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MEDA
ERME Topic Conference
Mathematics Education in the Digital Age

JKU LINZ SCHOOL OF EDUCATION

10th ERME TOPIC CONFERENCE (ETC10)

Mathematics Education in the Digital Age (MEDA)

16-18 September 2020 in Linz, Austria

PROCEEDINGS

Edited by:

Ana Donevska-Todorova, Eleonora Faggiano, Jana Trgalova, Zsolt Lavicza, Robert Weinhandl, Alison Clark-Wilson, and Hans-Georg Weigand

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Developing an analytical tool of the processes of justificational mediation

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Within the Instrumental Approach (IA) the newly developed notion of justificational mediation (JM) describes mediations that aim at establishing truth of mathematical statements in the context of CAS-assisted proofs in textbooks. Here we study JM with the intent to broaden the notion to the context of informal justification processes of early secondary students interacting with GeoGebra. Seeing JM as a process that has the objective of changing the status of a claim, we use Toulmin's model and combine it with the IA to unravel the structure of the process through an analytical tool. The study is part of a broader project on the interplay between reasoning competency and GeoGebra with lower secondary students.

Keywords: digital environment, Instrumental Approach, justificational mediation, reasoning competency, Toulmin's model.

REASONING COMPETENCY AND JUSTIFICATIONAL MEDIATION

During the last decades, the use of digital technologies in mathematics education has increased, as well as the body of research in this area (e.g., Hoyles & Lagrange, 2010). In Denmark, this development has coincided with the promotion of *mathematical competencies*, seen in the KOM-framework as "…someone's insightful readiness to act appropriately in response to a specific sort of mathematical challenge in given situations." (Niss & Højgaard, 2019, p. 6). In the wake of this development, a need has arisen for understanding the interplay of students' enactment and development of the specific mathematical competencies and their use of digital technology (Geraniou & Jankvist, 2019). What might "readiness to act appropriately" mean in the context of digital technology? How can such readiness be identified and nurtured? These are examples of broad questions that gave rise to this study.

We follow Geraniou and Jankvist (2019) who took some first steps in weaving together the KOM framework with the Instrumental Approach (IA), which is also widely used in the European research community. The IA suggests that the use of tools involves *pragmatic mediation,* concerning the subject's actions on objects and *epistemic mediation*, concerning how the subject gains knowledge of objects' properties through the tool (Rabardel & Bourmaud, 2003). However, Jankvist and Misfeldt (2019) suggest that a third form of mediation, *justificational mediation* (JM), may be useful in the context of CAS in proofs and proving activities. JM concerns how the status (e.g. probable, likely, true or false) of statements for a student is modified through the use of a digital environment (Jankvist & Misfeldt, 2018; 2019). However, the authors have advanced the notion of JM within the context of CAS-assisted proofs in textbooks in upper secondary school, which touches on the more formal part of the *reasoning competency*. Still the authors ponder whether students think about justification, insight and performing mathematical labor as different things and how (Jankvist & Misfeldt, 2019). So, other situations relating to the less formal side of the reasoning competency spectrum should be considered and studied separately within this frame.

Within the KOM-framework's reasoning competency, we study students' mathematical informal argumentations that take place within the digital environment GeoGebra, focussing on the processes through which an uttered statement changes status: it may either be rejected or believed to be true to a greater degree than in its initial form. The ways in which students justify their claims within an environment like GeoGebra can assume forms that are closely related to the environment itself, as well as to the underlying mathematical theory within which the objects are placed. Hence we ask: *how can we analyze JM and what insight into it can we gain*?

Seeing JM as a process of argumentation, our analytical tool is derived from Toulmin's model, and, because JM occurs in a digital environment, we make use of constructs from the IA. We now explain how the theoretical frame is set up.

THEORETICAL FRAMEWORK: CONSTRUCTING A TOOL OF ANALYSIS

Although the original intention of Toulmin's model was to analyze finalized argumentations (Toulmin, 2003), there are numerous examples in mathematics education where it is used to analyze students' processes of argumentation (e.g. Pedemonte, 2008; Simpson, 2015), also in the context of digital environments (eg. Hollebrands, Conner & Smith, 2010). These studies, however, do not usually situate the model within the research field of educational use of digital technologies in mathematics, and hence do not draw on the theories used in this field. In this study, we suggest an analytic tool that does exactly that.

With respect to the IA, we consider GeoGebra as an *instrument*. Such a notion arises from the use of an *artefact* and the development of *scheme.* In this context the artefact is GeoGebra itself, but in other cases it could be a specific tool within it (such as dragging, or a slider). *Schemes of utilization* are developed by a solver to accomplish a specific task (Rabardel, 2002). *Scheme* is understood according to Vergnaud's construct: "the invariant organization of activity for a certain class of situations" (Vergnaud, 2009, p. 88), that relates an "invisible part" to a student's visible actions. Schemes are made up of various aspects, including a *generative aspect:* rules to generate activity; namely the sequences of actions; information gathering; and controls and an *epistemic aspect:* operational invariant; namely concepts-in-action; and theorems-in-action, with the function to pick up and select the relevant information and infer from it goals and rules.

In the following, we will introduce elements from Toulmin's model and explain how we interpret them within the IA and with respect to JM.

JM through Toulmin's model in the context of GeoGebra

In Toulmin's model, the *claim* is a statement of the speaker, uttered with a certain indication of likelihood (*qualifier*); the claim is justified through other elements of the argument (data, warrant, backing). The first utterance of the claim indicates the start of the JM process, in which the *aim* is to change the qualifier. In younger students' informal argumentation, the aim is seldom to construct rigorous mathematical proofs but rather to convince themselves of the existence of mathematical relations and facts (Jeanotte & Kieran, 2017). Hence, a change in the status of the qualifier will often be from likely to more likely, and less often from likely to true. We recognize such a change of status of a claim by students' restatement of the claim accompanied by a new qualifier. The change of the status is reached through the generation of *data* that for the solver constitutes evidence and facts supporting the claim, and through the *warrant* that consists of inference rules that allow the solver to connect the generated data to the claim (Toulmin, 2003). The warrant is often implicit, in which case, it must be inferred from the utterances and gestures of the students. We can infer the warrants and analyze the generation of data through the notion of scheme introduced above. The *generation of data* is the product of the generative aspect of the schemes used (e.g., dragging, creating objects on the screen and interacting with them, utterances and other hand-gestures) that are carried out by students. Warrants are the epistemic aspect of the schemes used. One last element remains; *backing*. This element requires some careful consideration, which we elaborate in the next section.

Toulmin describes the *backing* of a warrant as "…other assurances, without which the warrants themselves would possess neither authority nor currency" (Toulmin, 2003, p. 96). However, Simpson (2015) identifies three different uses of backing in mathematics education research. In the context of JM, we consider the backing to be an explanation of why the warrant is relevant (Simpson, 2015). Central is, that the aim of JM is to change the status of the claim, so the backing must explain why the warrant is relevant for generating data that allows the change in the status of the claim. Thus, the backing becomes fundamental to the JM process. Currently, we have reached the following formulation of *backing* in JM processes:

If the claim is true, I can generate data, within the specific instrument, that is consistent with the claim.

This seems closely related to Vergnaud's (2009) notion of *theorem-in-action*, a sentence that the solver believes to be true, but that may in fact be false. Though it can be, it is not a mathematical theorem, and it can bridge domains of different natures. In our case it bridges the phenomenological domain of GeoGebra with the theoretical domain of algebra (also see Baccaglini-Frank, 2019). We recognize, that there might be variations of such a formulation, but we are currently studying this form.

METHODOLOGY

The task we analyze in this paper comes from a broader project, in which a series of tasks were designed by the first author and assigned to students in three classrooms of grade 7 students (in all 61 students). All students had prior experience using GeoGebra's geometric tools, as well as constructing points and sliders in the algebra view, but they had never used the slider to vary points, which is central in this task. The students worked in pairs for two 90-minute sessions while being video recorded. All together 17 pairs was recorded. The video recordings captured the screens and the students, both from the computer's camera and from a second handheld camera controlled by the first author, who was present during all the sessions.

The example below, is of a pair students, Lilly and Mia, who were described by their teacher as a particularly "talkative" pair, who usually participated with confidence to math class, even though they were not considered to be "the best" students. The tasks was posed and solved in Danish. The task as well as the excerpt have been translated to english for this paper.

We selected this example because of its short length and the fact that it contains many aspects of the process of JM. Indeed, in these 75 seconds the students changed the status of an initial claim from likely to more likely. This episode, therefore, constitutes a unit of analysis.

AN EXAMPLE AND ANALYSIS OF JM

We use the following transcript to illustrate the analytical tool and how it is applied in an analysis of students' justification processes. The two students are working on a task, where they are asked to predict how two given points $A = (1,s)$ and $B = (s,1)$ will move in the coordinate plane in GeoGebra. If the two points are constructed in the algebraic view, a slider for the interval [-5,5] will appear for the variable *s,* the slider can either be dragged or animated, and its movement induces the points to move in the coordinate plane as *s* varies. To ensure that the students predict, rather than construct and animate/drag the slider, the GeoGebra interface in this specific task is limited to the graphics view, showing a coordinate plane along with the cursor, the point tool, and the pen tool. An orange textbox also appears with the coordinates of the given points.

Lilly and Mia make a conjecture about a *line* through *AB* and discuss it, despite the task does not mention any lines. Lilly holds the mouse throughout the excerpt.

1.	Lilly:	[Reads out the task] Show in the coordinate system how you think point *A* and *B* move as *s* changes value.
2.	Mia:	I have the feeling they are making such a slanted line like this (Fig. 1a).
3.	Lilly:	Yes.
4.	Mia:	That is what I imagine.

5. Lilly: If I make a point now called *A* right? [Places point with point tool in (2, 2.98)] So this, this is *A*.

6. Lilly: And then we can say that ehm, *A* is equal to one comma *s*, right? [moves the curser to point at the coordinate sets in the orange text box (Fig. 1b)]

7. Mia: What should *s* be?

8. Lilly: One here, and then *s* could be… [Moves *A* towards (1,0)]

9. Mia: Four.

10. Lilly: Four, so it will be here then [Moves *A* to (1,4) (Fig. 1b) along *x*=1]

11. Mia: Yes.

12. Lilly: Then we do *B*.

13. Mia: [Points to approx. (4,1) with her index finger (Fig. 1c)] Yes, that is what I said, then it becomes such a slanted line.

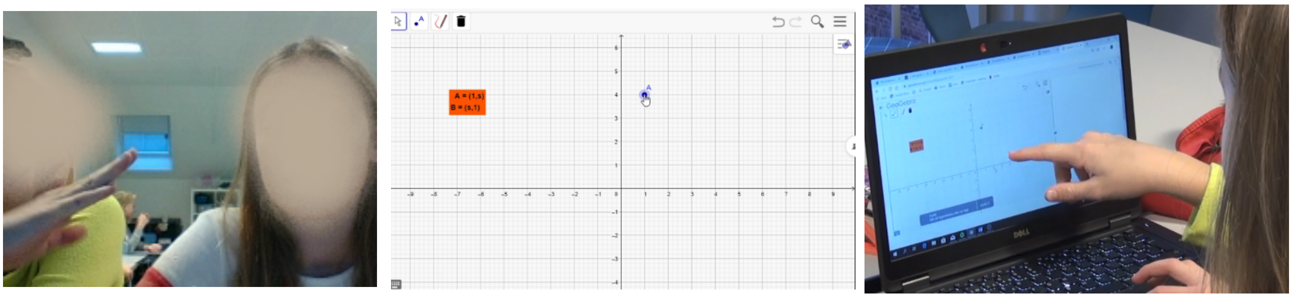

Figure 1. a, b, and c:
a) Mia's gesture, b) screenshot of Lilly's placement of point A, c) Mia points to screen approximately at (4,1)

Analysis of the example

In the analysis, we identify the structural elements and relate them to JM. Figure 2 on the next page visually illustrates Lilly's and Mia's JM process.

A process of JM starts in Lines 2-4 when the following claim (C_1) is stated and gestured: "they [*A* and *B*] are making such a slanted line like this" along with the qualifier "feeling" which indicates likelihood, not certainty. Lilly seems to base her claim on the initial data consisting of the algebraic expressions $A = (1,s)$ and $B = (s,1)$; moreover, she describes the line in her claim through a gesture (Fig. 1a), identifying certain geometrical features of such a line, possibly its "slant". Now the students go on to generate data for the claim using the instrument with the aim of changing the status of the claim, as we are about to show.

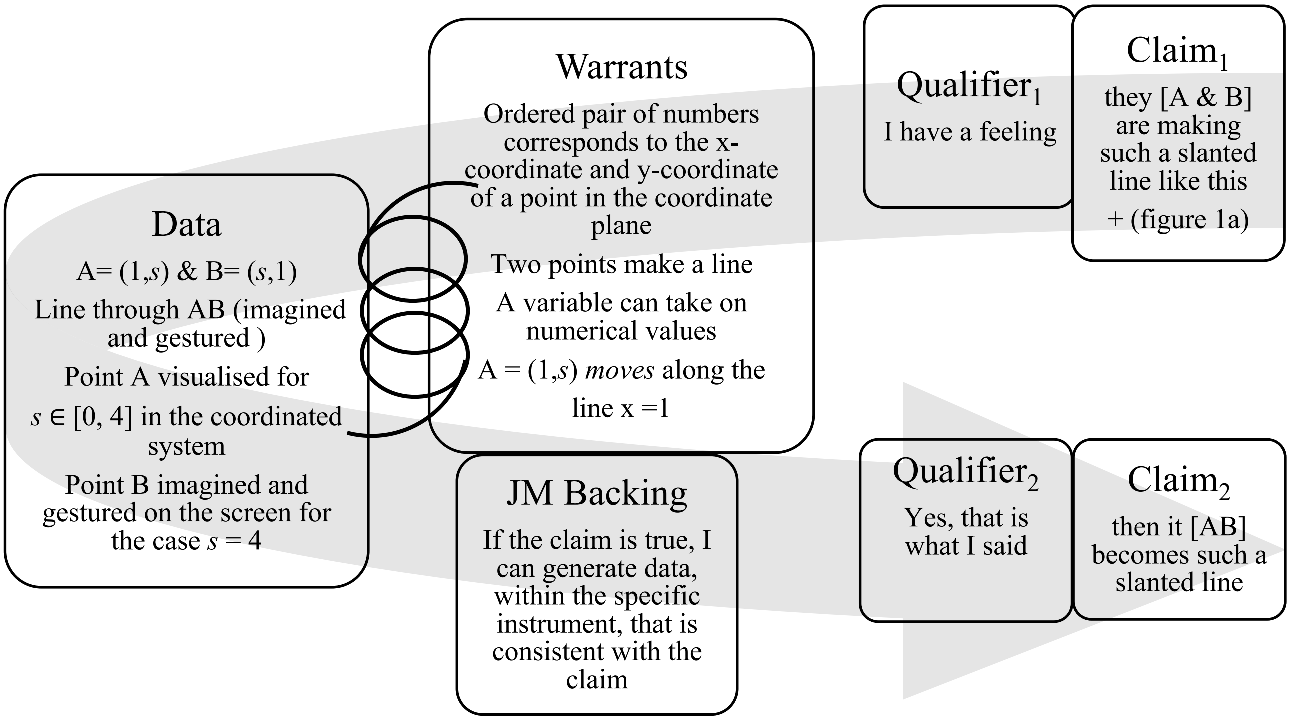

Figure 2: Illustration of Mia's and Lilly's JM process

Throughout lines 5-11 new data is generated by using the instrument. In line 5 and 6, the students create a point and establish the relationship between the algebraic notation of A and the created point. Throughout lines 7-11 data on this relationship is expressed by moving the point on $(1,s)$ from (1,0) to (1,4). On this basis we infer the warrants, schemes, used by the students to connect the data to the claim: an ordered pair of real numbers corresponds to the x-coordinate and y-coordinate of a point in the coordinate plane; two points makes a line; a variable can take on any real number; and $A = (1,s)$ moves along the line $x = 1$. We note that the third warrant depends on the instrument, as the *movement* of points only exists tacit within the instrument. This is an example of how warrants can contain both theoretical elements and phenomenological elements, linking the algebraic domain to the GeoGebra environment, as we discussed earlier. We infer the backing to be what we conjectured: If the claim is true, I can generate data, dependent on the specific, instrument that is consistent with the claim.

In lines 12 and 13, the students generate data regarding point B that is imagined and gestured on account of the same warrant and backing as lines 5-11. In addition, the restatement of the claim in line 13 indicates a change in its status of the claim: the utterance "Yes, that is what I said" suggests that the qualifier has changed from likely to more likely. Overall, to reach the change in status the students drew on their conceptual knowledge, as well as their knowledge about how variables are expressed within the tool. The restatement of the claim and change in its status also concludes a unit of analysis for the process of JM.

CONCLUDING DISCUSSION OF OUR ANALYSIS

In this study we seek to gain insight into informal argumentation processes as part of what we consider the students the reasoning competency and how this interplay with their use of GeoGebra. We do this with a focus on a particular form of mediation - justificational mediation (Jankvist & Misfeldt, 2019), arising from research that combines the KOM-framework and the IA (Geraniou & Jankvist, 2019). Here we designed an analytical tool inspired by Toulmin's model and grounded within the IA. In the following we will discuss and reflect upon the insights we have gained of this endeavour.

The use of Toulmin's argumentation model has allowed us to identify and amplify the importance of the qualifier as indication of change of status of a claim. This has served as a structure for identifying a unit of analysis of what can be considered a processes of JM. This supports that such a mediation is governed by the aim of changing this status of a claim; it has also allowed us to connect the generative aspects and epistemic aspects of schemes (Vergnaud, 2009) to the structure of an argument.

However, there are also limitations with this approach that relate to Toulmin's argumentation model. We do not yet find that this tool appropriately captures the crux of the matter, which is the interplay between theoretical and phenomenological components in students' informal argumentations. Aspects of this interplay can be seen through the notions of scheme and theorem-in-action, that we have adapted to the warrants and backing of the model. This adaptation feels like a long "stretch" with respect to what Toulmin's model has been previously used for in mathematics education (Simpson, 2015). Moreover, we have transformed Toulmin's model into a structure with two claims (or rather a first claim and then its restatement) and two qualifiers, to highlight the process of change in status of the claim and how it occurs. These stretches seem to be leading rather far from the initial model, and we wonder how appropriate it might be to still refer to Toulmin's model at all, also considering a posteriori how we have sort of "substituted" elements from the IA to parts of the model. Moreover, we have not yet been able to explicitly interweave the KOM-framework with the theoretical lenses used. To sum up, has referring to the IA and to Toulmin's argumentation model together supported us in understanding JM? To some extent yes, as it has provided some insight into students' instrumented activity involved in changing the status of a claim; however, it does not yet completely satisfy us.

ACKNOWLEDGMENTS

Supported by Independent Research Fund Denmark [Grant no. 8018-00062B].

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