example is in exercise 5 in Table 3. An individual with developed spatial orientation is expected to pair cameras and figures on the right according to the leftmost figure. It was further indicated that these factors are related, and therefore, cannot be strictly distinguished (Maier, 1996).

In my opinion, spatial visualization, mental rotation, spatial relation, and spatial orientation are dominant in the section of congruence of triangles. In the congruence section, students need to visualize the triangles and establish the congruence relationship between triangles with different orientations.

Numerous studies have documented a positive relationship between spatial visualization ability and mathematics achievement. It is affirmed that spatial ability is important in mathematics achievement since it helps students to: make sense of figures, shapes, or graphs; to interpret the visual representations; to notice the links between different concepts easily; to make generalizations about complex concepts; and to connect different concepts (Guzel & Senera, 2009).

Teachers who lack spatial visualization abilities or who have spatial anxieties may have difficulties in providing their students with effective learning opportunities through visual materials (Erkek et al., 2017). The students' interaction between the mechanical (spatial) and the theoretical (geometrical) supports the development of spatial reasoning (Erkek et al., 2017). The interconnected nature of spatial-graphical and theoretical reasoning is made explicit through the active manipulation of objects. The three-dimensional Dynamic Geometric Environments encourage the development of spatial/theoretical understanding (Crompton et al., 2018).

Additionally, they can create mathematical models of real-world objects and their dynamic movements, that clarify correspondence between the real phenomenon and mathematical structure (Crompton et al., 2018). Several studies have demonstrated that dynamic geometry software enhances the development of spatial ability and conceptual understanding (Güven & Kosa, 2002; Kuo-En et al., 2016; Jelatu, 2018; Pfeiffer, 2017). Jelatu (2018) examined the effect of GeoGebra-aided REACT (Relating, Experiencing, Applying, Cooperating, and Transferring) strategy on the understanding of geometry concepts. Table 4 shows REACT strategy and the description of each step. The REACT strategy is relevant to the proposed use of mobile technologies in teaching and learning mathematics. It allows exploration and the discovery of concepts. As students see, touch, and manipulate shapes; they begin to develop spatial reasoning skills (Pfeiffer, 2017).

Table 4

REACT strategy and descriptions of each step

Step	Description
Relating	learning in the context of one's life experiences or preexisting knowledge
Experiencing	learning by doing, or through exploration, discovery, and invention
Applying	learning by putting the concepts to use
Cooperating	learning in the context of sharing, responding, and communicating with other learners.
Transferring	using knowledge in a new context or novel situation—one that has not been covered in class

2.9 Technology in teaching and learning mathematics

In the field of mathematics education, technology has already been integrated; however, its implementation seems slow (Agyei & Voogt, 2011; De Witte & Rogge, 2014; Safdar et al, 2011, Mendezabal & Tindowen, 2018). Moreover, studies that focus on the integration of technology in mathematics teaching and learning present divergent results. Some researchers posit that the use of technology in mathematics teaching and learning has not led to any discernible improvements (Goodison, 2002). Others believe the use of Microsoft mathematics in teaching and learning differential calculus improves students' conceptual understanding, procedural skill, and attitude toward learning the subject; and is equally effective as the traditional approach. With the Microsoft mathematics embedded activities, students are afforded the opportunities to learn calculus concepts and processes by exploration and discovery allowing them to be more engaged in learning (Mendezabal & Tindowen, 2018).

I adopted a socio-constructivist theoretical approach, which assumes that students learn mathematics and develop conceptual understanding through interacting with material and technological artifacts while participating in discourses (Zahner et al., 2012).

2.10 GeoGebra in teaching and learning mathematics

This section expands the literature that was briefly mentioned in section 1.4. The literature review shows that there have been numerous studies investigating the use of GeoGebra in teaching and learning mathematics. The many scientific publications about GeoGebra and its applications are evidence of its widespread use and popularity (Korenova, 2017). Google Scholar currently registers approximately 17 500 publications with the keywords: GeoGebra, mathematics (Korenova, 2017). In the subsequent paragraphs, I discuss studies carried out on GeoGebra software on various aspects of teaching and learning mathematics in primary, secondary, and high schools, and at the tertiary level (Arbain & Shukor, 2014; Diković, 2009). A review of research focusing on the use of GeoGebra applets is also included.

Polásek and Sedlácek (2015, p.47) focused on GeoGebra software as a cognitive tool. In their research, they used GeoGebra to examine the validity of the hypothesis (a presumption that could become a theorem) by various modifications of construction. Due to the dynamic nature of GeoGebra, students were able to experiment with changing values and achieve verification of formulated hypothesis, validity, as well as its refutation of finding a counterexample. As part of their findings, they attested that dynamic geometry systems provide a virtual environment that allows students to experiment, explore mathematical knowledge, generate hypotheses, visualize their ideas, and thus, build a constructive way to the cognitive process (Polásek & Sedlácek, 2015). Additionally, they affirmed that, in contrast to the lecture method of mere transmission of information, using GeoGebra in teaching and learning can develop deeper cognitive abilities of students, and enhance understanding of geometric concepts (Polásek & Sedlácek, 2015). Furthermore, they also indicated that students are motivated if placed in situations where they become "discoverers" of new knowledge (Polásek & Sedlácek, 2015).

Similarly, Getenet (2015) used GeoGebra to teach the estimation of the decimal value of π. It was affirmed that most pre-service teachers managed to discover the estimated value of π (Getenet, 2015). In addition, they took a more independent and active role in answering questions that were designed by the teacher educator as a guide to achieving the intended outcome (Getenet, 2015). Furthermore, the use of GeoGebra helped the teacher educator to shift to a more student-centered approach (which is the main principle of constructivism) (Getenet, 2015). In their study, a few pre-service teachers were unable to use the software efficiently to construct the inscribed polygon and sliders and found the lesson boring and time-consuming. In line with this, it is affirmed that the full potential of GeoGebra to facilitate the discovery approach can be effectively realized if learners have prior knowledge of how to use GeoGebra (Getenet, 2015). To alleviate the challenges of lack of software knowledge, this research proposes the use of applets that can be mastered with very little software knowledge, and without training (Morphett et al.,2015). The idea of using applets is also supported by Žilinskiene & Demirbilek (2015), Morphett et al., Nisiyatussani et al., (2018), and Radović et al., (2018). One of the concluding remarks in a case study by Žilinskiene & Demirbilek (2015) was that teachers were not ready to use GeoGebra as a tool for creating. Instead, they were enthusiastic to use ready-made objects prepared by it, for example, applets and animations.

The benefits of using GeoGebra mentioned by Getenet (2015) concur with the benefits cited by Polásek & Sedlácek (2015) in the previous paragraph. The research design of Getenet (2015) aligns with the design of this study in many aspects. First, in Getenet (2015, p.213), questions were designed to guide pre-service teachers to the intended outcome. Likewise, in this study, applets were used in conjunction with a worksheet that guided learners to the discovery of cases of congruence. Second, in both pieces of research, sliders were used to experiment with different values. Third, pre-service teachers' perceptions on using GeoGebra were sought through interviews. Similarly, in this study, I sought learners' views on the use of GeoGebra applets and mobile technology. Though most pre-service teachers had positive views on the use of GeoGebra, the issue of software knowledge was of concern. In contrast to that, learners that were interviewed in this research were of the view that applets are very easy to use, and expected other learners to be able to use the same applets without any problems. The use of applets to enhance conceptual understanding in teaching and learning mathematics and other learning areas (Dimitrov & Slavov, 2018) is supported in the literature by several other researchers (Korenova, 2017; Nisiyatussani et al., 2018; Morphett et al., 2015).

It is fascinating to note that other researchers investigated the effect of GeoGebra applets on students' motivation (Akçakın, 2018; Denbel, 2015; Radović et al., 2018). Generally, there is a tendency of focusing on cognitive aspects and overlooking affective aspects of learning (Akçakın, 2018). Motivation, which is an affective feature, is one of the preconditions for learning since high motivation and success are strongly correlated(Akçakın, 2018; Radović et al., 2018). In his / her findings, a positive impact of GeoGebra Applets on students' goal motivation (Akçakın, 2018). Denbel (2015) investigated the effects of using worksheets and GeoGebra applets on aspects of (1) motivation, (2) interactions and discussions, (3) student-centered learning, (4) conceptual understanding, and (5) problem-solving strategies. In their finding, students were well-motivated, but discussion and interaction were limited (due to time limitations). Results on students' 'conceptual understanding' and problem-solving strategies were only partly satisfactory but improved during the intervention (Denbel, 2015). Radović et al. (2018) investigated the effects of GeoGebra applets on mathematics learning using interactive mathematics textbooks (e-book) in primary school. The e-book was designed to meet the pedagogical and didactic needs of learners, allowing a high degree of interactivity and feedback during the learning process (Radovik et al., 2018).

The idea of tailored applets to meet specific needs concurs with suggestions made by other researchers like Korenova (2017), Morphett et al. (2015), and Nisiyatussani et al. (2018), mentioned earlier. Radović et al. (2018) claim that the eBook offered increased opportunities for instructional design; enhanced mathematics learning; and fostered interaction between learning material, teachers, and students. Additionally, they also affirmed that the use of interactive GeoGebra applets encouraged pupils, enhanced motivation, and allowed greater engagement (Radović et al., 2018). However, they picked some concerns from the learners' perceptions. According to some learners, opportunities that the eBook offered were sufficient for understanding mathematics, but not enough for solving tasks, if they needed extra help (Radović et al., 2018). Some researchers that using applets facilitated learning in the succeeding classes because students would still have mental images of the concepts discovered using the applets (Radović et al., 2018).

In a similar study, applets were used in developing the concept of differentiation. The experimental tests that were carried confirmed that there was a positive effect on both the understanding and knowledge of the students. Salleh and Zakaria (2013, p.147) found that integrating mathematical software in learning integral calculus has a positive effect on both conceptual and procedural understanding.

Inspiring is the project at the University Of Melbourne in Australia at the School of Mathematics & Statistics which focused on producing applets tailored precisely to meet specific learning and teaching needs of the school, and to enhance conceptual understanding (Morphett et al., 2015). In their 2016 progress report, they indicated that 32 applets were in use in at least 9 University of Melbourne subjects (Morphett et al., 2016). It is also important to note that not all 32 applets were designed from scratch; some of the applets were derived from existing freely available GeoGebra resources (Morphett et al., 2016). To add to the advantages of using GeoGebra mentioned earlier from a developer's perspective building applets in GeoGebra is quick, and does not require a programmer's expertise or other specialist skills (Morphett et al., 2015).

Apart from the advantages, Morphett et al. (2015, p.11) also noted some limitations that were encountered in using GeoGebra applets. First, the main obstacle that they encountered with GeoGebra as a platform for applet development was the poor responsiveness of certain applets when running in HTML mode (Morphett et al. 2015). In some cases, animations would be jerky or a noticeable lag would occur between the user interaction (for instance, clicking a button) and the response appearing on screen (Morphett et al. 2015). They noted that this problem mainly occurred with larger applets with many components, or extensive computational requirements, or complex animations, and generally when the applet was displayed in HTML mode (Morphett et al. 2015). In instances where the challenges were related to the complexity of the applet, they redesigned aspects of the construction to be more efficient (Morphett et al. 2015). To curb the issue of poor response time, one author suggested minimal use of text in the applet, and minimal fancy styling (Morphett et al. 2015). In cases where the performance in HTML mode was unacceptable or very slow, they elected the Java version of the applet by default (Morphett et al. 2015). Even though the Java version generally has better performance, it comes at the cost of potentially higher technical barriers such as Java security warnings, update notifications, or plugin problems (Morphett et al. 2015).

Second, throughout the project, they ran into several bugs; where constructions would cause a crash or other software issues with GeoGebra. Examples are cases where a function would not work as expected (Morphett et al. 2015). In a small number of cases, an applet had to be modified or redesigned to avoid bugs or technical limitations (Morphett et al. 2015). However, the GeoGebra developers were quick to respond to reports of such issues on the forum, often providing a fix within a day or two (Morphett et al. 2015). The existence of this ongoing GeoGebra developer's support is an added advantage for using GeoGebra (Morphett et al. 2015).

Third, they also noted that GeoGebra's library of built-in mathematical functions, although sufficient for most needs in introductory calculus and statistics, is not as extensive as that of more specialized mathematical software such as Mathematica or Maple; which can be a limitation for more advanced applications (Morphett et al. 2015). It is very encouraging to note that research in this area is ongoing.

In line with the use of applets, Korenova (2017, p.159) attested that teachers found ready-made GeoGebra applets very attractive and easy to use. Lack of software knowledge may make teachers and learners reluctant to adopt the technology approach as alluded to in Getenet's (2015) study. Keronova (2017, p.159) also adds that GeoGebra is suitable for primary school mathematics instruction. I am greatly inspired by the observation that GeoGebra has great potential for use in the form of m-learning when students use mobile devices like smartphones and tablets because this points to the main focus of this study (Korenova, 2017). It was also added that, in the future, more material should be created and reviewed by experts at portals available for teachers, and classified according to topic units and student age (Korenova, 2017). This research is aligned with this recommendation.

As a case study, I investigated the use of GeoGebra applets on mobile technology in the discovery of cases of congruence of triangles by grade 9 learners. The congruence section is part of the grade 9 mathematics curriculum in South Africa (Department of Education, 2011). In addition, the applets were tailored specifically to achieve the outcome of discovering necessary and sufficient conditions to establish the congruence of triangles. Furthermore, the worksheet used was adopted from one of the recommended textbooks in the Curriculum Assessment Policy Statements (Department of Education, 2011). Recent research by Nisiyatussani et al. (2018) also focused on the applets tailored for the Indonesian curriculum. In the previous paragraphs, I discussed a project that is underway at the University of Melbourne, that is producing applets targeted at specific learning and teaching needs (Morphett et al. 2015). Nisiyatussani et al. (2018, p.33) also pointed out the use of languages that learners are not familiar with as an obstacle from adopting ready-made applets into the Indonesian curriculum (Nisiyatussani et al., 2018). It is important to consider the language of learning and teaching and the terminology used in the curriculum when designing the applets. As cited previously in section 1.1, language is one of the factors that influence performance in mathematics (Hatakka, 2017; Pfeiffer, 2017; Radović et al. 2018). Therefore, using the language that the learners are not familiar with may have detrimental effects on the usefulness of the applets (Nisiyatussani et al., 2018). This explains the need to design applets that used the desired language and were also compatible with the curriculum (Nisiyatussani et al., 2018). Though applets in this research used English, the use of text was very minimal. Instead, most pictures of the GeoGebra tools relate to their function and some tools are also familiar to learners from other apps. For example, the tool to measure the length has a line segment and an acronym of a centimeter (cm). By looking at the picture, one can discern that the tool is for measuring length.

In South Africa, efforts to integrate technology, especially GeoGebra, are apparent. Collette (2014) proposed a GeoGebra workbook with 42 Applets created to help visualize a range of key concepts in Curriculum Assessment Policy Statements in South Africa from grade 10 to grade 12, particularly in the areas of functions and Euclidean geometry. In the literature, I did not find evidence of the use of this workbook in other schools other than the schools that took part in the research. This may be an indication that technology integration is still minimal in South African schools, especially in public schools. In a similar approach, the Govan Mbeki Mathematics Development Center (GMMDC) set up a GeoGebra focus group, which conducted staff development workshops in 2017 and 2018 (Govan Mbeki Mathematics Development Center, 2018). The center designed GeoGebra applets aligned to the FET curriculum. In all these initiatives, there was less or no focus on the use of mobile devices by the learners, specifically smartphones. In all instances, all the resources were provided; yet in real life, it would take long to provide the resources to everyone, even those who can afford them.

2.11 GeoGebra applets on mobile devices

With the fast-growing GeoGebrause and introduction of smartphones, touchscreen devices, and tablet computers; the GeoGebra developer group was challenged to get GeoGebra applets to work on these devices (Ancsin et al., 2009). This group embarked on the project GeoGebraMobile in 2009, aiming to bring GeoGebra applets in modern web browsers both on computers and new mobile devices (Ancsin et al., 2009). The growing presence of open-source tools in mathematics classrooms on an international scale calls for in-depth research on the instructional design of GeoGebra-based curricular modules, and the impact of the dynamic mathematics resources on teaching and learning (Hohenwarter & Lavic, 2011). GeoGebra has become a tool that can help teachers to design effective instructional lessons (Arbain & Shukor, 2014).

The following subsection discusses the elements of the research paradigm and how these elements were considered in this research.

2.12 Research paradigm

A research paradigm is a philosophical way of thinking, or a research culture with a set of beliefs, values, and assumptions that a community of researchers has in common, that informs the meaning or interpretation of research data (Kivunja and Kuyini, 2017). Relatedly, Mackenzie and Knipe (2006) describe it as a set of shared beliefs. Lather (1986, p.1) sees it as a reflection of the researcher's beliefs about the world that s/he lives in. In other words, paradigm constitutes the beliefs and principles, that shape how a researcher sees the world, and how s/he interprets phenomenon and acts within that world (Kivunja & Kuyini, 2017). Gunbayi and Sorm (2018, p.57) use an analogy of the lens of colored glasses to explain the term paradigm. The color you see when looking at something depends on the color of the lens of glasses you are putting on (Gunbayi & Sorm, 2018). If you put on red glasses, everything looks red and if you put on green, the world looks green and everything around looks green (Gunbayi & Sorm, 2018). I would relate this to general human experience and practice. When a Christian is faced with a problem, he/she seeks counsel from God through prayer. All the activities done and materials used will be in line with Christian beliefs. One can read the Bible or visit a pastor of the same belief, and/or engage in fasting and prayer. However, if one believes in ancestors' assistance, one visits a sangoma or a person whom one believes can communicate with the ancestors. After consultations, instructions are given and suggested rituals are performed in honor of the ancestors. Therefore, the actions one takes to solve a problem are guided by one's belief. Kivunja and Kuyini (2017, p.26) cite four paradigms that are mainly used in education research: positivism (a belief that knowledge exists), interpretivism, critical paradigm, and pragmatic paradigm.

It is usually a norm that specific research methods are associated with specific paradigms. For example, the quantitative research method is normally associated with positivism, while the qualitative research method is normally associated with the interpretive paradigm. However, the alignment between the paradigm and the mixed research method is not so obvious. There has been a recurring debate involving paradigms and mixed-method research (McChesney & Aldridge, 2019). To overcome the problem associated with paradigms and mixed-method research, researchers have taken different stances including, a-paradigm, dual-paradigm (dialectical), pragmatist, and single-paradigm approaches. The a-paradigm stance ignores the importance of a paradigm in the inquiry process, while the dual paradigm combines two or more paradigms in one research (Shannon-Baker, 2016; McChesney & Aldridge, 2019). The pragmatic paradigm is more open and accepts whatever works to get answers to the research problems (McChesney & Aldridge, 2019). On the other hand, the holistic or single-paradigm stance claims that both qualitative and quantitative methods can be accommodated within a mixed-method study using a single overarching paradigm (McChesney & Aldridge, 2019). It has been noted in the literature that several mixed-method research is guided by the positivist paradigm. McChesney and Aldridge (2019) illustrated in their study, how the interpretive paradigm could inform the entire mixed-method research. The same approach was adopted in this study, which aimed to get participants' views on the use of mobile technology in learning the concept of congruence. The focus reflects the interpretive principle, where understanding is the fundamental aim of the research. In the following subsection, I explain the interpretive paradigm which guided this research.

2.12.1 Interpretive paradigm

The central focus of the interpretive paradigm is to understand the individual/s (their behavior, views, experiences, and emotions) and their interpretation of the world around them (Gunbayi & Sorm, 2018; Pfeiffer, 2017). This research investigates the use of mobile technologies in the teaching and learning of congruence of triangles. It is important to understand the learners' views on the use of GeoGebra applets on mobile devices, hence; this research being predominantly interpretive. Interpretivism believes that there is no single reality. Instead, the reality is constructed, that is why it is also called the constructivist paradigm (Kivunja & Kuyini, 2017). This study sought to understand reality from learners' views with regard to using GeoGebra applets on mobile technology. Additionally, interpretivism makes an effort to understand different subjects being studied and interpret what they think or the meaning they make of the context and the phenomenon (Kivunja & Kuyini, 2017; Gunbayi & Sorm, 2018). This research sought participants' perceptions on the use of mobile technology in the learning of the congruence of triangles. In the interpretive paradigm, the theory does not precede research, but follows it, so that it is grounded on the data generated from the research (Kivunja & Kuyini, 2017). I sought learners' views on the benefits and perceived challenges of using GeoGebra applets in teaching and learning mathematics. As was mentioned earlier, successful teaching requires an understanding and appreciation of the learners' needs, backgrounds, interests, and learning styles (Roberts et al., 2012). Instead of first proposing the use of mobile technology in teaching and learning, I sought to get the point of view of learners, since the process of effective teaching and learning is centered on the learner. Additionally, the mobile technology strategy proposed in this study directly involves learners. It is, therefore, essential to understand the learners' learning preferences and solicit their reality about them.

A paradigm comprises four elements, namely; epistemology, ontology, methodology, and axiology. Since this research is predominantly interpretive, these terms are explained in line with the interpretive paradigm discussed in the succeeding subsection.

2.12.2 Epistemology, ontology, methodology, and axiology

Epistemology originated from the Greek word episteme, which means knowledge (Kivunja & Kuyini, 2017). In research, epistemology is concerned with the bases of knowledge; its nature, how it is acquired, how it is comprehended for use to broaden and deepen understanding in the field under study, and how it is communicated to other human beings (Kivunja & Kuyini, 2017; Scotland, 2012; Dudovskiy, 2012). It helps researchers position themselves in the research context (Kivunja & Kuyini, 2017). In establishing the epistemology of research, several questions need to be asked. For example, is knowledge going to be acquired or is it personally experienced? What is the relationship between the researcher, participant, and knowledge? How do we know what we know? What is the source of knowledge? To answer these questions, a researcher looks at the source of knowledge (Kivunja & Kuyini, 2017). In the literature, four basic sources of knowledge are mentioned as; intuitive knowledge (based on intuition, faith, beliefs, and human feelings), authoritarian knowledge (which relies on information that has been obtained from books, research papers, experts, supreme powers, and other existing sources), logical or rational knowledge (which emphasizes reason as the surest path to knowing the truth), and empirical knowledge (that relies on objective facts that have been established and can be demonstrated) (Kivunja & Kuyini, 2017; Dudovskiy, 2011). This research is based on a subjective epistemology that relies on intuitive knowledge. A first questionnaire was administered, and it probed the learners' attitudes towards mathematics. It also explored their perspectives of what teachers could do to enhance understanding. Additionally, participants had to experience the use of GeoGebra applets on mobile devices. The second questionnaire and an interview were administered. These mainly focused on learners' perceptions of the use of applets on mobile devices in teaching and learning mathematics.

Ontology is the study of being (Dudovskiy, 2011). It deals with the nature of reality (Dudovskiy, 2011; Scotland, 2012). In establishing the ontology of research, the following questions are asked: is there reality out there or it is constructed by individuals? What is the nature of reality? Is the reality of an objective nature or is it subjective? What is the nature of the situation being studied? Ontology is a belief that reflects an interpretation by an individual about what constitutes a fact (Dudovskiy, 2011). It constitutes the assumptions made when an individual or researcher believes that something is real (Kivunja & Kuyini, 2017; Gunbayi & Sorm, 2018). These assumptions help the researcher's thinking about the problem, its significance, and how to approach it to contribute to the solution of the problem (Kivunja & Kuyini, 2017). It also helps a researcher to conceptualize the form and nature of reality, and what can be believed and known about that reality (Gunbayi & Sorm, 2018; Kivunja & Kuyini, 2017). This research is embedded within a relativist ontology. Views are relative to individuals. Reality is subjective and is constructed by learners. The problem of low performance in mathematics is one where learners are the main stakeholders, as they are the central actors in the teaching and learning system. Perceptions were sought from learners on the proposed use of GeoGebra applets to discover mathematical concepts using mobile technology.

Axiology refers to the value attached to reality (Dudovskiy, 2011). It attempts to clarify whether one seeks to explain, predict, or understand the world (Dudovskiy, 2011). In this research, I sought to understand the world of mathematics learning. With the advancement in technology, I believe our teaching methods ought to capitalize on the existing technologies. In another perspective, axiology is concerned with defining, evaluating, and understanding concepts of right and wrong behavior when conducting research (Kivunja & Kuyini, 2017). It focuses on the values attached to different aspects of the research, such as participants, data collected, and the audience to which results shall be communicated (Kivunja & Kuyini, 2017). Additionally, it is concerned with the right procedures that need to be followed to respect participants' rights and secure their goodwill as they take part in the research (Kivunja & Kuyini, 2017). It also involves the consideration of moral, cultural, and intercultural issues that may arise when conducting research, and how these may be dealt with (Kivunja & Kuyini, 2017). Furthermore, axiology also includes how research is carried out, to ensure procedures are socially just, respectful and peaceful (Kivunja & Kuyini, 2017). It also looks at whether research may result in harmful effects, be they physical, psychological, legal, social, economic, or other. Axiology also addresses questions like: What is the regard for human values or of everyone that will be involved with or participate in your research? What value guides you as you conduct the research? What needs to be done to respect all participants' rights? What are the moral issues and characteristics that need to be considered? Which cultural, intercultural, and moral issues arise, and how do you address them? How will the goodwill of participants be secured? How will the research be conducted in a socially just, respectful, and peaceful manner? How will the harm be avoided; whether physical, psychological, legal, social, economic, or other? The following paragraph explains how participants' rights were considered.

In this research, I sought to understand the point of view of learners in using mobile technology. All learners who took part in the research were under the age of 18 years, therefore; informed consent was sought from the parents/ legal guardian, and assent was sought from the participants. I provided each learner who participated in the study with two letters. Each participant was required to provide their written assent, and their parents/ legal guardians were also required to provide written consent. I also explained the procedures that would be followed during the research process. Additionally, I provided timeframes and relevant contact details of personnel at the university. Included in these letters were the right to withdraw, confidentiality, time involved, and activities to be completed. Furthermore, it was also indicated that there was no financial benefit for taking part in the study. A similar letter which also sought permission to use school premises and property was issued to the school principal. A copy of these letters is found in Appendix H. Ethical clearance certificates were issued by both the university and KwaZulu-Natal department of education, to carry out the study.

The following two subsections discuss qualitative and quantitative methods and how they fit in this research. I also explain the case study approach.

2.13 Qualitative Research

Qualitative research is strongly associated with the interpretive paradigm (Johnson & Onwuegbuzie, 2004; Pfeiffer, 2017; Cresswell & Cresswell, 2018). Qualitative research seeks to understand the ways people experience events, places, and how they construct reality (McGuirk & O'Neill, 2016; Cresswell & Cresswell, 2018). It draws on methods aimed at recognizing the complexity of everyday life, the degree of meaning-making in the changing world, and the multitude of influences that shape human lived experiences (McGuirk & O'Neill, 2016; Creswell & Poth, 2018). Furthermore, qualitative research allows the researcher to get a deep understanding of the problem (Creswell & Poth, 2018; McGuirk & O'Neill, 2016). It allows the researcher to figure out the knowledge, skills, and attitude of the studied phenomena (McGuirk & O'Neill, 2016; Cresswell & Cresswell, 2018). In the preceding paragraph, I discuss the common qualitative approaches.

There are five common qualitative approaches used in education research (Creswell & Poth, 2018). First, the narrative approach explores the experiences of individuals, describes a life experience, and discusses the meaning of the experience with the individual (Cresswell & Cresswell, 2018). Second, the phenomenological approach describes the common meaning for several individuals, of their lived experience of a concept or a phenomenon (Creswell & Poth, 2018). Third, unlike narrative and phenomenological approaches, grounded theory moves from description to the discovery of theories or unified theoretical explanations (Creswell & Poth, 2018). Fourth, ethnography focuses on describing the activities of a specific group, and the shared patterns of behavior it develops over time. It seeks to understand the meaning of behavior, the language, and the interaction among members of the culture-sharing group. Fifth, the case study approach explores a real-life, contemporary bounded system through in-depth data collection involving multiple sources of information (Creswell & Poth, 2018). This research adopted the case study design. It explored the views of grade 9 learners at a school in Durban, on the use of mobile technology to investigate cases of congruence of triangles. Therefore, it was an in-depth investigation of one unit, based on grade 9 learners at a specific school.

2.13.1 Case study

Case study research begins with the identification of a specific case that will be described and analyzed (Creswell & Poth, 2018). In this study, I explore the case of grade 9 learners' perspectives on using GeoGebra applets on mobile devices. Another key characteristic of a case study is that the identified case has to be bounded or defined within recognizable parameters (Creswell & Poth, 2018). This study focused on the current digitally proficient generation of learners. Trying to gather perspectives from all learners would practically be costly and time-consuming. Therefore, the study was confined to grade 9 participants at the selected school. Case study research involves an in-depth understanding of the case using different forms of data collection (Creswell & Poth, 2018). In line with this, questionnaires and interviews were administered. In the following subsection, I briefly explain the classification of case studies and the type of case study that was adopted in this research.

2.13.1.1 Classification of Case Study

Case studies can be distinguished using different aspects. They can be described using the focus of analysis for the bounded case as particularistic, descriptive, and heuristic (Gay et al., 2009).

The particularistic approach seeks to understand a particular situation, event, or problem (Gay et al., 2009). Three main characteristics reflect the particularistic nature of a study. One, by reading the research, a reader can get insights on what to do in similar situations (Ilham, 2009). Two, as the study examines a particular instance, general problems or scenarios may be illuminated. Three, it may or may not be biased by the researcher.

However, the main objective of the descriptive approach is to give a detailed narrative of the case under study (Gay et al., 2009). Many characteristics reflect a descriptive case study. Although it focuses on understanding an event that has passed, it can be relevant to current events. It can cover many years and describe how the preceding decades led to a situation (Gay et al., 2009). It can uncover differences of opinion on an issue and suggest how these differences influenced results (Gay et al., 2009). The descriptive study can show the influence of the passage of time on issues (Gay et al., 2009). It can obtain information from a variety of sources and can present information in a variety of ways and from the viewpoints of different groups (Gay et al., 2009).

Finally, the heuristic approach focuses on illuminating the reader's understanding of the case being studied (Gay et al., 2009). It is characterized by its ability to explain; the reason for a problem, the background of a situation, what happened, and why it happened (Gay et al., 2009). It also focuses on why innovation worked or failed. It can evaluate, summarize, and conclude happenings (Gay et al., 2009).

This study proposes the use of GeoGebra applets in teaching and learning congruence of triangles and sought the participants' views on using these applets on mobile devices. Thus, the research focused on a sample from one particular grade in a particular school. Because learners in the same generation are likely to have similar characteristics, the research could be repeated using different grades or schools. Thus, this study possesses particularistic approach characteristics.

Case studies can also be classified according to their overall intent (Gay et al., 2009; Creswell & Poth, 2018). Three major variations exist; a single instrumental case study, a collective or a multiple case study, and an intrinsic case study (Creswell & Poth, 2018). In a single instrumental case study, the researcher focuses on an issue or concern and then selects one bounded case to illustrate this issue (Creswell & Poth, 2018). This resembles the focus of the particularistic approach. However, in a collective or multiple case study, one issue or concern is selected but the inquirer selects multiple case studies to illustrate the issue (Creswell & Poth, 2018). The other type is an intrinsic case study in which the focus is on the case itself (Creswell & Poth, 2018). This approach is mainly used for cases that present unusual or unique situations (Creswell & Poth, 2018). This research fits well with the single instrumental case study. Grade 9 participants from a school in Durban were used as the selected bounded case to explore this issue.

Regardless of the criteria of classification, a qualitative case study seeks to describe the issue of concern in depth, holistically, and in context (Creswell & Poth, 2018). Similarly, this study sought the learners' views on using mobile technology in the learning of the congruence of triangles.

2.13.1.2 Advantages and disadvantages of Case Study

There are many advantages of using case studies. First, case studies are often conducted in a real-life environment (Cresswell & Cresswell, 2018). This allows data to be collected and examined within the context of its use (Gay et al., 2009). In this study, the participants had to complete a task using GeoGebra applets to get the real-life experience of using the applets. Case studies allow for both qualitative and quantitative data collection analysis methods. Additionally, it allows for extensive data collection that helps explore and explain the complexities of a real-life situation that may not be captured through experiments or survey research (Gay et al., 2009; Cresswell & Cresswell, 2018).

Despite these advantages, case studies have received criticism. In the literature, there are two main arguments that I have noted. First, it is argued that case studies lack rigor (Zainal, 2007; Pfeiffer, 2017), and they lack thoroughness and precision. There is room for biased views that influence the direction of the findings and conclusions (Zainal, 2007; Pfeiffer, 2017). Second, it is also contended that case studies provide very few bases that allow the generalization of results due to the small sample size that is often used (Zainal, 2007). However, in my view, this study satisfies the four general criteria of trustworthiness: credibility, transferability, dependability, and confirmability (see section 4.5.4).

2.14 Quantitative Research

Quantitative research is strongly associated with the positivist paradigm (Pfeiffer, 2017; Cresswell & Cresswell, 2018). However, in this study, quantitative data sources and methods used were viewed from an interpretive perspective. This involved seeing data as emerging from participants' conceptions of reality. Quantitative data were generated from the two questionnaires. The questionnaires had close-ended and open-ended items. The quantitative results were compared with the results from qualitative data to establish a solid conclusion. The deductive characteristic of quantitative research came into play on how these questionnaires were designed and analyzed. Several items in both questionnaires were adapted from previous studies and other standard questionnaires that have been proven to be valid and reliable. Underlying quantitative research has the philosophical belief or assumption that the world is relatively stable, uniform, and coherent (Gay et al., 2009). Positivists also believe that we can measure, understand and generalize rules, values, and regulations that govern the society, while qualitative researchers argue that all meaning is situated in a particular perspective or context (Gay et al., 2009). In this study, results are not generalized but can be transferred to similar contexts. For example, applets for other mathematics sections in different grades can be designed. Quantitative researchers are concerned with objective reality, whereas qualitative researchers believe that the world has many different meanings because different people and groups often have different perspectives and contexts (Gay et al., 2009; Pfeiffer, 2017). This interpretive notion guides the conclusion of this research. I believe that learners in the 21st century have similar perspectives, therefore results from this study would be transferrable to similar contexts. I believe that the participants' perceptions on the use of GeoGebra applets on mobile devices in discovering cases of congruence will be more or less the same for different mathematical content, grade, or different participants.

Chapter Three Theoretical Framework

This study proposes the use of mobile technology in the teaching and learning of congruence of triangles. In the previous chapter, literature informing this study was introduced and discussed. This chapter discusses the theoretical framework within which this study was located. I begin by presenting the background to the Framework for the Rational Analysis of Mobile Education (FRAME). Next, I present a detailed discussion of the different aspects of the FRAME model, and the coherence between them. Lastly, I discuss some research that utilized the FRAME model.

3.1 Background

The FRAME model was originally developed as a basis for assessing the effectiveness of mobile devices for distance learning (Koole & Ally, 2006). It offers insights on how to fully benefit from mobile experiences (Koole & Ally, 2006). This includes guidance on implementing mobile learning in both formal and informal settings (Koole & Ally, 2006). It was the first comprehensive theoretical model that described mobile learning by amalgamating mobile technologies, human learning capacities, and social interaction (Koole & Ally, 2006). In other words, it was the first to put into consideration the social, practical, and individual aspects of mobile learning (Hlagala, 2015; Koole et al., 2010; Koole, 2009). Additionally, it supports constructivist beliefs. Learners are actively involved in the process of knowledge construction (Koole et al., 2010). In addition to having constructivist characteristics, the FRAME model also draws insights from the activity theory.

In the activity theory, emphasis is on the doing of the activity in a rich social matrix of people (Kaptelinin & Nardi, 2009). Similarly, the FRAME model stresses both learner-centered knowledge construction and the socializing aspect (Hlagala, 2015). While the FRAME model focuses more on the aspect of technology, activity theory puts more emphasis on any tools that aid in achieving the desired outcomes (Hlagala, 2015; Kaptelinin & Nardi, 2009).

In addition to these characteristics, the FRAME model addresses contemporary pedagogical issues of information overload, knowledge navigation, and collaborative learning (Koole & Ally, 2006). It is used to guide designers of future mobile devices and instructors when preparing learning materials destined for mobile learning, and the proper selection of teaching and learning strategies for mobile education (Koole & Ally, 2006). That is why it forms the basis for this study.

This research proposes the use of mobile technology in the teaching and learning of mathematics; particularly to complete activities that focus on concept development, and which would need more time to carry out manually. This study also sought to capitalize on learners' digital skills in the teaching and learning of mathematics.

The FRAME model is characterized by three interconnected aspects; device (D) aspect, learner (L) aspect, and social (S) aspect (Koole, 2009; Hatakka, 2017; Koole & Ally, 2006). The three aspects overlap as shown in the Venn diagram in Figure 7. The intersections where two circles overlap contain attributes that belong to both aspects, such as device usability (DL), social technology (DS), and interaction learning (LS) (Koole & Ally, 2006; Asiimwe et al., 2017). The intersection of all three aspects defines a conducive environment for mobile learning (DLS) (Koole & Ally, 2006; Koole, 2009; Asiimwe et al., 2017).

3.2 Three aspects

The device aspect describes characteristics unique to electronic and networked mobile technologies. The learner aspect focuses on the characteristics of learners. The social aspect describes the mechanisms of interaction among learners (Koole, 2009).

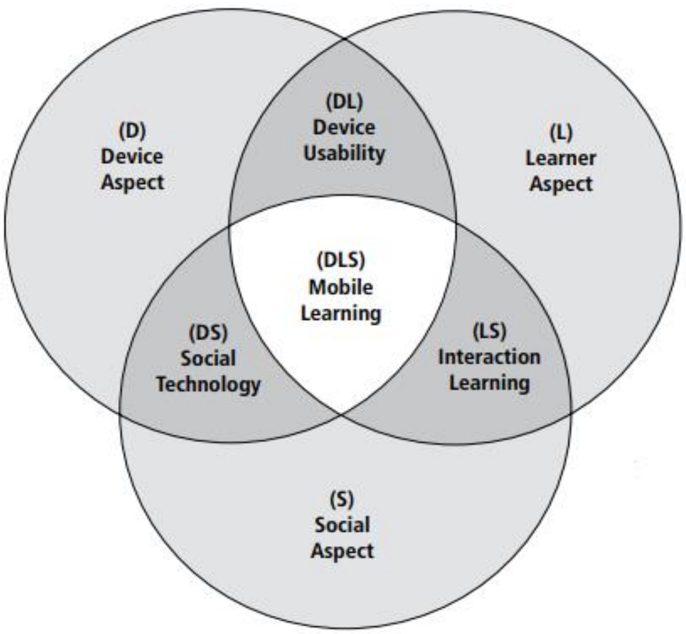
3.2.1 Device aspect

The device aspect refers to the physical, technical, and functional characteristics of mobile devices (Koole & Ally, 2006). These characteristics directly affect learners and their performance in a mobile learning environment (Koole & Ally, 2006). In other words, the device aspect describes the medium through which mobile learning takes place (Koole & Ally, 2006; Asiimwe et al., 2017). Mobile devices serve as a bridge between the human being and technology (Koole, 2009). Therefore, they should be constructed to support the high physical and psychological comfort levels of learners (Koole, 2009). The functional and technical characteristics include input and output capabilities, storage capabilities, power, processor speed, compatibility, and expandability (Koole, 2009). The functional and technical characteristics are a result of hardware and software design.

The FRAME model serves as a guide during the development of mobile devices (Koole & Ally, 2006; Koole, 2009). The device characteristics have a significant impact on usability (Koole, 2009). Learners equipped with well-designed mobile devices focus more on completing the task at hand, rather than on the device (Asiimwe et al., 2017; Koole, 2009). Moreover, the device should make the completion of the task faster and easier (Asiimwe et al., 2017; Koole, 2009).

Figure 7

The FRAME model



Note. From " A Model of Framing Mobile Learning ", by M. L. Koole., 2009, p.27. Copyright 2009 by Athabasca University.

The physical characteristics include size, weight, and composition of the mobile device, placement of buttons and keys on the mobile device, right/left-handed requirements, one or two-hand operability (Koole, 2009). This affects how the user manipulates the device and mobility while using the device (Koole, 2009). For a device to be portable, for example; the size, weight, structure, and composition must match the physical and psychological capacities of the individual users (Koole, 2009). In the literature, I did not come across barriers related to the size of the device, except the issue of screen size (Cheon et al., 2012). The one learner – one tablet drive, gives hope to the integration of mobile technology in teaching and learning in South Africa (Presidency, 2019). The proposed strategy does not require learners to be mobile, even though they can still use the strategy everywhere and at any time if the size of the device permits.

Input capabilities allow the selection and positioning of objects or data on the device. Though mobile devices are often criticized for inadequate input mechanisms, the advent of touch screen devices alleviates this challenge and allows learners to easily interact with the devices in a productive manner (Geist, 2012), and in the process, capitalizing on the digital expertise of learners (Thompson, 2013). Similarly, the capacity and speed of the device's memory, processor, file storage, and file exchange, require error-free response rates appropriately timed to the human user's needs and expectations (Geist, 2012). This research proposes the use of applets with minimal use of text and styling.

GeoGebra applets are compatible with most mobile devices on the market, regarding processor capacity, speed, memory, file storage, and file exchange methods to eliminate known technical barriers (Asiimwe et al., 2017) This was attested in this research. Two out of 25 participants indicated a challenge with speed when they were logging in.

It is important to assess these characteristics because mobile learning devices provide the interface between the mobile learner and the learning task(s) as described later in the device usability intersection (DL).

In this study, I focused on the use of smaller mobile devices that can connect to the internet, can be carried anywhere and everywhere, and can also be used in a palm. Examples are smartphones, tablets, and i-Pads.

3.2.2 Learner Aspect (L)

The learner aspect (L) is grounded in the belief that learners' prior knowledge, intellectual capacity, motivation, and emotional state have a significant impact upon retaining and transferring information (Koole, 2009). This same belief forms the basis of constructivism. It was attested in section 2.4.1 that a learner's previous knowledge, beliefs, and attitudes are of paramount importance in the construction of new knowledge. Cognitive structures already in memory affect how easily a learner comprehends new concepts (Koole, 2009). In this study, I expect learners to know that two triangles are congruent if all corresponding sides and angles are equal. Additionally, the knowledge and experience that the learners have on using mobile devices form part of the prior knowledge. Technology proficiency, in this study, refers to the knowledge and experience of using mobile technology, that the learners already possess.

The learner aspect also points to the situations and tasks in which the learner wishes or needs to succeed (Koole & Ally, 2006; Koole, 2009; Hatakka, 2017). The use of mobile technology offers a different approach from the usual pencil and paper method. This provides distinct stimuli that help learners understand and retain concepts more easily (Koole, 2009). This research proposes a strategy that capitalizes on the digital expertise of the current generation of learners. The study explores the use of mobile technology in discovering the conditions necessary and sufficient for the congruence of two triangles. Learners are likely to retain discovered knowledge and transfer it to varied contexts (Koole, 2009; Bada, 2015).

Koole (2009, p.34) posits that mobile learning helps learners use episodic memory. Episodic memory is a category of long-term memory that involves the recollection of specific events, situations, and experiences (Cherry, 2020). Relating concepts to real-life help learners connect them to their experiences, which makes them remember the concepts. Remembering a concept is largely dependent on remembering its use (Koole, 2009). Additionally, relating a concept to real-life also aids the learner in transferring the concept into other contexts (Koole, 2009). Since most of today's learners are digitally-oriented, it is hoped that the use of mobile technology would boost attention and enhances learners' desire to accomplish a task (Koole, 2009).

Mobile learning allows learners to access content in multiple formats, thus enhancing their ability to assimilate, recall, and transfer information to different contexts (Koole, 2009).

3.2.3 Social Aspect (S)

The social aspect takes into account features required for conversation, cooperation, and social interaction (Koole & Ally, 2006; Asiimwe et al., 2017). Individuals must follow the rules of cooperation to communicate (Koole & Ally, 2006; Asiimwe et al., 2017). These rules allow the exchange of information and knowledge assimilation while sustaining cultural practices (Asiimwe et al., 2017). Cooperative communication requires that contributions made are informative, accurate, relevant, and sufficiently clear. The use of familiar signs and symbols allows easy exchange of information. To reinforce this, GeoGebra tools include signs and symbols that are task-oriented. Additionally, most learners are familiar with some of the tools that have the same uses in other apps, thus; enhancing understanding.

3.3 Intersections

As mentioned in section 3.2, the intersections contain attributes that belong to both aspects, such as, device usability (DL), social technology (DS), and interaction learning (LS) (Koole & Ally, 2006; Asiimwe, Åke, & Hatakka, 2017).

3.3.1 Device Usability Intersection (DL)

The emerging device usability (DL) aspect describes the relationship between a student and a device (Asiimwe et al., 2017). This refers to how intuitive the device is, or how quickly a learner can understand and begin using the device (Koole, 2009). Device usability also explains technology characteristics that make it easy, or not, to use technology (Asiimwe et al., 2017). Characteristics such as device portability, accessibility, and ergonomics are inferred (Asiimwe et al., 2017). Device portability is dependent upon the physical attributes of the device, such as size and weight, the number of peripherals, and the materials used in the construction of the device (Koole, 2009). All these characteristics have an effect on psychological comfort (Asiimwe et al., 2017). Devices that are easy to use allow the easy and quick accomplishment of tasks. This allows users to enjoy the benefits, and experience the advantages of using mobile technology over other strategies (Koole, 2009). Additionally, easy-to-use devices direct users' concentration to tasks at hand, and not to the manipulation of the device itself. This reduces the cognitive load and increases the task completion rate (Koole, 2009). Furthermore, one's ability to access information, the speed with which users can perform tasks, and the ability to physically move devices to different physical and virtual locations, are considered (Koole, 2009). With the use of mobile devices, learners can access stored information anytime or anywhere; making just-in-time learning possible (Asiimwe et al. 2017).

This was attested in this study. Some participants indicated that mobile technology allowed them to complete work at their own pace in the comfort of their homes. Considering responses from the participants, this research complies with most aspects of device usability.

3.3.2 Social Technology Intersection (DS)

The social technology (DS) aspect describes how mobile devices enable communication and collaboration amongst multiple individuals and systems (Koole, 2009). Contrary to the device usability which describes the relationship between users and the device (Koole, 2009), in the case of the proposed model, applets and related outputs are the primary data to be shared. These applets are shared online, which gives access to everyone with the login details. In this study, communication and collaboration in-built features on mobile devices that include WhatsApp, Share-It, Bluetooth, and email applications; can be used to share login details and the worksheet.

3.3.3 Interaction Learning Intersection (DS)

This intersection is balanced between interaction, situated cognition, and learning communities (Koole, 2009). Different kinds of interaction can stimulate learning to varying levels of effectiveness, depending on the situation, learner, and task (Koole, 2009). Some of the interactions are learner to learner, learner to content, and learner to the instructor (Koole, 2009; Koole et al., 2010). The interactions rely heavily upon the philosophy of social constructivism (Koole et al., 2010). As mentioned earlier, the effective use of mobile devices blends teaching and learning within the new mobile lifestyle (Figueiredo et al., 2016). It is also inferred that, as learners carry out a task using technology, they observe the author's method, and can later try the techniques in a similar situation (Koole, 2009). Although mobile technology enhances the above-mentioned interactions in various ways, the main focus of the proposed strategy is bringing the content closer to learners in a way that interests them.

In line with situated cognition, learning tasks must be situated within authentic contexts (Koole, 2009). The intersection between the device and social aspects embodies the synthesis of learning and instructional theories, thus; creating environments conducive for learning (Koole et al., 2010).

It is hoped that the proposed strategy would provide an environment that enhances learning with understanding.

3.3.4 Mobile Learning Process (DLS)

Effective mobile learning results from the integration of the device (D), learner (L), and social (S) aspects (Koole, 2009). Mobile learning provides enhanced collaboration among learners, access to information, and a deeper contextualization of learning (Koole, 2009). This is where technology mediates learning processes to create knowledge (Asiimwe et al., 2017). The mobile learning process is characterized by the task artifact cycle, information access and selection, and knowledge navigation (Asiimwe et al., 2017). In keeping with the concept of mediation, the task-artifact cycle captures the idea that tasks and artifacts co-evolve (Asiimwe et al., 2017). The artifacts themselves introduce possibilities and constraints that redefine the uses for which they were originally intended (Asiimwe et al., 2017). Due to the advancement of technology, the process of mobile learning is continuously redefined and reshaped (Kumar et al., 2011). Besides using mobile technology mainly for information access and dissemination, this research hopes to explore mobile technology use to acquire a deep understanding of mathematical concepts and apply them to problem-solving in higher grades. Learners are actively involved in knowledge production, thus increasing their retention and application capabilities (Bada, 2015). Effective mobile learning facilitates information access, selection, and navigation; so that appropriate information is accessed and applied in accordance with contextual relevance (Kumar et al., 2011).

3.4 FRAME as a theoretical framework in other researches

Researchers and practitioners have used FRAME to explain the dynamics of a mobile learning environment. Some researchers used it to assess the effectiveness of mobile approaches and others used it as the foundation for other theoretical frameworks (Bachman & Gannod, 2011). In their study, Bachman and Gannod (2011) developed Augmented FRAME, a modified version of the FRAME model. In other instances, the FRAME model has been used in the development of learning materials designed for mobile learning (Vahed & Singh, 2015). For example, Vahed and Signh (2015) used it as a guide in the construction of the internal structure that was used to design the epistemic frame of the Muscle Mania mobile game. Likewise, in their study, Levene and Holly

(2015) were guided by the FRAME model in the designing of instruction for mobile devices. Asiimwe et al. (2017) used the FRAME model to evaluate the mobile learning environment (the mobile app) that was developed by the Mobi-Class project. Similarly, Hlagala (2015) used it to evaluate the usefulness of mobile digital technology in the Northern Academy. In a similar study, Kumar et al. (2011) used the FRAME model to analyze the effect of mobile device intervention for student support services. In this research, the FRAME model guided the design of GeoGebra applets that were used to discover the cases of congruence.

3.5 The relevance of the FRAME model

Several reasons are behind the selection of the FRAME model as the theoretical framework for this research. First, the model focuses on mobile learning. Similarly, the thrust of this research is on using mobile devices to learn the congruence of triangles. Second, the model supports learner-centered knowledge construction and in this study, learners are expected to discover the concept of congruence of triangles. Third, issues of information overload were considered during the construction of the applets and the worksheet.

Chapter Four Research Design and Methodology

4.1 Introduction

In this chapter, I discuss the research methodology guiding this study. This includes research methods and research design (sampling methods, data collection methods, and data analysis procedures that were implemented).

4.2 Problem Statement and critical questions

As mentioned earlier in section 1.1, low performance in Mathematics is still a challenge in both South Africa and abroad (Pfeiffer, 2017; Aric & Aslan-Tutak, 2015; Ünlü & Ertekin, 2017; Naidoo, 2011). Even though there has been a steady increase in learner performance in mathematics over the years, there are still concerns that the majority of learners struggle to answer questions that demand deep conceptual understanding (Department of Basic Education-d, 2018; Department of Basic Education_d, 2019; Department of Education_d, 2020; Pfeiffer, 2017). This research seeks to explore the use of mobile technology as a strategy for promoting conceptual understanding of the teaching and learning of mathematics. Specifically, it serves to answer the following questions:

- What are learners' learning preferences?
- Are learners' affection or performance in mathematics associated with technology proficiency or technology association?
- Is it practical to use GeoGebra applets on mobile devices to teach and learn the congruence of triangles?
- What are the learners' views on the benefits of using GeoGebra applets in the learning of congruence of triangles?
- What are the challenges anticipated by learners when they use GeoGebra applets for learning mathematics?

In my view, responses to the questions posed in this section would address the general statement of the problem, prescribing a strategy that promotes conceptual understanding of mathematics at the same time matching the learners' learning style. The following paragraph discusses the research methodology that guided this study.

4.3 Research Methodology

Research methodology encompasses the general principles that guide the research, or the foundation upon which the research is based (Cresswell & Cresswell, 2018; Pfeiffer, 2017). The methodology is guided by the research paradigm; a philosophical way of thinking that guides research action (Pfeiffer, 2017; Cresswell & Cresswell, 2018). The following subsection discusses the definition, components, and common paradigms in educational research.

4.4 Research methods

A research method is made up of various tools of inquiry and strategies (Naidoo, 2011). In this subsection, I discuss mixed-method research and how it fits in this research.

4.4.1 Mixed Methods Research

Mixed methods research is one in which the research incorporates both qualitative and quantitative data collection methods and analysis in a single study (Cresswell & Cresswell, 2018). This type of study enables researchers to understand complex phenomena qualitatively, as well as explain the phenomena through numbers, charts, and statistical analysis. Using different avenues helps draw the strengths and simultaneously minimize the weaknesses of both, qualitative and quantitative research methods in a single study (Gay et al., 2009). Additionally, it helps understand a phenomenon more fully than when a single method is used (Gay et al., 2009). It is contended in the literature that differences in epistemological beliefs should not prevent a qualitative researcher from utilizing data collection and analysis methods more typically associated with quantitative research, and vice-versa (Gay et al., 2009; Cresswell & Cresswell, 2018). Relatedly, this study is predominantly qualitative but also implements quantitative data collection and analysis methods as indicated in 4.4.1 and 4.4.2.

4.4.1.1 Types of Mixed Methods Research Designs

Johnson and Onwuegbuzie (2004) note that there are two major divisions of mixed methods research; mixed model and mixed-method. In the mixed model, the qualitative and quantitative data collection and analysis approaches are used within or across the stages of the research (Johnson & Onwuegbuzie, 2004). In the mixed method scenario, distinct phases of qualitative and quantitative approaches are implemented. In an across-stage model design, the mixing of qualitative and quantitative data collection and analysis takes place across the stages of the research process (Cresswell & Cresswell, 2018; Gay et al., 2009). A within-stage mixed model arises in research where, for example, a questionnaire with predominantly closed-ended and either one or more open-ended questions are used for data collection (Johnson & Onwuegbuzie, 2004). Correspondingly, in this research, I implemented the within-stage mixed model. Both questionnaires that I administered had closed and open-ended items. Both questions and statements in the questionnaires are referred to as items in this research.

There are three common types of mixed methods research designs in education research (Gay et al., 2009; Cresswell & Cresswell, 2018). The QUAL-quan model, the QUAN-qual model, and the QUAL-QUAL model. Uppercase letters (e.g., QUAL, QUAN) are used to emphasize the priority of data collection, and lower case letters (quan, qual) implies less emphasis (Johnson & Onwuegbuzie, 2004).

In the QUAN-qual (explanatory) mixed method, the quantitative research method is implemented in the first phase of the study (Gay et al., 2009; Creswell & Poth, 2018). Quantitative data have more weight than qualitative data. This approach may be used to identify outliers or extreme cases (Gay et al., 2009). Qualitative methods are then implemented in the second phase, to explore the characteristics of these cases (Gay et al., 2009). Similarly, it can be employed in cases where the quantitative phase is used to identify how two or more groups compare on the question (Gay et al., 2009). Then the qualitative phase explores the results from the first phase. Alternatively, it can also be implemented to come up with a typology (Gay et al., 2009). This involves identifying factors in the quantitative phase, which are then used to identify themes in qualitative data (Gay et al., 2009). Relatedly, it can be used to examine multi-levels (Gay et al., 2009). For example, factors obtained from the quantitative data in the first phase of the research are compared with the themes extracted from the qualitative data in the second phase (Gay et al., 2009).

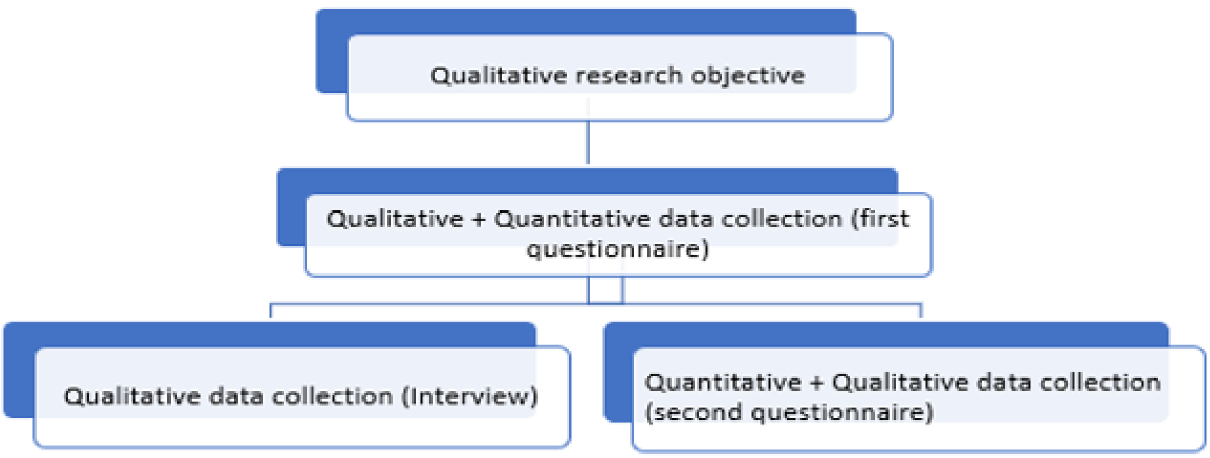
In the QUAL-quan (exploratory) mixed-method qualitative data is collected first and has more weight. This type can be implemented to identify suitable instruments. In this type of research, themes extracted from the qualitative data in the first phase are used to establish factors for quantitative instruments (Gay et al., 2009), which will then be tested with a sample of the population. Similarly, categories can be gathered in the first phase, which then become questions in the second phase of a correlational or regression study (Gay et al., 2009). Correspondingly, qualitative data cases that are extreme in a comparative analysis are followed in a second phase by the quantitative survey (Gay et al., 2009). This study was not carried out in phases, therefore, it does not qualify as an exploratory mixed method.

In a QUAN-QUAL (triangulation) mixed methods design, quantitative and qualitative data are equally weighted and are collected concurrently (Gay et al., 2009). Johnson and Onwuegbuzie (2004) classify the mixed methods research designs into two main categories: mixed-model and mixed-method. In the mixed method, the quantitative and the qualitative studies are done separately or in different phases. Thereafter, they are combined into the overall research study. However, the mixed model combines qualitative and quantitative methods in the same research study. This is further classified into across-stage and within-stage mixed model designs. In the across-stage mixed model, quantitative and qualitative data are collected at different stages of the research (Johnson & Onwuegbuzie, 2004). Alternatively, in the within-stage mixed model, quantitative and qualitative data are simultaneously collected using the same instrument. This study implemented a within-stage mixed model because data were collected concurrently throughout the same study, and not in different phases. Furthermore, the two questionnaires used had both closed and open-ended items allowing the collection of both quantitative and qualitative data using the same instrument. The mixed model can be implemented to quantify qualitative data. For example, qualitative data collected is coded, and frequencies recorded, and then analyzed using quantitative methods. Similarly, in this research, categorical responses in the first questionnaire were coded and percentage frequencies calculated. Alternatively, the mixed model can be used to qualify quantitative data. In this instance, quantitative data from questionnaires were factor analyzed.

These factors were then compared with the themes extracted from the qualitative data. The mixed model can be used to compare results, where qualitative data are directly compared with results from quantitative data analysis. Statistical trends are supported by qualitative themes or vice-versa. Analogously, the mixed model can also be used to consolidate data. Quantitative data and qualitative data are combined to form new questions. Original quantitative data is compared with qualitative themes to form new quantitative questions.

Figure 8

Mixed model used in this research



Note. Adapted from "Mixed Methods Research: A Research Paradigm Whose Time Has Come", by B. R. Johnson and A. J. Onwuegbuzie., 2004, *Educational Researcher, Vol. 33*(7), p.21. Copyright 2004 by Educational Researcher.

Figure 8 shows the mixed model design that was implemented in this research. This study did not implement quantitative and qualitative data in phases, therefore, it fits well with the QUAN-QUAL mixed model. Even though the research's philosophical orientation is interpretive, quantitative and qualitative data were given equal importance. This model was adopted for triangulation purposes. Factors that emerged from the questionnaires were compared with themes from the interview, thus; establishing the credibility and consistency of the research findings. Furthermore, using different approaches helped reduce the bias inherent in using one method of data collection (Gay et al., 2009). The first questionnaire was administered before participants' exposure to GeoGebra applets (before completing the task). The second questionnaire was administered upon completion of the task using GeoGebra applets. The interview was scheduled after the completion of the second questionnaire. In some studies, questionnaire results are considered in the construction of interview questions (Gay et al., 2009). However, in this study, the results of the questionnaire were not used to design the interview. Instead, the questionnaires and the interview were designed concurrently. Johnson and Onwuegbuzie (2004) assert that one of the advantages of adopting mixed method research is its flexibility. Researchers are free to create user-specific designs that effectively answer their research questions.

4.5 Research design

Research design is a detailed outline of how an investigation took place. In other words, it elaborates the plan of sampling participants and data collection and analysis strategies. Research design can be divided into empirical and non-empirical (Pfeiffer, 2017). Empirical research is based on observed and measured phenomena. It derives knowledge from experience. Its primary data comes mainly from surveys, experiments, case studies, and program evaluation (Johnson & Onwuegbuzie, 2004; Pfeiffer, 2017). However, non-empirical focuses mainly on philosophical analysis, conceptual analysis, theory-building, and literature review (Johnson & Onwuegbuzie, 2004; Pfeiffer, 2017). This research is empirical. It is a case study and data were collected from the participants. The preceding sections give a detailed outline of the sampling methods, instruments used for data collection, and data analysis strategies.

As discussed earlier, this study is grounded in an interpretive paradigm, where both the qualitative and quantitative research methods were combined in a mixed-method research design.

4.5.1 School background

The selected school is an all-girls school situated in the Umlazi district in the KwaZulu-Natal province. It has students from grades 8 to 12 who have very diverse cultural and religious backgrounds. The normal daily routine at this school is structured as follows: every morning, class teachers meet with their respective classes during morning registration. In a day there are two breaks; a 35-minute break and a 20-minute break. During these breaks, the students engage in different activities, ranging from going to the toilet or tuck-shop, relaxing in the sports field, or attending any scheduled meetings for extra-mural activities. When the last siren to mark the end of the day goes, the learners go to their respective register classes for afternoon registration. Thereafter, those without any afternoon activities go home, while those with afternoon commitments attend their afternoon activities.

4.5.2 Sampling and sample

In qualitative research, many sampling techniques can be used to recruit participants. The most popular methods are purposeful (judgment or purposive), convenience, quota, and snowball sampling (www.statisticssolutions.com, n.d.; Creswell & Poth, 2018; Gay et al., 2009).

Convenience sampling recruits participants who are easily accessible and convenient to the researchers (www.statisticssolutions.com, n.d.; Creswell & Poth, 2018). I conveniently selected a school in Durban since I was working there during the data collection period. Working with participants from this school had several pros. First, I had easy access to the participants, which facilitated the data collection process. Rescheduling interview times in instances of unforeseen circumstances was easier. It was also easy to collect completed questionnaires. Second, I was familiar with the participants since that was my fourth year working at the school. I hoped this would reduce the issue of mistrust and thus, influence the participants to take part and to give genuine responses. Any grade would qualify to participate in the research, but grade 9 was purposefully selected.

Purposeful sampling involves selecting a sample that is believed to be representative of a population under investigation (Gay et al., 2009). This sampling technique requires the researcher to decide on the qualifying criteria of each participant. (www.statisticssolutions.com, n.d.; Creswell & Poth, 2018). To come up with the criteria, the researcher may be required to spend time in the research setting to obtain background information on the phenomenon (Gay et al., 2009). According to the National Curriculum Statement (CAPS), in South Africa, the section of Euclidean geometry (referred to as Space and Shape in the CAPS document) is introduced at grade 9 level, thus, making this grade’s learners more appropriate participants for the topic that was used in the research.

I was also well acquainted with the curriculum and knew that the section of cases of congruence is dealt with in grade 9. This served as a qualifying criterion for selecting both the grade and investigation task that was used to illustrate how GeoGebra applets could be used in learning mathematics. Besides the fact that congruence is dealt with in grade 9, Euclidean geometry is one of the sections that needs attention in the Further Education and Training phase, and congruence is applied in some of the sections such as properties of quadrilaterals and circle geometry (Department of Basic Education_d, 2017; Department of Basic Education_d, 2018).

4.5.3 Gaining access and inviting participants

To research in KwaZulu-Natal schools, permission was sought from the Department of Education, University of KwaZulu-Natal ethical clearance office, and respective schools. After getting the ethical clearance from the university, I approached the deputy principal with my request of using both the school premises and the learners at the school. This request was taken forward to the school management team where permission was granted. Initially, I wanted participants to bring mobile devices to school but this was against the school policy. Therefore, participants had to complete the worksheet using a mobile device at their own time at home.

The school had seven, grade 9 classes. After the pilot study, I visited all the classes except the one that I was teaching. These visits were done during morning or afternoon registration. It took me a week to visit all the classes. The purpose of these visits was to give a brief description of what the study entailed and to invite learners to take part in the study. Additionally, a detailed explanation of the duties of the participants was presented, and the approximate amount of time for completing questionnaires and interviews was indicated. It was also made clear that participation was voluntary, that there were no financial benefits, and that participants were free to withdraw from taking part at any time. I also indicated that the participants would use their data to access the internet. Two letters were issued to the 25 learners who were interested in taking part. One letter was for parents’ consent and the other letter was for participants’ consent. Sample letters are provided in Appendix H. Signed consent letters were collected the following day, and the first questionnaire (see Appendix A) was issued to participants to complete over the weekend. I then met the participants 2 days after collecting the completed questionnaires. The meeting took place during the first break in the classroom. It was convenient to use the first break to illustrate the use of the applets because it was longer than the second break. I used the mathematics classroom for the head of the department for my meetings and interviews. This classroom was used with the school and head of department's consent. During the meeting, participants were put into three groups. Each group was given a mobile device, worksheet, and user documentation. In this context, user documentation refers to a document with instructions to access and navigate through the GeoGebra book. In addition, explanations on measuring lengths and angles were also included in the user documentation (see Appendix E). One group had an iPad, the other group had a smartphone (iPhone SE) and the third group had a tablet. These devices were mine and were not used for any other reason. Using a data projector, I illustrated how to; access the applets, use the slide option, measure the lengths and angles, as well as the navigation and reconstruction tools. The meeting was successful and what impressed me the most was that the participants started showing me better ways of navigating through the GeoGebra book. This was confirming their proficiency in technology. After the illustration, I gave the participants the worksheet that they had to complete using the applets at their convenient time. I did not want to exert pressure on the participants by setting a deadline.

4.5.4 Worksheet

Participants were required to complete an investigation task using GeoGebra applets. An investigation task is an activity that is designed to discover rules or concepts, identify or test patterns or relationships, draw conclusions, or establish general trends (Department of Basic Education_sp, 2011). In this study, the investigation task was used to establish the four cases of congruence of triangles. It was a modification of the investigation task in the grade 9 mind action series textbook (Phillips et al., 2014). The investigation task in the textbook was designed to be completed using manual construction of triangles and measuring, yet in this study, GeoGebra applets provided constructed triangles and allowed users to measure using GeoGebra tools. The investigation task was designed in the form of a worksheet with instructions, incomplete tables, and statements. There were six tables and some incomplete statements under each table (see appendix B). I will refer to the table and its corresponding statements as an activity. Each applet was tailormade for each activity.

The first activity led to the establishment of the case of congruence, where three corresponding sides are known to be equal (SSS). The second activity was a counterexample to illustrate that having three corresponding angles equal does not necessarily mean that the triangles are congruent. The third activity was used to discover the case where two corresponding sides and an included angle are known to be equal. The fourth activity was a counterexample to illustrate that if the angle is not between the two sides that are known to be equal, then the triangles may not be congruent. The fifth activity was used to establish the case of congruence where two corresponding angles and a side are known to be equal. The sixth activity was used to discover the case of congruence in right-angled triangles, where the hypotenuses and a pair of corresponding sides are equal.

One of the focus areas of the research was to seek learners' views on the benefits of using GeoGebra applets in investigating cases of congruence of triangles, with the hope of transferring results to similar contexts. The investigation was aimed at giving the learners an idea of GeoGebra applets and how they could be used in learning mathematics, basing on the assumption that the participants had never used GeoGebra for learning mathematics. This assumption was supported by learners' responses to the item that probed their use of technology in mathematics lessons.

4.5.5 Design of GeoGebra applets

In line with the focus of this research study, of exploring learners' technological proficiency in using mobile technology in the teaching and learning of mathematics, I designed GeoGebra applets that the participants used to investigate the cases of congruence of triangles. This was done so that the participants could get exposure to using GeoGebra applets on a mobile device in learning mathematical content since they were required to give their views and anticipated challenges in the process.

An applet is a small piece of software that serves a specific task and can be executed from within a web browser (Morphett et al.,2015). As mentioned earlier, teachers found ready-made GeoGebra applets very attractive and easy to use (Korenova, 2017). It is also inferred in the literature that using applets is faster and does not need any software expertise (Morphett et. al, 2015). For this reason, this research proposed the use of applets on mobile devices, mainly to remove the software knowledge barrier for both teachers and students. Besides being cumbersome and time-consuming, constructing triangles from scratch would require the installation of GeoGebra into mobile devices, which would lead to memory problems (Hohenwarter & Lavic, 2011). Moreover, construction would take time and would add to the constraint of completing the syllabus.

The participants completed an investigation task where they used GeoGebra applets to discover the four cases of congruence of triangles (see Task in Appendix B). Triangles are congruent when they have exactly three sides and exactly three angles (Phillips et al., 2014). In this study, a case of congruence defines a set of three minimum and sufficient corresponding attributes required to conclude that two triangles are congruent. In this context, an attribute refers to either a side or an angle of a triangle. This task was aimed at giving the learners an idea of GeoGebra applets and how they could be used in learning mathematics.

4.5.5.1 Design guidelines

When designing applets, some aspects should be put into consideration. In this subsection, I briefly discuss some of the guidelines that were put into consideration during the design of the applets used in this study. It is suggested that designed applets should; be interactive, be easy to use, have the right size, and not be cluttered with unnecessary text (Radović et al., 2018).

It is highly recommended that objects that are designed should be interactive (Radović et al., 2018); that is, the objects should be movable or changeable. This provides freedom for learners to explore the mathematical relations for the objects, and discover mathematical concepts. Similarly, the applets designed are dynamic in the sense that, if one measurement of either the length of the side of the triangle or the angle is changed, all the other measurements are updated instantly. In each applet, the measurement of the three attributes is given. The question of which attributes are given depends on the case of congruent under investigation. For example, in the case of SSS, the measurements of all three sides in each triangle were given. A slider was then used to adjust the length of two sides, and all the other angles and lengths were automatically adjusted. This allowed the participants to quickly complete the task as they were not required to start the process of measuring again to get adjusted measurements. To complete the task, a set of three measurements was required, which was then used to establish the case of congruence or, in the case of counterexamples, the set of measurements was used to illustrate that the two triangles were not congruent.

A significant strength of GeoGebra applets is that they remove potential technological barriers from learners and teachers (Morphett et. al, 2015). Therefore, it is a requirement to have applets that are easy to use. In line with this, triangles were constructed, and participants were only required to measure the missing lengths and angles.

It is suggested that the objects and labels should be large enough to allow all intended manipulations, but small enough to fit on one screen (Radović et. al, 2018). In support of this, the triangles can be adapted to smaller screens, which means that the size can be changed without changing the measurements of the attributes. It is advised to try out the applets on the same type of device that the users are going to use before using the applets. This was accomplished the time I showed the participants how to use the applets. Three devices were used, and the iPhone SE had the smallest screen. All triangles and labels were visible.

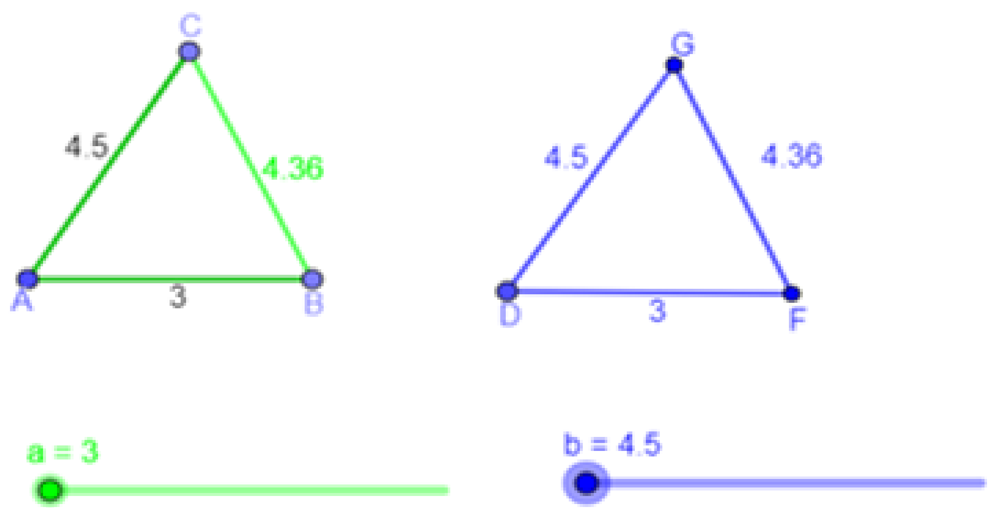
It is also recommended that a lot of writing be not made in the applets as this affects the speed at which the applets open or give results. Where text is used, it should be placed close to the corresponding object in the applet. Additionally, all labels and measurements should be clear. Following the guidelines, I created an online GeoGebra book with four chapters. It is saved in the GeoGebra online space. At the moment, this book is not open to public use, I set it as private so that it cannot be accessed without putting in the user name and password. A chapter contains one or two applets that are used to discover a particular case of congruence. The following sections give detailed explanations of the purpose of each applet.

4.5.5.2 Discover SSS and Does-AAA-work applets

Chapter one is entitled SSS and contains two applets, which I named, ‘Discover SSS’ and ‘Does AAA work’. Discover SSS applet is used to complete the activity that leads to the establishment of the case of congruence, where three corresponding sides are known to be equal, and is referred to as the SSS case (Phillips et al., 2014). On opening this applet, two triangles are displayed, each showing the lengths of the three sides as shown in Figure 9. Participants then measured the angles, either by clicking inside the respective vertex or by clicking the three points of the triangle clockwise, with the required angle included. In this context, an angle is included when its vertex is clicked second between the other two points (see user documentation in Appendix E). After recording the first set of measurements in the table provided in the worksheet, the participants used the slider action-handler to set the measurements for new pairs of triangles as indicated in the worksheet. Upon recording measurements for all three different pairs of triangles, participants filled in the blanks of the statements. By so doing, they would conclude that if three corresponding sides of triangles are known to be equal, then it is concluded that the two triangles are congruent without checking if the three angles are equal.

Figure 9

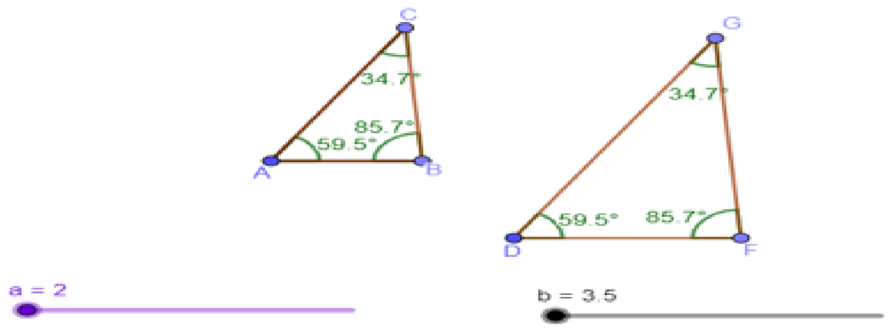
The first page of applet SSS



Similarly, Does AAA work applet is used to complete the counterexample that illustrates that if three corresponding angles are equal it does not necessarily mean that the triangles are congruent. In this applet, all the angles are given as shown in Figure 10. The participants were required to measure the three sides and complete the respective section of the worksheet. All the tables and fill-in-the-blank statements were designed similarly. By using the slider action-handler, to change the measurements, the participants completed the measurements for three sets of triangles. After that, they were to conclude that if all three corresponding angles in a set of triangles are equal, does not necessarily mean that the triangles are equal.

Figure 10

The first page of the applet for counter-example of congruent case SSS



4.5.5.3 Discover-SAS and Does-SSA-work applets

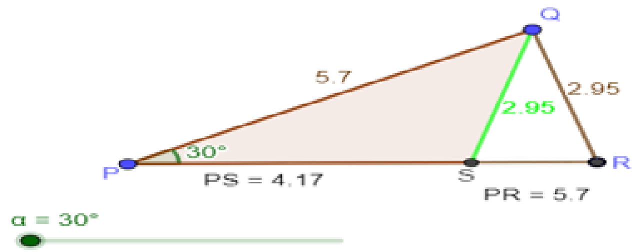
Chapter two of the designed online GeoGebra book is entitled SAS. It contains two applets, which I named Discover-SAS and Does-SSA-work. Discover-SAS is used to complete the activity that establishes the case where two corresponding sides and an included angle are known to be equal. On opening the applet, two triangles are displayed, each showing the lengths of the two sides and the included angles that are equal as shown in Figure 11. Participants measured the missing length and two angles. After recording the measurements in the table and completing the blank spaces, the participants were expected to establish that, if two corresponding sides and the included angle are equal, then it can be concluded that the triangles are congruent without verifying if the other three pairs of attributes are equal. Does SSA work is used as a counterexample to illustrate that, if it is given that two corresponding sides are equal and a non-included pair of angles are also equal, we can not conclude that the triangles are congruent.

Figure 11

The first page of applet Discover SAS

Figure 12

The first page of the applet Does SSA works

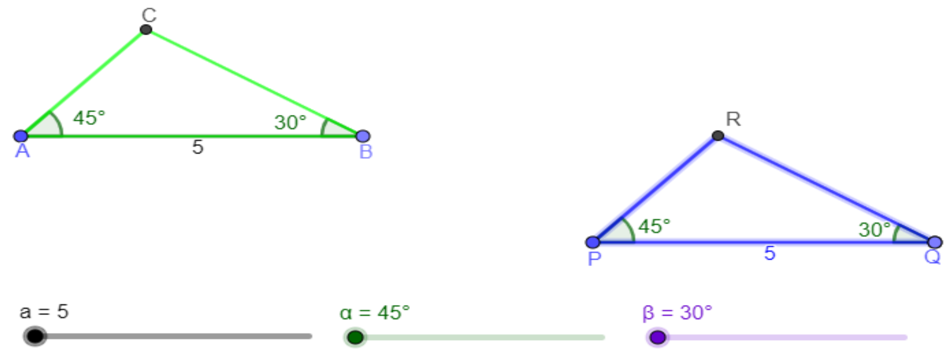


4.5.5.4 SAA applet

Chapter three has applet SAA. SAA applet illustrates the case in which the two triangles displayed have a pair of corresponding sides and two pairs of corresponding angles equal. The order of the equal attributes does not matter, that is, as long as there are two corresponding angles and an equal side, then it can be concluded that the triangles are congruent without measuring the missing attributes. On opening the applet, two triangles are displayed, each showing the two equal angles and a side that is equal as shown in Figure 13.

Figure 13

The first page of the applet Does SAA works

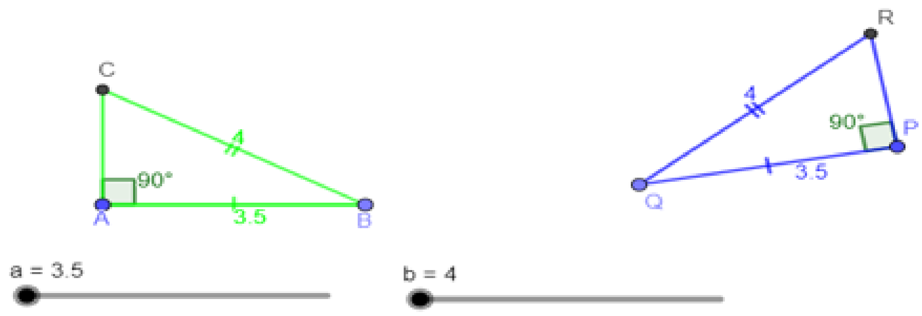


4.5.5.5 RHS applet

Chapter four contains applet RHS. The RHS applet illustrates the case of investigating congruence between two right-angled triangles, where the hypotenuses and a pair of corresponding sides are equal. On opening the applet, two right-angled triangles are displayed as shown in Figure 14.

Figure 14

The first page of the applet Does RHS works



4.6 Pilot study

I conveniently selected the class that I was teaching for the pilot study. Three learners volunteered to take part in the pilot study. The three learners were in the grade 9 mathematics class that I was teaching. Pilot studies are useful for various reasons. They allow researchers to refine research questions or methods, hypotheses, or questionnaire items, before embarking on the main study. Similarly, in this research, some items of the questionnaire were modified after the pilot study. That improved the reliability and validity of research instruments. Relatedly, I also modified user instructions as well as the sampling strategy. Initially, I thought I was going to get participants from three randomly selected grade 9 classes. But, after getting three volunteers from one class, I realized that I was supposed to include all six classes, otherwise I was going to end up with very few participants. Researchers can also estimate the time and cost required for the main study. In this study, I noted that participants had homework daily. Therefore, they needed more time to complete the worksheet than I anticipated. Additionally, I also realized that not all participants were willing to be interviewed. All three learners who took part in the pilot study were not willing to be interviewed.

4.7 Quantitative and Qualitative Data Collection using questionnaires

This research is grounded in the interpretive methodology as indicated earlier. In a review of the literature on research methods, it was noted that questionnaires are frequently used in mixed-method research (McGuirk & O'Neill, 2016). Similarly, in this study, quantitative and some qualitative data were generated from two questionnaires.

A questionnaire is a formalized set of items for obtaining information from respondents (Malhotra, 2015). Questionnaires are used to enable the collection of information in a standardized manner which, when gathered from a representative sample of a defined population, allows the inference of results to the wider population (Rattray & Jones, 2007). Nonetheless, since purposeful and convenience sampling was implemented in this study, the results cannot be inferred to the entire population of high school learners. In a questionnaire, the researcher's needs are transformed into a set of specific items that respondents are willing and able to answer (Malhotra, 2015). Therefore, the content of the questionnaire must relate to the research questions. To reduce the redundancy of items, each item should have a purpose. There are four main types of item content (McGuirk & O'Neill, 2016). First, there are attribute items that focus on establishing respondents' characteristics. Second, are behavior items aimed at discovering what people do. Third, are attitude items that seek to discover what people think is desirable or undesirable. Fourth, are items about belief, aimed at establishing what people believe to be true, false, or prefer. Though all types of items were featured in both questionnaires, the first questionnaire was dominated by attribute and behavior items, while the second questionnaire was dominated by attitude and belief items. The first questionnaire was completed by the participants at the beginning of the research before they used the applets. The second questionnaire was administered after the participants had completed a task using the applets. Apart from item content, items can also be classified according to the nature of the response; closed-ended or open-ended items. Some define closed-ended items as items that can be answered by a single word or short phrase (Gay et al., 2009). In this research, I define closed-ended items as any item for which a researcher provides research participants with options from which to choose a response. However, in open-ended items, possible answers are not suggested, and respondents answer in their own words (Gay et al., 2009). Closed-ended items which are commonly used may restrict the depth of the participant's response (Bowling 1997) and thus, the quality of data collected may be diminished or incomplete. To curb this, open-ended items were also included in both questionnaires. Questionnaire-based methods are, therefore, not the method of choice where little is known about a subject or topic area. In such an instance, qualitative methods may be more appropriate (Rattray & Jones, 2007). Closed-ended items collect different types of data. It is important to consider the type of data collected because it relates to the statistics that can be used to analyze the data (Guy, 2019).

There are four basic data measurement scales; nominal, ordinal, interval, and ratio (Guy, 2019; Bhat, 2020). In this research, I used nominal and interval measurement scales. The nominal scale uses labels, categories, or named responses that cannot overlap (Bhat, 2020). There is no numeric significance between the categories, and the order of response options is not important. In this study, the first questionnaire comprised 22 nominal response items; of which 17 had dichotomous (yes/no) responses, 4 items had a three-point response scale, and 1 had a four-points-response scale. It is recommended to use items with the same number of possible responses if quantitative methods will be used to analyze the data (Bhat, 2020). The impact of not adhering to this recommendation had an effect when I performed confirmatory factor analysis. I had to perform factor analysis on the 4 items that had a three-point scale separately.

One of the drawbacks of nominal data is its limitation in performing statistical analysis. The distribution of nominal data is described by the mode, the frequency of the categories, and bar graphs and cross-tabulations. Cross-tabulation is a method used to quantitatively analyze the relationship between multiple categorical items (Aprameys, 2019). Cross-tabulation was used to assess the association between liking mathematics or performance and technology appreciation or proficiency using first questionnaire data.

The second questionnaire had 28 items on an interval measurement rating scale and three open-ended items. Interval rating scales are convenient for collecting information about political beliefs or customer satisfaction or for evaluating a product (Bhat, 2020; Guy, 2019). They are highly recommended for generating mathematical data from opinion and feedback surveys (Guy, 2019). Additionally, they give more precision in measurement and are flexible in analysis options. The following statistics can be determined using data from interval rating scales; mode, median, mean, correlation, regression, standard deviation, variance, analysis of variance, and factor analysis (Guy, 2019). In this study, the mean for each of the 28 items was calculated. Since the research was guided by the interpretive paradigm, the main focus was not on statistical summaries, but the response of every participant. For that reason, I used the percentage frequencies of the scores to describe the distribution of the data. Factor analysis was used to assess the validity of the constructs. Another positive attribute of the interval measurement scale is that it allows participants to evaluate something using a numerical scale. Its numeric nature helps reduce confusion and distractions while selecting response options. Bearing in mind the issue of confusion and distraction, I chose the 10-point interval rating scale because participants are familiar with assessments that are evaluated out of 10. Therefore, I found it easier for them to use the 10-point interval rating scale.

The evaluation of a construct by the participants was determined by calculating the mean of each construct using the scores of the individual items. The constructs that were established from the responses of closed-ended items were then compared with the themes that emerged from the open-ended items and the interview.

The main benefits of using questionnaires for data collection are that they are usually quick to complete, are relatively economical, and are usually easy to analyze (Rattray & Jones, 2007). In this study, participants were given the questionnaires to complete at their own convenient time.

The first questionnaire was administered before the participants completed the task of investigating cases of congruence using GeoGebra applets on mobile devices. It elicited learners' learning preferences, the association between learners' affection or performance with mobile technology proficiency and appreciation, and the affordance of mobile devices. Consequently, the second questionnaire probed students' perceptions on the use of GeoGebra applets on mobile devices in discovering cases of congruence. There are two common strategies for constructing questionnaires; deductive and empirical.

The empirical approach is based on a statistical analysis of data from the completed instrument. The instrument is administered, and data is tested for the existence of relationships. This may be done by constructing correlation matrices. Items with a high correlation are then estimated to contribute towards the same construct.

Deductive construction is guided by some belief or theory that gives an idea of items that contribute to the construct. Alternatively, if constructs are easily operationalized using observable behavior, logic may be used to frame the content of the instrument. As indicated earlier, the deductive strategy guided the design of the two questionnaires. Items that were included in the first questionnaire were adapted from previous research studies. Similarly, the design of the second questionnaire was guided by Purdue Usability Testing Questionnaire (PUTQ) (Talirongan & Hernandez, 2017), Standardized User Experience Percentile Rank Questionnaire (SUEPRQ) (Talirongan & Hernandez, 2017), and User Questionnaire (Lund, 2001). The items were not extracted verbatim from these questionnaires but were constructed using ideas borrowed from them.

Microsoft Excel was used for data capturing, calculating percentage frequency, and bar graph construction. This program was chosen because it was readily available to me, and I am also comfortable in using it. SPSS was used for cross-tabulation analysis and for assessing the validity and reliability coefficiency. R was used to assess the validity of the first questionnaire.

4.7.1 Data collection process

After collecting signed consent letters from the participants who volunteered to take part in the research, I gave them the first questionnaire to complete on their own time. The questionnaire was completed before the participants used the designed applets on mobile devices to investigate cases of congruence of triangles. This questionnaire was collected the day I illustrated how to access and use GeoGebra applets.

During the meeting, participants were put into three groups. I gave each group a mobile device, worksheet, and user documentation. In this context, user documentation refers to a document with instructions to access and navigate through the GeoGebra book that I designed. An explanation on measuring the lengths and angles was included in the user documentation (see Appendix E). I repeatedly explained the measuring of angles because it had emerged as a challenge in the pilot study.

One group had an iPad, the other group had a smartphone (iPhone SE), and the third group had a tablet (Vodaphone). These devices had different screen sizes, with the iPhone having the smallest screen size. It is advised to check how the applets show on different screen sizes before you give them to the users. Besides availability, I also wanted to check if there were challenges with the screen size. Everything worked smoothly. Using a data projector, I showed the learners how they were going to use the mobile device to complete the worksheet. This included the login process and measuring of length and angles. The meeting was successful, and what impressed me the most was that the participants started showing me better ways of navigating between the applets in the GeoGebra book. This confirmed the digital proficiency of today's generation of learners. At the end of the illustration, each participant was given the user documentation and the investigation task that they had to complete using the designed GeoGebra applets. This task was designed as a worksheet with incomplete tables and spaces to fill in.

As each participant brought back the worksheet, a second questionnaire was issued. Again, the students completed the second questionnaire at their convenient time. As the students returned the second questionnaire, an interview date was set for those who were willing to be interviewed. Even though the participants had indicated in the assent letter that they would not mind being interviewed, I had to ask them again to confirm this. I also reminded them that taking part in the interview was completely voluntary and there were no implications of not getting interviewed. Initially, I wanted to interview 10 participants, but unfortunately, only six participants were willing to be interviewed. I thought one of the main reasons why participants did not want to be interviewed was that they had commitments during most of the breaks and got exhausted after completing the task and the second questionnaire. The other possible reason was that they had tests after break most of the days, and they would use break times for final test preparations. The interviews took place during either the first or second break. Most interviews were rescheduled.

The data analysis for the questionnaires started with data preparation and screening. These processes are explained in the subsequent paragraphs.

4.7.2 Data Preparation

This section explains how the data from both questionnaires were captured and screened. It is a requirement to screen data before performing most statistical analysis procedures because most procedures have underlying assumptions of the distribution of data (Kline, 2011). In the following paragraphs, it is explained how data from both questionnaires were checked for transcription errors and missing data. Also included is an explanation of the assessment of normality and the presence of collinear items, and outliers of the second questionnaire data. The first questionnaire data did not require the former assessment because it was nominal scale data, with mostly yes/no responses.

The data from both questionnaires was captured using excel. The closed-ended items for the first questionnaire were coded, and responses were assigned to numerical values as shown in Table 5. Reverse coding was used for items 5 and 7. Item 5 asked participants if they preferred memorizing concepts and formulae, and a yes would measure against the learning style that promotes understanding. Similarly, item 7 checked if a participant easily gets distracted in class, and a yes would not promote understanding.

Table 5

Coding for the responses of the closed-ended items in the first questionnaire

Item number	Response	Code
1a, 2a, 4, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20 and 21	Yes	1
	No	2
1b	I just hate Maths	1
	Maths is no for me	2
	I do not understand explanations	3
5 and 13 (Reverse coding was used)	Yes	2
	No	1
6	Yes	1

	No	3
	Sometimes	2
7 (Reverse coding was used)	Yes	3
	No	1
	Sometimes	2
15	Yes	1
	No	3
	Sometimes	2
Responses for items 2b, 22, 23, and 24 were typed.		

The preparation started the moment I received the completed questionnaires from the participants. Since I had the privilege of collecting the questionnaires directly from the participants, it was possible to check the legibility of the responses, and if all items were answered. Fortunately, there were no problems in that regard, as all responses were legible and there were no missing data.

To minimize transcription mistakes, I checked for typing mistakes immediately after capturing data from each questionnaire. After entering data from all questionnaires, I checked entry by entry for transcription mistakes, and no mistakes were found. After this process, I screened the second questionnaire data as explained in the following paragraphs.

Data should be screened before any statistical procedures are undertaken because most of these procedures are based on assumptions (Kline, 2011). For example, the confirmatory factor analysis that was used in this study assumes normality. The second questionnaire data were screened for outliers, collinearity, and normality.

In the next section, I discuss different types of validity and reliability and how these two characteristics were assessed in this study.

4.7.3 Reliability and Validity of questionnaires

Central to the understanding of results derived from questionnaires are the issues of reliability and validity (Rattray & Jones, 2007). It is essential to test for reliability and validity, to ascertain valuable interpretations (Rattray & Jones, 2007). Some researchers indicated that it is essential to establish reliability before establishing validity (Gay et al., 2009). Contrary to this, I consider checking whether the instrument measures what it is intended to measure before verifying reliability. Validity is discussed in the next sub-section.

4.7.3.1 Validity

Validity measures the degree to which an item or a test measures what it is intended to measure (Gay et al., 2009; Creswell & Poth, 2018). It refers to the degree to which collected data accurately gauges what the researcher is trying to measure, or answers the research items. It is advised to adopt or modify an approved reliable instrument to avoid the hassles of testing for reliability and validity (Gay et al., 2009). Unfortunately, some of these instruments do not focus on the key concepts detailed within specific research items (Rattray & Jones, 2007). Moreover, I did not find existing instruments that would gather enough data to answer the research questions in this study. Therefore, items in the first questionnaire were adapted from previous research studies, and items in the second questionnaire were adapted from the pre-existing and validated scales (the Purdue Usability Testing Questionnaire (PUTQ), Standardized User Experience Percentile Rank Questionnaire (SUEPRQ), and the User questionnaire Validity require the collection of sources of evidence to support the desired interpretation (Gay et al., 2009). A point to note is that an instrument used for different purposes must be validated for each purpose (Gay et al., 2009). Similarly, in this research, I had to assess the validity of each construct. Validity is specific to the interpretation being made and to the sample being tested (Gay et al., 2009). There are four general types of validity: content, criterion-related, construct, and consequential (Gay et al., 2009). These are interrelated aspects of validity and are not dependent on each other (Gay et al., 2009). The four general types of validity are discussed in the succeeding sub-sections.

4.7.3.1.1 Content Validity

Content validity is the degree to which a test measures an intended content area. Even though content validity is of paramount importance for achievement tests, it is viewed from a different perspective in this study. To assess content validity, the level of content is considered. It includes assessing whether the instrument measures what the participants have learned, or are expected to know. The wording of all items in both questionnaires was simple. The response rate was also 100%, which ascertains that participants could complete the instruments. The students were also given a task to investigate cases of congruence of triangles using GeoGebra applets on mobile devices. The purpose was not to test the achievement level, but for students to see how GeoGebra applets could be used in learning mathematics. The congruence section dealt with in the study is part of the grade 9 curriculum, and the participants had already covered the section. Therefore, content validity was strong for the two questionnaires and the task.

4.7.3.1.2 Criterion Validity

Criterion-related validity is determined by relating performances from different tests (Gay et al., 2009). It has two forms; concurrent validity and predictive validity.

Concurrent validity is the degree to which scores on one test are related to scores on a similar, pre-existing test administered at the same time frame (Gay et al., 2009). This study sought to explore learners' technological proficiency towards using mobile technology in the teaching and learning of mathematics. Therefore, I assessed the concurrent validity of the items related to learners' proficiency in using mobile technology. In the first questionnaire, 92% of participants indicated that they were proficient in using mobile technology. This was attested by a 10 out of 10 rating score by all the participants on item 6 (*My cellphone skills helped me to use the applets*) in the second questionnaire. Additionally, I calculated the Pearson correlation coefficient between items in the second questionnaire that I presumed to be correlated, even though they were not evaluating the same construct. A strong correlation affirms concurrent validity. In this study, a correlation coefficient of 0,8 and above is considered strong. I expected that, if one enjoyed completing the worksheet using a mobile device, then they would also like to use mobile technology more often. This was affirmed by the high correlation (0,85) between item 13 (*I enjoyed completing the worksheet using my cell phone/tablet/laptop, etc*) and item 28 (*I would like to use mobile* *technology (cell-phone, i-pad, tablet, laptop, etc) more often when learning Maths*). I also expect technology proficiency to strongly correlate with easy navigation when using a mobile device. This was affirmed by the high correlation (0,89) between item 5 (*It was easy to move from one applet to the other*) and item 6 (*My cellphone skills helped me to use the applets*).

Additionally, the type of device that the participants mentioned in the first questionnaire matched with the device that was used to complete the task. This exhibited a strong correlation, thus; confirming that the items were measuring an intended construct. Responses to two questionnaires and the interview were consistent.

The above discussion affirms the concurrent validity of the data.

4.7.3.1.3 Predictive Validity

Predictive validity is the degree to which a test can predict how well an individual will perform in a future situation. This is measured by calculating the correlation coefficient between two sets of scores. A high correlation coefficient indicates good validity, with the number close to one being the best (Gay et al., 2009). Even though this study did not focus on achievement, one of the constructs probed the acceptance of mobile technology. The participants' responses regarding this construct were consistent in both questionnaires. In the first questionnaire, 96% of the participants indicated that they appreciated the use of mobile technology in learning. The average rating of item 28 (*I would like to use mobile technology more often when learning maths*) in the second questionnaire, was 9.92 out of 10, with a standard deviation of 0,194. Also, the mean rating of item 24 (*I would recommend these applets to my friends*) was 9,44 out of 10, with a standard deviation of 0.190. The values of the standard deviation showed that participants would embrace and appreciate the use of mobile technology when learning mathematics. To further affirm this, 40% of the participants suggested that the use of mobile technology would help in the teaching and learning of mathematics. This was in response to an open-ended item (*What do you think your teacher should do that can help you understand Maths better?*) in the first questionnaire. Additionally, all participants who were interviewed indicated that some mathematics sections could be explored using mobile technology.

4.7.3.1.4 Construct Validity

Construct validity reflects the degree to which a test measures an intended construct (Gay et al., 2009). A construct is a non-observable trait or characteristic. It cannot be measured directly. Rather, it underlies items that can be measured (Gay et al., 2009). I used factor analysis and inter-item correlations to assess the construct validity of the 2 questionnaires. The validity was assessed per construct. The first questionnaire had four constructs which I named affection, learning preference, proficiency, and affordance. The second questionnaire had 5 constructs called access, easy-to-learn, legibility, satisfaction, and easy-to-use. Factor analysis is described in detail in the subsequent subsection.

Factor analysis

Factor analysis was used to assess the validity of the constructs. It is a statistical tool that investigates if items measure the constructs that they purport to be measuring. Alternatively, we can say that it is used to identify underlying constructs that are difficult to measure directly. It can be applied to either explore or to confirm the items that are used to measure the construct. Some researchers call them questions and others call them factors. A construct is a factor that cannot be measured directly but is measured using two or more questions. In this study, I referred to questionnaire questions as items. The results from factor analysis serve to assess whether the proposed items measured the construct. There are two types of factor analysis; exploratory and confirmatory factor analysis.

Exploratory factor analysis is used to identify items that measure the constructs (Kline, 2011). When performing exploratory factor analysis, the researcher does not indicate the number of factors nor the items that these constructs comprise. Instead, all items are used to perform the analysis, and the items that measure each construct emerge from the analysis (Kline, 2011). High factor loading is used as an indicator that the item contributes to the measurement or evaluation of the construct. A factor loading measures the relationship of the item to the construct. It can be referred to as the correlation between the item and the construct. Exploratory factor analysis would have worked if the sample size was large, and also if the scoring on the items were normally distributed. In this study, the scoring on all items had the same pattern (negatively skewed). Therefore, performing exploratory factor analysis would result in all items loading on one construct. I used confirmatory factor analysis to assess the construct validity of both questionnaires. The reason was that items in the first questionnaire were adapted from previous researches and items in the second questionnaire were constructed based on the pre-existing and validated scales (the Purdue Usability Testing Questionnaire (PUTQ), User questionnaire, and Standardized User Experience Percentile Rank Questionnaire (SUEPRQ).

Contrary to exploratory factor analysis, confirmatory factor analysis is performed using items that are believed to measure a specific construct (Kline, 2011). Therefore, when performing confirmatory factor analysis, items that are proposed as measuring the construct are indicated, then the analysis assesses whether the proposed items contribute towards measuring the respective constructs. These items would have been used in past research, or would have been taken from questionnaires that were used and validated before, or were grounded in some pre-existing theories. It is, however, recommended to start with exploratory factor analysis in the early stages of the scale development, especially if the research's focus is to develop a measuring scale. The Lavaan package in R was used to perform the confirmatory factor analysis for the first questionnaire and the second questionnaire. I also performed confirmatory factor analysis for the second questionnaire, using the AMOS package to confirm the results obtained from R.

R is open-source software that is used for statistical analysis. It uses several packages developed by different authors. Because it is open-source software, anyone is free to develop a package and contribute towards R software. The Lavaan package and the Polycor package are used for Confirmatory factor analysis. The Lavaan package was developed by Yves Rosseel to provide users, teachers, or researchers a free open-source package that performs confirmatory factor analysis. The Lavaan package assumes that data is interval or ratio scale, therefore, I used it for the second questionnaire. The Polycor package was used to perform confirmatory factor analysis using dichotomous categorical data from the first questionnaire. Polycor package was used to find the heterogeneous correlation matrix for binary-coded categorical responses. Heterogenous correlation is a measure of the relationship between different types of data. The heterogeneous correlation matrix was then inputted in the function that performs confirmatory factor analysis. Unlike when using interval or ratio scale data, there is no need to calculate the correlation separately as it is calculated within the function that performs confirmatory factor analysis. Hopefully, with time, a function is going to be derived, that can check the type of data and then compute the appropriate correlation matrix to use. I could not verify confirmatory factor analysis results for the first questionnaire using AMOS, because it does not have a function to calculate the heterogeneous correlations.

R displays several statistics from the confirmatory factor analysis, such as the ratio of chi-square, comparative fit index (CFI), the goodness of fit index (GFI), normalized fit index (NFI), root mean square residual (RMSR), root mean square error of approximation (RMSEA) (Kline, 2011). A number of these statistics are sensitive to sample size, to the extent that the researcher may fail to reject an inappropriate model in small samples. The root mean square error approximation and comparative fit index avoid issues of sample size. The RMSEA ranges between 0 and 1, where a value close to zero is an indication that there is evidence to support that the items proposed to measure the construct are plausible, with a threshold set at less than 0,06 (Kyriazos, 2018). Thus, the value of RMSEA less than 0.06 confirms the validity of the construct (Kyriazos, 2018). The CFI also ranges from 0 to 1, where, contrary to RMSEA, larger values close to 1 indicate that the items proposed to measure the construct are acceptable.

Besides these indices, many researchers use factor loadings. Similarly, in this research, I used factor loadings because most statistical indices from the confirmatory factor analysis are sensitive to small samples. Factor loadings are expected to be between -1 and 1 (Kline, 2011). A factor loading outside this range is called an ultra-Heywood case. Possible causes of an ultra-Heywood case include too many or too few items measuring a construct, or the sample size being too small to provide stable estimates, or an item being inappropriate to measure the construct (Kyriazos, 2018; Kline, 2011) It is recommended not to include items that give rise to ultra-Heywood case because they make the factor model suspicious or illegitimate. However, in this study, I considered two statistics; the value of the factor loading and the inter-item correlations of items in each construct. High loading confirms that the item contributes towards measuring the item or evaluates the construct. The common threshold for factor loadings is 0,6. However, in this study, the threshold for factor loadings was set on 0,4 because the sample size was small (Kim et al., 2016).

It is recommended that the minimum ratio of the number of items to the sample size required to perform confirmatory factor analysis be around 1:10 (Arifin & Yusoff, 2016). However, the ratios in this research were larger because the sample size was small. The ratios for the constructs in the second questionnaire are shown in Table 6.

Table 6

The ratio of number of items to sample size

Construct	Number of items: Sample size
Access	$3{:}25 \approx 1{:}8$
Navigation	$3{:}25 \approx 1{:}8$
Easy-to-learn	$4{:}25 \approx 1{:}6$
Legibility	$3{:}25 \approx 1{:}8$
satisfaction	$5{:}25 \approx 1{:}5$
Easy-to-use	8:25

After assessing validity, I assessed reliability using the items that were considered to be valid. The annotated syntaxes of confirmatory factor analysis that was performed using R are given in appendix C. In the next subsection, I discuss the common types of reliability, and how reliability was assessed in this study.

4.7.3.2 Reliability

Reliability measures the quality of an instrument (Rattray & Jones, 2007). A reliable instrument gives the same or similar results when administered again (Cresswell & Cresswell, 2018; Gay et al., 2009). There are four common types of reliability (Glen, 2016).

First, inter-rater reliability compares the rating of judges on every subject. Some of the statistics that can be used to establish inter-rater reliability are Cohen's Kappa, Fleiss Kappa, and Krippendorff's alpha (Glen, 2016). I could not use Cohen's Kappa alpha because it measures agreement between two participants, and I had 25 participants. Similarly, I could not apply the Fleiss Kappa alpha even though it extends to measure agreement between more than two participants. The challenge was that it assumes a random selection of participants from a larger population, yet in this study, I used convenient sampling. Krippendorff's alpha is a reliability coefficient developed to measure the agreement among observers, coders, judges, or raters. Krippendorff's alpha is robust, can handle various sample sizes and any number of categories, including dichotomous responses (Glen, 2016). Additionally, it can be applied to any measurement level (nominal, ordinal, interval, and ratio). Furthermore, it can handle missing data. Krippendorff's alpha ranges from 0 to 1, where 0 is perfect disagreement and 1 is a perfect agreement, with 0.667 being the lowest plausible limit (Glen, 2016). This looked like it could work for the second questionnaire but there was a hiccup. When assessing inter-rater reliability, raters should be trained to do the judging or evaluations. Additionally, there are set standards to be followed such that if the rating is perfectly done, each item would have the same score from all the judges or the participants. By contrast, the scoring on closed-ended items was subjective. The evaluation was based on participants' experiences using the applets.

Second, for the test-retest method, an instrument is administered twice (after a few days) to the same group of participants. The main challenge with this would be to convince participants to complete the same questionnaire twice. I did not opt for this method to avoid overloading the participants.

Third, the test parallel-form reliability items that measure the same construct are split and given to different groups of participants. Getting the same or very similar results establishes reliability. I could not use this method in the research because I did not have many items per construct.

Fourth, the internal consistency reliability is the extent to which items in a single test are consistent among themselves and with the test as a whole (Gay et al., 2009). It is commonly used because it requires one sample of data. This eliminates sources of measurement error such as differences in testing conditions thus, increasing its strength in measuring reliability (Gay et al., 2009). Three common approaches to measuring internal consistency reliability are split-half, Kuder-Richardson, and Cronbach’s alpha (α).

Split-half involves dividing the test into halves and then calculating the Pearson product-moment correlation of the scores in the halves. Therefore, the split-half coefficient represents the reliability of half the actual test. To cater for this, the Spearman-Brown formula adjusts this coefficient to get an estimated reliability measure for the actual. The two formulae are shown below:

Pearson product-moment correlation coefficient

$$r_{split} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{[\sum(x-\bar{x})^2][\sum(y-\bar{y})^2]}};$$

Where,

$x = one\ participant's\ score\ on\ the\ first\ half\ of\ items$

$\bar{x} = the\ mean\ score\ on\ the\ first\ half\ of\ items$

$y = one\ participant's\ score\ ont\ he\ second\ half\ of\ items$

$\bar{y} = the\ mean\ score\ on\ the\ second\ half\ of\ items$

Spearman-Brown formula

$r_{total\ score} = \frac{2r_{split\ half}}{1+r_{split\ half}};$

Where,

$r_{total\ score} = the\ reliability\ estimate\ for\ the\ actual\ test$

$r_{split\ half} = split\ half\ reliability\ coefficient$

In this study, I could not apply Spearman-Brown because of two main reasons: (1) sample size was small, and (2) each questionnaire was divided into four constructs, which resulted in few items per construct. Using few items reduces the reliability score (Gay et al., 2009).

Kuder-Richardson is a special case of Cronbach's alpha and is highly recommended for assessing the reliability of instruments with dichotomous scores (Gay et al., 2009). I could not use it because both instruments had some items with more than 2 scores. Cronbach's alpha coefficient was used to ascertain the internal consistency reliability of the two instruments.

Cronbach's alpha gives a high-reliability score if all items are related or are measuring the same construct (Zaiontz, 2019). In cases where multiple constructs exist in one instrument, it is recommended to calculate Cronbach's alpha for each set of items measuring the same underlying construct (Glen, 2016). In my study, the first questionnaire had four constructs (affection, learning preference, proficiency, and affordance) and the second questionnaire had five constructs (access, easy-to-learn, legibility, satisfaction, and easy to use). Cronbach's alpha coefficient was calculated for each construct.

Reliability was assessed using SPSS. Some statistics from the reliability analysis output are Cronbach's alpha coefficient, Cronbach's alpha coefficient when the respective item is deleted, corrected item-total correlation, inter-item correlations, item mean, and standard deviation. Cronbach's alpha coefficient greater than 0.7 reflects the high internal consistency of the construct (Library). Inter-item correlation is the correlation between pairs of items measuring the same construct. Items measuring the same construct should correlate well together. The corrected item-total correlation tells how much each item correlates with the overall questionnaire. A corrected item-total correlation less than 0.3 indicates that the item may not belong to the questionnaire or construct if the reliability analysis is done per construct, as in this study. The item means and standard deviation of items measuring the same construct are expected to be similar. If Cronbach's alpha coefficient increases when the item is excluded, that may be an indication that the item may not be measuring the same concept as the other items. Excluding it may improve the consistency of the questionnaire. In this research, I considered Cronbach's alpha coefficient to assess the reliability of the constructs.

One of the advantages of using Cronbach's alpha is that it can be used with any number of items. Additionally, it is not sensitive to the distribution of data and can be used for both dichotomous and multi-point questionnaires (Gay et al., 2009). Theoretically, Cronbach's alpha results should give you a number from 0 to 1, but you can get negative numbers as well. A negative number may be an indication of scores that need to be reversed. The general rule of thumb is that a Cronbach's alpha of 0.70 and above is good; 0.80 and above is better; and 0.90 and above is best (Glen, 2016).

Cronbach's alpha does come with some limitations, namely; constructs that have a low number of items associated with them tend to have lower reliability, and sample size can also influence your results for better or worse. Similarly, in this study, some individual constructs had low Cronbach's alpha compared to the respective questionnaires. The low values were accepted because of the sample size constraint. For example, when calculating Cronbach's alpha of a construct with three items, a minimum sample size of 31 is required to achieve Cronbach's alpha of 0.7 (Bujang, Omar, & Baharum, 2018) yet in this study, the sample size was 25.

4.7.3.3 Validity and Reliability of the first questionnaire

The first questionnaire investigated the learners' learning preferences. It was also used to assess the association between: liking mathematics (item 1a) and technology appreciation (item 17); proficiency in using technology (item 16) and technology appreciation (item 17); getting good marks in mathematics (item 2a) and proficiency in using technology (item 16); and getting good marks in mathematics (item 2a) and technology appreciation (item 17). It also helped to investigate the affordance of resources required to use mobile technology in learning mathematics, thus, checking the practicability of using mobile technology in learning mathematics.

Inter-item correlation coefficient and factor loadings were used to confirm the validity of the closed items in the questionnaire. SPSS was used to calculate the correlation coefficients and R was used to calculate the factor loadings.

The common threshold for factor loadings is 0.6. However, in this study, the threshold for factor loadings was set at 0.4 because the sample size was small. An item was accepted as contributing to or evaluating a construct if it had inter-item correlations greater than 0.3, or if the factor loading was 0.4 or more.

4.7.3.4 Validity and Reliability for the second questionnaire

The second questionnaire sought participants' evaluation and perceptions on the usability of GeoGebra applets on mobile technology in the discovery of cases of congruence of triangles. A technological product should comply with the user's ability, needs, and expectations (Torun & Tekedere, 2015; Hariyanto et al., 2020). Therefore, it was essential to evaluate the usability of GeoGebra applets on mobile devices (Torun & Tekedere, 2015; Hariyanto et al., 2020; Pal & Vanijja, 2020). There are two common approaches for evaluating usability; usability testing and user testing (Pal & Vanijja, 2020). Usability testing is done by experts who test the product against a set of rules and principles (Pal & Vanijja, 2020). On the contrary, user testing involves the testing of the product by intended users. Usability is evaluated by having the implementation team observe while the product is being used, or by having the users complete a usability questionnaire, or by interviewing the users after they use/test the product. User testing was used in this study. After completing a task using GeoGebra applets, the participants completed a questionnaire, and seven participants were interviewed. I chose the user testing approach mainly because it involved the intended users. Additionally, this research was guided by the interpretive paradigm which gives more value to the views of the participants. I also opted for user testing because the views of the experts differed from the views of the end-users (Pal & Vanijja, 2020). The five common and basic elements that are considered when evaluating usability are learnability, effectiveness, memorability, errors, and satisfaction (Torun & Tekedere, 2015).

Learnability is concerned with how easy it is to learn to use the app or software. It also assesses whether the app would allow users to finish the task quicker than they would do using other methods. Effectiveness is concerned with the productivity of the product, for example; assessing if using GeoGebra applets would help learners understand mathematics. Memorability focuses on how easy it would be to remember to use a product. Users should not start learning everything over again whenever they want to use the product. The errors aspect is concerned with how easy it would be to correct a mistake. Then the satisfaction aspect focuses on whether the users appreciate and enjoy using the product.

The second questionnaire evaluated five constructs that I named, access, easy-to-learn, legibility, satisfaction, and easy-to-use. The validity and reliability were assessed per construct. Kendal's tau b correlation coefficient and factor loadings were used to confirm the validity of the closed items in the questionnaire. SPSS was used to calculate Cronbach's alpha and correlation coefficients. Both coefficients were rounded off to one decimal place. The correlation coefficient above 0.3 implied that the pair of items were measuring the same construct. The same threshold for Cronbach's alpha (0.5) was used for the second questionnaire. Factor loadings were calculated using AMOS and R. The two software were used to confirm the validity of the items. Factor loadings from R are in table format and results from Amos are presented in a path diagram. A path diagram shows the items, the construct, and the factor loadings. The loadings are indicated on the arrows that link the constructs to the items. Factor loading greater than or equal to 0.4 implied that the item was contributing to the evaluation of the construct (Rahn, 2018).

This approach to data generation is not without criticism. It assumes that the researcher and respondents share underlying assumptions about language, and similarly interpret statement wording. The following sections explain how the collected data were analyzed.

4.7.4 The first questionnaire focus and analysis strategy

As mentioned in the preceding paragraph, the content of the questionnaire must be in line with the research items. Following this, the first questionnaire probed the learners' learning preferences, affection towards mathematics, current use of technology during the teaching and learning process, mobile technology appreciation, technology proficiency and affordance of mobile technology resources (devices and data), and views on how they think they should be taught and learn mathematics to enhance understanding.

In designing the first questionnaire, I modified some of the items from previous research studies, and from instruments that had been validated and used before. Since not all items were adopted from previous research, it was, therefore, necessary to assess validity and reliability. Confirmatory factor analysis was used to assess construct validity, while reliability was confirmed using Cronbach's alpha. This is discussed in detail in section 4.5.8.

The first questionnaire had 23 closed-ended items and 3 open-ended items. Out of the 23 closed-ended items, items 1 and 2 were filter items. A filter item is an item administered to determine if the respondent meets the criteria required to answer the next item (Verhoeven, 2011). Even though I had filter items, it is not recommended to use filter items that will be analyzed quantitatively (Starkweather, 2014). The first parts of these items were captured as 1a and 2a. Similarly, the corresponding filtered parts were captured as 1b and 2b. 1b was a closed-ended item, while 2b was an open-ended item. It could have been better if 1b was constructed as an open-ended item. The reason is that the responses I provided were not exhaustive and one respondent ended up writing her response because she could not get the response she wanted from the given list.

Item 1b was to be answered by students who indicated that they did not like mathematics. They were required to indicate whether this negative feeling was because they just hated mathematics, or they felt mathematics was not for them, or they did not understand explanations in mathematics. These three responses were constructed using the phrases that I used to hear the students saying during my period as a mathematics teacher. For that reason, I considered item 1b to be valid since it was constructed based on past experiences.

Item 2b was to be answered by participants who indicated that they get low marks in mathematics. Items 1a, 1b, 2a, 3, and 4 focused on learners' affection towards mathematics. The construct was validated using confirmatory factor analysis. Even though the loadings were low, the items were accepted as assessing the construct because they were adopted from previously validated instruments. These items were analyzed using bar graphs and proportions.

Since the focus of this research was exploring learners' technological proficiency towards using mobile technology in the teaching and learning of mathematics, I found it essential to check if there was an association between the following pairs of items: liking mathematics (item 1) and technology appreciation (item 17); as well as proficiency in using technology (item 16) and technology appreciation (item 17). Additionally, I assessed the association between getting good marks in mathematics (item 2a) and proficiency in using technology (item 16); and getting good marks in mathematics (item 2a), and technology appreciation (item 17). SPSS was used to assess these associations. Among the SPSS cross-tabulation results are the Chi-square and Fisher's exact statistics and the percentage of cells with theoretical frequencies less than 5. If the percentage of these cells is greater than 20, the chi-square test is considered to be violated, and Fisher's exact statistic should be used. Fisher's exact test can be used for any sample size (Aprameys, 2019). For Chi-square results to be valid, the theoretical frequencies should be at least 5. For this reason, I used Fisher's exact test since the percentage of cells with theoretical frequencies exceeded 20% in all cases. The Chi-square and Fisher's exact test of independence was used to affirm the null hypotheses that the items were not associated in cross-tabulations.

The nature of the first questionnaire data fulfilled the basic requirements for performing the Fisher exact test. The data was categorical and had two categories for each item. The participants' responses were independent and the category responses were not paired. The null hypothesis for Fisher's exact test is that there is no association between the items, such that the probability of selecting a response for one item is not influenced by the response of the other item (Freeman & Campbell, 2007; Aprameys, 2019). The null hypothesis was rejected for p-values less than the statistical significance ($\alpha = 0{,}05$). The test also produced clustered bar charts of the pairs of respective items. Bars of approximately the same height within a category insinuate that there is no association between the items (University K. S., 2021).

As mentioned earlier, learning styles affect conceptual understanding (Kusedade, Degeng, & Nur Ali, 2019). One of the indications of understanding is the student's active engagement during the learning process (Kade et al., 2019). This is displayed when students ask questions, participate in discussions, and share ideas with teachers or other classmates (Kade et al., 2019). Following this, items 5, 6, 7, 8, 9, 10, 12, 13, 14 assessed the learning-style construct. All these items were dichotomous except for items 6 and 7. This posed a challenge during validation using factor analysis, which involved the calculation of correlations between pairs of item responses. These calculations require items that have the same number of levels, that is; the same number of possible responses.

Items 6 and 7 were, therefore, validated separately. This research proposes the use of GeoGebra applets on mobile technology in the teaching and learning of mathematics. I found it essential to consider the learners' desire to use technology. This was the focus of items 17 and 18. As mentioned earlier in this section, the focus of this questionnaire was on the research question: How can we explore learners' technological proficiency towards using mobile technology in the teaching and learning of mathematics, to enhance the understanding of congruence? In alignment with this, items 16, 19, 20, 21, and 22 explored the learner's proficiency in using mobile devices and affordances. Items 21, 22, and 23 were also used for validity checking of item 19.

The open-ended item 2b was supposed to be answered by participants who were getting low marks in mathematics. It was included to check if some of the reasons for getting low marks that were mentioned could be alleviated using mobile technology in learning mathematics. The open-ended item 24 was included to probe learners' perspectives on what teachers could do to enhance their understanding of mathematics, to see whether the use of technology is in the learners' minds. The third open-ended item required the participants to state the type of phone they owned. The type of mobile device mentioned in the first questionnaire was compared with the type of mobile device that was used to complete the investigation task.

An inductive thematic approach was used to analyze these open-ended items. This is a bottom-up approach dependent on the data (Braun & Clarke, 2012). Codes and themes were derived from the content of the responses. Concurrent validity was assessed by checking if responses to the item that requested suggestions on how teachers could enhance understanding of mathematics (item 24) were addressing the reasons for not getting good marks stated in item 2b. This was done for responses of participants who were getting low marks because they are the ones who answered item 2b.

Descriptive statistics were used to analyze the data from closed-ended items. Clustered bar graphs and percentage frequencies were used to represent responses from closed-ended items. Items that were not included in any construct were analyzed separately, while the items that were included in specific constructs were jointly analyzed. That is, clustered bar graphs were constructed for the three constructs (learning-style, proficient, and affordance) and the frequencies were calculated. The results were used to affirm and validate some of the responses to the open-ended items.

4.7.5 The second questionnaire focus and analysis strategy

It was assumed that the participants were never exposed to the teaching and learning of mathematics using GeoGebra applets on mobile devices before. Therefore, they were given a task to investigate the four cases of congruence of triangles using designed applets on a mobile device in their possession. The second questionnaire was administered after the participants completed the task.

The second questionnaire focused on the learners' evaluation of the usability of GeoGebra applets on mobile devices in teaching and learning mathematics. Additionally, it investigated the learners' views on the benefits of using GeoGebra applets in the learning of mathematics, and the anticipated challenges thereof. The second questionnaire had 28 closed-ended questions and two open-ended items. In line with the second research question, participants were required to evaluate the usability of the applets. The usability of the applets was evaluated using the 10-point numeral rating scale with verbal anchors (strongly disagree and strongly agree) at the extreme ends on each of the 28 closed-ended items. The open-ended items explored both the usability of GeoGebra applets and anticipated challenges thereof. The usability of the applets was evaluated using five constructs: access, easy-to-learn, legibility, easy-to-use, and satisfaction. These constructs are discussed in the next paragraphs.

Items 1, 2, and 3 were proposed to measure the access construct. Item 1 focused on the aspect of logging into the page with the workbook, while item 2 probed whether it was easy to open the workbook, and item 3 was concerned with the speed of opening the applets. It was considered that if participants could easily log in to and open both, workbooks and applets, then I could conclude that it was easy to access the applets.

Items 4, 5, 9, 11, 15, 16, 17, and 18 were used to measure the easy-to-use construct. An individual who has proficiency in using mobile technology is expected to use it comfortably, easily, and to quickly learn new applications on the device, and quickly develop expertise in using new software. Similarly, if an individual is proficient in doing something, it is expected that they do it easily and with confidence. Therefore, the reason why items 17 and 18 were included was to check whether the participants were comfortable and confident while using the applets. Item 4 checked the effectiveness of the touch screen. This item was included with the assumption that all participants would use touch screen devices. In this study, a touch screen device refers to a mobile device that accepts input on the screen. I hoped that participants would not put a response on item 4 if they did not use a touch screen device. Since there was no missing data, it was, therefore, assumed that all participants used a touch screen device to access the applets that were used to complete the activity. Item 5 assessed how easy it was to move from one applet to the other, item 9 rated how easy it was to use the applets, while item 11 assessed how easy it was to undo a mistake. Items 14 and 15 checked whether it was easy to get the required answers from the applets, that is; the missing lengths and angles. Item 16 assessed the speed at which the activity was completed using the applets. Item 17 probed whether the participants were comfortable in using the applets, while item 18 asked if they were confident in using the applets. Item 6 checked on whether the participants felt that their cellphone navigation skills helped them to understand how to use the designed applets.

If an individual is well acquainted with using application software, it would be easy to learn new software. The easy-to-learn construct was initially measured by the items 7, 8, 9, 10, and 12. Item 7 checked how easy it was to learn to use the applets. Item 8 checked if it was easy to remember using the applets. Item 9 checked whether it was easy to use the applets, and item 10 enquired whether one considered themselves skillful in using the applets. Item 12 probed participants to rate their prediction on whether learners in the same grade would find it easy to use the applets. However, after validation checks, item 9 was dropped because it gave rise to an ultra-Heywood case, that is; a factor loading greater than 1.

Items 19, 20, and 21 were proposed to measure the legibility construct. Item 19 was concerned with the display of the triangles on the screen while items 20 and 21 checked on the visibility of the triangle labels and measurements respectively. It was hoped that if learners could see the triangle and all the writings on the triangle then it can be concluded that the applets were legible on the devices that were used.

For a user to be satisfied with the software, it should meet their expectations. This is exhibited when the software is accurate and gives expected solutions. To show satisfaction users can recommend the software to others. Items 22, 23, 24, and 25 were used to measure satisfaction. These items were selected after validation assessment. Item 22 probed on whether the applets gave correct expected answers, while items 25 and 26 checked on learners' evaluation on whether the applets would help understand and identify cases of congruence respectively. Item 13, 23, 24, and 28 probed the participants' acceptance of GeoGebra applets on mobile devices. Item 13 enquired whether the participants enjoyed completing the activity using the designed applets. Item 23 evaluated the satisfaction of participants on the applets that they used, while item 24 assessed how likely they would recommend the applets that they used to their friends. Item 28 checked if they would like to use mobile technology more often in learning mathematics. Item 29 asked for the type of mobile device that the participants used to complete the worksheet. This was essential to include because this research is guided by the FRAME model which stipulates that the device used plays a role in the successful implementation of mobile learning (Koole & Ally, 2006). Open-ended item 30 explored the learners' perspectives on the negative aspects of using GeoGebra, while item 31 focused on the positive aspects.

4.7.5.1 Analysis strategy

Since I did not use a standardized questionnaire, I used factor analysis to assess validity. Similarly, Cronbach's alpha was used to assess reliability. Factor analysis and reliability are explained in detail in section 4.7.3. Descriptive statistics were used to analyze data from the second questionnaire. The analysis of the closed-ended items involved four steps. First, I calculated the mean for each item of the construct. Second, I calculated the overall mean for each construct. The overall mean for each construct was calculated by averaging the mean of all the items measuring the construct. Third, I calculated the frequency for each score of the respective items. Since this research was guided by the interpretive paradigm, every response from every participant was important. I also checked the responses to the open-ended item 30 for the participants who did not score a 10 on an item to see if their responses included challenges related to the respective item. The responses to open-ended item 31 and interviews were used to support the results from the closed-ended items.

The mean and the frequency of the scores were used to describe the distribution of the responses for each item. The numerical rating scale that I used consisted of numbers 1 to 10 with strongly disagree or strongly agree anchored on both ends. For example, item 1 was, "It was easy to login". The participants were supposed to choose a numerical score from 1 to 10. I considered not scoring a 10 to indicate the existence of some challenge on the respective aspect. To interpret these scores, I applied the idea that generally, a mean score of 50% is considered a pass or success. Similarly, some researchers consider 50% as an acceptable limit of a positive evaluation (Hariyanto et al., 2020). In this research, 5 was considered as the minimum positive evaluation on an item. Below 5 would indicate unacceptable or unsatisfactory or a negative response to the item. I considered a score below 10 to indicate the existence of some negative sentiments or challenges on the respective item. I also calculated the percentage frequency of the scores for each item. On these frequencies, I was interested in the percentage of participants who scored a 10 on each item. Percentage frequency of the maximum rating below 50% meant that participants had high negative perspectives on the item, percentage frequency from 50% to 69% indicated that a fair number of participants were satisfied, 70% to 79% indicated higher satisfaction, and 80% and above was indicative of very high positive rating with very few negative sentiments on the item. Regardless of the value of the percentage frequency, I checked the responses to the open-ended item 30 of the participants who gave a rating score of less than 10 on a respective item to see if they made any comments concerning the respective item that could help identify the reason for not giving the maximum rating. I also checked for related positive comments from the responses to the open-ended item 31 of the participants who scored a 10 on the respective item. In addition to the mean and the percentage frequencies of the scores on each item, the mean for each construct was calculated using the scores of all items included in the respective construct. The mean of the constructs was interpreted as follows: below 5 (construct not acceptable), 5 to 6.99 (fair positive rating), 7 to 8.99 (good positive rating), 9 to 10 (high positive rating).

Item 29 required the participants to indicate the type of device that they used to complete the task. This was used to investigate if there was any device effect on some of the responses, for example; on the items that were asking the participants if it was easy to access and open the applets.

Correlations between items in each validated construct were calculated. Positive correlations implied that the items contribute towards the construct in the same direction, while negative correlation implied that the items had opposite effects on the construct and were excluded from the construct. The correlation coefficients greater than 0,3 were considered to support the notion that the items measured the same construct.

NVivo was used to perform the thematic analysis for the responses to two open-ended items, 30 and 31. Item 30 probed the anticipated challenges when using GeoGebra applets, and item 31 was focusing on the usability of the applets. Initially, codes were automatically detected. Codes are common words or phrases that appear in the data set, that are then used to group responses expressing the same or similar ideas. Besides automatic detection of the codes, I also manually selected more codes by looking at the familiar words and phrases in the responses. The detected codes were then used in the text search queries to identify the participants who had the same or similar words in their responses. The text search query also picked similar words. For example, if the search word was fast, then the text search query would also include results with words like quick, speedy, immediate, prompt, etc.

4.8 Data collection from Interviews

To further probe the usability of the proposed applets, six participants were interviewed. An interview is a process of constructing knowledge through the interaction between the interviewer and the interviewee (Creswell & Poth, 2018). It can also be described as an attempt to understand the world from the subjects' point of view, to unfold the meaning of their experiences, and to uncover their lived world (Creswell & Poth, 2018). In this study learners' perspectives on using GeoGebra applets on mobile devices, were sought. Therefore, knowledge was constructed between the researcher and the learners. This is in line with research questions two and three in this study. Research question two focused on learners' views on the benefits of using GeoGebra applets on mobile devices in the learning of mathematics. Research question three focused on the predicted challenges cited by learners on learning mathematics using GeoGebra applets on mobile devices. In both instances, participants' points of view based on the experiences of using GeoGebra applets and mobile devices were sought.

Researchers commonly use structured and semi-structured interviews. A structured interview is a type of quantitative interview that makes use of standardized sequencing of questioning to gather relevant information about research subjects (Blog, 2020). However, in a semi-structured interview, the researcher uses predetermined questions but has a leeway to divert from those questions or ask more questions depending on the participant's response but still maintaining the scope of the research. I used a semi-structured interview since participants were asked the same set of questions and in the same order.

An interview can be classified into three: face-to-face, telephone, and survey questionnaire interviews. I used face-to-face interviews which allow for the collection of more in-depth, detailed, and more accurate information from the participants. It is documented that face-to-face interviews are time-consuming, may be costly as the researcher may incur traveling costs to meet the participants, and is limited to small samples. The issue of cost was alleviated in this research because I conveniently selected the school where I was working at the time of the research. Therefore, I did not incur traveling costs to meet the participants since the interviews were conducted during break times while I was at work. I never made a trip that was only dedicated to meet the participants. I only interviewed 6 people, therefore the time constraint was also limited. The face-to-face interview is also susceptible to social desirability bias. This bias occurs when the participants give responses that they think the researcher wants, or responses that they think the public would want, instead of giving their own opinions. To minimize social desirability, I encouraged the participants to give genuine answers and made it clear that genuine responses would help come up with more reliable research answers.

Interaction between interviewer and interviewee can take place in different ways (Creswell & Poth, 2018). Interviews can be one-on-one, focus group, or interaction in writing. A variation of a one-on-one interview is where both interviewer and interviewee are located in the same room, talking face-to-face using technology, or talking over the phone (Creswell & Poth, 2018). In this study, 6 participants volunteered to take part in the interview. One-on-one interaction in the same room was employed, except with one participant who opted to write her responses. One of the drawbacks of this method arises when a researcher is in the presence of a shy interviewee. In this instant, the researcher may get less than adequate data. However, the method also comes with some advantages. It is easier to reschedule appointments and participants do not feel intimidated or influenced by other participants' presence or responses. In the next paragraph, I present the stages involved in interviewing as described by Creswell and Poth (2018).

First, the open-ended questions used should be focused on answering the central phenomenon of the study. In other words, they should give answers to postulated research questions. This was considered in this research where eight open-ended items were asked.

To answer the research question: What are learners' views on the benefits of using GeoGebra applets on mobile devices in the learning of mathematics? The following questions were asked: What can you say about the mobile strategy that you used to complete the worksheet? What would you consider as advantages of using mobile technology in learning Mathematics? What did you enjoy the most when you were completing the worksheet? What would you consider as advantages of using mobile technology in learning Mathematics? What improvements or alternatives on the applets do you wish to suggest? Are there other maths sections in which you think mobile technology may assist in learning and understanding? Is there anything else you would want to share or say about using mobile technology for learning Mathematics?

Similarly, the research question on the challenges that learners could face when using GeoGebra applets for learning mathematical content was answered through the following questions: What would you consider as challenging in using mobile technology for learning Mathematics? Are there other challenges you think fellow learners would encounter when using mobile technology?

Second, after constructing open-ended items directed towards specific research questions, interviewees were identified. Even though the participants had indicated in the consent letter that they wouldn't mind taking part in an interview, I asked them again if they were still willing to be interviewed. Only 6 participants volunteered to be interviewed, and among them, one participant opted to write down the responses.

Third, the type of interview should be determined considering the nature of the research. I employed a one-on-one interview in a classroom that was free from distractions.

Fourth, adequate recording procedures should be used to collect data. In this study, I used my smartphone for voice recording. Fifth, it is advisable to design and use an interview protocol or interview guide. I used 8 open-ended questions.

The interview focused on two research questions. It probed participants' views on using GeoGebra applets on mobile devices and anticipated challenges. The interviews were administered on the agreed dates and times after the participants submitted the second questionnaire. I had hoped to interview 10 participants, but unfortunately, six were willing to be interviewed and the other participant opted to put the responses in writing. The participant is referred to as WR (meaning written response).

The results from interview data were used to affirm the results from the post questionnaire. Thematic analysis was used to analyze interview data. The responses were coded and extracts affirming usability of the applets were identified.

Chapter Five Results and Data Analysis

This chapter presents the results and describes the analysis of data. Included in the chapter are the results for the validity and reliability assessment. Data were obtained from two questionnaires and interviews. The first questionnaire was administered before participants used GeoGebra applets. The second questionnaire was administered after participants had completed a task on investigating cases of congruence using GeoGebra applets on mobile devices. The response rate for both questionnaires was 100%. A face-to-face semi-structured interview was administered to 6 participants. The 7th participant opted to put the responses of the interview in writing. I use WR to represent written responses. The words mean and average are used interchangeably.

5.1 Assessment of reliability and validity for the first questionnaire

The first questionnaire had four constructs. The reliability and validity were assessed per construct and the results are presented in this subsection.

5.1.1 Affection construct

The affection towards mathematics was measured by items 1a, 2a, 3, and 4. The correlations and factor loadings are shown in Tables 7 and 8. From the results in Table 8, item 1 resulted in an ultra-Heywood case (factor loading greater than 1) and item 2 had a very low loading. I, therefore, used the correlation coefficients to establish the validity of the items measuring the affection construct. The correlations of items 1 (*I like Maths*), 2 (*I get good marks in Maths*), and 3 (*I feel comfortable when learning mathematics*) were positive, thereby confirming that these items were measuring the affection construct. Item 4 (*I understand the importance of learning Maths in my life*) was negatively correlated with item 1 and positively correlated with items 2 and 3. Considering the two positive correlations, I concluded that item 4 also contributed towards measuring the affection construct. The negative correlation is explained by the fact that 28% of the participants did not like mathematics even though they knew the importance of learning mathematics.

Table 7

The correlation coefficient between items of affection construct

Inter-Item Correlation Matrix

	Q1a	Q2a	Q3	Q4
Q1a	1.000	.442	.531	-.221
Q2a	.442	1.000	.402	.241
Q3	.531	.402	1.000	.022
Q4	-.221	.241	.022	1.000

Table 8

The factor loadings of items on construct affection

Item	Factor Loadings
Q1	0.79
Q2a	0.58
Q4	-0.05

5.1.2 Learning preference construct

Items 5, 6, 7, 8, 9, 10, 12, 13, and 14 were initially proposed to measure the learning preference construct (see questionnaire 1 in Appendix A). The respective correlations and factor loadings are shown in Tables 9 and 10. Items 7, 9, and 13 were negatively correlated with the other items. Considering this, and looking at the fact that logically, learners with different learning preferences are likely to enjoy answering the easy items, item 13 was excluded from measuring the learning preference construct. Some learners may be shy to go and present solutions on the board even though they prefer learning with understanding. Therefore, I excluded items 7, 9, and 13 from measuring the learning preference construct.

Table 9

Correlation coefficient between items of learning preference construct including item 13

Inter-Item Correlation Matrix

	Q5	Q6	Q7	Q8	Q9	Q10	Q12	Q13	Q14
Q5	1.000	.272	-.123	.306	-.068	.204	.238	-.408	.076
Q6	.272	1.000	.082	.226	.132	.167	.215	-.375	.164
Q7	-.123	.082	1.000	-.120	.396	-.264	.232	-.113	.120
Q8	.306	.226	-.120	1.000	.031	.187	.164	-.421	.316
Q9	-.068	.132	.396	.031	1.000	-.042	.215	.042	.164
Q10	.204	.167	-.264	.187	-.042	1.000	.250	-.250	.281
Q12	.238	.215	.232	.164	.215	.250	1.000	-.042	.031
Q13	-.408	-.375	-.113	-.421	.042	-.250	-.042	1.000	-.047
Q14	.076	.164	.120	.316	.164	.281	.031	-.047	1.000

Table 10

Factor loadings of items on construct learning preference

Item	Factor Loadings
Q5	0.92
Q6	0.21
Q7	0.64
Q8	0.83
Q9	0.24
Q10	0.59
Q12	0.86
Q13	-0.79
Q14	0.11

Finally, items 5 (*I prefer memorizing Maths concepts and formulae without understanding*), 6 (*I enjoy hearing my Maths teacher explain new concepts*), 8 (*I enjoy hearing the thoughts and ideas of my peers in maths class*), 10 (*If I get wrong answers I always want to identify my mistakes*), 12 (*I am interested in discovering cases of congruence*), and 14 (*I feel confident in my ability to answer questions that say: Prove or show*) were used to measure the learning preference construct. The correlation coefficients and factor loadings are shown in Tables 11 and 12. The correlation coefficients were positive. Similarly, the factor loadings were positive and the smallest loading was 0,4; confirming that the items contributed to measuring the learners' preference construct.

Table 11

Correlation coefficient between items of learning preference construct exdcluding item 13

Inter-Item Correlation Matrix

	Q5	Q6	Q8	Q10	Q12	Q14
Q5	1.000	.272	.306	.204	.238	.076
Q6	.272	1.000	.226	.167	.215	.164
Q8	.306	.226	1.000	.187	.164	.316
Q10	.204	.167	.187	1.000	.250	.281
Q12	.238	.215	.164	.250	1.000	.031
Q14	.076	.164	.316	.281	.031	1.000

Table 12

Factor loadings of items on construct learning preference excluding item 13

Item	Factor Loadings
Q5	0,50
Q6	0,51
Q8	0,80
Q10	0,75
Q12	0,40
Q14	0,81

5.1.3 Proficiency construct

Proficiency was measured by items 16 (*I am proficient in using mobile technology*), 17 (*I embrace/appreciate the use of mobile technology in learning*), and 18 (*Mobile technology may enhance our learning of mathematics*).

The correlation coefficients and the factor loadings of the items measuring the proficiency construct are shown in Tables 13 and 14. The correlation for item 18 gave a #DIV/0! error because the responses were the same. All participants indicated that mobile technology would enhance the learning of mathematics. The correlation coefficient between item 17 and item 16 was almost zero. This is logical because, if technology proficiency was correlated with technology appreciation, that would imply that the use of technology would only be embraced by technologically proficient learners, yet there is room to learn and also become proficient. All the 3 items were highly loaded on the construct. Considering the factor loadings, I concluded that the three items measured the proficiency construct.

Table 13

Correlation coefficient between items of proficient construct

Inter-Item Correlation Matrix

	Q16	Q17	Q18
Q16	1.000	-.042	NA
Q17	-.042	1.000	NA
Q18	NA	NA	1.000

Table 14

Factor loading of items on proficient construct

Factor Loadings	
Q16	0.96
Q17	0.96
Q18	1.04

5.1.4 Affordance construct

The affordance construct was measured by items 19, 20, and 21. The correlation coefficient and factor loadings are shown in Tables 15 and 16. Item 19 (*We can afford gadgets required for mobile learning interventions*) had the highest loading. Item 21 resulted in an ultra_Heywood case. The three items were positively correlated which implied that they all contributed positively to the affordance construct.

Table 15

Correlation coefficient between items of the affordance construct

Inter-Item Correlation Matrix			
	Q19	Q20	Q21
Q19	1.000	.07	.41
Q20	.07	1.000	.24
Q21	.41	.24	1.000

Table 16

Factor loadings for items of the affordance construct

Factor Loadings	
Q19	0.91
Q20	0.29
Q21	1.03

Cronbach's alpha coefficient was used to assess the reliability of each construct. SPSS was used to calculate Cronbach's alpha coefficient. The coefficients are shown in Table 17.

Table 17

Reliability coefficients for the first questionnaire

Construct	Cronbach's alpha
Learners' affection	0.6
Learners'learning preference	0.6
Proficiency	0.7
Affordance	0.5

The common threshold value for Cronbach's alpha is 0.7. Because of the sample size constraint in this study, the threshold was set at 0.5. The reliability coefficients of the four constructs were 0.5

or more. I concluded that the first questionnaire was reliable and valid. Thus, the data were sufficient to establish the participants' affection towards mathematics, their learning preference, technology proficiency, and affordance of mobile technology resources (devices and data).

5.2 Learners' technological proficiency when using mobile technology in the learning of mathematics to enhance conceptual understanding

The first questionnaire provides data for this section. The following constructs were considered: affection towards mathematics (which was assessed by items 1a, 2a, 3, and 4), learning style (which was measured using items 5, 6, 7, 8, 9, 10, 12, 13, and 14), proficiency and appreciation (which was measured using items 16, 17, and 18), and affordance (which was measured using items 19, 20, and 21). Item 1b was answered by the participants who did not like mathematics, and item 11 asked if the participants could easily identify cases of congruence. Learning style was considered to check if the present practices of learners showed the desire and effort to understand mathematics. This was per the learner aspect of the FRAME model which focuses on how learners wish or need to accomplish a task (Koole & Ally, 2006; Koole, 2009; Asiimwe et al., 2017). It is necessary to assess learners' expectations to avoid coming up with teaching and learning strategies that they may not embrace.

Proficiency and appreciation constructs were included to find out if the participants were proficient in using mobile technology. I believe that technologically proficient learners have less anxiety about using mobile devices and have enhanced psychological comfort. The FRAME model emphasizes the psychological comfort of learners while using mobile technology (Koole, 2009). Leaners who are comfortable with using technology complete tasks quicker because they concentrate on doing the task rather than on how to operate the device. The same construct also probed learners' willingness to use mobile technology in learning mathematics, and if they thought it could enhance their understanding of mathematics. This construct emerged from the FRAME model aspect that emphasizes the consideration of learners' wishes and needs required to succeed or complete a task (Koole, 2009; Koole et al., 2010).

The affordance construct was included to check whether the learners would afford the devices and data that would be needed to use GeoGebra applets in learning mathematics. According to the FRAME model, affordance is one of the aspects that should be put into consideration to ensure the successful implementation of mobile learning.

The descriptive statistics of closed-ended questionnaires are given in the following subsections. Open-ended items 2b and 24 were analyzed using NVivo.

5.2.1 Learners' affection for mathematics

Learners' beliefs and attitudes are of paramount importance in the construction of new knowledge (refer to section 2.4.1). Hence, the reason for considering the item that asked whether the learners liked mathematics.

Results for item 1a (*I like mathematics*) showed that 64% of the participants liked mathematics while 36% indicated that they did not like mathematics. Item 1b (*I do not like mathematics because*) was a filter item, which was answered by participants who had indicated that they did not like mathematics in response to item 1a. Consequently, 64% of the participants did not respond to item 1b. This synchronism added to the validity of the data since participants who liked mathematics were not supposed to respond to item 1b. The distribution of the responses was as follows: 22,2% of participants who did not like mathematics also indicated that they hated mathematics, 33,3% chose an option that mathematics was not meant for them, while 44,4% selected an option that they did not like mathematics because they did not understand explanations. These results show that negative affection towards mathematics was mainly caused by a lack of understanding. This supports the need for teaching and learning strategies that enhance conceptual understanding. In my teaching career, I noted that students tend to like a topic or a subject that they understand more. In line with this notion, Kade et al. (2019) indicated that learners' affection towards a subject influences their success in the respective learning area. Relatedly, Confessore and Park (2004) state that students are likely to work hard, persevere and set goals to get high marks if they like the subject.

Item 2a (*I get good marks in Maths*) probed learner performance in mathematics. Only 40% of the participants indicated that they were getting good marks in mathematics. The other 60% were asked to give reasons why they were not getting good marks in mathematics. I labeled this as item 2b (*I do not get good marks because*). The responses were coded, and three constructs were extracted: lack of understanding, negative attitude, and application challenges. The text search facility in NVIVO was used to locate the extracts that were related to each construct. In all three cases, the generalization option was applied. The words ‘understand’ and ‘concentrate’ was used as the codes for the lack-of-understanding construct. The words ‘hard’ and ‘hate’ were used as codes for the negative-attitude construct. The words ‘tricky’, ‘challenging’, ‘long’, and ‘different’ were used as codes for the application challenges construct.

The responses to lack of understanding and concentration during mathematics lessons were echoed by some participants. Participant 6 indicated that she sometimes had challenges understanding the teacher’s approaches. She stated, “*The teacher that is teaching maths, I sometimes fail to understand his/her teaching*”. This statement reflects that there is a need for integrating other teaching strategies to enhance understanding. A similar sentiment was shared by participants 15, 16, 19, 20, 23, 24, and 25.

Participant 15 stated, “*…I sometimes do not understand the formula…”* This response shows one of the reasons why the participant and other learners in general, got low marks in mathematics. Most learners end up memorizing what they do not understand. It seems that learners would not be able to apply memorized formulae to answer unfamiliar and complex items. This is in line with the conclusion made by Wiest and Amankonah (2019). I believe that learners would understand and remember more if they took part in deriving the formulae. This can be achieved through the use of mobile technology.

Participant 16 said, *“...because I can't concentrate and I don’t understand explanations …”* It is evident in this response that the participant struggled with concentration which is caused by the lack of understanding. This is the root cause of indiscipline during mathematics lessons (Silva et al., 2017; Mji & Makgato, 2006). The use of GeoGebra applets on mobile technology would boost learner concentration and enhance understanding of concepts. This was alluded to by Žilinskiene and Demirbilek (2015).

Participant 19 stated, "*I don't understand and I do not try hard enough."* This response reflects the need for teaching strategies that arouse a learner's interest in the subject. The use of GeoGebra applets on mobile technology would provide a platform for learners to explore and engage more with the content. Responses by participants 20, 23, 24, and 25 further support the need for strategies that enhance understanding. Their responses are indicated in the following extracts:

Participant 20: *"…I don't understand…"*
Participant 23: *"…I don't get good marks in maths because I don't understand it…"*
Participant 24: *"…I do not understand most of the time…"*
Participant 25: *"…I sometimes don't understand certain aspects of Maths when my teacher " explains it…"*

Related to understanding, participant 7 stated, "…*geometry is hard*…" and participant 13 said, *"It is not always easy to see the stuff".* Participant 17 stated, "*Some sections are difficult."* Comparably, participant 18 stated, "*I am very bad with algebra and expressions…"* It is evident from these statements that there is a need to bring the concepts closer to the learners in ways that help them visualize the concepts and understand more.

The use of mobile technology would help learners visualize content that is difficult to see (Andraphanova, 2015; Polásek & Sedlácek, 2015). This could be achieved through using GeoGebra applets on mobile devices.

Participant 9 said that she did not understand the items that were asked in assessments. Participant 25 echoed the same sentiment and stated, " ..*sometimes what the exam asks is different from what I had learned as the questions can be tricky and challenging."* Participant 12: *"…Sometimes the questions are too long and I stop in the middle…"* Usually in assessments, the first subquestions are routine questions (level 1 and level 2) This response reflects that participant 12 was comfortable with answering easy subquestions but had challenges answering complex subquestions. I relate the challenge of answering complex and unfamiliar questions to the lack of understanding of formulae and concepts. As was alluded by Wiest and Amankonah (2019), students with a lack of understanding of formulae and concepts struggle to answer unfamiliar and complex questions. Therefore, the use of mobile technology may help explore formulae and concepts and increase conceptual understanding.

Some participants expressed a negative attitude towards mathematics. This is evident from the following excerpts:

Participant 16: *"…Maths is not for me and I also hate it…"*

Participant 18: *"…I am very bad with algebra and expressions, I hate it as if it won't help me in future…*"

Participant 19: *"…I do not try hard enough…"*

It is evident from these expressions that these students had lost hope. There is need to explore teaching strategies that arouse students' interest in the subject, especially strategies that incorporate technology to match the digital nativeness of the current generation of learners. This makes the proposed strategy of using mobile technology in the teaching and learning of mathematics more relevant.

On the same item, some participants gave responses that showed positive affection towards mathematics. These responses are expressed in the following extracts:

Participant 14: *"I do not get bad marks nor do I get good marks, I get average marks, which do not satisfy me."*

Participant 22: *"I don't get good marks*, not bad marks. *I get average mark."*

The findings from the responses to item 2b (*I do not get good marks because*) support the notion that students with lack of conceptual understanding struggle to answer unfamiliar or complex questions (level 3 and level 4 questions). These results are directly in line with what Wiest and Amankonah (2019) concluded in their research. Therefore, there is a need for teaching and learning innovations that actively engage the learners to improve their concentration and understanding.

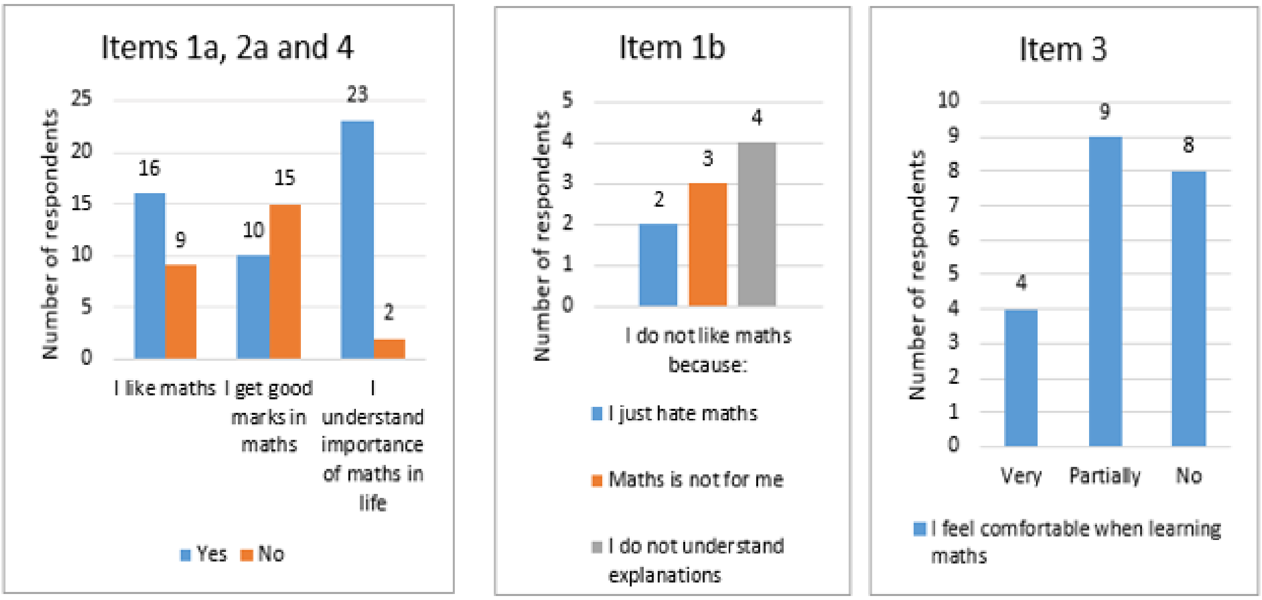
To further check learners' affection towards mathematics, item 3 (*I feel comfortable when learning Maths*) probed whether learners felt comfortable during mathematics lessons. In response to this, 16% indicated that they felt very comfortable when learning mathematics, while 72% felt partially comfortable, and 12% did not feel comfortable. These results suggest that some of the teaching strategies being implemented may be contributing to the negative affection towards mathematics. This view is consistent with Kade et al.'s (2019) findings where they established that improper teaching strategies demotivate students towards the subject.

There is a need for teaching strategies that promote a conducive and comfortable environment for learning mathematics. The FRAME model stipulates that mobile learning provides a different kind of learner-content interaction, which boosts motivation and stimulates learning. Thus supporting the proposed use of GeoGebra applets on mobile devices.

Item 4 (*I understand the importance of learning Maths in my life*) was used to check if the learners knew the importance of learning mathematics. Most participants (92%) indicated that they understood the importance of mathematics in life, while 8% did not know the value of mathematics in life. Considering that understanding the importance of the subject in the future boosts intrinsic motivation and interest to learn it (Froiland et al., 2012; Confessore & Park, 2004), I expected these results to be similar to the results for Item 1a (*I like mathematics*). On the contrary, the percentage of participants who liked mathematics was 64%, which was less than 92%. This implied that 28% of the participants knew the importance of mathematics but still did not like it. This result further supports the implementation of teaching strategies that boost learners' motivation towards mathematics. This supports the proposed use of applets on mobile technologies in teaching and learning. Clustered bar graphs in Figure 15 show the responses for items 1a, 1b, 2a, 3, and 4.

Figure 15

Responses to items for the affection construct



5.2.2 Learners' learning style.

Items 5, 6, 7, 8, 9, 10, 12, 13, and 14 were included to probe learners' learning styles. Results for these items are shown in Figure 16.

Several participants (60%) preferred understanding the concepts to memorizing them. This percentage reflects that many participants preferred learning with understanding. Therefore, they were more likely to embrace teaching and learning strategies that they think may enhance understanding of concepts, for example; the proposed use of GeoGebra applets on mobile devices. From my experience, I noted that learners resort to memorizing concepts if they do not understand them.

Fewer participants (36%), enjoyed hearing explanations of new concepts while 64% sometimes enjoyed listening to the teacher explaining new concepts. These results coincide with what I noted during my teaching career, that some students get bored with lengthy explanations while others embrace the explanations. The fact that none of the respondents selected the option of not enjoying listening to explanations was an indication that the learners' learning style supported conceptual understanding. These results also show the relevance of using mobile technology in teaching and learning mathematics. The use of GeoGebra applets allows visualization of concepts, thereby reducing the time required to explain the concepts. Very few participants (4%) never got distracted, 64% sometimes got distracted, and 32% of the participants indicated that they got distracted with long explanations. Students usually get distracted if they are not interested in the section or they are confronted with challenging activities (Kade et al., 2019). As mentioned earlier, the use of mobile technology is more engaging and helps learners to focus more on their school work.

To further support the notion that more learners wished to learn with understanding, 76% enjoyed listening to explanations from peers. These results are displayed in Figure 17.

Few participants (36%) indicated that they enjoyed sharing ideas with peers on the board, while 64% indicated otherwise. The majority of the participants (80%) always wanted to identify their mistakes when they got something wrong.

Several participants (64%) indicated that they were interested in discovering cases of congruence of triangles. This percentage indicates the need to explore teaching strategies that give learners a chance to explore and discover new concepts. Therefore, it is essential to explore strategies that promote discovery learning and prioritize reflection, thinking, experimentation, and exploration; and enable students to make connections among mathematical ideas (Balım, 2009). Additionally, in discovery situations, learners do not only comprehend concepts and rules but also learn the construction and connections among them (Tran et al., 2014).

The majority of the participants (84%) stated that they enjoyed answering items that required simple recalling and routine procedures, while 16% indicated otherwise. Few participants (24%) felt confident in answering questions that required proofs, while 76% indicated that they did not feel confident. This was an indication that participants did not have confidence in their understanding. This notion aligns with the view by Kade et al. (2019) who indicated that lack of understanding is commonly displayed by students' failure to answer non-routine questions (Kade et al., 2019).

Item 15 probed on the current use of technology by teachers, specifically computers and data projectors. None of the participants affirmed this with a yes. Few participants (28%) indicated that teachers were not using a data projector for teaching while 72% indicated that they would sometimes use it. These results are shown in Figure 18. The fact that no one confirmed yes implies that the use of technology was minimal. This makes the proposed strategy relevant.

Figure 16

Clustered bar graphs of items measuring the learning-style construct

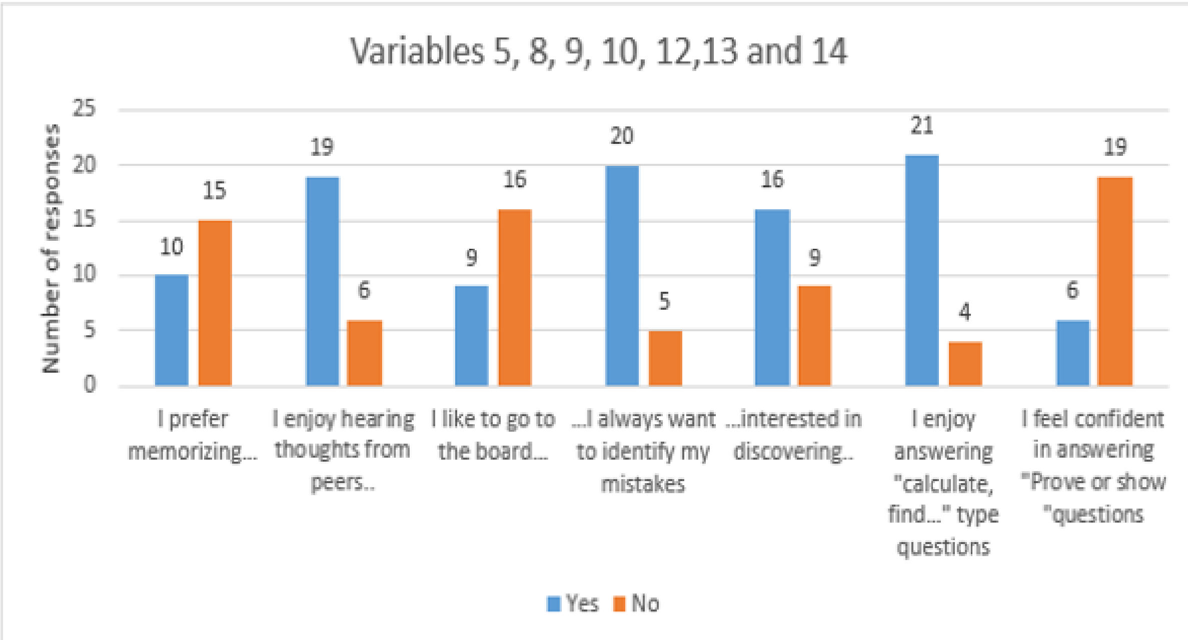


Figure 17

Clustered graphs for items 6 and 7

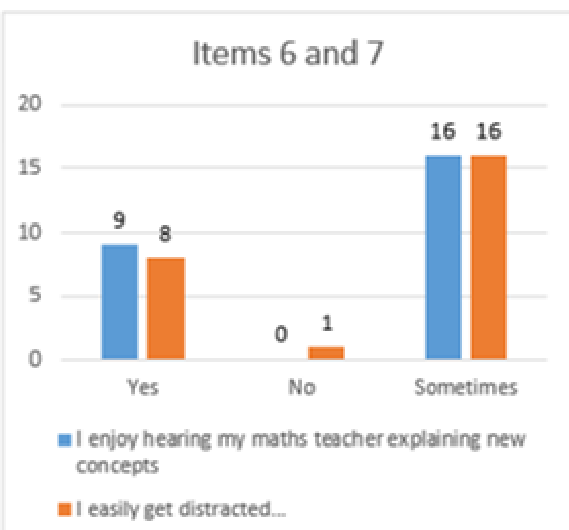
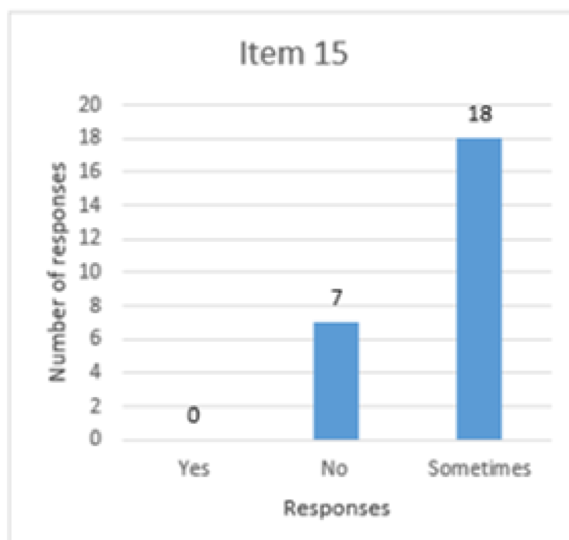


Figure 18

Use of data projector



5.2.3 Learners' proficiency, affordance, and acceptance

Learners of today are digital natives (Franklin & Peng, 2008; Thompson, 2013). This research sought to explore learners' technological proficiency in using mobile technology in the teaching and learning of mathematics, to enhance conceptual understanding. It was, therefore, important to probe learners' competence in using mobile devices. This aspect falls in the device usability region (the intersection of the learner and device aspect) of the FRAME model (refer to section 3.1). Almost all participants (96%) indicated that they were proficient in using mobile technology. This high percentage suggests that learners would be psychologically comfortable in using mobile devices when learning mathematics. As stipulated in the FRAME model, high psychological comfort reduces cognitive load. In other words, instead of learners worrying about manipulating and operating the mobile device, they would focus on the task to be completed (Koole, 2009), thereby increasing the speed with which they complete the task.

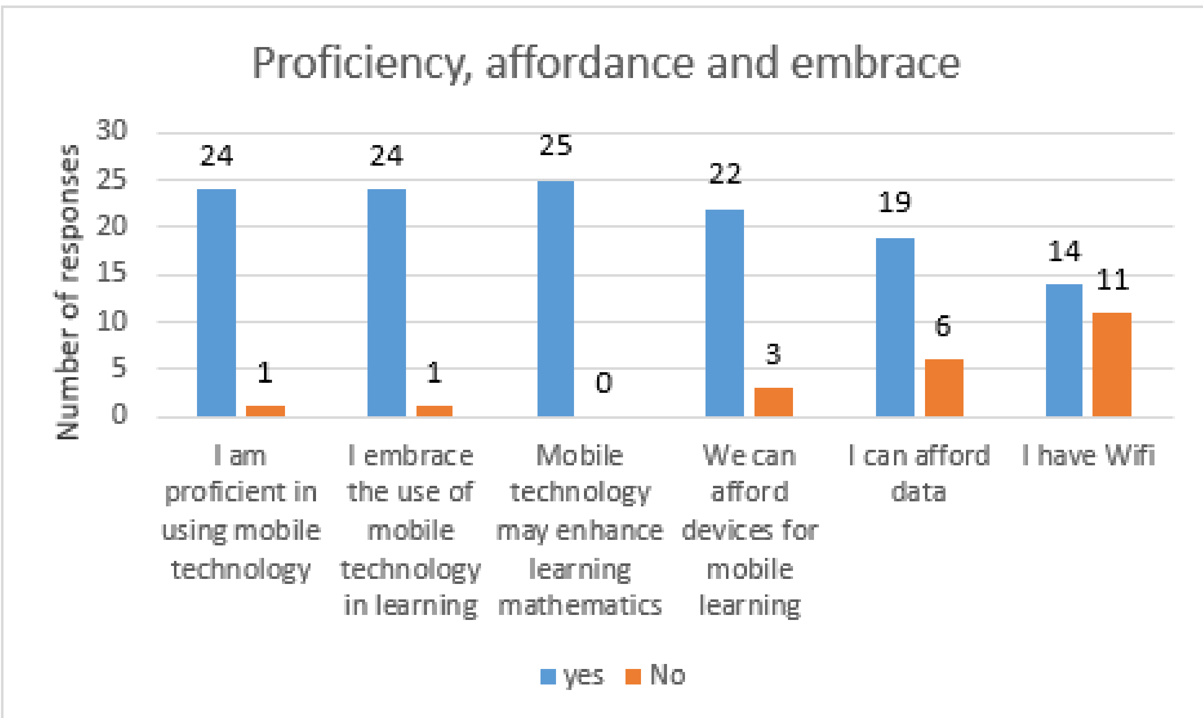
The results show that learners would embrace the use of technology in learning mathematics. It is important to note that these results were based on the prior knowledge that the learners had about mobile technology. The results show that learners were satisfied with the nature and functionality of mobile devices, and they believed that the devices can be used in the teaching and learning of mathematics. It is also specified in the FRAME model, that some of the attributes that contribute to learner satisfaction of the device are the interface design, the physical appearance, and functionality.

GeoGebra applets proposed in this study require connectivity to the internet, which requires the user to have data or Wi-Fi. Regarding this, three items were included to check the learners' affordance of mobile devices and data. A higher percentage of the participants (88%) responded that they could afford mobile devices. Correspondingly, 76% indicated that they could afford data and 56% of the participants indicated that they had Wi-Fi at home. Even though these percentages were high, I presume that the results are transferrable to schools with a similar setting to this particular case (refer to section 4.5.1 for the school background). Considering what I noticed during the COVID-19 pandemic, some schools embarked on online learning during the lockdown, while for other schools it was a non-starter because of lack of resources. It would, therefore, be essential to check the affordability of mobile devices and data in different school settings before making a general conclusion that a higher percentage afforded mobile devices and data required for learning purposes. I think affordability is one of the important aspects that was not mentioned in the FRAME model.

The bar graphs representing the distribution of responses from items related to proficiency, affordance, and acceptance are displayed in Figure 19.

Figure 19

Proficiency, affordance, and embrace



5.2.4 Analysing item association

This study sought to explore the technological proficiency of learners in learning mathematics. I, therefore, found it essential to check if there was an association between the following pairs of items: liking mathematics (item 1a) and technology appreciation (item 17), proficiency in using technology (item 16), and technology appreciation (item 17). Additionally, I assessed the association between getting good marks in mathematics (item 2a) and proficiency in using technology (item 16) and getting good marks in mathematics (item 2a), and technology appreciation (item 17). I used Fisher's exact test because the sample size was small (see section 4.7.4. for the explanation of Fisher's exact test).

For the first pair of items (liking mathematics and technology appreciation), the p-value was equal to 1, which was greater than the chosen level of significance ($\alpha = 0.05$). Therefore, there was not enough evidence to suggest an association between liking mathematics and technology appreciation. The results show that even learners who do not like mathematics still appreciate the use of technology. This was affirmed by the higher percentage (92%) of participants who embraced the use of mobile technology compared to the 64% of participants who liked mathematics. Therefore, the results support the proposed use of mobile technology in teaching and learning mathematics since both groups (those who like and those who do not like mathematics) embrace the use of technology.

Similarly, there were no grounds to suggest an association between proficiency in using technology and technology appreciation since the p-value was equal to 1, which is greater than the chosen level of significance ($\alpha = 0.05$). The p-value was also 1 for the remaining pairs of items. Therefore, there was not enough evidence to suggest an association between getting good marks in mathematics and proficiency in using technology, and between proficiency in using technology and technology appreciation. These results also support the proposed use of mobile technology since technology is embraced by non-technologically and technologically proficient learners. The results also reflect that learners' performance in mathematics does not influence their technological proficiency nor their ability to embrace the use of mobile technology. See tables with these results in Appendix F.

These results promote the proposed strategy's feasibility. It could have been challenging if there was an association between the pairs of items discussed above. The existence of association would imply that the proposed use of applets would be embraced by a particular group of learners; learners who like mathematics or learners who are technologically proficient.

5.2.5 What can teachers do to improve understanding?

Before analyzing an open-ended item that probed learners' views on what teachers could do to enhance understanding of mathematics, concurrent validity was assessed by checking if the responses given to item 24 were aligned to the challenges that were indicated in item 2b. Item 2b asked participants who were getting low marks in mathematics to indicate reasons for the low performance. In the next paragraph, I compare the responses to items 2b and 24 for each participant who responded to item 2b (Not all participants responded to this item because it was a filtered item).

5.2.5.1 Assessing Concurrent validity of responses

Participant 6 indicated that at times she did not understand when the teacher was teaching. In response to item 24, she said that the teachers should explain briefly and give activities to consolidate the new work, and further explain what students get wrong in the activity. These two responses were aligned.

Participant 7 indicated that she was getting low marks because geometry was hard and indicated that it would help if her teacher was involved in Saturday classes. I considered these responses to be aligned because the participant suggested getting more contact time with her respective teacher to enhance understanding. I said respective teacher because there were classes that were offered on Saturdays, but students were allocated to educators who were available for those lessons. So, some students were taught by their normal school mathematics teachers, and others were allocated to other teachers who were only teaching them on Saturdays.

Participant 12 stated that some of the questions that were asked were too long and she sometimes did not complete them. She then suggested the use of tutorials with pictures and noted that she found it challenging to sit and listen. This response shows that she did not enjoy doing the same activity for a long time.

Responses for participant 13 were coherent since she indicated challenges of seeing things and then suggested that the educators should find ways of visualizing concepts.

In item 2a, participant 14 indicated that she got low marks and then went on to explain that she was not getting good or bad marks, but she was not happy with her average performance. This showed that the responses for 2a were not exhaustive. In item 24, she suggested the use of incentives and data projector to boost learners' interest in the subject. She also indicated that teachers should dwell more on what learners do not understand. In as much as the learner did not indicate a problem in 2a, her responses to item 24 show that she was giving suggestions she thought could enhance her understanding of mathematics. I, therefore, considered the two responses to be articulated.

Participant 15 noted the challenge of understanding the formulae, and in her suggestion to improve understanding, she mentioned the use of phones and stated that phones would allow them to learn from home.

Participant 16 indicated that she was not for mathematics. She hated it and she did not understand the explanations and she had no suggestions on how the challenge could be addressed. This was also aligned because the first response showed that the participant had lost hope, so she could not suggest anything to alleviate the problem.

Participant 17 echoed that some sections were difficult, and she proposed the use of easier methods to teach. She suggested proper explanation and the use of examples. She also mentioned the use of phones. The alignment of the two responses is also evident.

Participant 18 indicated that she hated algebra and expressions, and did not see their future applications. In her suggestions, she echoed that mobile and digital use could help learners' concentration. I consider the two responses to be aligned.

Participant 19 indicated that she did not understand and was not putting in the effort to understand. In her suggestions on what teachers could do to improve understanding, she indicated that the use of mobile devices could help the students to focus more and minimize writing. The two responses were aligned.

Participant 20 stated that she did not understand most of the work and was getting low marks regardless of the effort she was putting in. She suggested the use of a smartboard to demonstrate shapes, angles, and equations. She added that the use of video to explain concepts could be more interesting and could help learners understand more. Alignment is evident between the two responses.

Participant 22 indicated that she was getting average marks. She suggested that sweets could be used to motivate learners. She further indicated that the use of phones could enhance the understanding of mathematics. I cannot assess coherence in this case because the participant did not give reasons for the average performance.

Participant 23 said that she had challenges in understanding mathematics. She suggested that the use of mobile devices could help learners focus better. Participant 24 indicated the same challenge. She, however, suggested that teachers should give examples that are similar to questions that come in the tests. The alignment was evident in the pair of responses for these two participants.

Similarly, Participant 25 stated that she had challenges understanding some of the teacher's explanations. She also added that they were getting tricky and difficult exam questions that looked different from what they learnt in class. To enhance understanding, she suggested that teachers could show YouTube videos on a data projector and use examples and activities that relate to everyday life. The two responses were aligned.

Almost all pairs of responses were aligned thus confirming the concurrent validity of the open-ended items in the first questionnaire. The analysis of open-ended item 24 that explored participants' views on what teachers could do to enhance understanding of mathematics follows in the next paragraph.

5.2.5.2 Analysis of the responses

Responses to item 24 were coded into five constructs; technology use, examples, detailed explanations, more time, and different methods.

Several participants called for educators to be innovative and use various existing technologies to present lessons in an exciting and fun way. After scanning through the data, I noted that participants used the following terms related to technology: apps, videos, phones, mobile, and smartboard. I then used these words in the text-search criteria to find the participants that had made sentiments about using technology in their responses. I also chose the text-search option that considers synonyms. The results of the text search query showed that 76% of the participants suggested the use of technologies in the teaching and learning of mathematics.

Participant 1 suggested the use of videos and the internet. She stated, "*Show us videos explaining different concepts and rules using the internet."* It is clear in this statement that the participant preferred the use of technology to aid teachers in explaining the concepts. This would avoid lengthy explanations by the educators. This suggests that the learners would be more receptive to the proposed strategy that explore the use of mobile technology. The fact that the participant mentioned the use of the internet which requires data implies that she could afford the data required to access the internet. Participants 14, 20, and 25 shared similar sentiments as participant 1. Participant 20 stated, *"She could use a smartboard we have here at school to demonstrate the angles, shape, and equation much clear. And she could find videos on the internet that explains each section we are doing as some videos tend to be interesting after all making maths fun will help us understand the subject better."* It is evident in this statement that the participant was suggesting the use of existing technologies in aiding educators explain mathematical concepts. She added that the use of videos would make the learning of mathematics more interesting. This concurred with participant 14's response. She said, *"Use the projector more often so we can be interested."* Likewise, participant 25 stated, *"She should use data projector often as I can understand better after watching a YouTube video on a concept than being taught on the board. She should explain the concepts more in depth…"* These responses show that there was a call for educators to integrate technology to assist the explanation of concepts, and capture learners' attention and boost their interest in the subject. It is also clear from the responses that the participants wished to understand the concepts and hoped that this could be achieved by integrating technology. Students who held similar sentiments were more likely to accept strategies that allowed exploration and visualization of concepts like the use of GeoGebra applets proposed in this study.

Participants 9, 11, and 13 suggested that the educators should use teaching strategies that allow visualization of concepts. Participant 9 stated, *"…must implement apps more as they allow us to visualize…"* Relatedly, participant 13 said, *"Find a visual way to show us new concepts."* Similarly, participant 11 called for more illustration of concepts. She stated, *"…show more instead of explaining all the time."* It is evident from these statements that the learners who shared these sentiments would accept and appreciate the use of GeoGebra applets that allowed the exploration and visualization of concepts.

Participants 5 and 12 called for educators to use strategies that engaged learners more. Participant 12 stated, *"...should do some kind of tutorials with pictures that you can change the size of the sides with because it is hard to sit and listen all the time."* It is apparent in this statement that the learners wanted to be involved in the construction of new knowledge. This supports the implementation of the proposed strategy that allows learners to explore and discover new concepts. Relatedly, participant 5 said, *"...make lessons more practical and easier."* I believe that using GeoGebra applets reduces cognitive overload because learners get actively engaged in the construction of new knowledge and can visualize concepts, which makes the lessons easier and understandable.

Some participants mentioned the use of mobile devices/phones. Participants 22 and 17 just indicated that they should be allowed to bring phones but did not give the reasons. It is not clear whether they wanted to bring phones for the learning of mathematics or to use them for other activities. Faloon (2017) warns that when using mobile technology, it is sometimes unclear whether the learners' enthusiasm is in learning or just completing the task using the device. Participant 17 stated, "*Use different easier methods to teach us, ... Bring phones.*" I presume that this participant believed that teaching and learning using mobile phones would serve as an alternative and easier method of teaching. This would support the proposed strategy of using mobile technology in teaching and learning mathematics.

Responses by participants 16, 18, 19, and 23 included the reasons why they thought they should be allowed to bring phones. Participant 16 stated, *"To let us use our phones so that we will understand better and we can also go and learn from home with more understanding."* This participant hoped that the use of phones would give the learners more time to engage with the content, thus; enhancing understanding. This response further supports the use of mobile technologies, that would allow learners to revisit the content learned anytime and everywhere.

Participant 18 stated, *"I think a mobile would allow us to be more concentration as we will be able to see and have the work digital in front of us."* The participant's views were in concordance with the advantages of using mobile technology mentioned by Radović et al. (2018). It is apparent that the participant would appreciate the use of GeoGebra applets on mobile devices. Similarly, participant 19 stated, *"I think if we use a mobile device so that we can focus more on what we are doing than doing so much writing."* Again, the participant would prefer to be involved in knowledge construction. Therefore, I presumed that the learners who shared the same view would appreciate the proposed strategy of using GeoGebra applets on mobile devices. A similar response was given by participant 23. She said, *"She must let us use our phones because we focus better on mobile devices."* The participant's view that using mobile devices would enhance concentration concurs with the findings by Denbel (2015) and Radović et al. (2018).

From the above extracts, it is evident that the learners presumed that using technology would increase concentration and boost their interest in the subject. Additionally, they indicated that the use of mobile technology would help them visualize the content and enhance their conceptual understanding. This presumption is consistent with the stipulation in the learner aspect of the FRAME model, that mobile learning enhances the ability to assimilate and understand the content (Koole, 2009). These findings show that learners would embrace the proposed use of mobile technologies in teaching and learning mathematics. These interview results concur with the high percentage (92%) of participants that indicated in the questionnaire findings, that they would embrace the use of mobile technology; in response to item 17 (*I embrace/appreciate the use of technology in learning*).

Participant 25 mentioned that educators should make lessons easier by giving more practical examples that relate to real life. She stated, *"...give us more examples/activities that relate to everyday life, eg if she is speaking about how to find x, instead could say in maths imagine x is..."* These participants' presumption of using real-life examples to enhance conceptual understanding concurs with one of Killen's (2015) suggestions for quality teaching and learning. Killen indicated that giving learners examples within their contextual life allows them to see the application and relevance of the content. By so doing, their understanding and retention of content would be enhanced. Even though the applets proposed in this study did not incorporate real-life examples, I recommend the inclusion of real-life examples in future applet designs. The participants' suggestions reflected that they wished to learn with understanding. Therefore, they would embrace teaching strategies that enhance conceptual understanding.

Participant 24 indicated that educators should give examples that are similar to assessment questions by saying, *"Give more examples similar to questions that come in the test."* This response reflects the need to explore strategies that enhance conceptual understanding. Learners who lack conceptual understanding face challenges in answering questions that they are not familiar with, or questions that require the application of concepts. This view coincides with what was mentioned in the National Senior Certificate diagnostic report. It was reported that performance in the 2019 examination showed a deficiency in the understanding of basic concepts across some topics in the curriculum (DBE, 2019).

Participants 2, 7, 14, 17, and 25 echoed similar sentiments in the following verbatim:

Participant 2: "*Explain more detail. …*"
Participant 7: *"She must be able to explain more briefly because some of us are not on the same page as everyone else."*
Participant 14:"*…Explain what we don't understand in details…."*.
Participant 17 said, "*…Explain properly…*"
Participant 25 stated, "*…She should explain the concepts more in-depth…*".

For each of the responses, the common word was 'explain' showing that learners needed more elaboration of the content. This was expressed using the words 'properly, 'in-depth', in detail with only participant 7 noting that the explanation could be done 'briefly' but the participants referred to further explanation of new content to enhance understanding. Therefore, using applets would allow visualization and assist in the explanation of concepts. This notion was affirmed by some interview and post questionnaire responses.

The above responses further emphasize the need for teaching and learning strategies that aid the illustration and explanation of mathematical concepts. This supports the proposed use of applets on mobile technologies in the teaching and learning of mathematics. It is affirmed in the literature that GeoGebra applets allow for visualization of concepts, facilitates the explanation, and promotes swift understanding of concepts (Polásek & Sedlácek, 2015; Dimitrov & Slavov, 2018; Denbel D. G., 2015).

Some participants noted that they needed more time to understand new concepts and that they needed extra lessons. These suggestions were expressed by three participants. Participant 2 suggested having extra classes, and Participant 7 indicated that she would want her mathematics teacher to attend Sartuday lessons. There were Sartuday lessons at the time of data collection. The lessons were conducted by some educators who were teaching grade 9 mathematics at the school and hired educators. Therefore, it is either one was taught by her mathematics teacher or another teacher. Participant 7 response indicates that she was not taught by her mathematics teacher and wished to have more time with the teacher, which explains why she wanted her mathematics teacher to conduct Saturday lessons so that she could be in her group. Participant 7 also echoed the use of more class activities to consolidate new content. Relatedly, participant 11 stated, "*Give us more time to understand new work*" This statement indicates that the participant had challenges in understanding new work and thought that could be alleviated by spending more time on the new content. This reflects the need for teaching strategies that aid explanations and reduce the time required to elaborate concepts. As indicated earlier, the use of GeoGebra applets enables learners to see and explore mathematical relations and concepts that are difficult to illustrate and explain using conventional teaching methods (Diković, 2009). This reduces the time required to explain and understand concepts. The use of GeoGebra applets also allows learners to revisit the content anywhere and anytime.

Some participants recommended that educators should illustrate different methods of finding solutions so that they can choose the method easier for them. This suggestion was expressed by participants 4, 17, and 21. Participant 4 stated, "*Give different methods of finding answers.*" Similarly, participant 21 said, "*Use different methods to get answers so that we can choose what's easier for us.*" These two responses called for different methods to solve problems. However, this research did not focus on problem-solving but on exploring and discovering the concepts. I would recommend the design of applets that focus on the consolidation of concepts, and include feedback with different approaches.

Participants 14 and 22 said that educators should bring sweets as an incentive to motivate them. Similarly, participant 17 suggested the use of various teaching methods to make the lessons more fun. These responses reflect the need for teaching strategies that arouse learners' interests in the subject. Learners who shared the same sentiments would be more likely to be receptive to the proposed strategy of using GeoGebra applets on mobile devices.

The responses above show that learners wished to have engaging and interesting lessons. This further supports the incorporation of GeoGebra applets in teaching and learning mathematics. The presumption that using applets boost learners' motivation in learning mathematics is supported in the literature (Denbel D. G., 2015; Radović et al., 2018; Akçakın, 2018). It is also evident that most participants had faith in mobile technology, and wished educators could use methods that help them understand mathematics better. The learners' views are in line with the grounding of the learner aspect of the FRAME model that supports the use of mobile technology in learning. This shades a green light for the proposed use of GeoGebra applets. It is evident from the responses that learners would embrace the use of applets on mobile devices for learning mathematics.

5.3 Second questionnaire data screening

This subsection presents the data screening processes that were done, checking of missing data, identification of outliers, and assessment of collinearity and normality.

Even though missing data was checked as the participants were submitting the completed questionnaires, I had to affirm this using excel. I used the excel function, COUNTBLANK, to calculate the number of blank cells for each item, including the open-ended items. The number of blank cells for all items was equal to zero. This meant that all cells had something captured, therefore, there was no missing data. After that, the second questionnaire data were examined for the existence of outliers.

Outlies are scores that are different from the rest (Kline, 2011). They should be identified because they influence the outcome of statistical analysis. In the event of identifying an outlier, it is advisable to check the possible causes. Some of the possible causes of outliers are either data were entered incorrectly or the inclusion of a participant that did not belong to the target population. I checked if there were any transcription mistakes but found that the susceptible outliers were indeed the correct scores that were put by the participants. There was no issue with regards to the target population because participants were all in grade 9, and the research focused on learners in grade 9. There are several suggestions in the literature on how to deal with outliers. One of the easiest methods to deal with outliers is to change the value of the outlier to the next extreme value that is three standard deviations from the mean (Kline, 2011). An alternative way involves transforming the outlier, using a mathematical operation, to a new score that lies within three standard deviations from the mean. Outliers have a negative impact if the research depends on statistics that are sensitive to outliers. In this study, I used the mean to evaluate the construct. Therefore, it was essential to check the existence of outliers and their effect on the mean. I was not only interested in the mean, but also in the reasons that caused respective participants to score extreme values. The detailed analysis was done in section 5.3

Scores that are more than three standard deviations from the mean are suspected outliers (Kline, 2011). To identify outliers in this study, I calculated the absolute value of the number of standard deviations from the mean, of the minimum value for each item in excel, using the following formula:

$$\frac{min - \bar{x}}{s}; \ where$$

$\min is\ the\ minimum\ score\ of\ the\ variable,$

$\bar{x}\ is\ the\ mean\ of\ the\ scores\ of\ the\ variable,$

$s\ is\ the\ standard\ deviation\ of\ the\ scores\ of\ the\ variable$

Similarly, the number of standard deviations of the maximum score for each item was calculated by replacing the minimum value with the maximum value in the formula above. These deviations are shown in Tables 6 and 7. Item 6 (*My cellphone skills helped me to use the applets*) gave a #DIV/0! Because all the participants scored a 10, therefore the standard deviation was zero.

As can be seen in Tables 18 and 19, most values are within three standard deviations from the mean, except the values for items 1, 14, 18, 19, 22, 25, and 28. All these values came from the calculations using the minimum values. It was interesting to note that all values that were calculated using the maximum values were less than 1. This was an indication that the means were high and very close to the maximum values. I was curious to see why the number of standard deviations for these seven items was more than 3. I established that the minimum value for item 1 was 5, and the standard deviation was small. This resulted in a wide range thus, giving the result to 3,811 deviations from the mean. Similarly, the minimum value for items 14, 19, and 25 was 6. This also gave a wide range resulting in a number of standard deviations greater than 3. For items 22 and 28, I established that the number of standard deviations from the mean was high (equal to 3,323) because the standard deviation for the scores was very small (0,277) since the minimum score was 9, and the maximum was 10. Thus, there was very little variation in the scores. It was either a 9 or a 10. Because this research was guided by the interpretive principle, original responses had to be considered, the main focus was not on the summary of results. During data analysis, all these possible outliers were considered and all responses were checked, in a bid to establish why participants gave those extreme scores.

Table 18

Absolute value of Number of standard deviations of extreme values from the mean for items 1 to 14.

Item Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of the standard deviations from the minimum value	3,811	2,805	2,2	-2,70	1,776	#DIV/0!	1,292	2,337	2,83	2,398	2,85	1,94	1,96	3,82
Number of the standard deviations from the maximum value	0,52	0,64	0,8	0,57	0,75	#DIV/0!	0,94	0,54	0,54	0,68	0,438	1,15	0,49	0,378

I checked if these minimum scores were coming from the same participant and found that they were not all associated with one participant. The minimum score for item one came from participant 6. The minimum scores for items 14, 18, 19, and 25 came from participant 4; except item 19, where participant 18 also scored a 6. The minimum scores for items 22 and 28 came from participants 16 and 23, and, participants 5 and 19 respectively.

Table 19

Number of standard deviations of extreme values from the mean for items 15 to 28

Item Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Number of the standard deviations from the minimum value	1,67	1,44	2,80	3,25	3,22	2,81	2,33	3,32	-2,80	-2,34	-4,01	-2,34	-2,89	-3,32
Number of the standard deviations from the maximum value	0,89	0,81	0,456	0,44	0,36	0,58	0,73	0,29	0,46	0,54	0,40	0,514	0,55	0,29

It was, however, noted that participant 4 had a minimum score on four of these items. I then calculated the mean of the scores for each participant. Instead of labeling participants with the low average scores as participants who were generally stingy with scores, I had to check their responses for possible reasons of giving low scores compared to the other participants. Since the mean value was established to evaluate the constructs, I found it essential to check the effect of excluding the suspected outliers when calculating the mean for items 1, 14, 18, 19, 22, 25, and 28. This was done by calculating the absolute values of the difference between the mean of the scores, including and excluding the suspected outliers as follows:

$$Mean\ difference\ = \frac{sum\ -\ suspected\ outlier}{n-1} - \bar{x}, where:$$

$sum\ is\ the\ total\ of\ the\ scores\ of\ the\ varable$

$suspected\ outlier\ is\ is\ the\ minimum\ variable$

$\bar{x}\ is\ the\ mean\ for\ the\ variable$

$n\ is\ the\ number\ of\ participants.$

Where two participants scored the susceptible outlier, the formula was adjusted as follows:

$$Mean\ difference\ =\ \frac{sum\ \ - 2(suspected\ outlier)}{n-2} - \bar{x}.$$

The mean differences for items 1, 14, 18, 19, 22, 25, and 28 above are shown in Table 20. These differences, according to my view, were small. Therefore, they had less effect on the mean of the respective item. Besides, excluding those scores was going to be against the interpretive principle which values every bit of information from the participant.

Table 20

The mean differences for items that were suspected to have outliers

Item	Mean difference
1	0.183
14	0.152
19	0.313
22	0.080
25	0.152
28	0.08

As an alternative to the method described above, the kurtosis coefficient could also be used to check the existence of outliers in a data set. Kurtosis tells us about extreme values. It is a measure of whether the data is heavily tailed or lightly tailed relative to a normal distribution (NIST/SEMATECH, 2013). In other words, it indicates the possible presence of outliers. That is, data sets that have high kurtosis tend to have heavy tails or outliers (NIST/SEMATECH, 2013). The kurtosis coefficient for normally distributed data is three (NIST/SEMATECH, 2013). However, other formulae subtract 3 to make the coefficient of normally distributed data equal to zero (Zaiontz, 2019). Excel uses the following adjusted formula to calculate the kurtosis of a sample:

$$\frac{n(n+1)\sum_{i=1}^{n}(x_i-\bar{x})^4}{(n-1)(n-2)(n-3)s^3} - \frac{3(n-1)^2}{(n-2)(n-3)};where$$

$n\ is\ the\ sample\ size,$

x_i *is the score,*

s *is the standard deviation of the sample* (Zaiontz, 2019).

Kurtosis coefficients are shown in Tables 21 and 22.

Table 21

Kurtosis coefficient of items 1 to 14

Item Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Kurtosis	8,286	3,378	0,178	0,983	1,275	#DIV/0!	1,732	0,638	3,744	0,507	4,194	-0,903	0,593	8,991

Table 22

Kurtosis coefficients of items 15 to 28

Item Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Kurtosis	-0,908	-1,495	3,539	4,494	8,120	3,547	0,412	9,641	3,539	0,638	11,084	1,606	1,915	9,641

Results in Tables 21 and 22 were consistent with the results that were obtained using the number of standard deviations from the mean. The kurtosis coefficients for items 1, 14, 19,22, 25, and 28 are high compared to the rest of the coefficients. It is evident from Figures 20 and 21 that these items had long tails thus, affirming the possible existence of outliers. Positive kurtosis implies a distribution with a long tail, that is; it indicates the possibility of having outliers in the data. After checking for outliers, I then investigated the existence of collinear items.

Collinearity occurs when two or more items overlap so much in what they measure, to the extent that their effects cannot be distinguished (Martin, n.d). That is, the items appear different but will be measuring the same thing (Kline, 2011). The bivariate correlation was used to detect collinearity. This entailed calculating correlations between pairs of items proposed to be measuring the same construct. Correlation coefficients equal to 0.8 or more are considered to be high and to confirm collinearity between the items (Martin, n.d). In addition to detecting collinearity, the same correlations were used to verify if the items were positively contributing to the same construct. Therefore, the positive correlation coefficient below 0.8 was considered as an indication that the items were not overlapping (Zaiontz, 2020).

Figure 20

Distribution for item 1 and item 14

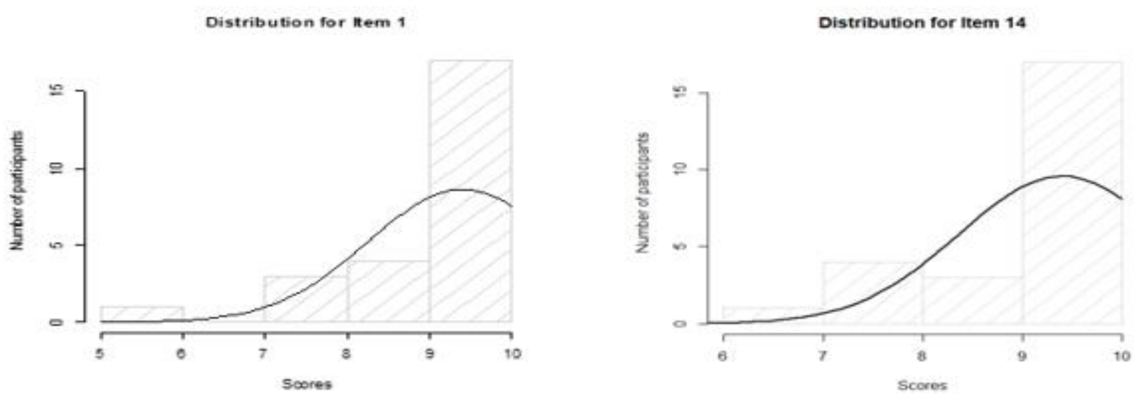
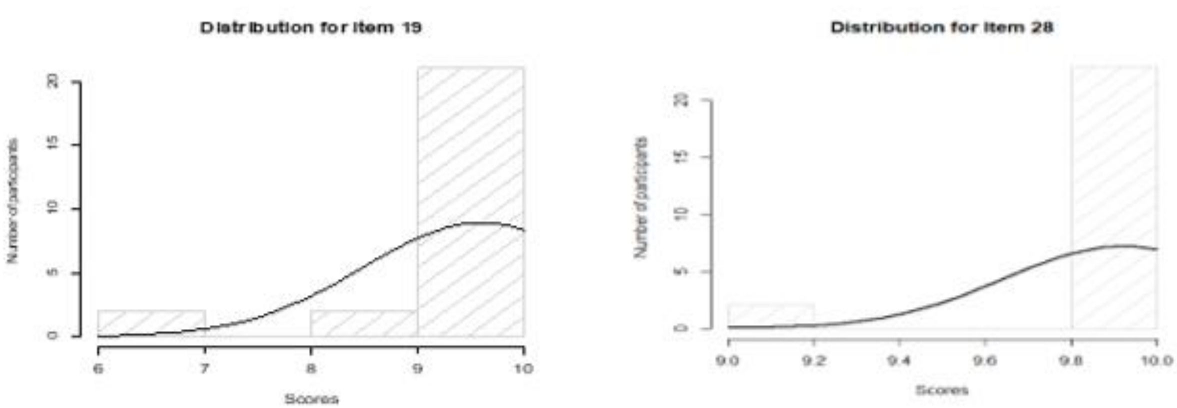


Figure 21

Distribution for item 19 and item 28



It was also essential to check for normality because I used confirmatory factor analysis to assess the validity of the questionnaires. Some of the results obtained from confirmatory factor analysis are suspicious if data does not follow a normal distribution. Data is said to be normally distributed if it is evenly spread around the mean. In other words, if the probability of every score was calculated and represented graphically, the graph would be bell-shaped and symmetrical about the mean, as shown in Figure 22. In contrast, non-normal distributions are skewed. Skewness is a measure of symmetry or precisely a measure of non-symmetry (NIST/SEMATECH, 2013). The value of skewness is zero for normally distributed data. The shape of the skewed distribution is bell-shaped like the normal distributing one, but it is asymmetrical about the mean. In a positively skewed distribution, most of the scores are below the mean, and the median is less than the mean, and the distribution is skewed to the right (Kline, 2011; Zaiontz, 2019). Contrary, in a negatively skewed distribution, most scores, including the median, are greater than the mean; and the distribution is skewed to the left (Kline, 2011; Zaiontz, 2019). Excel calculates the adjusted Fisher-Pearson coefficient of skewness using the following formula:

$$\frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)s^3}; where$$

$\bar{x}$ *is the mean,*

s is the standard deviation of the sample,

n is the sample size.

I used skewness to assess normality. Additionally, the relationship between the mean and the median was used to affirm the results from the skewness assessment. Skewness is equal to zero for normally distributed data. It is positive for positively skewed data, and if the distribution has a long tail on the right side of the distribution. Alternatively, it is negative if the data has a long tail on the left side of the distribution. This is illustrated in Figure 22.

As can be seen in Figure 22, generally the mean is greater than the median, which is also greater than the mode, if data is positively skewed (Dugar, 2018). The mean is equal to the median and mode if data is normally distributed (Dugar, 2018). The mean is less than the median, which is less than the mode if the data is negatively skewed (Dugar, 2018). Skewness coefficients were also obtained from the summary of the descriptive statistics generated in excel. These are shown in Tables 23 and 24.

Table 23

Skewness coefficients for items 1 to 14

Item Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Skewness Coefficient	-2,648	-1,865	-0,978	-1,515	-0,801	#DIV/0!	-0,337	-1,499	-2,080	-1,227	-2,286	-0,336	-1,597	-2,955

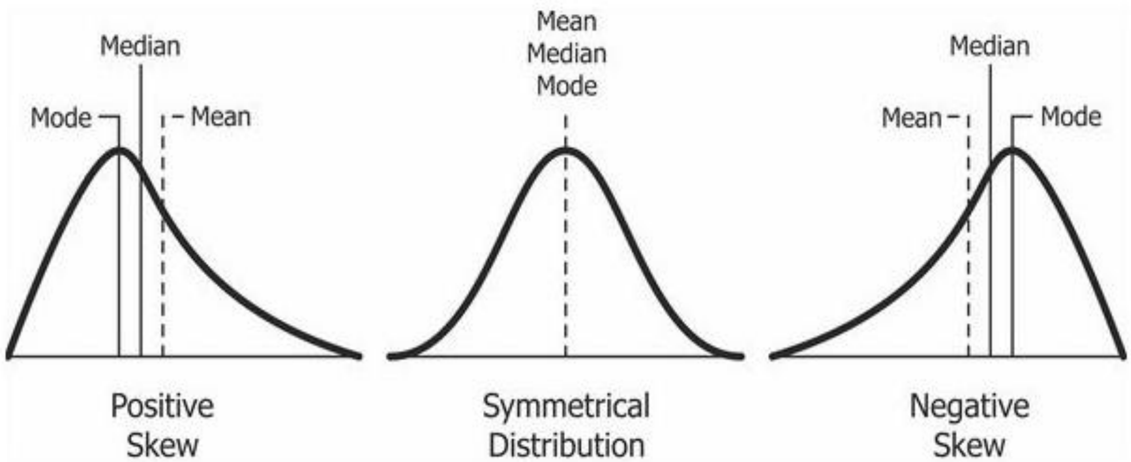
Table 24

Skewness coefficients for items 15 to 28

Item Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Skewness Coefficient	-0,761	-0,611	-2,127	-2,282	-2,994	-2,008	-1,199	-3,298	-2,127	-1,499	-3,182	-1,734	-1,707	-3,298

Figure 22

Diagram to illustrate symmetric and skewed distribution



Note. From "Skew and Kurtosis: 2 important statistics terms you need to know in Data Science", by D. Dugar, 2018, *(https://codeburst.io/2-important-statistics-terms-you-need-to-know-in-data-science-skewness-and-kurtosis-388fef94eeaa). Reprinted with permission.*

The data is not normally distributed. Rather, it is negatively skewed. The general rule of thumb is that data is fairly symmetrical if the coefficient of skewness is between -0,5 and 0,5 (Dugar, 2018). Skewness coefficients between -1 and -0,5 imply a moderate negative skew, while coefficients between 0,5 and 1 imply a positive moderate skew (Dugar, 2018). Data is highly skewed if the absolute value of the skewness coefficient is greater than 1 (Dugar, 2018). Skewness coefficients for all 28 items were negative. This was consistent with the relationship between the mean and the median of the items. As can be seen in Tables 25 and 26, 27 out of 28 items have their mean less than their corresponding median. The median for all items is either less or equal to the corresponding mode. That affirms the negative skewness that was portrayed by the coefficients of skewness. This is an indication that the usability of the applets was rated high.

Table 25

Mean, median, and mode for items 1 to 14

Item Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mean	9,4	9,44	9,2	9,48	8,52	10	9,16	9,44	9,36	9,56	9,6	8,88	9,8	9,64
Median	10	10	10	10	10	10	9	10	10	10	10	9	10	10
Mode	10	10	10	10	10	10	10	10	10	10	10	9	10	10

Table 26

Mean, median and mode for items 15 to 28

Item Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Mean	8,96	9,28	9,72	9,64	9,6	9,32	9,28	9,92	9,44	9,44	9,64	9,64	9,52	9,92
Median	9	10	10	10	10	10	10	10	10	10	10	10	9	10
Mode	10	10	10	10	10	10	10	10	10	10	10	10	10	10

If the distribution of data is non-normal and the research is based on some statistics that require data to be normally distributed, it is advised to transform the data to near-normality (Kline, 2011). In this study, I used the mean to affirm the constructs. Even though the mean has no conditions of normality, I had to assess normality because I also used confirmatory factor analysis, some of whose results may not be reliable if data does not follow a normal distribution. Several formulae can be used to transform the data to near normality. There are suggestions on the type of formulae to apply depending on the nature of non-normality. It is advisable to try several transformations to identify one that yields the best transformation (Kline, 2011). The process of transformation involves the conversion of an original value to a new one using a mathematical formula. Some common mathematical formulae used are the square root $\left(x^{\frac{1}{2}} \, or \, \sqrt{x}\right)$, logarithmic $\left(\log_{y} x\right)$, inverse function $\left(\frac{1}{x}\right)$, cube root $\left(x^{\frac{1}{3}} \, or \, \sqrt[3]{x}\right)$, sine function $\left(sin(x)\right)$ and box-cox function (Kline, 2011). The square root, logarithmic, and inverse functions can be used to improve normality when the distribution of data is either positively or negatively skewed (Kline, 2011). However, if the non-normality cannot be categorized as negative skew or positive skew, it is better to try the cube root and sine functions that reduce the gap between the extreme values and the mean (Kline, 2011). The Box-Cox transformation is, however, recommended to work better than the rest (Kline, 2011). It draws its strength on the fact that it uses an optimal value of λ, which maximizes the correlation between the original data and the transformed scores (Kline, 2011). There are existing computer algorithms for finding the value of λ.

The basic Box-cox transformation is given by the split function:

$$X^{\lambda} = \begin{cases} \frac{X^{\lambda - 1}}{\lambda}, & if\ \lambda \neq 0 \\ \log X, & if\ \lambda = 0 \end{cases}$$

where;

λ is a constant.

This formula works for positive scores. An adjusted formula that can be used for negative values is:

$$X^{\lambda} = \begin{cases} \frac{(X+\lambda_2)^{\lambda_1 - 1}}{\lambda_1}, & if\ \lambda_1 \neq 0 \\ \log(X + \lambda_2), & if\ \lambda_1 = 0 \end{cases}$$

where;

λ_1 is a constant

λ_2 is a constant that makes the minimum data value positive.

In the following paragraph, I explain the general steps to be followed when applying the square root transformation.

First, if the minimum value of the data set is less than one, a constant should be identified that gives one when added to the minimum value in the data set. This constant will then be added to all the values in the data set. There are two main reasons for having the minimum value of the data set greater or equal to one. Since we are using the real number system, and the square root of a negative number is non-real, all values should be positive before applying the transformation. One may ask why choose one to be the minimum and not zero? The reason is that the square root of a number between zero and one is smaller than the square root of the original number, whereas the square root of a number greater than one is greater than the original value (Kline, 2011) . Consequently, if the data set has values between zero and one and other values greater or equal to one, the effect of transformation will be different in the same data set, making the values either greater or smaller than the original values (Kline, 2011). However, data should be reflected first before adding the constant in the presence of negative skewness. This is achieved by multiplying all data scores by negative one before applying the transformation.

Considering the recommendation that two or more transformations should be applied to get one transformation that gives new values that are more close to normality, I successfully applied the square-root and inverse transformation. I attempted using the Box-Cox transformation but because of the nature of the data, there was a problem with singular matrices, and it could not calculate new values. The distribution of data obtained from the second questionnaire was negatively skewed. Therefore, I had to reflect the data scores before identifying the constant to add in step one described in the preceding paragraph. I used 11 as the constant because, before reflecting the data, the maximum was 10 and was then reflected and became the minimum, -10. To adjust it to 1, I had to add 11. So, 11 served as the constant in both transformations. The skewness coefficients before and after transformations are shown in Tables 27 and 28.

Using the rule of thumb by Dugar (2018), I constructed the color-coded scale in Figure 23. I set the threshold for adopting the transformation at 50%. That is, for a transformation to be used, it was supposed to have at least 50% of the items with a skewness coefficient between -1 and 1 (within the yellow and green region). The number of items in each region of the color-coded skewness scale for each transformation is displayed in Table 27. The total frequency in the table is 27 instead of 28. I could not calculate the skewness of item 6 since the standard deviation was zero and dividing by zero yielded a #DIV\0 error.

Table 27

Skewness coefficients before and after transformations for items 1 to 14

Item Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Skewness coefficients before transformation	-2,648	-1,865	-0,978	-1,515	-0,801	#DIV/0!	-0,337	-1,499	-2,080	-1,227	-2,286	-0,336	-1,597	-2,955
Skewness coefficients after square root transformation	1,969	1,423	0,724	1,368	0,708	#DIV/0!	0,241	1,415	1,722	1,026	2,112	0,036	1,597	2,613
Skewness coefficients after inverse transformation	1,007	0,630	-0,252	-1,130	-0,422	#DIV/0!	-0,024	-1,313	-1,023	-0,714	-1,743	0,572	-1,597	-2,101

Table 28

Skewness coefficients before and after transformations for items 15 to 28

Item Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Skewness coefficients before Coefficient	-0,761	-0,611	-2,127	-2,282	-2,994	-2,008	-1,199	-3,298	-2,127	-1,499	-3,182	-1,734	-1,707	-3,298
Skewness coefficients after square root transformation	0,537	0,530	1,9762	2,075	2,806	1,611	0,929	3,298	1,924	1,415	2,683	1,620	1,503	3,298
Skewness coefficients after inverse transformation	0,019	-0,349	-1,701	-1,720	-2,225	-0,836	-0,427	-3,298	-1,639	-1,313	-1,81	-1,39	-1,16	-3,29

Before the transformation, 6 out of 27 items had a skewness coefficient within the interval -1 < skewness coefficient < 1. After the square root transformation, the coefficients were less than the original coefficients, and 7 out of 27 items had skewness coefficients within the -1 and 1 interval.

Similarly, skewness coefficients for the inverse transformation yielded coefficients that were less, compared to the square root transformation. It was also noted that most coefficients that were negative before the transformation were positive on the transformed data. Because the data was not following a normal distribution, some statistics from the results of factor analysis were not considered. Factor analysis is explained in detail in section 4.7.3. I referred to the mean of all the items measuring the same construct as the overall mean. A mean equal to or greater than 9 was considered as a threshold for a good evaluation.

Figure 23

Color-coded skewness scale

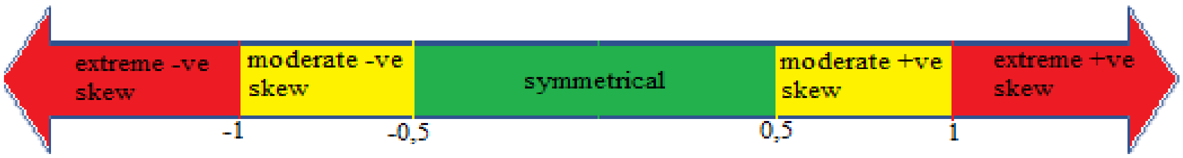


Table 29

Number of items in each region of the color-coded skewness scale for the original and skewed data

	Red	Yellow	Green
Original	21	4	2
Square root	20	5	2
Inverse	17	4	6

5.4 Assessment of reliability and validity for the second questionnaire

For all constructs, loadings from AMOS output were higher than loadings from R. However, the results from both software led to the same conclusion for all the constructs. I could not use AMOS to calculate the factor loadings of the items in the first questionnaire because the data was dichotomous. In the following sections, I discuss the validity and reliability of each construct.

5.4.1 Access construct

Items 1 (*It was easy to log in*), 2 (*It was easy to open the workbook*), and 3 (*The applets open quickly*) were used to evaluate the access construct. The factor loadings in Table 30 and Figure 24 were greater than 0.4. Even though the factor loadings calculated using AMOS or R are not the same, they both affirm that the three items were contributing to the evaluation of the access construct.

All inter-item correlations in Table 31 were greater than 0.3, which supports the notion that the three items evaluated the same construct. The Cronbach's alpha was 0.7. Therefore I concluded that the access construct was valid and reliable.

Table 30

Factor loadings for items contributing towards Access construct (R output)

Item	Factor loading
Q1	0.7
Q2	0.6
Q3	0.7

Figure 24

Factor loadings for items contributing towards Access construct (Amos output).

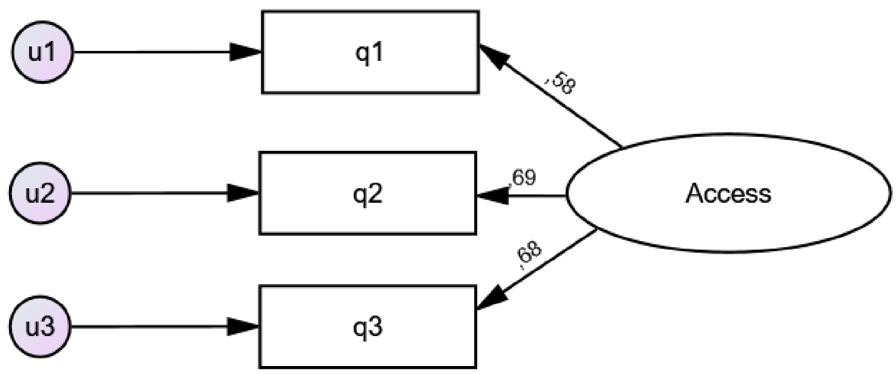


Table 31

Correlations and Cronbach's alpha coefficient for items contributing towards access construct

Inter-Item Correlation Matrix			
	Q1	Q2	Q3
Q1	1.0	0.4	0.4
Q2	0.4	1.0	0.5
Q3	0.4	0.5	1.0
Cronbach's alpha coefficient			0.7

The next subsection explains the validation and reliability analysis of the easy-to-learn construct.

5.4.2 Easy-to-learn construct

Computer application software that is easy to learn has a higher chance of being accepted. End users usually resist change if new software is difficult to learn. This is why I considered this construct.

Items 7 (*I learnt to use the applets quickly*), 8 (*I easily remember how to use the applets*), 9 (*It was simple to use these applets*), 10 (*I find it easy to become skilful in using these applets*), and 12 (*I would find it easy for learners in my grade to use these applets*) evaluated the easy-to-learn construct. The factor loadings in Table 32 and Figure 25were greater than 0.4. Thus, affirming that these items contributed to the evaluation of the easy-to-learn construct.

All inter-item correlations in Table 33 were greater than 0.3, thus, supporting the notion that they were measuring the same construct. The easy-to-learn construct reached acceptable reliability, α = 0.848. This value further supported that the five items evaluated the same construct.

Table 32

Factor loadings for items evaluating the Easy-to-learn construct (R output)

Item	Factor loading
Item 7	0.6
Item 8	1.0
Item 9	0.9
Item 10	0.5
Item 12	0.6

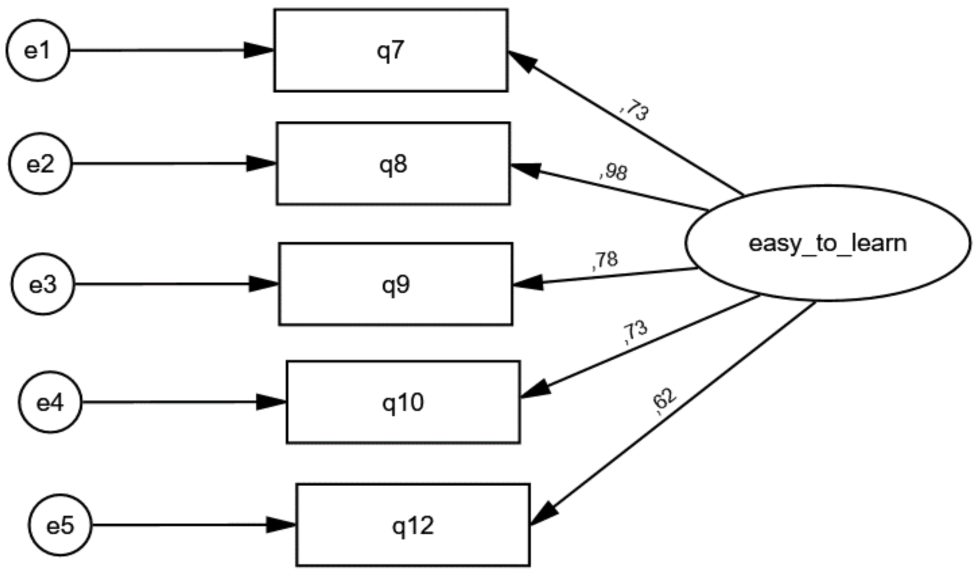
Table 33

Correlations and Cronbach's alpha coefficient for items evaluating the easy-to-learn construct

Inter-Item Correlation Matrix				
	q7	q8	q10	q12
q7	1.0	0.7	0.5	0.5
q8	0.7	1.0	0.7	0.6
q10	0.6	0.7	1.0	0.4
q12	0.5	0.6	0.4	1.0
Cronbach's alpha coefficient				0.8

Figure 25

Factor loadings for items evaluating the Easy-to-learn construct (Amos output)



5.4.3 Legibility construct

According to the FRAME model, it is important to consider the device aspect when using mobile technology in learning and teaching. The device aspect includes the physical, technical, and functional characteristics of the mobile device that is used in learning and teaching. It was indicated in section 3.3.1 that screen size was one of the limitations of mobile technology (Cheon et al., 2012). Therefore, I found it essential to include the construct that measured legibility. The legibility construct was evaluated by items 19 (*Triangles were displayed clearly*), 20 (*Triangle labels were clear*), and 21 (*Triangle measurements were easy to read*). The factor loadings in Table 34 and Figure 26 were greater than 0.4. The factor loading for item 20 from the R-results was outside the acceptable range. The item was accepted because the factor loading from AMOS was acceptable. Additionally, item 20 was highly correlated with items 19 and 20 (see Table 35). Thus, affirming that these items contributed to the evaluation of the legibility construct which supported the notion that they were measuring the same construct. The easy-to-learn construct reached acceptable reliability, $\alpha = 0.899$. This value further supported that the three items evaluated the same construct.

Table 34

Factor loadings for items evaluating the legibility construct

Item	Factor loading
Item 19	0.9
Item 20	1.1
Item 21	0.7

Figure 26

Factor loadings for items evaluating the legibility construct (Amos output)

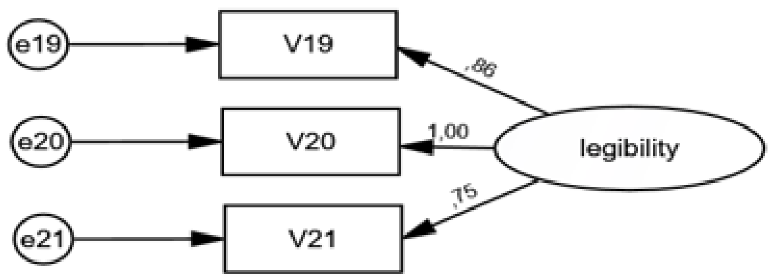


Table 35

Correlations and Cronbach's alpha coefficient for items evaluating the easy-to-learn construct.

Inter-Item Correlation Matrix			
	V19	V20	V21
V19	1.0	0.9	0.6
V20	0.9	1.0	0.7
V21	0.6	0.7	1.0
Cronbach's alpha coefficient			0.9

5.4.4 Satisfaction construct

It is believed that if software produces accurate and expected answers, users will get satisfied and are likely to recommend its use to colleagues (Lund, 2001; Torun & Tekedere, 2015; Hariyanto et al., 2020). The construct was evaluated using items 22 (*I got expected answers*), 23 (*I am satisfied with these applets*), 24 (*I would recommend these applets to my friends*), 25 (*I feel the applets help* *to understand cases of congruence*), 26 (*I feel using the applets will help identify/see cases of congruency of triangles*), and 28 (*I would like to use mobile technology (cell_phone, iPad, tablet, laptop, etc) more often when learning Maths*).

The factor loadings and correlations are shown in Table 36, Figure 27, and Table 37. Considering that these items were adapted from validated questionnaires, I concluded that the six items were evaluating the satisfaction construct even though some items had correlations less than 0.3 and factor loadings less than 0.4.

Table 36

Factor loadings for items evaluating the satisfaction construct (R output)

Item	Factor loading
Item 22	0.6
Item 23	1.03
Item 24	0.9
Item 25	0.9
Item 26	-0.3
Item 28	0.0

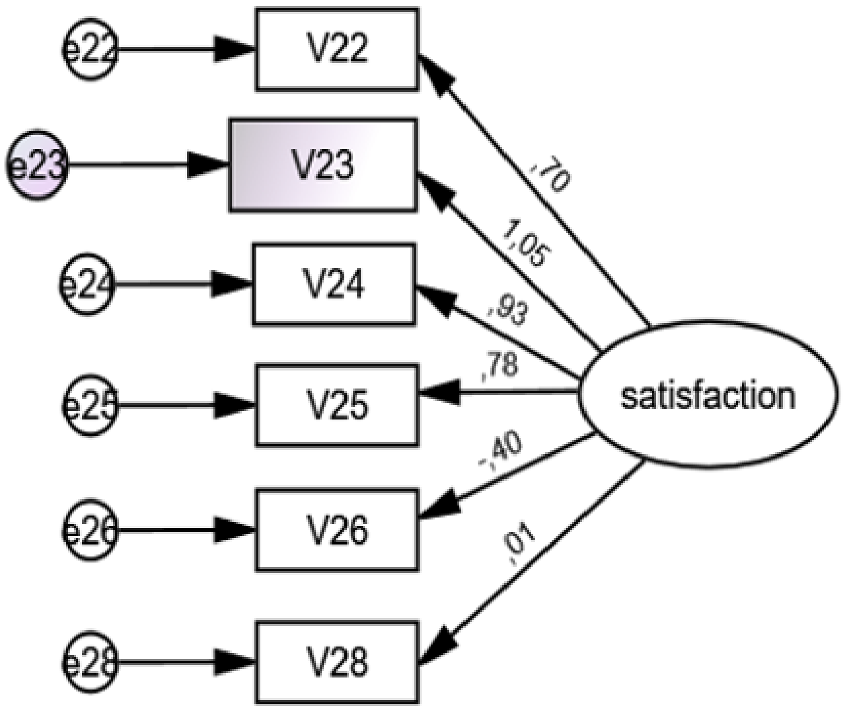
Table 37

Correlations and Cronbach's alpha coefficient for items evaluating the satisfaction construct

Inter-Item Correlation Matrix						
	V22	V23	V24	V25	V26	V28
V22	1.000	-.137	-.161	-.119	-.155	-.087
V23	-.137	1.000	.916	.485	.047	.353
V24	-.161	.916	1.000	.570	-.002	.415
V25	-.119	.485	.570	1.000	.378	.046
V26	-.155	.047	-.002	.378	1.000	.060
V28	-.087	.353	.415	.046	.060	1.000
					Cronbach's alpha coefficient	0.678

Figure 27

Factor loadings for items evaluating the satisfaction construct (Amos output)



5.4.5 Easy-to-use construct

Items 4, 5, 9, 11, 14, 15, 16, 17, and 18 evaluated the easy-to-use construct. Items 4 and 5 focused on the navigation within the applet and between the applets, while items 9 and 11 were concerned with how easy it was to manipulate the applets. Items 14 and 15 were concerned with how easy it was to get the missing lengths and angles respectively. Item 16 requested participants to evaluate if the applets helped them to quickly complete the worksheet. Items 17 and 18 focused on how comfortable and confident the participants felt when they were using the applets.

The results of the confirmatory factor analysis are shown in Table 38 and Figure 28. All items had a factor loading greater than 0.4 from either of the results. All items had high pairwise correlation coefficients except item 15. The item was kept because half of its correlation coefficients were greater than 0.3 (see Table 39) and its factor loading was 0.5. Considering the factor loadings, I concluded that all 8 items were measuring the easy-to-use construct.

Table 38

Factor loadings and variances for items evaluating the Easy-to-use construct (R output)

Item	**Factor loading**
Item 4	0.5
Item 5	0.7
Item 9	0.9
Item 11	0.7
Item 14	0.6
Item 15	0.5
Item 16	0.5
Item 17	0.4
Item 18	0.6

Figure 28

Factor loadings for items evaluating the easy-to-use construct (Amos output) output).

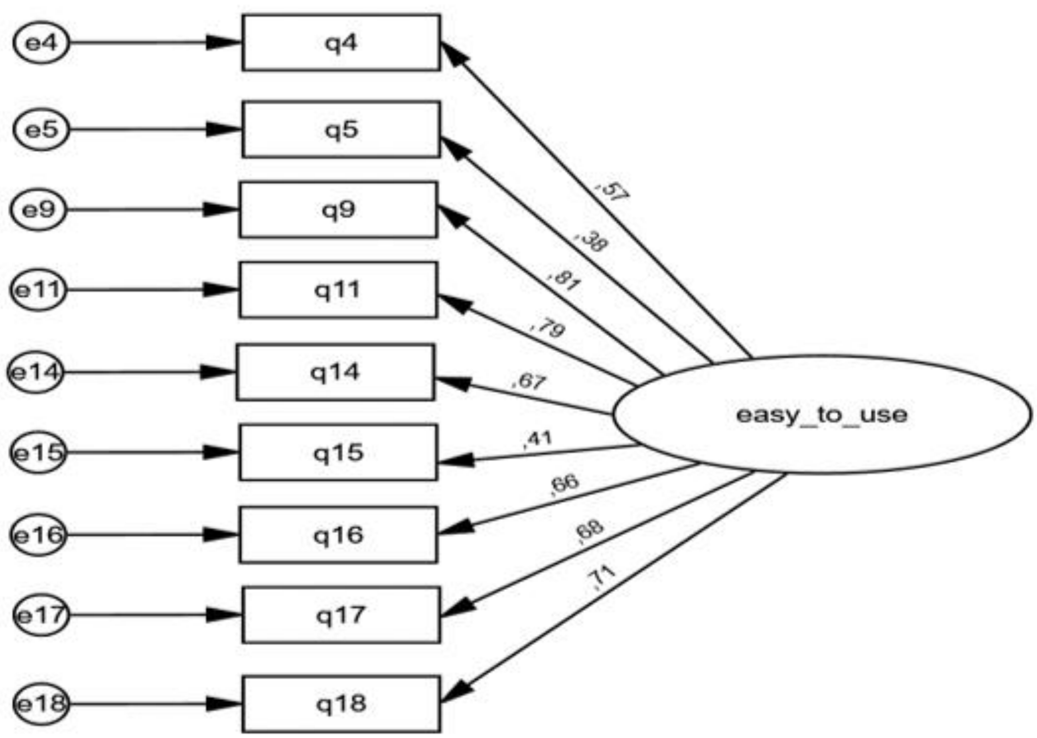


Table 39

Correlations and Cronbach's alpha coefficient for items evaluating the easy-to-use construct.

Inter-Item Correlation Matrix									
	q4	q5	q9	q11	q14	q15	q16	q17	q18
q4	1.000	.063	.332	.586	.349	-.059	.236	.618	.522
q5	.063	1.000	.254	.511	.302	-.080	.363	.090	.225
q9	.332	.254	1.000	.639	.599	.610	.611	.488	.531
q11	.586	.511	.639	1.000	.499	.179	.451	.461	.642
q14	.349	.302	.599	.499	1.000	.360	.320	.462	.527
q15	-.059	-.080	.610	.179	.360	1.000	.410	.331	.072
q16	.236	.363	.611	.451	.320	.410	1.000	.607	.434
q17	.618	.090	.488	.461	.462	.331	.607	1.000	.459
q18	.522	.225	.531	.642	.527	.072	.434	.459	1.000
							Cronbach's alpha coefficient		0.798

Responses to Item 6 were the same for the 25 participants, therefore they yielded a standard deviation of zero, which led to division by zero while calculating the correlation coefficients, resulting in undefined values. Item 27 (*I feel confident to answer questions on congruence of triangles.*) was excluded because it was evaluating a characteristic that was not focusing on the use of the applets.

5.5 Practicability and Learners' views on the benefits of using GeoGebra applets on mobile devices in the learning of mathematics

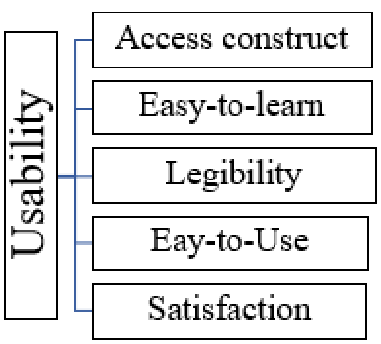
The second questionnaire and interviews focused on answering this third question (*What are the challenges anticipated by learners when they use GeoGebra applets for learning mathematics?)* In this section, I focus on the analysis of the aforementioned research question.

The second questionnaire had closed-ended items that assessed the usability of GeoGebra applets on mobile devices. It also had two open-ended items. One probed the advantages and the other probed the negative aspects of using GeoGebra applets on mobile devices. The interviews explored learners' views on the benefits of using GeoGebra applets on mobile devices in the learning of mathematics. Additionally, it probed the challenges anticipated by learners when using GeoGebra applets on mobile devices. Interview responses also served to validate some results from the closed-ended items.

The second questionnaire was administered after the participants used the GeoGebra applets to complete the task of investigating cases of congruence. It had 28 closed-ended items and three open-ended items. The analysis of closed-ended items was done per construct as explained in section 4.7.5.1. A construct is a characteristic that cannot be measured directly, but can be evaluated using one or more items (Gay et al., 2009). The second questionnaire evaluated 5 constructs that I named access, easy-to-learn, easy-to-use, legibility, and satisfaction. The names of the constructs were based on some of the characteristics that were used to assess the usability of application software (Talirongan & Hernandez, 2017). These five constructs measured the usability of the applets. In other words, they focused on the usability theme. The structure of the theme is displayed in Figure 29. The second questionnaire was assessed for validity and reliability in section 5.4. It was found to be valid and reliable.

Figure 29

Theme structure of closed-ended questions in the post questionnaire



The second questionnaire assessed the plausibility of using mobile technology in the learning of mathematics. Additionally, it sought the participants' perspectives on the usability and perceived benefits of using GeoGebra applets on mobile technology, particularly in the discovery of cases of congruence of triangles. Furthermore, it probed the challenges anticipated by learners when using GeoGebra applets. In the following subsections, I explain the results from the analysis of each construct.

5.5.1 Access construct analysis

The access construct assessed if logging in and opening the applets was easy. The means of the three items were greater than 9. This is an indication that participants did not experience serious challenges with logging in and accessing the applets. In the next paragraphs, I present the results of each of the three items that were used to evaluate the access construct.

The mean for item 1 (*It was easy to log in*) was 9.4. Many participants (68%) scored a 10, 16% scored a 9, 12% scored an 8, and 4% scored a 5. I presume that logging in was not a problem for the participants who scored a 10. I then checked the comments on the open-ended item 30 to see if the participants who had a score of less than 10 indicated some concerns with logging in. Participant 6, who scored a 5, noted that there were challenges in using the applets at the beginning but expected it to become easier with practice: *"Sometimes you may not understand, but if you use them all the time and practice, then maybe it comes easier to you.* This comment shows that the participant hoped to become skillful in using the applets with more practice. Similarly, participant 14 expressed concerns about the instructions that were given by saying *"Logging in instructions did not exactly match the options on my phone. I had to guess but I managed."* Likewise, participants 10 and 25 expressed that it was slow at first. Participant 10 stated, *"It was a bit slow starting off. ..."* Participant 25 said, *"It was slow at first but later on it got better and I managed."* I assumed these two participants were referring to logging in since they mentioned that it was slow when they started using the applets. From the extracts, it is evident that the challenges were encountered only at the beginning. A related sentiment was echoed by the interviewee (voice20) in the following quote, "*They might not know eeh the, for example, login; and they might not even understand what to do so it becomes more difficult for them to even do the work…"* This comment indicates the need for illustrating the login process and how to use the applets. Therefore, it would be necessary to show the learners how to get started. The high value of the mean indicates that there were no major challenges with logging in., thus supporting the usability of the applets.

Item 2 probed how easy it was to open the workbook. The mean was 9.44. Several participants (60%) opened the workbook easily, while 32% scored a 9 out of 10, and 8% scored a 7; again indicating slight glitches for other participants. I checked the comments for the 10 participants that did not score a 10 to see if they had issues with opening the workbook. 7 out of 8 participants who scored less than 10 for variable 1 did the same for item 2. Therefore, the same concerns indicated in the preceding paragraph may also apply here, except for participant 14 who indicated that it was a bit slow starting. Participant 7 indicated a rare challenge when she said: *"The website would shut down for a small period of time."* The other participants did not give comments directly related to opening the workbook. This again, reflects that the applets were easy to open.

Item 3 enquired how fast it was to open the applets. Several participants (52%) indicated that the applets opened quickly, while 24% scored a 9, 16% scored an 8, and 8% scored a 7. Participants 4, 6, 10, 18, and 25 scored less than 10 for the 3 items in the access construct. Even though the items were specific, responses for participants 6, 10, and 25 were general and could be applied to all 3 items. Therefore, I do not repeat the comments that were expressed in items 1 and 2. Participant 21 indicated that it was slow to open a new applet when she said: *"It took a bit of time to load a new applet."* The fact that she scored 9 out of 10 indicates that the challenge was not serious.

Looking at these responses, it can be seen that there were no major challenges with accessing the applets. To further support this, the overall mean for the access construct was 9.35. The high value of the mean reflects that the participants could easily log in, and open the workbook and applets quickly. This was confirmed by participant 2 who indicated in her response to open-ended item 31 that the applets were easy to access, they provided information effectively and they also loaded quickly. Interviewee voice07 affirmed this in the following extract, "*…it doesn't take much of your time because, as the pages would load faster….*" The other participant even indicated in her response that the speed got better. All the participants who cited the problem used different phones, therefore, I could not associate this problem with the type of phone. I recommend future studies on the association between the performance of the applets and the type of phone. The issue of log-in speed could be attributed to the performance of the phone and/or strength of internet connectivity, though further investigations would be needed to substantiate this assumption. I believe that the high value of the mean was attributed to the learners' prior knowledge (technology proficiency). The learner aspect in the FRAME model stipulates that the learners' prior knowledge helps learners to easily comprehend new content. Similarly, I believe that the participants' technological proficiency helped them to log in and access the applets easily. The high value of the mean indicates that the applets adhered to the focus of the device usability intersection of the FRAME model. It reflects that mobile devices were easy to use, and allowed easy and quick accomplishment of tasks.

5.5.2 Easy-to-learn construct analysis

Learnability is the degree to which the user can quickly learn to use a new system (applets), remember how to use it, and quickly become skillful in using it (Lund, 2001; Ryu, 2005). In this subsection, I present the results and the analysis of the construct that assessed how easy it was to adapt and learn to use the applets. The construct was evaluated using 4 items. The means for three out of the four items were greater than 9, except for the mean for item 12 which was equal to 8.88. In the following paragraphs, I discuss the results of each of the four items and related responses to open-ended items and interviews.

Item 7 focused on how easy it was to learn to use the applets. The mean score for item 7 was 9.16. The percentage of the participants who scored a 10 was 48% and the minimum score on this item was an 8. Even though the percentage of participants who scored a 10 was low, I believe that a minimum value of 8 implies that the challenges encountered were manageable. This postulation was supported by participants 9, and 11 who noted, in their responses, that it was easy to learn. Also, in response to the open-ended item 31, participant 3 said, "…*The use of technology is quite popular in the modern-day world therefore I am comfortable using this applet as it is easy to manage. …"* This participant associated her skills with her prior knowledge of using mobile devices. In one of her interview responses, Voice12 supported that the applets were easy to learn in the following extract, "*Well some people don't have the privilege to use, they don't have phones or anything, so they might not be able to know how to control it but it is easy to learn.*" This interviewee believed that even learners who were not technologically proficient could easily learn to use the applets, thus supporting the practicability of using applets on mobile devices for teaching and learning. In their responses to item 31, participants 9 and 11 also indicated that the applets were easy to learn. Participant 1 also added, *"They were easy to use and understand. They were straightforward."* The word straightforward implied that the applets were easy to learn. Interviewee 20 stated that she understood what to do, the instructions, and the terminology that was needed to complete the worksheet. This supports the notion that the use of mobile technology boosts psychological comfort and reduces cognitive overload while completing tasks.

However, on the question that probed the anticipated challenges, some interviewees were of the view that the use of applets could be challenging to learners who are not technologically proficient. Voice19 also echoed that other students could take longer to understand. *"Some of them may take longer to understand and …"*

The results from the closed-ended items were consistent with most responses to the open-ended items and interviews. From the results, it was clear that the applets were easy to learn. Therefore, the learners would enjoy using the applets and concentrate more on the task. This would create a conducive environment for understanding concepts. These basic findings were consistent with the research by Torun and Tekedere (2015) who indicated that users normally enjoyed software that is easy to learn since it enabled them to complete tasks quickly. Software that is easy to learn is usually easy to remember, and hence; easy to develop some expertise in using it. These results affirmed the establishment by Morphett et al. (2015). They indicated in their study that applets are easy for users to master, with very little software knowledge and without training (Morphett et al., 2015). The fact that no comment directly said it was difficult to learn or understand how to use the applets further supports that the applets were easy to learn.

Item 8 rated how easy it was to remember using the applets. The mean score was 9.44 and the minimum score was 7. Most participants (76%) scored a 10. These high values reflect that the applets were easy to remember. A minimum score of 7 implies that the participants could remember to use most aspects of the applets. In her response to an open-ended item, participant 18 stated that she forgot how to get the angles, but only remembered how to get them after reading the instructions. This response supports the assertion that the participants could remember using most aspects of the applets. To further support this, participants 9 and 11 indicated (in their responses to the open-ended item 31) that they could easily remember how to use the applets.

The high values of the mean and the percentage frequency of the participants that scored a 10, and the minimum score of 7 reflected that these applets did not cause cognitive overload. Thus, the learners did not have to learn everything over again, when they return to use the software after a while. Therefore, the applets met some criteria emphasized in the learner aspect of the FRAME model.

Similarly, item 10 assessed whether it was easy to become skillful in using the applets. The mean was 9.56. Several participants (64%) scored a 10 and the minimum score was 8. These high values of the mean reflect that it was easy to develop expertise in using the applets. The following comments support that it was easy to become skillful in using the applets:

Participant 6: "*Sometimes you might not understand, but if you use them more all the time and practice, then maybe it comes more easier to you*." It is clear that the participant anticipated to become more skillful with time.

Participant 18: "*Angles were a bit trick at first.*"

Participant 24: "*I got wrong angles at first, but read instructions and everything was working well.*" The two statements show that the participants had challenges at first, but developed the skill as they were using the applets.

The results of item 7 (*I learnt to use the applets quickly*) were in concordance with the results of item 10 (*I find it easy to become skillful in using these applets*). Therefore, software that is easy to learn is usually easy to remember and to become skillful in using it. This concurs the findings of Torun and Tekedere (2015).

Item 12 asked participants if their fellow learners would find it easy to use the applets. The mean value was 8.88. Few participants (32%) scored a 10, 32% scored a 9, 28% scored an 8, and 8% scored a 7. There was no comment related to this item from responses to the open-ended items. This item had the lowest mean and lowest number of 10s. I got curious why it was not presumed to be easy for other learners in the same grade to use the applets. The participants who scored less than 10 on item 12 had either scored less than 10 on items 7, 8, or 10. This indicated that they had faced some kind of challenge in using the applets, therefore, they would also expect the learners in the same grade to have the same experience. That is why they had a score of less than 10.

The overall mean for the construct easy-to-learn was 9.26. This was an indication that the applets were not difficult to learn. To sum up the analysis of the easy-to-learn construct, the participants posited that the applets were easy to learn and remember. They also presumed that their fellow learners, who were technologically proficient, would easily learn to use the designed applets. This presumption is supported by another research's findings, which indicated that the group that used applets (constructed figures) did not face technological challenges as the group that had to construct the geometrical figures (Mavani, Mavani, & Schäfer, 2018). I could not rule out the fact that some learners were not proficient in using mobile technology. I would expect today's generation of learners to quickly adapt and explore the features of mobile devices, and that one session to illustrate the use of the applets would suffice.

5.5.3 Legibility construct analysis

The legibility construct assessed the visibility of the diagrams, labels, and measurements. This construct was evaluated using three items.

Item 19 asked whether the triangles were clear. The mean was 9.6. Most participants (84%) scored a 10, while 8% scored a 9, and another 8% scored a 6. No challenge was mentioned concerning the visibility of the triangles. The high value of the mean and the percentage of the participants who scored a 10 indicates that it was possible to display triangles on mobile devices. Therefore, the mobile devices that were used by the participants satisfied the required physical characteristics that were stipulated in the FRAME model that promotes mobile learning. This confirms the practicability of exploring mobile technology in the teaching and learning of mathematics.

Item 20 probed the visibility of the labels. The mean was 9.32. Many participants (64%) scored a 10, 8% of the participants scored a 6, another 8% scored an 8, and 20% scored a 9. Participant 13 echoed a general comment on visibility concerning the color of the measurements in the following statement: " *The light green can be hard to see over the white background*." The issue of color should be taken into consideration in future designs. Light font color should be used on a dark background and vice versa. The meaning of colors should also be considered when selecting colors to use.

Item 21 focused on the visibility of the measurements (the angles and the lengths and the angles). The mean value was 9.28. Several participants (56%) scored a 10, 8% of the participants scored a 7, 12% scored an eight, and 24% scored a 9. In line with the visibility of measurements, participant 9 stated that angles were not always very clear, and that there were screen compatibility issues. I assumed that the screen compatibility statement referred to the screen size that made the angle measurements difficult to read on a small screen. In the design guidelines, it is stipulated that applets should be tested at the same type of devices that the users are going to use, so that they can be adjusted for specific devices. This was put into consideration because the applets were tested on four devices of different sizes, where the i-phone SE was the smallest device with a diagonal screen length of 4.7 inches (11.94 cm). The visibility problem could have been attributed to the font color that was mentioned by participant 13. The fact that participant 9 scored an 8 on item 21 (*Triangle measurements were easy to read*) shows that the screen size challenge was manageable. The high value of the mean and the minimum value of 7 support the suitability of the existing mobile technology in teaching and learning mathematics.

The overall mean for the legibility construct was 9.4. This was an indication that the challenges were manageable. I would have expected the mean of the construct to be less than 5 if the challenges had a huge impact on completing the task. The high mean rating indicates that there were no major concerns about legibility. This point was supported by interviewee Voice17 who commented that she could see the triangles and dimensions clearly when she was using the applets. WR also said that the applets showed the angles clearly. It is clear from these results that the output capabilities of mobile devices support the use of mobile devices in the teaching and learning of mathematics. However, Voice10 cited a possible challenge with screen size in the following statement, "*… if the screens are small it could be difficult for them to see properly."* Because the other participants who used phones with small screens did not indicate this challenge shows that the issue of screen size was manageable.

In the next paragraph, I explain the results of the construct that focused on how easy it was to use the applets.

5.5.4 Easy-to-use construct analysis

The easy-to-use construct assessed how participants felt about the use of the applets. For example, if they could easily undo a mistake, or if they could easily measure the missing lengths and angles. Items 4, 5, 9, 11, 14, 15, 16, 17, and 18 measured the easy-to-use construct. The following paragraphs present the analysis of the nine items that evaluated the easy-to-use construct.

Item 4 assessed if the touch screen was effective. This item assumed that all participants were going to use a device with a touchscreen. The assumption was based on the responses to the item in the first questionnaire that asked the participants the type of mobile device available to them. The mean for item 4 was 9.48. The majority of the participants (76%) scored a 10, indicating that they were happy with their touchscreens. Two participants (8%) scored a 9, while 16% scored an 8 and 4% scored a 7. In response to the open-ended item 30, participant 1 stated that her touchscreen was sometimes ineffective but still worked well. This was expressed in the following response: *" The touch screen was sometimes ineffective, but it still worked well"*. This statement indicates that the touch screen challenge was manageable. Likewise, participant 2 had a similar notion which she expressed in the following response: *"...Touch screen was not very effective"*. Similarly, participant 10 echoed that points for measuring the sides were difficult to click in the following response: "… *It is a bit hard to click on points measuring sides*". I considered this challenge to be related to the touchscreen since the participant used a Samsung Galaxy S8, which is a touchscreen device. Based on the fact that 76% of the participants scored a 10 and those who had a score of less than 10 on item 4 did not state specific challenges related to the touch screen, I presumed that the challenges they faced were minor, and they considered them not worth mentioning.

The high values of the mean and the percentage of participants that scored a 10 showed that the input capabilities of most mobile devices satisfied the requirements of the device aspect stipulated in the FRAME model as providing a conducive mobile learning environment.

The applets were put in one online GeoGebra book with four chapters (refer to 4.5.5.1 for the description of the GeoGebra book). Each chapter focused on investigating a specific case of congruence. The chapter had either one or more applets. Item 5 enquired on how easy it was to move from one applet to the other. The mean was 8,52. Many participants (56%) scored a 10, 8% of the participants scored a 5, then 36% of the participants scored a six, and 12% scored a 9. Out of the 11 participants who scored less than 10, only participant 7 made a statement that could be related to the challenge of moving from one applet to the other when he said: "*The website would shut down for a small period of time*". The other 10 participants did not mention anything related to the challenge of moving between the applets. The fact that the participants did not pinpoint the challenge of moving between applets in their responses to the open-ended item that requested the negative aspects encountered while using the applets, was an indication that they did not consider it a serious challenge that was worth mentioning.

These results indicate that the functional characteristics of mobile devices allow the use of mobile devices in teaching and learning mathematics. The next item assessed whether it was simple to use the applets.

Item 9 assessed how simple it was to use the applets. One may be tempted to think that this was a redundant item since it was representing the construct itself. However, it was included to try and probe further on the aspect. Users usually do not appreciate new application software if it is difficult to use. The mean for this item was 9,36. Many participants (68%) scored a 10 on item 9, 16% scored a 9, and 8% scored a 6. The high value of the mean was an indication that the applets were easy to use. This presumption was supported by participants 7, 13, 14, and 24 in their responses to the open-ended item 31. They all included the phrase "easy to use" in their responses. These results showed that using these applets on mobile devices provided a conducive environment that promoted psychological comfort, and allowed learners to focus more on the task. This then promoted the understanding of the concept and also increased the task completion rate. The comment by participant 23 also supports the notion that the applets were easy to use. She stated that: "*It was easy to complete the worksheet."* These results from the post questionnaire concurred with the interviewee responses. Most interviewees believed that the applets were easy to use. Voice07, Voice10, Voice12, Voice19, and WR included the phrase, "*It is/was easy to use"* in their responses. Voice12 even added an adjective in her response, to emphasize how easy it was to use the applets. She stated, *"It was very easy to use…"* Voice17 shared the same sentiment in her responses. She said, "*It was more easier for me to get the answer …*" Additionally, in her response to the question that probed the expected challenges that the fellow learners could have, she affirmed the same sentiment in the following comment *" No I don't think so because it is very easy and fun to use."* To further support her sentiment, she also said *"I enjoyed the fact that when I pressed the part for the triangle, it gave me fast responses and I was able to fill in the paper quickly because they gave out quickly."*

However, on the question that probed the anticipated challenges, some participants were of the view that the use of applets could be challenging for learners who were not technologically proficient. Voice019 also echoed that other students could take longer to understand. *"Some of them may take longer to understand and …"* Considering the high value of the mean and some interviewee responses, I believe that even those who are not technologically proficient would manage to use the applet after an illustration session. Participant 11 stated that there was a lack of written explanation. According to my view, the participant was referring to lack of written explanation on the applet pages. I did not include any writing on the applet pages because, in the design guidelines, it was advised not to include a lot of writing on the applet pages, as the writing would affect the speed of the applets.

Participant 21 noted something related to using the applets in the following extract: *"I did not understand at the start, but later it became easier".* I linked this challenge to the lack of a written explanation that was pointed out by participant 11. Other challenges included logging in issues, measuring angles, and lack of speed in accessing the applets. Looking at the mean value and also at the fact that no one indicated that the applets were difficult to use, I would say that it was simple to use the applets. All interviewees supported this presumption. Voice07, Voice10, Voice12, Voice19, and WR indicated that GeoGebra applets were easy to use. Voice07 added that they were effective. Voice19 included that she felt comfortable in using the applets. These statements reflect that both the physical characteristics and the applets complied with the technical characteristics stipulated in the device usability intersection (DL) of the FRAME model. It is evident from the participants' comments that the use of mobile devices promoted psychological comfort, which in turn, enhance understanding and retention of new content.

Item 11 asked the participants to evaluate if it was easy to undo a mistake. The mean was 9.6. Most participants (80%) scored a 10, 8% scored a 9, the other 8% scored a 7, and 4% scored an 8. All five participants who had scored less than 10 did not indicate any challenge related to reversing action, which further affirmed that it was easy to undo a mistake. This showed that the applets were user-friendly. This was in line with what was indicated by Torun and Tekedere (2015) who stated that one of the characteristics of user-friendly software is a facility to reverse actions or undo a mistake. Based on the high values of the mean and the percentage that scored a 10, I supposed that it was easy to undo the mistakes. The fact that there was no complaint about reversing an action further supports this. I also did not anticipate any problem with this because the GeoGebra tool to reverse action is similar to tools of other application programs. I expected participants who were proficient in using technology to know the purpose of the tool.

The participants were given a task where they measured the missing sides and angles. Item 14 probed if it was easy to measure the missing sides of the triangles. The mean was 9.64. Most participants (84%) scored a 10, and 4% scored a 9, the other 8% scored an 8, and 4% scored a 6. Participant 10 is the only one who indicated that there was a challenge in measuring the sides of the triangles in the following response: *"... It is a bit hard to click on points measuring sides*". The high values of the mean and the percentage that scored a 10 indicated that most participants could easily measure the missing lengths of the triangles. These high values support the usability of the applets on mobile devices in teaching and learning mathematics.

Item 15 assessed whether participants could easily measure the missing angles. The mean was 8,96. The minimum score was 5 and 44% of the participants scored a 10 on this item. A few participants (28%) scored a 9, then 8% scored an 8, followed by 20% who scored a 7, and 4% who scored a 5. Participants were supposed to read the instructions on the first page of the worksheet on how to measure the angles. It is evident from some of the comments that some participants tried to get the angles without following the given instructions, that is why they initially had challenges. Participants 12, 17, 18, and 24 indicated that they initially got the wrong answers before they read the instructions. This shows that the instructions to measure the angles were clear, and that it was also easy to measure the angles. Participant 24 confirmed that, after reading instructions, everything worked well. This implies that after reading the instructions, there were no challenges in measuring the angles. This was confirmed by participants 15, and 19 who also indicated that it was tricky to measure the angles at first. The inclusion of the word "at first" implied that the challenge did not persist. To further support this, the participants had correct angles in the worksheet. Participant 6 was also amazed by how easy it was to get the angles. She stated, *"Easy to find angles on a touch of a screen."* Participant 21 stated that she got all measurements easily. These comments indicate that the standards of mobile learning stipulated in the FRAME model were achieved.

Item 16 assessed whether participants were able to quickly complete the worksheet using the applets. The mean was 9,26. Several participants (56%) scored a 10 and the minimum score was an 8. A comment was made that the worksheet was long. Considering the minimum score, I would say that even though the worksheet was long, the applets helped the participants to quickly complete the worksheet. To support this, several participants stated that applets helped them to quickly complete the worksheet. Participants 4, 5, 13, 15, 16, 20, and 25 had responses indicating the fast rate of either getting the measurements or completing the task. Participant 4 stated, "*The answers can be found quickly…".* Relatedly, participant 5 said, "*It is more accurate and is very helpful as you can find answers quicker."*

Since the worksheet involved measuring missing angles and missing lengths, getting measurements quickly implied an increased worksheet completion rate. Participant 16 said that: "*Even though the worksheet was long, I managed to complete quickly using the applets. I don't know how long it was going to take if I was using ruler and protractor.*" This response reflects the participant's appreciation of the proposed strategy and its benefit of increasing task completion rate compared to the use of the ruler and protractor. Participants 15 and 20 indicated that the applets helped them to quickly complete the worksheet. Participant 15 stated, "*It helped to quickly complete the worksheet.*" She also complained that the worksheet was too long and she had to use more data. These two responses indicate that, even if the worksheet was long, the applets improved the task completion rate. Participant 13 said, "*It makes maths go much faster.*" I believe that this statement referred to both understanding and completing the task faster.

The results of item 17 reveal whether the participants felt comfortable in using the applets. The mean for item 17 was 9,72. Most participants, (80%) scored a 10, followed by 8% who scored an 8 and 12% who scored a 9. Considering the high mean value, the percentage that scored a 10, and the minimum value of 8, I concluded that most participants felt comfortable when they were using these applets. These results from the closed-ended item were confirmed by some interviewees. Voice19 and Voice20 noted, in their responses, that they felt comfortable using the applets. Voice 19 stated, "*It was really comfortable…*". Voice20 said, "*It was very comfortable…*" The use of the adverbs "really" and "very" add some emphasis on how easy it was to use the applets, thus, confirming the notion that the applets were easy to use.

Item 18 checked whether the participants felt confident when they were using applets. Most participants (80%) scored a 10, followed by 4% who scored a 7, then 8% who scored an 8, and another 8% who scored a 9. The mean was 9.64. The high values of the mean and the percentage of participants who scored a 10 showed that they felt confident when they were using the applets. These results coincide with the results for item 17. This indicates that feeling comfortable and confident were highly correlated.

The overall mean for construct 'easy-to-use was 9.36, and 7 out of 9 items had a mean greater than 9. Items 4 and 15 had a mean of less than 9. These items assessed the effectiveness of the screen and whether participants could easily measure the angles respectively. Considering the overall mean, the item means, and the responses to open-ended item 30, I concluded that the applets were easy to use. This conclusion supports the practicability of the proposed strategy.

5.5.5 Satisfaction construct analysis

Satisfaction is closely related to the actual or predicted usage of the system (Lund, 2001). In his study, Lund (2001) established that users get satisfied with the software if it meets their expectations and if they get expected results. They will enjoy using it and find it fun to use. In this study, applets were expected to help learners quickly complete the worksheet and understand the concept of congruence. The satisfaction construct was assessed using items 22, 23, 24, 25, 26, and 28.

Item 22 checked whether the participants got the expected answers. The mean was 9.84. Almost all participants (92%) got the expected answers. The high values of the mean and the percentage of participants that scored a 10 indicate that the applets met the learners' expectations. "*…Provides information efficiently….*"

This was also confirmed by participant 6 in one of her open-ended responses. She stated, "*Lengths and sides were the same as expected.".* Relatedly, participant 24 also stated, *"The applets worked and …"* This was also supported by interviewee 17 in one of her responses, that, *"… my answers were accurate and right…"* Participant 2 also mentioned that the applets provided information efficiently. The adverb "efficiently" means that the applets produced expected results. Surprisingly, participant 5 scored a seven on this item but in the comments, she had nothing related to the expected answers. She indicated that the applets gave all the answers and did not give the learners room to think of the answers. I expected 100% of the participants to score a 10 but 92% did.

Surprisingly, the two participants who did not score a 10 on this item never mentioned anything concerning the measuring of the missing lengths or angles. Answers on the worksheet were all correct, reflecting that participants got expected answers. Therefore, mobile technology could be used in completing tasks that could take much longer to accomplish using pen and paper.

Item 23 probed whether the participants felt satisfied. The mean for this item was 9.44. Most participants (80%) were satisfied with the applets, while 12% of the participants scored an 8 and 8% scored a 6. It is evident from the high values of the mean and the frequency of the participants who scored a 10, that the participants were satisfied with the applets. To affirm this, there were no statements that expressed dissatisfaction among the responses to open-ended item 30. Satisfaction-related sentiments were expressed by most interviewees. In her response, Voice17 indicated that she enjoyed getting quick answers. *"I enjoyed the fact that when I pressed the part for the triangle it gave me fast responses and I was able to fill in the paper quickly because they gave out quickly."* WR also indicated that she enjoyed working with her phone to complete the worksheet and that she got fast responses. She also added in the other response that technology would make mathematics enjoyable and easy to understand. Voice010 testified that the applets were working fast when she said, *"I enjoyed that the results came quickly and …"* Voice07 asserted the same sentiment in the following excerpt. *"it doesn't take much of your time because, as the pages would load faster…"* She also indicated in another statement that the applets were effective and efficient. In her responses, Voice19 said that she got expected answers that were accurate and precise. She expressed these sentiments in the statement *"… I got the results that I wanted."* She further emphasized this in another statement, "*It's more precise and give you exact answers…"* She also added that *"It is more …accurate and .. gives you the best answers…"* These statements reflect that the participant was satisfied. The results tell us that the participants would be more receptive to using mobile technology in learning mathematics.

Some participants and interviewees were, however, concerned that the applets did not give room to figure out the answers. Participant 5 stated, *"Gives answers too easily"* From this response, I infer that the participant wanted some activity that would allow them to figure out the answers. Similarly, Voice17 stated, *"I suggest that maybe they don't give the answer immediately, they have a part where they let the learner attempt the question first, then they tell them when they are wrong* *or right and if you are wrong they explain why and what you were supposed to do in that part."* This response suggests the use of the tutorial type of applets which was, however, not the focus of this study. Relatedly, Voice19 stated, *"Maybe you shouldn't make it like.. solve quickly. Sometimes you need to think a little more because then … like get the answer."* These responses enlighten other ways the participants expected to use GeoGebra applets besides the exploration of the concepts. All in all, they acknowledged the potential of using mobile technology in learning mathematics.

Considering the high value of the mean and the responses that were shared by the interviewees, I concluded that the participants were satisfied with the applets. That affirms the practicability of using GeoGebra applets on mobile devices in teaching and learning mathematics. Users that are happy with a product are most likely to recommend it to the next person.

Item 24 inquired if participants would recommend the same applets to their friends. In his study, Wong (2015) established that users are encouraged to use a product and are likely to recommend it to others if they perceive it to be useful. Considering the results from item 25 (*I feel the applets help to understand cases of congruence*) and item 26 (*I feel the applets help to identify/see cases of congruence of triangles*), the applets were useful. The mean for item 24 was 9,52. The majority of the participants (80%) indicated that they would recommend the applets to their friends. Few participants (8%) scored a 7 and 12% scored an 8. The 5 participants who scored less than 10 on this item were the same participants mentioned in the previous paragraph.

The applets were designed to discover and understand the cases of congruence of triangles. Item 25 was included to probe whether the applets would help to understand cases of congruence. The mean score was 9,64. The majority of the participants (80%) scored a 10, followed by 4% who scored a 6, then 4% who scored an 8, and 12% who scored a 9. The high values of the mean and the percentage of participants that scored a 10 indicate that the applets helped to understand the cases of congruence of triangles. This was affirmed by interviewee Voice12 who stated that, *"…I learnt more maths .. and it was easy to understand the concept using the app."* Similarly, Voice20 stated that she understood the terminology that was needed to complete the worksheet. *"you understood what to do and the rules that you had to follow and understanding terminology that was needed to complete the worksheet."*

On the contrary, some participants thought that the applets gave them easy answers and did not allow them to think or get the answers. Participant 4 stated, "*It gives answers away too easily, the user doesn't really think or attempt the problem on their own*." As stated earlier, this was not the focus of the proposed strategy, which instead, focused on discovering cases of congruence.

Item 26 probed participants' views on whether the applets would help to establish the cases of congruence. The majority of the participants (84%) scored a 10, while 8% scored an 8 and the other 8% scored a 9. The mean for this item was 9.67. The high value of the mean and the percentage of participants who scored a 10 indicates that the applets helped to see the cases of congruence. This notion was supported by some participants in response to the open-ended item. Participant 8 stated, "*It gave me a visual representation to assist me in my investigation.*" Participant 13 said, *"I get to see what I am working with easily…."* Similarly, participants 10 and 14 said that GeoGebra applets helped them to see the cases of congruence. Participant 10 added in her response that the understanding was enhanced by the use of different measurements. She stated, *"It helps to see the cases of congruence using different measurements."* These responses support that the use of GeoGebra applets allows visualization of concepts and promotes understanding, thus making the implementation of the proposed strategy relevant.

Item 28 asked the participants if they would like to use mobile technology more often when learning mathematics. This item had a high mean of 9.92. Almost all participants (92%) scored a 10 and 8% scored a 9. This indicates that the participants would like to use mobile technology more often when learning mathematics. This notion was supported by some interviewees who indicated that they would like to use mobile technology and they wished the school could adopt the use of mobile technology in learning mathematics in response to the question "*Is there anything else you would want to share or say about using mobile technology for learning mathematics.*" Voice12 stated, *"I think that we should use it more often and I really wish that our school offered using mobile technology in maths and the stuff."* It is evident from this statement that the participant would embrace the proposed strategy of using GeoGebra applets on mobile devices. Voice10 said, *"I think it should be used in schools because it will spark an interest in the learners."* It is encouraging to note that the participant pointed out the benefit that would come from using mobile technology in teaching and learning mathematics, thus; showing that she would appreciate the proposed strategy.

Voice07 stated, *"In this day in age as technology is improving you can easily teach yourself maths on the applet, it will help you because you can keep it like your pace, it gives you, efficient …"* This statement included the benefits of using mobile technology. From the statement, it is evident that the participant believes that using applets on mobile devices would allow more content-learner interaction time. Additionally, it would also allow learners to engage with content anywhere and anytime. Learners would be able to learn at their own pace thus, giving them more time to understand the concepts. Voice17 even suggested the use of GeoGebra applets to learn other sections. She said, "*I wish it could help for the algebra part, algebra yes, like the factorisation and all of that ..they show you the steps you need to follow to make it easier to follow like what we did for congruence and the rest of geometry*." This statement reflects that the participant was satisfied to the extent that she wished the applets could be used in learning other sections. From the statement, one can tell that the participant would be receptive to using mobile technology in teaching and learning.

Several sections were mentioned in response to the question, "*Are there maths sections in which you think mobile technology may assist in learning and understanding?*" Geometry was mentioned the most, possibly because they had seen the use of applets on learning congruence of triangles which falls under the Geometry section. Algebra was also mentioned by many interviewees. Some suggested it could be used to understand algebraic expressions and equations. One interviewee suggested that mobile technology could be used to understand patterns. Interviewee Voice20 mentioned three other specific topics that could benefit from the use of GeoGebra applets in the following verbatim, "*Graphs, understanding gradient, understanding equations, all those things, maybe technology will help us understanding how to find the gradient, how to find the equation so that it does not become hard as it is right now."* It is evident from this statement that the interviewee believed that the use of mobile technology would assist learners to understand mathematics. The participants saw the potential of using GeoGebra applets on mobile devices in learning many different sections. This shows that they would embrace the proposed strategy in learning mathematics and also rendered the proposed strategy feasibility and relevance.

The overall mean of the satisfactory construct was 9.72. The high value of the mean indicates that the participants were satisfied with the applets. The mean of item 23 (*I was satisfied with these applets*) was 9.44. This mean was also high and it supports the validity of the overall mean.

5.6 Anticipated challenges by learners when using GeoGebra applets for learning mathematics

The anticipated challenges in using mobile technology in the teaching and learning of mathematics were picked from the responses to item 30 of the post questionnaire and the interviews.

The issue of data availability was indicated by many interviewees. Voice07 stated, *"It is easy to use, provided you have data…"* The same interviewee in response to the question, *Are there other challenges you think fellow learners would encounter when using (the same) mobile technology?* said, "*Regarding that, they have data for a smartphone then it would be easy for them to access, but if they don't have those two then I think it will very difficult for them to use it*." She even suggested how the issue could be alleviated in the following response, *"I suggest that instead of having it as a page, maybe the site could be an app so that you can download it and use it offline which will give an advantage to children who can't necessarily afford data to use this page all the time."* Voice10 shared the same view as Voice07 in the following statements, "*I think the only challenge would be data"* and *"An alternative would be making it an application (where) you could work offline."* Relatedly, Voice12 stated, *"Ehh, could use the app offline so that we don't have to use the internet every time you use the app, …"*

Besides the affordability of data, the non-availability of mobile devices and learners' lack of ability to use them were also mentioned as expected challenges. Voice20 said, *"Well some people don't have the privilege to use, they don't have phones or anything, so they might not be able to know how to control it…"* Relatedly, Voice19 said, *"...you might not even have access like to a phone or actual mobile…"* Similarly, Voice07 indicated the need for mobile devices as a disadvantage by saying, "…*and you also require smartphones in most cases…"* The same participant mentioned that there could be problems with the touch screen which could cause one to keep tapping on it. Voice10 said that small screens could make it difficult to see properly. Voice20 remarked that some learners may not know how to use technology and thus, can face challenges of logging in and not understanding how to use the applets. Such learners would be disadvantaged since they will not be able to help themselves with mathematics. Relatedly, WR shared the same opinion saying, "*As much as teenagers use technology, not everyone is equipped…*" It was not clear whether the word equipped meant having the mobile devices or the skills to use mobile devices. I presumed that the statement meant both. Additionally, Voice19 said that some learners could take longer to understand how to use the applets. Voice17 suggested that it would be better to first teach the learners how to use the technology before they use it for mathematics. The participant meant that the learners should be taught the basics of operating mobile technology, for example; logging in and accessing different applications. As indicated earlier, I presumed that one session (less than an hour) would suffice to show the learners how to log in and to use any designed applets.

Data affordability emerged as one of the major anticipated challenges in using GeoGebra applets. However, some participants indicated that it would be better if the applets could be downloaded and used offline

5.6.1 Summary of the responses to item 30

Item 30 focused on the negative aspects of using the applets. Even though the responses to item 30 were referred to in previous subsections, I also summarise them in Table 40.

Table 40

The summary of responses to item 30 of the second questionnaire

Concern	% of participants who echoed the same sentiment
Touch screen not very effective	8%
Issues of buying data	16%
Using electricity (when using a personal computer)	4%
The opening of applets was a bit slow	16%
Applets gave answers easily	8%
It was difficult to understand how to use the applets at first	8%
The site would shut down while working	4%

Logging in was a bit slow	4%
Visibility of measurements (due to the color used and screen size)	16%
Measuring length	4%
Measuring angles	24%
Lack of instructions on the applets	4%
Logging in steps not matching	4%

It is encouraging to note that there were no issues mentioned by 50% or more of the participants. This further supports the practicability of using mobile technology in learning mathematics. It also indicates that learners with the same characteristics would embrace the proposed strategy.

5.7 Summary

In this subsection, I summarise the usability evaluation results. The means for the five constructs that assessed usability were 9.35 (access construct), 9.26 (easy-to-learn construct), 9.4 (legibility construct), 9.36 (easy-to-use construct), and 9.67 (satisfaction construct). According to the interpretation guide that was explained in section 4.7.5.1section, all five constructs had a high positive rating. The usability mean was 9.43. The high value of the mean suggests that the applets were easy to use. The fact that all participants completed the worksheet supports the usability of the applets. Responses to the open-ended items also affirmed this. Furthermore, there was no score below 5, and there was no single response to the open-ended items that expressed the impossibility of using the applets. What was most encouraging was that the participants were able to resolve most of the challenges that involved the use of the applets. For example, logging in instructions and options that were given did not match one participant’s options but she managed to figure out for herself and logged in. Another participant indicated that she had problems measuring angles before she read the instructions but she could get the angles thereafter. This ability for the participants to maneuver by themselves affirms that the current generation of learners was technologically proficient.

It was encouraging to note that most of the suggestions that were indicated by the participants in response to the first questionnaire item (*What do you think your teacher should do that can help you understand Maths better?)* were met by the proposed strategy. They indicated that teachers should use teaching strategies that are motivating, engaging, that allow the visualization of concepts, facilitate explanations, and improve understanding of concepts. It is evident from the discussion in this section that all these expectations were met.

The mixed-method (QUAN-QUAL) model implemented in this study strengthened the credibility of the research. It is apparent in the discussion of the results that the findings from the closed-ended items, open-ended items, and the interviews were similar. These findings were dependable. I believe that similar findings can be obtained if the analysis is repeated. The non-availability of mobile devices, internet access, and data were the common challenges that were likely to surface if the research was repeated. Besides that, the current generation of learners was receptive and could quickly learn how to use technology, which made the use of mobile technology in learning mathematics more practical and supported the transferability of the research. The GeoGebra applets are easy to learn and use without the need to know the software. A thick description of the research processes carried out in this study was given in chapter four. This would allow other researchers/readers to determine if the context in this study matches their situation, or if the findings could be transferred to their context. A detailed explanation of data collection, data capture, and data analysis makes it possible for other researchers or readers to confirm the results.

Chapter Six Research Findings

The research study investigated the use of mobile technology in teaching and learning to enhance the understanding of the congruence of triangles. The results from the second questionnaire showed the potential feasibility of using GeoGebra applets on mobile devices in teaching and learning the congruence of triangles. A similar mobile technology approach could be implemented in the teaching and learning of many sections of the curriculum. For example, the similarity of triangles and the section of graphs. Considering the applets' usability results and participants' views on advantages and anticipated challenges when using GeoGebra applets, the study proposed the use of tailor-made GeoGebra applets on mobile devices to complete tasks that are designed to explore, derive, or discover mathematical knowledge (For example, definitions, concepts, theorems, axioms, formulae, or concepts). As learners explore and discover new knowledge, conceptual understanding is consequently enhanced.

Even though in this study the participants were grade 9 girls at a specific school, I believe that these results can be transferred to other learners in schools with similar contexts regardless of gender and grade. This belief is based on the fact that all learners belong to the same digita1 native generation. This does not rule out the reality that some learners may not be technologically proficient as was pointed out by some participants. Taking this into consideration, I suggest the inclusion of clear instructions on using the applets in the task. Before the learners use the applets, the teacher should explain these instructions and illustrate how the applets should be used to complete the task. Some participants noted that using mobile technology would arouse learners' interest. So, I expected learners to put more effort into getting the skill of using the applets. The use of applets on mobile devices would explore both the learners' technological proficiency and the learners' time spent on mobile devices. The digital native generation quickly learns to use mobile devices. If the study is repeated with learners from any school the results would be similar. The results are based on the responses to two questionnaires and an interview.

The first questionnaire probed learners' learning preferences, their affection towards mathematics, current use of technology during the teaching and learning process, mobile technology appreciation, proficiency and affordance, and views on what teachers should do to enhance understanding of mathematics. The second questionnaire focused on the learners' evaluation of the usability of GeoGebra applets on mobile devices in teaching and learning mathematics. Additionally, it investigated the learners' views on the benefits of using GeoGebra applets in the learning of mathematics and the anticipated challenges thereof. The semi-structured interview investigated the learners' views on using GeoGebra applets on mobile devices, for teaching and learning mathematics.

6.1 The learners' learning preferences

The results showed that most learners preferred learning with understanding rather than memorizing. All participants indicated that they either enjoyed or sometimes enjoyed listening to explanations of new concepts from their teachers. No one said that she did not enjoy listening to explanations of new concepts. Most participants also indicated that they enjoyed listening to the explanations of their peers. The findings also showed that most learners were interested in discovering cases of congruence. These results showed that most participants preferred learning with understanding, regardless of the challenges. Considering these findings, I conclude that the learners would appreciate and embrace the use of GeoGebra applets on mobile devices.

6.2 Learners' affection for mathematics and how the understanding of mathematics could be improved

Learners put effort to understand and get good results in a subject if they know its importance and relevance. The results showed that the majority (92%) of the participants knew the importance of learning mathematics. The findings showed that even though some participants knew the importance of learning mathematics, they still did not like the subject; mainly because of its complexity to understand. Geometry and algebra were specified as among the difficult sections. One participant indicated that she hated algebra and did not see its relevance in real life. Some indicated that mathematics was not meant for them. From my own experience, this saying is common for learners who would have lost hope of performing better. Those learners stop putting effort to understand. Lack of concentration during lessons was also cited as one of the reasons for getting low marks. All these responses showed the need for teaching strategies that enhance understanding.

From the responses to the open-ended question that probed what educators should do to enhance conceptual understanding, I gathered that learners want educators to use innovative and interesting strategies that enhance visualization of concepts, and that also increase concentration span. The use of youtube videos on a smartboard or data projector was indicated. Some mentioned the use of mobile devices. The participants' suggestions supported the relevance of the strategy proposed in this study. Based on the interview and questionnaire responses, I conclude that the use of GeoGebra applets on mobile devices meets most of the participants' expectations. After using the GeoGebra applets to discover cases of congruence, the participants indicated that GeoGebra applets helped them to visualize and understand the cases of congruence of triangles.

6.3 Learners' mobile technology appreciation, proficiency, and affordance

The results from both questionnaires and the interview showed that most participants would embrace the use of technology in learning mathematics. The use of online learning during the pandemic increased technology appreciation and proficiency for both learners and teachers. The findings showed that many participants were proficient in using mobile technology. Additionally, it boosted the use of mobile technology in learning. Many parents had to buy mobile devices for learners to use for online learning during the lockdown. The findings also indicated that the participants could afford mobile devices and data. Since I used convenient sampling, it was possible that the participants who volunteered had mobile devices and could afford data. Concerning data affordability, learners could access the internet at public libraries in their vicinity, as was done by one participant. These results support the feasibility of using mobile technology in teaching and learning mathematics. The concern about the affordability of data and devices was being addressed at the national level.

The results also showed that getting good marks did not affect either technology proficiency or technology appreciation. There was also no association between liking mathematics and technology appreciation, nor between proficiency in using technology and technology appreciation. The lack of association between these variables indicated that the proposed strategy could be appreciated by learners of different abilities, and at different levels of technology proficiency.

6.4 The feasibility of using GeoGebra applets on mobile devices in learning mathematics

Based on the participants' usability evaluation, I found it feasible to use GeoGebra applets on mobile devices for learning mathematics. The overall usability score was 9.42. To get to the conclusion, I considered the evaluation of the five constructs that measured usability, access construct, easy to learn construct, legibility construct, easy-to-use construct, and satisfaction construct. The rating of the access construct was 9.35. This reflected that the participants could easily log in and open the workbook and applets quickly. The rating of the construct that measured how easy it was to learn and remember the applets was 9.26. The legibility construct got a rating of 9.4. The high value of the mean reflected that the display of triangles, labels, and measurements was all legible. It was possible to display figures and diagrams on mobile devices with smaller screens. The easy-to-use construct had a rating of 9.36. This high score reflected that it was easy to use the touch screen, undo mistakes, and get the missing measurements (lengths of triangles and angles). It also showed that the applets helped to quickly complete the worksheet. Most participants were comfortable and felt confident using the applets. The applets met most of the participants' expectations. The satisfaction construct had a rating of 9.72.

6.5 Learners' views on the benefits of using GeoGebra applets

From the responses to the second questionnaire and interview questions, several advantages of using GeoGebra applets on mobile devices were indicated. From my own experience, tasks that allow learners to discover and explore new mathematical knowledge require more time. Educators end up using other approaches that barely promote conceptual understanding. Learners usually get distracted and sometimes do not put effort to complete challenging or time-consuming tasks. Using the applets helped the participants to quickly complete the worksheet. Therefore, mobile technology can facilitate discovery learning without impacting syllabus coverage.

It was also pointed out that using applets would help learners to self-study, thus allowing more learning time outside the formal learning hours. The use of applets gave accurate and expected answers, at the same time, avoiding mistakes that could obstruct the discovery of new knowledge. For example, in this study, if learners had to construct the triangles and measure the missing lengths and angles manually, it would take more time to complete the worksheet, and any mistakes would lead to making wrong conclusions. Using tailor-made applets reduces cognitive overload and increases psychological comfort because the applets are easy to use, thereby allowing learners to focus more on the new content, which enhances conceptual understanding. It was indicated that using applets on mobile devices can arouse learners' interest in learning mathematics. Based on the participants' responses, the use of applets helped learners to understand and visualize the cases of congruence.

6.6 Anticipated challenges when using GeoGebra applets on mobile devices

The major anticipated challenge was the affordability of data required to access and use the applets online. Another possible challenge was the non-availability of mobile devices and infrastructure. Some learners could be residing in areas where there is no network or where the network could be weak. The shift towards online learning during the COVID-19 pandemic led many people to purchase mobile devices that could connect to the internet. It also prepared learners and stakeholders to accept the use of mobile devices in teaching and learning, thus, paving a way for the proposed use of GeoGebra applets in teaching and learning mathematics. Even though no one complained about the screen size, some learners may have had challenges in using devices with a small screen. Apart from the size, there could be input challenges when the touch screen was not effective. Some students may not have been technologically proficient. Therefore, they could have needed guidance in operating the mobile devices, accessing and using the applets.

Chapter Seven Conclusions

This chapter concludes the study by summarising the chapters, the key research findings, the implications, and the contributions of the study. It will also review the limitations and recommendations for future work.

7.1 Summary of Chapters

Chapter 1 provided the introduction to the study. It included the percentage pass rates of mathematics at National Senior Certificate exams which reflected low performance in mathematics. Factors affecting low performance were also discussed. The lack of conceptual understanding was seen as contributing to low performance (DBE, 2018). This study investigated the use of mobile technologies in the teaching and learning of congruence in mathematics. Specifically, it sought learners' perceptions when using GeoGebra applets to discover the cases of congruence.

Chapter 2 discussed the literature review on theories underlying geometry understanding, as well as technology integration in teaching and learning mathematics. The incorporation of GeoGebra and the advantages of using GeoGebra applets were also discussed.

Chapter 3 reviewed the literature on the FRAME model. The design of the applets was guided by the FRAME model. The construction of questionnaire items that assessed the usability of GeoGebra applets on mobile devices followed the requirements stipulated in the FRAME model.

Chapter 4 discussed the study methodology and the research design adopted. The research was a case study of grade 9 learners at a specific school. The mixed-method was used. Semi-structured interviews and open-ended items in the second questionnaire were used to gather the perceptions of learners on using mobile technology in learning mathematics. Closed-ended items in the first questionnaire were used to check if the learner’s learning characteristics would support learning with understanding, while the closed-ended items in the second questionnaire were used to assess the usability of GeoGebra applets on mobile devices. The data analysis strategies were also discussed.

Chapter 5 included the assessment of reliability and validity of the questionnaires, data screening, the presentation of results and the analysis.

Chapter 6 focused on the discussion of the research findings.

7.2 Overal findings

This study aimed to investigate the use of mobile technology in the teaching and learning of congruence. The results indicate that it is feasible to use mobile technology in learning congruence. It was further revealed that learners appreciate the use of technology in learning congruence and mathematics regardless of their affection or their level of performance in the subject.

7.3 Emerging model

The research findings reveal that mobile technology can be integrated into the teaching and learning of mathematics using curriculum-tailored GeoGebra applets, designed to explore or discover mathematical concepts. The research was grounded in the FRAME model. However, the FRAME model does not specify the type of content in mobile learning. Therefore, a modified frame model emerged in this study, the FRAME_applet model, that emphasizes the use of curriculum tailored GeoGebra applets on mobile devices to enhance conceptual understanding. Effective integration of mobile devices in the teaching and learning of mathematics is centered on the use of curriculum-tailored applets. This research focused on GeoGebra applets but the framework can be extended to applets designed using any other software. Although GeoGebra can be used in different ways, the use of tailored applets removes some technological barriers from the learners and the teachers (Morphett et al., 2016; Pfeiffer, 2017). There is no need to have experience with the software, a factor which increases the psychological comfort of both the teachers and learners (Radović et al., 2018; Taleba & Hassanzadeh, 2015), as well as the satisfaction and acceptance rate of integrating mobile technology in teaching and learning.

The similarities between the FRAME model and the FRAME_Applet model (see Figure 30) is that they both have all the three aspects of the FRAME model (device, learner, and social aspect), and all the pairwise overlapping sections (device usability, social technology, interaction learning). I refer to pairwise overlaps as the intersection of two aspects/sets. There are two differences between the FRAME model and the emerging model. First, the emerging model has an applet aspect included inside the three aspects, thus, forming three extra pairwise overlaps (DAp, LAp, SAp) and three intersections containing three aspects (DLAp, DSAp, LSAp). Second, the mobile learning sector was formed by the intersection of the three aspects in the FRAME model, but it contains four aspects in the emerging model (Device, Learner, Social, and Applet). Instead of calling it a mobile learning aspect, it is referred to as the mobile applet integration aspect in the emerging model. An explanation of the six extra overlaps and the mobile applet integration aspect is given in the next paragraphs.

Device and Applet compatibility (DAp)

The device and applet compatibility focuses on the performance of the applets on mobile devices. This includes the speed of accessing the applets, easy navigation between/within the applets, legibility of the diagrams and labels on the screen, and the rate at which the applets give results/answers. Both the device and the applets contribute towards the successful integration of technology in teaching and learning mathematics.

Learner and Applet Interaction (LAp)

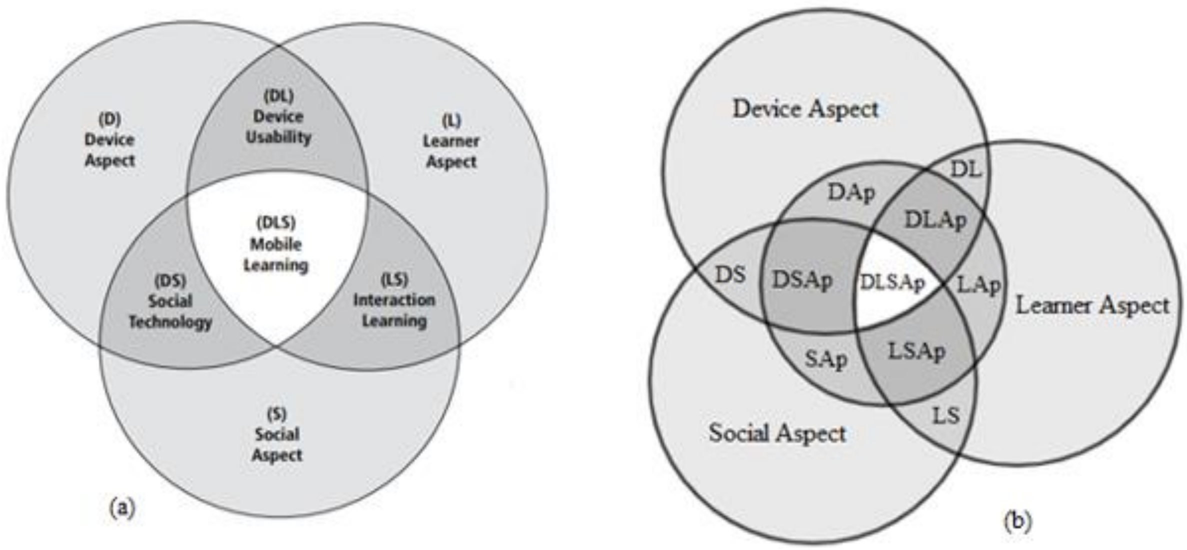
The learner and applet interaction refers to how quickly learners learn to use the applets. Drawing from the constructivist beliefs, learners construct new knowledge based on what they already know and believe (Crawford, 2001). Prior knowledge affects how a learner can easily comprehend new concepts. This intersection emphasizes the knowledge of using applications on mobile devices that the learners already have. Meanings of symbols and tools that the learners have from other applications affect how they can quickly learn how to use the applets.

Social and Applet (SAp)

This intersection takes into account the communication and cooperation between learners as they use the applets. Deriving from the work of Vygotsky, as cited by Amineh & Asl (2015), students can learn from someone or a more knowledgeable peer (Amineh & Asl, 2015; Pfeiffer, 2017). This can happen in various ways. Learners can share the applets or the details to access the applets. Additionally, learners can also assist each other virtually or face-to-face, on how to use the applets. For example, one learner could communicate with a colleague and get assistance in measuring the angles. Mobile technology allows learners to communicate from anywhere, which networks them.

Figure 30

The FRAME and FRAME_Applet model



Note. Diagram (a) is reprinted from " A Model of Framing Mobile Learning ", by M. L. Koole., 2009, p.27. Copyright 2009 by Athabasca University. Diagram (b) represents the emerged FRAME_applet model.

Applet Usability (DLAp)

The applet usability aspect refers to how easy it is for the learners to use the applets on mobile devices. It emphasizes the satisfaction of the learners with the performance of the applets. In other words, it refers to how the applets meet the expectations of the learners. The design of the applets contributes to its performance. For example, applets with a lot of text may take time to download, thus, negatively affecting the usability of the applets (Radović et al., 2018). Similarly, malfunctioning devices can affect the overall rating of the applets by the learners.

Applet Learning Interaction (SLAp)

This aspect focuses on the interaction between the learner and the applet. The interaction through a mobile device may stimulate learning. Through the interaction with the applets, learners are expected to gain new knowledge, for example; progressing from one Van Hiele's level to the next level of understanding. The applets should be aligned to the curriculum and be on the cognitive level of the learners.

Social Applet Technology (DSAp)

This intersection focuses on the learner-to-learner connection and learner-to-system connection. Availability of network, data, and mobile devices to access the internet is a dormant characteristic of the intersection. The learners within the same vicinity can hotspot and share data while using the applets. The learners can access WiFi from a different system, at schools or public libraries.

Mobile Applet Integration (DLSAp)

The integration of the device, learner, social aspect, and applet aspects contributes to the effective incorporation of mobile technology into the teaching and learning of mathematics. The performance of the applets on the mobile device influences learners' satisfaction. Relatedly, the proficiency of the learners in using both the device and the applets contributes to the fast and comfortable accomplishment of the task. The interaction of the learner with the device, the applet, and the content also contributes to the effective integration of technology in the teaching and learning of mathematics. From the social constructivist viewpoint, the learner's background, culture, and learning style also determine the acceptance or rejection of technology integration in learning (Amineh & Asl, 2015; Mvududu & Thiel-Burgess, 2012). Relatedly, it is essential to use mobile devices for teaching and learning, since learners of today are living in a mobile society.

Applets on mobile devices can be used to complete activities that lead to discovering and exploring new knowledge. Discovered knowledge is more likely to be retained and transferred to other scenarios (Bada, 2015). Consequently, learners can remember concepts that they discovered in previous grades and apply them in successive grades (Bada, 2015). This alleviates the most common problem of lack of understanding of basic concepts in mathematics as cited in the National Senior Certificate diagnostic reports (Department of Basic Education_dr, 2019). Table 41 gives a summary of the characteristics of the FRAME_Applet model.

Table 41

The aspects and intersections in the FRAME-Applet Model

Intersection/Aspect in the FRAME model	**Characteristics**	**Additional FRAME_Applet Intersections**	**Specified Characteristics**
Device Aspect (D)	Physical components Input/output capabilities File storage and retrieval Processor speed Error rates	**Device and Applet compatibility (DAp)**	Applet behavior across mobile devices. Legibility of the diagrams and the labels Touchscreen performance
Learner Aspect (L)	Prior Knowledge Memory Context and transfer Learning proclivities	**Learner and Applet interaction (LAp)**	Knowledge of using other applications on mobile devices Learners' interest in using the applets Motivation to complete tasks using the applets
Social Aspect (S)	Conversation Cooperation Interaction	**Applet interaction (SAp)**	Software tools and symbols provide the means of communication.
Device usability (DL)	Portability Information availability Psychological comfort	**Applet Usability (DLAp)**	Learner expectation Expected solutions Learner's satisfaction

	Satisfaction		Learner's technological proficiency
Social technology (DS)	Networking System connectivity Collaboration/interaction tools	**Social and learner technology (SLAp)**	Internet and data availability Sharing data through hot spotting Assistance on the usability of the applets
Interaction learning (LS)	Interaction (learners, instructors, content, computers) Situation cognition Learning communities Pedagogical practices Curriculum	**Applet and learning interaction (DLSAp)**	Alignment of applets to the curriculum Applets and cognitive level Discovery learning Inductive approach
Mobile learning	Mediation, mediators, translators Information access and selection Knowledge navigation	**Mobile applet integration**	Discovering and exploring mathematical concepts Inductive approach Use of tailored applets

7.4 Implications

Drawing from the findings, I recommend the use of tailor-made GeoGebra applets to complete worksheets designed to discover and explore mathematical knowledge (concepts, definitions, axioms, and theorems). From the time I started my studies, I have seen an increase in literature on integrating technology in teaching and learning, in South Africa and the world at large, specifically on the use of GeoGebra, (University N. M., 2018). Few studies have focused on the use of GeoGebra applets on mobile devices.

The use of GeoGebra applets on mobile devices can help teachers to implement the inductive approach when introducing new mathematical knowledge, without impacting syllabus coverage. Additionally, the use of curriculum-tailored GeoGebra applets enhances the visualization of concepts, thereby reducing the amount of time required to explain the concepts. Apart from the advantage of saving time, the use of GeoGebra applets enhances conceptual understanding and retention of content. The use of mobile technology helps the learners to complete the work anywhere and anytime. This allows more learning time without impacting syllabus coverage.

7.5 Contributions

This study responds to the call by the Department of Basic Education in South Africa for the integration of technology in teaching and learning, with a focus on improving the achievement of learning outcomes in the Action plan to 2024 (Department of Basic Education., 2020).

In this study, I designed applets following the manual method of discovering cases of congruence in one of the recommended grade 9 textbooks for the South African Curriculum. Therefore, the study can directly benefit the grade 9 educators and learners in South Africa. Educators can use the applets and the worksheet designed in this study when teaching the cases of congruence. The integration of mobile technology allows students to complete tasks much quicker and increases psychological comfort.

To the board of knowledge of social sciences, the research contributes a new theoretical framework that could be considered in mobile technology research. It can be argued that there is a gap in research on the use of GeoGebra applets on mobile devices in teaching and learning mathematics, with a focus to enhance conceptual understanding. There is not much research done with GeoGebra applets on mobile devices in South Africa. Hence, this study makes a valuable contribution.

7.6 Limitations

This research is limited to one school and one grade. The school was conveniently selected and the grade was purposefully selected. These sampling techniques that were used limit the generalization of the findings of the study to all schools nor grades. However, based on the digital nativeness of the current generation of learners, I believe that the research can be transferred to other grades and schools within a similar setting. The other limitation is that the research was conducted in a school in an urban area. Therefore, the results may not be consistent if the research is conducted in a rural setting.

The other limitation is the small sample that was used. The study was mixed-method research. The sample size was large enough for qualitative processes but there was a constraint on the quantitative processes, specifically, on the validation of both questionnaires. The recommended minimum ratio of the number of items to the sample size required to perform confirmatory factor analysis is 1:10 (Arifin & Yusoff, 2016). However, the ratios in this research were larger than 1:10 because the sample size was small. This could have impacted the factor loadings of the items and reduced the validity of the questionnaires. However, the validity of the questionnaires was enhanced by the fact that most items were adopted from previously validated questionnaires. I also ended up interviewing seven participants (six interviews were recorded and one participant wrote the responses to the interview questions) instead of ten participants that I wanted to interview. I think with the six participants, I managed to get to the saturation point because their responses were almost the same.

Another limitation was on some of the types of items that were used in the first questionnaire. Three closed-ended items had three possible responses while the rest were dichotomous. This posed a challenge when I was assessing the validity of the questionnaire. I had to assess the contribution of these items to the measuring of their respective constructs separately. Item 1b (*…I do not like mathematics because:*) was closed-ended with three responses. I think more than three different responses were going to be gathered if it was an open-ended item. The three possible responses that were provided were common responses that I had heard from learners. However, the inclusion of the open-ended item, "*What do you think your teacher should do that can help you understand* *maths better"* covered up the weakness of item b. I believe that the responses to this open-ended item would also focus on improving the affection for mathematics, which somehow addressed the reasons for not liking mathematics.

Another limitation could be the school policy which did not allow learners to bring mobile devices to school. They had to use mobile devices at home. This was a limitation in this study which later turned out to support the usability of the applets because the participants managed to use the applets on their own without my assistance.

To add more rigor to my research, I could have observed the learners' reactions while I was illustrating the use of the applets.

7.7 Future Work

The participants in this case study were grade 9 volunteers at a particular school. The research focused on the discovery of cases of congruence of triangles, which is part of the geometry section in grade 9. Considering the characteristics of the current generation of learners, I believe that the findings are transferable to other grades and schools. Therefore, I recommend the broader application of the research that focuses on the entire mathematics curriculum.

The findings showed that GeoGebra applets are easy to learn and use regardless of whether one has minimal or no knowledge of GeoGebra. Therefore, I recommend the integration of GeoGebra applets on mobile devices in learning and teaching mathematics. The results showed that GeoGebra applets promote visualization of concepts, thereby enhancing the understanding of concepts. Even though there is an increase in the integration of mobile technology in learning and teaching mathematics, there is a limited focus, if any, on conceptual learning. Therefore, I recommend the integration of mobile technology in teaching and learning mathematics in all schools and all grades.

There are many applets available, but these applets may not use the language and terminology that the learners are familiar with. GeoGebra allows users to construct applets with relatively low technical demands and moderate time and resource requirements. Therefore, I recommend a project that focuses on the design of applets and worksheets tailored to learn and teach selected mathematical content in the South African curriculum, particularly in the senior phase. The project would be one where the applets are used to complete the activities that help explore and discover new content. Before the design, a needs assessment should be conducted to identify the content in all grades that can be taught using GeoGebra applets. After that, the applets can be designed that allow the implementation of the inductive strategy. Additionally, the designed applets can use the terminology that the students are familiar with from the prescribed mathematics textbooks that they use. Teachers and learners are not expected to know the design of the applets, but rather use the designed applets, thus eliminating the technical and software knowledge barriers. The designed applets can then be shared on the resources site of the Department of Basic education for easy access.

I also recommend that teacher training institutions adopt the mobile technology inductive strategy as one of their teaching strategies. This has a two-fold benefit. Firstly, newly trained teachers would exit college equipped to use mobile technology in teaching and learning mathematics. Secondly, these new graduates would then go and share their expertise with their colleagues at their respective workplaces. Schools and the department of education are also encouraged to conduct training workshops where teachers would be taught the implementation of the designed applets.

I also recommend that higher institutions incorporate GeoGebra in the curriculum, and also train the teachers to design applets. The benefit of having teachers design the applets is that they would design the applets based on the pedagogical content that they have. They would also design applets that implement their chosen teaching strategies.

Even though applets can be publicly shared without the need for username and password, I opted for restricted access because it was taking long to open them with public access. I recommend public sharing on the GeoGebra community to be explored. One of the drawbacks of this strategy is that the applets are only available online. Work is also in progress to make applets available offline on mobile devices, without the need to install GeoGebra.

7.8 Final Word

The triangulation of the results from the first questionnaire, the second questionnaire, and the semi-structured interviews showed learners' willingness to use mobile technology in learning mathematics. The results also showed that it is practical to use GeoGebra applets on mobile devices when learning mathematics, to enhance conceptual understanding. The use of tailored GeoGebra applets helps to bridge the technological gap. Additionally, tailored applets can be adapted to discover and explore specific mathematical knowledge without impacting syllabus coverage. The use of mobile technology in teaching and learning mathematics could enhance conceptual understanding. This study is relevant in the current situation of the COVID-19 pandemic, where the time for face-to-face learning and teaching has been reduced. Learners can self-learn new content through the use of curriculum-tailored GeoGebra applets.

References

Applet (n.d.). *Applet definition*. https://www.webopedia.com/definitions/applet/

Abdullah, A. H., & Zakariab, E. (2012). The effects of Van Hiele's phases of learning geometry on students' degree of acquisition of Van Hiele levels. Proceedings of the 6th International Forum on Engineering Education (IFEE 2012) (pp. 251-266). Malaysia: Elsevier Ltd. https://doi.org/10.1016/j.sbspro.2013.10.740

Akçakın, V. (2018). Teaching mathematical functions using geometric functions approach and its effect on ninth-grade students' motivation. *International Journal of Instruction*, *11*(1), 17-32. https://doi.org/10.12973/iji.2018.1112a

Alex, J. K., & Mammen, K. J. (2016). Lessons learnt from employing Van Hiele Theory-Based instruction in senior secondary school geometry classrooms. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(8), 2223-2236. https://doi.org/10.12973/EURASIA.2016.1228A

Alfieri, L., Brooks, P. J., & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology, 103*(1), 1-18. https://doi.org/10.1037/a0021017

Alqahtani, M. M., & Powell, A. B. (2015). Instrumental appropriation of a collaborative, dynamic-geometry environment and geometrical understanding. *International Journal of Education in Mathematics, Science and Technology*, *4*(2), 72-83. https://doi.org/10.18404/ijemst.38054

Amineh, R. J., & Asl, H. D. (2015). Review of constructivism and social constructivism. *Journal of Social Sciences, Literature, and Languages*, *1*(1), 9-16.

Ancsin, G., Hohenwarter, M., & Kov´acs, Z. (2009). GeoGebra goes Mobile. *The Electronic Journal of Mathematics and Technology, 1*(1), 1-10.

Andraphanova, N. V. (2015). Geometrical Similarity Transformations in Dynamic Geometry Environment GeoGebra. *European Journal of Contemporary Education*, *12*(2), 116-128. https://www.learntechlib.org/p/158427/

Arbain, N., & Shukor, N. A. (2014). The effects of GeoGebra on students' achievement. *GCBSS-2014.* Proceedings of the Global Conference on Business & Social Science, Kuala Lumpur (pp. 208-214). https://doi.org/10.1016/j.sbspro.2015.01.356

Aric, S., & Aslan-Tutak, F. (2015). The effect of origami-based instruction on spatial visualization, geometry achievement, and geometric reasoning. *International Journal of Science and Mathematics Education*, *13*(1), 179-200.

Asiimwe, E. N., Åke, G., & Hatakka, M. (2017). Practices and challenges in an emerging m-learning environment. *International Journal of Education and Development Using Information and Communication Technology*, *13*(10), 103-122.

Aszalos, L., & Bako, M. (2004). How can we improve the spatial intelligence? Proceedings of the 6th International Conference on Applied Informatics, Eger (pp 1-7)

Bachman, K. M., & Gannod, G. C. (2011). A critical analysis of m-learning initiatives. *Mobile Learning.* Proceedings of the IADIS International Conference, Avila, Spain (pp. 310-312).

Bada, S. O. (2015). Constructivism Learning Theory: A paradigm for teaching and learning. *Journal of Research & Method in Education*, *5*(6), 66-70. https://doi.org/10.9790/7388-05616670

Balım, A. G. (2009). The Effects of Discovery Learning on Students' Success and Inquiry Learning Skills. *Eurasian Journal of Educational Research*, *1*(35), 1-20.

Bartell, T. G., Webel, C., Bowen, B., & Dyson, N. (2013). Prospective teacher learning: recognizing evidence. *Journal of Mathematics Teacher Education, 16*(4), 57-79. https://doi.org/10.1007/s10857-012-9205-4

Beilock, S. L. (2015). Math Anxiety: A Factor in Math Achievement Not to Be Ignored. *Behavioral and Brain Sciences*, *2*(1)4-12. https://doi.org/10.1177/2372732215601438

Bhat, A. (2020, January 4). *Rating Scale: Survey Question Types and Examples*. https://www.questionpro.com/blog/rating-scale/

Blackwell, C. K., Lauricella, A. R., & Wartella, E. (2014). Factors influencing digital technology use in early childhood education. *Computers & Education*, *77*(8), 82-90. https://doi.org/10.1016/j.compedu.2014.04.013 Blog, F. (2020, September 23). *Structured Interviews: Definition, Types + [Question Examples].* https://www.formpl.us/blog/structured-interview#:~:text=In%20a%20structured%20interview%2C%20the,interview%20or%20a%20standardized%20interview

Braun, V., & Clarke, V. (2012). Thematic Analysis. In V. Braun, & V. Clarke, *APA handbook of research methods in psychology, Vol. 2: Research designs: Quantitative, qualitative, neuropsychological, and biological* (pp. 57-71). American Psychological Association. https://doi.org/10.1037/13620-004

Bruner, J. S. (1966). *Toward a theory of instruction.* Cambridge: Harvard University Press.

Buckley, J., Seery, N., & Canty, D. (2018). A heuristic framework of spatial ability: a review and synthesis of spatial factor literature to support its translation into STEM education. *Educational Psychology Review*, *29*(2), 947-972. https://doi.org/10.1007/s10798-018-9446-3

Cheon, J., Lee, S., Crooks, S. M., & Song, J. (2012). An investigation of mobile learning readiness in higher education based on the theory of planned behaviour. *Computers & Education*, *59*(3), 1054-1064. https://doi.org/10.1016/j.compedu.2012.04.015

Ciampa, K. (2014). Learning In A Mobile Age: An investigation of student motivation. *Journal Of Computer Assisted Learning*, *30*(1), 82-96. https://doi.org/10.1111/jcal.12036

Collette, P. (2014). Dynamic Visualisations. A GeoGebra workbook for selected topics in the mathematics curriculum, Grades 10-12. Port Elizabeth, Eastern Cape, South Africa. https://www.GeoGebra.org/u/philipcollett#materials/created

Collins, H. (2018). *COBUILD Advanced English Dictionary.* HarperCollins.

Contributors, W. (2019, February 10). *List of countries by smartphone penetration*. T. F. Wikipedia. https://en.wikipedia.org/w/index.php?title=List_of_countries_by_smartphone_penetration&oldid=882718285

Crawford, M. L. (2001). *Teaching contextually: Research, rationale, and techniques for improving student motivation and achievement in mathematics and science.* Texas: CCI Publishing.

Cresswell, J. W., & Cresswell, D. J. (2018). *Research Design: Qualitative, Quantitative and Mixed Methods Approaches* (5th ed.). Los Angeles: SAGE.

Creswell, J. W., & Poth, C. N. (2018). Five Qualitative Approaches to Inquiry. In J. W. Creswell, & C. N. Poth, *Qualitative Inquiry and Research Design: Choosing Among Five Approaches* (pp. 67-107). Croydon: SAGE .

Crompton, H. (2016). Using Mobile Learning to Support Students' Understanding in Geometry: A Design-Based Research Study . *Educational Technology & Society*, *20*(3), 207-219. http://www.jstor.org/stable/26196131

Crompton, H., Grant, M. R., & Shraim, K. Y. (2018). Technologies to Enhance and Extend Children's Understanding of Geometry: A Configurative Thematic Synthesis of the Literature. *Journal of Educational Technology & Society*, *21*(1), 59-69. https://digitalcommons.odu.edu/teachinglearning_fac_pubs/45

Daichendt, L. (2019, January 16). *Mobile Technology Facts*. http://www.strategicgrowthconcepts.com/growth/increase-productivity--profitability/mobile-technology-facts.html

Department of Basic Education_s. (2011). *Curriculum and Assessment Policy Statement Grades 7 - 9.* https://www.education.gov.za/Curriculum/CurriculumAssessmentPolicyStatements(CAPS)/CAPSSenior.aspx

Department of Basic Education_d. (2017). *National Senior Certificate Diagnostic Report (Part 1).* https://www.education.gov.za/Portals/0/Documents/Reports/2017%20NSC%20Diagnostic%20Report%20Part%201.pdf?ver=2018-01-30-140924-883

Department of Basic Education_d. (2018). *National Senior Certificate Diagnostic Report (Part 1).* *https://www.education.gov.za/Portals/0/Documents/Reports/FINAL%20DIAGNOSTIC%20REPORT%20NON%20LANGUAGES%202018.pdf?ver=2019-01-24-125928-000*

Department of Basic Education_l. (2010). *The Status of The Language of Learning and Teaching in South African Public Schools.* https://www.education.gov.za/Portals/0/Documents/Reports/The%20Status%20of%20Learning%20and%20Teaching%20in%20South%20African%20Public%20Schools.pdf?ver=2015-05-18-141132-643

Department of Basic Education_cfet. (2011). *Curriculum Assessment Policy Statement. Further Education and Training phase, Grades 10-12*. https://www.education.gov.za/Portals/0/CD/National%20Curriculum%20Statements%20and%20Vocational/CAPS%20FET%20_%20MATHEMATICS%20_%20GR%2010-12%20_%20Web_1133.pdf?ver=2015-01-27-154314-253

Department of Basic Education_ci. (2011). *Curriculum Assessment Policy Statement Grades 4-6 mathematics.* https://www.education.gov.za/Portals/0/CD/National%20Curriculum%20Statements%20and%20Vocational/CAPS%20IP%20%20MATHEMATICS%20GR%204-6%20web.pdf?ver=2015-01-27-161430-553

Department of Basic Education_cf. (2011). *Curriculum Assessment Policy Statement, Grades 1-3 mathematics.* https://www.education.gov.za/Portals/0/CD/National%20Curriculum%20Statements%20and%20Vocational/CAPS%20MATHS%20%20ENGLISH%20GR%201-3%20FS.pdf?ver=2015-01-27-160947-800

Department of Basic Education_f. (2011). *Integrated Strategic Planning Framework for Teacher Education and Development in South Africa, 2011–2025*. https://www.education.gov.za/Portals/0/Documents/Reports/Integrated%20Strategic%20Plan%20Teacher%20Dev%202011-2025.pdf?ver=2015-02-03-110453-033

Department of Basic Education_a. (2015). *Action Plan to 2019 Towards the Realisation of Schooling 2030*. https://www.education.gov.za/Portals/0/Documents/Publications/Action%20Plan%202019%20Final.pdf?ver=2016-01-20-155119-060

Department of Science and Technology. (2019). *White Paper on Science, Technology and Innovation.* White Paper on Science, Technology and Innovation (www.gov.za)

Department of Basic Education_ict. (2016). *Information Communication and Technology in Schools*. https://www.education.gov.za/ArchivedDocuments/ArchivedArticles/InformationCommunicationandTechnologyinschools.aspx

Department of Basic Education_g. (2017). *Mathematics, Science, and Technology Conditional Grant registers significant progress in provinces*. https://www.education.gov.za/ArchivedDocuments/ArchivedArticles/MSTGrant0517.aspx

Department of Basic Education_w. (2018). *2018 Winter School Programme*. https://www.education.gov.za/WinterClasses04072018.aspx

Department of Basic Education_ar. (2018). *Annual Report 2017/2018*. https://www.education.gov.za/Portals/0/Documents/Reports/201718%20Annual%20Report.pdf?ver=2018-09-26-125541-803

Department of Basic Education_m. (2018). *Moodle teacher training for teachers and subject specialists on using MOODLE*. https://www.education.gov.za/Portals/0/Documents/Manuals/MOODLE%20TEACHER%20TRAINING.pdf?ver=2018-07-25-094841-030

Department of Basic Education_pr. (2019). *National Senior Certificate Performance Report.* https://www.education.gov.za/Portals/0/Documents/Reports/NSC%202018%20School%20Performance%20Report%20WEB.pdf?ver=2019-01-03-084932-000

De Villiers, M. (2010, 30 June –2 July 2010). *Some reflections on the Van Hiele theory* [Plenary presentation]. 4th Congress of teachers of mathematics of the Croatian Mathematical Society, Zagreb. https://www.researchgate.net/publication/264495589_Some_Reflections_on_the_Van_Hiele_theory

Dempster, E. R., & Vijay, R. (2007). Question readability and science achievement in TIMSS 2003 in South Africa. *Science Education*, *91*(6), 906-925. https://doi.org/10.1002/sce.20225

Denbel, D. G. (2015). Students' Learning Experiences When using a Dynamic Geometry Software Tool in a Geometry Lesson at Secondary School in Ethiopia. *Journal of Education and Practice, 6*(1), 23-38. https://files.eric.ed.gov/fulltext/EJ1083857.pdf

Dev, P. C. (1997). Intrinsic Motivation and Academic Achievement. What Does Their Relationship Imply For The Classroom Teacher? *Remedial and Special Education*, *18*(1), 12-19. https://doi.org/10.1177/074193259701800104

Diković, L. (2009, December 1). Applications GeoGebra into teaching some topics of mathematics at the college level. *Computer Science and Information Systems, 6*(2), 191-203. https://doi.org/10.2298/CSIS0902191D

Dimitrov, D. M., & Slavov, S. D. (2018). Application of GeoGebra software into teaching mechanical engineering courses. *MATEC Web of Conferences 178* (pp. 1-6). Bulgaria: EDP Sciences. https://doi.org/10.1051matecconf201817807008

Drijvers, P. (2013). Digital technology in mathematics. education: why it works (or doesn't). *Proceedings of the 12th International Congress on Mathematics Education*, 485-501. https://doi.org/10.1007/978-3-319-17187-6_8

Dudovskiy. (2011, September 4). *Ontology*. Research Methodology. https://research-methodology.net/research-philosophy/ontology/

Dudovskiy, J. (2011, September 9). *Epistemology*. Research Methodology. https://research-methodology.net/research-philosophy/epistomology/

Dudovskiy, J. (2011, September 4). *Research Methodology*. https://research-methodology.net/research-philosophy/axiology-2/

Dudovskiy, J. (2012). *Epistemology-Research Methodology*. Research Methodology. https://research-methodology.net/research-philosophy/ontology/

Engelbrecht, J. C., Kågesten, O., & Bergsten, C. (2017). Conceptual and procedural approaches to mathematics in engineering. *European Journal of Engineering Education*, *42*(5), 1-20. https://doi.org/10.1080/03043797.2017.1343278

Erdogan, E. O., & Dur, Z. (2014). Preservice mathematics teachers' personal figural concepts and classifications about quadrilaterals. *Australian Journal of Teacher Education, 39*(6), 107-133. http://dx.doi.org/10.14221/ajte.2014v39n6.1

Erkek, Ö., Işıksal, M., & Çakıroğl, E. (2017). A study on pre-service teachers' spatial visualization ability and spatial anxiety. *Kastamonu Education Journal*, *25*(1), 33-50. https://dergipark.org.tr/tr/download/article-file/296345

Erşen, Z. B., Ezentaş, R., & Altun, M. (Evaluation of the teaching environment for improving the geometric habits of tenth-grade students. European Journal of Education Studies, 4(6), 47-65. http://dx.doi.org/10.5281/zenodo.1239

Falloon, G. (2017). Mobile devices and apps as scaffolds to science learning in the primary classroom. *Journal of Science Education and Technology, 26*(4), 613-628. http://dx.doi.org/10.1007/s10956-017-9702-4

Feza, N. N. (2016). Basic numeracy abilities of Xhosa Reception year students in South Africa: Language policy issues. *Issues in Educational Research*, *26*(4), 576-591. https://www.researchgate.net/publication/309769033_Basic_numeracy_abilities_of_Xhosa_reception_year_students_in_South_Africa_Language_policy_issues

Figueiredo, M., Godejord, B., & Rodrigues, J. (2016). The development of an interactive mathematics app for mobile learning. Proceedings of the 12th International Conference Mobile Learning, Algarve (pp. 75-81). IADIS Press. https://core.ac.uk/download/pdf/216333629.pdf

Fischbein, E. (1993). The Theory of Figural Concepts. *Educational Studies in Mathematics, 24*(2), 139-162. http://www.jstor.org/stable/3482943

Franklin, T., & Peng, L.-W. (2008). Mobile math: math educators and students engage in mobile learning. J *Comput High Educ*, *20*, 69-80. http://dx.doi.org/10.1007/s12528-008-9005-0

Froiland, J. M., Oros, E., Smith, L., & Hirchert, T. (2012). Intrinsic Motivation to Learn: The Nexus between Psychological Health and Academic Success. *Contemporary School Psychology, 16*, 91-102. http://dx.doi.org/10.1007/BF03340978

Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon. *Journal of Mathematical Behavior, 31*(1), 60-72. http://dx.doi.org/10.1016/j.jmathb.2011.08.003

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education, 9*(1), 3-20. http://dx.doi.org/10.1080/14794800008520167

Gay, L. R., Mills, G. E., & Airasian, P. (2009). *Educational Research* (9th ed.). Garamond: Pearson.

Geist, E. A. (2012). A qualitative examination of two-year-olds interaction with tablet-based interactive technology. *Journal of Instructional Psychology*, 26-35. https://www.learntechlib.org/p/88310/.

Getenet, S. T. (2015). Using dynamic software to teach mathematical concepts: The cases of GeoGebra and Microsoft mathematics. *Journal of mathematics and technology*, *12*(1), 210-229. https://www.researchgate.net/publication/323240095_Using_Dynamic_Software_to_Teach_Mathematical_Concepts_The_Cases_of_GeoGebra_and_Microsoft_Mathematics

Glen, S. (2016, July 17). *Krippendorff's Alpha*. Statistics how to. https://www.statisticshowto.datasciencecentral.com/krippendorffs-alpha/

Goodwin, D., & Webb, M. A. (2014). Comparing teachers' paradigms with the teaching and learning paradigm of their state's teacher evaluation system. *Research in Higher Education Journal*, 1-11.

Govan Mbeki Mathematics Development Center, (2018, January 1). GeoGebra Support Programme for Teacher Development. https://mbeki-maths-dev.mandela.ac.za/Projects/GeoGebra-Support-Programme-for-Teacher-Development

Govender, R., & De Villiers, M. (2004, June). A dynamic approach to quadrilateral definitions. *Pythagoras*, pp. 34-45. http://dx.doi.org/10.4102/pythagoras.v0i59.130

Gürbüz, M. Ç.; Ağsu, M.; Güler, H. K. (2018), Investigating geometric habits of mind by using paper folding. *Acta Didactica Napocensia*, 11(3), 157-174. http://dx.doi.org/10.24193/adn.11.3-4.12

Gunbayi, I., & Sorm, S. (2018). Social paradigms in guiding social research design: The functional, interpretive, radical humanist, and radical structural paradigms. *International Journal on New Trends in Education and Their Implication,* *9*(2), 57-76. https://www.researchgate.net/figure/Social-Paradigms-in-guiding-Social-research-design-Social-Paradigm-Guiding-Social_tbl1_328052660

Gunhan, B. C. (2014). A case study on the investigation of reasoning skills in geometry. *South African Journal of Education*, *34*(2), 1-19. http://dx.doi.org/10.15700/201412071156

Güven, B., & Kosa, T. (2002). The effect of dynamic geometry software on student mathematics teachers' spatial visualization skills. *The Turkish Online Journal of Educational Technology*, *7*(4), 100-107. https://files.eric.ed.gov/fulltext/EJ1102930.pdf

Guy, M. R. (2019, September 3). *Types of Data & Measurement Scales: Nominal, Ordinal, Interval, and Ratio*. My Market Research Methods. https://www.mymarketresearchmethods.com/types-of-data-nominal-ordinal-interval-ratio/

Guzel, N., & Senera, E. (2009). High school students' spatial ability and creativity in geometry. Proceedings of the *World Conference on Educational Sciences,* Turkey (pp. 1763-1766). Elsevier.

Han, S. C., & Capraro, M. M. (2014). How science, technology, engineering and mathematics (STEM) project-based learning (PBL) affects high, middle, and low achievers differently: The impact of student factors on achievement. *International Journal of Science and Mathematics Education*, *13*(5), 1089-1113. http://dx.doi.org/10.1007/s10763-014-9526-0

Hanus, M. D., & Jesse, F. (2014). Assessing the effects of gamification in the classroom: A longitudinal study on intrinsic motivation, social comparison, satisfaction, effort, and academic performance. *Computers and Education*, *80*, 152-161. http://dx.doi.org/10.1016/j.compedu.2014.08.019

Harcourt, H. M. (2010). *Webster's New World College Dictionary.* Houghton Mifflin Harcourt.

Hariyanto, D., Triyono, M.B., & Köhler,T. (2020). Usability evaluation of personalized adaptive e-learning system using USE questionnaire. *Knowledge Management & E-Learning, 12*(1), 85-105. https://doi.org/10.34105/j.kmel.2020.12.005

Hatlevik, O. E., Throndsen, I., Loi, M., & Gudmundsdottir, G. B. (2018). Students' ICT self-efficacy and computer and information literacy: Determinants and Relationships. *Computers & Education, 118*, 107–119. https://doi.org/10.1016/j.compedu.2017.11.011

Hlagala, A. R. (2015). *Mobile Educational Technologies Currently Used As a Means to Enhance Teaching.* [Masters thesis, University of South Africa]. http://uir.unisa.ac.za/bitstream/handle/10500/19206/dissertation_Hlagala_AR.pdf?sequence=1&isAllowed=y

Hohenwarter, J., (2016, November 23). Design Guidelines for Dynamic Activities. https://www.geogebra.org/m/e9Z6UDu4#material/Ch3PPkEq

Hohenwarter, M., & Lavic, Z. (2011). The Strength Of The Community: How GeoGebra Can Inspire Technology Integration In Mathematics. *MSOR Connections*, 9(2), 3-5. https://doi.org/10.11120/msor.2009.09020003

Howie, S. J. (2003). Language and other background factors affecting secondary pupils' performance in mathematics in South Africa. *African Journal of Research in SMT Education*, *7*(1), 1-20. https://doi.org/10.1080/10288457.2003.10740545

Jelatu, S. (2018). Effect of GeoGebra-Aided REACT strategy on the understanding of geometry concepts. *International Journal of Instruction, 11*(4), 325-336. https://doi.org/10.12973/iji.2018.11421

John, M. F., Smith, L., & Hirchert, T. (2012). Intrinsic motivation to learn: The nexus between psychological health and academic success. *Contemporary School Psychology*, *16*, 94-102. https://doi.org/10.1007/BF03340978

Johnson, B. R., & Onwuegbuzie, A. J. (2004). Mixed Methods Research: A Research Paradigm Whose Time Has Come. *Educational Researcher, 33*(7), 14-26. https://doi.org/10.3102/0013189X033007014

Jones, K. (1998). *theoretical frameworks for the learning of geometrical reasoning.* Proceedings of the British Society for Research into Learning Mathematics, London, University of Southampton (pp. 29-34).

Joubert, M. (2007). *A framework for examining student learning of mathematics: tasks using technology,* University of Nottingham.

Kade, A., Degeng, I. N., & Nur Ali, M. (2019). Effect of jigsaw strategy and learning style on conceptual understanding on senior high school students. *International Journal of Emerging Technologies in Learning (iJET), 14*(19), 4-15. https://doi.org/10.3991/ijet.v14i19.11592

Kaptelinin, V., & Nardi, B. A. (2009). *Acting with technology: Activity theory and interaction design.* The MIT Press.

Kaufmann, H. (2004). *Geometry education with augmented reality.* Vienna University of Technology.

Kennedy, T. J., & Kennedy, O. M. (2014). Engaging students in stem education. *Science Education International*, 246-258.

Kivunja, C., & Kuyini, A. B. (2017). Understanding and applying research paradigms in educational contexts. *International Journal of Higher Education, 6*(5), 26-41. https://doi.org/10.5430/ijhe.v6n5p26

Koohang, A., Riley, L., & Smith, T. (2009). E-Learning and Constructivism: From Theory to Application. *Interdisciplinary Journal of E-Learning and Learning Objects*, *5*(1), 91-109.

Koole, M. L. (2009). *A Model of Framing Mobile Learning.* Athabasca University.

Koole, M., & Ally, M. (2006). *Framework for the Rational Analysis of Mobile Education (FRAME) Model: Revising the ABCs of Educational Practices.* Athabasca University.

Koole, M., McQuilkin, J. L., & Ally, M. (2010). Mobile learning in distance education: Utility or Futility? *Journal of Distance Education*, *24*(2), 59-82. https://www.learntechlib.org/p/54733/

Korenova, L. (2017). GeoGebra in the teaching of primary school mathematics. *International Journal of Technology in Mathematics Education*, *24*(3), 155-160. https://doi.org/10.1564/tme_v24.3.08

Kösa, T. (2016). Effects of using dynamic mathematics software on preservice mathematics teachers' spatial visualization skills: The Case Of Spatial Analytic Geometry. *Educational Research and Reviews*, *11*(7), 449-458. https://doi.org/10.5897/ERR2016.2686

Koyuncu, I., Akyuz, D., & Cakiroglu, E. (2015). Investigating pane geometry problem-solving strategies of prospective mathematics teachers in technology and paper-and-pencil environments. *International Journal of Science and Mathematics Education*, *13*(4), 837-862. https://www.learntechlib.org/p/161691/

Kumar, L. S., Jamatia, B., Aggarwa, A. K., & Kannan, S. (2011). Mobile device intervention for student support services in distance education context – FRAME model perspective. *European Journal of Open, Distance, and E-Learning*, *14,* 1-10.

Kuo-En, C., Lin-Jung, W., Shing-Chuang, L., & Sung, Y.-T. (2016). Using mobile devices to enhance the interactive learning for spatial geometry. *Interactive Learning Environments*, *24*(4), 916-934. https://doi.org/10.1080/10494820.2014.948458

Lai, K., & White, T. (2012). Exploring quadrilaterals in a small group computing environment. *Computers & Education*, *59*(3), 963-973. http://dx.doi.org/10.1016/j.compedu.2012.04.006

Larkin, K., & Calder, N. (2015). Mathematics education and mobile technologies. *Math Education Research Journal*, *28*(1), 1-7. https://doi.org/10.1007/s13394-015-0167-6

Leung, A. (2012). Variation and mathematics pedagogy. *Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, Singapore* (pp. 433-440). Mathematics Education Research Group of Australasia.

Leung, A., & Bolite-Frant, J. (2015). Designing mathematics tasks: The role of tools. In A. Watson, & M. Ohitan (Eds), *Task Design In Mathematics Education* (pp. 191-225). https://doi.org/10.1007/978-3-319-09629-2_6

Levene, J., & Holly, S. (2015). Evaluation of Mobile Learning: Current Research and Implications for Instructional Designers. *TechTrends, 59*(6), 46-52. https://doi.org/10.1007/s11528-015-0904-4

Mabona, S. (2018, January 4). *Gauteng Education Congratulates the Class Of 2018.* Retrieved from www.gov.za: https://www.gov.za/speeches/gauteng-department-education-achieves-8792-2018-nsc-examinations-4-jan-2019-0000

Magwa, S., & Ngara, R. (2014). Learner Indiscipline In Schools. *Review of Arts and Humanities*, *3*(2), 79-88. https://doi.org/10.15640/rah

Maier, P. H. (1996). SPATIAL geometry and spatial ability - How to make solid geometry solid? *Annual Conference of Didactics of Mathematics*, pp. 69-81.

Malhotra, N. K. (2006, January). *Questionnaire design and scale development.* Georgia Institute of Technology. https://www.researchgate.net/publication/266864633_Questionnaire_design_and_scale_development

Manuguerra, M. (2011). Promoting student engagement by integrating new technology into Tertiary Education: The role of the iPad. *Asian Social Science* *7*(11), 61-65. https://doi.org/10.5539/ass.v7n11p61

Marginson S., Tytler R., Freeman B., Roberts K., (2013). *STEM: Country comparisons: international comparisons of science, technology, engineering, and mathematics (STEM) education.* Final Report. Melbourne: Australian Council of Learned Academies.

Mayer, R. E. (2004). Should there be a three-strikes rule against pure discovery learning? The case for guided methods of instruction. *American Psychological Association, 59*(1), 14-19. https://doi.org/10.1037/0003-066X.59.1.14

McGee, M. G. (1979, September 29). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin, 86*(5), 889-916.

McGuirk, P. M., & O'Neill, P. (2016). Using questionnaires in qualitative human geography. In I. Hay (Eds.), *Qualitative Research Methods in Human Geography* (pp. 246-273). Oxford University Press.

Mendezabal, M. J., & Tindowen, D. J. (2018). Improving students' attitude, conceptual understanding, and procedural skills in differential calculus through Microsoft Mathematics. *Journal of Technology and Science Education*, *8*(4), 385-397. http://dx.doi.org/10.3926/jotse.356

Mji, A., & Makgato, M. (2006). Factors associated with high school learners' poor performance: A spotlight on mathematics and physical science. *South African Journal of Education, 26*(2), 253-266.

Mobinizad, M. M. (2018). The use of mobile technology in learning English language. *Theory and practice in language studies*, *8*, 1456-1468. https://doi.org/10.17507/TPLS.0811.10

Morphett, A., Gunn, S., & Maillardet, R. (2015). Developing interactive applets with GeoGebra: processes, technologies. In Blignaut, R. & Kizito, R (Eds.), *Elephant Delta*. Proceedings of Elephant Delta: 10th Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics, Port Elizabeth, South Africa. (pp. 1-13).

Morphett, A., Maillardet, R., & Gunn, S. (2016). *Learning and Teaching Initiatives Project Report Template*. https://melbapplets.ms.unimelb.edu.au/wp-content/uploads/2015/12/LTI-project-final-report-Conceptual-Learning-with-Interactive-Applets.pdf

MUDALY, V. (2004). *The Role and Use of Sketchpad as a Modeling Tool in Secondary Schools.* [Ph.D., University of KwaZulu_Natal]. https://researchspace.ukzn.ac.za/handle/10413/9464

Mulligan, J. (2015). Looking within and beyond the geometry curriculum: connecting spatial reasoning to mathematics learning. *ZDM Mathematics Education*, 47, 511–517. https://doi.org/10.1007/s11858-015-0696-1

Murphy, E. (1997). Characteristics of Constructivist Learning & Teaching. http://www.ucs.mun.ca/~emurphy/stemnet/cle3.html

Mvududu, N., & Thiel-Burgess. (2012). Constructivism in Practice: The Case for English language learners. *International Journal of Education*, *4*(3) 108-118. https://doi.org/10.5296/ije.v4i3.2223

Naidoo, J. (2011). *Exploring master teachers use of visuals as tools in mathematics classrooms [Ph.D., University of KwaZulu_Natal]. https://researchspace.ukzn.ac.za*

University of KwaZulu_Natal (2019). *Undergraduate Prospectus.* https// https://eduloaded.com/blog/ukzn-prospectus-2019-pdf/

NEEDU. (2018, November 22). *Making subjects relevant and interesting*. Policy Brief 13: https://www.education.gov.za/Portals/0/Documents/Publications/NEEDU%20POLICY%20BRIEF%20SERIES/Policy%20Brief%2013%20-%20Making%20subjects%20relevant%20and%20interesting_.pdf?ver=2018-11-22-153051-387

NEEDU, N. E. (2018, February 27). *NEEDU, Schools that Work II. Lessons from the Ground*. Retrieved from Department of Education: https://www.education.gov.za/Portals/0/Documents/Reports/NEEDU%20School%20that%20work%20II%202018.pdf?ver=2018-02-27-135155-300

Nisiyatussani, Ayuningtyas, V., & Fathurrohman, M. N. (2018). GeoGebra applets design and development for junior high school students to learn quadrilateral mathematics concepts. *Journal on Mathematics Education, 9(1)*, 27-40. https://doi.org/10.22342/JME.9.1.4162.27-40

O'Malley, P., Jenkins, S., Wesley, B., Donehower, C., Rabuck, D., & Lewis, M. (2013, April 3-6). *Effectiveness of using ipads to build math fluency [Meeting Presentation].* Council for Exceptional Children Annual Meeting, San Antonio, Texas.

Olkun, S. (2003). Making connections: Improving spatial abilities with engineering drawing activities. *International Journal of Mathematics Teaching and Learning*, 1-10. https://doi.org/10.1501/0003624

Ota, M. A., & de Araujo Jr, C. F. (2016). M-learning challenges in teaching crosscutting themes in the education of young people and adults. Proceedings of the *12th International Conference On Mobile Learning, Vilamoura* (pp. 19-156). IADIS Press.

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students, *Procedia - Social and Behavioral Sciences.* Proceedings of International Conference On New Horizons In Education INTE2012 (pp. 720-729). https://doi.org/10.1016/j.sbspro.2012.09.557

Padayachee, P. (2010). *A Case Study: Exploring a DVD driven approach for teaching and learning mathematics, at secondary school level, with a framework of blended learning [PhD, Nelson mandela Metropolitan University].* http://hdl.handle.net/10948/1384

Pademetri-Kachrimani, C. (2012). Revisting Van Hiele. *For The Learning Of Mathematics, 32*(3), 2-7. http://www.jstor.org/stable/23391967

Pfeiffer, C. (2017). *A study of the development of mathematical knowledge in a geogebra focused learning environment [Ph.D., Stellenbosch University].* Stellenbosch University Library and Information Services. https://scholar.sun.ac.za

Phillips, M. D., Basson, J., & Odendaal, J. (2014). *Mind Action Series.* Johannesburg: Allcopy.

Pillay, M. (2013). *Executive summary: The status of the language of learning and teaching (LoLT) in South African public schools: a quantitative review, 2010.* Johannesburg: Education Policy Research Unit.

Plant, T. (2014, August). *Trending: Mobile devices in the classrooms*. ECESSA.com.

https://www.ecessa.com/blog/mobile-devices-classroom/

Polásek, V., & Sedlácek, L. (2015). Dynamic geometry environments as cognitive tool in mathematic education. *Journal of Technology and Information Education*, *7(*2), 45-54. https://doi.org/10.5507/jtie.2015.017

Pournara, C., Hodgen, J., Adler, J., & Pillay, V. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in mathematics. *South African Journal of Education*, *35*(3), 1-10. https://doi.org/10.15700/saje.v35n3a1083

Powell, K. C., & Kalina, C. J. (2009). Cognitive and social constructivism: Developing tools for an effective classroom. *Education, 130*(2), 241-250.

Presidency, T. (2019, February 7). *President Cyril Ramaphosa: 2019 State of the Nation Address*. Retrieved from South African Government: https://www.gov.za/speeches/president-cyril-ramaphosa-2019-state-nation-address-7-feb-2019-0000

Radović, S., Radojičić, M., Veljković, K., & Marić, M. (2018). Examining The Effects Of GeoGebra Applets On Mathematics Learning Using Interactive Mathematics Textbook. *Interactive Learning Environments*, *28*(1), 1-17. https://doi.org/10.1080/10494820.2018.1512001

Ramey, K. (2013, December 12). *What Is Technology – Meaning of technology and its use*. Use of technology.com. https://www.useoftechnology.com/what-is-technology/

Rattray, J., & Jones, M. C. (2007). Essential elements of questionnaire design and development. *Journal of Clinical Nursing, 16*(2), 234-243. https://doi.org/10.1111/j.1365-2702.2006.01573.x

Reddy, V., Visser, M., Winnaar, L., Arends, F., Juan, A., Prinsloo, C., & Isdale, K. (2016). *TIMSS 2015: Highlights of mathematics and science achievement of grade 9 South African learners.* Human Sciences Research Council.

Rittle-Johnson, B., & Schneider, M. (2014). Developing conceptual and procedural knowledge of mathematics. In R. C. Kadosh, & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 1102-1118). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199642342.013.014

Roberts, D. H., Newman, L. R., & Schwartzstein, R. M. (2012). Twelve tips for facilitating millennials' learning. *Medical teacher*, *34*(4), 274-278. https://doi.org/10.3109/0142159X.2011.613498

Romero Albaladejo, I. M., del Mar García, M., & Codina, A. (2015). Developing mathematical competencies in secondary students by introducing dynamic geometry systems in the classroom. *Education and Science*, *40*(177), 43-58. https://doi.org/10.15390/EB.2015.2640

Rossing, J. P., Miller, W. M., Cecil, A. K., & Stamper, S. E. (2011). iLearning: The future of higher education? Student Perceptions on Learning with Mobile Tablets. *Scholarship of Teaching and Learning*, *12*(2), 1-26.

Rouse, M. (2011, November). *Definition App*. SearchMobileComputing. https://searchmobilecomputing.techtarget.com/definition/app

Sariyasa. (2017). Creating dynamic learning environment to enhance students' engagement in learning geometry. *Journal of Physics: Conference Series*, *824*(1), 1-5. https://doi.org/10.1088/1742-6596/824/1/012057

Saylan, A., Onal, N. T., & Onal, N. (2018). Using Technology In Education From The Pre-service Science And Mathematics Teachers' Perspectives. *International Education Studies*, *11*(10), 28-41. https://doi.org/10.5539/ies.v11n10p28

Scotland, J. (2012). Exploring the philosophical underpinnings of research: relating ontology and epistemology to the methodology and methods of the scientific, interpretive, and critical research paradigms. *English Language Teaching, 5*(9), 9-16. https://doi.org/10.5539/elt.v5n9p9

Silva, A., Negreiros, F., & Albano, R. (2017). Indiscipline at public school: Teachers' conceptions on causes and intervention. *International Journal of Research in Education and Science*, 3(25311), 1-10. https://doi.org/10.21890/ijres.267354

Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016, August). Recent research on geometry education: an ICME-13 survey team report. *ZDM Mathematics Education, 48*(5), 691-718. https://doi.org/10.1007/s11858-016-0796-6

Smartphone (2018, February 7). *Smartphone definition*. English Oxford Living dictionary. https://en.oxforddictionaries.com/definition/smartphone

Soobrayan, P. B. (2012). *Action Plan To 2014 Towards the Realisation of Schooling 2025.* Department of Basic Education.

Spaull, N. (2013). *South Africa's Education Crisis: The quality of education In South Africa 1994-2011.* Centre for Development and Enterprise.

Stoilescu, D. (2016). Aspects of Theories, Frameworks, and Paradigms in Mathematics Education Research. *European Journal Of Science And Mathematics Education, 4*(2), 140-153. https://doi.org/10.30935/scimath/9460

Stumbles, R. (2018). Dynamic Geometry Software Within The Van Hiele Teaching Framework. *Australian Primary Mathematics Classroom*, *23*(1), 25-30. https://files.eric.ed.gov/fulltext/EJ1175543.pdf

Swoboda, E., & Vighi, P. (2016). *Early Geometrical Thinking in the Environment of Patterns, Mosaics, and Isometries.* Springer International Publishing. https://link.springer.com/chapter/10.1007/978-3-319-44272-3_1

Takači, D., Stankov, G., & Milanovic, I. (2015). Efficiency of learning Environment Using GeoGebra When Calculus Contents Are Learned In Collaborative Groups. *Computers & Education*, *82*, 421-431. https://doi.org/10.1016/j.compedu.2014.12.002

Taleba, Z., & Hassanzadeh, F. (2015). Toward Smart School: A Comparison Between Smart School and Traditional School for Mathematics Learning. *Procedia - Social and Behavioral Sciences 171, Iran* (pp. 90-95). Elsevier Ltd. https://doi.org/10.1016/j.sbspro.2015.01.093

Durban University of Technology (2019). *Prospectus.* https://eduloaded.com/blog/dut-prospectus/

Thompson, P. (2013). The digital natives as learners: Technology use patterns and approaches to learning. *Computers & Education*, *65*, 12-33. https://doi.org/10.1016/j.compedu.2012.12.022

Tran, T., Nguyen, N. G., Bui, M. D., & Phan, A. H. (2014, August). Discovery learning with the help discovery learning with the help. *International Journal of Learning, Teaching and Educational Research, 7*(1), 44-57.

Ünlü, M., & Ertekin, E. (2017). A Structural Equation Model for Factors Affecting Eighth Graders' Geometry Achievement. *Educational Sciences: Theory & Practice*, *17*(5), 1815-1847. https://doi.org/10.12738/estp.2017.5.0545

Vahed, A., & Singh, S. (2015). Developing an Epistemic Game: A Preliminary Examination of the Muscle Mania Mobile Game. Proceedings of the European Conference on Information Management & Evaluation (pp. 516-522). Academic Conferences & Publishing International Ltd.

Venkat, H., & Spaull, N. (2007). What Do We Know About Primary Teachers' Mathematical Content Knowledge In South Africa? An Analysis Of SACMEQ 2007*. International Journal of Educational Development,* 41, 121-130. http://dx.doi.org/10.1016/j.ijedudev.2015.02.002

Verhoeven, N. (2011). *Doing Research. The Hows and Whys of Applied Research.* Eleven International Publishing.

Visser, M., Andrea, J., & Feza, N. (2015, February). Home and School Resources as Predictors of Mathematics Performance in South Africa. *South African Journal of Education, 35*(1), 1-10. https://doi.org/10.15700/201503062354

Vojkuvkova, I. (2012). The van Hiele Model of Geometric Thinking. *WDS'12.* Proceedings of Contributed Papers, Part I, Prague (pp. 72-75). MATFYZPRESS.

Webmaster. (2009, November 24). *National Assembly_Teacher Laptop Initiative*. https://www.education.gov.za/Portals/0/Media/Parliamentary%20Questions/WQ_1886_NA_Teacher_Laptop_Initiative.pdf?ver=2009-11-24-151011-227

Webmaster, D. (2009, October 13). *Dinaledi Schools Programme*. https://www.education.gov.za/Portals/0/Media/Parliamentary%20Questions/WQ%2068%20NCOP%20Dinaledi%20schools%20programme%20180909.pdf?ver=2009-10-13-095352-650

Wiest, L. R., & Amankonah, F. O. (2019). Conceptual Versus Procedural Approaches to Ordering Fractions. *European Journal of Science and Mathematics Education, 7*(1), 61-72. https://doi.org/10.30935/scimath/9534

Statistics Solutions. (n.d.). Retrieved from Qualitative Sampling Techniques: https://www.statisticssolutions.com/qualitative-sampling-techniques/

Yılmaz, G. K., & Koparan, T. (2016). The effect of designed geometry teaching lesson to the candidate teachers' Van Hiele geometric thinking level. *Journal of Education and Training Studies*, *4*(1), 129-141. https://doi.org/10.11114/JETS.V4I1.1067

Zahner, W., Velazquez, G., Moschkovic, J., Vahey, P., & Lara-Meloy, T. (2012). Mathematics teaching practices with technology that support conceptual understanding for Latino/ a students. *The Journal of Mathematical behavior, 31*(4), 431-446. http://dx.doi.org/10.1016/j.jmathb.2012.06.002

Zakaria, E., Salleh, T., S. (2015). Using Technology in Learning Integral Calculus. *Mediterranean Journal of Social Sciences, 6*(5), 144-148. https://doi.org/10.5901/mjss.2015.v6n5s1p144

Zainal, Z. (2007). Case study as a research method. *Jurnal Kemanusiaan bil.9*, 1-6. https://www.researchgate.net/publication/41822817_Case_study_as_a_research_method

Zaiontz, C. (2019). *Cronbach's Alpha Basic Concepts*. Real Statistics Using Excel. www.real-statistics.com/reliability/internal-consistency-reliability/cronbachs-alpha/cronbachs-alpha-basic-concepts/

Zenkin, Y., Furkan, H., & Kutluca, T. (2011). The effect of dynamics mathematics geogebra on student achievement in teaching of trigonometry. *Procedia - Social and Behavioral Sciences 31, Turkey* (pp. 183-187). Elsevier Ltd.

Žilinskiene, I., & Demirbilek, M. (2015). Use of geogebra in primary math education in lithuania: an exploratory study from teachers' perspective. *Informatics in Education*, *2015*(1), 127-142. https://doi.org/10.15388/infedu.2015.08

Zulnaidi, H., & Zakaria, E. (2012). The effect of using geogebra on conceptual and procedural knowledge of high school mathematics students. *Asian Social Science*, *8*(11), 102-106. https://doi.org/10.5539/ass.v8n11p102

Appendices

Appendix A

Interview questions and responses

1. **What can you say about the mobile strategy (GeoGebra applets for 010) that you used to complete the worksheet?**

   Voice007: It is easy to use provided you have data, it is effective and efficient and for a lot of children its helpful if you want to self study.

   Voice010: It is easy to use and understand and everything….

   Voice012: It was very easy to use and I think I leant more maths .. and it was easy to understand the concept using the app

   Voice 017: I enjoyed the fact that when I pressed the part for the triangle it gave me fast responses and I was able to fill in the paper quickly because they gave out quickly.

   Voice019: It was really comfortable and easy to use and I got the results that I wanted.

   Voice20: It was very comfortable, you understood what to do and the rules that you had to follow and understanding terminology that was needed to complete the worksheet.

2. **What would you consider as advantages of using mobile technology in learning Mathematics?**

   Voice007: If you are behind in class, you can use it to help you when you are at home. It doesn't take much of your time because, as the pages would load faster, it gives you the information that you want, at the same time its teaching you what you adore

Voice010: I think students in schools are comfortable in using technology…so using it for mathematics will spark an interest ..

Voice012: Well, you understand more, and … most learners use cellphones and technology, so using technology for maths is actually making it more easier to learn and is more fun

Voice017: They gave answers easily and you can see clearly the triangles and be able to … dimensions

Voice 019: It's more precise and give you exact answers so it will be easier for the learners to understand.

Voice20: It's something that aa, .. apply our general knowledge of technology.

3. What would you consider as challenging in using mobile technology for learning Mathematics?

Voice007: If you don't have data, then you can't easily access the page, and you also require smartphones in most cases, like or, modern devices and sometimes, the touch screen is not as effective, so you may have problems of tapping on.

Voice010: I think the only challenge would be data

Voice012: Well, some people don't have the privilege to use, they don't have phones or anything. So, they might not be able to know how to control it but it is easy to learn.`

Voice 017: For the learners to learn how to use the mobile technology, it would be hard for them, like to get the, how to figure out the answer it would be hard for them. So, they have to be taught first on how to use the technology before they are able to use it for maths.

Voice019: Sometimes like it might not work, you might not even have access like to a phone or actual mobile..

Voice20: Some students do not know how to use technology, so they have a disadvantage , they can't use it so they cant help themselves with maths

4. What improvements or alternatives on the applets do you wish to suggest?

Voice007: I suggest that instead of having it as a page, maybe the site could be an app, so that you can download it and use it offline which will give an advantage to children who can't necessarily afford data to use this page all the time.

Voice010: An alternative would be making it an application, (where) you could work offline.

Voice012: Ehh, could use the app offline so that we don't have to use internet everytime you use the app, and maybe add more sections about maths

Voice 017: I suggest that maybe they don't give the answer immediately. They have a part where they let the learner attempt the question first, then they tell them when they wrong or right and if you are wrong, they explain why and what you were supposed to do in that part.

Voice019: Maybe you shouldn't make it like.. solve quickly. Sometimes you need to think a little more because then … like get the answer.

Voice20: Like wee, there isn't much because everything was really good you could actually conclude that this …was equal to the one that was given

5. What did you enjoy the most when you were completing the worksheet?

Voice007: You can go at your own pace, and as you are completing the worksheet you are learning at the same time and……and you can control what your applet can do to fit what you are working in.

Voice010: I enjoyed that the results came quickly and …

Voice012: I enjoyed going on the app and clicking onto the sides and finding the measurements and getting the results.

Voice017: It was more easier, for me to get the answer and my answers were accurate and right ..and everytime I checked…

6. **Are there other challenges you think fellow learners would encounter when using (the same) mobile technology?**

   Voice007: Regarding that, they have data for a smartphone, then it would be easy for them to access, but if they don't have, those two then I think it will very difficult for them to use it.

   Voice010: I think besides the data if the screens are small it could be difficult for them to see properly.

   Voice012: (This was skipped by mistake and could not come back to it because break time was up)

   Voice 017: No, I don't think so because it is very easy and fun to use.

   Voice019: Some of them may take longer to understand and actually be able to use the thing but then it ..more easier.

   Voice20: They might not know eeh the, for example login, and they might not even understand what to do so it becomes more difficult for them to even do the work.

7. **Is there anything else you would want to share or say about using mobile technology for learning mathematics?**

   Voice007: In this day and age, as technology is improving you can easily teach yourself maths on the applet, it will help you because ….like it gives you, its efficient and… for you.

   Voice010: I think it should be used in schools because it will …

   Voice012: I think that we should use it more often and I really wish that our school offered using moble technology in maths and ..

   Voice017: I wish it could help for the algebra part, algebra yes, like the factorization and all of that ..they show you the steps you need to follow to make it easier to follow like what we did for congruence and the rest of geometry.

Voice019: It is more easier and accurate and like it gives you the best answers and ya basically that's it.

Voice20: It's good to use mobile technology, us the learners… we are used to using mobile technology but at the same time we should try to encourage and teach those who can't use mobile technology to try and use it in order to help them with maths. It's actually one way to understand and get good...

8. **Are there maths sections in which you think mobile technology may assist in learning and understanding?**

Voice007: It's mostly geometry with angles, and algebra expressions and equations.

Voice010: I think geometry as a whole and algebra.

Voice012: Algebra, algebra expressions

Voice 017: Yes, the geometry section….

Voice019: Geomerty, specifically geometry , it makes it easier to find all the .. angles, patterns.

Voice20: Graphs, understanding gradient, understanding equations, all those things, maybe technology will help us understanding how to find the gradient, how to find the equation so that it does not become hard as it is right now.

First Questionnaire

QUESTIONNAIRE

I, Elamo Blessing Chibaya, student number: 213573538, am a Ph.D. student at the University of Kwazulu-Natal under the supervision of Professor Vimolan Mudaly. The main purpose of this research is to investigate the use of mobile technology in the teaching and learning of the concept of congruency of triangles in Mathematics. This questionnaire will help us collect views regarding ways and better practices in learning and understanding this particular concept in mathematics. Thank you for agreeing to take part in this research. All responses to this questionnaire will remain confidential. I will be grateful if you could spare some of your valuable time to answer the questions here posed.

---

1. I like Maths. ☐ Yes ☐ No

   If your answer is No please answer this question: I do not like Maths because:

   ☐ I just hate Maths. ☐ Maths is not for me. ☐ I do not understand explanations

2. I get good marks in Maths ☐ Yes ☐ No

   If your answer is No, answer this question: I do not get good marks in Maths because:

____

____

____

3. I feel comfortable when learning Maths: ☐ Very ☐ Partially ☐No

4. I understand the importance of learning Maths in my life. ☐ Yes ☐No

5. I prefer memorizing Maths concepts and formulae without understanding: ☐ Yes

   ☐No

6. I enjoy hearing my Maths teacher explaining new concepts. ☐ Yes ☐ No

   ☐Sometime

7. I easily get distracted by a long explanation of a concept. ☐ Yes ☐ No

   ☐Sometime

8. I enjoy hearing the thoughts and ideas of my peers in class. ☐ Yes ☐No

9. I like to go to the board or share my answers with peers in maths class. ☐ Yes

   ☐No

10. If I get wrong answers, I always want to identify my mistakes. ☐ Yes ☐No

11. I know the cases of congruency of triangles (SAS, SSS, SAA, RHS) ☐ Yes

    ☐No

12. If No, I am interested in discovering the cases of congruency of triangle. ☐ Yes

    ☐No

13. I enjoy answering questions that say: Calculate, find, what is the value of… ☐ Yes ☐No

14. I feel confident in my ability to answer questions that say: Prove or show: ☐ Yes ☐No

15. My teacher uses a computer (data projector) to explain concepts. ☐ Yes ☐No ☐Sometime

16. I am proficient in using mobile technology. ☐ Yes ☐No

17. I embrace/appreciate the use of mobile technology in learning. ☐ Yes ☐No

18. Mobile technology may enhance our learning of mathematics ☐ Yes ☐No

19. We can afford gadgets required for mobile learning interventions ☐ Yes ☐No

20. I can afford data for my mobile device ☐Yes ☐No

21. I have WiFi at home ☐Yes ☐No

22. What mobile device do you have? ☐ Tablet ☐Smart-phone ☐ Laptop ☐ other

23. State the type of phone you have e.g. Samsung Galaxy S5___________________________

24. What do you think your teacher should do that can help you understand Maths better.

____
____

____
____

Thank you

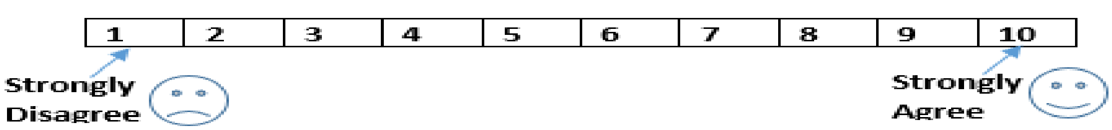
Second Questionnaire

I, Elamo Blessing Chibaya, student number: 213573538, am a PhD student at the University of Kwazulu-Natal under the supervision of Professor Vimolan Mudaly. The main purpose of this research is to investigate the use of mobile technology in the teaching and learning of the concept of congruency of triangles in Mathematics. This questionnaire will help me collect your views regarding the use of Geogebra applets to discover cases of congruency.

Thank you for agreeing to take part in this research. All responses to this questionnaire will remain confidential.

I will be grateful if you could spare some of your valuable time to answer the questions here posed.

Please indicate your level of agreement or disagreement with each of these statements regarding the activity that you just completed. Place a "✓" mark in the box of your answer. The rating is weighed as follows:



Item	Response rating									
	1	2	3	4	5	6	7	8	9	10

1. It was easy to log in.										
2. It was easy to open the workbook.										
3. The applets open quickly.										
4. Touchscreen was working effectively.										
5. It was easy to move from one applet to the other.										
6. My cellphone skills helped me to use the applets.										
7. I learnt to use these applets quickly.										
8. I easily remember how to use the applets.										
9. It was simple to use these applets.										
10. I find it easy to become skillful in using these applets.										
11. It was easy to UNDO a mistake.										
12. I would find it easy for learners in my grade to use these applets.										
13. I enjoyed completing the worksheet using my cell _phone / tablet / laptop etc.										
14. It was easy to measure the missing lengths.										

15. It was easy to measure the missing angles.										
16. I was able to quickly complete the worksheet using these applets.										
17. I feel comfortable using these applets.										
18. I felt confident using these applets.										
19. Triangles were displayed clearly.										
20. Triangle labels were clear.										
21. Triangle measurements were easy to read.										
22. I got expected answers										
23. I am satisfied with these applets										
24. I would recommend these applets to my friends.										
25. I feel the applets help to understand cases of congruence.										
26. I feel using the applets will help identify / see cases of congruency of triangles										
27. I feel confident to answer questions on congruence of triangles.										
28. I would like to use mobile technology (cell_phone, i-Pad, tablet , laptop etc) more										

often when learning Maths.										

29. State the type of device you used (e.g. Samsung Galaxy S5)______________________________

30. List the most negative aspect(s) of using these applets

____
____
____
____

31. List the most positive aspect(s) of using these applets.

____
____
____
____

Appendix B

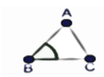
Worksheet

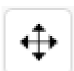
Investigation on congruence of triangles

Two triangles are congruent if they are identical in size and shape. This means that the corresponding sides and angles of the two congruent triangles are equal.

Whenever two triangles are congruent, we can state the following: $\Delta ABC \equiv \Delta FGD$

Note: An included angle in a triangle is formed by two sides of a triangle.


Angle $\hat{B}$ is included between sides BC and AB. Angle__ is included between sides ___and__

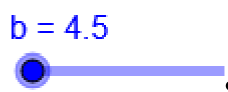

use to move the diagrams


use to drag labels (eg length and angles)


use to measure length

Slider a


use to measure angles



Slider b


Use to get the original triangles

Activity 1i.

a. Use the applet “Discover SSS” to complete the table below:

Triangle	**Slider a = 3 and b = 4.5**					
ΔABC (Green)	AB = 3	AC= 4.5	BC= 4.36	$\hat{A}$=	$\hat{B} =$	$\hat{C} =$
ΔFGD (Blue)	DF = 3	DG = 4.5	FG = 4.36	$\hat{D} =$	$\hat{F} =$	$\hat{G} =$
	Set Slider a = 3.8 and b = 5					

ΔABC (Green)	AB = 3.8	BC = 5.01	AC = 5	$\hat{A}$=	$\hat{B}$=	$\hat{C}$=
ΔFGD (Blue)	DF = 3.8	FG = 5.01	DG = 5	$\hat{D}$=	$\hat{F}$=	$\hat{G}$=
	Set Slider a = 4.6 and b = 5.5					
ΔABC (Green)	AB = 4.6	BC = 5.68	AC = 5.5	$\hat{A}$=	$\hat{B}$=	$\hat{C}$=
ΔFGD (Blue)	DF = 4.6	FG = 5.68	DG = 5.5	$\hat{D}$=	$\hat{F}$=	$\hat{G}$=

b. Using the measurements in the table above and the applet, fill in the gaps:

AB _______ DF; AB is opposite angle ______ and DF is opposite angle_______; and $\hat{C}$_____$\hat{G}$_

BC _______FG; BC is opposite angle _______ and FG is opposite angle _____and $\hat{A}$_______$\hat{D}$

AC _______DG; AC is opposite angle______ and DG is opposite angle _______ ; and $\hat{B}$______$\hat{F}$

c. What can you say about the corresponding angles in each case?____

d. What can you conclude about ΔABC and ΔFGD in each case?____

e. If the corresponding sides of two triangles are ______; then the triangles are __________. This is written as ._______

Activity 1ii.

a. Use applet "Does AAA work" to complete the table below:

Triangle	**Slider a =2 and b = 3.5**					
ΔABC	AB =	BC=	AC=	$\hat{A}$=	$\hat{B}$=	$\hat{C}$=
ΔFGD	DF =	FG =	DG =	$\hat{D}$=	$\hat{F}$=	$\hat{G}$=
Slider a = 3 and b = 4						

ΔABC	AB =	BC=	AC=	$\hat{A}$=	$\hat{B}$=	$\hat{C}$=
ΔFGD	DF =	FG =	DG =	$\hat{D}$=	$\hat{F}$=	$\hat{G}$=

b. Using the measurements in the table above, complete the gaps

$\hat{C}$________$\hat{G}$ but AB ____________________ DF

$\hat{A}$_______$\hat{D}$ but BC _____________________FG

$\hat{B}$______$\hat{F}$ but AC ____________________DG and

c. What can you conclude about ΔABC and ΔFGD in each case?____________________________

d. In two triangles, three corresponding angles may be equal but _____________________________

Activity 2i

a. Use applet "Discover SAS" to complete the table below:

Triangle	**Slider a = 3.5 and α = 45°**					
ΔABC (Green)	AB = 3.5	BC=	AC=4.87	$\hat{A}$= **45°**	$\hat{B}$=	$\hat{C}$=
ΔPQR (Pink)	PQ = 3.5	QR =	PR = 4.87	$\hat{P}$=**45°**	$\hat{Q}$=	$\hat{R}$=
	Set Slider a = 4.5 and α = 60°					
ΔABC (Green)	AB = 4.5	BC=	AC=6.26	$\hat{A}$= **60°**	$\hat{B}$=	$\hat{C}$=
Δ PQR (Pink)	PQ = 4.5	QR =	PR = 6.26	$\hat{P}$= **60°**	$\hat{Q}$=	$\hat{R}$=
	Set Slider a = 5.5 and α = 75°					
ΔABC (Green)	AB =	BC=	AC=	$\hat{A}$=	$\hat{B}$=	$\hat{C}$=

Δ PQR (Pink)	PQ =	QR =	PR =	$\hat{P}$=	$\hat{Q}$=	$\hat{R}$=

b. What can you conclude about the corresponding sides and angles of ΔABC and ΔPQR in each case.

____

c. If, in two triangles, two pairs of corresponding sides are ________________and the corresponding pair of included angles are _____________. Then the two triangles are ______________. This is written as ______________.

Activity 2ii

a. Use applet "Does SSA work" to complete the table below:

Triangle						
ΔPQS	PQ =	QS=	PS=	$\hat{P}$=	$P\hat{Q}S$=	$P\hat{S}Q$=
ΔPQR	PQ =	QR =	PR =	$\hat{P}$=	$P\hat{Q}R$=	$P\hat{R}Q$=

b. What sides and angles are equal in both triangles?____

____

c. Are the triangles congruent?

d. What can you conclude if, in two triangles, two pairs of corresponding sides are equal but the pair of corresponding equal angles are non-included angles:_______________________________________.

Activity 3

a. Use applet "SAA" to complete the table below:

Triangle	**Slider a = 5 ; α = 45° and γ = 30°**

ΔABC (Green)	AB = 5	BC=	AC=	$\hat{A}$= **45°**	$\hat{B}$= **30°**	$\hat{C}$=
ΔPQR (Pink)	PQ = 5	QR =	PR =	$\hat{P}$=**45°**	$\hat{Q}$= **30°**	$\hat{R}$=
	Slider a = 6 ; α = 50° and γ = 35°					
ΔABC (Green)	AB = 6	BC=	AC=	$\hat{A}$= **50°**	$\hat{B}$= **35°**	$\hat{C}$=
Δ PQR (Pink)	PQ = 6	QR =	PR =	$\hat{P}$= **50°**	$\hat{Q}$= **35°**	$\hat{R}$=
	Slider a = 7 ; α = 55° and γ = 40°					
ΔABC (Green)	AB = 7	BC=	AC=	$\hat{A}$= **55°**	$\hat{B}$= **40°**	$\hat{C}$=
Δ PQR (Pink)	PQ = 7	QR =	PR =	$\hat{P}$= **55°**	$\hat{Q}$= **40°**	$\hat{R}$=

b. What can you conclude about the corresponding sides and angles of the triangles?

____

c. What can you conclude about ΔABC and ΔPQR in each case.__________________________________

d. If, in two triangles, one pair of corresponding sides are ________________ and two pairs of corresponding angles are ______________________, then the triangles are __________________.This is written as ____________.

Activity 4

a. Use applet "RHS" to complete the table below:

Triangle	**Slider a = 3,5 ; b = 4**					
ΔABC (Green)	AB = 3.5	BC = 4	AC =	$\hat{A}$= **90°**	$\hat{B}$=	$\hat{C}$=

ΔPQR (Pink)	PQ = 3.5	QR = 4	PR =	$\hat{P}$=**90°**	$\hat{Q}$=	$\hat{R}$=
	Slider a = 4 ; b = 5					
ΔABC (Green)	AB = 4	BC= 5	AC=	$\hat{A}$= **90°**	$\hat{B}$=	$\hat{C}$=
Δ PQR (Pink)	PQ = 4	QR = 5	PR =	$\hat{P}$= **90°**	$\hat{Q}$=	$\hat{R}$=
	Slider a = 4.5 ; b = 6					
ΔABC (Green)	AB = 4.5	BC= 6	AC=	$\hat{A}$= **90°**	$\hat{B}$=	$\hat{C}$=
Δ PQR (Pink)	PQ = 4.5	QR = 6	PR =	$\hat{P}$= **90°**	$\hat{Q}$=	$\hat{R}$=

b. What can you conclude about the corresponding sides and angles of the triangles?

____

c. What can you conclude about ΔABC and ΔPQR in each case.___________________________________

d. If, in two right-angled triangles, the hypotenuse of each triangle is equal and a pair of corresponding sides are ________________, then the triangles are __________________. This is written as _______

Appendix C

Confirmatory Factor Analysis of the first questionnaire

Annotated syntaxes for affection, learning preference, proficiency, and affordance constructs and the tables of the loadings are presented here. From the tables, I only considered the factor loadings in column 1 with the heading MR1.

Calculating factor loadings of affection construct

```
> setwd("C://Users//Admin//Documents/Research")

>  datamatrix=read.table("data.txt", header=FALSE)

> datam=sapply(datamatrix,as.factor)

> library(polycor)

> library(mvtnorm)

> library(sfsmisc)

> library(psych)

> hetmat = hetcor(datam)

> hetmat=as.matrix(hetmat)

> Affection <- fa(r = hetmat[,c(1, 2, 32) ], nfactors = 1, n.obs = nrow(datam), rotate = "varimax")

datamatrix=read.table("binary2.txt", header=FALSE)

>Affection

   MR1   h2    u2 com

Q1 -0.36 0.13  0.87   1

Q2  0.37 0.14  0.86   1
```

Q3 1.06 1.12 -0.12   1

Calculating factor loadings of affection construct

> preference <- fa(r = hetmat[,c(5, 6, 8, 10, 12, 14)], nfactors = 1, n.obs = nrow(datam), rotate = "varimax")

> preference

   MR1   h2   u2 com

Q5 0.50 0.25 0.75   1

Q6 0.51 0.26 0.74   1

Q8 0.80 0.64 0.36   1

Q10 0.75 0.56 0.44   1

Q12 0.40 0.16 0.84   1

Q14 0.81 0.66 0.34   1

Calculating factor loadings for the proficiency construct

> proficiency <- fa(r = hetmat[,c(16,17,18)], nfactors = 1, n.obs = nrow(datam), rotate = "varimax")

   MR1   h2   u2 com

Q16 0.96 0.832 0.168   1

Q17 0.96 0.085 0.915   1

Q18 1.04 1.052 -0.052   1

Calculating factor loadings for the affordance construct

> affordance <- fa(r = hetmat[,c(19,20,21)], nfactors = 1, n.obs = nrow(datam), rotate = "varimax")

   MR1   h2   u2 com

Q19	0.91	0.832	0.168	1
Q20	0.29	0.085	0.915	1
Q21	1.03	1.052	-0.052	1

Reliability of affection, learning preference, proficiency, and affordance construct

Reliability Statistics of the Affection Construct

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.612	.553	4

Reliability Statistics of the Learning Preference Construct

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items

.565	.610	6

Reliability Statistics of the Proficiency Construct

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.734	.709	3

Reliability Statistics of the Affordance Construct

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.498	.502	3

Confirmatory Factor Analysis of the second questionnaire

Annotated syntaxes for access, easy-to-learn, easy-to-use, legibility, and the satisfactory constructs and the tables of the loadings are presented here. From the tables, I only considered the factor loadings in column 1.

Calculating factor loadings for the access construct

```
> setwd("C://Users//Admin//Documents/Research")

> datamatrix=read.table("q2.txt", header=FALSE)

> library(lavaan)

> f1='f=~V1+V2+V3'

> Acc=cfa(f1,data=org,std.lv=TRUE)

> summary(Acc,fit.measures=TRUE,standardized=TRUE)

 f =~

  V1        0.657   0.259   2.531   0.011   0.657   0.580

  V2        0.585   0.204   2.860   0.004   0.585   0.686

  V3        0.670   0.235   2.854   0.004   0.670   0.684
```

Calculating factor loadings for the easy-to-learn construct

```
> setwd("C://Users//Admin//Documents/Research")

> datamatrix=read.table("29Juneq2.txt", header=FALSE)

> library(lavaan)
```

```
> f3='f=~V7+V8+V9+V10+V12'

> learn = cfa(f3,data=datamatrix,std.lv=TRUE)

> summary(learn,fit.measures=TRUE,standardized=TRUE)

Latent Variables:

                Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all

 f =~

   V7              0.646    0.153    4.214    0.000    0.646    0.734

   V8              1.007    0.152    6.604    0.000    1.007    0.984

   V9              0.912    0.197    4.620    0.000    0.912    0.785

   V10             0.465    0.111    4.175    0.000    0.465    0.729

   V12             0.586    0.174    3.365    0.001    0.586    0.616
```

Calculating factor loadings for the easy-to-use construct

```
> f4='f=~V4+V5+V9+V11+V14+V15+V16+V17+V18'

> Use = cfa(f4,data=datamatrix,std.lv=TRUE)

> summary(Use,fit.measures=TRUE,standardized=TRUE)

 f =~

   V4              0.515    0.173    2.970    0.003    0.515    0.573

   V5              0.735    0.396    1.857    0.063    0.735    0.379

   V9              0.941    0.200    4.709    0.000    0.941    0.810

   V11             0.711    0.155    4.579    0.000    0.711    0.795

   V14             0.629    0.172    3.647    0.000    0.629    0.674

   V15             0.470    0.232    2.023    0.043    0.470    0.410

   V16             0.577    0.162    3.556    0.000    0.577    0.661

   V17             0.407    0.111    3.666    0.000    0.407    0.677
```

V18	0.567	0.144	3.934	0.000	0.567	0.714

Calculating factor loadings for the legibility construct

```
> f4='f=~V19+V20+V21'

> leg=cfa(f4,data=org,std.lv=TRUE)

> summary(leg,fit.measures=TRUE,standardized=TRUE)
```

f =~

V19	0.938	0.181	5.174	0.000	0.938	0.857
V20	1.160	0.174	6.648	0.000	1.160	1.003
V21	0.716	0.168	4.275	0.000	0.716	0.746

Calculating factor loadings for the satisfaction construct

```
> f5='f=~V22+V23+V24+V25+V26+V28'

> satsf=cfa(f5,data=org,std.lv=TRUE)

> summary(satsf,fit.measures=TRUE,standardized=TRUE)
```

f =~

V22	0.664	0.145	4.574	0.000	0.664	0.779
V23	1.03	0.142	6.072	0.000	1.051	0.921
V24	0.961	0.158	6.089	0.000	0.961	0.939
V25	1.079	0.177	6.082	0.000	1.079	0.939
V26	0.285	0.136	2.096	0.036	0.285	0.412
V28	0.071	0.055	1.292	0.196	0.071	0.261

Constructing a histogram with an overlay Curve for item 1

The histograms for items 14, 19 and 28 are constructed using the same commands. Changes should be made on the lines in italics. For example, for item 14, instead of "*>g = data[,1]*" it will be "*>g = data[,14]*" and in the heading *"Distribution for item 1"*, the 1 is replaced with 14.

```
> data=read.table("cfanalysis.txt",header=FALSE)

> data2=as.matrix(data)

 >g=data[,1]

h <- hist(g, breaks = 2, density = 5, main = "Distribution for Item 1",ylab="Number of
participants",xlab="Scores")

xfit <- seq(min(g), max(g), length = 40)

> yfit <- dnorm(xfit, mean = mean(g), sd = sd(g))

> yfit <- yfit * diff(h$mids[1:2]) * length(g)

> lines(xfit, yfit, col = "black", lwd = 2)
```

Appendix D

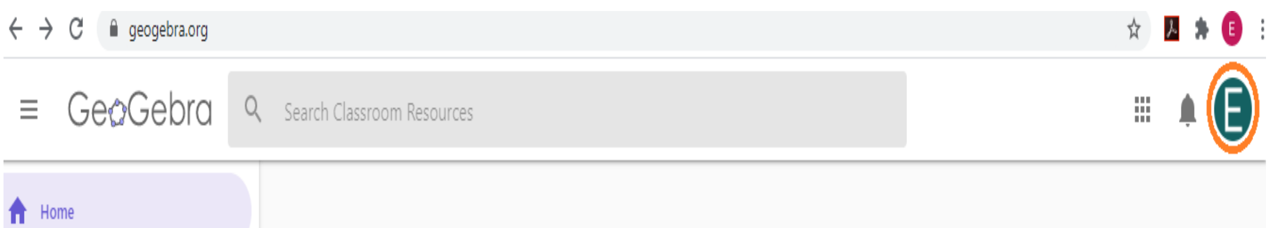
Steps to create the workbook

Step 1: In the text box of your browser type GeoGebra.org and press enter.

Step 2: When GeoGebra site opens, select sign in.

Step 3: If you have a GeoGebra account you enter your credentials, if not you select create account and enter the required information.

Step 4: After signing in, click on your initial icon on the right top (circled in orange).



Step 5: Click create option and then choose book from the menu list.

Step 6: The book information window will be displayed and then you enter the following information: title, book description, target age group, tags (keywords that help with searching), and choose visibility information. There are three visibility options, public (allows other users to view and use the book), shared with link only allows users with the lick to view and access the book. It

does not appear in the search results of other users. Private allows the owner to access the book by entering username and password.

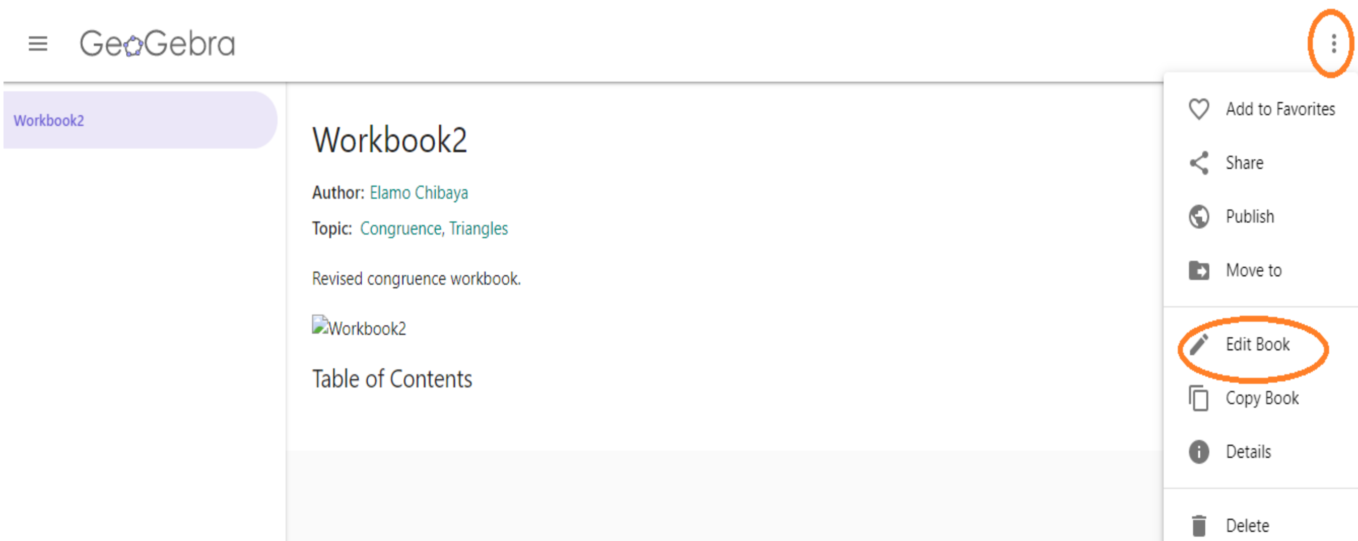
Step 7: Save the book.

Steps to create applet sss

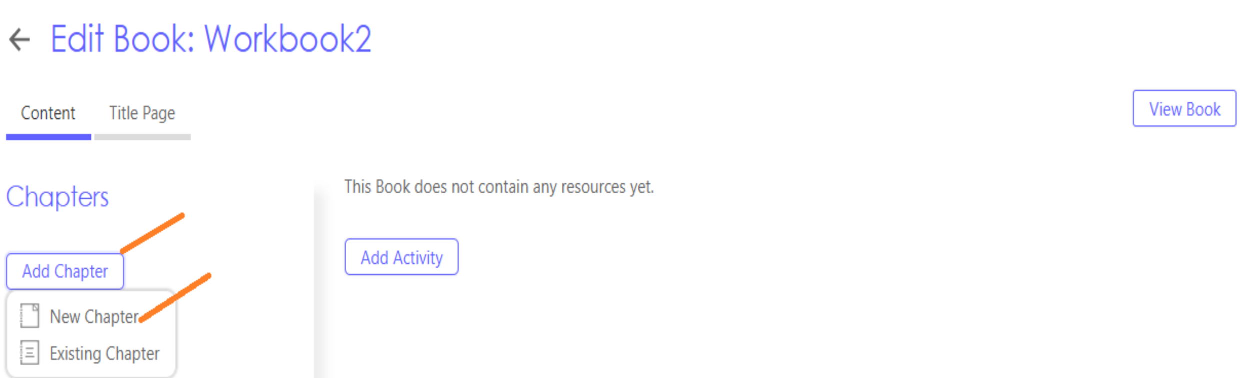
Step 1: To access the book click the initial icon.

Step 2: Click on the book.

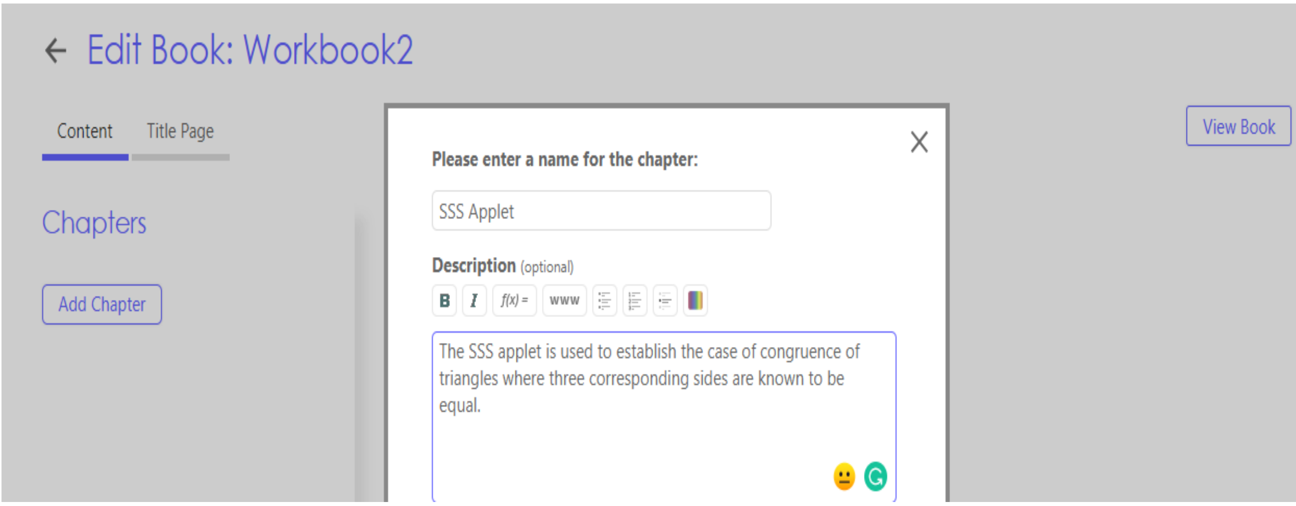
Step 3: Click on the three dots, then click edit book.



Step 4: On the new window that appears, click add a chapter, and then click new chapter.

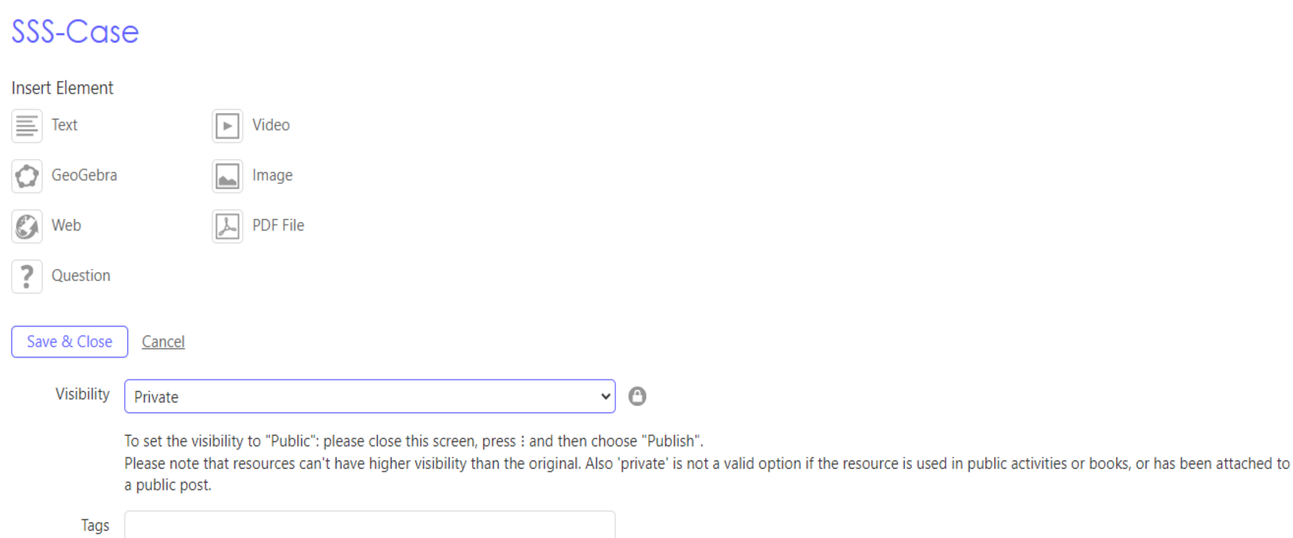


Step 5: In the new window that appears, enter the name of the chapter, for example, SSS and a short description of the purpose of the applet, and click save.

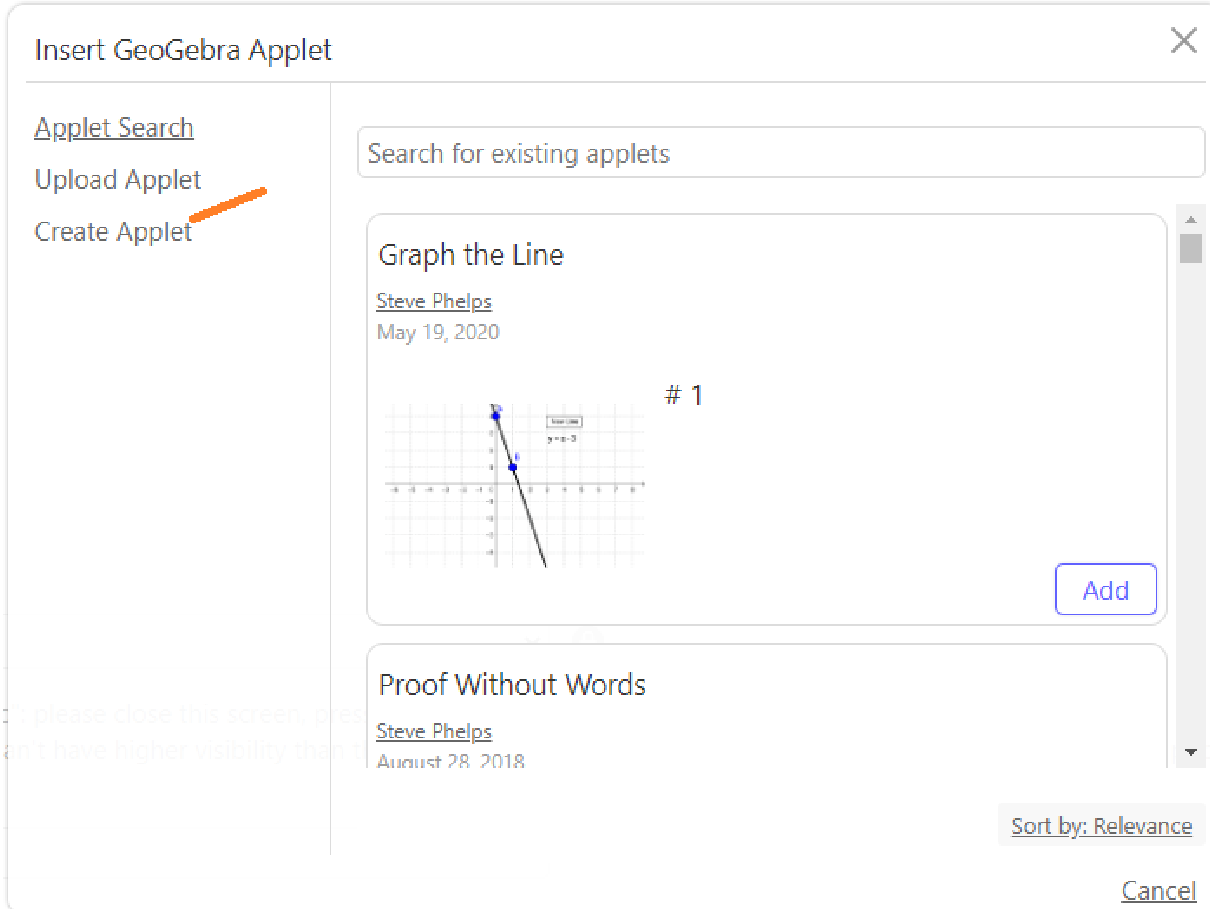


Step 6: Click add activity, and then on the new window that appears click new activity.

Step 7: In the new window that appears, under insert elements, select the visibility option, public, private, or shared with a link, enter your keywords in the tags text box, then select GeoGebra.



Step 8: In the insert GeoGebra Applet window that appears select Create Applet.



Step 9: On the list of options presented, select Geometry, and the construction window is displayed.

Step 10: Create two sliders that will be used to adjust the length of AC and AB.

Step 11: Click on the tenth tool and select the slider option. On the slider window displayed, in the name text box you can enter the desired name for the slider, on the checklist that follows select option number, then enter the minimum, maximum, and increment values. For example, slider a is used to change the length of AB. I set the minimum at 3 cm, maximum at 4.6 and increment is equal to 0.8

Step 10: Click the third tool and choose Segment with Given length.

Step 11: Select the position of vertex A, and then for the length of AB, you enter the slider name, a, because the length of AB depends on the value set on the slider.

Step 12: Click the 6th tool and select option Circle: Centre & Radius.

Step 13: Click point A and then enter slide name b as, the radius. Length of side AC depends on the value set on slider b.

Step 14: Click point B and then enter the radius 4,36 cm.

Step 15:Vertex C is the point of intersection of the two circles constructed in steps 13 and 14. To mark the point of intersection, click the second tool, then choose option intersect. Click anywhere on the circumference of the two circles. There are two intersection points C and D, but I chose to use C because I want the base of the triangle to be at the bottom.

Step 16: After getting point of intersection, hide the circles by right-clicking on each circle and then uncheck the show object. Do the same for point D.

Step 17: To join point A and C and point B and C, select the third tool, then option segment, then click point A, then point C, and do the same for B and C.

Step 18: To display the length, click the length icon and then select the distance or length option and then click on each of the three sides.

Step 19: If the measurement labels are not well positioned, click the first tool and select option move. Then move the measurements to desired positions. Measurements should not be displayed over the lines (Hohenwarter, 2016).

Step 20: To put the stokes to show equality, right-click side AB, select option settings, then click tab style and then click Decoration, then choose the option with one stroke. Repeat the same steps for side AC and BC.

Step 21: Translate triangle ABC to get the congruent image and label it ∆FGD. This is achieved by creating a translation vector first. Click the line and vector menu (third tool), then select vector option. Select the transformation menu (ninth tool) and click Translate by Vector option.

Step 22: Hide all unwanted objects and labels (right click on the object and uncheck show object)

Steps to create applet Does AAA work

Step 1 to Step 18 is the same as the steps for Applet SSS.

Step 19 Enlarge/Dilate to get a similar triangle. To enlarge, choose a center of enlargement point preferably on the top left of the first triangle depending on which side you want the image to appear. Then, on the transformation menu, select tool dilates from point and use factor 1.5.

Step 20 Hide all unwanted objects and labels (right-click and uncheck)

Step 21 If there is a need to change the name of the label, right-click on the label and select the option rename.

Steps to create applet SAS

Steps 1 to 18 are almost the same in Applet SSS. For this applet create one slider to change the length of side AB and a slider to change the angle between the two given sides.

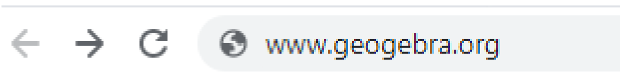
You can use rotation, translation, or reflection to get the image. I used reflection. To reflect, draw a line of reflection using the line option on the third tool. Then, choose the option to reflect on a point on the transformation menu.

Appendix E

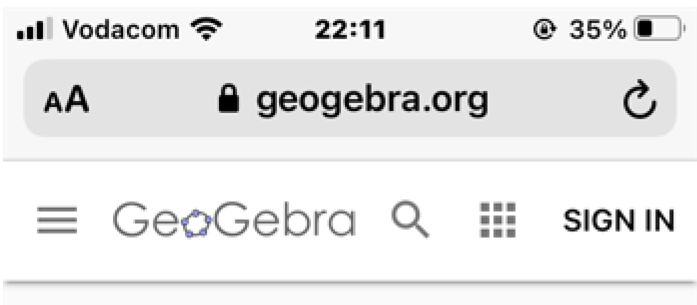
User Documentation

This section has instructions to access and navigate through the GeoGebra book. In addition, explanations on measuring lengths and angles are included.

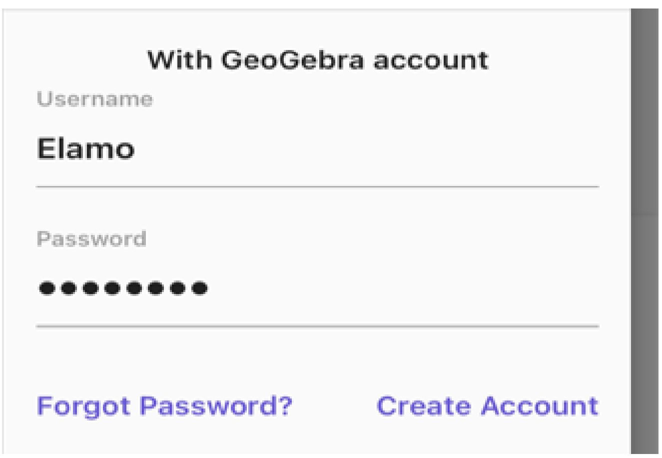
Step 1: On the internet browser visit www.geogebra.org



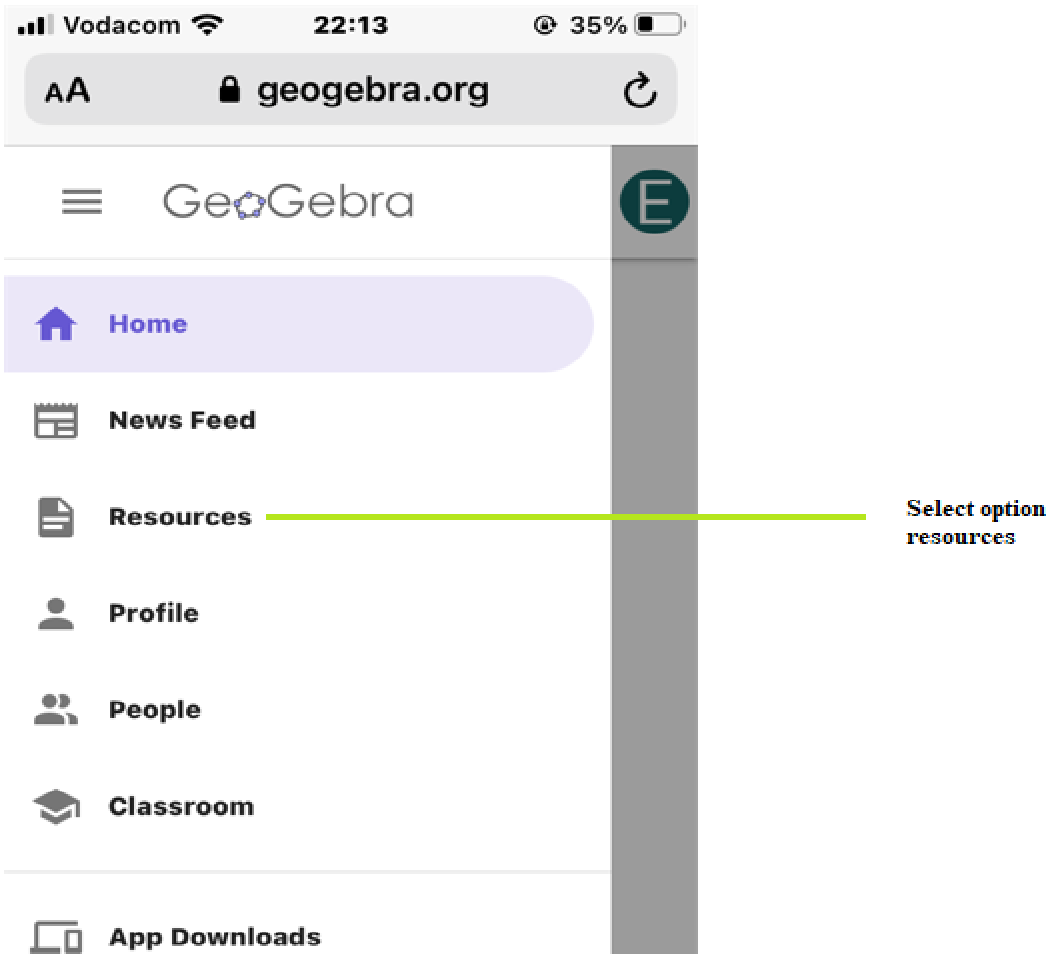
Step 2: On the new window select sign in



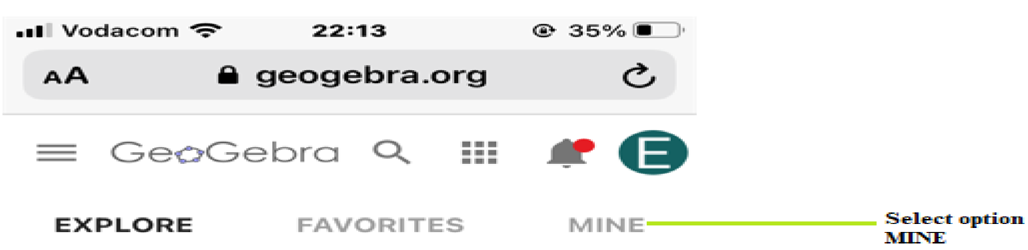
Step 3: Enter the username **Elamo** and password **Elamo123**



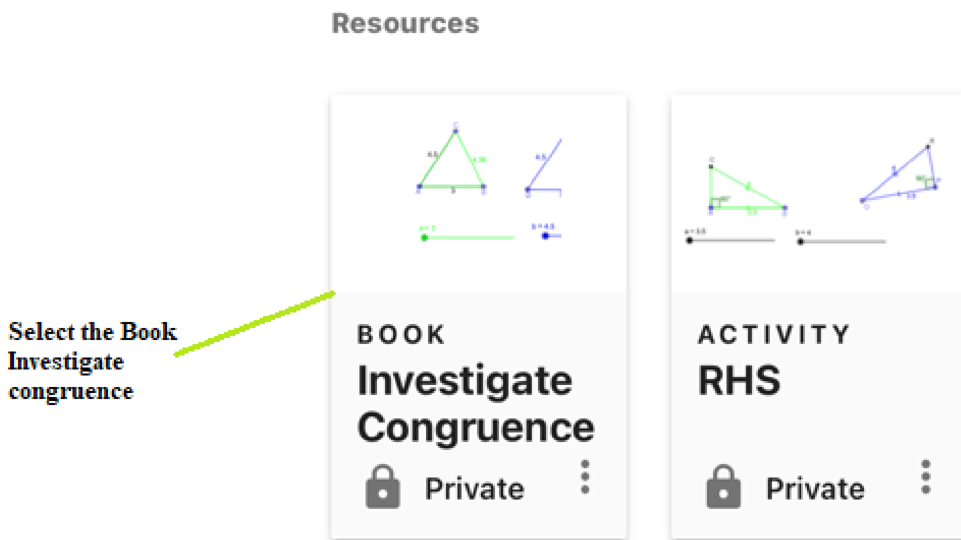
Step 4: Select the more option ≡ to get the drop down menu if its not showing. Then select option **Resources.**



Step 5: Select option **MINE.**



Step 6: Many activities may show up but, select the **Book Investigate Congruence.**



Step 7: On the new window you will see a table of contents as shown below.

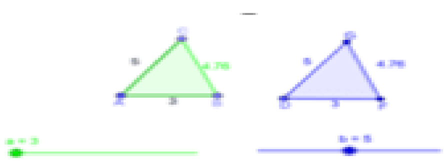


Table of Contents

Step 8: Select the applet.

Explanation of the Tools

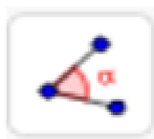
Measuring length

To measure the length you select this tool,  , then you click on the line segmet you want to measure.

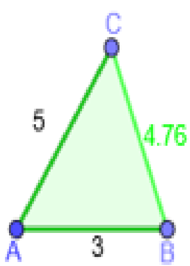
Measuring Angles

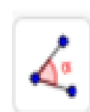
You can get all three angles same time or measure one angle at a time.

Measuring three angles same time

Select the angle tool,  , then click inside the triangle.

Measuring one interior angle

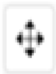


If in ΔABC, you want to measure $\hat{B}$, select angle tool ,  then, click C", "B", then "A" in that order (clockwise). If you select the points in an anti-clockwise direction.

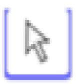
Using the slider option

Drag the circle to change the value of "b"

Movement Tools



The move graphics view tool is used to move the objects (for example, the triangles or sliders). You select the tool first, then you drag the object.



The move tool is used to move the labels (For example, the measurements and the vertices labels of the triangles.

Reconstruction Tool

The reconstruction tool is used to rest to the original triangles.

Reverse process tool

Undo tools are used to reverse a process.

Appendix F

SPSS Association results

This section has the SPSS results for assessing the association between the following pairs of items: liking mathematics (item 1a) and technology appreciation (item 17), proficiency in using technology (item 16), and technology appreciation (item 17). Additionally, I assessed the association between getting good marks in mathematics (item 2a) and proficiency in using technology (item 16) and getting good marks in mathematics (item 2a), and technology appreciation (item 17).

Fisher's exact test for association between liking mathematics and technology appreciation.

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.233[a]	1	.629	.682	.501
Continuity Correction[b]	.000	1	.985		
Likelihood Ratio	.238	1	.626	.682	.501
Fisher's Exact Test				1.000	.501
N of Valid Cases	25				

a. 2 cells (50.0%) have expected count less than 5. The minimum expected count is 2.52.

b. Computed only for a 2x2 table

Fisher's exact test for association between proficiency in using technology and technology appreciation.

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.586[a]	1	.444		
Continuity Correction[b]	.000	1	1.000		
Likelihood Ratio	.916	1	.339		
Fisher's Exact Test				1.000	.640
Linear-by-Linear Association	.562	1	.453		
N of Valid Cases	25				

a. 2 cells (50.0%) have expected count less than 5. The minimum expected count is .36.

b. Computed only for a 2x2 table

Fisher's exact test for association between getting good marks in mathematics and proficiency in using technology

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.043[a]	1	.835		
Continuity Correction[b]	.000	1	1.000		
Likelihood Ratio	.083	1	.773		
Fisher's Exact Test				1.000	.960
Linear-by-Linear Association	.042	1	.838		
N of Valid Cases	25				

a. 3 cells (75.0%) have expected count less than 5. The minimum expected count is .04.

b. Computed only for a 2x2 table

Fisher's exact test for association between getting good marks in mathematics and technology appreciation

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.694[a]	1	.405		
Continuity Correction[b]	.000	1	1.000		

Likelihood Ratio	1.049	1	.306		
Fisher's Exact Test				1.000	.600
Linear-by-Linear Association	.667	1	.414		
N of Valid Cases	25				

a. 2 cells (50.0%) have expected count less than 5. The minimum expected count is .40.

b. Computed only for a 2x2 table

Appendix G

Ethical Clearance

UNIVERSITY OF KWAZULU-NATAL™
INYUVESI YAKWAZULU-NATALI

26 June 2018

Mrs Ilamo Blessing Chibaya (213573538)
School of Education
Edgewood Campus

Dear Mrs Chibaya,

Protocol reference number: HSS/0421/018D
Project Title: An exploration of the use of mobile technologies in the teaching and learning of Mathematics: A case study of the teaching and learning of Congruency of Triangles

Approval Notification – Expedited Application

In response to your application received 03 May 2018, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment /modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

[redacted]

Dr Shamila Naidoo (Deputy Chair)

/ms

Cc Supervisor: Professor Vimolan Mudaly
Cc Academic Leader Research: Dr SB Khoza
Cc School Administrator: Ms Tyzer Khumalo

Humanities & Social Sciences Research Ethics Committee
Professor Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** [illegible] / [illegible] / [illegible]
Website: www.ukzn.ac.za

1910 - 2010
100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses: Edgewood, Howard College, Medical School, Pietermaritzburg, Westville

Originality Report

213573538FinalThesis_Excluding Reflist

ORIGINALITY REPORT

16%	13%	5%	7%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	link.springer.com Internet Source	1%
2	hdl.handle.net Internet Source	1%
3	melbapplets.ms.unimelb.edu.au Internet Source	<1%
4	researchspace.ukzn.ac.za Internet Source	<1%
5	docslide.us Internet Source	<1%
6	Submitted to University of KwaZulu-Natal Student Paper	<1%
7	mafiadoc.com Internet Source	<1%
8	files.eric.ed.gov Internet Source	<1%
9	ro.ecu.edu.au Internet Source	<1%