On different strategies to solve problems involving algebraic expressions

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Introduction and research questions

This study is part of a larger project that involves Italian teachers and students in grades 6 and 7, with the objective of designing inclusive mathematical activities, through cycles of implementation and revision of the designed materials, in order to promptly address known difficulties or overcome emerging difficulties in mathematical learning.

The literature shows that an important source of difficulty in mathematics - often leading to failure in algebra - is students' lack of number sense (Boaler, 2015). For example, Gray & Tall (1994) found that many students labelled as low achievers did not actually "know less" than others, but they did not use numbers flexibly. Our conjecture is that in most cases such mathematical behavior depends on teaching approaches based on memorization and not on building flexibility in interacting with numbers. Here we refer to this flexible way of working with numbers as "number sense". Moreover, the research shows that a lack of number sense contributes to difficulties in the perception of number structure, which is a key aspect for the development of pre-algebraic thinking (Mulligan & Mitchelmore, 2013).

The activities that we designed, and that are proposed to the students in the experimental classes involved in the project, are thought to support students' development of number sense; and they emphasize the importance of reasoning to have success in math, rather than memorizing. Here we focus on some examples: the work we designed on algebraic expressions asked students to find ways to transform one expression in an equivalent one (e.g., $9 + 5 \cdot 3$ into $6 \cdot 4$), without calculating "the result" (in the example "24"), by using algebraic properties that had been previously discussed and by flexibly interacting with the numbers. Moreover, to better see the *structure* of expressions, we asked students to represent algebraic expressions by using *trees* (Maffei & Mariotti, 2010). This study is aimed at qualitatively analyzing students' different strategies adopted to solve problems involving algebraic expressions. In particular, we focus on specific strategies used by the students who typically experience difficulties and have a history of low achievement in mathematics. In particular, we want to study how they make sense of different strategies and reason about the comparison of such strategies. Indeed, literature shows that being able to compare strategies is a fundamental cognitive process that can support learning in a variety of domains, including mathematics (Alfieri et al., 2013; Gentner et al., 2003).

In order to investigate these issues, we set out an interview that we used with some students involved in the project. The interview consists of activities similar to those in the experimental materials. For example, different representations of algebraic expressions are included: the standard mathematical representation, the Italian language representation (e.g., "the sum of nine with the product of five and three") and the tree graphical representation. The task asks the students to "translate" an algebraic expression represented in one of these forms into another representation of the same expression. Other tasks ask to compare different strategies used to transform one expression into an equivalent one. For example, we present a dialogue of two imaginary students attempting to pass from an expression to an equivalent one, without calculating its value but manipulating numbers and we ask the interviewed students to compare the two strategies and choose the best one for that context (Durkin et al., 2017).

Methodology

For this study, we used clinical interviews as the main methodological tool (Piaget, 1929; Goldin, 2000). The interviews were conducted by one of the authors of this paper. The students were chosen by the interviewer together with their regular teachers, in order to have a population with different cognitive characteristics with respect to mathematical learning: both students with a history of low achievement in mathematics and high achieving students.

Preliminary outcomes

In line with the literature, we expected students who usually experience difficulties in learning mathematics to struggle, but also to be able to become more confident with particular strategies or representations. For example, we observed that the tree graphical representation of expressions helped them to see the structure, and this allowed some students to not memorize the order of precedence of operations as a rule. Therefore, visualizing this type of representation seems to have the potential to lighten the load on working memory.

The task that we designed for the interview, in which we ask the interviewed students to compare the strategies used by two imaginary students to pass from an expression to an equivalent one, provided us with a valuable approach for exploring students' understanding of the mathematical meanings involved. In particular, we gained insights into their use of algebraic properties and their flexibility in manipulating numbers (Zazkis & Zazkis, 2014). Moreover, we found that the work on number flexibility and number sense carried out with the students, through the experimental materials that we designed, resulted in a mathematically appropriate management and manipulation of numbers and algebraic expressions during the interview.

Relation to the theme chosen

This study investigates students' understanding of arithmetic and number systems. In particular, we are interested in analyzing students' flexibility with manipulating algebraic expressions. At a broader level, we also address the issue of design of approaches for fostering the development of flexibility and number sense in inclusive education contexts.

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