Numerical computation of the poloidal magnetic field created by a toroidal current in a torus. Applications in nuclear fusion.

Topic: T12 - Electromagnetic Simulations in Advanced Applications.

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Tokamak fusion reactors try to confine the particles to be fused -the plasma- in the shape of a torus. Planar loops of poloidal current are located around the toroidal vacuum vessels of the reactors to generate a toroidal magnetic field that confines the particles. However, the curvature and non-uniformity of the toroidal field produce forces which cause radially outward "drift" motions of the charged particles. These forces must be compensated to prevent the charged particles to hit the vessel walls, which would quickly cool the plasma. A poloidal magnetic field must be superimposed upon the toroidal magnetic field in order to compensate these drifts, resulting in a helical magnetic field which is entirely contained within the toroidal confinement chamber. The charged particles will spiral around the helical magnetic field lines. In tokamak reactors, the poloidal magnetic field is produced by a toroidal current induced in the plasma. (W. M. Stacey, "*Fusion: An Introduction to the Physics and Technology of Magnetic Confinement Fusion*", Wiley-VCH, 2010). The accurate computation of the helical magnetic field inside the reactor is crucial to check if the charged particles will be efficiently confined. Whereas the determination of the magnetic field of a planar loop carrying poloidal current is relatively simple, the determination of the magnetic field of the plasma carrying a toroidal current is not trivial.

In this paper we present an efficient algorithm for the computation of the static vector potential and the poloidal magnetic field of a toroidal current in a torus. To achieve that goal, first the vector potential is expressed in integral form in cylindrical coordinates, the Green's function of the integrand is expanded in terms of toroidal harmonics (J. P. Selvaggi et al., *IEEE Trans. Magnetics,* 43, pp. 3833-3839, Oct. 2007), and integration with respect to the azimuthal coordinate is carried out in closed-form. Then, the resulting double integral is numerically computed. This numerical computation cannot be carried out brute force since the integrand of the double integral has a logarithmic singularity coming from complete elliptic integrals when source and field points coincide if the magnetic field is to be computed inside the torus, and a quasi-singularity when source and field points are close if the magnetic field is to be computed outside the torus. And both the singularity and the quasi-singularity have a deleterious effect on the numerical integration process. To avoid this problem, first a change of variables to local polar coordinates is carried out in the double integrals to isolate the logarithmic singularity, and then, Ma-Rokhlin-Wandzura quadrature rules (J. Ma et al., *SIAM J. Numer. Anal.*, 33, pp. 971-996, June 1996) are iteratively used to deal with the logarithmic singularity of the integrands. Once the vector potencial is obtained, the magnetic field components are obtained in terms of the vector potential by means of numerical derivation. The numerical results obtained for the magnetic field have been validated by comparing those of a torus with a sufficiently large revolution radius with analytical results for the magnetic field of an infinitely long cylinder carrying axial current, and by checking the fulfillment of Ampère's law. Also, numerical simulations have shown that the superposition of the poloidal field of the toroidal plasma current and that of the planar poloidal loops in a real tokamak makes it possible the particle confinement.