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# Microscopic Disruption Management: Energy Consumption and Passenger Compensation Optimisation

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## Abstract

Rail operations are often disrupted by accidents that cause traffic to diverge from the scheduled operations, rendering it difficult to run the schedule as planned. In such a case, the operator must change the schedule to return to the original schedule. If passengers are delayed, a train operator may have a policy of economically compensating them (e.g., refunding ticket fare). Compensation amounts are usually determined by the length of the delay. As a result, it is critical to have a smart way of determining whether to accelerate trains to absorb delays, thus increasing energy usage, or to compensate passengers. This paper presents a mathematical model for determining the speed profile while taking passenger usage into account. The model determines the best sequence of operating regimes and switching points between them for a variety of different situations and train types, all while accounting for delays and passenger compensation policies. The aim is to reduce both the amount of energy consumed and the amount of compensation paid to passengers. There are constraints on traction and braking forces, train velocity, forces induced by vertical and horizontal track profile, and passenger compensation policy. The results of computational tests performed on practical problem instances of the Spanish rail operator RENFE are showed. The suggested approach is capable of producing strategies that strike an excellent balance between different managerial objectives.

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## 1. Introduction

When operating railway networks, some unexpected or inadvertent events can cause railway traffic to deviate from the planned or regular scenarios. Some of these incidents will make it difficult, if not impossible, to follow the original

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schedule. In such a case, the operator must change the planned schedule for the time interval of the incident, as well as perform additional recovery steps, in order to return to the original state.

Following the classification of the type of incidents made in Cadarso et al. (2015) and Cacchiani et al. (2014), on the one hand, infrastructure blockages, failing rolling stock, and crew shortages may all cause large-scale service interruptions. The first task after one of these events is to decide the level of needed active intervention, and arrangements must be developed to recover from the disrupted situation. We will refer to this type of event as a *disruption*. In the works by D'Ariano et al. (2007) and D'Ariano and Pranzo (2009), strategies for recovery actions and resolution of conflicts are presented. In Espinosa-Aranda and García-Ródenas (2013) and Cadarso et al. (2013), novel approaches are presented in which the demand of the system is also taken into account.

On the other hand, small-scale incidents occur when minor-to-moderate train delays occur. We will refer to these incidents as *perturbations* and they will be the type of incident dealt with in this research. As proposed in Cadarso et al. (2013) and Cadarso et al. (2015), these incidents can be fixed allowing delays to be mitigated by timetables' buffer times, but some services requiring active intervention such as velocity changes to re-accommodate the service.

Optimisation of train movements and control methods have received increasing attention in recent years by researchers and engineers. These methods may be employed to efficiently re-schedule service during perturbations, which do not require heavy intervention. Dealing with delay propagation may be an example. Efficient optimal control strategies can be categorised into three major classes: proportional-integral-derivative (PID) control techniques, fuzzy control methods and optimisation techniques (Yang et al., 2012). The review in Scheepmaker et al. (2017) provides extensive literature on train control oriented to efficient energy management. Optimal control-based driving strategies, which minimise energy consumption, can be found in several studies, such as for example Khmel'nitsky (2000), Liu and Golovitcher (2003), Albrecht et al. (2016a), and Albrecht et al. (2016b). The solutions are all compromises between driving times and energy-consumption, the two minimisation objectives concurrently optimised.

However, energy consumption may not be the only economic indicator. On top of minimisation of passenger inconvenience, a train operator may have the policy of economically compensating (e.g., refunding ticket fare) passengers when they incur in delays. This may be especially important if it is impossible to maintain schedule punctuality and costly actions are needed to obtain a new schedule as close as possible to the original one. Note that passenger inconvenience and compensation levels usually depend on the amount of delay. With this in mind, it seems plausible to take into account passenger's compensation as a component of an overall recovery strategy combining both, passenger compensation and actions for alleviating the perturbations that the timetable will experience after a disruption.

With this approach, this research presents a model to decide the extent to which energy consumption (in actions such as accelerating or braking a set of trains) must be jointly combined with an economic compensation to the set of passenger, accordingly to a set of compensation rules adopted by the operators. The aim is to obtain a prompt response once it is recognised that a set of services have experienced a level of delay such that intervention is required. Accordingly, a model and solution algorithm are developed with the previous objectives, being formulated as an optimal control problem which considers a set of trains, with known initial states. It will determine a set of control parameters to obtain optimal arrival times of those trains so that the total amount of energy plus economical compensations is minimal. A suitable method for approximating the optimal control model by discretization is developed. And because the resulting optimization problem is a mixed integer non-linear programming one, an algorithmic scheme based on the Benders decomposition is proposed, such that much better computational performance is obtained as compared to known commercial optimization packages.

The outline of the structure of this paper is as follows. Section 2 describes the problem in detail. The mathematical model is developed and discussed in Section 3. Section 4 presents the solution algorithm. Section 5 contains computational experiments. Finally, we draw some conclusions.

## 2. Problem description

Disruptions, such as those caused by infrastructure blockages, collapsing rolling stock, and crew shortages, necessitate large-scale adjustments to the schedule in order to recover. Leaving aside these major actions, when the effects of an incident on the rail network result in a significant delay of some services, there exists the possibility that these delays cannot be absorbed by the timetable buffers, which means recovery actions are needed resulting in incremental costs. In addition, according to European regulations on the railway sector, in case of delays, operators must compensate economically the passengers. And, on top of that, if the operator considers it is convenient, it is possible

Table 1. Compensations to passengers in case of delays for different service types by the main Spanish rail operator, RENFE.

Delay(")	AVE&AVANT	ALVIA	LD	MD	REG	CT <sub>1</sub>	CT <sub>2</sub>	SF
15	50	50	-	25	-	50	-	-
30	100	50	-	50	25	100	50	-
45	100	50	-	50	50	100	50	-
60	100	100	50	100	100	100	100	50
90	100	100	100	100	100	100	100	100

to improve the protection level to passengers. As an example, the compensations by the main Spanish rail operator, RENFE, are shown in Table 1. They are determined by: a) the length of the delay, b) the type of train service, and c) the fare charged to the passengers. The first column in Table 1 shows the delay thresholds over which passengers are entitled to compensation, which is offered as a percentage of the paid fare depending on train service types. The rest of the columns in the table, one per train service type (AVE, AVANT, ALVIA, LD, MD, REG, CT<sub>1</sub> and CT<sub>2</sub>), show the percentages of the fares to be reimbursed for each delay threshold. The last column in Table 1, SF, shows special compensation rules for discounted ticket fares. Consequently, there is an open question when looking for recovery plans, which asks for the trade-off between the following counter-posed key performance indexes: energy costs for recovering the schedule and the cost of compensations to passengers.

This research assumes that the set of services, which are (potentially) affected by delays, are defined on a line defined by its terminal stations and the path through the infrastructure connecting them. The considered rescheduling actions will affect the speed profiles of the train services. Note that the studied incidents here are perturbations, which do not usually require high intervention. It will be assumed that, a) no overtaking between trains may occur along their trajectories, and b) no energy is returned to the power line. The aim is to decide optimum speed profiles to reschedule the system at the lowest total cost, taking into account operational costs, such as energy consumption and passenger compensations, and passenger goodwill cost. The problem is stated as an optimal control problem, where speeds, positions and arrival times of each of the trains play the role of state variables with known initial values, whereas braking forces, acceleration forces and compensation to passengers play the role of control variables. Constraints on trains' dynamics, kinematic relationships, maximum velocities and on headway observation between consecutive trains are taken into account. Initial value conditions are stated for speeds and positions, which are assumed to be known at the starting time of the perturbation and/or recovery period. Also, because delays may propagate impacting to other trains, the problem needs to consider a subset of trains which can be potentially affected by delay propagation.

### 3. Mathematical model

This section describes the Train REScheduling and PASSenger COmpensation Model (TRESPACOM), which is a mixed integer nonlinear mathematical model. Prior to delving into the objective function and constraints, sets, parameters and variables are introduced.

#### 3.1. Sets and parameters

- $T$  is the set of trains indexed by  $t$ .
- $Z$  is the set of delay levels indexed by  $\zeta$ .
- $\Upsilon$  is the set of passenger types indexed by  $\nu$ .
- $d_t^\nu, \varphi_t^\nu$  are the number of passengers of type  $\nu$  and the fare that they pay traveling in train  $t$ , respectively.
- $c_{t,\nu}^\zeta$  is the percentage of the fare paid by a passenger of type  $\nu$  in train service  $t$  to be compensated with when suffering a delay of level  $\zeta$ .
- $c_\delta$  is the cost/loss of goodwill per unit of delay and passenger.
- $a_\zeta$  is the upper bound to the delay of level  $\zeta$  at arrival under which passengers do receive compensation, if and only if the delay at arrival is above  $a_0$ .
- $F_t, B_t$  are the maximum traction and braking force, respectively, per unit of mass for train  $t$ .
- $r_0, r_1, r_2$  are the independent, linear and quadratic terms, respectively, of the running resistance per unit of mass.

- $V$  is the speed limit, assumed to be constant on the line.
- $srtat_t, sata_t$  are the relative and absolute scheduled arrival times for train  $t$ , respectively. Relative time refers to local time for each train such that scheduled relative departure time is set to zero for every train.
- $ht_{t,t'}$  is the headway time between train  $t$  and train  $t'$ .
- $s_{ft}, s_t^0$  are the final and initial position for train  $t$ , respectively.
- $\xi$  is a penalty for the rate of change of the controls.
- $\sigma$  is a constant to convert energy consumption to expended money.

### 3.2. Variables

- $\alpha_t^\zeta \in \{0, 1\}$  is 1 if train  $t$  is delayed such that arrival time activates the delay of level  $\zeta$ ; 0, otherwise.
- $f_t(\tau) \in \mathbb{R}^+, b_t(\tau) \in \mathbb{R}^+$  are the relative traction and braking force, respectively, of train  $t$  at time instant  $\tau$ .
- $v_t(\tau) \in \mathbb{R}^+, s_t(\tau) \in \mathbb{R}^+$  are the speed and position, respectively, of train  $t$  at time instant  $\tau$ .
- $\tau_{ft} \in \mathbb{R}^+$  is the relative arrival time for train  $t$ . The delay for train  $t$  is then  $\delta_t = \tau_{ft} - srtat_t$ .

### 3.3. Objective function

The objective function (1) minimizes the amount of money to be paid at every compensation level and the energy consumption of all the trains  $t \in T$ , which is calculated as the product of power and time. The riding comfort is also taken into account, which is expressed as a function of the change in the control variable  $f_t(\tau)$ , since lowering its rate of change may increase passenger comfort.

$$\min z = \sum_{t \in T} \sum_{v \in V} \sum_{\zeta \in Z} d_t^v c_{t,v}^\zeta \varphi_t^v \alpha_t^\zeta + \sum_{t \in T} c_\delta \delta_t + \sigma \sum_{t \in T} \int_0^{\tau_{ft}} \left( F_t f_t(\tau) v_t(\tau) + \xi \frac{df_t(\tau)^2}{d\tau} \right) d\tau \quad (1)$$

### 3.4. Delay and compensation constraints

Constraints (2) state that only one compensation level can be active at most at any given time. Constraints (3) activate compensation levels for each train  $t \in T$  based on the amount of delay  $\delta_t$  experienced.

$$\sum_{\zeta \in Z} \alpha_t^\zeta \leq 1 \quad \forall t \in T, \zeta \in Z \quad (2)$$

$$\delta_t \leq a_0 + \sum_{\zeta \in Z} (a_\zeta - a_0) \alpha_t^\zeta \quad \forall t \in T \quad (3)$$

### 3.5. Train dynamics constraints

Constraints (4) are the dynamics equations of motion for all of the trains in the system; they show that the real acceleration of each train is equal to the traction force minus the braking force and resistance to motion. Constraints (5) are the kinematic constraints that apply to all of the trains in the system. Constraints (6) place an upper bound on the trains' speed, relative traction force, and relative braking force. Constraints (7) specify the initial and final conditions for each train in the system, and calculate the delay of the trains as  $\tau_{ft} - srtat_t$ . Note that for calculating delays for each train, local times are employed. This means  $srtat_t$  is used, which is the scheduled arrival time as measured from the origin of time for each train, i.e., the scheduled trip time.

$$\frac{dv_t(\tau)}{d\tau} = F_t f_t(\tau) - B_t b_t(\tau) - (r_0 + r_1 v_t(\tau) + r_2 v_t^2(\tau)) \text{sgn}(v_t(\tau)) \quad \forall t \in T \quad (4)$$

$$\frac{ds_t(\tau)}{d\tau} = v_t(\tau) \quad \forall t \in T \quad (5)$$

$$0 \leq v_t(\tau) \leq V, 0 \leq f_t(\tau) \leq 1, 0 \leq b_t(\tau) \leq 1 \quad \forall t \in T \quad (6)$$

$$v_t(0) = 0, v_t(\tau_{ft}) = 0, s_t(0) = s_t^0, s_t(\tau_{ft}) = s_{ft}, \delta_t = \tau_{ft} - srtat_t \quad \forall t \in T \quad (7)$$

### 3.6. Headway constraints

It is assumed that trains  $t \in T$  are initially subject to some ordering accordingly to their initial position, i.e.,  $s_{t_1}^0 \leq s_{t_2}^0 \leq \dots \leq s_{|T|}^0$ . Also assuming that overtaking is not an admissible maneuver, constraints (8) state that running times of different trains in the same infrastructure must respect headway times. Note that for ensuring headway times are respected, absolute times are employed. This means  $sata_t$  is used, which is the scheduled arrival time as measured from the origin of time of the overall problem. Without loss of generality, this modelling approach ensures headway times are maintained at given point in the railway infrastructure. However, the formulation could be easily extended to respect separation times at multiple points in the network. The problem shall be divided into multiple optimal control problems, which shall be coupled by means of initial and final conditions. Consequently, headway times could be ensured at the entry and exit of each of the optimal control problems. Due to space limitations, the mathematical exposition of this feature is left aside.

$$sata_{t_i} + \delta_{t_i} \geq sata_{t_{i+1}} + \delta_{t_{i+1}} + ht_{t_i,t_{i+1}} \quad \forall i = 1, \dots, |T| - 1, \quad (8)$$

The TRESPACOM is then composed of the objective function in (1) subject to constraints (2)-(8).

## 4. Solution approach

The challenge to solve the optimization model in the previous section consists in dealing with integral and differential equations, and also with integer and continuous variables. To discretize the functional that defines the objective function as well as the differential equations that define the constraints in the previous optimal control problem, we have used the third-degree Gauss Lobatto collocation rule, which, for example, applied to  $\dot{x} = f(x)$  results in  $x_{j+1} - x_j = \frac{\Delta\tau_j}{6} [f(x_j) + 4f(x_C) + f(x_{j+1})]$ , where  $f(x_C) = \frac{1}{2}(x_{j+1} + x_j) + \frac{\Delta\tau_j}{8}(f(x_j) - f(x_{j+1}))$ . Note that  $j$  is the index for the nodes in the time mesh, and  $\Delta\tau_j$  is the time step size. The discretization yields a non-linear integer programming problem with a difficult computational tractability. Because of that, we will present two approaches to solve it. Firstly, an approach based on available commercial software. Secondly, a Benders Decomposition-based approach following that of Barea et al. (2019). The objective is to provide optimal or near optimal solutions in a reasonable budget of computational time. The commercial software-based approach directly solves the mixed integer non-linear model using BONMIN. For the the Benders Decomposition-based method, that is described next, CPLEX and CONOPT are used.

Once the values of variables  $\alpha_t^\zeta$  are known or fixed, the rest of the problem consists of solving a traditional optimal control problem. Benders' decomposition strategy is iterative and it is based on divide-and-conquer: the original problem's variables are split into two subsets, belonging the first set of variables to a first-stage master problem or model (MM) and the second set of variables to a second-stage subproblem or submodel (SM) for a given first-stage solution. Here, the first-stage variable are given by variables  $\alpha_t^\zeta$ , and depending on their values the optimal control problem must be solved. Note that if they are not set to appropriate values in the master model, the submodel might be infeasible due to the need of big delays. Consequently, a new set of variables is defined:  $\lambda_t$ . These variables are used to enable dummy delays for the submodel in case the  $\alpha_t^\zeta$  variables are set such that delays must be above the given thresholds. Note that  $\lambda_t$  is heavily penalized ( $M$  is a big number) in order to eliminate it from the solution. Therefore, feasibility cuts are not needed, and only optimality cuts will be used.

The master model is given by (9)-(10), where  $\ell$  represents the iteration of the algorithm,  $\theta$  is a free variable,  $\mu^\ell$  is the value of the objective function of the submodel at each iteration ( $z_{SM}^*$ ), and  $\gamma_t^\ell$  is the dual variable of (discretized) constraints (3). The submodel is composed of the objective function in (11), constraints (4)-(8), and constraints (12). Note that the integral and derivative expressions in the submodel must be discretized following the previous Gauss-Lobatto collocation scheme.

$$\textbf{Master Model:} \quad \min z_{MM} = \theta + \sum_{t \in T} \sum_{v \in Y} \sum_{\zeta \in Z} a_t^v c_{t,v}^\zeta \varphi_t^v \alpha_t^\zeta \quad (9)$$

$$\text{Subject to: Constraints (2) and } \theta \geq \mu^\ell + \sum_{t \in T} (a_0 + \sum_{\zeta \in Z} (a_\zeta - a_0) \alpha_t^\zeta + \bar{\lambda}_t^\ell - \bar{\delta}_t^\ell) \gamma_t^\ell \quad \forall \ell \in L \quad (10)$$

$$\text{Sub-Model: } \min z_{SM} = \sum_{t \in T} c_\delta \delta_t + \sigma \sum_{t \in T} \int_0^{\tau_{f_t}} \left( f_t(\tau) v_t(\tau) F_t + \xi \frac{df_t(\tau)^2}{d\tau} \right) d\tau + \sum_{t \in T} M \lambda_t \quad (11)$$

$$\text{Subject to: Constraints (4) – (8) and } \delta_t \leq a_0 + \sum_{\zeta \in Z} (a_\zeta - a_0) \bar{\alpha}_t^\zeta + \lambda_t \quad \forall t \in T \quad (12)$$

The iterative solution approach based on the Benders algorithm is shown in Algorithm 1.

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**Algorithm 1** Benders Decomposition-based approach
 

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- 1: **Initialization**  $UB^\ell \leftarrow \infty$ ;  $LB^\ell \leftarrow -\infty$ ; Let  $\epsilon$  be a tolerance parameter; set the iteration counter  $\ell = 1$ .
  - 2: Solve the **master model**  $MM^\ell$  and let  $z_{MM^\ell}^*$  be its optimal value.
  - 3: Update the lower bound  $LB^\ell = \max \{ LB^{\ell-1}, z_{MM^\ell}^* \}$
  - 4: Update the submodel parameters:  $\bar{\alpha}_t^\zeta \leftarrow \alpha_t^{\zeta*}$  and solve the **sub-model**  $SM^\ell$ . Let  $z_{SM^\ell}^*$  be its optimal value.
  - 5: Update the master model parameters  $\bar{\gamma}_t^\ell \leftarrow \gamma_t^*$ ;  $\bar{\delta}_t^\ell \leftarrow \delta_t^*$ ;  $\bar{\lambda}_t^\ell \leftarrow \lambda_t^*$ ;  $\mu^\ell \leftarrow z_{SM^\ell}^*$
  - 6: Update the upper bound  $UB^\ell$  and evaluate the GAP:  $UB^\ell = \min \{ UB^{\ell-1}, z_{MM^\ell}^* - \theta^* + z_{SM^\ell}^* \}$ ,  $GAP = \frac{UB^\ell - LB^\ell}{LB^\ell}$
  - 7: **If**  $GAP < \epsilon$  **then** { **return**  $(\alpha_t^\zeta, \delta_t, f_t, b_t, v_t)$ ; **Stop** }
  - 8: Update the iteration counter  $\ell \leftarrow \ell + 1$ . **Go to 2**
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## 5. Computational experiments

The studies are focused on practical cases taken from the network of the main Spanish train operator, RENFE. Several case studies are discussed to demonstrate the applicability of the presented strategy. Firstly, an Optimal Control Model (OCM) is used to solve the trajectory of one train without delays. Secondly, the proposed mathematical model (TRESPACOM) is employed to solve several one train instances featuring different delays. Finally, TRESPACOM is also applied to several two train instances running the same infrastructure and featuring different delays.

All the mathematical models were coded in GAMS. Their sizes for presented case studies are shown in Table 2, where the number of discrete and continuous variables, constraints, and non-zero elements are provided. The proposed solution approach is tested against available mixed integer non-linear software, i.e., Bonmin, CPLEX and CONOPT are used in the the Benders Decomposition-based approach. A personal computer with an Intel Core i9 CPU at 3.7 GHz and 32 GB of RAM, running under Windows 10 64-Bit, was used for the tests.

Table 3 compares the solutions provided for the different case studies and approaches. Case studies are denoted in the first column by OCM and TRESPACOM( $x, y$ ), where  $x$  refers to the number of trains running consecutively and  $y$  to the minutes of delay of the first train. Note that the OCM case study consists of solving the optimal control problem for one train without any perturbation, i.e., delay. The second column shows the employed solution approach, namely, commercial software (CONOPT or BONMIN) and the proposed solution strategy (Decomposition). The rest of the columns show the values for the objective function, energy consumption, delay in seconds, monetary compensation and solution time in seconds. When the case study involves two trains, energy consumption, delay, and compensation are shown separately for each train.

When addressing problems featuring one train, it is worth noting that both approaches find the same solution, the decomposition doing it in lower computational times. Note that for rescheduling problems computational times are crucial. But, when the case study includes two trains, the commercial software based approach fails to obtain feasible solutions, while the presented approach can balance energy consumption with delays and compensations. It looks to minimize energy consumption in combination with compensation and loss of goodwill. When dealing with the case study of two trains, it is also remarkable that due to the delay of the first train and to headway times, the second train must be also delayed. Figure 1 gives more insights to these results. It shows the optimal speed profiles for TRESPACOM(2,30) and TRESPACOM(2,60).

Table 2. Number of variables, constraints and non-zeros

Item	OCM	TRESPACOM (1 train)	TRESPACOM (2 trains)
Discrete variables	0	2	4
Continuous variables	1,314	1,317	2,837
Constraints	1,307	1,310	2,824
Non-zero elements	5,320	5,330	11,071

Table 3. Computational results for several case studies

Case Study	Solution Approach	Objective Function	Energy Consumption	Delay	Compensation	CPU time
OCM	CONOPT	14,417.83	14,277.14	–	–	0.30
TRESPACOM (1, 30)	BONMIN	20,045.72	19,792.11	0.00	0.00	34.60
TRESPACOM (1, 30)	Decomposition	20,045.72	19,792.11	0.00	0.00	0.57
TRESPACOM (1, 45)	BONMIN	36,310.33	20,230.14	744.82	0.00	8.36
TRESPACOM (1, 45)	Decomposition	36,310.33	20,230.14	744.82	0.00	0.56
TRESPACOM (1, 60)	BONMIN	84,310.33	20,230.14	1,644.82	30,000	51.64
TRESPACOM (1, 60)	Decomposition	84,310.33	20,230.14	1,644.82	30,000	0.85
TRESPACOM (1, 75)	BONMIN	132,310.33	20,230.14	2,544.82	60,000	13.74
TRESPACOM (1, 75)	Decomposition	132,310.33	20,230.14	2,544.82	60,000	0.80
TRESPACOM (2, 30)	BONMIN	–	–	–	–	–
TRESPACOM (2, 30)	Decomposition	34,414.27	$t_1=19,778.01$ $t_2=14,258.79$	$t_1=0.00$ $t_2=0.00$	$t_1=0.00$ $t_2=0.00$	1.8
TRESPACOM (2, 45)	BONMIN	–	–	–	–	–
TRESPACOM (2, 45)	Decomposition	50,612.83	$t_1=20,180.88$ $t_2=14,258.79$	$t_1=756.92$ $t_2=0.00$	$t_1=0.00$ $t_2=0.00$	0.5
TRESPACOM (2, 60)	BONMIN	–	–	–	–	–
TRESPACOM (2, 60)	Decomposition	105,973.06	$t_1=20,343.22$ $t_2=15,874.11$	$t_1=1,610.41$ $t_2=260.41$	$t_1=30,000$ $t_2=0.00$	0.58
TRESPACOM (2, 75)	BONMIN	–	–	–	–	–
TRESPACOM (2, 75)	Decomposition	201,973.06	$t_1=20,343.22$ $t_2=15,874.11$	$t_1=2,710.41$ $t_2=1,360.41$	$t_1=60,000$ $t_2=30,000$	1.51

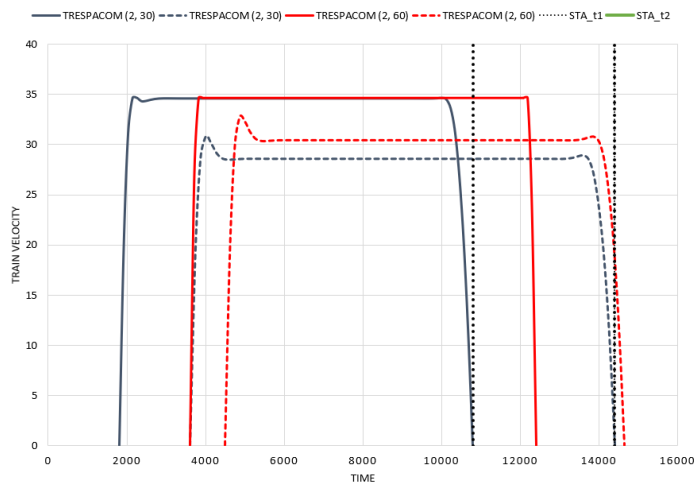


Fig. 1. Optimal speed profiles for different delays



## 6. Conclusions

In this paper, a novel approach to dealing with small-scale disruptions in rail transportation networks is discussed. The presented method determines the speed profile, the optimum sequence of operating regimes, and the switching points between them for a variety of different circumstances and trains, all while considering the train operator's delays and passenger compensation policies. A comprehensive review of the relevant literature revealed that this field of research has been scarcely addressed. The computational experiments on practical examples demonstrate the applicability of the proposed method. The aim of reducing energy usage while also reducing compensation costs is met. Furthermore, using a Benders Decomposition based solution strategy allows for near-real-time solution times, which is critical for rescheduling problems.

Future research will concentrate on case studies involving additional challenges, but also on developing some general rules that could allow practitioners to make informed decisions without running the whole approach for every disruption. Investigation of at what delay level the decision changes from speeding up and reducing delay to slowing down and using up all of the available time before an additional delay charge is incurred. Multiple types of trains with varying passenger loads, different route characteristics, and stopping patterns will be also addressed to better illustrate the model's capabilities. Furthermore, the focus will be posed on passengers, also addressing connecting passengers.

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