Mechanical model for the shear strength of steel fibers reinforced concrete beams (SFRC) without stirrups

Modelo mecánico para la resistencia a cortante de vigas de hormigón armado reforzado con fibras, sin armadura de cortante

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ABSTRACT

Despite the numerous studies made showing that the addition of steel fibres to concrete enhances the shear strength of RC beams, current design formulations are still empirical and present large scatter in front of the test results. In this paper, the previously developed Multi-Action Shear Model is extended to SFRC beams without stirrups, adopting an analytical formulation to evaluate the residual tensile stress of SFRC and incorporating the effects of the crack bridging capacity of SFRC in the shear resisted trough the different shear-transfer mechanisms. The proposed model predicts the tests results included in a recently published database with 448 shear tests with less scatter than any of the existing models.

RESUMEN

A pesar de que numerosos estudios realizados muestran que la adición de fibras al hormigón aumenta su capacidad a cortante, las formulaciones normativas están basadas en estudios empíricos y presentan gran dispersión frente a los resultados experimentales. En esta ponencia, el modelo mecánico MASM, previamente desarrollado para resistencia a cortante, se ha extendido para incorporar el efecto de la resistencia residual a tracción que proporcionan las fibras, sobre los distintos mecanismos resistentes a cortante. Se han comparado las predicciones del modelo con los resultados de 488 ensayos incluidos en una reciente base de datos, obteniendo menor dispersión que cualquiera de los modelos existentes.

KEYWORDS: Steel fibre-reinforced concrete (SFRC), shear strength, reinforced concrete, fibres, mechanical model, tests.

PALABRAS CLAVE: Hormigón reforzado con fibras, SFRC, resistencia, cortante, hormigón armado, fibras, modelo mecánico, ensayos.

1. Introduction

Over the past three decades, there has been a wide interest in using Fibre Reinforced Concrete (FRC) to improve the performance of Reinforced Concrete (RC) elements, especially in tension and shear or torsion. The fibres help to delay crack initiation and to control the post-cracking behaviour in tension of the composite material. Then, fibres enhance the mechanical properties and the deformation capacity of the new material. Therefore, the effectiveness of fibres in improving the shear resistance is widely recognized in the scientific community, as indicates the large number of experimental studies, most of which deal with steel fibres, Lantsoght [1] and Cuenca [2].

Recent design codes allow to use fibres as shear reinforcement (RILEM TC-162-TDF 2003 [3]; Fib Model Code 2010 [4]). RILEM TC 162-TDF suggested to consider the fibre contribution to beam shear strength as a separate term, which is calculated as function of the FRC post-cracking residual stress. By contrast, the FIB Model Code 2010 considers the positive effect of fibres in shear as an enhancement of concrete contribution, then SFRC is considered as a unique composite material characterized by significant toughness properties after cracking, due to the bridging effect of fibres. The contribution of fibres is modelled in MC2010 by analytical equations based on the post-cracking residual strength provided by SFRC. Such behaviour is a function of the residual flexural tensile strength corresponding to Crack Mouth Opening Displacement (CMOD) = 0.5 and 2.5 mm (f_{R1} and f_{R3}, respectively) measured on a three-point bending test. In the same way, the Spanish Concrete Code EHE-08 [5], requires the estimations of the stresses characteristic points by a four points bending test on a prismatic notched beam.

A critical issue, which is still remaining an argument of debate, is the comprehension of the size-effect mitigation in FRC beams. Several investigations with large-scale specimens have been carried out by several researchers (Dinh et al. [6].; Minelli et al. [7].; Shoaib et al. [8]; Zarrinpour and Chao [9]. Recently, Zarrinpour and Chao [9] carried out an experimental campaign to investigate the shear-enhancement and failure mechanisms behind the ultimate shear strength of FRC full-scale slender beams by using the Digital Image Correlation (DIC) technology. They observed that the greater shear strength in FRC beams was due to the fibre bridging effect, which delays the propagation of the cracks into the compression zone, whose shear strength is enhanced by the compressive stresses induced by the higher load. It is a new way to take into account the beneficial effects of fibres to the shear strength of beams, which is in partial contrast with most of the existing models, which only introduce the residual tensile stress, across the shear critical crack, as FRC shear strength contribution.

Recently, some theoretical models originally proposed to predict the shear strength of RC beam, have been updated to the case of FRC specimens through the addition of the fibres contribution to the shear strength of RC beams. These kind of formulations are based on sophisticated theory, as the Modified Compression Field Theory (MCFT) (Foster [10]; Spinella et al. [11]), the plasticity equations for concrete (Voo et al. [12]; Spinella et al. [13]), the critical shear band concept, Tung and Tue [14], and the Cracked Membrane Model (CMM) Kaufmann et al. [15].

In this scenario, Marí et al. [16] proposed a theoretical model for the prediction of the shear-flexural strength of slender RC beams with and without transverse reinforcement, namely Multi-Action Shear Model (MASM). The model incorporates the contributions to shear strength provided by: the un-cracked compressed concrete chord (V_c), the diagonally cracked web (V_{cw}), the stirrups crossing the critical shear crack (V_s), if they are present and, in that case, the dowel action (V_l). It assumes that failure occur when the stresses, at any point of the concrete compression chord, reach the Kupfer's biaxial stress failure envelope. The MASM and a simplified version, called Compression Chord Capacity Model [17], have proved to predict the shear strength of plain and Prestressed Concrete (PC) beams, with and without stirrups, having different cross section shapes, and reinforced with steel or Fibre Reinforced Polymers (FRP), Cladera et al. [18].

In this paper, the MASM has been extended to the case of FRC beams without stirrups. The ability of FRC to maintain a residual tensile stress after shear cracking allows to take into account: the reduction of damage in the compression chord, by suitably estimating the neutral axis depth; the bridging effect of the fibres across the critical shear crack; and the enhancement of the dowel action in the longitudinal reinforcement.

The residual tensile strength of the FRC, essential in the proposed formulation, is evaluated in a simple manner by using the drop-down constant tensile stress model, as proposed by Lim et al. [19] and adapted by Spinella et al. [13]. This simplified approach allows to avoid intricate calculations and, especially, to carry out any preliminary experimental test to estimate the residual tensile stress.

2. Residual tensile stress of SFRC

As mentioned in the previous section, some of the formulations suggested by the codes require to preliminary carry out experimental tests for the estimation of the SFRC residual tensile strength. Then, from the design viewpoint it is clear that a handy way to overcome this drawback is needed.

Some analytical models, to reproduce the tensile stress constitutive behaviour, are available in the literature (Voo and Foster [20]; Lee et al. [21] [22]), and they need the knowledge of the shear crack width at failure. However, the tensile force transferred across the shear critical diagonal crack through fibre tension can be estimated by assuming a constant tension stress, avoiding the calculation of its overall distribution.

In this work, a simple analytical equation for the estimation of the average residual tensile stress (σ_{tu}), constant along the shear crack, is used and described next

2.1 Analytical formulation

The SFRC post-cracking tensile stress can be assumed proportional to the volumetric percentage of fibers V_f and the fibre aspect ratio L_f/d_f being L_f and d_f the length and the diameter of the fibres, respectively. Introducing the bond factor β_τ, which depends on the type of fibre, the fibre factor F_τ is calculated as:

$$F_\tau = \beta_\tau V_f \frac{l_f}{d_f} \quad (1)$$

In this work, the fibre bond factor is the ratio between the mean fibre-matrix shear stress and the strength under direct tension of the concrete matrix ($\beta_\tau=\tau_f/f_{ct}$). In turn, the shear stress τ_f is calculated as function of: f_{ct}, shape of fibre (hooked, plain or other), and type of matrix (concrete or mortar), Voo and Foster [20]. In the usual case of concrete matrix, τ_f/f_{ct} is 2.5 for hooked fibres and 1.2 for straight fibres.

The SFRC average residual tensile stress follows a horizontal branch, and it is estimated as:

$$\frac{\sigma_{tu}}{f_{ct}} = 2\eta_0\eta_l F_\tau \qquad (2)$$

where $\eta_0 = 0.405$ is the fibre orientation factor; η_l is the fibre length efficiency factor, which depends on the critical length $l_c = (\sigma_{sy}\, d_f)/(2\tau_f)$. If the fibre length, L_f, is $\leq l_c$ then $\eta_l = 0.5$, else $\eta_l = 1 - l_c/(2L_f)$. Such value is determined without the need of any tensile characterization tests, what is a great practical advantage.

3. The Multi-Action Shear Model

3.1. Brief overview of MASM

It is generally recognized that as the load increases in a RC beam failing in shear, the damage is concentrated around the shear critical crack. It is originated by a flexural crack, which arrives to the neighbourhood of the flexural neutral axis. Then, a second branch of the crack propagates inside the un-cracked concrete chord, and when the first branch of the crack and the load application point are joined each other, the beam get the failure.

The main assumption of the MASM is that when the second branch of the shear critical crack develops, the load does not significantly increase, as softening of the concrete in the compression zone initiates. During the crack propagation inside the flexural compression zone, redistribution of internal forces may occur, affecting the relative importance of the different shear resisting actions. The compression chord is subjected to a plane state of stresses, then a failure criterion is needed. It depends on the compression (f_{cm}) and tensile strength (f_{ct}) of concrete.

The MASM proposes explicit equations for each shear transfer contributions considering that the tip of the shear critical crack has propagated until the flexural neutral axis. The shear strength, V_u, is the sum of the shear resisted by the transverse reinforcement, if it exists, V_s, and by the shear resisted in the un-cracked compression chord, V_c, the shear transferred across web cracks, V_{cw}, and the dowel action in the longitudinal reinforcement, V_l (Fig. 1), Cladera et al. [17].

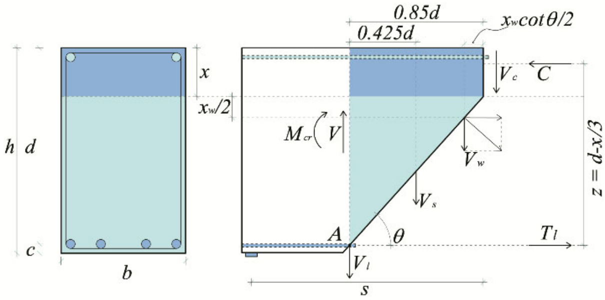

Figure 1. Shear strength contributions in a RC beam according to MASM.

In order to derive V_c, stresses distributions at the un-cracked concrete chord level are assumed. Since the shear failure takes place for moderate stresses in the critical section (where the critical crack reaches the flexural neutral axis), a linear distribution of compression stresses (σ_x) is assumed. Furthermore, it is assumed that shear stresses (τ) are fully concentrated in the un-cracked concrete chord, and having a parabolic distribution with zero values at top fibre and at the neutral axis level. Moreover, the horizontal projection of the first branch of the flexural-shear critical crack is considered to be equal to 0.85*d*, where *d* is the effective depth of the beam, Marí et al. [16].

Therefore, V_c is obtained by relating forces and stresses in the compression chord when failure takes place, according to Kupfer's criterion, and setting the equilibrium between forces, moments and the shear resisting components in the portion of the beam placed above the first branch of the critical crack (Fig. 1).

The shear resisted along the web, V_{cw}, is obtained by integrating the residual tensile stresses transferred across the critical crack, in function of the strain in the longitudinal reinforcement, among other factors.

The shear transferred by dowel action, V_l, is neglected in beams without stirrups, due to the low capacity of the plain concrete cover to resist the push-off force due to shear.

4. Influence of fibres on the shear resisting mechanisms

The capacity of FRC to provide a residual tensile stresses affects the shear resisting mechanisms adopted by the MASM for beams without stirrups. This influence is illustrated in Figure 2 and described in the following sections.

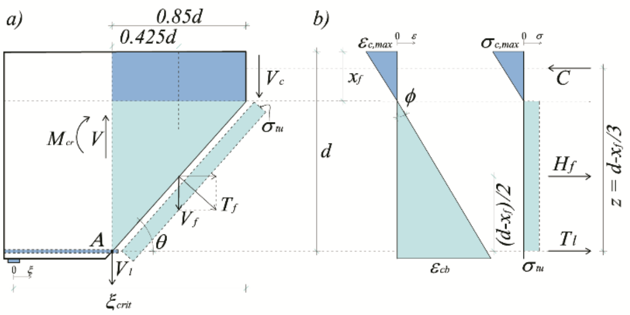

Figure 2. Contribution of steel fibers to the shear resisted in the web and by dowel action.

4.1. Shear transferred along the critical crack

The bridging effect of the steel fibres allows transferring tensile stresses across the critical shear crack, providing a resultant force T_f, whose component in the direction normal to the beam axis is the shear force V_{cf}. T_f is due to the increment of shear strength provided by the fibres, the crack width at failure is large enough to reduce the aggregate interlock, as experimentally observed by Zarrinpour and Chao [9]. Herein, a constant tensile stress σ_{tu} [Eq. (2)] along the whole first branch of the critical crack is assumed (Fig. 2). Then, the non-dimensional shear strength contribution v_{cf} can be obtained as 0.85 σ_{tu}/f_{ct}.

4.2. Dowel action

In beams without stirrups, the resistance offered by the plain concrete to prevent the outwards displacement of the longitudinal reinforcement is small. Furthermore, once the tensile strength is reached at any point, a quick propagation of a horizontal crack takes place driving to a brittle failure.

However, in the case of SFRC, the residual tensile stresses normal to the reinforcing bar, in a certain length, providing a non-negligible shear resistance (σ_{tu}). According to the MASM, the critical shear crack initiates where, for the load producing the ultimate shear force V_u, the bending moment reaches the cracking moment, $M_{cr} = 0.2 f_{ct}\, b\, d^2$. Therefore, the non-dimensional shear resisted by dowel action is $v_{lf} = (0.2/v_u)\,(\sigma_{tu}/f_{ct})$. v_{fl} depends on the total shear resisted v_u, which is not known *a priori*, so an iterative procedure must be implemented.

4.3. Shear resisted by the un-cracked compression chord

The horizontal component of force, $T_f = V_{cf}/cot\theta$, increases the neutral axis depth, and, then, the shear capacity of the un-cracked concrete chord.

The neutral axis depth in FRC, $x_f > x$, requires to perform a flexural analysis of cross section, incorporating a constant value of the residual tensile stress (σ_{tu}). The equilibrium of forces and moments provides a four degree polynomial equation on x_f/d, which is solved numerically. The numerical solution can be approached (with an error about of 3%) by the following linear equation:

$$\frac{x_f}{d} \simeq \frac{x}{d} + 0.42\frac{M_{cr}}{M}\left(1-1.66n\rho\right)\frac{\sigma_{tu}}{f_{ct}} \qquad (3)$$

being M_{cr}/M the bending moment ratio at the abscissa ξ_{crit} (Fig. 2); $n = E_s/E_c$ the homogenization factor; and $\rho = A_s/bd$ the geometrical percentage of longitudinal reinforcement.

Once x_f/d is known, the shear resisted by the compression chord is obtained by integration of the shear stresses. At this aim, the parabolic distribution of the shear stresses at the compression chord level must be determined.

The shear stress (τ) at any level of the compression chord can be estimated by rearranging the Mohr's Circle stresses equations, and in function of the normal (σ_x) and principal tensile stresses (σ_1, tension positive):

$$\tau = \sigma_1 \sqrt{1 - \frac{\sigma_x}{\sigma_1}} \tag{4}$$

The critical point, where failure of the compression chord initiates, is assumed to be placed at a distance $y = 0.425x_f$ from the neutral axis. Such position was determined by Mari et al. (2015) as the point where Kupfer's failure envelop is first reached. Then, the principal tensile stress of the critical point in the compression chord can be obtained as:

$$\sigma_1 = \left(1 - 0.8\frac{\sigma_2}{f_{cm}}\right) f_{ct} = R_t f_{ct} \tag{5}$$

where R_t is the tensile strength reduction factor due to the presence of the compressive principal stress, σ_2. By substituting Eq. (5) into Eq. (4), the maximum shear stress, τ_u, is:

$$\tau_u = R_t f_{ct} \sqrt{1 - \frac{\sigma_x}{R_t f_{ct}}} \tag{6}$$

Once the value of τ_u and its position along the compression chord are known, it is possible to define the equation of the parabolic distribution of the shear stresses inside the un-cracked chord. Then, by integrating the shear stresses, the non-dimensional shear resisted by the un-cracked concrete chord is obtained:

$$v_c = 0.682 \frac{xf}{d} R_t \sqrt{1 - \frac{\sigma_x}{R_t f_{ct}}} \tag{7}$$

being the integration factor = 0.682. It depends by the above defined position of the critical point.

5. Shear strength equation for SFRC beams

The normal stress at the critical point depth, $\sigma_x = \sigma_{c,crit}$, can be expressed in function of the compression force in the un-cracked chord. Because it is assumed linear along the neutral axis depth, the normal stress is $\sigma_{c,crit} = 0.425\sigma_{c,max}$, being $\sigma_{c,max}$ the compression stress of concrete top fiber. Then, the following expression for the non-dimensional shear resisted by the un-cracked concrete chord is obtained:

$$v_c = 0.682 \frac{xf}{d} R_t \sqrt{1 - \frac{0.85\left[0.2 + v_c + 0.5 v_{cf}\left(1 + \tan^2\theta\right)\right]}{R_t f_{ct} \frac{xf}{d}\left(1 - \frac{xf}{3d}\right)}} \tag{8}$$

where a non-dimensional cracking moment μ_{cr}= 0.2 and a flexural lever arm z= $d - x_f/3$ have been introduced. Eq. (8) is a second order polynomial equation in v_c, which needs to be solved iteratively, since R_t is a function of the principal stress σ_2 [Eq. (5)] which is not known *a priori*.

Once v_c is known, the shear strength of a FRC beam without stirrups is obtained by summing the contributions of the different shear mechanisms: $v_u = v_c + v_{cf} + v_{lf}$.

In Figure 3 the total non-dimensional shear resisted is plotted in function of the neutral axis depth x/d obtained by means of Eq. (9) - without considering the effect of fibers - for different values of σ_{tu}/f_{ct}.

$$\frac{x}{d} = n\rho\left(-1 + \sqrt{1 + \frac{2}{n\rho}}\right) \approx 0.75(n\rho)^{1/3} \tag{9}$$

where $n=E_s/E_c$ and $\rho=A_s/(bd)$.

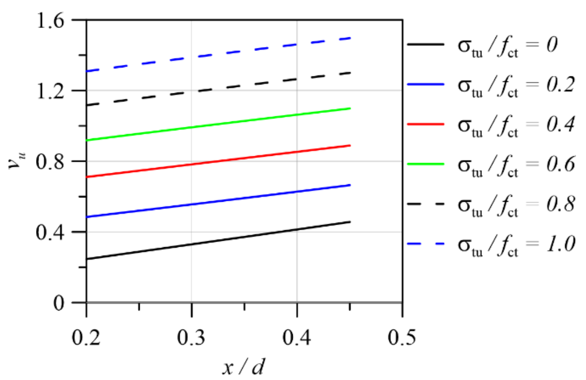

Figure 3. Shear strength vs. neutral axis depth x/d.

A linear dependency between the shear strength and the neutral axis depth is observed, so the theoretical solution is very closely approached by linear equation (10), in which the size effect given by Eq. (11) Bazant et al. [23], Pérez et al. [24] has been included:

$$V_u = \left[\xi\left(0.84 - 0.10\frac{\sigma_{tu}}{f_{ctm}}\right)\frac{x}{d} + 0.08 + 1.10\frac{\sigma_{tu}}{f_{ctm}}\right] f_{ctm}\, b\, d \qquad (10)$$

$$\xi = \frac{2}{\sqrt{1+\frac{d}{200}}}\left(\frac{d}{a}\right)^{0.2} \nless 0.45 \;\; d \textit{ and } a \textit{ in } mm \qquad (11)$$

6. Experimental validation

Recently, Lantsoght [1] compiled a database of 488 experiments reported in the literature. The structural parameters covered by the database vary over a wide range, however the mechanical characterization of SFRC was not reported. For this database, the fibre contribution has been calculated by using the σ_{tu} in Eq. (2) for all formulations, since f_{Ftu} is not available in this database.

Contextually, Cuenca et al. [2] presented a material-performance-based shear database for SFRC elements. The database is composed by 93 SFRC specimens with a broad range of selected parameters, and the post-cracking resistance ($f_{R,1}$ and $f_{R,3}$) is also given according to EN 14651 standard.

Method	Lantsoght (1)		Cuenca (1)		Cuenca (2)	
	Mean	CoV	Mean	CoV	Mean	CoV
Proposed	1.15	0.22	1.17	0.24	1.04	0.22
Dinh [6]	1.70	0.50	1.50	0.33	1.18	0.29
MC1 [4]	1.58	0.49	1.19	0.24	1.04	0.23
MC2 [4]	1.43	0.39	1.28	0.24	0.99	0.24

An analysis of the predictions provided by the proposal and also by the formulation suggested by Dinh et al. [6] and two different analytical models recently included in MC2010 [4] have been used in the predictions of the tests results. The choice of these formulations between the several ones available in literature is due to the fact that the model of Dinh et al. [6] is similar to the proposed one, and, as for the MC2010 models, the fiber contribution is provided by the experimental result of a bending test.

In Figure 4, the values of V_{Exp}/V_{Pre} for the studied models are presented as a function of σ_{tu}/f_{ct}, for the specimens of both databases considered. It highlights that the proposal of this paper presents good results for the two databases, and that the dispersion is very limited. By contrast, the other formulations used to predict the tests results provide too conservative results and higher dispersion. Finally, Figure 4 looks to indicate that the ratio V_{Exp}/V_{Pre} is independent on the ratio σ_{tu}/f_{ct}.

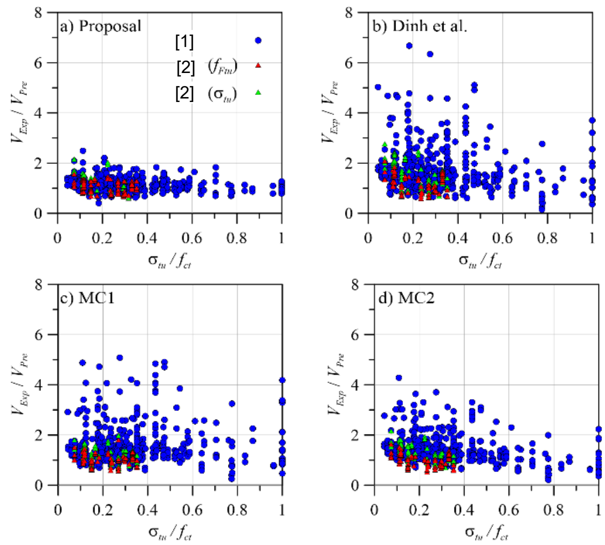

Figure 4. Values of V_{Exp}/V_{Pre} as a function of σ_{tu}/f_{ct}.

Table 1 summarizes the statistical results for all models considered, using both the proposed simplified residual strength σ_{tu} (1) and the experimental residual stress (2)

Table 1. Statistical results of V_{exp}/V_{pred}.

Again, it can be observed that the proposed method provides the best results in terms of mean value and coefficient of variation in all cases, but especially when using the simplified residual strength.

7. Conclusions

The following main conclusions of the work done can be drawn:

1) A simple and compact formulation has been derived for the shear strength of SFRC beams without stirrups, including the contributions of the un-cracked concrete chord, the shear transferred across the critical shear crack and the shear resisted by dowel action
2) The main contribution of the SFRC is the capacity to transfer residual stresses across the cracks. Such tensile residual stresses σ_{tu} contribute directly to resist shear in the web, to increase the dowel action in the longitudinal reinforcement by bridging the horizontal bond crack and, indirectly, to increase the shear resisted in the compression concrete chord through the increment of the neutral axis depth.
3) A uniform ultimate tensile strength σ_{tu}, not depending on the crack width, has been used, whose value σ_{tu} is a function of the aspect ratio, shape, volumetric percentage, and the plain concrete tensile strength, which are values known by the designer.
4) The model has been verified by comparing its predictions with the results of shear tests included in two different already published databases: a) a large data based recently published, containing 488 tests; and b) a selected database containing only those tests where SFRC post-cracking was characterized by means of flexural beam tests. Very good correlation has been obtained in terms of mean and CoV of V_{exp}/V_{pred}. In addition, the results of the proposed model are more accurate and have less scatter than those obtained by other formulations, some of which need an iterative procedure.
5) Since each component can be obtained separately, the relative contribution of each component can be studied as a function of design parameters, such as the longitudinal reinforcement or the volume or type of fibers, etc., thus becoming a useful tool for design.

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