Effect of rear walls on the rocking response of rock blocks under seismic excitations

To cite this article: M Mennitti *et al* 2021 *IOP Conf. Ser.: Earth Environ. Sci.* **833** 012085

Effect of rear walls on the rocking response of rock blocks under seismic excitations

M Mennitti, R M S Maiorano and S Aversa

Parthenope University of Naples, Italy

Abstract. The rocking response of rigid free standing bodies subjected to seismic excitation has been studied by many researchers interested in different slender elements such as ancient stone columns, tombstones, rigid building structures. The extension of this model to rock mechanics has been proposed by a few authors. The rocking response of rectangular free standing bodies subjected to horizontal accelerations of natural recorded motions showed that the pseudo-static approach, based on Peak Ground Acceleration (PGA), permits only the determination of the uplift conditions and the beginning of rocking. It does not permit to evaluate the overturning of the blocks. The combined effect of vertical and horizontal seismic motions is negligible and, in some cases, beneficial. This paper presents a new mechanical model, called "one-sided rocking", that takes into account the presence of a rear rigid wall, that is a typical scenario for the rock blocks completely detached from the cliff but close to it. The dynamic response of a great number of rectangular rigid blocks, subjected to 62 recorded earthquake motions on rock soil (from US, Europe and Asia), has been analysed considering only the horizontal acceleration. The results show that the presence of the wall is detrimental for the rocking stability. However, there is still a safety reserve more significant for large blocks and rich frequency content time histories. This reserve could be taken into account in simplified (pseudo-static) analyses through reductive coefficient of PGA.

1. Introduction

During an earthquake, different slender elements such as ancient stone columns, tombstones, furniture, reservoirs, electrical equipment may slide, rock or slide-rock and overturn. *Rocking* motion of a rigid block on a rigid plane subjected to dynamic actions, presented in this work, has been focused on toppling.

Starting from the pioneering work of [1] a number of contribution may be found in literature. The first studies approached the problem by analyzing the dynamic behaviour of the slender elements defining the equations and the parameters affecting the motion by means of deterministic approaches, based on the integration of motion ([2], [3], [4], [5], and [6]) or probabilistic approaches, based on fragility curves ([7], [8], [9]). Other Authors proposed numerical and approximate closed form solutions limited to rectangular and sinusoidal pulses of half-cycle duration ([10], [11], [12], [6], [13]) and also analytical formulations based on natural recorded motions ([3], [5], [14], [15], [16], [17], [18], [19]). Just a few Authors extended this model to rock mechanics ([20], [21]).

Maiorano *et al.* [22] analysed the rocking response of rigid rectangular bodies free to rotate in both sides (*two-sided rocking*) and subjected both to simple pulses, such as sinusoidal pulses of one-cycle duration, and to natural earthquakes by considering only the horizontal accelerations. The analyses were carried out by means of a mechanical model developed in SIMULINK extension in Matlab [26]. The results showed that the dynamic response of a rigid body on a stiff horizontal plane is very complex and it is not directly related to the Peak Ground Acceleration (PGA), but it is strongly affected from both size and slenderness parameters of blocks. The authors proposed a seismic reduction coefficient β for pseudo-static analysis, based on the results of dynamic analysis carried out for different rectangular blocks subjected to natural earthquakes. Maiorano *et al.* [22] defined the coefficient β as the dimensionless block base ratio b/b_{ps}, where b is the minimum stable base obtained from dynamic analysis and b_{ps} is the minimum stable base obtained from pseudo-static analysis.

In the present paper the model developed by Maiorano *et al.* [22] is extended to consider the effects of the presence of a rear wall, that is the typical scenario for the rock blocks completely detached from the cliff but close to it.

2. Problem definition

2.1. Rocking motion of rigid block against the wall

Unlike free-standing blocks, which can oscillate on both sides at their base (*two-sided rocking*), there are some situations where the rocking occurs only on one side (*one-sided rocking*). This scenario is very common for the rock blocks adjacent to the cliff. Very few studies have addressed on *one-sided rocking* problem unlike to *two-sided rocking.*

A similar problem was studied by Hogan [23]: he investigated the rocking response of household objects, such as slender furniture placed against walls, during earthquakes. Winkler *et al.* [24] studied *one-sided rocking* by means a series of shacking table tests and numerical analysis with Distinct Element Method observing that for lower frequencies, *one-sided rocking* is more stable than *two-sided*, while for higher frequencies is the opposite. Sigurdsson *et al.* [25] resumed this study for household furniture, like the IKEA BRIMNESS bookshelf. They showed that the *one-sided rocking* is less safe than *two-sided rocking*. Other works on *one-sided rocking* are related to masonry structures to study the out-of-plane behaviour of masonry walls (Giresini *et al.* [9]).

The configuration of the problem is illustrated in figure 1. The rigid block on a stiff horizontal plane has a rectangular shape with uniformly distributed mass m and dimension $2b$ times $2h$. The block is characterised by a dimensionless parameter $a=b/h$, or the equivalent critical angle of rotation $\alpha=\tan^{-1}(b/h)$, and a size parameter $R = (h^2 + b^2)^{0.5}$ that is the radial distance from the centre of gravity. Depending on the characteristic of the horizontal acceleration of the stiff ground and the properties of the frictional interface, the block may rest, slide, rock, or slide-rock.

Assuming that the coefficient of friction is large enough to prevent sliding, the rigid block, subjected to a horizontal acceleration $\ddot{u}_g(t)$ of the stiff ground, rests until the overturning moment is less than the restoring moment and begin to rotate around points O or O' as soon as:

$$\left|\frac{\ddot{u}_g}{g}\right| \geq a \tag{1}$$

With reference to figure 1, the equations of motion for the rotation θ of a rigid block with rotational inertia I_0 subjected to a horizontal acceleration $\ddot{u}_g(t)$ when rocking respectively around O and O' are:

$$I_0\ddot{\theta}(t) + mgR\sin\left(-\alpha - \theta(t)\right) = -m\ddot{u}_g(t)R\cos\left(-\alpha - \theta(t)\right) \qquad \theta(t) < 0 \tag{2}$$

$$I_0\ddot{\theta}(t) + mgR\sin\left(\alpha - \theta(t)\right) = -m\ddot{u}_g(t)R\cos\left(\alpha - \theta(t)\right) \qquad \theta(t) > 0 \tag{3}$$

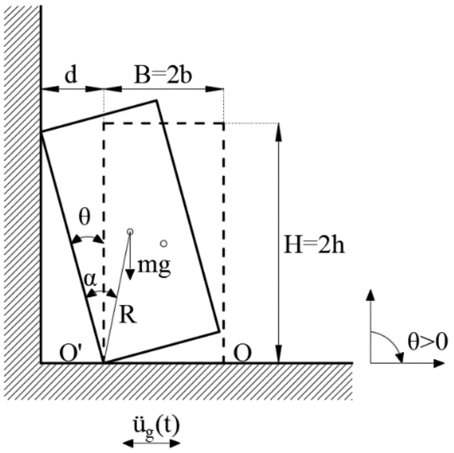

Figure 1. Rigid rectangular block under rocking motion close to a wall

For a uniform rectangular block, the polar moment of inertia I_0 about corner point is:

$$I_0 = \frac{4}{3} m R^2 \tag{4}$$

Introducing the sign function, equations (2) and (3) can be expressed in the compact form:

$$\ddot{\theta}(t) = -p^2 \left\{ \sin\left[\alpha \operatorname{sgn}\left(\theta(t)\right) - \theta(t)\right] + \frac{\ddot{u}_g(t)}{g} \cos\left[\alpha \operatorname{sgn}\left(\theta(t)\right) - \theta(t)\right] \right\} \tag{5}$$

where:

$$p = \sqrt{\frac{3g}{4R}} \tag{6}$$

is a frequency parameter (rad/s) of the block and is an expression of its size. During rocking motion, when the block hits on the ground, it loses a part of its kinetic energy. Its angular velocity after each impact (at time t^+_0) is a fraction of that just prior to impact (at time t^+_0):

$$\dot{\theta}^2\left(t_0^+\right) = r \cdot \dot{\theta}^2\left(t_0^-\right) \tag{7}$$

Conservation of angular momentum before and after the impact gives the maximum coefficient of restitution [1]:

$$r = \left(1 - \frac{3}{2} \sin^2 \alpha\right)^2 \tag{8}$$

When the block hits the rear wall, an elastic impact is assumed without the change of the centre of rotation. This implies a simple reverse of the velocity.

2.2. Numerical model

Maiorano *et al.* [22] developed a two dimensional mechanical model (RHA model: Rocking with Horizontal Accelerations) in the SIMULINK extension of Matlab extension [26] to analyze *two-sided rocking*. The non-linear equation (5) is solved with a numerical integration by means of standard Ordinary Differential Equations (ODE3) solver.

In the present paper the mechanical model was extended to include the presence of the adjacent wall with a limit rotation (with reference to figure 1, is a counterclockwise or negative rotation), related to the case study.

3. One-sided rocking response to a one-sine pulse

Analyses of *one-sided rocking* subjected to dynamic actions have been carried out varying the amplitude and the frequency of a simple one-sine pulse for a rigid rectangular block, of dimensions b=0.5 m and h=2.5 m. The excitation, modelled as a sinewave of one-cycle duration, is similar to a natural record because it starts from zero amplitude. Unlike *two-sided rocking*, in this case the problem is asymmetric and therefore all toppling simulations have been doubled by changing the sign of the initial acceleration in order to take into account the different position of the wall, in both sides.

With reference to figure 1, if the motion starts from positive accelerations, the block initially rotate with negative rotations and it always hits against the wall. If the motion starts from negative accelerations, the block rotate with positive rotations and four scenarios are possible:

- the block overturns without impact;
- the block has counterclockwise rotations without impact against the wall, the velocity decreases with time after each impact against the ground and it overturns;
- the block has counterclockwise rotations, impacts the wall and overturns;
- the block does not overturn.

Considering two distances, d, between the block and the wall of 1 and 10 cm, a wide range of accelerations between 0 and 5 g and a range of frequencies between 0 and 2.5 Hz, the overturning acceleration spectra for the slender block investigated are illustrated in figures 2(a), 2(b), 3(a) and 3(b). The spectra in figures 2(a) and 2(b) report the toppling potential for initial positive accelerations. It is possible to define two different areas: “safe” region (blue area), where the rocking rotations are damped until expiring and “overturning with impact” region (yellow area). On the other hand, the spectra in figures 3(a) and 3(b) for initial negative accelerations show another region between the previous: the “overturning with no impact” region.

The results show that the overturning area increases when the block always hits against the wall (for positive accelerations) and in particular a distance of 10 cm between the block and the wall is detrimental for the block’s stability. It means the presence of wall makes less stable the blocks.

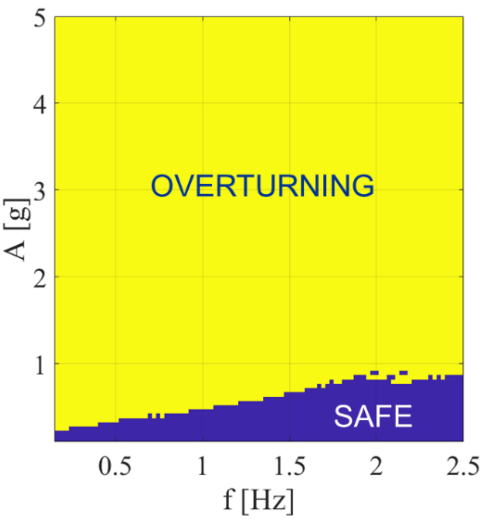

Figure 2(a). Overturning accelerations spectrum of slender block (p=1.70 rad/s, α=0.197 rad, r=0.89, b=0.5 m, h=2.5 m, d=1 cm) under one-sine pulse with positive acceleration

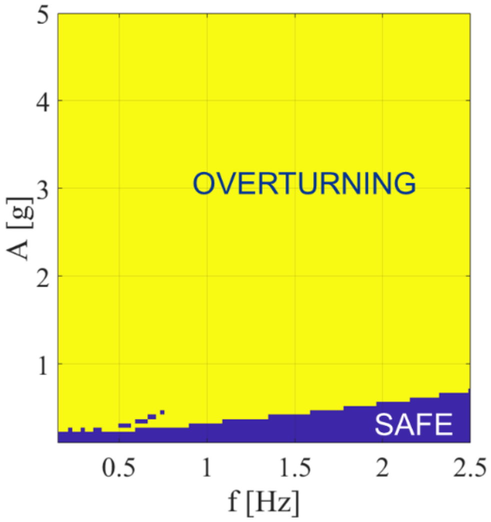

Figure 2(b). Overturning accelerations spectrum of slender block (p=1.70 rad/s, α=0.197 rad, r=0.89, b=0.5 m, h=2.5 m, d=10 cm) under one-sine pulse with positive acceleration

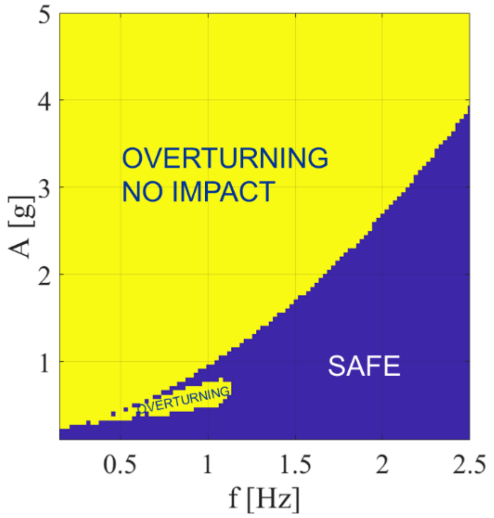

Figure 3(a). Overturning accelerations spectrum of slender block (p=1.70 rad/s, α=0.197 rad, r=0.89, b=0.5 m, h=2.5 m, d=1 cm) under one-sine pulse with negative acceleration

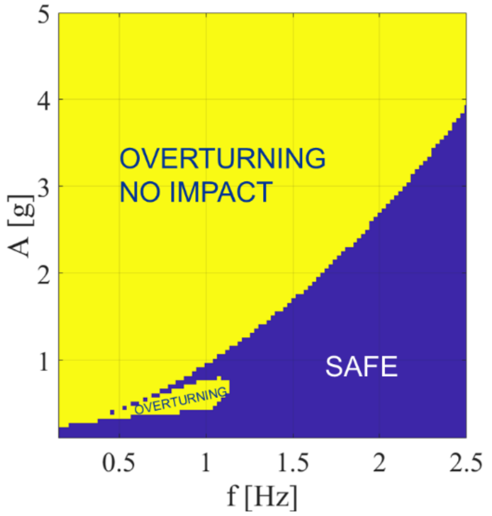

Figure 3(b). Overturning accelerations spectrum of slender block (p=1.70 rad/s, α=0.197 rad, r=0.89, b=0.5 m, h=2.5 m, d=10 cm) under one-sine pulse with negative acceleration

4. One-sided rocking response to earthquake ground motions

This section provides the numerical results of the *one-sided rocking* due to seismic excitation. The seismic database used in the analysis has been taken from Maiorano *et al.* ([22], [27]), considering the horizontal component of acceleration. These records are representative of significant events and cover a wide range of strong motion parameters [28] and are collected from International, European and Italian earthquakes.

For each earthquake the rocking response has been computed for 400 rectangular blocks, meaning for 20 values of slenderness parameter a, ranging between 0.1 and 0.5, and 20 values of b, ranging from 0.1 m to 1 m. The blocks investigated were placed at a distance d of 1 and 10 cm from the wall.

The stability analyses have been carried out for all the investigated blocks and the earthquakes of the seismic database of Maiorano *et al.* ([22], [27]). The results are reported on the β-T_p/T_m plane, where β is the reductive coefficient of the seismic actions, proposed by Maiorano *et al.* [22]:

$$\beta = \frac{b}{b_{ps}} = \frac{PGA}{b \cdot h} \tag{9}$$

and T_p/T_m a dimensionless parameter, representative of frequency characteristics of the ground motion and block dimensions. T_p is the inverse of the frequency parameter p:

$$T_p = \frac{2\pi}{p} = 2\pi\sqrt{\frac{4R}{3g}} \tag{10}$$

while T_m is the mean period proposed by [29] of the horizontal component of the acceleration time history.

The upper bound curve found by Maiorano *et al.* [22] for the *two-sided rocking* and given by the equation

$$\beta = 1.35 \cdot \exp[-0.12 \cdot (T_p/T_m)] \tag{11}$$

has been superposed on the rocking spectrum reported in figures 4 and 5.

The plot in figure 4 shows the results of the dynamic analyses for the blocks placed at a distance *d* of 1 cm from the wall. The rocking spectrum highlights a reduction of the safety reserve compared to *two-sided rocking*, as shown by the points above the curve: this means that the presence of the wall is detrimental for the rigid block stability. In particular, we observe an increase of the reductive coefficient β for the larger blocks (T_p/T_m>10).

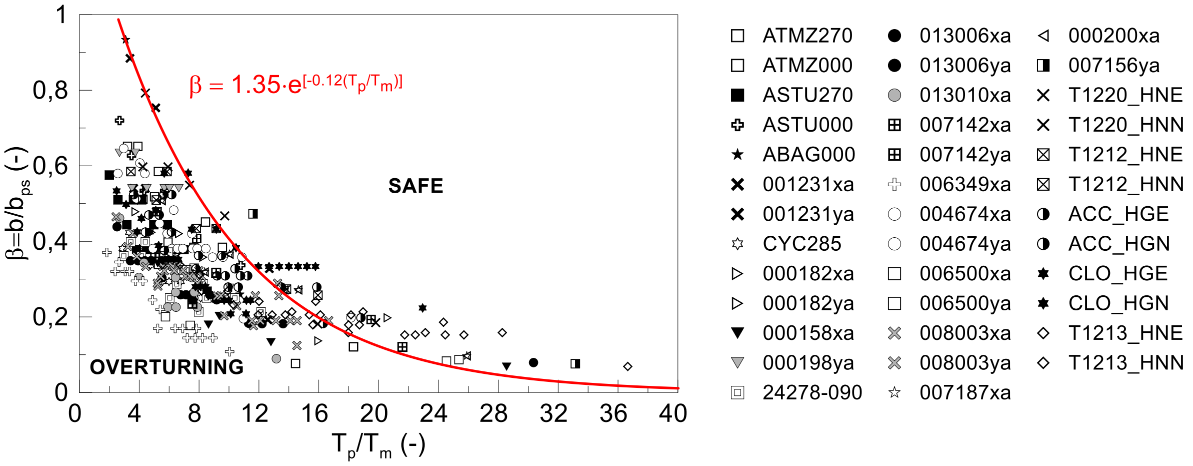

Figure 4. Rocking spectrum of rigid blocks placed at a distance *d* of 1 cm from the wall

This effect becomes more important for the rigid blocks placed at a distance *d* of 10 cm from the cliff. It can be observed an increase of the point above the upper bound curve that corresponds to a reduction of the safety reserve (figure 5). These results show a good agreement with those carried out by simple excitations. In particular, short distances between the rigid block and the wall are more safe than the greater ones. For higher distances the rocking tends to the two-sided case.

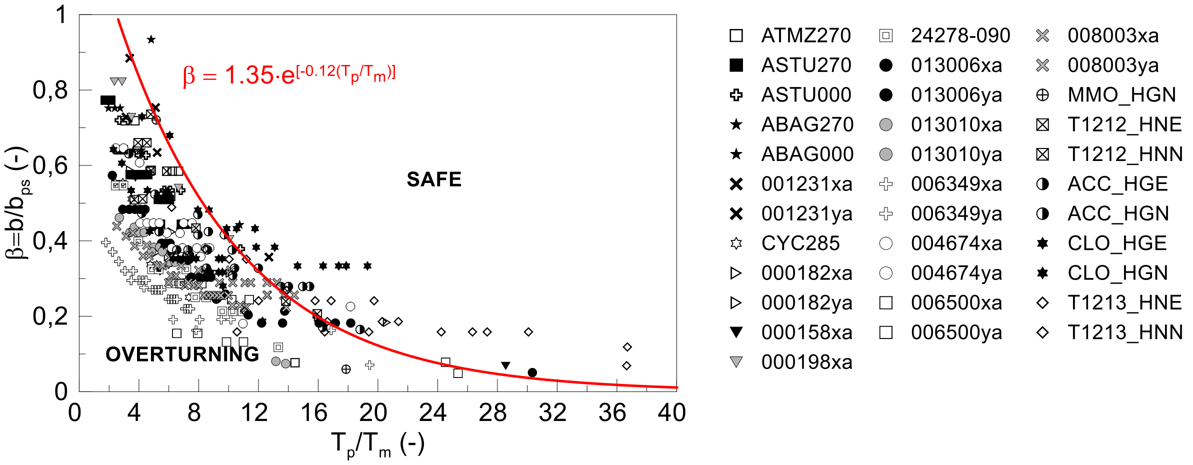

Figure 5. Rocking spectrum of rigid blocks placed at a distance *d* of 10 cm from the wall

5. Concluding remarks

The paper investigates the rocking response of rigid rectangular blocks placed close to a rigid wall subjected to simple excitations and to natural earthquakes.

When the blocks are subjected to a simple sinewave of one-cycle duration, the distance between the block and the wall has a strong effect on the toppling potential. The stability of the blocks decreases when the distance increases from 1 to 10 cm. For larger distances, not illustrated in the present paper, the rocking tends to the two-sided case.

In order to compare the *two-sided rocking* with *one-sided rocking*, the upper bound curve, found by Maiorano *et al.* [22], has been superposed on the dynamic results of the analysis with natural records considering the horizontal component of the acceleration. The blocks investigated are placed at a distance *d* of 1 and 10 cm from the cliff and when they are subjected to natural earthquakes, the presence of the wall is detrimental for the rocking stability, in particular for larger blocks.

The negative effect of the wall is greater for the blocks placed at a distance *d* of 10 cm from the wall, as already observed in the analyses with simple sine excitations.

References

[1] Housner G W 1963 *The behaviour of inverted pendulum structures during earthquakes* Bulletin of the Seismological Society of America vol 53 n 2 pp 403-417

[2] Ishiyama Y 1983 *Motions of rigid bodies and criteria for overturning by earthquake excitations* Third South Pacific Regional Conference on Earthquake Engineering (Wellington)

[3] Shi B Anooshehpoor A Zeng Y Brune J N 1996 *Rocking and Overturning of Precariously Balanced Rocks by Earthquakes* Bullettin of the Seismological Society of America vol 86 n 5 pp 1364-1371

[4] Zhang J Makris N 2001 *Rocking response of free standing blocks under cycloidal pulses* Journal of Engineering Mechanics vol CXXVII (Reston: American Society of Civil Engineers) pp 473-483

[5] Makris N Konstantinidis D 2003 *The rocking spectrum and the limitations of practical design methodologies* Earthquake Engineering and Structural Dynamics vol 32 pp 265–289

[6] Kounadis K 2010 *On the overturning instability of a rectangular rigid block under ground excitation* The Open Mechanics Journal vol 4 pp 43-57

[7] Yim C S Chopra K A Penzien J 1980 *Rocking response of rigid blocks to earthquakes* Earthquake Engineering and Structural Dynamics vol 8 pp 565-587

[8] Dimitrakopoulos E G Paraskeva T S 2015 *Dimensionless fragility curves for rocking response to near fault excitations* Earthquake Engineering and Structural Dynamics vol 44 n 12 pp 2015-2033

[9] Giresini L Casapulla C Denysiuk R Matos J Sassu M 2018 *Fragility curves for free and restrained rocking masonry façades in one-sided motion* Engineering Structures vol 164 (Amsterdam: Elsevier) pp 195-213

[10] Spanos P D Koh A S 1984 *Rocking of rigid blocks due to harmonic shaking* Journal of Engineering Mechanics vol 110 (Reston: American Society of Civil Engineers) pp 1627-1642

[11] Makris N Roussos Y 1998 *Rocking response and overturning of equipment under horizontal pulse-type motions* Report Pacific Earthquake Engineering Research Center 1998/05 (Berkley: College of Engineering, University of California)

[12] Makris N Zhang J 1999 *Rocking response and overturning of anchored equipment under seismic excitations* Report Pacific Earthquake Engineering Research Center 1999/06 (Berkley: College of Engineering, University of California)

[13] Voyagaki E Psycharis I N and Mylonakis G 2013 . *Rocking response and overturning criteria for free standing rigid blocks to single - lobe* Soil Dynamics and Earthquake Engineering vol 46 (Amsterdam: Elsevier) pp 85-95

[14] Kaneko M Hayashi Y G 2004 *A proposal for simple equations to express a relation between overturning ratios of rigid bodies and input excitations* Proceedings of the 13th World Conference on Earthquake Engineering

[15] Apostolou M Gazetas G Garini E 2007 *Seismic response of slender rigid structures with foundation uplifting* Soil Dynamics and Earthquake Engineering vol 27 (Amsterdam: Elsevier) pp 642-654

[16] Arredondo C A Reinoso E 2008 *Influence of Frequency Content and Peak Intensities in the*

Rocking Seismic Response of Rigid Bodies Journal of Earthquake Engineering vol 12 pp 517-533

[17] Chatzis M N Smyth A W 2012 *Robust Modeling of the Rocking Problem* Journal of Engineering Mechanics vol 138 (Reston: American Society of Civil Engineers) pp 247-262

[18] Vassiliou M F Mackie K R Stojadinovic B 2013 *Rocking response of slender flexible columns under pulse excitation* COMPDYN 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering

[19] Vassiliou M F Mackie K R Stojadinovic B 2013 *Rocking and sliding of unanchored bodies subjected to seismic load according to conventional and nuclear rules* COMPDYN 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering

[20] Schurch P, Becker A 2005 *Studies on 'precarious rocks' in the epicentral area of the AD 1356 Basle earthquake, Switzerland* Geophisical Journal International vol 163 (Oxford. Oxford University Press) pp 689-697

[21] Brune J N Pruvance M D and Anooshehpoor A 2005 *Gauging Earthquake Hazards with Precariosly Balanced Rocks* American Scientist vol 95

[22] Maiorano R M S Adinolfi M Aversa S 2015 *Rocking of slender rock blocks under seismic excitation* Rivista Italiana di Geotecnica vol 49 pp 87-101

[23] Hogan S 1992 *On the motion of a rigid block, tethered at one corner, under harmonic forcing* Proceedings of the Royal Society of London: Mathematical and Physical Sciences vol 439 pp 35-45

[24] Winkler T Meguro K Yamazaki F 1995 *Response of rigid body assemblies to dynamic excitation* Earthquake Engineering and Structural Dynamics vol 24 pp 1389-1408

[25] Sigurdsson G Ö Rupakhety R Ólafsson S 2017 *A study of rigid blocks rocking against rigid wall* International Conference on Earthquake engineering and Structural Dynamics, Reykjavík, Iceland, 12-14 June 2017

[26] MATLAB R2019a. The language of Technical Computing. The Mathworks, Inc.: Natick, MA, R2019a

[27] Adinolfi M Maiorano R M S Aversa S 2019 *On the stability of slender blocks subjected to horizontal and vertical seismic accelerations* International Conference on Earthquake and Geotechnical Engineering (VII ICEGE), Rome, 17-20 June 2019.

[28] Kramer S L 1996 *Geotechnical earthquake engineering* (Upper Saddle River: Prentice Hall)

[29] Rahje M E Abrahamson N A Bray J D *Simplified frequency content estimates of earthquake ground motions* Journal of Geotechnical and Geoenvironmental Engineering vol 124 (Reston: American Society of Civil Engineers) pp 150-159