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Statistical evaluation of the critical distance in the finite life fatigue regime

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Abstract

The procedure to evaluate the critical distance with an optimized V-notched specimen is initially reviewed in the paper. This procedure was devised by the authors, and another numerical methodology was recently proposed to evaluate the uncertainty of the critical distance assessment. The input of the analysis is the combination of the statistical distribution of the fatigue properties from which the critical distance is deduced. After assuming the specimen fatigue strengths as Gaussian (normal) distributions, the critical distance turns out to be well represented by a Skew-normal distribution. This statistical assessment is extended to the finite fatigue life, in the present paper, showing experimental results for the aluminium alloy 7075-T6 at two load ratios. The fatigue strength of other specimens are finally evaluated, reconsidering the critical distance deviation, thus providing a complete uncertainty analysis of the critical distance assessment, and a successful comparison with the experimental scatter is obtained.

Keywords: Critical distance; V-notched specimen; Skew-normal distribution; Probability density function; Finite fatigue life.

1. Introduction

The Theory of Critical Distances (TCD) offers effective engineering criteria for the fatigue and the brittle fracture assessments of any notched component. In order to properly use a method of the TCD, a material length is required, which is the critical distance itself. This length, for the fatigue analysis, can be in principle obtained from the fatigue threshold stress intensity factor range, which is however quite challenging to be experimentally determined. This length can be alternatively obtained on the basis of similar material and/or from data available in the literature. On the other hand, an inverse search determination can be performed by combining the plain specimen and a sharp notched specimen. This approach has been developed by Santus et al. (2018a) and an experimental validation was provided by the same authors Santus et al. (2018b), implementing both the Line Method (LM) and the Point Method (PM).

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Nomenclature

D	V-notched specimen outer diameter.
R	Notch radius of the V-notched specimen.
L	Fatigue critical distance.
$\Delta\sigma_{fl}$	Plain specimen fatigue limit range.
S	Standard deviation of the plain specimen fatigue limit range.
$\Delta\sigma_{N,fl}$	Notched specimen fatigue limit range.
S_N	Standard deviation of the notched specimen fatigue limit range.
N_f	Number of cycles to failure.
$\Delta\sigma$	Plain specimen fatigue strength at N_f.
$\Delta\sigma_N$	Notched specimen fatigue strength at N_f.
R	Fatigue load ratio.
CV	Coefficient of variation.
Σ	Equivalent CV for the critical distance determination.
ν	Normalized CV of the critical distance with respect to Σ.
PDF	Probability density function of the critical distance.
α, β, γ	Shape, location and scale parameters, respectively, of the skew-normal PDF of the critical distance.
μ, δ, sk	Mean value, standard deviation and skewness, respectively, of the critical distance PDF.
R_{lim}	Limit radius (maximum allowed) for an accurate critical distance assessment.

An optimized notched specimen was proposed to have the highest stress gradient at the notch tip, however, still having a well controlled notch radius. A similar procedure was then applied to strain energy density (SED) criterion for the analogous determination of the related length parameter, namely the control radius Benedetti et al. (2019). The same data was interpreted according to the most common multiaxial criteria and the related material parameters were determined by Benedetti and Santus (2018). Moreover, this critical distance determination procedure was applied to the Ti-6Al-4V ELI additively manufactured, via selective laser melting. Two different machining sequences were compared: turned notch from a plain bar, and notch produced as additive, though still finished with turning, Benedetti and Santus (2019).

The approach combining plain and notched specimens, bypassing the fracture mechanics properties, was already proposed in several literature contributions, such as by Cicero et al. Cicero et al. (2012), about the brittle fracture (static) critical distance determination, considering the PM. Moreover, the fatigue threshold and the fracture toughness properties were determined by Susmel and Taylor (2010), initially finding the critical distance length according to this methodology, and then by reversing the critical distance formula.

The proposed procedure for the critical distance determination is summarized in Fig. 1. A specific value of the notch depth was imposed, and just two notch opening angles were considered, namely 60° and 90°, and in the experimental activity 90° was applied. In this way the remaining geometry parameter is just the local notch radius R, which plays a key role in this research. As investigated later, it is indeed recommended to manufacture a quite sharp notch radius. However, it is also required that this radius is well known, even better if measured with a microscope after specimen sectioning.

The authors recently proposed a statistical analysis of this inverse critical distance determination, Benedetti and Santus (2020). The two fatigue strengths of the plain specimen and the (sharply) notched specimen are assumed not just as two deterministic values, but on the contrary two probability distributions. As a consequence, the deduced critical distance features a probability distribution too, which is affected by the uncertainty of both the plain and the notched specimens. This procedure was just applied to the fatigue limit in the previous work, while in the present paper it is extended to the finite fatigue life, allowing a complete statistical assessment of the fatigue strength on the entire high cycle fatigue regime. A similar approach was also applied to the SED criterion by Benedetti et al. (2020) to both the aluminium 7075-T6 and the titanium Ti-6Al-4V ELI additively manufactured, obtaining a quite accurate prediction of the S-N curves and their scatter.

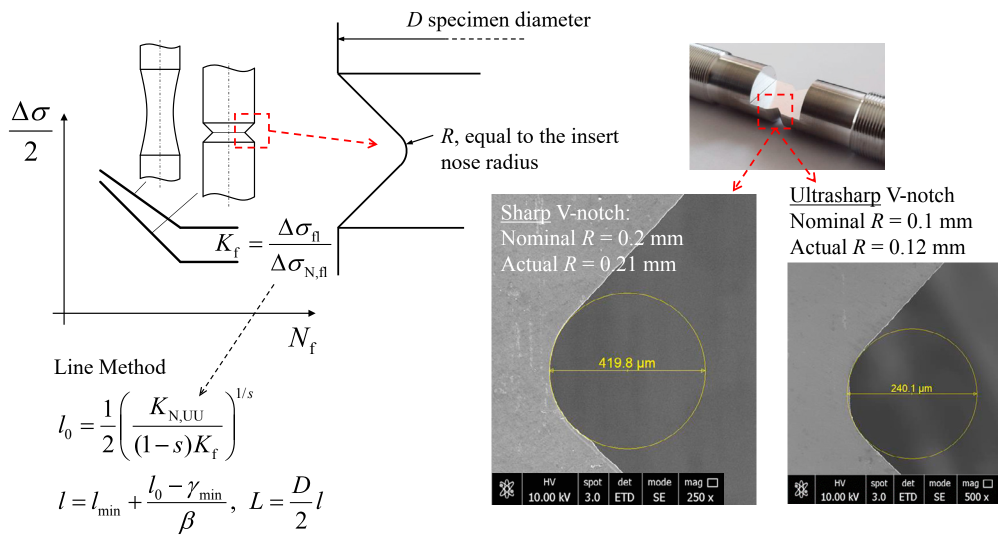

Fig. 1. Summary of the proposed procedure for the critical distance determination, and definition of the specimen sharpness.

2. Statistical analysis of the critical distance determination

As discussed in the Introduction, the proposed critical distance determination requires as input the fatigue limit of the plain specimen and that of the notched (optimized) specimen. It is common to assume the fatigue strength as normal, or Gaussian Lemaire (2009), thus this assumption is applied as well in this research. Under this hypothesis, after running a Monte Carlo simulation campaign, it was found that the derived distribution of the resulting critical distance, can be quite successfully approximated by a Skew-normal distribution (right-skewed), Fig. 2.

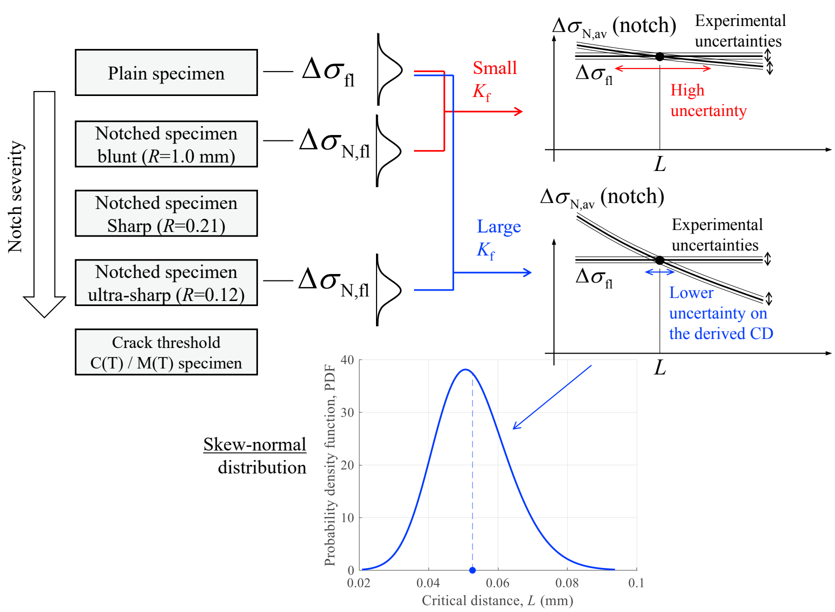

Fig. 2. Combination of plain specimen and V-notched specimen for the critical distance determination, and more accurate assessment with a sharper notch, providing a lower standard deviation of the obtained skew-normal distribution.

The probability density function (PDF) of this distribution can be presented in an analytical form Azzalini and Capitano (2013):

$$PDF(L) = \frac{1}{\sqrt{2\pi}\,\gamma}\left(1 + \text{erf}\left(\frac{\alpha(L-\beta)}{\sqrt{2}\,\gamma}\right)\right)\exp\left(-\frac{(L-\beta)^2}{2\,\gamma^2}\right) \tag{1}$$

From this equation it is evident that the location and scale parameters β and γ, respectively, are of the same unit as the critical distance L (mm), while the shape parameter α is dimensionless. These parameters α, β, γ, can be converted into the mean value μ, the standard deviation δ and the skewness sk, according to the following equations:

$$\mu = \beta + \frac{\sqrt{\frac{2}{\pi}}\,\alpha\,\gamma}{\sqrt{1+\alpha^2}}, \qquad \delta = \gamma\sqrt{1 - \frac{2\alpha^2}{\pi(1+\alpha)}}, \qquad sk = \frac{\sqrt{2}\,(4-\pi)\,\alpha^3}{(\pi + (\pi-2)\,\alpha^2)^{3/2}} \tag{2}$$

The parameters μ and δ similarly share the same unit as the critical distance, while the skewness sk is dimensionless. By following the procedure by Benedetti and Santus (2020), these parameters of the critical distance PDF can be put into relationship with the equivalent coefficient of variation (equivalent CV) which combines the ratios between standard deviation to mean value of the plain and the notched specimens:

$$\Sigma = \sqrt{\frac{(S/\Delta\sigma_{\text{fl}})^2 + (S_{\text{N}}/\Delta\sigma_{\text{N,fl}})^2}{2}} \tag{3}$$

Obviously, the higher the equivalent CV, the larger the critical distance distribution. However, besides the uncertainty of the two fatigue limits, the V-notch radius has a significant role too. As evident in Fig. 2, if a blunt notch is combined with the plain specimen, the local gradient of the stress distribution is relatively similar to the (zero gradient) plain specimen stress. On the contrary, if a sharp notch is employed, or even better a very sharp notch, the stress distribution gradient contrast is higher, and then the shape of the critical distance distribution obtained is narrower. A more reliable assessment of the critical distance is thus obtained in this way, provided that an accurate control of the radius itself can be ensured.

3. Extension of the critical distance statistical assessment to the finite fatigue life

The procedure briefly described above can be easily and effectively applied to the finite fatigue life. Regardless the models taken into account for the finite life strengths of the plane and the notched specimens, the evaluation of the critical distance and its distribution can be obtained for any value of the number of cycles to failure. This is obtained just by combining the fatigue strengths and the related standard deviations of the plain specimen and the sharp (or very sharp) notched specimen, both evaluated at the same number of cycles to failure.

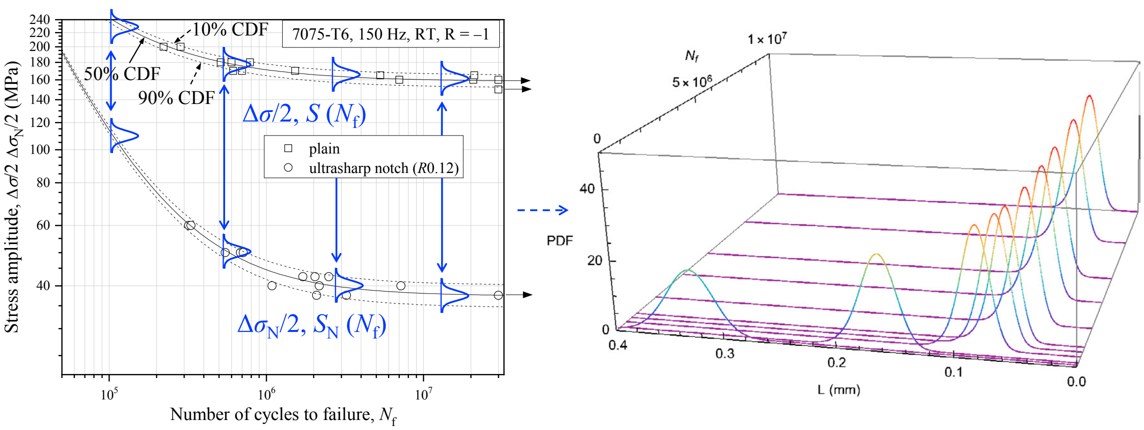

Fig. 3. Determination of the critical distance statistical distributions at several numbers of cycles to failure.

An accurate resolution is easily obtained, since the proposed procedure is quite effective being based on analytical fit models, and thus it can be quickly performed for several N_f values. In Fig. 3 the aluminium alloy 7075-T6 at load ratio R = −1 is presented as an example. A small number of N_f values are pointed out in the figure, for the sake of clarity, while a higher resolution was actually implemented.
The results in terms of the critical distance distribution evolution are reported in Fig. 4 for the same aluminium alloy at ratio R = −1 and also load ratio R = 0.1. In this representation, the shape of the PDF is not shown. The main lines of the cumulative distribution function (CDF) are reported instead, to show both the mean value evolution and its statistical uncertainty, as dependent on the number of cycles to failure. It is worth noting that the critical distance mean value and the 50% CDF are not coincident, because of the skewness of the distribution, however, these two values are actually quite similar.

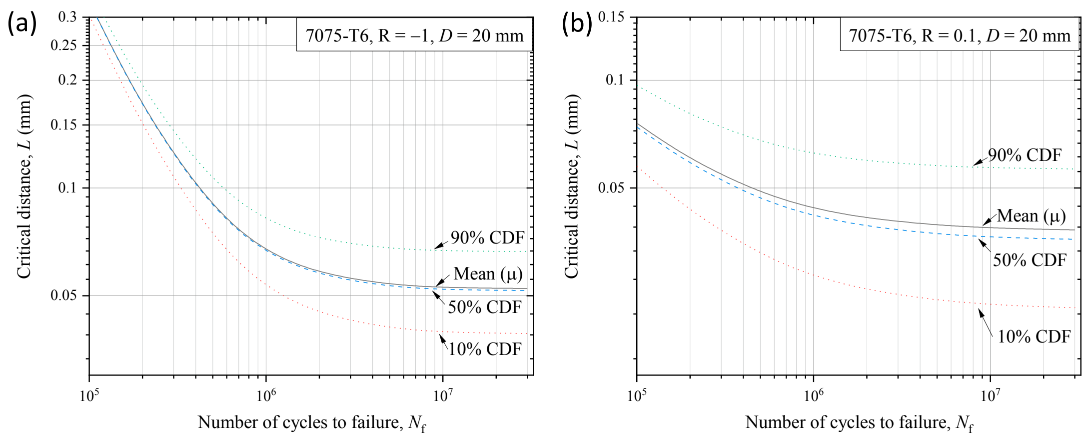

Fig. 4. Critical distance distributions dependent on the number of cycles to failure, for aluminium alloy 7075-T6 at load ratios −1 (a) and 0.1 (b).

The PDF parameters μ, δ, sk are reported in Table 1 for the investigated aluminium alloy, at the two mentioned load ratios, in the high cycle fatigue regime.

Table 1. Critical distance distribution parameters at several numbers of cycles to failure for aluminium alloy 7075-T6 and load ratios −1 and 0.1.

R = −1				R = 0.1			
N_f	μ, mm	δ, mm	sk	N_f	μ, mm	δ, mm	sk
10^5	0.3320	0.02204	0.2135	10^5	0.07583	0.01556	0.6091
2×10^5	0.1658	0.01581	0.2532	2×10^5	0.05997	0.01455	0.6728
5×10^5	0.08682	0.01216	0.2957	5×10^5	0.04840	0.01377	0.7360
10^6	0.06612	0.01113	0.3168	10^6	0.04378	0.01348	0.7688
2×10^6	0.05759	0.01075	0.3297	2×10^6	0.04114	0.01333	0.7913
5×10^6	0.05362	0.01064	0.3386	5×10^6	0.03934	0.01327	0.8098
10^7	0.05271	0.01066	0.3420	10^7	0.03867	0.01326	0.8184
2×10^7	0.05245	0.01069	0.3438	2×10^7	0.03831	0.01327	0.8238
3×10^7	0.05241	0.01071	0.3445	3×10^7	0.03818	0.01328	0.8261

The trends of the mean value, the standard deviation and the skewness are also graphically reported in Fig. 5, for the two load ratios investigated, in a normalized form after dividing by the maximum value. The general trend of the mean value is descending with the number of cycles to failure. Thus the critical distance is larger for lower fatigue lives, and this is in agreement with other data in the literature. A similar trend can also be observed for the standard deviation, and the ratio δ/μ is asymptotically increasing with the number of cycles to failure. The skewness also follows an increasing trend, which means that at lower lives the critical distance shape is more resembling a normal distribution. Lastly, by comparing the two load ratios, it is quite evident that for R = 0.1 the variations are lower, which means that the distribution is more uniform.

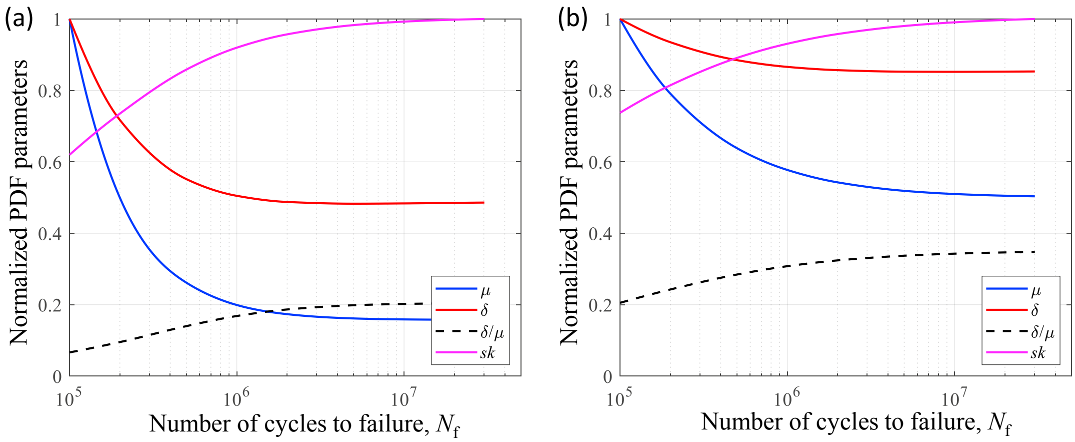

Fig. 5. Trends of the PDF parameters for the aluminium alloy 7075-T6 and load ratio R = −1 (a) and 0.1 (b).

3.1. Minimum notch radius required for accurate critical distance assessment

A normalized coefficient of variation (NCV) is considered a key factor by Benedetti and Santus (2020) defined as the ratio between the mean over standard deviation, of the critical distance, and the equivalent CV of the two fatigue strength input values:

$$\nu = \frac{\delta/\mu}{\Sigma} \tag{4}$$

This ratio defines how the critical distance assessment is affected by the introduced specimen combination procedure, regardless the experimental standard deviations of the two specimens. It was found that ν depends on the notch radius and also on the critical distance itself. The best scenario is a specimen with a very sharp notch (small radius) to assess a large critical distance. On the other hand, a certain value of this NCV can be prescribed, and then an appropriate radius is obtained to be intended as the limit value, not to be exceeded. This analysis is again extended in this work to the entire high cycle fatigue regime, for the aluminium alloy 7075-T6 at the two mentioned load ratios. As evident in Fig. 6, the lower the value of ν, the smaller the limit radius. Moreover, for lower N_f values this maximum allowed radius is larger, so less demanding from the manufacturing side, since the critical distance to be assessed is higher.

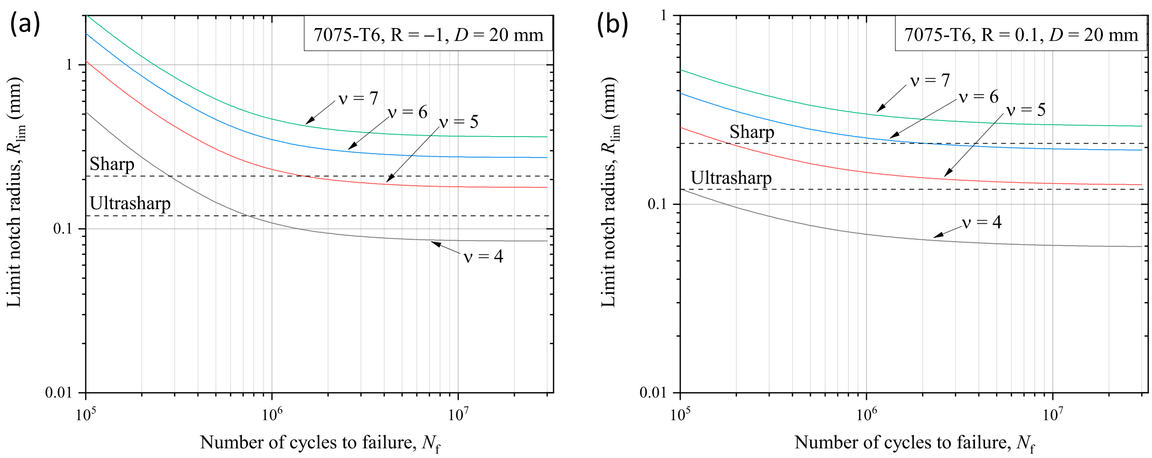

Fig. 6. Limit radius required for prescribed NCV values, to be intended as maximum, dependent on the number of cycles to failure, for load ratio R = −1 (a) and 0.1 (b).

3.2. Strength assessment validation on blunter notches

A final validation is provided by proposing the assessment of the fatigue strength, on the entire high cycle fatigue regime, of notched specimens which are blunter than that considered for the critical distance determination and its distribution. This validation takes into account not only the statistical distribution of the plain specimen, but also the statistical properties of the critical distance, as deeply investigated in this paper, instead of being considered just a single value dependent on N_f. The results of this analysis are reported in Fig. 7, again for the two load ratios R = −1 and R = 0.1, and for the notch radii R = 1.0 mm and R = 0.21 mm. This latter specimen radius, shown in Fig. 1 and referred to as Sharp, could be alternatively considered for the critical distance determination itself. However, as described above, a lower NCV is obtained with the Ultra-sharp specimen with a lower radius. The prediction of a sharp specimen is quite challenging, because with a high stress gradient, the uncertainty on L is more affecting the assessment result. As evident in Fig. 7, the mean value trend and the scatter of the experimental data is well reproduced for both specimens and both load ratios, except for the sharp specimen at R = −1, for which this calculation provides a slight underestimation of the observed fatigue strength.

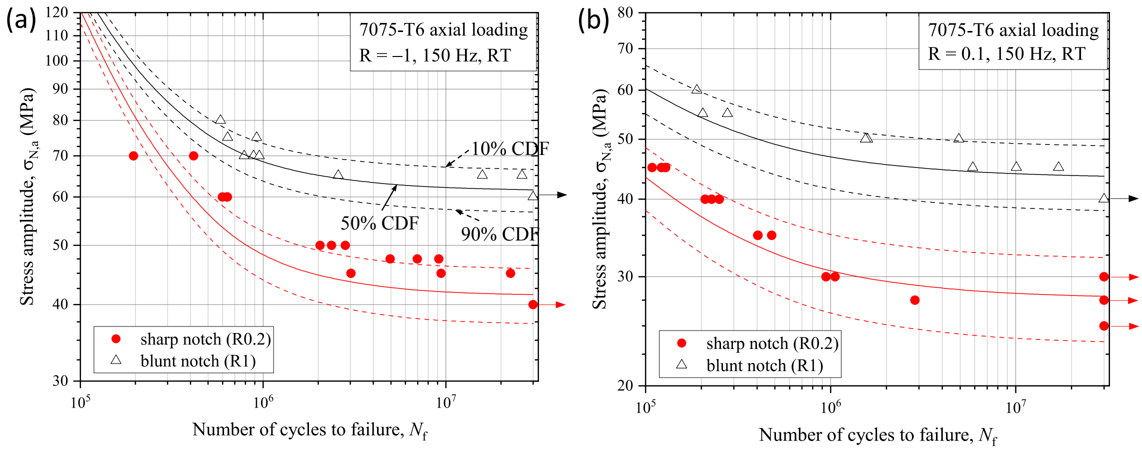

Fig. 7. Fatigue strength assessment of validating notched specimens, sharp and blunt, at load ratios R = −1 (a) and R = 0.1 (b).

4. Conclusions

In this paper the critical distance is no longer considered a single value, or just a deterministic function dependent on the number of cycles to failure, while it is treated as a statistically distributed variable. The distribution of the critical distance is found by combining the plain and the notched specimen strengths, assumed as normal distributions, and a skew-normal is obtained. The trend of this distribution is obviously dependent on the material and the fatigue load ratio. However, as a general remark, the mean value significantly increases for the lower numbers of cycles to failure, while the standard deviation and the skewness are relatively more uniform. In particular, the skewness reduces for lower fatigue lives. A final validation is provided on blunter specimens, confirming that taking into account the statistical property of the critical distance is a correct approach for the assessment of the fatigue strength and its scatter.

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