The Edge Investment Problem: Upgrading Transit Line Segments with Multiple Investing Parties

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Abstract

Bus Rapid Transit (BRT) systems can provide a fast and reliable service to passengers at lower costs compared to tram, metro and train systems. Therefore, they can be of great value to attract more passengers to use public transport, which is vital in reaching the Paris Agreement Targets. However, the main advantage of BRT systems, namely their flexible implementation, also leads to the risk that the system is only implemented partially to save costs. This paper focuses therefore on the Edge Investment Problem: Which edges (segments) of a bus line should be upgraded to full-level BRT? Motivated by the construction of a new BRT line around Copenhagen, we consider a setting in which multiple parties are responsible for different segments of the line. Each party has a limited budget and can adjust its investments according to the benefits provided to its passengers. We suggest two ways to determine the number of newly attracted passengers, prove that the corresponding problems are NP-hard and identify special cases that can be solved in polynomial time. In addition, problem relaxations are presented that yield dual bounds. Moreover, we perform an extensive numerical comparison in which we evaluate the extent to which these two ways of modeling demand impact the computational performance and the choice of edges to be upgraded.

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1 Introduction

Public transport plays an important role in the transition towards a more sustainable transportation network in cities. In order to convince people to choose public transport over other modes, many cities opt to build Bus Rapid Transit (BRT) networks. Such networks are characterized by a high average speed and frequent service, to a large extent achieved through separation from other traffic. As the construction of BRT networks is expensive, careful planning is needed to choose the final design of the system before investments are made.

In this paper, we study a problem that is motivated by the development of a new BRT line in the Capital Region of Denmark (Region Hovedstaden). This new BRT line will form a radial around Copenhagen and will connect multiple municipalities surrounding the city [12]. A first assessment has defined five feasible route alternatives, each describing a possible route of the new BRT line. The next steps in the process consist of determining the final route and the investments made on this route. We focus on the second step: How to determine the best investments in a line with respect to budget and return on investment constraints?

In our case, investments along a line cover, e.g., the costs needed to construct a separate bus lane as well as the upgrading of intersections and traffic installations to allow for priority of the BRT line. As these investments contribute to the quality of the journey for the passengers, they have a direct impact on the passenger potential of the line. In particular, these investments decrease the travel time along the route and at the same time increase the reliability of the route. Our main focus is to find a set of upgrades along the line that attracts the largest amount of passengers.

A complicating factor in constructing the BRT line in the Capital Region of Denmark is that each municipality that is crossed by the line is responsible for investments for those segments that lie within its borders. For the investments, a certain budget is available per municipality. Moreover, municipalities have to compare the investment costs to the benefits for their passengers. We incorporate this into our problem by introducing constraints that limit the investments that municipalities are willing to make according to the number of their passengers that are attracted.

1.1 Related Literature

Rapid transit network design, including the determination of stations, lines and frequencies, has widely been studied in the literature. For a survey on the problem, the models and the solution methods used to solve them, we refer to [9, 8]. Some more recent work has focused on better modeling the interaction with the existing public transport system, e.g., in the design of feeder-bus networks [3] and in computing the modal split between metro and bus transit systems [10]. Moreover, [6] propose an integrated approach for both the design of a new rapid transit network and the adaptation of the existing bus network. A different perspective is taken by [2], who incorporate spacial and social equity principles in the transit network design problem.

Another related problem is the network improvement problem. This problem consists of choosing edges (and nodes) in a network to be upgraded while minimizing costs or satisfying budget constraints and has, e.g., been studied by [7, 22, 13, 11, 1].

Literature on transit network design usually assumes that all upgrade decisions are made by one central authority. In contrast to that, [20] consider local authorities that can only make upgrade decisions for their own subgraphs, i.e., parts of the network. In a game-theoretic setting, they formulate the interaction of the local authorities among others in a cooperative, competitive and chronological way. Assuming a fixed demand, the travel time depends on the capacity and the amount of flow on each edge. Each local authority aims to minimize the travel time by increasing the capacity of an edge restricted by a budget.

Note that the Edge Investment Problem introduced here differs from these settings because the route and stations of the BRT line are already given. Instead, we are interested in attracting new passengers, i.e., in maximizing the demand, which is modeled by two different objective functions, through infrastructure improvements.

1.2 Contribution

The contribution of this paper is to model the Edge Investment Problem for BRT lines as required in the Capital Region of Denmark. We present two variants to model the decision process of the municipalities, a collaborative version EIP and a version focusing on the return on investment for each municipality ROI, as well as two variants to compute the number of newly attracted passengers, Linear and MinImprov. We analyze the complexity of the four resulting problems, identifying both NP-hard and polynomially solvable cases. Additionally, we perform an extensive experimental evaluation both on artificial instances and on a case study based on the BRT line in the Capital Region of Denmark. Here, we analyze the influence of collaboration between municipalities and of the budget split on the number of newly attracted passengers. We further assess the potential to attract new passengers for five different route alternatives given in the case study.

2 Model and Problem Formulation

In the Edge Investment Problem, we assume that investing in the upgrade of edges attracts new passengers. Before we model the different ways to determine the number of attracted passengers, we formulate the general setting.

The BRT line is described as a line graph $G = (V, E)$, where $V = \{1, \ldots, n\}$ for $n \in \mathbb{N}_{>0}$ denotes the set of stations and $E = \big\{e_i = \{i, i+1\} : i \in \{1, \ldots, n-1\}\big\}$ the set of direct connections between the stations. Let $D \subseteq \{(i, j) : i, j \in V, i < j\}$ be the set of undirected potential origin-destination (OD) pairs. For $d = (i, j) \in D$, let W_d be the set of edges of the path from station i to station j, namely $W_d = \{e_k : k \in \{i, i+1, \ldots, j-1\}\}$, and let $a_d \in \mathbb{N}_{>0}$ be the maximum number of passengers that can be attracted.

Different parts of the graph are under the responsibility of different municipalities. We denote the set of municipalities by M. For each municipality $m \in M$, let $E_m \subseteq E$ be the subset of edges that lie within the responsibility of municipality m such that $\bigcup_{m \in M} E_m = E$. As in our application, we assume that the sets E_m for $m \in M$ are pairwise disjoint and contain only consecutive edges, i.e., for all $m \in M$ there is some $i, j \in V$, $i < j$ such that $E_m = \{e_k : k \in \{i, i+1, \ldots, j-1\}\}$. By $D_m \subseteq D$, $D_m \neq \emptyset$, we denote the subset of OD pairs that municipality m is interested in. Here, a municipality wants to increase the number of passengers that start or end in their municipality but does not care whether passengers in a different part of the network are attracted. Note that setting $|M| = 1$ represents the case of one general budget, which is not separated into budgets for multiple municipalities.

The costs for upgrading an edge $e \in E$ are $c_e \in \mathbb{R}_{>0}$. We consider two types of constraints for the municipalities. For each municipality $m \in M$, the amount of upgrades is restricted by a budget $B_m \in \mathbb{R}_{>0}$ and by a return on investment factor $b_m \in \mathbb{R}_{>0}$ per newly attracted person. While the budget constraints model a general budget on the investments, the return on investment constraints guarantee that the costs are smaller than the gain through upgrades measured in the number of newly attracted passengers multiplied with the investment factor.

Upgrading edges of a bus line to BRT standard by implementing additional exclusive infrastructure for BRT decreases the travel time and increases the reliability as it is less dependent on congestion caused by regular (individual) traffic. Hence, more people are attracted to the BRT line. We denote the infrastructure improvement achieved by upgrading an edge $e \in E$ by $u_e \in \mathbb{R}_{>0}$. In the Edge Investment Problem, we aim at maximizing the number of newly attracted passengers. The remaining question is how many upgraded edges or which level of infrastructure improvements is necessary to attract new passengers. In this paper, we propose two ways of modeling that: In the Linear model, we assume that the number of attracted passengers increases linearly with the amount of realized infrastructure improvements in proportion to the total amount of possible infrastructure improvements. The maximum number of passengers is only attracted when all edges of the path of an OD pair are upgraded, otherwise only a share is attracted. In the MinImprov model, we assume that all potential passengers of an OD pair $d \in D$ are attracted when a certain threshold of infrastructure improvements $L_d \in \mathbb{R}_{>0}$ is reached on their path. Otherwise, no new passengers are attracted for this OD pair. This is formally introduced next.

▶ **Definition 1.** *Let $F \subseteq E$ be the set of upgraded edges, and let an OD pair $d \in D$ be given. In* Linear*, the number of newly attracted passengers of OD pair d is determined by*

$$p_d(F) := \frac{\sum_{e \in F \cap W_d} u_e}{\sum_{e' \in W_d} u_{e'}} \cdot a_d.$$

In MinImprov*, the number of newly attracted passengers of OD pair d is determined by*

$$p_d(F) := \begin{cases} a_d & \text{if } L_d \leq \sum_{e \in F \cap W_d} u_e, \\ 0 & \text{otherwise.} \end{cases}$$

We are now in the position to formally describe the Edge Investment Problem, to which both objective functions from Definition 1 can be applied, in the following definition:

▶ **Definition 2** (EIP and ROI)**.** *Given are*

- *a line graph (V, E) and a set of OD pairs $D \subseteq \{(i, j) : i, j \in V, i < j\}$,*
- *costs $c_e \in \mathbb{R}_{>0}$ and infrastructure improvements $u_e \in \mathbb{R}_{>0}$ for all $e \in E$,*
- *a set of municipalities M,*
- *a set of edges $E_m \subseteq E$, a set of OD pairs $D_m \subseteq D$, $D_m \neq \emptyset$, a budget $B_m \in \mathbb{R}_{>0}$ and an investment factor $b_m \in \mathbb{R}_{>0}$ for all $m \in M$ such that $\bigcup_{m \in M} E_m = E$,*
- *a maximum number of potential passengers $a_d \in \mathbb{N}_{>0}$ for all $d \in D$,*
- *a lower bound $L_d \in \mathbb{R}_{>0}$ with $L_d \leq \sum_{e \in W_d} u_e$ for all $d \in D$ (needed only for* ROI *in the* MinImprov *case).*

The aim of the basic version EIP *of the* Edge Investment Problem *is to determine a subset $F \subseteq E$ of edges to be upgraded such that the budget constraints*

$$\sum_{e \in F \cap E_m} c_e \leq B_m \text{ for all } m \in M \tag{1}$$

are met and the number of newly attracted passengers $\sum_{d \in D} p_d(F)$ is maximized, where $p_d(F) \in [0, a_d]$ denotes the number of passengers of OD pair $d \in D$ that are newly attracted depending on F according to Definition 1.

In order to take into account that a municipality might require a certain return on investment, we introduce return on investment constraints

$$\sum_{e \in F \cap E_m} c_e \leq b_m \cdot \sum_{d \in D_m} p_d(F) \text{ for all } m \in M \tag{2}$$

in addition to the budget constraints (1) *and obtain the model* ROI*.*

In the following, we address both models EIP and ROI in combination with both objective functions Linear and MinImprov given in Definition 1, yielding a total of four problems:

- EIP-Linear (only constraints (1), Linear objective),
- EIP-MinImprov (only constraints (1), MinImprov objective),
- ROI-Linear (constraints (1) and (2), Linear objective) and
- ROI-MinImprov (constraints (1) and (2), MinImprov objective).

We remark that the EIP-MinImprov model contains the special case in which we only consider unit costs $c_e = 1$ and unit infrastructure improvements $u_e = 1$ for all edges $e \in E$. In this case, we are allowed to upgrade at most B_m edges in municipality $m \in M$, and for OD pair $d \in D$ all potential passengers are attracted if at least a number of L_d edges is upgraded, and no passengers otherwise.

3 Theoretical Analysis

In this section, we study EIP and ROI as well as relaxations. We analyze both problems with both objectives regarding their complexity and show that all four problems are NP-hard but admit polynomial special cases.

The first lemma gives an indication about the budget and the investment factor per passenger that are sufficient such that it is optimal to upgrade all edges.

▶ **Lemma 3.** *In* ROI*, we can omit constraints in the following cases:*

(a) *Let $m \in M$. The budget constraint* (1) *regarding m is redundant if one of the following assumptions is satisfied:*

  1. $B_m \geq \sum_{e \in E_m} c_e$,

  2. $B_m \geq b_m \sum_{d \in D_m} p_d(E)$.

(b) *The set $F := E$ is an optimal solution to* ROI *if $B_m \geq \sum_{e \in E_m} c_e$ and $b_m \geq \frac{\sum_{e \in E_m} c_e}{\sum_{d \in D_m} p_d(E)}$ for all $m \in \{1, \ldots, M\}$.*

Proof. In case (a1), the assumption clearly yields that the corresponding constraint is always satisfied. Hence, it is redundant and can be omitted. In case (a2), we have for any $F \subseteq E$ satisfying the return on investment constraint (2) that also the budget constraint (1) is satisfied because

$$\sum_{e \in F \cap E_m} c_e \leq b_m \sum_{d \in D_m} p_d(F) \leq b_m \sum_{d \in D_m} p_d(E) \leq B_m.$$

Hence, it is again redundant and can be omitted.

Case (b) implies that case (a1) is satisfied for all $m \in M$. Hence, all budget constraints (1) can be omitted. This yields that $F := E$ is a feasible solution because the return on investment constraints (2) are also satisfied for all $m \in M$ by assumption:

$$b_m \sum_{d \in D_m} p_d(E) \geq \frac{\sum_{e \in E_m} c_e}{\sum_{d \in D_m} p_d(E)} \sum_{d \in D_m} p_d(E) = \sum_{e \in E \cap E_m} c_e.$$

Finally, $F = E$ is an optimal solution as most passengers are attracted when all edges are upgraded. ◀

3.1 The Linear Case

We start the theoretical analysis by giving a linear integer programming (IP) formulation of ROI-Linear in IP (3). For all $e \in E$, we introduce a binary variable $x_e \in \{0,1\}$ which satisfies that $x_e = 1$ if and only if edge e is upgraded. Simplifying the notation by setting $\mu_d := \frac{a_d}{\sum_{e' \in W_d} u_{e'}}$, we get:

$$
\begin{aligned}
\max_{x_e} \quad & \sum_{d \in D} \left(\mu_d \sum_{e \in W_d} u_e x_e \right) & \\
\text{s.t.} \quad & \sum_{e \in E_m} c_e x_e \leq B_m & \text{for all } m \in M \\
& \sum_{e \in E_m} c_e x_e \leq b_m \cdot \sum_{d \in D_m} \left(\mu_d \sum_{e \in W_d} u_e x_e \right) & \text{for all } m \in M \\
& x_e \in \{0,1\} & \text{for all } e \in E.
\end{aligned}
\tag{3}
$$

By pre-computing

$$
\tilde{u}_e := \sum_{\substack{d \in D: \\ e \in W_d}} \mu_d u_e \quad \text{and} \quad \tilde{u}_e^m := \sum_{\substack{d \in D_m: \\ e \in W_d}} b_m \mu_d u_e
$$

for all $e \in E$ and $m \in M$, this problem can equivalently be reformulated as follows:

$$
\begin{aligned}
\max_{x_e} \quad & \sum_{e \in E} \tilde{u}_e x_e & \\
\text{s.t.} \quad & \sum_{e \in E_m} c_e x_e \leq B_m & \text{for all } m \in M \\
& \sum_{e \in E_m} (c_e - \tilde{u}_e^m) x_e - \sum_{e \in E \setminus E_m} \tilde{u}_e^m x_e \leq 0 & \text{for all } m \in M \\
& x_e \in \{0,1\} & \text{for all } e \in E.
\end{aligned}
$$

Hence, EIP-Linear and ROI-Linear are multidimensional 0-1 knapsack problems. Note that negative weights occur in the reformulated return on investment constraints. Moreover, both problems are NP-hard by a reduction from 0-1 knapsack as Theorem 4 shows.

▶ **Theorem 4.** EIP-Linear *and* ROI-Linear *are both NP-hard.*

Proof. As EIP-Linear is a special case of ROI-Linear, it suffices to prove that they are both in NP and that the decision version of EIP-Linear, which we call EIP-Linear again for the sake of simplicity, is NP-complete. Given a solution to EIP-Linear or ROI-Linear, we can check in polynomial time whether the budget constraints and (if applicable) the return on investment constraints are satisfied and a certain value in the objective function is reached.

We reduce (the decision version of) 0-1 knapsack to EIP-Linear. Let k elements with rewards $r_i \in \mathbb{Z}_{>0}$ and weights $w_i \in \mathbb{Z}_{>0}$ for all $i \in \{1, \ldots, k\}$, a budget B and a bound S' be given. We construct an instance of EIP-Linear as follows: We set $S := S'$, $n := k+1$, this means $V := \{1, \ldots, k+1\}$, $E := \{e_i : i \in \{1, \ldots, k\}\}$, $D := \{(i, i+1) : i \in \{1, \ldots, k\}\}$, $c_{e_i} := w_i$ and $u_{e_i} := 1$ for all $i \in \{1, \ldots, k\}$, $M := \{1\}$, $B_1 := B$ and $a_d := r_i$ for all $d = (i, i+1)$ with $i \in \{1, \ldots, k\}$. We show that every feasible solution $F' \subseteq \{1, \ldots, k\}$

of 0-1 knapsack with an objective value of at least S' corresponds to a feasible solution $F \subseteq E$ of EIP-LINEAR with an objective value of at least S. The solutions F' and F correspond to each other as follows: $i \in F'$ if and only if $e_i \in F$. Then the claim holds because $\sum_{i \in F'} w_i = \sum_{i \in F'} c_{e_i} = \sum_{e \in F} c_e$ and

$$\sum_{i \in F'} r_i = \sum_{e_i \in F} a_{(i,i+1)} = \sum_{\substack{d=(i,i+1): \\ i \in \{1,\ldots,k\}}} \left(\frac{\sum_{e \in F \cap \{e_i\}} 1}{1} \cdot a_d \right) = \sum_{d \in D} \left(\frac{\sum_{e \in F \cap W_d} u_e}{\sum_{e \in W_d} u_e} \cdot a_d \right). \qquad ◀$$

Note that EIP-LINEAR can be decomposed into $|M|$ independent knapsack problems and hence can be solved in pseudo-polynomial time by dynamic programming. In the following, we identify a case in which it is even polynomially solvable. To this end, we review the consecutive ones property, which is well known in the literature (see, e.g., [15, 18, 5, 4]).

▶ **Definition 5** (Consecutive Ones Property). *A matrix $A \in \{0,1\}^{k \times l}$ satisfies the consecutive ones property (C1P) on the rows if for all rows $i \in \{1, \ldots, k\}$ it holds: If $A_{i,j} = 1$ and $A_{i,j'} = 1$ for some $j, j' \in \{1, \ldots, l\}$, $j < j'$, then $A_{i,\bar{j}} = 1$ for all $j \leq \bar{j} \leq j'$.*

▶ **Lemma 6** ([21]). *If a matrix $A \in \{0,1\}^{k \times l}$ satisfies C1P, then A is totally unimodular.*

▶ **Lemma 7.** EIP-LINEAR *can be solved in polynomial time if $c_e = 1$ for all $e \in E$.*

Proof. We sort the edges and municipalities from one end of the line to the other. Let $A \in \mathbb{R}^{|M| \times |E|}$ be the coefficient matrix of the budget constraints, i.e., for all $m \in M$ and $e \in E$, we have $A_{m,e} = 1$ if $e \in E_m$, and $A_{m,e} = 0$ otherwise. Because of the assumption that the municipalities contain only consecutive edges, the matrix A satisfies the consecutive ones property. By Lemma 6, it is totally unimodular and the linear programming relaxation of IP (3) yields an integer solution. Therefore, the problem can be solved in polynomial time [21]. ◀

3.2 The MinImprov Case

We present a linear IP formulation of ROI-MINIMPROV in IP (4). For all $e \in E$, we introduce a binary variable $x_e \in \{0,1\}$ which satisfies that $x_e = 1$ if and only if edge e is upgraded. Additionally, we need a binary variable $y_d \in \{0,1\}$ for all $d \in D$ which satisfies in each optimal solution that $y_d = 1$ if and only if $L_d \leq \sum_{e \in F \cap W_d} u_e$ for the set $F \subseteq E$ of upgraded edges due to the maximization. This yields the following IP:

$$\begin{aligned}
\max_{x_e, y_d} \quad & \sum_{d \in D} a_d y_d & & \\
\text{s.t.} \quad & \sum_{e \in E_m} c_e x_e \leq B_m & & \text{for all } m \in M \\
& \sum_{e \in E_m} c_e x_e \leq b_m \cdot \sum_{d \in D_m} a_d y_d & & \text{for all } m \in M \\
& L_d y_d \leq \sum_{e \in W_d} u_e x_e & & \text{for all } d \in D \\
& x_e \in \{0,1\} & & \text{for all } e \in E \\
& y_d \in \{0,1\} & & \text{for all } d \in D.
\end{aligned} \tag{4}$$

As before, we prove NP-hardness of EIP-MINIMPROV by a reduction from 0-1 knapsack.

▶ **Theorem 8.** EIP-MinImprov *and* ROI-MinImprov *are both NP-hard, even if* $u_e = 1$ *for all* $e \in E$ *and* $L_d = 1$ *for all* $d \in D$*.*

Proof. As in the proof of Theorem 4, EIP-MinImprov is a special case of ROI-MinImprov and both problems are in NP.

Further, we apply the same reduction from 0-1 knapsack to EIP-MinImprov and additionally choose $L_d := 1$ for all $d \in D$. It remains to show that the objective value is the same for solutions that correspond to each other. We have that

$$\sum_{\substack{d \in D:\\ L_d \leq \sum\limits_{e \in F \cap W_d} u_e}} a_d = \sum_{i \in F'} r_i,$$

because

$$\begin{aligned}\{d \in D : L_d \leq \sum_{e \in F \cap W_d} u_e\} &= \{(i, i+1) : i \in \{1, \dots, k\} \text{ and } 1 \leq \sum_{e \in F \cap \{e_i\}} 1\}\\ &= \{(i, i+1) : i \in \{1, \dots, k\} \text{ and } e_i \in F\} = \{(i, i+1) : i \in F'\}.\end{aligned}$$ ◀

Exploiting C1P, we again obtain a polynomial special case in the following lemma:

▶ **Lemma 9.** EIP-MinImprov *can be solved in polynomial time if* $c_e = 1$*,* $u_e = 1$ *for all* $e \in E$ *and* $L_d = 1$ *for all* $d \in D$*.*

Proof. We again sort the edges and municipalities from one end of the line to the other. The considered special case yields the following simplified formulation:

$$\begin{aligned}\max_{x_e, y_d} \quad & \sum_{d \in D} a_d y_d\\ \text{s.t.} \quad & \sum_{e \in E_m} x_e \leq B_m && \text{for all } m \in M\\ & \sum_{e \in W_d} -x_e + y_d \leq 0 && \text{for all } d \in D\\ & x_e \in \{0, 1\} && \text{for all } e \in E\\ & y_d \in \{0, 1\} && \text{for all } d \in D.\end{aligned}$$

The coefficient matrix of the budget and return on investment constraints is of the form $A = \begin{bmatrix} A^1 & \mathbf{0} \\ -A^2 & I \end{bmatrix}$, where $I \in \mathbb{R}^{|D| \times |D|}$ is the unit matrix, $A^1 \in \mathbb{R}^{|M| \times |E|}$ denotes whether an edge belongs to a municipality, and $A^2 \in \mathbb{R}^{|D| \times |E|}$ denotes whether an edge is on the path of an OD pair. Formally, we have for all $m \in M$, $d \in D$ and $e \in E$ that

$$A^1_{m,e} = \begin{cases} 1 & \text{if } e \in E_m, \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad A^2_{d,e} = \begin{cases} 1 & \text{if } e \in W_d, \\ 0 & \text{otherwise.} \end{cases}$$

The matrix A^1 has C1P because of the assumption that municipalities contain only consecutive edges, and A^2 has C1P because the considered graph is a line graph. As multiplying a row of a matrix by -1 only influences the sign of the determinant of the matrix and its submatrices, the matrix $\begin{bmatrix} A^1 \\ -A^2 \end{bmatrix}$ is totally unimodular by Lemma 6. This yields that the coefficient matrix A, which we obtain by appending a part of a unit matrix to the TU matrix, is also totally unimodular. Therefore, the linear programming relaxation yields an integer solution in this special case, and the problem can be solved in polynomial time [21]. ◀

3.3 Relaxations and Dual Bounds

Because ROI and EIP are NP-hard with both objective functions, we study different relaxations and bounds on the objective value of the Edge Investment Problem. The trivial lower and upper bounds are 0 and $\sum_{d\in D} a_d$, respectively.

First, it is easy to see that EIP is a relaxation of ROI because the return on investment constraints (2) are omitted in EIP, which expands the feasible set, but the objective stays the same. Hence, EIP yields an upper bound on the number of newly attracted passengers in ROI. However, EIP is NP-hard itself for both objective functions. Therefore, we consider the special cases of Lemmas 7 and 9, which are relaxations of EIP-Linear and EIP-MinImprov, respectively, as the following results show.

▶ **Lemma 10.** *Let $m \in M$. If $F \subseteq E$ satisfies budget constraint* (1) *regarding m, then it also satisfies $|F \cap E_m| \leq \frac{B_m}{\min\{c_e : e\in E_m\}}$.*

Proof. By assumption, it holds that $B_m \geq \sum_{e\in F\cap E_m} c_e \geq \sum_{e\in F\cap E_m} \min\{c_e : e \in E_m\}$. Hence, we also have that $\frac{B_m}{\min\{c_e : e\in E_m\}} \geq \sum_{e\in F\cap E_m} 1 = |F \cap E_m|$. ◀

▶ **Lemma 11.** *Let $F \subseteq E$ and $d \in D$. If $L_d \leq \sum_{e\in F\cap W_d} u_e$, then we also have $1 \leq |F \cap W_d|$.*

Proof. By assumption, it holds that $L_d \leq \sum_{e\in F\cap W_d} u_e \leq \sum_{e\in F\cap W_d} \max\{u_e : e \in W_d\}$. Hence, we also have that $|F \cap W_d| = \sum_{e\in F\cap W_d} 1 \geq \frac{L_d}{\max\{u_e : e\in W_d\}}$. Integer rounding yields $|F \cap W_d| \geq \left\lceil \frac{L_d}{\max\{u_e : e\in W_d\}} \right\rceil \geq 1$. ◀

From Lemmas 10 and 11, we obtain the following relaxations:

▶ **Corollary 12.** *The following problem is a relaxation of* EIP*:*

$$\begin{aligned}
\max_{F\subseteq E} \quad & \sum_{d\in D} p_d(F) \\
\text{s.t.} \quad & |F \cap E_m| \leq \frac{B_m}{\min\{c_e : e \in E_m\}} \quad \text{for all } m \in M.
\end{aligned}$$

Considering EIP-Linear, the relaxation in Corollary 12 is of the same form as the problem considered in Lemma 7 and can, hence, be solved in polynomial time.

▶ **Corollary 13.** *The following problem is a relaxation of* EIP-MinImprov*:*

$$\begin{aligned}
\max_{F\subseteq E} \quad & \sum_{\substack{d\in D:\\ 1\leq |F\cap W_d|}} a_d \\
\text{s.t.} \quad & |F \cap E_m| \leq \frac{B_m}{\min\{c_e : e \in E_m\}} \quad \text{for all } m \in \{1, \ldots, M\}.
\end{aligned}$$

The relaxation in Corollary 13 is of the same form as the problem considered in Lemma 9 and can, hence, be solved in polynomial time.

4 Computational Study

In the computational study, we evaluate the impact of the model variants, the budgets and the investment factors on the choice of edges to be upgraded. We first present the results for a set of artificial instances and afterwards for the proposed BRT line in Copenhagen.

4.1 Artificial Instances

We evaluate the models on a set of artificial instances, where each instance is determined by a graph scenario $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and a budget scenario $\beta = (\beta_1, \beta_2, \beta_3)$ as given in Table 1. The data is available at `https://doi.org/10.11583/DTU.c.6130014`.

Table 1 Parameters for generating artificial instances.

Parameter	Value	Explanation
α_1 size	10 25	10 stations with 2 municipalities 25 stations with 5 municipalities
α_2 segment costs	UNIT MIDDLE ENDS	unit costs of $c_e = 1$ for all $e \in E$ more expensive towards the middle of the line more expensive towards the end stations of the line
α_3 demand pattern	EVEN CENTER END	same amount per OD pair centered around a number of large stations strong demand between end stations of the line
β_1 budget limit	1 0.8 0.6	determines available overall budget as fraction of the costs for upgrading all edges, i.e., $B = \beta_1 \sum_{e \in E} c_e$
β_2 budget split	even cost pass	budget B evenly distributed to municipalities B_m proportional to the costs of the edges in municipality m B_m proportional to the number of passengers interesting for municipality m
β_3 scaling factor	1.2 1	investment factor per passenger given by $b_m = \beta_3 \frac{\sum_{e \in E} c_e}{\sum_{d \in D} a_d}$

Evaluation

For each combination of a graph α and a budget scenario β, we determine an optimal solution using both of the proposed objectives: LINEAR and MINIMPROV. For the latter, we require that 75% of the edges on the path of an OD pair are upgraded before the passengers corresponding to that OD pair are attracted. For LINEAR, the infrastructure improvement u_e of an edge $e \in E$ is drawn at random. Moreover, we vary for both objectives which constraints are enforced: only one overall budget for a global decision maker (SOC), budget constraints (1) for all municipalities (EIP), and both constraints (1) and (2) for all municipalities (ROI). The models are solved by means of the commercial solver CPLEX 22.1.

Runtime

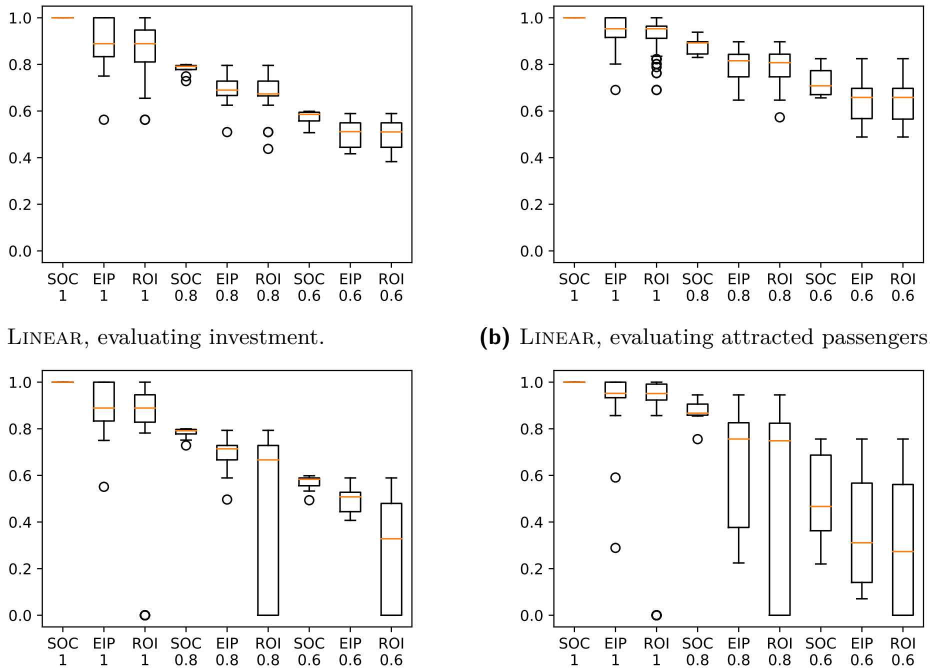
Table 2 shows the obtained average runtimes in milliseconds, split out over the different objectives, cost types and budget variants. Next to the influence of the number of stations, the results show that the model SOC with only a global budget constraint is (often) the hardest to solve for objective MinImprov. Moreover, in most cases ROI is harder to solve than EIP, and ROI often turns out to be the hardest model to solve for objective Linear. Considering the different cost types for objective Linear shows that the polynomially solvable special case of UNIT costs for SOC and EIP is indeed solved much faster than MIDDLE or ENDS. For MinImprov, there is no clearly easiest cost type as UNIT would only be polynomially solvable if the lower bound L_d would be chosen as 1. The overall low runtimes suggest that specialized polynomial-time algorithms are not necessary for realistically sized instances.

Table 2 Average runtime in milliseconds.

		$\alpha_1 = 10$			$\alpha_1 = 25$		
Objective	α_2	SOC	EIP	ROI	SOC	EIP	ROI
Linear	UNIT	**3.09**	**3.00**	**4.11**	**9.09**	**8.54**	**19.20**
Linear	MIDDLE	13.37	5.96	6.96	28.54	18.26	33.76
Linear	ENDS	18.78	7.02	10.91	18.11	18.43	31.41
MinImprov	UNIT	30.61	**15.46**	**15.35**	**508.63**	106.33	**128.30**
MinImprov	MIDDLE	**17.87**	21.33	22.87	2622.13	**76.48**	149.94
MinImprov	ENDS	23.74	21.70	24.03	591.54	79.02	159.87

What is gained by collaborating?

In Figure 1, we see the investment and the number of attracted passengers for the models SOC, EIP and ROI for objective functions Linear and MinImprov. As expected, SOC results in the highest investments for each budget limit $\beta_1 \in \{1, 0.8, 0.6\}$ as well as the highest number of attracted passengers. Similarly, EIP results in higher (or equal) investments and attracted passengers than ROI, as ROI is the more restrictive model. For all budget limits, especially the lower ones, the median share of newly attracted passengers is higher for the objective Linear than for the objective MinImprov. Moreover, for the objective Linear, the median share of newly attracted passengers is always higher than the median share of investments. This is also true for objective MinImprov with a budget limit $\beta_1 \in \{1, 0.8\}$, while it is distinctly lower for $\beta_1 = 0.6$. This shows that in the distributed setting EIP and especially in the benefit-oriented setting ROI, it is more difficult to upgrade 75% of the edges of the path of an OD pair. Note that in the benefit-oriented case ROI, the 25th percentile sometimes reaches zero, i.e., in the MinImprov case, the municipalities relatively often do not invest at all.



(a) LINEAR, evaluating investment. **(b)** LINEAR, evaluating attracted passengers.

(c) MINIMPROV, evaluating investment. **(d)** MINIMPROV, evaluating attracted passengers.

Figure 1 Box plots showing the share of investment and attracted passengers compared to upgrading all segments for varying budget limits. The orange line marks the median, the box the 25th- to 75th-percentile.

How does changing the budget split β_2 influence the passengers?

Table 3 shows the influence of the budget split β_2 in EIP and ROI on the number of attracted passengers for different passenger demand patterns α_3. For all three demand patterns, splitting the demand according to the costs of the municipalities' segments yields the highest number of attracted passengers. While for the objective function LINEAR, switching between EIP and ROI has almost no influence, there is a considerable difference between EIP and ROI for MINIMPROV. This is especially the case for demand pattern END and, to a lesser extent, for demand pattern EVEN. The reduction in passenger potential is considerably lower for demand pattern CENTER, which is the demand pattern that has the highest passenger potential for both models.

Table 3 Influence of the budget split β_2 on the attracted passengers.

		EIP			ROI		
Objective	α_3	β_2 = cost	β_2 = even	β_2 = pass	β_2 = cost	β_2 = even	β_2 = pass
LINEAR	CENTER	**85.96%**	83.61%	83.65%	**85.53%**	83.61%	83.65%
LINEAR	END	**79.74%**	75.55%	72.23%	**77.00%**	74.98%	72.23%
LINEAR	EVEN	**80.31%**	76.92%	76.92%	**80.31%**	76.92%	76.92%
MINIMPROV	CENTER	**79.83%**	77.51%	77.49%	**78.18%**	77.04%	76.98%
MINIMPROV	END	**60.29%**	54.41%	45.07%	**47.64%**	33.67%	26.12%
MINIMPROV	EVEN	**66.06%**	61.33%	61.33%	**60.62%**	54.52%	54.52%

Upgrading segments in a dynamic setting

When increasing the budget, more segments can be upgraded in order to attract more passengers. This is especially important when a fixed budget is available now, but more budget might be available in the future. In our experiments, we see that the segments upgraded for a low budget are almost always also upgraded for a higher budget: When increasing the budget limit β_1 from 0.6 to 0.8 and from 0.8 to 1, respectively, only 2.4% of the segments are upgraded for the lower budget limit and would not be upgraded for the higher one. Thus, we conclude that implementing an optimal solution for a low budget allows for an optimal solution when increasing the budget later on in the vast majority of cases. In this sense, a greedy heuristic seems to be a good solution approach here. For an example, see Figure 4 in Appendix A.

4.2 Copenhagen Case Study

The analyzed problem is motivated by the plans to build a set of new BRT lines in the Copenhagen Region. One of these lines will run foremost along the route of the current bus line 400S. The line runs through several municipalities, that each individually need to decide on the route approval, investment budget and upgrading of the segments. A pre-assessment study was conducted for the line that calculated the expected costs, travel durations and number of passengers per station for five different route alternatives, see Figure 2.

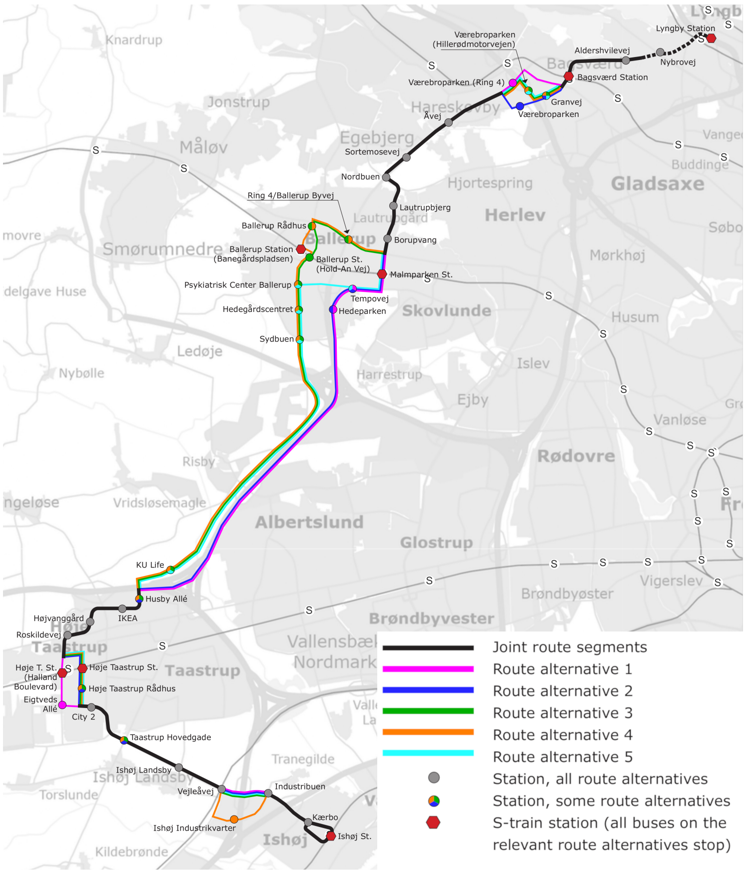


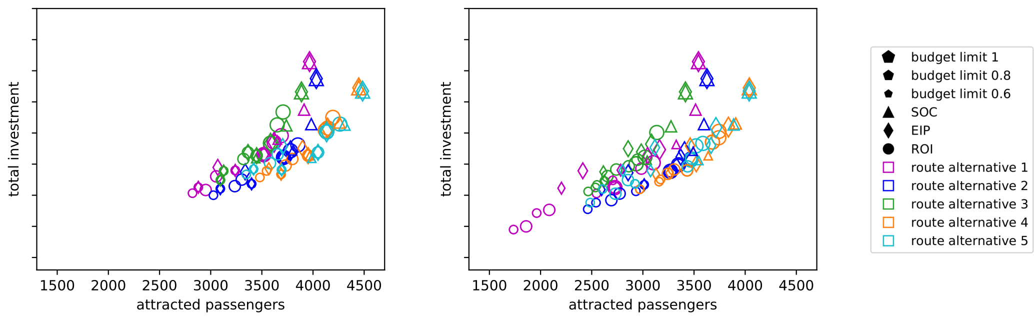
Figure 2 Route alternatives for a new BRT line in the Copenhagen Region. Adapted from [19].

We use the data about the five route alternatives from the pre-assessment study to construct instances for EIP and ROI. These instances contain between 24 and 32 stations depending on the route alternative, with the restriction that there are two edges that are not upgradable. To obtain OD-data, we translate station passenger demand information to OD pair demands according to the classical gravity model [14]. Moreover, as the municipalities still have to decide on the investments that they are willing to make, we create budget scenarios $\beta = (\beta_1, \beta_2, \beta_3)$ as in the artificial instances according to Table 1.

Evaluation

For all five route alternatives, considering the model SOC or EIP with the same budget limit β_1, the investment is almost the same for the objectives LINEAR and MINIMPROV (see, e.g., Figures 5 and 6 in Appendix A). However, the numbers of passengers that are attracted are considerably lower for MINIMPROV. Note that particularly fewer passengers can be attracted in the benefit-oriented problem ROI-MINIMPROV for some of the route alternatives. A reason might be that it is difficult to achieve an upgrade of 75% of the edges on the path of an OD pair, especially because there are two segments on the routes that are not allowed to be upgraded.

When comparing the various route alternatives, the goal is to determine which one has the highest potential to attract new passengers without leading to high investment costs. Figure 3 shows that for both passenger behavior patterns investigated here, i.e., for the objective functions LINEAR and MINIMPROV, route alternatives 4 and 5 have the highest potential to attract new passengers for all budget limits and all models SOC, EIP and ROI. These two route alternatives are therefore to be given preference. A peculiarity of route alternatives 1 and 2 is that one municipality contains a costly highway segment in the middle (see Figure 5 in Appendix A). An investment would be very advantageous for passengers in general, but the investing municipality would not benefit as much because there are no stations along this costly highway segment that can attract new passengers. Therefore, this segment is only upgraded in SOC and EIP:cost with $\beta_1 = 1.0$, and in particular never in ROI. Note that this costly segment is not contained in the preferable route alternatives 4 and 5 (see Figure 6 in Appendix A).



(a) LINEAR. **(b)** MINIMPROV. **(c)** Legend.

Figure 3 Comparing investment costs and attracted passengers for the different route alternatives. Note that the x- and the y-axes are scaled the same in both plots.

5 Conclusion

In this paper, we introduced the Edge Investment Problem, which is motivated by the construction of a new BRT line around Copenhagen and which aims to capture a maximum amount of new passengers. We modeled the problem mathematically, developed linear integer programming formulations and analyzed the complexity. Additionally, we evaluated both the Copenhagen case study as well as related artificial instances concerning the investment and the newly attracted passengers.

The presented models can also be applied to general graphs, which is considered in ongoing research. Here, an upgrade of one edge can affect several lines such that the problem structure gets more involved. Future work could also consider the connectivity of upgraded edges in addition to the gained infrastructure improvements, as their relative arrangement might have an impact on the attractiveness to passengers of a BRT line as well. For example, if the bus often switches between normal traffic and the dedicated BRT infrastructure, it might no longer be perceived as a BRT line by the passengers. Hence, a preferred solution would contain long consecutive sections of upgraded edges.

Further, a natural extension of the problem analysis is to model the Edge Investment Problem in a game-theoretic setting. In addition to the municipalities, it is interesting to consider a central authority that can either subsidize the investments of the municipalities or invest in any edges itself with respect to a budget constraint. This could give new incentives for the municipalities to invest.

When extending the problem to general graphs with multiple lines, it might also be beneficial to consider the Edge Investment Problem in an integrated setting, see [16]. Here, combinations with line planning, passenger routing and tariff planning based on [17] are especially promising.

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A Appendix: Further Evaluations

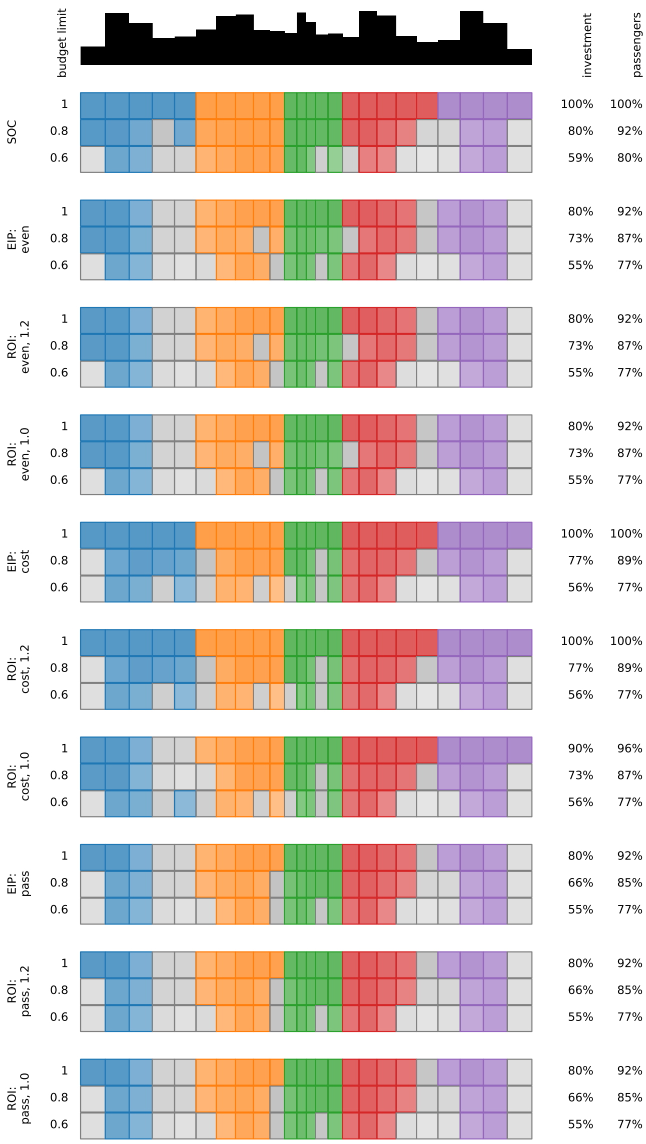
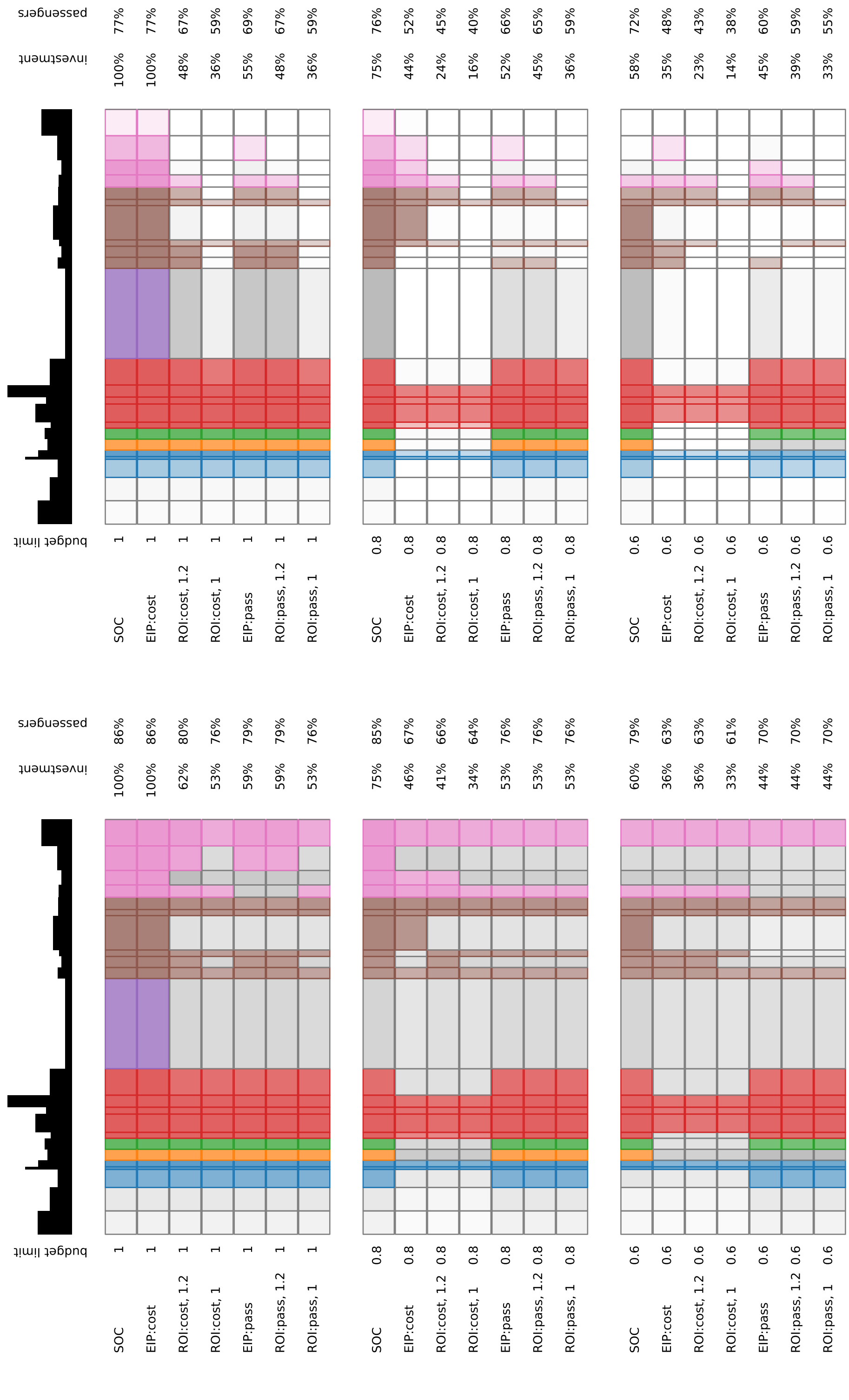


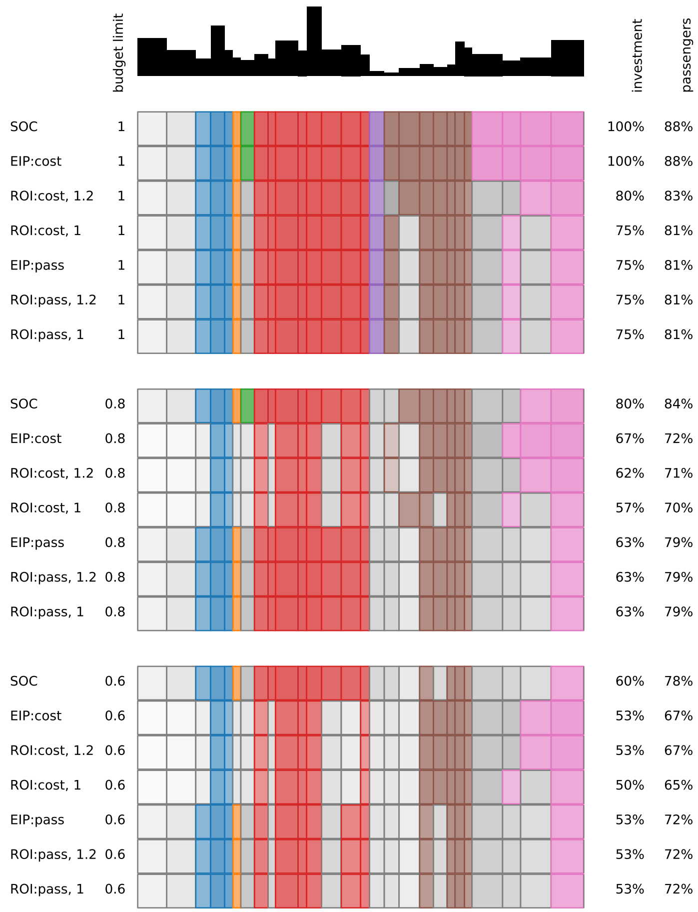
Figure 4 Example for upgraded line segments, $\alpha_1 = 25$, $\alpha_2 =$ ENDS, $\alpha_3 =$ CENTER for objective Linear. For each model SOC, EIP with β_2, ROI with β_2, β_3, the segments upgrades for budget limit $\beta_1 \in \{1, 0.8, 0.6\}$ are given. Segments are colored according to the corresponding municipality if they are upgraded and are gray if they are not upgraded. The shade of the color gives the share of the passengers using the segment compared to the total number of potential passengers. The width of a segment corresponds to its costs. The passenger distribution for the completely upgraded BRT line is given at the top. The investment is given as a percentage of the costs of the complete BRT line and the passengers attracted are given as a percentage of the potential of the completely upgraded BRT line.



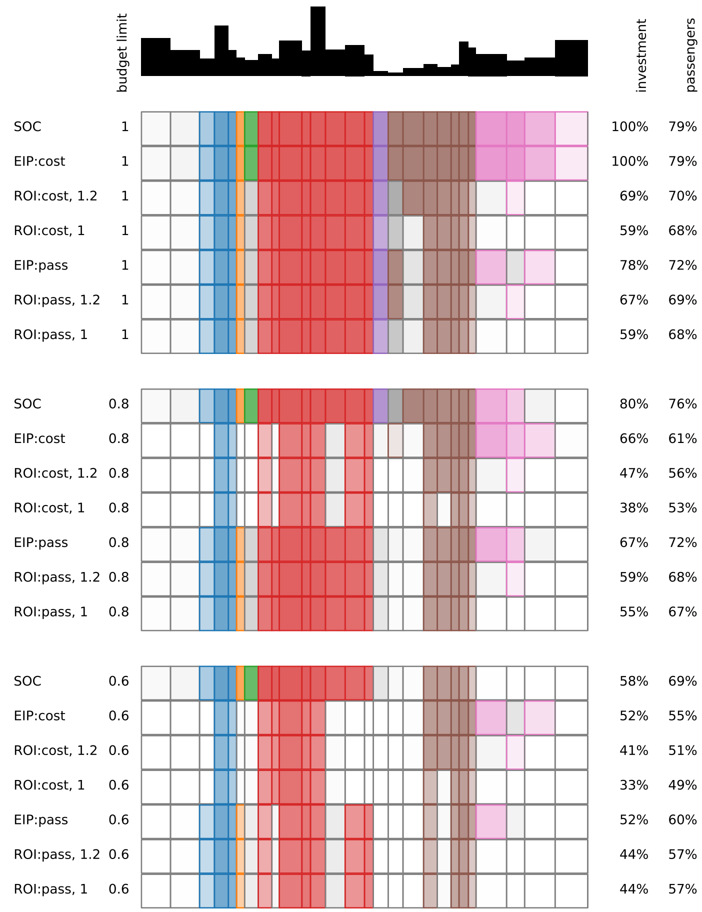
(a) Linear.

(b) MinImprov.

Figure 5 Overview of the upgraded segments for route alternative 1. The first two edges are not allowed to be upgraded.



(a) LINEAR.



(b) MINIMPROV.

Figure 6 Overview of the upgraded segments for route alternative 5. The first two edges are not allowed to be upgraded.