# Online Estimation of Continuous Gait Phase for Robotic Transtibial Prostheses Based on Adaptive Oscillators

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Abstract—Continuous gait phase plays an important role in wearable robot control. This study focuses on the online estimation of continuous gait phase based on robotic transtibial prosthesis signals. First, we adopt the prosthetic foot deformation information to detect the heel strike as the start timing (reset 0 rad) of one gait cycle. Then we conduct the gait phase estimation based on adaptive oscillators using the prosthetic shank angle signal as input. Three transtibial amputees were recruited in this study and they walked on the treadmill at different speeds (slow, normal and fast) and on different ramps  $(10^{\circ}, 5^{\circ}, 0^{\circ}, -5^{\circ})$  and  $(10^{\circ})$  in the experiment. The root-meansquare error between online estimation result and ground truth gait phase is calculated. The maximum and minimum errors are 0.147 rad and 0.058 rad, and the corresponding ratios in one gait cycle are 2.34% and 0.92%. This study achieves good performance and provides an effective method to estimate the continuous gait phase, which will instruct robotic transtibial prosthesis control.

## I. INTRODUCTION

Lower-limb robotic prostheses can help to improve the life quality for amputees by assisting their daily activities. In robotic prosthesis research, the control of prosthesis is one critical issue and has attracted a lot of attentions. The widely used control strategy for robotic prostheses is based on finite state machine [1]–[3]. This strategy divides each gait cycle into several discrete states, such as swing phase and stance phase, and then the output torque is formulated mapping from joint angle, velocity, *etc.* [4], [5]. Based on finite state machine, some studies have achieved a lot of improvements in walking metabolic economy [6], speed adaptation [7] and so on. Finite state machine control relies on the detection of gait events (heel strike, toe off and so on) to trigger state transitions, which is difficult to synchronize the correct sequence of discrete gait events over time to accurately

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detect the transition timing between two different gait phases, especially when the gait is disturbed [8]. Besides, the divided states may limit the smoothness of assistance control [8], [9]. All these can cause performance decline in prosthesis control.

To solve these issues and improve the lower-limb prosthesis control, researchers have developed some alternative methods, for example, neuromuscular controller [10]. The neuromuscular controller uses models of muscle dynamics and hypothesized reflexes, but it brings many parameters which are difficult to tune. Another alternative control method is based on the estimation of continuous gait phase [11]. Quintero et al. have used an adaptive Kalman filter based on Newton's and Euler's equations of motion to compute real-time Euler angles to conduct continuous-phase control of a powered knee-ankle prosthesis and has achieved some effects [12]. However, the phase estimation results still need further improvement. Seo et al. have conducted online continuous gait phase estimation to control ankle exoskeletons, but the estimation performance also need further improvement [13]. Adaptive oscillators (AOs) have also been used to continuous gait phase estimation on healthy people and exoskeleton [14], [15], and AOs have shown better performance than Kalman filter and RNN methods, since its inherent synchronization properties provided advantages in continuous gait phase estimation.

In order to extend the continuous gait phase control in robotic prosthesis with better performance, we conducted the study aiming at the online estimation of continuous gait phase based on adaptive oscillators for robotic prosthesis users. In this study, we used strain gauge to record the deformation information of prosthetic carbon-fiber foot to detect heel strike as the start point (reset 0 rad) of one gait cycle at first. Then, prosthetic shank angle signals (measured by inertial measurement unit (IMU)) were used as the input of AOs to conduct continuous gait phase estimation based on the detected gait event.

In this paper, we first introduce the related studies and research progress in Section I. Then, we introduce the robotic prosthesis, experimental protocol, signal processing and evaluation method in Section II. At last, result and conclusion are presented in Section III and IV, respectively.

#### II. MATERIALS AND METHODS

# A. Robotic Transtibial Prosthesis

1) Prosthesis Prototype: We use one commercialized robotic transtibial prosthesis (developed by Peking Univer-

sity) for this study. The prosthesis consists of one control circuit, one strain gauge, one IMU, one angle sensor and battery. Its weight is about 2 kg. More details can be found in [2], [5], [16]. The prototype of prosthesis and wearing diagram can be seen in Fig. 1.



Fig. 1. The prototype of robotic transtibial prosthesis and wearing diagram.

One full bridge of strain gauge is integrated in prosthetic carbon-fiber foot to record the deformation of carbon-fiber foot, as shown in Fig. 1. During stance phase interaction is between the prosthetic carbon-fiber foot and ground. During swing phase, no interaction is recorded between the prosthetic foot and ground. Therefore, strain gauges can be a viable solution to discriminate between swing and stance phases [17]. Control strategies of prosthesis are performed based on different gait phases and phases [2], [17]. Position control is adopted in swing phase and torque control strategy is adopted in stance phase. An angle sensor placed at the rotational joint of prosthesis is used to measure the prosthetic ankle angle. IMU can provide inclination angle (yaw, pitch and roll), tri-axis angular velocity and tri-axis acceleration information. The DC motor is used to drive the prosthesis with power of 150 W.

# B. Experimental Protocol

Three people with transtibial amputations were recruited in the experiments as subjects, and their detailed information are listed in Table I. In this study, the subjects wear their customized prosthetic sockets which would be mounted on the designed robotic prosthesis by adapters. Each subject will do some exercises before the formal experiments to adapt to the robotic prosthesis. All subjects have signed written informed consents and this study has been approved by the Local Ethics Committee of Peking University.

The designed experiment was comprised of two sessions in the study. The first experiment aimed at assessing continuous gait phase estimation at different walking speeds (speed experiment). The subjects were asked to walk on the treadmill at their self-selected three speeds (slow, normal and fast), as shown in Table II, and online estimation of continuous gait phase was conducted at the same time. The second experiment was to conduct continuous gait phase

estimation on different ramps (ramp experiment), as shown in Table II. All subjects walked on the treadmill with the ramps of different inclination angles (10°, 5°, 0°, -5° and -10°) at their normal walking speeds, as listed in Table II. Inclination angles (10° and 5°) were corresponding to ramp ascending, inclination angles (0°) were corresponding to level-ground walking and inclination angles (-5° and -10°) were corresponding to ramp descending. The online estimation of continuous gait phase was conducted while subjects were walking on ramps. In this study, each subject walked for about 3 minutes under each walking conditions and the estimation of continuous gait phase was conducted meanwhile.

TABLE II
THE DESIGNED EXPERIMENTAL TASKS.

		Speed/Inclination			
Speed Experiment		Sa/0°	N <sup>b</sup> /0°	F <sup>c</sup> /0°	
Ramp Experiment	N/10°	N/5°	N/0°	N/-5°	N/-10°

<sup>&</sup>lt;sup>a</sup> S denotes slow speed.

## C. Gait Phase Estimation Algorithm

We used Matlab platform to acquire real-time signals data from robotic prosthesis in wireless way and conduct online estimation of continuous gait phase at the same time. The sample frequency was 100 Hz. The framework of gait phase estimation was shown in Fig. 2, and it was comprised of (1) adaptive oscillators and (2) gait event detector. IMU could provide the prosthetic shank angle relative to the ground, and the angle signal was the input of AOs. The deformation information of prosthetic foot recorded by strain gauge was used to detect gait event (i.e. heel strike), which was used to reset the gait phase to 0 rad (i.e. the start of one gait cycle). First we would introduce the adaptive oscillators.

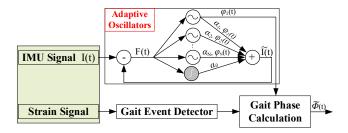


Fig. 2. The framework of continuous gait phase estimation based on adaptive oscillators and gait event detector. The IMU (prosthetic shank angle) and strain gauge signals are input to adaptive oscillators and gait event detector, and the final outputs are the estimated gait phases. Adapted from [9], [18].

Adaptive oscillators have been widely used in cyclical movements [19], for example gait phase estimation in some studies [9], [14]. In Fig. 2, the input signal (I(t)) of AOs were prosthetic shank angle measured by IMU and the output (I(t)) was the estimated prosthetic shank angle. The error

N denotes normal speed.

<sup>&</sup>lt;sup>c</sup> F denotes fast speed.

 ${\bf TABLE~I}$  Information of Three Transtibial Amputees as Subjects

	Gender	Age	Weight (kg)	Height (cm)	Years post-amputation	The amputation side	Residual limb length ratio
Subject 1	Male	30	72	171	9	Right	33%
Subject 2	Male	53	70	170	17	Left	40%
Subject 3	Male	56	81	170	10	Left	32%

F(t) (F(t)=I(t)-I(t)) between the output and input drives the evolution of the oscillator [9]. The I(t) could be calculated as follow:

$$\tilde{I(t)} = \alpha_0(t) + \sum_{i=1}^{N} \alpha_i(t) sin(\varphi_i(t))$$
 (1)

where i denoted the  $i_{th}$  harmonic (i=1,2,...,N and N=25 in the study). The other variables in Fig. 2 were as follows:

$$\dot{\varphi}_i(t) = \omega(t) \cdot i + \lambda F(t) \cos(\varphi_i(t)) \tag{2}$$

$$\dot{\omega}(t) = \gamma F(t) \cos(\varphi_1(t)) \tag{3}$$

$$\dot{\alpha}_i(t) = \eta F(t) sin(\varphi_i(t)) \tag{4}$$

$$\dot{\alpha}_0(t) = \eta F(t) \tag{5}$$

where the  $\lambda$ ,  $\gamma$  and  $\eta$  were the learning rates corresponding to phase  $(\varphi_i(t))$ , frequency  $(\omega(t))$  and amplitude  $(\alpha_i(t))$ .

For the lower-limb locomotion, we defined the continuous gait phase corresponding to the interval  $[0, 2\pi)$  rad linearly. The acquired phase  $\varphi_1(t)$  (in Fig. 2) based on AOs were normalized into the interval  $[0, 2\pi)$ , and the normalized phase was denoted as  $\varphi_{nor}(t)$  (in Fig. 2):

$$\varphi_{nor}(t) = mod(\varphi_1(t), 2\pi) \tag{6}$$

For the continuous gait phase estimation, we use the gait event (heel strike) as the start point, namely the 0 rad timing point, and the heel strike could be detected according to the deformation information of prosthetic foot.

The 0-rad phase should be matched with each heel strike at timing  $t_k$  in gait cycles, so there might exist phase error  $\tilde{e}(t_k)$  between the estimated phase  $\varphi_{nor}(t)$  at  $t_k$  and 0. The final estimated gait phase  $\widetilde{\Phi}(t)$  could be denoted as follow:

$$\widetilde{\Phi}(t) = \begin{cases} \varphi_{nor}(t) - \widetilde{e}(t_k), & \varphi_{nor}(t) - \widetilde{e}(t_k) > 0 \\ \varphi_{nor}(t) - \widetilde{e}(t_k) + 2\pi, & \varphi_{nor}(t) - \widetilde{e}(t_k) < 0 \end{cases}$$

Gait phase increases forward within one gait cycle, so we can revise the current estimated gait phase by comparing with the last estimated gait phase(s) to make sure the monotonic increasing feature of gait phase forward in each gait cycle.

## D. Evaluation Method

The root-mean-square error  $(\theta_{rms})$  between the estimated phase and the actual phase is used to evaluate the continuous

gait phase estimation performance, which can be formulated as follows:

$$\theta_{rms} = \sqrt{\sum_{i=1}^{m} \frac{(\widetilde{\Phi}(i) - \Phi(i))^2}{M}}$$
 (8)

where m denotes the sample number in one gait cycle,  $\widetilde{\Phi}(i)$  denotes the estimated  $i_{th}$  gait phase and  $\Phi(i)$  is the actual  $i_{th}$  gait phase (ground truth value) in one gait cycle. A small  $\theta_{rms}$  can reflect good online estimation performance for continuous gait phase. Besides, we also introduce the ratios (R) of the root-mean-square error  $(\theta_{rms})$  in one gait cycle. The R can be calculated as follows:

$$R = \frac{\theta_{rms}}{2\pi} \times 100\% \tag{9}$$

where  $2\pi$  is the gait phase length of one gait cycle.

#### III. RESULTS

#### A. Gait Event Detection

The deformation of prosthetic foot during gait cycle had some features. During stance phase, there existed interaction between the prosthetic foot and ground, and the strain gauge signal varied, as shown in Fig. 4. When the prosthesis was in swing phase, the strain gauge signal varied little as shown in Fig. 4. The heel strike of prosthesis was corresponding the transition timing point from swing phase to stance phase. By analyzing the stain gauge signals, we could detect the gait event (heel strike).

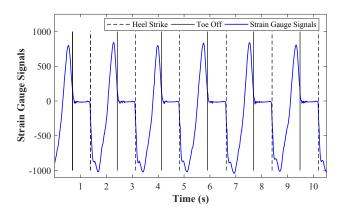


Fig. 4. The strain gauge signals (deformation of robotic transtibial prosthetic carbon-fiber foot) during level-ground walking. The black dashed and solid lines denote the gait events: heel strike and toe off, respectively. Data come from subject 2 who walks on the level ground (treadmill) at his normal walking speed.

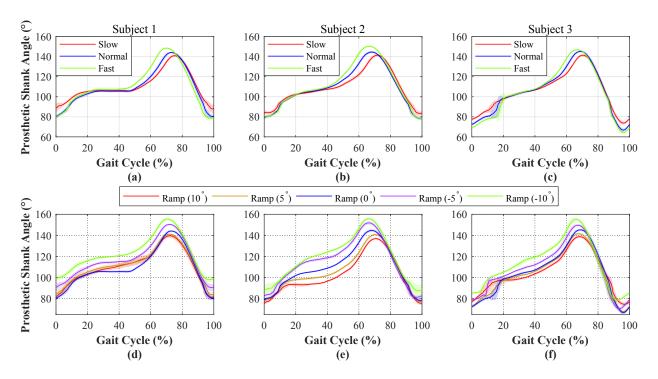


Fig. 3. The normalized angle signals of prosthetic shank relative to the ground for subjects walking at different speeds and on different ramps. (a)  $\sim$  (c): Normalized angle signals at slow, normal and fast speeds for level-ground walking corresponding to subject 1, 2 and 3. (d)  $\sim$  (f): Normalized angle signals on different ramps (inclination angle:  $10^{\circ}$ ,  $5^{\circ}$ ,  $0^{\circ}$ ,  $-5^{\circ}$  and  $-10^{\circ}$ ) at normal walking speeds corresponding to subject 1, 2 and 3. The colorful solid lines denote the mean values across 50 gait cycles, and the corresponding shade areas denote the standard deviations across 50 gait cycles.

## B. Normalized Prosthetic Shank Signals

The locomotion of lower-limb is periodical and quasiperiodical, which provides possibility to estimate the continuous gait phase estimation. In this study, one IMU was integrated in the prosthesis to record the locomotion information of prosthesis. The prosthetic shank angle (measured by IMU) relative to the ground was input to AOs to conduct online continuous gait phase estimation. In our study, three subjects were asked to walk at different speeds and on the ramps of different inclination angles. During the experiments, the prosthetic shank angle were recorded, and the normalized prosthetic shank angle could be seen in Fig. 3. Angle signals of prosthetic shank in gait cycle for the three participants were different from each other, as shown in Fig. 3(a)  $\sim$ (c) and (d)  $\sim$  (f). Angle signals of prosthetic shank were periodical and had small standard deviations which were shown as the shade areas in Fig. 3.

# C. Continuous Gait Phase Estimation

Adaptive oscillators were used to estimate the gait phase. Three main parameters of adaptive oscillators:  $\lambda$ ,  $\gamma$  and  $\eta$  corresponding to phase  $(\varphi_i(t))$ , frequency  $(\omega(t))$  and amplitude  $(\alpha_i(t))$ , were set 0.05, 0.04 and 2.5 for all the subjects initially during their walking at different speeds and on different ramps. The diagram of estimated gait phase and ground truth gait phase could be seen in Fig. 5.

The root-mean-square error  $\theta_{rms}$  between the online estimation result and the ground truth gait phase was shown in Fig. 6. For subject 1, the errors for each subjects walking at

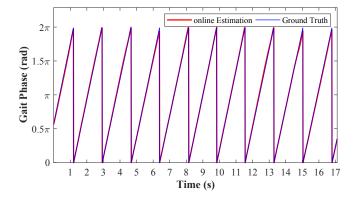
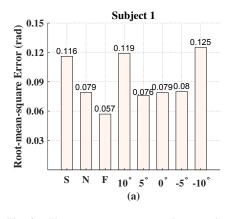
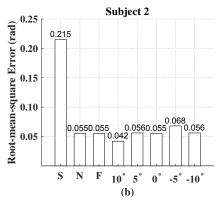


Fig. 5. The diagram of online estimation of continuous gait phase and the ground truth gait phase. One gait cycle is corresponding to the range of gait phase  $[0, 2\pi)$ , which is also the ground truth gait phase. The red and blue solid lines denote the online estimation and the ground truth gait phases, respectively. The gait cycle starts at heel strike corresponding to 0 rad of gait phase. Data come from subject 2 who walks on the level ground (treadmill) at his normal walking speed.

different speeds and on different ramps ranged from 0.079  $\sim 0.125$  rad, as shown in Fig. 6(a). For subject 2, the errors ranged from 0.042  $\sim 0.215$  rad, as shown in Fig. 6(b), and for subject 3, they ranged from 0.061  $\sim 0.11$  rad, as shown in Fig. 6(c). For the three subjects, the maximum and minimum errors were 0.215 rad and 0.042 rad corresponding to the subjects 2's slow speed and ramp (10°), respectively. The ratios were 3.42% and 0.67% of one gait cycle (2 $\pi$ ).

The average and standard deviation (SD) of root-meansquare errors and ratios between the online estimation results





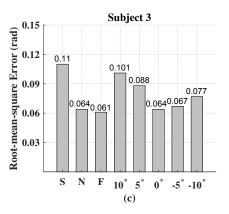


Fig. 6. The root-mean-square error between the online estimation result and the ground truth gait phase. (a)  $\sim$  (c) denote root-mean-square errors for subject 1  $\sim$  3. The text on the bar (and the bar's height) denotes the root-mean-square error value. The horizontal axis denotes the different walking speeds (S denotes slow speed, N denotes normal speed and F denotes fast speed) and ramps (inclination angle:  $10^{\circ}$ ,  $5^{\circ}$ ,  $0^{\circ}$ ,  $-5^{\circ}$  and  $-10^{\circ}$ ).

and the ground truth gait phases were listed in Table. III. The errors were 0.147  $\pm$  0.059 rad, 0.066  $\pm$  0.012 rad, 0.058  $\pm$  0.031 rad, 0.087  $\pm$  0.040 rad, 0.073  $\pm$  0.016 rad, 0.066  $\pm$  0.012 rad, 0.072  $\pm$  0.007 rad and 0.086  $\pm$  0.035 rad corresponding to walking at different speeds and on different ramps, as shown in Table. III. The ratios were also listed in the right column of Table. III.

TABLE III THE ROOT-MEAN-SQUARE ERROR  $(\theta_{rms})$  (Mean  $\pm$  SD) and ratio (R) (Mean  $\pm$  SD) between the online estimation and the ground truth gait phase.

		$\theta_{rms}$ (rad)	R (%)
	Slow	$0.147 \pm 0.059$	$2.34 \pm 0.94$
Speed	Normal	$0.066 \pm 0.012$	$1.05\pm0.19$
	Fast	$0.058 \pm 0.031$	$0.92\pm0.05$
	10°	$0.087 \pm 0.040$	$1.39 \pm 0.64$
	5°	$0.073\pm0.016$	$1.17 \pm 0.26$
Ramp	0°	$0.066 \pm 0.012$	$1.05\pm0.19$
	-5°	$0.072\pm0.007$	$1.14 \pm 0.12$
	-10°	$0.086 \pm 0.035$	$1.37 \pm 0.56$

The maximum and minimum errors (and ratios) were 0.147 rad (2.34%) and 0.058 rad (0.92%) corresponding to slow and fast speeds, respectively. Except the slow speed condition, we could achieve errors no more than 1.40%. The results of the study are comparable with the study [14], who have conducted the gait phase estimation based on AOs on healthy people. Compared with the study [11] which adopted the extended Kalman filter to estimate gait phase and study [13] which adopted RNN-Based method to estimate gait phase, our results show better performance. In addition, AOs have little parameters to tune, and the parameters was adaptive for different subjects.

Though the study has got some preliminary results and obtained good performances, we still need to deepen and extend the study. As we can see this study focuses more on online estimation of continuous gait phase, next we need to combine continuous gait phase estimation with the prosthesis

control to provide improvements for the amputee wearers next, and more transtibial amputees need to be recruited in the future study to analyze the effect and robustness.

# IV. CONCLUSION

The study focuses on the online estimation of continuous gait phase for robotic transtibial prosthesis. First, we adopted the prosthetic foot deformation information to detect the heel strike as the start points (reset 0 rad) of one gait cycle. Then we conducted the gait phase estimation based on adaptive oscillators. The study were conducted on three transtibial amputees walking at different speeds (slow, normal and fast) and on different ramps (10°, 5°, 0°, -5° and -10°) to validate the study's feasibility. The maximum and minimum root-mean-square errors were 0.147 rad and 0.058 rad, and the corresponding ratios in one gait cycle were 2.34% and 0.92%. This study provide an effective method to estimate the continuous gait phase for robotic transtibial prosthesis users.

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