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Chiral anomaly, induced current, and vacuum polarization tensor for a Dirac field in the presence of a defect

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ABSTRACT

We evaluate the vacuum polarization tensor (VPT) for a massless Dirac field in 1+1 and 3+1 dimensions, in the presence of a particular kind of defect, which in a special limit imposes bag boundary conditions. We also show that the chiral anomaly in the presence of such a defect is the same as when no defects are present, both in 1+1 and 3+1 dimensions. This implies that the induced vacuum current in 1+1 dimensions due to the lowest order VPT is exact.

Chiral and other anomalies have been objects of intense research, since the pioneering derivation of the anomalous divergence of the axial current [1]. Their relevance to diverse theoretical and phenomenological aspects of Quantum Field Theory (QFT) can hardly be emphasized. This is one of the reasons why the chiral anomaly has been derived in the context of many diverse models, and by following different approaches [2–4] (for a comprehensive review of chiral and other types of anomalies see, for example [5]).

In physical terms, anomalies have their origin in the existence of UV divergences: short-distance fluctuations that require the use of a regulator. This, either implicitly or explicitly, implies the introduction of a *mass* scale Λ, which results in the breaking of a symmetry that depends on the absence of any such scale. This breaking manifests itself in the violation of a classical conservation law, a violation which survives the removal of the regulator, i.e., $\Lambda \to \infty$.

On the other hand, introducing nontrivial boundary conditions in QFT models is a rather explicit breaking of (at least) translation symmetry. This leads to many interesting effects, a noteworthy example of which being the Casimir effect [6,7], as well as many other related phenomena [8]. In the case of fermionic fields, nontrivial boundary conditions, and the resulting Casimir effect, has been studied extensively within the bag model of QCD [9]. Besides, interesting applications of the fermionic Casimir effect to carbon nanotube models have been presented [10,11]. It is the aim of this letter to study the interplay between the two phenomena, anomalies and non trivial boundary conditions, in a concrete system: fermions in the presence of a potential, which may be used to impose bag boundary conditions. As shown in [12], in order to impose that kind of condition, the potential has to be a singular, space-dependent mass term.

We analyze the results of that interplay on two objects: the chiral anomaly and the vacuum polarization tensor (VPT), a correlator between current fluctuations, which one should expect to exhibit a strong dependence with the distance to the boundary.

The structure of this work is as follows: in Sect. 1 we introduce the model; then, in Sect. 2 we evaluate the chiral anomaly for that model, and in Sect. 3 the induced vacuum current and the VPT. Finally, in Sect. 4 we present our conclusions.

1. The model

We consider a quantum fermionic field $(\psi, \bar{\psi})$ in $D = d+1$ dimensions ($d = 1, 3$), endowed with an Euclidean action $S(\bar{\psi}, \psi; A)$, with A an external Abelian gauge field. We impose non-trivial boundary conditions on the fermionic field at $x_d = 0$, by coupling it to a singular scalar potential:

$$\mathcal{S}(\bar{\psi}, \psi; A) = \int d^{d+1}x\, \bar{\psi}(x) \left[\not{D} + g\delta(x_d) \right] \psi(x) \,, \tag{1}$$

with $\not{D} = \not{\partial} + ie\, \not{A}(x)$, and g a dimensionless constant. It is worth noting that in [13] a $1+1$ dimensional model has been studied, where a singular potential is part of the gauge potential; namely, $A_\mu(x) = B_\mu(x) + s_\mu(x_0)\delta(x_1)$, with B_μ a bulk contribution of the gauge potential, and s_μ a singular term.

Recalling the approach of [14,15], one sees that $g = 2$ imposes *bag* boundary conditions [14], namely, the normal component of the vector current J_μ due to the Dirac field, vanishes on the interface $x_d = 0$. We assume that the fermions are confined to the region which, in our choice of coordinates, corresponds to $x_d > 0$.

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For $g = 2$, it becomes disconnected (independent) from its complement. For $g \neq 2$, however, that is no longer true, as part of the current may cross the interface between them.

In our conventions, both $\hbar$ and the speed of light are equal to 1, spacetime coordinates are denoted by x_μ, $\mu = 0, 1, \ldots, d$, and the metric tensor is $g_{\mu\nu} \equiv \mathrm{diag}(1, 1, \ldots, 1)$. Regarding Dirac's γ-matrices, for $d = 1$ they are chosen in the representation:

$$\gamma^0 \equiv \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 \equiv \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{2}$$

and

$$\gamma^5 \equiv \gamma_5 \equiv -i\gamma^0\gamma^1 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3}$$

with σ_i ($i = 1, 2, 3$) representing the standard Pauli's matrices. On the other hand, for $d = 3$:

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu^\dagger \\ \sigma_\mu & 0 \end{pmatrix}, \quad \gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} \mathbb{I}_{2\times 2} & 0 \\ 0 & -\mathbb{I}_{2\times 2} \end{pmatrix}, \tag{4}$$

where $\sigma_0 \equiv i\mathbb{I}_{2\times 2}$.

The boundary conditions that this system imposes on the fields are as follows: the singular potential in the Dirac equation implies a discontinuity in ψ which, following [16], may be replaced by the average of the two lateral limits:

$$\gamma_d\big(\psi(x_\shortparallel, \epsilon) - \psi(x_\shortparallel, -\epsilon)\big) + \frac{g}{2}\big(\psi(x_\shortparallel, \epsilon) + \psi(x_\shortparallel, -\epsilon)\big) = 0, \tag{5}$$

with $x_\shortparallel \equiv (x_0, x_1, \ldots, x_{d-1})$.

Setting $g = 2$, and introducing the (orthogonal) projectors: $\mathcal{P}^\pm \equiv \frac{1\pm\gamma_d}{2}$,

$$\mathcal{P}^+\psi(x_\shortparallel, \epsilon) = -\mathcal{P}^-\psi(x_\shortparallel, -\epsilon). \tag{6}$$

Thus, the orthogonality of these projectors leads to:

$$\mathcal{P}^+\psi(x_\shortparallel, \epsilon) = 0, \quad \mathcal{P}^-\psi(x_\shortparallel, -\epsilon) = 0. \tag{7}$$

Each one of these conditions implies the vanishing of $\bar\psi(x)\gamma_d\psi(x)$, the normal component of the vacuum current (see next Section below) for $g = 2$, approaching the border, or 'wall' either from $x_d > 0$ or from $x_d < 0$.

2. Chiral anomaly in the presence of a defect

Let $j^5_\mu(x) \equiv \langle J^5_\mu(x)\rangle$ the vacuum expectation value of the axial current $J^5_\mu(x) \equiv ie\bar\psi(x)\gamma_\mu\gamma_5\psi(x)$, where the average symbol $\langle \ldots \rangle$ is defined as follows:

$$\langle \ldots \rangle \equiv \frac{\int \mathcal{D}\bar\psi\mathcal{D}\psi \ldots e^{-S(\bar\psi,\psi;A)}}{\int \mathcal{D}\bar\psi\mathcal{D}\psi\, e^{-S(\bar\psi,\psi;A)}}, \tag{8}$$

with S as in (1). A *naive* evaluation of the divergence of $j^5_\mu(x)$, using the equations of motion satisfied by the Dirac field, yields the wrong (classical) result: $\partial_\mu j^5_\mu(x) = 2ieg\delta(x_d)\langle\bar\psi(x)\gamma_5\psi(x)\rangle$. This is wrong because of the ill-defined nature of the fermion bilinear. A possible way to tackle this in a gauge invariant manner is to use a (single), bosonic, Pauli-Villars regulator field ϕ, with a mass Λ, having a Dirac action with the same couplings as the Dirac field. This produces for the divergence[1]:

$$\partial_\mu j^5_\mu(x) = \lim_{\Lambda\to\infty}\big\{2ieg\delta(x_d)\big[\langle\bar\psi(x)\gamma_5\psi(x)\rangle + \langle\bar\phi(x)\gamma_5\phi(x)\rangle\big] + \Lambda\langle\bar\phi(x)\gamma_5\phi(x)\rangle\big\}. \tag{9}$$

Therefore

$$\partial_\mu j^5_\mu(x) = \mathcal{A}(x) + 2ieg\delta(x_d)\,\langle\bar\psi(x)\gamma_5\psi(x)\rangle \tag{10}$$

where

$$\mathcal{A}(x) = 2ie\lim_{\Lambda\to\infty}(\Lambda + g\delta(x_d))\mathrm{tr}\Big[\gamma_5\langle x|\big(\not{D} + g\delta(x_d) + \Lambda\big)^{-1}|x\rangle\Big] \tag{11}$$

where Dirac's bra-ket notation has been used to denote matrix elements of the inverse of the Dirac operator (which includes the singular potential and the gauge field). At this point we note that, for $g = 0$ (i.e., no defect), one can show that:

$$\mathcal{A}|_{g=0}(x) \equiv \mathcal{A}_0(x) = 2ie\lim_{\Lambda\to\infty}\mathrm{tr}\Big[\langle x|f(-\frac{\not{D}^2}{\Lambda^2})|x\rangle\Big] \tag{12}$$

where $f(x) \equiv \frac{1}{1+x}$. Since f is a function which satisfies $f(0) = 1$, and $\lim_{x\to\infty} f^{(k)}(x) = 0$ for all k, this reproduces the proper result for the anomaly, namely,

$$\mathcal{A}_0(x) = \frac{2ie^{1+\frac{D}{2}}}{(4\pi)^{\frac{D}{2}}(\frac{D}{2})!}\epsilon_{\mu_1\nu_1\ldots\mu_{\frac{D}{2}}\nu_{\frac{D}{2}}}F_{\mu_1\nu_1}(x)\ldots F_{\mu_{\frac{D}{2}}\nu_{\frac{D}{2}}}(x), \tag{13}$$

where $\epsilon_{\mu_1\nu_1\ldots\mu_{\frac{D}{2}}\nu_{\frac{D}{2}}}$ is the Levi-Civita Symbol in $D = d + 1$ dimensions.

The anomaly $\mathcal{A}$ can also be obtained in terms of the anomalous Jacobian J_ϕ due to the (infinitesimal version of the) transformation $\psi(x) \to e^{ie\phi(x)\gamma_5}\psi(x)$, $\bar\psi(x) \to \bar\psi(x)e^{ie\phi(x)\gamma_5}$, as follows:

$$\mathcal{A}(x) = 2ie\frac{\delta\log[J_\phi]}{\delta\phi(x)}, \tag{14}$$

where

$$\log[J_\phi] \equiv \lim_{t\to 0^+}\sum_{k=0}^{\infty} t^{\frac{k-D}{2}}a_k(\phi\gamma^5, \not{D}^2), \tag{15}$$

where a_k are functions of matrix-valued arguments. Let us consider now the form of the anomaly for two cases: bag boundary conditions and the general (g not necessarily equal to 2).

2.1. Bag boundary conditions

For the case of bag boundary conditions (in our set-up: $g = 2$), and more general geometries, the calculation of the coefficients in (15) has been presented in [17]. For a planar wall, the first 5 coefficients reduce to:

$$a_0(\phi\gamma^5, \not{D}^2) = (4\pi)^{\frac{-D}{2}}(\int_{x_d>0} d^Dx\,\mathrm{tr}(\phi(x)\gamma_5)), \tag{16}$$

$$a_1(\phi\gamma_5, \not{D}^2) = \frac{1}{4}(4\pi)^{\frac{-d}{2}}(\int d^dx_\shortparallel\,\mathrm{tr}(\chi\phi(x_\shortparallel, 0)\gamma_5)), \tag{17}$$

$$a_2(\phi\gamma_5, \not{D}^2) = \frac{1}{6}(4\pi)^{\frac{-D}{2}}(\int_{x_d>0} d^Dx\,\mathrm{tr}(6\phi(x)\gamma_5[\gamma_\nu, \gamma_\mu]\frac{F_{\mu\nu}(x)}{4}) + \int d^dx_\shortparallel\mathrm{tr}(3\chi\,\partial_d\phi(x_\shortparallel, 0)\gamma_5)), \tag{18}$$

$$a_3(\phi\gamma_5, \not{D}^2) = \frac{1}{384}(4\pi)^{\frac{-(D-1)}{2}} \times (\int dx_\shortparallel\mathrm{tr}(\phi(x_\shortparallel, 0)\gamma^5(96\chi[\gamma_\nu, \gamma_\mu]\frac{F_{\mu\nu}(x_\shortparallel, 0)}{4}) + 24\chi\,\partial_d\partial_d\phi(x_\shortparallel, 0)\gamma^5)), \tag{19}$$

[1] The average symbol for the regulator field is defined in an entirely analogous fashion as for the original field, except for its mass and opposite statistics.

$$
\begin{aligned}
&a_4(\phi\gamma^5,\not{D}^2)\\
&=\frac{1}{360}(4\pi)^{\frac{-D}{2}}\Big(\int_{x_d>0} d^D x\,\mathrm{tr}(\phi(x)\gamma^5(60\partial_i\partial_i[\gamma_\nu,\gamma_\mu]\frac{F_{\mu\nu}(x)}{4}\\
&\quad+180E^2(X)-30F^2_{\mu\nu}))\\
&\quad+\int dx_{\shortparallel}\,\mathrm{tr}(\phi(x)\gamma_5((240\sqcap_+ -120\sqcap_-)\partial_d[\gamma_\nu,\gamma_\mu]\frac{F_{\mu\nu}(x)}{4})\\
&\quad+\partial_d\phi(x)\gamma_5(180\chi[\gamma_\nu,\gamma_\mu]\frac{F_{\mu\nu}(x)}{4})+30\partial_i\partial_i\partial_d\phi(x)\gamma_5\chi)\Big),
\end{aligned}
\tag{20}
$$

where $\sqcap_+ \equiv \frac{1}{2}(1+i\gamma^5\gamma_d)$, $\sqcap_- \equiv \frac{1}{2}(1-i\gamma^5\gamma_d)$, $\chi \equiv \sqcap_+ - \sqcap_-$ and ∂_i denotes derivation with respect to all coordinates except x_d. Higher-order coefficients are multiplied by a positive power of t in the expansion (15), so they do not contribute.

In $D=2$, just the coefficients (16), (17) and (18) are relevant. Given that the trace of an odd number of γ matrices vanishes [18], and taking into account the Dirac algebra, one sees that the contribution from a_1 vanishes.

The coefficient $a_2(\phi\gamma^5,\not{D}^2)$ contains both boundary ($x_1=0$) and bulk ($x_1>0$) terms. Only the latter is non-vanishing, and it produces the anomaly in 2 dimensions

$$
\mathcal{A}(x)=\frac{ie^2}{2\pi}\epsilon_{\mu\nu}F_{\mu\nu}(x) \tag{21}
$$

where $\epsilon_{\mu\nu}$ is the 2-dimensional Levi-Civita tensor. Thus, there is no quantum correction to the anomaly in 2 dimensions when confining fields to the half-line.

In 4 dimensions, a_0 and a_1 are zero, as well as the boundary term in a_2. The bulk term is proportional to $tr(\gamma^5\gamma_\mu\gamma_\nu)$, which is a trace of 6 γ matrices. This trace is evaluated in more detail in [19], and is equal to zero. Similarly to the precedents coefficients, a_3 has two terms and they are proportional to $tr(\gamma^5\chi\gamma_\mu\gamma_\nu)$ and $tr(\gamma^5\chi\gamma^5)$ respectively. Both of them are zero because they are traces of 11 and 13 gamma matrices.

We are left with the contributions of the coefficient a_4. The boundary contribution has 4 terms. These are all proportional to the trace of an odd number of γ matrices or proportional to $tr(\gamma^5\gamma_\mu\gamma_\nu)$, so there is no quantum boundary contribution. The bulk term yields the usual result

$$
\mathcal{A}(x)=\frac{ie^3}{16\pi^2}\epsilon_{\mu\nu\alpha\beta}F_{\mu\nu}(x)F_{\alpha\beta}(x) \tag{22}
$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor in 4 dimensions.

Thus, so far we have proved that axial anomaly has no boundary contributions when bag boundary conditions are imposed at $x_d=0$.

2.2. General case

In this case, we come back to the general expression (11) for $\mathcal{A}$, and note that the inverse operator appearing there may be rendered as follows:

$$
\begin{aligned}
&\langle x|\big(\not{D}+g\delta(x_d)+\Lambda\big)^{-1}|y\rangle=\langle x|\big(\not{D}+\Lambda\big)^{-1}|y\rangle\\
&-g\int d^d z_{\shortparallel} d^d z'_{\shortparallel}\,\langle x|\big(\not{D}+\Lambda\big)^{-1}|z_{\shortparallel},0\rangle M(z_{\shortparallel},z'_{\shortparallel})\langle z'_{\shortparallel},0|\big(\not{D}+\Lambda\big)^{-1}|y\rangle
\end{aligned}
\tag{23}
$$

with

$$
M(z_{\shortparallel},z'_{\shortparallel})\;=\;\langle z_{\shortparallel},0|\big[1+g(\not{D}+\Lambda)^{-1}\big]^{-1}|z'_{\shortparallel},0\rangle\,. \tag{24}
$$

Now, we see that the first term in (23) reproduces the previous, bag model anomaly. The second term, whenever one considers points in the bulk $x_d=\varepsilon>0$ will produce a vanishing contribution when $\Lambda\to\infty$. The reason is that that contribution is UV finite for $\varepsilon>0$, since one needs to consider that term for $y=x=(x_{\shortparallel},\varepsilon)$, and there are no coincident points in any operator. Therefore, the whole contribution is finite, and, since it has an extra negative power of Λ, it vanishes in the limit.

3. Induced vacuum current and vacuum polarization tensor

3.1. Induced vacuum current

Let $j_\mu(x)\equiv\langle J_\mu(x)\rangle$ the vacuum expectation value of the (vector) current, which is given by

$$
j_\mu(x)\;=\;-e\,\mathrm{tr}\Big[\gamma_\mu\langle x|\big(\not\partial+ie\not{A}(x)+g\delta(x_d)\big)^{-1}|y\rangle\Big]\,. \tag{25}
$$

In order to express j_μ in terms of the VPT, we expand that inverse in powers of A:

$$
\begin{aligned}
&\langle x|\big(\not\partial+ie\not{A}+g\delta\big)^{-1}|y\rangle\\
&=\mathcal{S}_F(x,y)-\int d^{d+1}z\,\mathcal{S}_F(x,z)\,ie\not{A}(z)\,\mathcal{S}_F(z,y)+\ldots
\end{aligned}
\tag{26}
$$

where $\mathcal{S}_F(x,y)$ is the exact propagator in the presence of the defect, and no A. Therefore, to the lowest non-trivial order, we obtain:

$$
j_\mu(x)\;=\;i\,e^2\int d^{d+1}y\;\mathrm{tr}\Big[\gamma_\mu\,\mathcal{S}_F(x,y)\gamma_\nu\,\mathcal{S}_F(y,x)\Big]A_\nu(y)\,. \tag{27}
$$

Namely, to this order, the response to the external gauge field is linear, the proportionality being given by the VPT, $\Pi_{\mu\nu}$, defined by:

$$
\Pi_{\mu\nu}(x,y)\;=\;-e^2\,\mathrm{tr}\big[\gamma_\mu\,\mathcal{S}_F(x,y)\gamma_\nu\,\mathcal{S}_F(y,x)\big]\,, \tag{28}
$$

such that,

$$
j_\mu(x)\;=\;-i\int d^{d+1}y\;\Pi_{\mu\nu}(x,y)\,A_\nu(y)\,. \tag{29}
$$

3.2. Vacuum polarization tensor in the presence of the defect, in $1+1$ dimensions

Since the defect is static, $\mathcal{S}_F$, and therefore also $\Pi_{\mu\nu}$, will depend on the time arguments only through their difference. Using a mixed Fourier representation whereby we just transform the time coordinate,

$$
\begin{aligned}
\mathcal{S}_F(x_0,x_1;y_0,y_1)&=\mathcal{S}_F(x_0-y_0;x_1,y_1)\\
&=\int\frac{dp_0}{2\pi}e^{ip_0(x_0-y_0)}\,\widetilde{\mathcal{S}}_F(p_0;x_1,y_1)\,,
\end{aligned}
\tag{30}
$$

we see that:

$$
\begin{aligned}
&\widetilde{\Pi}_{\mu\nu}(k_0;x_1,y_1)\\
&=-e^2\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}\,\mathrm{tr}\big[\gamma_\mu\,\widetilde{\mathcal{S}}_F(p_0+k_0;x_1,y_1)\gamma_\nu\,\widetilde{\mathcal{S}}_F(p_0;y_1,x_1)\big]\,.
\end{aligned}
\tag{31}
$$

By an analogous procedure to the one applied in the previous Section, the form of $\widetilde{\mathcal{S}}_F$ may be obtained exactly, for example, by expanding in powers of g, taking into account the form of the singular term, and afterwards summing the resulting series. The outcome of this procedure may be put in terms of the free-space fermion propagator, $\mathcal{S}_F^{(0)}(x_1,y_1)$ (we omit, for the sake of clarity, writing the p_0 argument: it is the same in all the instances where it appears here) as follows:

$$\widetilde{\mathcal{S}}_F(x_1,y_1) = \widetilde{\mathcal{S}}_F^{(0)}(x_1,y_1) - \widetilde{\mathcal{S}}_F^{(0)}(x_1,0)\,\frac{g}{1+g\,\widetilde{\mathcal{S}}_F^{(0)}(0,0)}\,\widetilde{\mathcal{S}}_F^{(0)}(0,y_1)\,. \tag{32}$$

On the other hand, since $\mathcal{S}_F^{(0)}(x_1,y_1)$ may be shown to be given by:

$$\widetilde{\mathcal{S}}_F^{(0)}(p_0;x_1,y_1) = \frac{1}{2}\,[-i\gamma_0\sigma(p_0) + \gamma_1\sigma(x_1-y_1)]e^{-|p_0||x_1-y_1|}\,, \tag{33}$$

where σ denotes the sign function, we can render (32) into a form which will be more convenient in our study of $\Pi_{\mu\nu}$:

$$\begin{aligned}\widetilde{\mathcal{S}}_F(p_0;x_1,y_1) &= \frac{1}{2}\Big\{-i\gamma_0\sigma(p_0)\big[e^{-|p_0||x_1-y_1|} \\ &\quad - \frac{g^2(1-\sigma(x_1)\sigma(y_1))}{4+g^2}e^{-|p_0|(|x_1|+|y_1|)}\big] \\ &\quad + \gamma_1\big[\sigma(x_1-y_1)\,e^{-|p_0||x_1-y_1|} \\ &\quad - \frac{g^2}{4+g^2}(\sigma(x_1)-\sigma(y_1))e^{-|p_0|(|x_1|+|y_1|)}\big]\Big\} \\ &\quad + \frac{g}{4+g^2}\Big[1+\sigma(x_1)\sigma(y_1)+\gamma_5\sigma(p_0)(\sigma(x_1)+\sigma(y_1))\Big] \\ &\quad \times e^{-|p_0|(|x_1|+|y_1|)}\,.\end{aligned} \tag{34}$$

Introducing this expression for the propagator into (31), one can obtain $\widetilde{\Pi}_{\mu\nu}$. Note that because of the lack of explicit Lorentz invariance (time and space coordinates are treated differently), one should expect the calculation to miss a 'seagull term' [20]. Indeed, $\Pi_{\mu\nu}$ being the correlator between current operators:

$$\Pi_{\mu\nu}(x,y) \;=\; -\langle J_\mu(x)\,J_\nu(y)\rangle\,, \tag{35}$$

a seagull term $\tau_{\mu\nu}(x,y)$ should be concentrated on $x=y$. Or, for the partially Fourier transformed version, concentrated on $x_1=y_1$ and a local polynomial in k_0.

Coming back to the result for $\Pi_{\mu\nu}$, due to its dependence on the sign of x_1 and y_1, we have found it convenient to present the result according to the value of those signs:

- $x_1>0$ and $y_1>0$
  In this case, we obtain for $\widetilde{\Pi}_{\mu\nu}(k_0;x_1,y_1)$ a result that may be written explicitly:

$$\begin{aligned}\widetilde{\Pi}_{\mu\nu}(k_0;x_1,y_1) &= -\frac{e^2}{2\pi}\,|k_0|\,\Big\{\Big[e^{-|k_0||x_1-y_1|} \\ &\quad + \frac{g^2}{(1+\frac{g^2}{4})^2}e^{-|k_0||x_1+y_1|}\Big]\delta_{\mu0}\delta_{\nu0} \\ &\quad - \Big[e^{-|k_0||x_1-y_1|} - \frac{g^2}{(1+\frac{g^2}{4})^2}e^{-|k_0||x_1+y_1|}\Big]\delta_{\mu1}\delta_{\nu1} \\ &\quad + i\sigma(k_0)\Big[\sigma(x_1-y_1)\,e^{-|k_0||x_1-y_1|} \\ &\quad - \frac{g^2}{(1+\frac{g^2}{4})^2}e^{-|k_0||x_1+y_1|}\Big]\delta_{\mu0}\delta_{\nu1} \\ &\quad + i\sigma(k_0)\Big[\sigma(x_1-y_1)\,e^{-|k_0||x_1-y_1|} \\ &\quad + \frac{g^2}{(1+\frac{g^2}{4})^2}e^{-|k_0||x_1+y_1|}\Big]\delta_{\mu1}\delta_{\nu0}\Big\}\,.\end{aligned} \tag{36}$$

- $x_1>0$ and $y_1<0$

$$\begin{aligned}\widetilde{\Pi}_{\mu\nu}(k_0;x_1,y_1) &= -\frac{e^2}{2\pi}\,\frac{1-(\frac{g}{2})^2}{1+(\frac{g}{2})^2}\,e^{-|k_0||x_1-y_1|}\Big[|k_0|\big(\delta_{\mu0}\delta_{\nu0}-\delta_{\mu1}\delta_{\nu1}\big) \\ &\quad + ik_0\big(\delta_{\mu0}\delta_{\nu1}+\delta_{\mu1}\delta_{\nu0}\big)\Big]\,.\end{aligned} \tag{37}$$

Note that, for the particular choice $g=2$, $\widetilde{\Pi}_{\mu\nu}$ vanishes, exhibiting the decoupling from the current in the $x_1>0$ region from the gauge field at $x_1<0$. In other words, in this situation the induced current is insensitive to the existence of a gauge field in the $x_1<0$ region.

The *form* of the vacuum polarization function in this case is identical, albeit suppressed by a g-dependent factor, to the one for a fermionic field in free spacetime. Besides, there is a covariantizing seagull term missing, due to the lack of explicit Lorentz covariance in our calculation. Also, note that that term should indeed be missing from the result obtained for $x_1>0$ and $y_1<0$, which excludes $x_1=y_1$.

Denoting by $\widetilde{\Pi}^{(0)}_{\mu\nu}$ the VPT in the absence of the wall, we recall the result for the vacuum polarization tensor corresponding to the well-known result for the Schwinger model in the absence of borders, with both arguments Fourier transformed,

$$\widetilde{\Pi}^{(0)}_{\mu\nu} \;=\; \frac{e^2}{\pi}\,\big(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\big)\,, \tag{38}$$

we find that:

$$\begin{aligned}\widetilde{\Pi}^{(0)}_{\mu\nu}(k_0;x_1,y_1) &= -\frac{e^2}{2\pi}\,e^{-|k_0||x_1-y_1|}\Big[|k_0|\big(\delta_{\mu0}\delta_{\nu0}-\delta_{\mu1}\delta_{\nu1}\big) \\ &\quad + ik_0\,\sigma(x_1-y_1)\big(\delta_{\mu0}\delta_{\nu1}+\delta_{\mu1}\delta_{\nu0}\big)\Big] \\ &\quad + \frac{e^2}{\pi}\,\delta(x_1-y_1)\delta_{\mu0}\delta_{\nu0}\,,\end{aligned} \tag{39}$$

where the last term is the seagull. $\widetilde{\Pi}^{(0)}_{\mu\nu}(k_0;x_1,y_1)$ does of course satisfy the Ward identity:

$$ik_0\widetilde{\Pi}^{(0)}_{0\nu}(k_0;x_1,y_1) \;+\; \partial_{x_1}\widetilde{\Pi}^{(0)}_{1\nu}(k_0;x_1,y_1) \;=\; 0\,. \tag{40}$$

Indeed, it is just another version of $k_\mu\widetilde{\Pi}^{(0)}_{\mu\nu}=0$, which (38) clearly satisfies. When $g\neq0$, it is straightforward to verify that also the part of the VPT which depends on the presence of the wall, satisfies the Ward identity by itself. Therefore,

$$ik_0\widetilde{\Pi}_{0\nu}(k_0;x_1,y_1) \;+\; \partial_{x_1}\widetilde{\Pi}_{1\nu}(k_0;x_1,y_1) \;=\; 0\,. \tag{41}$$

3.3. Behaviour of the normal-current correlator

We study here the behaviour of $\widetilde{\Pi}_{\mu\nu}(k_0;x_1,y_1)$ when $\mu=1$ and one approaches the boundary with x_1: $x_1\to0$ (the situation would be identical if one considered $\nu=1$ and $y_1\to0$, instead). This is the correlator between the normal component of the current on the boundary, and both components of the current J_ν. We find, for $y_1>0$:

$$\widetilde{\Pi}_{1\nu}(k_0;0,y_1) \;=\; \frac{e^2}{2\pi}\,\Big[1-\frac{g^2}{(1+\frac{g^2}{4})^2}\Big]\,e^{-|k_0|y_1}\,(ik_0\delta_{\nu0}+|k_0|\delta_{\nu1})\,, \tag{42}$$

and

$$\widetilde{\Pi}_{1\nu}(k_0;0,y_1) \;=\; \frac{e^2}{2\pi}\Big[1-\frac{g^2}{(1+\frac{g^2}{4})^2}\Big]e^{|k_0|y_1}\,(-ik_0\delta_{\nu 0}+|k_0|\delta_{\nu 1})\,, \tag{43}$$

for $y_1 < 0$. In both cases, the result vanishes only if $g = 2$.

Finally, we note that $\widetilde{\Pi}_{\mu\nu}$ may be conveniently written in an equivalent way:

$$\widetilde{\Pi}_{\mu\nu}(k_0;x_1,y_1) = \widetilde{\Pi}^{(0)}_{\mu\nu}(k_0;x_1,y_1) + \frac{g^2}{(1+\frac{g^2}{4})^2}\,(-1)^\nu\,\widetilde{\Pi}^{(0)}_{\mu\nu}(k_0;x_1,-y_1)\,, \tag{44}$$

(no sum over ν) in terms of $\widetilde{\Pi}^{(0)}_{\mu\nu}$. The second term proceeds from the correlator between $J_\mu(x_0,x_1)$ and $J'_\nu(y_0,y_1) = (-1)^\nu J_\nu(y_0,-y_1)$ (no sum over ν), the current reflected on the wall.

3.4. Vacuum polarization tensor in the presence of the defect, in $3+1$ dimensions

Most of the previous results generalize in a rather straightforward way. Here, we use a Fourier representation where $x_{\shortparallel} \equiv (x_0,x_1,x_2)$ are transformed, since there is translation invariance in that hyperplane:

$$\mathcal{S}_F(x;y) \;=\; \int \frac{d^3p_{\shortparallel}}{(2\pi)^3}e^{ip_{\shortparallel}\cdot(x_{\shortparallel}-y_{\shortparallel})}\,\widetilde{\mathcal{S}}_F(p_{\shortparallel};x_3,y_3)\,, \tag{45}$$

and we see that:

$$\widetilde{\Pi}_{\mu\nu}(k_{\shortparallel};x_3,y_3) = -e^2\int\frac{d^3p_{\shortparallel}}{(2\pi)^3}\,\mathrm{tr}\big[\gamma_\mu\,\widetilde{\mathcal{S}}_F(p_{\shortparallel}+k_{\shortparallel};x_3,y_3)\gamma_\nu\,\widetilde{\mathcal{S}}_F(p_{\shortparallel};y_3,x_3)\big]\,. \tag{46}$$

In this case, we will just present the most relevant result for $\widetilde{\Pi}_{\mu\nu}(k_{\shortparallel};x_3,y_3)$, namely, for the case $x_3,y_3>0$. Taking into account that $\mathcal{S}^{(0)}_F(x_3,y_3)$ is given by:

$$\widetilde{\mathcal{S}}^{(0)}_F(p_{\shortparallel};x_3,y_3) = \frac{1}{2}\,[-i\gamma_{\shortparallel}\cdot\hat{p}_{\shortparallel} + \gamma_3\sigma(x_3-y_3)]e^{-|p_{\shortparallel}||x_3-y_3|}\,, \tag{47}$$

where $\hat{p}_{\shortparallel} \equiv (\frac{p_\alpha}{|p_{\shortparallel}|})$, $\alpha = 0,1,2$, we find that (for $x_3,y_3>0$)

$$\widetilde{\mathcal{S}}_F(p_{\shortparallel};x_3,y_3) = \widetilde{\mathcal{S}}^{(0)}_F(p_{\shortparallel};x_3,y_3) + \frac{g/2}{1+(\frac{g}{2})^2}\,\widetilde{\mathcal{S}}^{(0)}_F(p_{\shortparallel};x_3,-y_3)\gamma_3\,. \tag{48}$$

Again, as it happened in $1+1$ dimensions, $\widetilde{\Pi}_{\mu\nu}$ may be conveniently written in terms of the known, free-space result for the VPT:

$$\widetilde{\Pi}_{\mu\nu}(k_{\shortparallel};x_3,y_3) = \widetilde{\Pi}^{(0)}_{\mu\nu}(k_{\shortparallel};x_3,y_3) + \frac{g^2}{(1+\frac{g^2}{4})^2}\,e^{i\pi\delta_{\nu 3}}\,\widetilde{\Pi}^{(0)}_{\mu\nu}(k_{\shortparallel};x_3,-y_3)\,, \tag{49}$$

(no sum over ν) which may also be thought of as the combination of a direct term and a suppressed image contribution.

4. Conclusions and discussion

We have obtained the anomaly in the presence of a planar defect of a kind that can impose bag boundary conditions in a special limit, showing that the anomaly is independent of the value of the coupling constant g.

Since the chiral anomaly is the same, away from the defect, as the one for the free, no defect case ($g=0$). This has important consequences in the $1+1$ dimensional case. To that end, we recall here some well-known relations, which depend on that property, and apply them to the case at hand: The axial current, $J^5_\mu \equiv ie\bar{\psi}\gamma_\mu\gamma_5\psi$ is, in $1+1$ dimensions, completely determined by the vector current. Indeed, in terms of expectation values,

$$j^5_\mu \;=\; \epsilon_{\mu\nu}j_\nu\,. \tag{50}$$

Now, since the vector current is conserved and, because of the previous relation, its curl is the anomaly, we have:

$$\partial_\mu j_\mu \;=\; 0\;,\quad \epsilon_{\mu\nu}\partial_\mu j_\nu \;=\; \frac{ie^2}{\pi}\epsilon_{\mu\nu}\partial_\mu A_\nu\,. \tag{51}$$

Using the decomposition:

$$j_\mu = \partial_\mu\varphi + i\epsilon_{\mu\nu}\partial_\nu\chi\,, \tag{52}$$

one finds:

$$\partial^2\chi \;=\; -\frac{e^2}{\pi}\epsilon_{\mu\nu}\partial_\mu A_\nu\,. \tag{53}$$

The general solution to the previous equation for χ may be written as follows:

$$\chi \;=\; \chi_0 - \frac{e^2}{\pi}\,\frac{1}{\partial^2}\partial_\mu A_\nu \tag{54}$$

where, as usual, $\frac{1}{\partial^2}$ denotes the inverse of the Laplacian in $\mathbb{R}^2$, with null conditions at infinity, and χ_0 is a harmonic function that enforces the boundary conditions. When one considers the $g=0$ case, $\chi_0=0$, and inserting (54) into (52) one recovers the result for $\Pi^{(f)}_{\mu\nu}$. In the $g=2$ case, on the other hand, one considers the $x_1>0$ region, and a non-trivial χ_0 function is required in order to make $j_1=0$ on the boundary. The result for $\Pi_{\mu\nu}$ in this case may be interpreted as the one for $g=0$ plus a contribution which can be understood as due to an image source, outside of the region.

Note that the combination of two effects: invariance of the anomaly and boundary conditions, completely determine the form of the VPT.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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