# Performance Improvement for SAR Tomography Based on Local Plane Model 

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#### Abstract

Multilook approaches have been applied in synthetic aperture radar (SAR) tomography (TomoSAR), for improving the density and regularity of persistent scatterers reconstructed from multipass SAR images in both rural and urban regions. Multilook operations assume that all scatterers in a given neighborhood are similar in height, thereby providing additional data for recovering the position and reflectivity of a single scatterer, so that a higher signal-to-noise ratio can be achieved. This is equivalent to assuming that scatterers belonging to a local neighborhood of range-azimuth cells are located on horizontal planes. The present article generalizes this approach by adopting the so-called local plane (LP) model for TomoSAR imaging in urban areas, accounting for local variations in the height of scatterers that are not negligible. Furthermore, an LP-generalized likelihood ratio test (LP-GLRT) algorithm is developed to implement the previous idea. Compared with the multilook generalized likelihood ratio test algorithm, LPGLRT shows better performance in the case of urban structures and terrains in experiments based on both simulated data and TerraSAR-X images.


Index Terms-Generalized likelihood ratio test (GLRT), local plane (LP), multilook, synthetic aperture radar (SAR) tomography (TomoSAR).

## I. Introduction

SYNTHETIC aperture radar (SAR) tomography (TomoSAR) is an effective tool for unmixing overlapped reflectivity contributions in the same range-azimuth cells. It fully resolves the three-dimensional (3-D) position of the detected scatterers by exploiting the information inside multipass images over the same scene [1]-[4]. This technique and its extensions to higher dimensional cases [5], [6] have been implemented for several applications, including SAR imaging of urban and forested areas [7]-[12] and deformation monitoring

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[6], [13], [14]. Although TomoSAR can locate the position of scatterers in the 3-D space, the estimated results may be inaccurate owing to several factors, such as low signal-to-noise ratios (SNRs), low elevation resolutions, high sidelobes in the reconstructed elevation profile, and the decorrelation phenomenon, present in practical tomographic imaging because of spatiotemporal diversities in acquisitions [1], [2].

Different approaches have been introduced for solving the aforementioned issues and improving the reconstruction performance of the ground scene from multipass SAR images. These approaches exploit advanced statistical processing techniques which consider each pixel of the image stack independently from the others (single-look techniques), including the methods based on the generalized likelihood ratio test (GLRT), i.e., single-look GLRT (SL-GLRT) [15]-[18]. GLRT methods test the image pixels for detecting single and multiple scatterers, and determine the maximum likelihood (ML) estimates of their elevation and reflectivity.

To improve the point cloud density and further increase the position estimation accuracy, available a priori and contextual information can be considered [19]-[22]. In [20], a regularized approach using the Capon filter was proposed, which restricts the scene height variation by introducing an additional term in the array processing chain. In [21], an iterative method involving spatial regularization in the compressed sensing processing framework was proposed, imposing the spatial regularity of the reconstructed tomogram. Another method based on compressive sensing, which exploits the correlation between neighboring azimuth-range pixels and polarimetric channels, to achieve high super-resolution imaging and elevation estimation accuracy was presented in [22].

Multilook operations have been advantageous in promoting efficient detection in both rural and urban regions [23]-[25]. In rural regions, multilook is implemented to improve the coverage of monitored area, which is characterized by the presence of typically weak and decorrelating scattering mechanisms. Component extraction and selection SAR (CAESAR) [23] is a multilook-based method inspired by the SqueeSAR algorithm [26], which was developed for monitoring distributed scatterer deformation. In CAESAR, the principal component analysis is performed on the covariance matrix of the multilook data to filter interferograms or separate different scattering mechanisms. Other tomographic approaches exploit polarimetric data to separate different scattering mechanisms in
urban areas for examining the physical properties of reflectors [27]-[29].

The multilook GLRT (MGLRT) [24] has been introduced for the detection and monitoring of weak scattering at close-to-full resolution in urban areas. In this case, the multilook detection scheme (for weak scatterers) acts by increasing the probability of detection $\left(P_{D}\right)$ for a fixed probability of false alarm $\left(P_{\mathrm{FA}}\right)$. It achieves an improved detection efficiency of PSs with constant false alarm rates compared with SL-GLRT approaches [15], [16].

Multilook operations assume that all scatterers in a given neighborhood (looks) exhibit the same height. However, this assumption may fail in some special scenarios, such as sloped facades or roofs of buildings with evident height variations. In these cases, directly implementing multilook operations may fail in terms of detection, resulting in deteriorated reconstruction accuracy.

In this article, we analyze the performance of a tomographic method that exploits a parametric model of the unknown ground surface profile to improve the scatterer detection rate and reconstruction accuracy. In particular, a local plane (LP) model of the surface, which was already adopted to improve the performance of phase unwrapping (PU) for SAR interferometric processing [30], [31], is proposed.

Specifically, we focus on urban scenarios and extend the LP model from PU to TomoSAR, because building surfaces in urban regions can be modeled locally using planes in different orientations and extensions. By introducing this geometrical characteristic of urban scenarios in the processing algorithm, tomographic reconstructions are implicitly regularized, by introducing contextual information, thereby improving the performance of traditional approaches.

Contextual information may consist in applying deterministic or statistical constraints between nearby pixels [32]. As far as deterministic constraints are concerned, pixels are jointly processed in clusters, in which some geometrical relations are imposed. In particular, we impose the nearby pixels to belong to a plane [30]. The resulting tomographic reconstruction satisfying such constrain can be considered implicitly regularized.

By combining the LP estimation with GLRT methods, scatterers can be detected with improved efficiency and accuracy.

The proposed LP-GLRT method can be regarded as a generalization of the multilook methods, adaptable to more complex and general urban environments. Moreover, the proposed method is based on estimating the parameters of LP that best approximates the true urban geometry. Such parameters are estimated using the ML estimator, which is minimum variance and an asymptotically efficient, providing the best matching of the data model to the measured data.

The rest of this article is organized as follows. Section II presents the TomoSAR signal model and briefly introduces the GLRT tomographic method. Section III describes the surface model for multilook and LP representation. Section IV shows some results on simulated and real data. Section V presents conclusions, and the appendix section presents the derivation of GLRT.


Fig. 1. Range-elevation geometry, fixing the $x$ coordinate, for a single-antenna SAR configuration. No scatterer is present in $(r-x)$ cell A; one scatterer is present in $(r-x)$ cell B. The scatterer in $(r-x)$ cell B cannot be located along $s$.

## II. Tomosar Signal Model

TomoSAR fully recovers the 3-D reflectivity profile and exact position of the scatterers on the ground surface based on multibaseline data. In most TomoSAR techniques, the range-azimuth $(r-x)$ resolution cell is fixed and the 1-D reflectivity and height profiles along the direction orthogonal to the range $r$ and azimuth $x$ are estimated for each cell, which is denoted as elevation $s$.

To briefly illustrate TomoSAR principles, consider the geometry shown in Fig. 1. In this geometry, the 2-D profile in the $(r, s)$ plane of a 3-D scenario is obtained by fixing the $x$ coordinate for a single-antenna SAR configuration. Because the azimuth coordinate $x$ in Fig. 1 is constant, only the slant-range cells are highlighted, showing that each cell approximately covers a curved segment along the elevation $s$. If a scattering element is present in the $(r-x)$ cell, recovering its position along $s$ is impossible. In Fig. 1, no scatterers are present in $(r-x)$ cell A, while a single scatterer is present in $(r-x)$ cell B , with no possibility of determining its exact position inside the cell.

Consider the geometry in Fig. 2, where the 2-D profile in the plane $(r, s)$ of a 3-D scenario is obtained by fixing the $x$ coordinate for a multiple-antenna SAR configuration. In this case, the multiple-antenna system (dashed box) can detect the presence/absence of a scatterer inside the $(r-x)$ cell and recover its position along $s$ [see Fig. 2(a)] by coherently processing the returns from different antennas. As a result, a larger antenna size is synthesized along the elevation and a narrower antenna beam is obtained. Furthermore, Fig. 2(a) shows that no scatterers are present in the $(r-x)$ cell A, a single scatterer with backscattering signal coherent on different antennas is present in the $(r-x)$ cell B, and a single scatterer without coherent backscattering signal on different antennas is present in the $(r-x)$ cell C . In such a case, the multiple-antenna SAR system can detect and locate target $T_{1}$ but not target $T_{2}$.

For two or more scatterers present in the same ( $r-x$ ) cell, the multiple-antenna SAR system can discriminate between them and recover their positions [see Fig. 2(b)].

The capability of separating the responses of multiple scatterers along the elevation is related to the so-called Rayleigh resolution, that is given by $\rho_{s}=\lambda R_{0} / 2 b_{\mathrm{TOT}}$, where $R_{0}$ is


Fig. 2. Range-elevation geometry fixing the $x$ coordinate for multiple-antenna SAR configuration. (a) No scatterer is present in $(r-x)$ cell A, one coherent scatterer is present in $(r-x)$ cell B , and one incoherent scatterer is present in $(r-x)$ cell C (e.g., in vegetated areas). The scatterer present in $(r-x)$ cell B can be located along $s_{1}$. (b) No scatterer is present in $(r-x)$ cell A, two coherent scatterers are present in $(r-x)$ cell B, and two incoherent scatterers are present in $(r-x)$ cell C (e.g., in vegetated areas). The scatterers present in $(r-x)$ cell B can be located along $s_{1}$ and $s_{2}$, while the ones in $(r-x)$ cell C cannot be located.
the distance between the considered $(r-x)$ cell and the master antenna, $\lambda$ is the wavelength, and $b_{\text {TOT }}$ is the total size of the multiple-antenna system along $s$. A larger $b_{\mathrm{TOT}}$ signifies a better resolution along $s$. In any case, it is not possible to increase the overall size of the multiple antenna as much as one wants, since very large baselines introduce significant spatial decorrelations, due to the associated view angles variations [33].

In Fig. 2(b), no scatterers are present in the ( $r-x$ ) cell A, two scatterers $T_{1}$ and $T_{2}$ whose backscattering signals are coherent on different antennas are present in the $(r-x)$ cell B , and two scatterers $T_{3}$ and $T_{4}$ whose backscattering signals are incoherent (for instance, due to changes of position and/or reflectivity of the scatterers between different acquisitions, e.g., in vegetated areas), on different antennas are present in the $(r-x)$ cell C. In such a case, the multiple-antenna SAR system can detect and locate targets $T_{1}$ and $T_{2}$, but not targets $T_{3}$ and $T_{4}$.

Assume $b_{m}, m=1, \ldots, M$, is the orthogonal baseline between $M$ SAR antennas and a reference antenna (master antenna); then, TomoSAR methods based on GRLT [15], [16] can discriminate between two statistical hypotheses, namely $H_{0}$ (absence of scatterers in the $(r-x)$ cell) and $H_{1}$ (presence of at least one scatterer in the ( $r-x$ ) cell), thus facilitating the recovery of the position and backscattering coefficient of the scatterer. The presence of more than one scatterer in the same $(r-x)$ cell can be determined and their positions and backscattering coefficients can be estimated by iterating the GLRT and introducing an additional statistical hypothesis, as described in [16].

In this article, we assume that at most one coherent scatterer is present in each $(r-x)$ cell. Then, after a preprocessing step compensating the flat Earth phase contribution [34], the complex signals relevant to a fixed ( $r-x$ ) cell and obtained using $M$ baselines $b_{m}$ under the two statistical hypotheses can be expressed as

$$
\begin{array}{ll}
\boldsymbol{u}=\boldsymbol{w} & \text { Hypothesis } H_{0}  \tag{1}\\
\boldsymbol{u}=\boldsymbol{\varphi}(s) \gamma+\boldsymbol{w} & \text { Hypothesis } H_{1}
\end{array}
$$

with

$$
\begin{gather*}
\boldsymbol{u}=\left[u\left(b_{1}\right) u\left(b_{2}\right) \cdots u\left(b_{M}\right)\right]^{T} \\
\boldsymbol{w}=\left[w\left(b_{1}\right) w\left(b_{2}\right) \cdots \cdot w\left(b_{M}\right)\right]^{T}  \tag{2}\\
\boldsymbol{\varphi}(s)=\left[e^{j \frac{4 \pi}{\lambda R_{0}} b_{1} s} e^{j \frac{4 \pi}{\lambda R_{0}} b_{2} s} \cdots e^{j \frac{4 \pi}{\lambda R_{0}} b_{M} s}\right]^{T}
\end{gather*}
$$

where $\gamma$ is the reflectivity coefficient of the scatterer present in the $(r-x)$ cell along elevation $s, w\left(b_{m}\right)$ is the additive clutter and noise contribution to the $m$ th acquisition, and $\varphi(s)$ is the steering vector. Note that, after the above mentioned flat Earth removal operation, the elevation $s$ is related to the height $z$ by the relation $z=s \sin (\theta)$, where $\theta$ is the look angle introduced in Figs. 1 and 2 , so that the steering vector in (2) is depending on look angle.

Assume a deterministic reflectivity $\gamma$, and a circularly Gaussian white noise with zero mean and covariance matrix $\mathbf{C}=$ $\sigma^{2} \mathbf{I}$, where $\sigma^{2}$ is the (unknown) noise variance, and $\mathbf{I}$ is the $M \mathrm{x} M$ identity matrix. Consequently, the data vector $\boldsymbol{u}$ is still Gaussian, with covariance matrix equal to $\sigma^{2} \mathbf{I}$, and mean equal to zero in the hypothesis $H_{0}$, and equal to $\varphi(s) \gamma$, in the hypothesis $H_{1}$. Then, its probability density function (PDF) under the two hypotheses can be expressed as [16]

$$
\begin{align*}
f_{H_{0}}\left(\boldsymbol{u} ; \sigma^{2}\right) & =\frac{1}{\pi^{M} \sigma^{2 M}} e^{-\frac{u^{H} u}{\sigma^{2}}} \\
f_{H_{1}}\left(\boldsymbol{u} ; \gamma, s, \sigma^{2}\right) & =\frac{1}{\pi^{M} \sigma^{2 M}} e^{-\frac{(u-\boldsymbol{\varphi}(s) \gamma)^{H}(u-\boldsymbol{\varphi}(s) \gamma)}{\sigma^{2}}} \tag{3}
\end{align*}
$$

where apex $H$ represents Hermitian (conjugate transpose).
The GLRT to discriminate between the two hypotheses is expressed as [16]

$$
\begin{equation*}
\Lambda_{01}=\frac{\max _{\gamma, s, \sigma^{2}} f_{H_{1}}\left(\boldsymbol{u} ; \gamma, s, \sigma^{2}\right)}{\max _{\sigma^{2}} f_{H_{0}}\left(\boldsymbol{u} ; \sigma^{2}\right)} \stackrel{H_{H_{0}}}{>} \eta_{01} . \tag{4}
\end{equation*}
$$

The threshold $\eta_{01}$ is set using a constant false alarm rate approach, which consists in imposing the $P_{\mathrm{FA}}$ equal to an assigned value (e.g., $P_{\mathrm{FA}}=10^{-3}$ ), and evaluating the corresponding threshold by means of Monte Carlo simulation. The probability
of false alarm $\left(P_{\mathrm{FA}}\right)$, is defined as the probability that $\Lambda_{01}$ is larger than $\eta_{01}$, when the scatterer is absent. A sample size of $10^{5}$ realizations of noise signals, i.e., the data under hypothesis $H_{0}$, have been generated. Then, for the considered $P_{\mathrm{FA}}$ the threshold is evaluated such that the generalized likelihood ratio is over the threshold with a probability equal to the fixed $P_{\mathrm{FA}}$.

The expression of the ML estimates of $\gamma$ and $\sigma^{2}$ maximizing (4) can be found in a closed form. Consequently, after some mathematical manipulations, a simplified form of the test is obtained [24]

$$
\begin{equation*}
\Lambda_{\mathrm{SL}}=\frac{\max _{s} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}(s) \boldsymbol{u}\right|^{2}}{\boldsymbol{u}^{H} \boldsymbol{u}} \underset{H_{0}}{\stackrel{H_{1}}{>}} \eta_{\mathrm{SL}} . \tag{5}
\end{equation*}
$$

Test (5) is derived in the appendix section for the general case of a cluster of $L$ pixels. Test (5) can be easily obtained in the limit case of a single-pixel cluster $(L=1)$.

The above expression denotes the SL-GLRT, because in this case the scatterer detection and its reflectivity and elevation estimations are performed using a single $(r-x)$ pixel of the multibaseline image stack.

In terms of the probability of detection (for an assigned $P_{\mathrm{FA}}$ ) and the estimation accuracy of elevation, the performance of the previous test strongly depends on the scatterer coherence among the $M$ acquisitions and on SNR. In practice, the performance can be unsatisfactory when $M$ is too small. Hence, to improve the performance achieved with a fixed $P_{\mathrm{FA}}$, a cluster of $L$ pixels in a local neighborhood of the $(r-x)$ cell of interest can be considered, so that the number of data exploited to detect and locate the scatterer is increased of a factor $L$. To introduce a relation among the pixels of the cluster, an appropriate parametric function $s_{l}(\boldsymbol{\alpha})$ can be introduced, where $\boldsymbol{\alpha}$ is a parameter vector of size $P<$ $L$, for locally approximating the height profile in the considered neighborhood.

Assuming $\boldsymbol{u}_{l}$ is the $M \times 1$ data vector in each of the $L$ pixels of the neighborhood, the following clustered GLRT can be obtained (refer to the appendix for the derivation):

$$
\begin{equation*}
\Lambda_{C}=\frac{\max _{\alpha} \sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}(\boldsymbol{\alpha})\right) \boldsymbol{u}_{l}\right|^{2}}{\sum_{l=1}^{L} \boldsymbol{u}_{l}^{H} \boldsymbol{u}_{l}} \stackrel{H_{1}}{>} \eta_{0} . \tag{6}
\end{equation*}
$$

In this test, the number of parameters to be estimated is $P$ (the dimension of $\boldsymbol{\alpha}$ ), while the number of data is $L \times M$ (the overall dimension of data vector $\left.\boldsymbol{u}_{C}=\left[\boldsymbol{u}_{1}{ }^{T} \boldsymbol{u}_{2}{ }^{T} \cdots \boldsymbol{u}_{L}{ }^{T}\right]^{T}\right)$.

## III. Surface Representation

The pixel neighborhood for consideration in test (6) is selected based on a compromise between estimation accuracy and the compliance of the parametric model with the real surface. In other words, the neighborhood should be sufficiently large to increase the scatterer detection probability and estimation accuracy, and sufficiently small to approximate the real height surface profile accurately with a simple model depending on a few parameters.


Fig. 3. Local plane approximating the actual 3-D surface at the nine $(r-x)$ cluster cells

We consider two choices: neighborhood pixels with a constant elevation (multilook [24]) and LPs with arbitrary slopes.

## A. Multilook

Consider a case where different pixels of the cluster exhibit the same height. In other words, a neighborhood cluster of $L$ pixels is selected such that the surface is locally approximated by a horizontal (parallel to the ground) plane. The assumption of constant height (constant $z$ ) is equivalent to the consideration of the same $s$ for all the $L$ pixels in the cluster:

$$
\begin{equation*}
s_{l}=s_{0}, l=1, \ldots, L \tag{7}
\end{equation*}
$$

By substituting (7) into (6), the parameter vector $\boldsymbol{\alpha}$ is reduced to a scalar $s_{0}$ and test (6) becomes the MGLRT [24]:

$$
\begin{equation*}
\Lambda_{\mathrm{ML}}=\frac{\max _{s_{0}} \sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{0}\right) \boldsymbol{u}_{l}\right|^{2}}{\sum_{l=1}^{L} \boldsymbol{u}_{l}^{H} \boldsymbol{u}_{l}} \stackrel{{ }_{H_{0}}^{>}}{\stackrel{H_{1}}{>}} \eta_{\mathrm{ML}} . \tag{8}
\end{equation*}
$$

Note that, due to the adopted assumption (7), in (8) the same steering vector has been used for all the pixels in the considered neighborhood.

## B. Local Planes

Consider a case where different pixels of the cluster belong to a plane that is arbitrarily oriented in the 3-D space. In other words, the neighborhood cluster of $L$ pixels is selected such that the surface is locally approximated by an arbitrary plane.

Consider a neighborhood cluster represented by a window of $L=\left(2 L_{r}+1\right) \times\left(2 L_{x}+1\right)$ pixels, centered in the $(r-x)$ cell of interest. In Fig. 3, a neighborhood with $L_{r}=L_{x}=1$ is represented $(L=9)$. In this case, the point coordinates of the $L$ cells in the window are given by the discrete values $(p \Delta x, q \Delta r)$, with $p, q=-1,0,1$, where $\Delta x$ and $\Delta r$ are the azimuth and slant-range sampling intervals, respectively, which are constant for all image cells.


Fig. 4. Geomety of building façade with arbitrary orientation.

We constrain the elevation of the $L$ neighboring cells to belong to the LP

$$
\begin{equation*}
s_{p q}\left(s_{00}, k_{x}, k_{r}\right)=s_{00}+\Delta s_{p q}=s_{00}+k_{x} p \Delta x+k_{r} q \Delta r \tag{9}
\end{equation*}
$$

with $p \in\left(-L_{r}, \ldots, 0, \ldots, L_{r}\right)$, and $q \in\left(-L_{x}, \ldots, 0, \ldots, L_{x}\right), s_{p q}$ the elevation of the point with coordinates $(p, q), s_{00}$ the elevation of the central cell, $\Delta s_{p q}=k_{x} p \Delta x+k_{r} q \Delta r$ the variation of elevation when passing from the central cell of the cluster $(p=q=0)$ to the cell of indexes $p$ and $q$, and $k_{x}$ and $k_{r}$ the slope rates of the LP along the azimuth and slant-range directions, respectively.

The equation of the LP (9) is expressed in the $(x, r, s)$ system, while the ground surface slopes are usually expressed in the $(x, y, z)$ system. Subsequently, we consider the relation between the plane equations in the two coordinate systems.

For surface approximation (9), the parameter vector $\boldsymbol{\alpha}$ is reduced to

$$
\boldsymbol{\alpha}=\left[\begin{array}{lll}
s_{00} & k_{x} & k_{r} \tag{10}
\end{array}\right]^{T}
$$

and by substituting (9) into (6), test (6) becomes the LP-GLRT.
Note that, in (11), unlike (8), different steering vectors are used for each pixel in the neighborhood. Considering now tests (8) and (11), and considering for them the same cluster, they yield to the same results when

A rough criterion that can be applied for stating whether the condition (12) is approximately satisfied or not, is the Rayleigh criterion, used in optics for studying wavefront aberrations in imaging systems [35]. It imposes that the maximum variation of the phases of the steering vectors within the considered neighborhood has to be lower than $\pi / 2$

$$
\begin{gather*}
\frac{4 \pi}{\lambda R_{0}} b_{\mathrm{TOT}} 2 \Delta s_{\max }=\frac{4 \pi}{\lambda R_{0}} b_{\mathrm{TOT}}\left(\frac{2 \Delta z_{\max }}{\sin \theta}\right) \leq \frac{\pi}{2}  \tag{13}\\
\Delta z_{\max } \leq \frac{\lambda R_{0} \sin \theta}{16 b_{\mathrm{TOT}}}
\end{gather*}
$$

where $\Delta s_{\max }=\max \left|\Delta s_{p q}\right|$. This condition implicitly introduces a limitation on the patch size, since for planar surfaces the height difference between the central point and the patch border points increases when the patch size increases.

For all cases where the surface cannot be locally approximated by a plane satisfying (13), MGLRT may introduce considerable errors. Note that $\Delta z_{\max }$ is inversely proportional to the total orthogonal baseline span $b_{\mathrm{TOT}}$, and when $b_{\mathrm{TOT}}$ increases, the elevation resolution becomes finer, while the limitation of the patch size becomes stricter. Thus, for the sensors characterized by a larger baseline span, the LP-GLRT is supposed to achieve better performance than the MGLRT.

To investigate the range of applicability of the LP approximation, consider a simplified geometry of a building. The building's illuminated planar facade forms an angle $\beta$ with the azimuth/ground-range plane $(x, y)$ and its intersection with the $(x, y)$ plane forms an angle $\psi$ with the azimuth direction $x$ (see Fig. 4).

In Fig. 5, a building facade with angles $\beta=110^{\circ}$ and $\psi=$ $25^{\circ}$ is shown in green. The red planes represent two iso-azimuth planes spaced by $\Delta x=1.9 \mathrm{~m}$, and the blue planes represent two iso-(slant) range planes spaced by $\Delta r=0.9 \mathrm{~m}$ (these values are from the TerraSAR-X data used in the following sections). The look angle of the SAR system is assumed to be equal to $\theta=28.75^{\circ}$. The blue lines indicate the intersection of the facade with the iso-azimuth planes, and the red lines indicate the intersections with the iso-(slant) range planes. The quadrilateral with vertices $A, B, C$, and $D$ represents a sampling image cell.

The spatial extension of the facade portion in the $3 \times 3$ neighborhood of a given range-azimuth pixel as a function of the facade slopes is determined by considering the intersection of the facade with two iso-azimuth planes spaced by $\Delta x$ and two iso-(slant)range planes spaced by $\Delta r . \Delta x$ and $\Delta r$ are the image

$$
\begin{equation*}
\Lambda_{\mathrm{LP}}=\frac{\max _{s_{00}, k_{x}, k_{r}} \sum_{p=-L_{r}}^{L_{r}} \sum_{q=-L_{x}}^{L_{x}} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{p q}\left(s_{00}, k_{x}, k_{r}\right)\right) \boldsymbol{u}_{p q}\right|^{2}}{\sum_{p=-L_{r}}^{L_{r}} \sum_{q=-L_{x}}^{L_{x}} \boldsymbol{u}_{p q}^{H} \boldsymbol{u}_{p q}} \stackrel{H_{1}}{>} \eta_{\mathrm{LP}} \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{p=-L_{x}}^{L_{x}} \sum_{q=-L_{r}}^{L_{r}}\left|\varphi^{H}\left(s_{00}+\Delta s_{p q}\right) \boldsymbol{u}_{p q}\right|^{2} \cong \sum_{p=-L_{x}}^{L_{x}} \sum_{q=-L_{r}}^{L_{r}}\left|\varphi^{H}\left(s_{00}\right) \boldsymbol{u}_{p q}\right|^{2} \tag{12}
\end{equation*}
$$



Fig. 5. Building facade, iso-azimuth, and iso-(slant) range planes along with their intersections.


Fig. 6. Projection on the $(x, z)$ plane of the facade plane and its intersections with three iso-azimuth planes (red lines) and three iso-(slant) range planes (blue lines) for $\psi \neq 0^{\circ}$.
sampling lengths along the azimuth and slant-range directions, respectively.

For a facade with $\psi=0^{\circ}$, the sampling cell is approximately rectangular, $A$ and $B$ exhibit the same height, and $C$ and $D$ exhibit the same height. The vertical (height) dimension of the cell is $\Delta z=z_{A}-z_{D}$, which is also equal to $z_{B}-z_{C}$. The $\Delta z$ value can be also expressed as $\Delta z=\Delta r \sin (\beta) / \sin (\beta+\theta)$.

For a generic value of $\psi \neq 0^{\circ}$, the quadrilateral becomes a rhombus and consequently the heights of $A$ and $B$ differ from each other, as those between $C$ and $D$ do. The projection on the azimuth-height $(x, z)$ plane of the facade plane, and its intersections with the iso-azimuth and iso-(slant) range planes are shown in Fig. 6 for an arbitrary value of $\psi \neq 0^{\circ}$ and all 3 $\times 3(r-x)$ cells of the cluster. In this case, the vertical (height) dimension of the cell is achieved, in the general case, by the difference $\Delta z$ between $\max \left(z_{A}, z_{B}\right)$, and $\min \left(z_{C}, z_{D}\right)$.

In the particular case of Fig. 6, the vertical dimension of the cell is given by $\Delta z=z_{B}-z_{D}$, which is also equal to the height difference between the central points of cells (11), and (00), given by $z_{11}$ and $z_{00}$, respectively. This implies that the $3 \times$


Fig. 7. ( $r-x$ ) cluster cell vertical extension $2 \Delta z$ for $\psi=0^{\circ}$ (solid line), $\psi=15^{\circ}$ (dashed-dotted line), $\psi=30^{\circ}$ (dashed line), and $\psi=45^{\circ}$ (dotted line).

TABLE I
TSX CONFIGURATION PARAMETERS

| Wavelength $\lambda(\mathrm{m})$ | 0.0311 |
| :---: | :---: |
| View angle $\theta\left({ }^{\circ}\right)$ | 28.75 |
| Range $R_{0}(\mathrm{~km})$ | 579.4 |
| Number of images $M$ | 15 |
| Total Baseline $b_{\mathrm{TOT}}(\mathrm{m})$ | 751.6 |

$3(r-x)$ cluster over the facade exhibits an overall extension along $z$, equal to $2 \Delta z=z_{11}-z_{-1-1}$. The extension along $s$ of the cell and $3 \times 3$ cluster can be easily obtained as $2 \Delta z$ $\sin (\theta)$.

The behavior of the vertical extension of the $3 \times 3(r-x)$ cluster for four values of $\psi\left(0^{\circ}, 15^{\circ}, 30^{\circ}\right.$, and $\left.45^{\circ}\right)$, with $\beta$ varying from $0^{\circ}$ to $180^{\circ}$, are shown in Fig. 7. We also fix the look angle value to $\theta=28.75^{\circ}$. Note that the vertical extension can assume very large values for $\beta$ intervals such that the sampling ( $r-x$ ) cell tends to be aligned with the direction of $s$. This behavior occurs, for instance, when the entire facade is parallel to the azimuth direction ( $\psi=0^{\circ}$, solid line), and when it approaches to be perpendicular to $\operatorname{LoS}$ [for $\beta$ getting close to $151.25^{\circ}\left(180^{\circ}-\theta\right)$, namely]. Hence, based on different $\psi$ values, there are $\beta$ intervals where the vertical extension assumes large values such that (13) is not verified and the MGLRT approximation is no longer valid.

## IV. Experimental Results

## A. Simulation Results

The performance of the proposed LP-GLRT is first validated based on the results obtained using simulated data by employing the TerraSAR-X (TSX) parameters given in Table I. We consider 15 multipass images with their orthogonal baseline values given in Table II.

In all experiments, we consider a local neighborhood of $3 \times 3$ pixels. The performance of LP-GLRT was compared with those of two state-of-the-art tomographic approaches, namely, singlelook sup-GLRT [16] and MGRLT [24], in terms of detection probability $\left(P_{D}\right)$ achieved for a given value of the probability

TABLE II
TSX Orthogonal Baselines

| $\#$ | Orthogonal baseline |
| :---: | :---: |
| 1 | 0 |
| 2 | 42.8800 |
| 3 | -248.0900 |
| 4 | 204.7900 |
| 5 | 180.8100 |
| 6 | -121.6600 |
| 7 | 79.3100 |
| 8 | -100.7700 |
| 9 | -226.3600 |
| 10 | -166.1900 |
| 11 | -227.5100 |
| 12 | 328.4600 |
| 13 | -314.9400 |
| 14 | 436.6600 |
| 15 | 300.7300 |



Fig. 8. ROC curves obtained using LP-GLRT (solid line) and MGLRT (dashed line) with $\mathrm{SNR}=-6 \mathrm{~dB}$ in the case of a horizontal plane with $\beta=0^{\circ}$ (blue line), vertical plane with $\beta=90^{\circ}$ and $\psi=0^{\circ}$ (red line), and slanted plane with $\beta=130^{\circ}$ and $\psi=30^{\circ}$ (green line).
of false alarm $\left(P_{\mathrm{FA}}\right)$ and the root-mean-square error (RMSE) of the estimated elevation as a function of SNR.

Using the parameter and baseline values given in Tables I and II, the maximum height variation $\Delta z_{\max }$, which provides phase deviations smaller than $\pi / 2$ in the steering vectors of the considered $3 \times 3$ neighborhood, as shown by (13), is approximately 0.72 m . Hence, when the maximum extension of the LP $3 \times 3$ neighborhood along the vertical direction $z$ is smaller than $\Delta z_{\max }$, which depends on the plane slopes, MGLRT and LP-GLRT show the same performance.

Fig. 8 shows the receiver operating characteristic (ROC) curves for the $P_{D}$ values obtained under different $P_{\mathrm{FA}}$ values using MGLRT and LP-GLRT. The point target has been generated according to the statistical model under hypothesis $H_{1}$, belonging to a plane obtained fixing the angles $\beta$ and $\psi$, and with a specified SNR.

A moving window of $L=3 \times 3$ pixels is considered, so that the center point is automatically set, and the SNR is fixed to -6 dB . In particular, three planar surfaces have been considered: a horizontal plane with $\beta=0^{\circ}$ (blue lines), a vertical plane parallel to the azimuth direction with $\beta=90^{\circ}, \psi=0^{\circ}$ (red lines), and a slanted plane with $\beta=130^{\circ}, \psi=30^{\circ}$ (green lines).


Fig. 9. $\quad P_{D}$ and RMSE versus SNR obtained for an horizontal plane with $\beta=$ $0^{\circ}$ and $\psi=0^{\circ}$ (blue lines), vertical plane with $\beta=90^{\circ}$ and $\psi=0^{\circ}$ (red lines), and slanted plane with $\beta=130^{\circ}$ and $\psi=30^{\circ}$ (green lines), with $P_{F A}=10^{-3}$ : (a), (c), and (e) $\mathrm{P}_{\mathrm{D}}$ obtained using LP-GLRT (solid line), MGLRT (dashed line), and sup-GLRT (dotted black line). (b), (d), and (f) height RMSE using LP-GLRT (solid line), MGLRT (dashed line), and sup-GLRT (dotted black line).

The performance of LP-GLRT (solid lines) is basically the same in all three cases, because LP-GLRT adaptively estimates the plane slopes with an accuracy that is essentially independent of the slope values. As far as the performance of MGLRT are concerned (dashed lines), in the case of the horizontal plane the PD is slightly higher than that of LP-GLRT. This happens because, as MGLRT is based on the horizontal plane assumption exactly, it requires the estimation of only one parameter compared with LP-GLRT, which requires the estimation of three parameters. Conversely, in case of a deviation from the horizontal plane, a modeling error occurs for MGLRT, and its performance deteriorates, leading to a decrease in the PD with respect to LP-GLRT.

The decrease in performance of MGLRT is affected by the plane slopes and becomes obvious when the overall height variation within the considered neighborhood, which can be expressed as $\Delta z=2\left(k_{x} \Delta x+k_{r} \Delta r\right) \sin \vartheta$, considerably exceeds $\Delta z_{\max }=0.72 \mathrm{~m}$ [defined in (13)]. For the vertical and slanted planes considered in the simulations, $\Delta z$ assumes the values of 1.8 and 7.7 m , respectively, both exceeding $\Delta z_{\max }$.

To investigate the performance with respect to SNR, Fig. 9(a)(f) shows the $P_{D}$ obtained for $P_{\mathrm{FA}}=10^{-3}$, and the RMSE of the estimated elevation for different SNR values for the same three planes considered in Fig. 8. In Fig. 9(a)-(f), solid lines refer
to LP-GLRT, and dashed lines refer to MGLRT. Fig. 9(a)-(f) also shows the results obtained using sup-GLRT (dotted line), which are independent of the plane slopes, as a reference to indicate the significant improvements obtained using MGLRT and LP-GLRT. By comparing the plots presented in Figs. 9(a) and (b), obtained for a horizontal plane ( $\Delta z=0$ in the $3 \times$ 3 neighborhood), MGLRT and LP-GLRT exhibit comparable performance, with a slight improvement for MGLRT, as it requires the estimation of only one parameter compared with the three parameters to be estimated with LP-GLRT.

For a vertical plane [see Fig. 9(c) and (d)] the performance of MGLRT slightly deteriorates with respect to LP-GLRT, while for the chosen slanted plane [see Fig. 9(e) and (f)], the performance of MGLRT are significantly worse than the ones obtained with LP-GLRT.

Therefore, we can infer that LP-GLRT outperforms the other two algorithms in the case of slanted planes owing to the better capability of the LP approach to fit a real slope of the surface profile. Compared with sup-GLRT, both LP-GLRT and MGLRT achieve better performance because they use "context" information inside the local area, which implicitly facilitates the regularization of the problem solution. Note that the single-look GLRT-based detector is the basis for developing LP-GLRT and MGLRT.

It should be highlighted that the gain in terms of RMSE and $P_{D}$ obtained with LP-GLRT has been achieved at the expense of a higher computational cost. As it is clear from (9), in the LPGLRT approach three unknowns have to be estimated compared to one in the MGLRT approach, given the same amount of data $M \times L$.

The previous $P_{D}$ and RMSE results shown in Figs. 8 and 9 have been obtained using a local neighbourhood of $3 \times 3$ pixels. The choice of the neighbourhood patch size has to be made on the basis of a compromise between noise smoothing and accurate matching of the adopted planar model to the real surface profile. In fact, from one side it would be more convenient to choose a bigger patch for obtaining a better averaging over the noise, and then a better pixel detection rate $P_{D}$. From the other side, a bigger patch size can introduce a more remarkable displacement of the height profile within the patch from the horizontal plane model adopted in the MGLRT case and from the slanted plane model for the LP-GLRT case. Hence, the optimal choice would strongly depend on the surface profile under observation, which is unknown. Anyway, the use of a small patch that typically does not have a very extended size, for instance $3 \times 3$ or $5 \times 5$ pixels, is recommended for urban structures.

In order to better investigate the role of the neighbourhood patch size, let us examine the results obtained on simulated data for a slanted plane with $\beta=130^{\circ}$ and $\psi=30^{\circ}$, shown in Figs. 10 and 11. Fig. 10 shows the ROC curves for MGLRT and LP-GLRT, obtained using patches of size $3 \times 3$ or $5 \times$ 5. It appears that for both methods the detection probabilities significantly increase when considering a size $5 \times 5$. Anyway, if we look at the RMSE curves reported in Fig. 11, we notice that while in the case of LP-GLRT a reduction of the RMSE is observed when passing from $3 \times 3$ to a $5 \times 5$ size for all the SNR values, in the case of MGLRT an increase of the


Fig. 10. ROC curves for MGLRT and LP-GLRT, obtained using patches of size $3 \times 3$ or $5 \times 5$ (green for $3 \times 3$ patches, orange for $3 \times 3$ patches).


Fig. 11. RMSE curves for MGLRT and LP-GLRT, obtained using patches of size $3 \times 3$ or $5 \times 5$ (green for $3 \times 3$ patches, orange for $3 \times 3$ patches).

RMSE is observed when passing from $3 \times 3$ to $5 \times 5$. This behavior is due to the fact that while LP-GLRT fully adapts to the simulated plane, in the case of MGLRT the height displacement from the assumed horizontal plane increases when the patch size increases.

This shows that in the presence of model errors, obtaining a higher $P_{D}$ does not always correspond to a more accurate reconstruction of the height profile. The same problem could occur in the LP-GLRT case when the local surface deviates from a slant plane. The choice of a small patch size $(3 \times 3)$, guarantees a good fit of the LP model to almost all the surface profiles that can be encountered in practical cases.

## B. Results on Real Data

This section presents the experimental results on real data with respect to TSX images to further investigate the performance of the proposed algorithm.

The configuration parameters are the same as the simulation parameters given in Tables I and II. Fig. 12 shows the optical and SAR images of Piazza del Plebiscito, Naples, Italy. Fig. 13(a) presents the LiDAR ground truth, and Fig. 13(b)-(d) show the results obtained using sup-GLRT, MGLRT, and LP-GLRT, respectively, for $P_{\mathrm{FA}}=10^{-3}$. Regarding the results obtained using sup-GLRT in Fig. 13(b), the high density of detected scatterers can be appreciated, although no contextual information


Fig. 12. Images of Piazza del Plebiscito. (a) Optical image. (b) SAR image (Copyright DLR 2012-2014).


Fig. 13. Comparison of different GLRT-based method. (a) LiDAR data. (b) sup-GLRT. (c) Multilook generalized likelihood ratio test. (d LP-generalized likelihood ratio test.


Fig. 14. San Francesco di Paola church dome. (a) LiDAR height profile. (b) Optical image.
was used. Compared with Fig. 13(b), Fig. 13(c) and (d) shows significant improvements for MGLRT and LP-GLRT in terms of larger densities of detected scatterers.

Comparing the color bars of different images also shows that the reconstructions of the height values using the three algorithms are very accurate. To better compare the performance of MGLRT and LP-GLRT, the results of processing an image of the San Francesco di Paola Church dome, a part of the scenario in Fig. 12, are compared.


Fig. 15. San Francesco di Paola church dome. Scatterers detected using (a) multilook generalized likelihood ratio test and (b) LP-generalized likelihood ratio test.

Fig. 14 shows the zoomed LiDAR and optical images of the church dome. The reconstruction results of the church dome using MGLRT and LP-GLRT are presented in Fig. 15(a) and (b), respectively. LP-GLRT detects more scatterers on the dome surface than MGLRT.

In both cases some outliers are present, and the two algorithms exhibit similar visual performance. To quantitatively compare their performance, we introduce the two metrics employed in the literature [21], and [36], namely accuracy and completeness error, which are, respectively, defined as

$$
\begin{align*}
\mathrm{Acc} & =\frac{1}{N_{p}} \sum_{j=1}^{N_{p}} \min _{k} \operatorname{dist}\left(\hat{p}_{j}, p_{k}\right) \\
\mathrm{Comp} & =\frac{1}{N_{p}^{\prime}} \sum_{j=1}^{N^{\prime} p} \min _{k} \operatorname{dist}\left(\hat{p}_{k}, p_{j}\right) \tag{14}
\end{align*}
$$

where $N_{p}$ is the number of available ground truth points, provided by the LiDAR profile in this case, and $N^{\prime}{ }_{p}$ is the number of recovered scatterers using GLRT.

Accuracy error measures the average distance between each recovered scatterer and the closest point on the ground truth. This metric measures how accurate the estimated scatterer position is on average, a high accuracy error value is obtained when the reconstructed points are far from the actual surface. Completeness error measures the average distance between each point on the ground truth and the closest scatterer in the recovered three-dimensional (3-D) cloud. This metric characterizes the density of the reconstructed 3-D cloud, and a high completeness error value is obtained when few scatterers are recovered with respect to the number of the available ground truth points (for a sparse cloud of recovered scatterers). Lower completeness and accuracy error values signify better reconstruction performance.

Note that these metrics depend on the threshold values selected in the adopted GLRT. If the threshold is set at a lower


Fig. 16. Accuracy versus completeness by varying threshold values. By increasing the threshold, we move on the curves from the left to the right.
value, more scatterers are recovered; however, their reflectivity and position estimation accuracy error can decrease and some outliers can appear. Hence, a smaller threshold will yield a lower completeness error value and a higher accuracy error value. Alternatively, for high threshold values, the results of accuracy and completeness errors are opposite.

The accuracy and completeness errors for the two reconstructions in Fig. 15(a) and (b) are reported in Fig. 16, where dashed and solid lines represent MGLRT and LP-GLRT reconstructions, respectively. The two plots were obtained by changing the threshold values. In other words, by increasing the threshold value, the curves move toward the right. In fact, in such a case, a higher threshold value implies a less dense cloud of reliable reconstructed points; hence, the completeness error increases and accuracy error decreases. For high threshold values, MGLRT and LP-GLRT exhibit equivalent performance, whereas for low threshold values (left part of the two curves), LP-GLRT exhibits lower accuracy error than MGLRT. In conclusion, LP-GLRT shows an overall better performance than MGLRT because it achieves denser detected point clouds for given positioning accuracy.

## V. Conclusion

In this article, an LP-GLRT algorithm was proposed for improving the reconstruction performance of TomoSAR. The integration of the estimation of the LPs parameters into the reconstruction process increased the adaption of the multilook process to height-variations of urban structures, thus increasing the density of detected scatterers and the accuracy of their position estimates. The proposed algorithm is particularly valuable for the 3-D imaging of urban scenarios where significant height variations can exist.

Results obtained using simulated data show that LP-GLRT exhibits a higher detection probability for a fixed $P_{\mathrm{FA}}$ than MGLRT when the observed surface on the ground deviates from a horizontal plane. The gain in detection performance noticeably increases as the surface approaches a plane orthogonal to the range direction. Additionally, the performance improvement becomes more pronounced, the more the overall baseline increases, making LP-GLRT particularly suitable for high-heightresolution tomographic configurations.

Results obtained using real TSX data on the dome of San Francesco di Paola church, Naples, Italy, which exhibits a quite curved surface; confirmes the results obtained using simulated data because for a fixed threshold, LP-GLRT showed an increased number of detected scatterers exhibiting a lower accuracy error than MGLRT.

The proposed algorithm is particularly valuable for the 3-D imaging of urban scenarios where significant height variations can exist and/or when the SAR images are acquired with sensors characterized by a large baseline span, e.g., COSMO-SkyMed.

However, LP-GLRT has a high computational cost because it requires the numerical estimation of three parameters instead of just one parameter. This increased complexity is certainly acceptable for reconstructing and observing single structures and their immediate surroundings with a high degree of accuracy.

Another aspect to be investigated in the near future is the effect of the high height accuracy and higher scatterers' density on the detection and estimation of temporal and thermal deformations of the observed structures.

## APPENDIX

Consider an $M$ baseline SAR system and a cluster of $L$ pixels in a local neighborhood of the $(r-x)$ cell of interest. The signal received from the $(r-x)$ position $l$, whose elevation is $s_{l}$ and reflectivity is $\gamma_{l}$, at the $M$ baselines under the two hypotheses $H_{0}$ (absence of scatterer) and $\mathrm{H}_{1}$ (presence on a single scatterer) is

$$
\begin{array}{ll}
\boldsymbol{u}_{l}=\boldsymbol{w}_{l} & \text { Hypothesis } H_{0} \\
\boldsymbol{u}_{l}=\boldsymbol{\varphi}\left(s_{l}\right) \gamma_{l}+\boldsymbol{w}_{l} & \text { Hypothesis } H_{1} \tag{A.1}
\end{array}
$$

where

$$
\begin{gather*}
\boldsymbol{u}_{l}=\left[u_{l}\left(b_{1}\right) u_{l}\left(b_{2}\right) \cdots u_{l}\left(b_{M}\right)\right]^{T} \\
\boldsymbol{w}_{l}=\left[w_{l}\left(b_{1}\right) w_{l}\left(b_{2}\right) \cdots w_{l}\left(b_{M}\right)\right]^{T}  \tag{A.2}\\
\boldsymbol{\varphi}\left(s_{l}\right)=\left[e^{j \frac{4 \pi}{\lambda R_{0}} b_{1} s_{l}} e^{j \frac{4 \pi}{\lambda R_{0}} b_{2} s_{l}} \cdots e^{j \frac{4 \pi}{\lambda R_{0}} b_{M} s_{l}}\right]^{T}
\end{gather*}
$$

Collecting the $L$ data vectors in a single model, we achieve

$$
\begin{array}{ll}
\boldsymbol{u}_{C}=\boldsymbol{w}_{C} & \text { Hypothesis } H_{0} \\
\boldsymbol{u}_{C}=\boldsymbol{\Phi}_{C} \boldsymbol{\gamma}_{C}+\boldsymbol{w}_{C} & \text { Hypothesis } H_{1}
\end{array}
$$

where

$$
\begin{gather*}
\boldsymbol{u}_{C}=\left[\begin{array}{c}
\boldsymbol{u}_{1} \\
\boldsymbol{u}_{2} \\
\vdots \\
\boldsymbol{u}_{L}
\end{array}\right], \boldsymbol{\gamma}_{C}=\left[\begin{array}{c}
\gamma_{1} \\
\gamma_{2} \\
\vdots \\
\gamma_{L}
\end{array}\right], \boldsymbol{w}_{C}=\left[\begin{array}{c}
\boldsymbol{w}_{1} \\
\boldsymbol{w}_{2} \\
\vdots \\
\boldsymbol{w}_{L}
\end{array}\right] \\
{[L M \times 1]} \\
{[L \times 1] \quad[L M \times 1]} \\
\mathbf{\Phi}_{C}=\left[\begin{array}{cccc}
\boldsymbol{\varphi}\left(s_{1}\right) & \mathbf{0}_{M} & \cdots & \mathbf{0}_{M} \\
\mathbf{0}_{M} & \boldsymbol{\varphi}\left(s_{2}\right) & \cdots & \mathbf{0}_{M} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{0}_{M} & \mathbf{0}_{M} & \cdots & \boldsymbol{\varphi}\left(s_{L}\right)
\end{array}\right], \mathbf{0}_{M}=\left[\begin{array}{c}
0 \\
0 \\
\vdots \\
{[L M \times L]}
\end{array}\right] .  \tag{A.4}\\
{[M \times 1]}
\end{gather*}
$$

The PDFs under the two hypotheses $H_{0}$ and $H_{1}$ are expressed as

$$
\begin{align*}
f_{H_{0}}\left(\boldsymbol{u}_{C} ; \sigma^{2}\right) & =\frac{1}{\pi^{L M} \sigma^{2(L M)}} e^{-\frac{u_{C}^{H} u_{C}}{\sigma^{2}}} \\
f_{H_{1}}\left(\boldsymbol{u}_{C} ; \boldsymbol{\gamma}_{C}, \boldsymbol{s}_{C}, \sigma^{2}\right) & =\frac{1}{\pi^{L M} \sigma^{2(L M)}} e^{-\frac{\left(u_{C}-\boldsymbol{\Phi}_{C} \boldsymbol{\gamma}_{C}\right)^{H}\left(u_{C}-\boldsymbol{\Phi}_{C} \boldsymbol{\gamma}_{C}\right)}{\sigma^{2}}} \tag{A.5}
\end{align*}
$$

where $s_{C}=\left[\begin{array}{llll}s_{1} & s_{2} & \cdots & s_{L}\end{array}\right]^{T}$.
The GLRT is expressed as

$$
\begin{equation*}
\Lambda=\frac{\max _{C}, \boldsymbol{s}_{C}, \sigma^{2}}{\max _{\sigma^{2}} f_{H_{1}}\left(\boldsymbol{u}_{C} ; \boldsymbol{\gamma}_{C}, \boldsymbol{s}_{C}, \sigma^{2}\right)} \stackrel{H_{H_{0}}\left(\boldsymbol{u}_{C} ; \sigma^{2}\right)}{>} \underset{H_{0}}{>} \eta . \tag{A.6}
\end{equation*}
$$

We maximize the pdf under the null hypothesis $H_{0}$

$$
\begin{align*}
& \underset{\sigma^{2}}{\arg \max } f_{H_{0}}\left(\boldsymbol{u}_{C} ; \sigma^{2}\right)=\underset{\sigma^{2}}{\arg \max } \ln f_{H_{0}}\left(\boldsymbol{u}_{C} ; \sigma^{2}\right) \\
& =\underset{\sigma^{2}}{\arg \max }\left[-\ln \left(\pi^{L M} \sigma^{2(L M)}\right)-\frac{\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}}{\sigma^{2}}\right] . \tag{A.7}
\end{align*}
$$

Maximizing (A.7) with respect to $\sigma^{2}$ yields

$$
\begin{align*}
& \frac{d \ln f_{H_{0}}\left(\boldsymbol{u}_{C} ; \sigma^{2}\right)}{d\left(\sigma^{2}\right)}=-\frac{L M \sigma^{2(L M-1)}}{\sigma^{2(L M)}}+\frac{\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}}{\sigma^{4}}=0 \\
&-\frac{L M}{\sigma^{2}}+\frac{\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}}{\sigma^{4}}=0 \Rightarrow \hat{\sigma}^{2}=\frac{\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}}{L M}=\frac{\sum_{l=1}^{L} \boldsymbol{u}_{l}^{H} \boldsymbol{u}_{l}}{L M} \tag{A.8}
\end{align*}
$$

Consequently, the maximum of pdf under the null hypothesis $H_{0}$ is

$$
\begin{equation*}
\max _{\sigma^{2}} f_{H_{0}}\left(\boldsymbol{u}_{C} ; \sigma^{2}\right)=\frac{1}{\pi^{L M}\left(\frac{\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}}{L M}\right)^{L M}} e^{-L M} \tag{A.9}
\end{equation*}
$$

Next, we maximize the pdf under the hypothesis $\mathrm{H}_{1}$. Maximizing the pdf with respect to $\gamma_{C}$ [37] yields

$$
\begin{aligned}
& \underset{\boldsymbol{\gamma}_{C}}{\arg \max } f_{H_{1}} \Rightarrow \hat{\boldsymbol{\gamma}}_{C}=\left(\boldsymbol{\Phi}_{C}^{H} \boldsymbol{\Phi}_{C}\right)^{-1} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C} \\
& =\left[\begin{array}{cccc}
\boldsymbol{\varphi}^{H}\left(s_{1}\right) \boldsymbol{\varphi}\left(s_{1}\right) & 0 & \cdots & 0 \\
0 & \boldsymbol{\varphi}^{H}\left(s_{2}\right) \boldsymbol{\varphi}\left(s_{2}\right) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \boldsymbol{\varphi}^{H}\left(s_{L}\right) \boldsymbol{\varphi}\left(s_{L}\right)
\end{array}\right]^{-1} \\
& \quad \times\left[\begin{array}{c}
{\left[\begin{array}{c}
\boldsymbol{\varphi}^{H}\left(s_{1}\right) \boldsymbol{u}_{1} \\
\boldsymbol{\varphi}^{H}\left(s_{2}\right) \boldsymbol{u}_{2} \\
\vdots \\
\boldsymbol{\varphi}^{H}\left(s_{L}\right) \boldsymbol{u}_{L}
\end{array}\right]=\left[\begin{array}{cccc}
M & 0 & \cdots & 0 \\
0 & M & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & M
\end{array}\right]^{-1}\left[\begin{array}{c}
\boldsymbol{\varphi}^{H}\left(s_{1}\right) \boldsymbol{u}_{1} \\
\boldsymbol{\varphi}^{H}\left(s_{2}\right) \boldsymbol{u}_{2} \\
\vdots \\
\boldsymbol{\varphi}^{H}\left(s_{L}\right) \boldsymbol{u}_{L}
\end{array}\right]} \\
=\frac{1}{M}\left[\begin{array}{c}
{\left[\begin{array}{c}
\boldsymbol{\varphi}^{H}\left(s_{1}\right) \boldsymbol{u}_{1} \\
\boldsymbol{\varphi}^{H}\left(s_{2}\right) \boldsymbol{u}_{2} \\
\vdots \\
\boldsymbol{\varphi}^{H}\left(s_{L}\right) \boldsymbol{u}_{L}
\end{array}\right]}
\end{array}\right] \\
\hat{\boldsymbol{\gamma}}_{2} \\
\vdots \\
\hat{\gamma}_{L}^{H} \boldsymbol{u}_{C}
\end{array}\right]
\end{aligned}
$$

where we used the identity $\varphi^{H}\left(s_{l}\right) \boldsymbol{\varphi}\left(s_{l}\right)=M, l=1, \ldots, L$. Continuing the maximization with respect to $\sigma^{2}$ yields

$$
\begin{align*}
& \underset{\gamma_{C}, \sigma^{2}}{\arg \max } f_{H_{1}} \Rightarrow \frac{d \ln f_{H_{1}}\left(\boldsymbol{u}_{C} ; \hat{\boldsymbol{\gamma}}_{C}, s_{C}, \sigma^{2}\right)}{d\left(\sigma^{2}\right)} \\
& \quad=-\frac{\pi^{L M} L M \sigma^{2(L M-1)}}{\pi^{L M} \sigma^{2(L M)}}+\frac{\left(\boldsymbol{u}_{C}-\boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right)^{H}\left(\boldsymbol{u}_{C}-\boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right)}{\sigma^{4}} \\
& \quad=-\frac{L M}{\sigma^{2}}+\frac{\left(\boldsymbol{u}_{C}-\boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right)^{H}\left(\boldsymbol{u}_{C}-\mathbf{\Phi}_{C} \hat{\gamma}_{C}\right)}{\sigma^{4}}=0 \tag{A.11}
\end{align*}
$$

which provides

$$
\begin{equation*}
\hat{\sigma}^{2}=\frac{\left(\boldsymbol{u}_{C}-\mathbf{\Phi}_{C} \hat{\boldsymbol{\gamma}}_{C}\right)^{H}\left(\boldsymbol{u}_{C}-\mathbf{\Phi}_{C} \hat{\boldsymbol{\gamma}}_{C}\right)}{L M} \tag{A.12}
\end{equation*}
$$

Consequently, the maximum of pdf under the hypothesis $H_{1}$ with respect to $\gamma_{1}$ and $\sigma^{2}$ is

$$
\begin{align*}
& \max _{\boldsymbol{\gamma}_{C}, \sigma^{2}} f_{H_{1}}\left(\boldsymbol{u}_{C} ; \boldsymbol{\gamma}_{C}, \boldsymbol{s}_{C}, \sigma^{2}\right)  \tag{A.13}\\
& =\frac{1}{\pi^{L M}\left(\frac{\left(u_{C}-\boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right)^{H}\left(u_{C}-\boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right)}{L M}\right)^{L M}} e^{-L M} .
\end{align*}
$$

Next, considering the term

$$
\begin{align*}
&\left(\boldsymbol{u}_{C}-\boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right)^{H}\left(\boldsymbol{u}_{C}-\mathbf{\Phi}_{C} \hat{\gamma}_{C}\right) \\
&=\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\boldsymbol{u}_{C}^{H} \boldsymbol{\Phi}_{C} \hat{\gamma}_{C}-\hat{\gamma}_{C}^{H} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C}+\hat{\gamma}_{C}^{H} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{\Phi}_{C} \hat{\gamma}_{C}\right) \\
&=\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\boldsymbol{u}_{C}^{H} \boldsymbol{\Phi}_{C} \frac{1}{M} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C}-u_{C}^{H} \boldsymbol{\Phi}_{C} \frac{1}{M} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C}\right. \\
&\left.+\boldsymbol{u}_{C}^{H} \boldsymbol{\Phi}_{C} \frac{1}{M} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{\Phi}_{C} \frac{1}{M} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C}\right) \\
&=\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\frac{1}{M} \boldsymbol{u}_{C}^{H} \mathbf{\Phi}_{C} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C}-\frac{1}{M} \boldsymbol{u}_{C}^{H} \mathbf{\Phi}_{C} \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C}\right. \\
&\left.+\frac{1}{M^{2}} \boldsymbol{u}_{C}^{H} \boldsymbol{\Phi}_{C}\left(M \mathbf{I}_{L \times L}\right) \boldsymbol{\Phi}_{C}^{H} \boldsymbol{u}_{C C}\right) \\
&=\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\frac{1}{M} \boldsymbol{u}_{C}^{H} \mathbf{\Phi}_{C} \mathbf{\Phi}_{C}^{H} \boldsymbol{u}_{C}\right) \\
&=\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}\right) \boldsymbol{u}_{l}\right|^{2}\right) \tag{A.14}
\end{align*}
$$

and substituting it into (A.14) yields

$$
\left.\begin{array}{l}
\max _{\gamma_{C}, \sigma^{2}} f_{H_{1}}\left(\boldsymbol{u}_{C} ; \boldsymbol{\gamma}_{C}, s_{C}, \sigma^{2}\right)  \tag{A.15}\\
\pi^{L M}\left(\frac{1}{u_{C}^{H} u_{C}-\sum_{l=1}^{L} \frac{1}{M}\left|\varphi^{H}\left(s_{l}\right) u_{l}\right|^{2}}\right. \\
L M
\end{array}\right)^{-L M} .
$$

The last maximization of $f_{H 1}$ must be performed with respect $\boldsymbol{s}_{\mathrm{C}}$

$$
\begin{align*}
& \underset{s_{C}}{\arg \max } f_{H_{1}}\left(\boldsymbol{u}_{C} ; \hat{\gamma}_{C}, \boldsymbol{s}_{C}, \hat{\sigma}^{2}\right) \\
& =\underset{\boldsymbol{s}_{C}}{\arg \max } \frac{e^{-L M}}{\pi^{L M}\left(\frac{\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}\right) u_{l}\right|^{2}}{L M}\right)^{L M}} \\
& =\frac{e^{-L M}}{\underset{{ }^{s} C}{\arg \min } \pi^{L M}}\left(\frac{u_{C}^{H} u_{C}-\sum_{l=1}^{L} \frac{1}{M}\left|\varphi^{H}\left(s_{l}\right) u_{l}\right|^{2}}{L M}\right)^{L M}  \tag{A.16}\\
& =\frac{e^{-L M}}{\pi^{L M}\left(\frac{u_{C}^{H} u_{C}^{-\operatorname{argmax}} \sum_{l=1}^{L} \frac{1}{M}\left|\varphi^{H}\left(s_{l}\right) u_{l}\right|^{2}}{L M}\right)^{L M} .}
\end{align*}
$$

The GLRT can now be written as

$$
\begin{align*}
\Lambda & =\frac{\max _{C}, \boldsymbol{s}_{C}, \sigma^{2}}{\max _{\sigma^{2}} f_{H_{1}}\left(\boldsymbol{u}_{C} ; \boldsymbol{\gamma}_{C}, \boldsymbol{s}_{C}, \sigma^{2}\right)} \\
& \left.=\frac{\left(\boldsymbol{u}_{C} ; \sigma^{2}\right)}{\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}\right)^{L M}} \boldsymbol{u}_{C}-\max _{\boldsymbol{s}_{C}} \sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}\right) \boldsymbol{u}_{l}\right|^{2}\right)^{L M} \tag{A.17}
\end{align*}{ }_{H_{0}}^{H_{1}} \eta .
$$

After some manipulations, we achieve

$$
\begin{align*}
& \boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C} \stackrel{H_{H_{0}}^{>}}{>} \eta^{\prime}\left(\boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}-\max _{s_{C}} \sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}\right) \boldsymbol{u}_{l}\right|^{2}\right) \\
& \boldsymbol{u}_{C}^{H} \boldsymbol{u}_{C}\left(\frac{1-\eta^{\prime}}{\eta^{\prime}}\right) \stackrel{H_{1}}{>}-\max _{\boldsymbol{s}_{C}} \sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}\right) \boldsymbol{u}_{l}\right|^{2}  \tag{A.18}\\
& \frac{\max _{s_{C}} \sum_{l=1}^{L} \frac{1}{M}\left|\boldsymbol{\varphi}^{H}\left(s_{l}\right) \boldsymbol{u}_{l}\right|^{2}}{\sum_{l=1}^{L} \boldsymbol{u}_{l}^{H} \boldsymbol{u}_{l}} \stackrel{H_{1}}{>} \underset{H_{0}}{<}\left(\frac{\eta^{\prime}-1}{\eta^{\prime}}\right)
\end{align*}
$$

where $\eta^{\prime}=\eta^{1 / L M}$, and where result (A.8) was used.
Result (A.18) involves a maximization with respect to the vector $\boldsymbol{s}_{C}$, collecting the elevations of the cluster of $L$ pixels. Such a maximization provides the general clustered GLRT (6) for the elevation vector $\boldsymbol{s}_{C}$ generally parametrized using a parameter vector $\boldsymbol{\alpha}$, MGLRT (8) when all the elevations $s_{l}$ are equally set to a single value $s_{0}$, and LP-GLRT (11) when the elevation vector $\boldsymbol{s}_{C}$ is parametrized using the parameter vector $\boldsymbol{\alpha}=\left[\begin{array}{lll}s_{00} & k_{x} & k_{r}\end{array}\right]^{T}$, representing an LP. Note that in case of the LP-GLRT, the double summation on $p=-L_{x}, \ldots, L_{x}$, and $q=-L_{r}, \ldots, L_{r}$ presents in (11), is perfectly equivalent to the single summation on $l=1, \ldots, L$ presents in (A.18), as both allow to consider all pixels of the considered patch of dimension $L=L_{x} \times L_{r}$.
The threshold at the right-hand side of (A.18) is set by fixing an assigned $P_{\mathrm{FA}}$. The $P_{\mathrm{FA}}$ is the probability that the left-hand side is larger than the threshold when the scatterer is absent.

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