IAC-20-C1,1,12x57540

# Thrust Expenditure Feasibility Analysis for Rendezvous Operations in Cis-Lunar Space

**Giordana Bucchioni[a]*, Mario Innocenti[b]**

[a] *Department of Information Engineering , University of Pisa, Via Caruso 16 , Pisa, Italy 56122*, giordana.bucchioni@ing.unipi.it
[b] *Department of Information Engineering, University of Pisa, Via Caruso 16 , Pisa, Italy 56122*, mario.innocenti@unipi.it
* Corresponding Author

**Abstract**

In recent years, Moon exploration has become a primary objective within most space agencies worldwide. The Lunar Space Gateway program ARTEMIS (or LOP-G) is an example of mission proposal for technology feasibility in terms of autonomous (and later manned) operations of a space station orbiting the L2 Earth – Moon Lagrangian point. Rendezvous and docking (berthing) are tasks that are envisioned to be performed fully autonomously. The focus of the paper falls in this category, whereby an active module called Lunar Ascender Element (LAE), returning from the lunar surface, shall be able to operate an automatic rendezvous mission with the LOP-G station. The paper concentrates, in particular, with the feasibility analysis needed to assess the engines' thrust capabilities to provide appropriate propulsion for open loop and closed loop control during rendezvous. The capability of providing the desired amount of thrust is not only linked to the actual guidance commands, but also to the nature of the motors. The rendezvous maneuver sequence, dynamics and hold points are first defined, and the thrust distribution and configuration detailed for the specific mission. The guidance logics are described, and the implementation of a passively safe trajectory outlined. Based on the dynamic model of the system, and the assumed actuator model, the main causes of unfeasibility are listed. The paper continues by analyzing the sensitivity of the thrust profile at each motor with respect to the control allocation algorithm, the duration of the maneuver, the duration of each impulse (assuming a two-impulse maneuver), and the location of the berthing port within a selected near rectilinear halo orbit around the Moon. The tests take into account how the parameters influence the Delta V required to perform the mission. The authors wish to remark that this analysis is critical to the design of rendezvous and berthing (docking) operations, since feasibility is necessary for the success of the mission, and it provides a structured computation of a realistic parameter space in the relative motion in the presence of a third body perturbation.

**Keywords:** Rendezvous, LP3, Thrust Feasibility, Control allocation

**Nomenclature**

$[\dot{\omega}_{l/I}]_{\mathcal{L}}$ : angular velocity on the LVLH frame derived in the LVLH frame

$[\ddot{\rho}]_{\mathcal{L}}$ : chaser acceleration w.r.t. the target in the LVLH reference frame

$\boldsymbol{F}_{S_{min}}$ : minimum thrust provided by the thrusters

$\boldsymbol{B}_{S}$: control allocation matrix

$\omega_{l/I}$: angular velocity of the LVLH reference frame w.r.t. the Inertial reference frame

$\boldsymbol{r}_{em}$: Erath-Moon radius

$\boldsymbol{r}$: position of the Target w.r.t. the Moon

ρ: relative position of the Chaser w.r.t. the Target

$\boldsymbol{\tau}$: forces and torques along the axis of the geometrical reference frame

$\boldsymbol{F}_{S_{max}}$: maximum thrust provided by the thrusters

$[\dot{\rho}]_{\mathcal{L}}$: relative velocity of the chaser w.r.t. the target derived in the LVLH frame.

$\boldsymbol{F}_{S}$: vector of the thrusts at teach thruster

μ : Earth-Moon gravitational constant

**Acronyms/Abbreviations**

ER3BP: Elliptic Restricted 3 Body Problem
HP: Hold point
LAE: Lunar Ascent Element
LOP-G: Lunar Orbital Platform Gateway
TOF: Time of Flight
ATV: Automated Transfer Vehicle

## 1. Introduction

In recent years, the Moon has created new interest among all the space agencies around the world. In the near future NASA in planning to send an orbiting space station on an L2 Near Rectilinear Halo Orbit, called Lunar Space Gateway (LOP-G). This space station will also be used as arrival point for other missions to explore the Moon and its properties. Therefore the content of the proposed study has a particular importance. In particular, the paper focuses on ESA's EL3 mission, also called Heracles mission. This operation has the goal of proving the advancements in autonomous operations in space, this means to autonomously collect some samples from the south pole of the moon and bring them back to the orbiting space station. The study concentrates on the rendezvous phase in which the chaser vehicle – the Lunar Ascent Element – must safely approach the target and dock with it. The goal is then to understand how the design of the rendezvous (hold points location, manoeuvre duration, impulse duration, control allocation algorithm) can influence the feasibility of the manoeuvre, since the chaser vehicle has limited thrust assigned by the mission design.
Different studies are reported in literature relative to the rendezvous sequence, the best location of the hold points and the control allocation algorithms.
The state of the art of rendezvous strategies are is extensively explained in [9], in this book, two main strategies emerge: the rendezvous approach along V-bar or along R-bar and their variations. In [8] is considered the best approaching strategy for the rendezvous of the ATV with the ISS. Multiple references are present in the literature that discuss the best approaching manoeuvre, in terms of guidance (open-loop or closed-loop), energy saving or safety (active or passive) [4] [5] [7].
Another key aspect that influence the goodness of the approach is the control allocation algorithm, so the way in which the required forces and torques are located on the single thrust. Different control allocation algorithms are presented in [Fossen], the easiest way to allocate the control is just a Moon-Penrose pseudoinverse, but it does not take into account the thruster's saturations consequently other approaches must be studied. In [ankersen], for example, it is studied the best control allocator for the rendezvous of the ATV with the ISS.

In this paper we briefly explain and use two different procedures: Look-up tables, a standard technique suggested by the European Space Agency, and an optimal technique that is an extension of the one described in [3]. However, to the authors' knowledge, a lack of studies exists in the analysis of the feasibility on the closing manoeuvre in terms of thrust. In other words, the purpose of the paper is to study how the design of the rendezvous sequence and the control allocation algorithm may influence the feasibility of the trajectory, then it is fundamental to understand if the change of the algorithm may guarantee the accomplishment of the mission's goal. The paper is organised as follows: the structure of the dynamics are explained, then detailed afterwards in the theory and calculation section. The results of the experiments are reported in the results section followed by some comments and conclusions.

## 2. Manoeuvre and Structure

The paper presents a sensitivity analysis carried out starting from a specified rendezvous manoeuvre and analysing how feasibility properties change if some key parameters are varied.

The idea is to define a standard approaching manoeuvre and to vary one by one the parameters that may influence the feasibility of the manoeuvre and analyse how much they can be varied.

The peculiarity of this rendezvous manoeuvre is that the perturbation of the third body – the Earth – must be considered to obtain more realistic results, so the feasibility of the closing approach could change significatively form the state of the art.

**Figure 1 Chaser and Target vehicle**

Going into details the approaching trajectory is performed through a series of hold points (way points) that are defined in position and velocity with respect to the target vehicle - the LOP-G - in this case. The chaser vehicle must relatively stop with respect to the target at every hold-point and check is state to being able to decide if it is safe to proceed or not. In this context the safety (passive) is defined as the ability of avoiding the collision with the target without performing any active action.
The original passive safe sequence of hold points is:

- HP0: [-26.7759 -3.9550 54.1078] km
- HP1: [-13.5776 -4.5617 7.1301] km
- HP2: [-2.1936 -0.1622 0.0409] km
- HP3: [-0.15 0 0] km (fixed)
- HP4: [0 0 0] km (fixed)

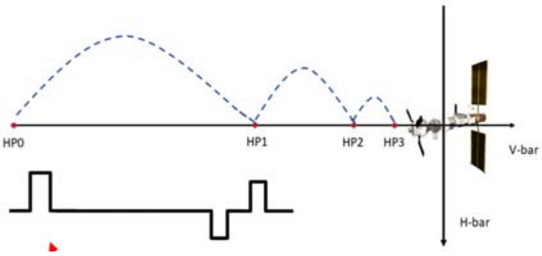
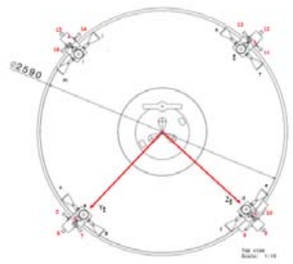
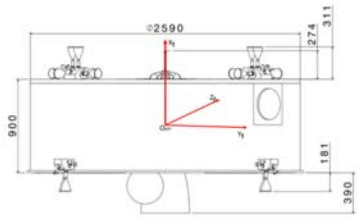



**Figure 2 Hold Point sequence in LVLH**

The impulses duration is 30s, and the length of the different Time of Flight (TOF) from an hold point to the next is 40h, 12h, 5h and 6h. The original control allocation method specified by the Agency is a Look-up table.
Once the standard mission is detailed, the scope of the experiments consist of varying one by one the location of HP0, the impulse duration and the TOF, with the goal of understanding if the mission should be aborted in case of replanning or there are some errors margins to continue. Another fundamental aspect that is modified in the tests is the control allocation algorithm, in fact the chaser vehicle architecture is composed by 16 thrusters, distributed as in the Figure 1, and the control effort required by the guidance algorithm must be allocated on each thruster. The allocation method influences the feasibility of the motion, that is why in this paper two different methods are proposed: Look-up Table and Optimal.



(a) up-view




(b) side-view.

**Figure 3 Chaser thrusters' distribution**

The details of these two methods are reported in [3], in the next section few notions to understand the importance of such control allocation algorithms are provided, for clarity's sake.

The dynamics that is propagated is based on the Elliptic Restricted Three Body Problem (ER3BP) and it was formulated in [6], the guidance algorithm instead is based on the adjoint method and it provides the amplitude of the required impulses to move from HP0 to HP1 and from HP1 to HP2, afterwards the control loop is closed, but this is not considered in this paper.

## 2. Theory and Computation Review

In this section is described the mathematical theory that was used to retrieve the results presented in this paper. Firstly, the reference systems and a quick overview of the equations of motion, then the two control allocation algorithms are explained and the structure of the simulations is provided.

### *3.1 Main reference frames*

The most useful reference frames are briefly explained herein. Firstly, an *Inertial* reference frame must be defined.

$$\mathcal{I}:\ \left\{O; \hat{\boldsymbol{I}}, \hat{\boldsymbol{J}}, \hat{\boldsymbol{K}}\right\}$$

In this paper is not necessary to refer to any particular inertial reference frame.

The most used reference frame is the LVLH frame, in which the relative propagated dynamics is written. The Local Vertical Local Horizon reference frame is defined as:

$$\mathcal{L}_i : \left\{ \boldsymbol{r}_{it}; \hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}} \right\}$$

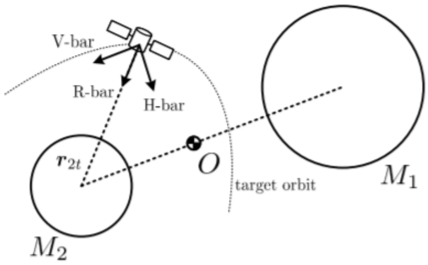



**Figure 4 LVLH reference frame**

The LVLH frame is defined with respect to the primary body around which the target is orbiting, Denoting with  the target position with respect to the primary *i*, with $[\boldsymbol{r}'_{it}]_{\mathcal{M}}$ the target velocity as seen from the primary, and with $\boldsymbol{h}_{it} = \boldsymbol{r}_{it} \times [\boldsymbol{r}'_{it}]$ *the specific angular momentum* with respect to the primary, the LVLH frame unit vector are defined as follows:

- $\hat{\boldsymbol{k}}$: points to the primary and is called *R-bar*
- $\hat{\boldsymbol{\jmath}}$ :is perpendicular to the target instantaneous orbital plane and is called *H-bar*
- $\hat{\boldsymbol{\imath}}$: completes the right-handed reference frame, and is called *V-bar*

Another important reference frame is the *Geometric* reference frame. In general, the definition of geometric frame is based on geometrical property of rigid body. Assuming the mass is uniformly distributed and a regular geometric form, as the cylinder, for spacecraft we can define the geometrical frame with the origin located on the geometrical centre of one of the chaser surface of body and the axes parallel to the principal axes of inertia.

$$\mathcal{G} : \left\{ O_{t/c_{com}}; \hat{\imath}_g, \hat{\jmath}_g, \hat{k}_g \right\}$$

In Figure 3 we can see the geometrical frame for chaser, but in the same way is defined the geometrical frame of target.

*3.2 Equations of motion*
The equations of motions that regulate the dynamics of the simulations are written under the hypothesis of Elliptic Restricted Three Body Problem, this means that only the Moon and the Earth are taken into account and the are revolving around the common centre of mass in an elliptical motion.

The equation of motion reported below describe the dynamics of the relative motion

$$[\ddot{\boldsymbol{\rho}}]_{\mathcal{L}} + 2\boldsymbol{\omega}_{l/i} \times [\dot{\boldsymbol{\rho}}]_{\mathcal{L}} + [\dot{\boldsymbol{\omega}}_{l/i}]_{\mathcal{L}} \times \boldsymbol{\rho} + \boldsymbol{\omega}_{l/i} \times (\boldsymbol{\omega}_{l/i} \times \boldsymbol{\rho})$$
$$= \mu \left( \frac{\boldsymbol{r}}{r^3} - \frac{\boldsymbol{r} + \boldsymbol{\rho}}{\|\boldsymbol{r} + \boldsymbol{\rho}\|^3} \right) + (1-\mu) \left( \frac{\boldsymbol{r} + \boldsymbol{r}_{em}}{\|\boldsymbol{r} + \boldsymbol{r}_{em}\|^3} - \frac{\boldsymbol{r} + \boldsymbol{\rho} + \boldsymbol{r}_{em}}{\|\boldsymbol{r} + \boldsymbol{\rho} + \boldsymbol{r}_{em}\|^3} \right)$$

Where $\boldsymbol{\rho}$ is the relative position of the chaser with respect to the target, $\boldsymbol{\omega}_{l/t}$ is the angular velocity of the LVLH reference frame, $\boldsymbol{r}_{em}$ position of the Moon with respect to the Earth, $\boldsymbol{r}$ position of the target with respect to the Moon, $[\ddot{\boldsymbol{\rho}}]_{\mathcal{L}}$ relative acceleration of the chaser derived in the LVLH reference frame, $[\dot{\boldsymbol{\omega}}_{l/t}]_{\mathcal{L}}$ is the angular acceleration of the LVLH reference frame derived in the LVLH reference frame, $[\dot{\boldsymbol{\rho}}]_{\mathcal{L}}$ is the relative velocity of the chaser with respect to the target derived in the LVLH reference frame.

The equations are then simplified under the assumption of elliptic restricted three body problem that implies that the two primaries are revolving on the same plane in an elliptical motion around the barycentre.

*3.2 Thrusters configuration*

The working framework of the proposed study is the Heracles mission, which defines a distribution of the thrusters and the thrust allocation matrix. The chaser vehicle has three different kind of thrusters: the main engine, 16 medium thrusters to accomplish the phasing manoeuvre and 16 smaller thrusters for the rendezvous. The latter are located to provide torques and forces independently along the three directions.
Formally, given [] the matrix B maps the trust provided by each motor with the forces and torques defined with respect to the *geometrical* reference frame.
The matrix Bs is defined as

$$F_s = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 & F_9 & F_{10} & F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \end{bmatrix}^T$$

$$\tau = \begin{bmatrix} F_x & F_y & F_z & N_x & N_y & N_x \end{bmatrix}^T$$

$$\tau = [B_s] \begin{bmatrix} F_s \end{bmatrix}$$

*3.3 Control allocation approaches*

Two different control allocation approaches are used in this paper: *Look-up table* and *Optimal control allocation*.

The first approach consists of a static method. Each column of the look-up table is optimised in the sense of energy consumption for a particular force/torque combination, but in general the resulting thrust on each motor in not optimal. Below is reported the look-up table used in this case.

$$B^{Force} = \begin{pmatrix} 0 & 0 & 0 & 0.0707 & 0 & 0 \\ 0 & 0.0707 & 0 & 0 & 0 & 0.0707 \\ 0 & 0 & 0.0707 & 0 & 0 & 0 \\ 0 & 0.0707 & 0 & 0 & 0.0707 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0707 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0707 & 0 & 0 & 0 & 0 & 0.0707 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0; \\ 0 & 0 & 0.0707 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0707 & 0 & 0 & 0 & 0.0707 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The second approach is a linear programming problem that minimize the ΔV consumption at each step (1Hz).

$$\min_{F_S} \sum_t \Delta V_t$$

$$\begin{cases} B_S F_S - \tau \\ F_S\_min \le F_S \le F_S\_max \end{cases}$$

This approach is less computationally efficient than the look-up tables, in fact at each step an optimization problem must be solved, however the reachable areas are larger, so some manoeuvre may result feasible using this allocation procedure. For clarity's sake the reachable area is defined as the set of torques and forces that can be obtained with this specific thruster's configuration.

*3.4 Feasibility definition and computation*

In this context is considered the feasibility is defined as:

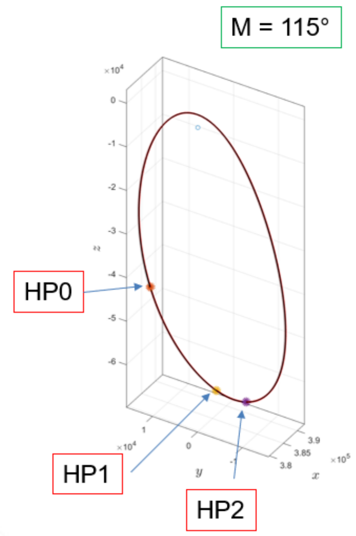



**Figure 5 Example of Hold point location sequence**

**Definition:** *A manoeuvre is considered feasible if the thrusters can provide the thrust required from the guidance algorithm.*

The experiments are performed varying one by one the already cited paraments:

- The location of HP0 in terms of mean anomaly: 235°, 226°, 215°, 202°, 197°, 180°, 163°, 148°, 135°, 124°, 115°.
- The duration of the impulses: 10s, 30s, 60s.
- The Time of flight at each manoeuvre:
    - HP0_HP1: 40h, 30h, 20h, 10h, 5h
    - HP1-HP2: 30h, 20h, 15h, 12h, 10, 5h, 3h, 2h

If a test results not feasible with the Look-up table approach, then the same simulation is run with the Hold point allocation algorithm optimal allocation algorithm??.

## 4. Results

In this section we describe some results of the simulations.

**Table 1**

| HP0 Location(°) | Feasibility | Delta (m/s) |
|---|---|---|
| **235°** | Yes | 1.7522 |
| **226°** | Yes | 1.6863 |
| **215°** | Yes | 1.5105 |
| **202°** | Yes | 1.3583 |
| **197°** | Yes | 1.3742 |
| **180°** | Yes | 1.4291 |
| **163°** | Yes | 1.4674 |
| **148°** | Yes | 1.5338 |
| **135°** | Yes | 1.5201 |
| **124°** | Yes | 1.4697 |
| **115°** | Yes | 1.4283 |

**Table 2**

| Duration (h) | Feasibility (Look-up table) | Feasibility (Optimal) | ΔV(m/s) |
|---|---|---|---|
| 2 | No | Yes | 4.76 |
| 3 | No | Yes | 4.59 |
| 5 | Yes | Yes | 2.44 |
| 10 | Yes | Yes | 1.67 |
| 12 | Yes | Yes | 1.55 |
| 15 | Yes | Yes | 1.22 |
| 20 | Yes | Yes | 1.29 |
| 30 | Yes | Yes | 1.18 |

Table 3 ??

In Table1 are reported the results obtained if the starting of the rendezvous on the Near Rectilinear Halo Orbit. In Table2, is studied how the time of flight of the first transfer phase of the manoeuvre, in Table 3 is studies how the feasibility changes if the duration of the second transfer phase is made to vary.

**5. Discussion**

The discussion of the results is reported in this section. From the Table 1 is possible to see that the position of the HP0 does not influence the feasibility of the manoeuvre, this means that the original rendezvous sequence is well designed. In fact, if some errors would happen in the location of HP0, the thrusters would still be able to provide the required thrust and accomplish the manoeuvre.

Table2 and Table 3 show the results obtained if Time of Flight varies, in this case it is possible to observe that in some cases, the ones with short durations are unfeasible if the control allocation is performed with the Look-Up table approach, But the control allocation based on the optimization procedure allows the thrusters to provide the required guidance profile, This means that in case of fast rendezvous it would be preferrable to change the control allocation algorithm to make the manoeuvre feasible. Even if this implies a higher computational cost to solve an optimization at each step. However, the proposed standard manoeuvre seems to be reliable also with respect to the variation of the impulse duration in fact it is always feasible. This can be considered a good result since the standard can be adjusted and modified in case of failure and non-idealises, but it will preserve its feasibility.

**6. Conclusions**

The reliability of the proposed standard rendezvous manoeuvre appears very good in fact its feasibility seems not to be particularly influenced by the parameters that were varied in this work. However, it proves that the control allocation algorithm is fundamental for the feasibility of the mission, especially when high ΔV are required, and it may make the difference in cases that are safety critical.

**Acknowledgements**

This work was partially supported by the European Space Agency under contract No.000121575/17/NL/hh. The view expressed herein can in no way be taken to reflect the official opinion of the European Space Agency.

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| **Duration (h)** | **Feasibility (Look-up table)** | **Feasibility (Optimal)** | **ΔV(m/s)** |
|---|---|---|---|
| 5 | No | Yes | 8.28 |
| 10 | Yes | Yes | 2.77 |
| 20 | Yes | Yes | 1.82 |
| 30 | Yes | Yes | 1.52 |
| 40 | Yes | Yes | 1.41 |