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A Mixed-integer Linear Program for Real-time Train Platforming Management

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Abstract

Unexpected events may perturb operations and generate conflicts that must be addressed promptly to limit delay propagation and other negative impacts on the network. The real-time railway traffic management problem deals with disruptions in railway networks, including tracks, junctions and stations. When they happen in station areas, new decisions involving train platforming, rerouting, ordering and timing must be made in real time. This paper explores a mesoscopic approach to deal with disruptions at rail stations. A mathematical programming-based model is proposed to determine re-routing and re-scheduling decisions for railway traffic in a station area. The key steps of the approach, which simulate what happens in real-time traffic management, are: i) an initial off-line preprocessing stage of the set of feasible routes originally planned, ii) a second preprocessing stage which analyses the disruption and sets the necessary parameters for the last step iii), which consists of an integer programming model that seeks solutions which minimise deviations from planned train schedules and assigns new and appropriate platforms (if necessary). Computational experiments show that realistic instances can be solved near to optimality using CPLEX in very short times. This allows to consider this methodology for solving real time traffic management problems.

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Keywords: Railway traffic; train platforming problem; real-time re-scheduling; real-time re-routing.

1. Introduction

Incidents can cause railway traffic to deviate from planned operations during daily operations on a dense railway network. If it becomes impossible to operate the schedule as originally intended, steps must be taken in accordance

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with recovery plans that must be structured in accordance with the degree of intervention. Cadarso et al. (2015) and Cacchiani et al. (2014) adopted a classification scheme based on the type of conflict to be managed, distinguishing between *disturbance* and *disruption*. Both of them call for re-scheduling operations and/or *resources*, such as crews and rolling stock. A disturbance is a primary delay that causes secondary delays. These incidents are small-scale and appear in case of minor train delays. They may be resolved by letting the delays propagate until the timetable's buffer times absorb them and/or also applying recovery strategies such as rescheduling timetable and re-routing trains, which may impose platforming changes at stations. Disruptions, such as those caused by infrastructure blockages, collapsing rolling stock, and crew shortages, necessitate large-scale adjustments to the schedule in order to recover. They may include, among others, train cancellations and/or extra trains (Almodóvar and García-Ródenas (2013)), train replacement (Cadarso et al. (2013), Mesa et al. (2013)), and changing planned stops Canca et al. (2016). In case of disruptions, dispatchers solve the real-time Railway Traffic Management Problem (rtRTMP) to re-schedule and re-route trains in an attempt to minimise deviations from the publicly available timetable. This paper considers the case of disruptions in station areas or, as it is known, *the real-time Train Platforming Problem* (TPP). This problem has been mainly studied from an off-line point of view. There are few exceptions, such as for example Chakroborty and Vikram (2008) or Zhang et al. (2020b). Existing approaches for railway traffic management problems at the operational level employ microscopic schemes that require a high degree of detail in the network definition, having a negative effect not only on the computational cost of the solution process but also on the time required to collect data. Mesoscopic schemes have been applied to integrated models that tackle jointly problems both tactical and operational problems, such as Timetabling and Platforming, Zhang et al. (2020a), Zhang et al. (2021). This allows the integration of the tactical level, usually under a macroscopic approach with the operational level at a microscopic approach.

This paper suggests a mesoscopic method for the TPP. Instead of developing a model based on *line sections* (Törnquist and Persson (2007), Tornquist Krasemann (2012)), *multi-block sections* are employed. A multi-block section based Mixed-Integer Linear Programming (MILP) formulation is proposed to determine rescheduling and rerouting actions. The goal is to minimise inconveniences caused by (i) delays, (ii) allocation of non-preferred platforms, and (iii) last minute changes in platform allocation. The MILP model relies on a previous off-line preprocessing, which analyses the set of feasible routes to detect all the potential conflicts and their effects on the network. The theoretical and practical contributions of this study mainly include the following three aspects:

– The novel modeling mesoscopic method based on multi-block sections.
– The train arrival and departure times and the train platform assignment are optimized simultaneously so that the negative influence of train delays can be minimized.
– Realistic computational experiments drawn from Atocha-RENFE, the main rail station in Spain, are presented. to show the efficiency and effectiveness of the proposed model.

The remainder of this paper is organized as follows: Section 2 describes the modelling approach. Section 3 proposes a mixed-integer linear programming model to TPP. Section 4 presents computational results by the commercial CPLEX solver. Conclusions and future research directions are presented in Section 5.

2. Problem statement: the train platforming problem in real time

This section introduces all the required concepts and notations related to train operations within a station. Figure 1 shows an example of a station, which is used to illustrate the problem description. Firstly, the elements of the infrastructure are introduced. Secondly, the necessary elements related to trains and routes on the station are described. Finally, definitions for the modeling of conflicts are provided.

2.1. Infrastructure

In many countries, the railway system is operated in a decentralized way (Schipper and Gerrits (2018)). This means the network is divided in different parts which are known as *rail traffic control areas*. In the case of the TPP, they are denominated *station areas*. Each of them are composed of *track sections*, *switches* and *platforms*, which may be also

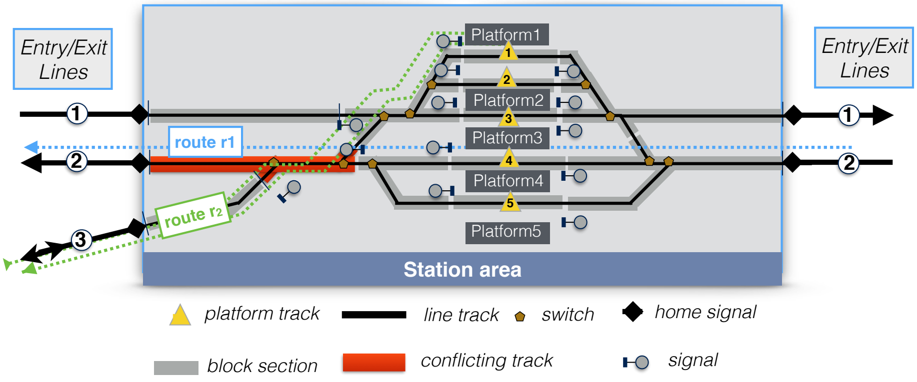

Fig. 1. Example of railway station layout.

generally denominated as *resources*. A *block section* is a set of *track sections* limited by adjacent signals. Due to safety reasons, no block section can be shared at the same time by two or more trains.

Trains may enter and exit stations through *line tracks*. The example in Figure 1 features five exit/entry lines. The rhombuses describe *home signals*. If a train reaches a *home signal* and all the tracks ruled by that signal are occupied, the train must wait. If any of the tracks is available and operational constraints are satisfied, it may continue. There might be several locations at stations where trains may stop in order to serve passengers. These locations are usually known as *platform tracks*, which may feature different lengths (it may also exist *dead-end platform tracks*). The occupancy time of platform tracks to serve passengers is known as *sojourn time*. The example in Figure 1 features five platform tracks. Departure of trains is controlled by exit signals located in the platform tracks. Switches, which are represented as pentagons in Figure 1, allow trains to proceed from track to track.

2.2. Trains and routes

Let T and R be the set of trains to be controlled in a given station area during a given time period and the set of routes within a station, respectively, being t an index running in T (i.e., a specific train) and r an index running in R (i.e., a specific route).

A train's route within a station is defined by an ordered sequence of block sections. Platform assignment to a given train t is implicitly given by the route r followed by the train. Note that it may occur that there is no feasible connection between some platform tracks and some entry/exit lines, or that the length of a train exceeds the length of a platform producing an infeasible platform assignment. Also note that *dead-end platform tracks* are only usable by trains which have a driver seat at both ends. These are examples of *eligibility restrictions*, which are modeled considering that each train t may only operate routes r belonging to a given and suitable subset of routes $R_t \subseteq R$.

Each route is decomposed in *route sections*, which can be considered as indivisible elements providing a way to state the granularity of the (mesoscopic) model. Thus, route sections are linear and ordered sequences of block sections. A train may run all the block sections in a route section without needing additional resources to the ones contained in the route section. In turn, a route r may be defined as a disjoint union of route sections $r = rs_1 \cup \cdots \cup rs_p$. The definition of each route section rs_p determines the dimensions of the problem. Note that two extreme definitions may be given for route sections: (i) biunivocal correspondence between routes and route sections and (ii) biunivocal correspondence between block sections/resources and route sections. A trade-off between (i) and (ii) may be achieved by decomposing routes in three route sections. The first route section would be the *in-route* section (from the entry of the station to the platform), the second one would be the *central section*, usually corresponding to a platform track, and the last one, the *out-route* section (from the platform to the exit of the station).

Each train $t \in T$ has a *planned entrance time* (pe_t^{in}) and a *planned exit time* (pe_t^{out}) to the station area. The first one is the time at which the train coming from external line tracks reaches the home signal. The second one indicates the time at which the train reaches the exit home signal to go to external line tracks. Additionally, the current/real entrance time of train t at the station area is rt_t.

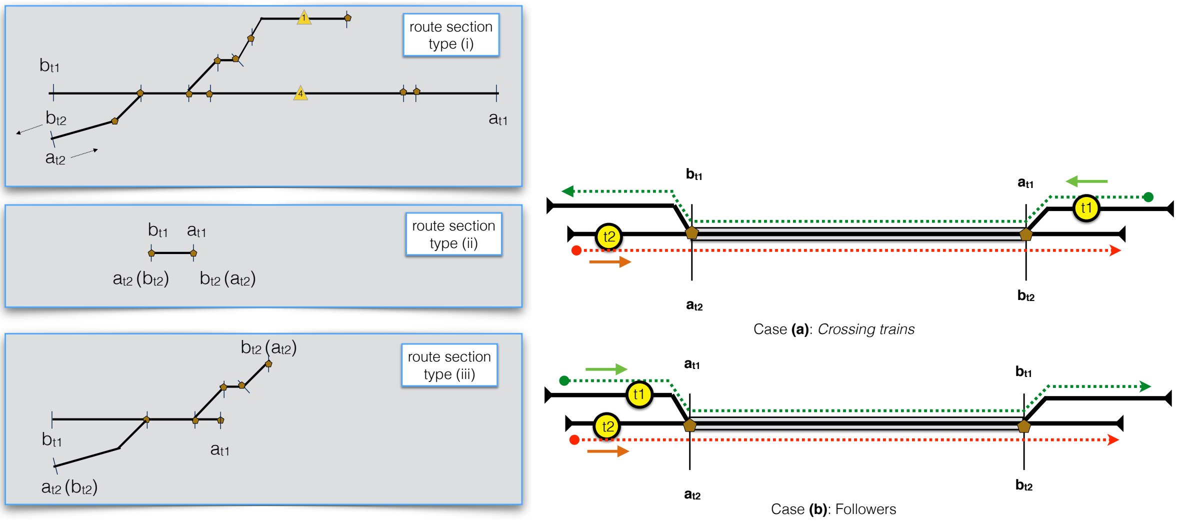

Fig. 2. On the left: example of multi-block section as a function of the type of route section. On the right: types of train conflicts.

2.3. Conflict modeling based on pairs of trains

Suppose the modeller makes a suitable choice of the route sections in the station area, accordingly to a desired granularity level in order to make the resulting model accurate enough. Then, the following definition for the *multi-block section* concept must be introduced to locate *conflicts* within the station area.

Definition 1 (Multi-block section). *Suppose that trains t_1 y t_2 follow routes r_1 and r_2 within a station area, respectively. Consider that rs_1 and rs_2 are route sections belonging to routes r_1 and de r_2, respectively. If route sections rs_1 and rs_2 share any resources, i.e., $rs_1 \cap rs_2 \neq \{\emptyset\}$, then $s = rs_1 \cup rs_2$ will be defined as a multi-block section.*

Figure 1 shows two routes featuring a conflict in a track section, which is highlighted in red. Depending on the granularity of the model, i.e., the definition of *route sections* (i) biunivocal correspondence between routes and route sections, (ii) biunivocal correspondence between block sections and route sections, (iii) decomposition of each route in in-route, platform track and out-route sections, the given conflicting track generates three types of multi-block sections. Figure 2 (on the left) shows the multi-block sections for each of the mentioned granularity levels. Let a_{t_i} be the time train t_i begins to occupy a given multi-block section and b_{t_i} the time it leaves it. Figure 2 (on the left) shows where these times are measured. Note the differences depending on the above definition for route sections.

There might be two types of conflicts in multi-block sections depending on the nature of the traffic. Consider Figure 2 (on the right). Case **(a)** shows two trains cannot simultaneously occupy the multi-block section, no-matter the number of resources that compose it, because otherwise, a deadlock would occur. This type of traffic will be denominated as *crossing trains*. Case **(b)** shows that two trains may occupy simultaneously the same multi-block section. A train precedes the other one in the multi-block section. The second train occupies resources as the first train leaves them. This type of traffic will be denominated as *follower trains*. Therefore, two main rules may appear to process traffic in multi-block sections:

- *Rule* **(a)**. Two trains cannot simultaneously occupy the same multi-block section.
- *Rule* **(b)**. No train overrunning may occur in a multi-block section, i.e., two trains using the same multi-block section enter and exit it in the same order.

For crossing trains, only rule **(a)** applies, while for follower trains both rules **(a)** and **(b)** apply. Two routes may originate a set of conflicts at the same multi-block section. For instance, considering Figure 1 and type (iii) route sections, the simultaneous circulation of a train on route r_1 and of another train on route r_2 may originate two conflicts, which are associated to the input/output to/from the train station circulating on route r_2. For model purposes we will consider them as two different multi-block sections s and s'.

A *conflict* is given by the affected multi-block section s, the trains running on it and the routes they are running. The set of conflicts is given by

$$C := \{\ c = (t_1, t_2, r_1, r_2, s) : \begin{array}{l} c \text{ is a conflict that may occur between trains } t_1 \text{ and } t_2 \text{ running routes } r_1 \text{ and } r_2 \text{ in} \\ \text{multi-block section } s. \end{array} \}$$

Conflicts are classified depending on the rules ((**a**) or (**b**)) they are processed with:

$$C^a := \{\ c = (t_1, t_2, r_1, r_2, s) \in C : c \text{ imposes rule (a) } \}, \quad C^b := \{c = (t_1, t_2, r_1, r_2, s) \in C : c \text{ imposes rule (b) } \}$$

In case of conflict, the available control strategies are: i) delay the entrance time at the station area of train t w_t minutes, and ii) change the route of the train within the station. Due to safety reasons, once a train enters the station area, it performs its route according to a speed profile and stopping times, which depend on the route and train type. This means that no new scheduling decision may be made in this regard.

No matter the control strategy followed, separation or headway times must be respected. The way it is modeled depends on the type of conflict. Next subsections introduce constraints ensuring that traffic conflicts are avoided.

2.3.1. Conflicts governed by rule a

Suppose that trains t_1 and t_2 run routes r_1 and r_2, respectively. Assume these trains may have a *potential conflict* $c \in C^a$ in the multi-block section s. Trains have a *head* and a *tail*. The time instant at which a train's head t_i arrives at a multi-block section is denominated *starting occupancy time* ($a'_{t_i,c}$) and the time instant at which a train's tail leaves the multi-block section *ending occupancy time* ($b'_{t_i,c}$). The headway (i.e., the separation or safety time gap between two trains running the same multi-block section) is $\varepsilon_{t,c}$, and it is the time that train t must wait to occupy the multi-block section s of conflict c once the other train in the conflict has left it. To simplify notations, let $b_{t,c}$ be equal to $b'_{t,c} + \varepsilon_{t,c}$ (note that $a_{t_i,c} = a'_{t_i,c}$ in this type of conflict). Index c will be skipped to avoid confusion.

Conflicts processed with rule (**a**) may be modeled by the following disjunctive constraints.

$$\textbf{Rule (a)} \quad w_{t_1} + a'_{t_1} \geq w_{t_2} + b_{t_2} \quad \vee \quad w_{t_1} + b_{t_1} \leq w_{t_2} + a'_{t_2}\,, \quad \text{for each conflict } c = (t_1, t_2, r_1, r_2, s) \in C^a \tag{1}$$

where w_{t_1} and w_{t_2} are the delay times for trains t_1 y t_2 respectively. These constraints model the fact that multi-block sections managed with rule (**a**) cannot be simultaneously occupied by trains t_1 and t_2.

2.3.2. Conflicts governed by rule b

For each conflict $c = (t_1, t_2, r_1, r_2, s) \in C^b$, it must be fulfilled that trains t_1 and t_2 enter and exit in the same order. Moreover, headway times are imposed: $\varepsilon^a_{t,c}$ and $\varepsilon^b_{t,c}$ for the entering and exiting train t, respectively. $\varepsilon^a_{t,c}$ is the time the other train must wait to occupy the multi-block section and $\varepsilon^b_{t,c}$ is the time that it must wait to abandon the multi-block section. Therefore:

$$a_{t,c} = a'_{t,c} + \varepsilon^a_{t,c}\,, \text{ for all } c \in C^b \qquad \text{and} \qquad b_{t,c} = b'_{t,c} + \varepsilon^b_{t,c}\,, \text{ for all } c \in C^b.$$

For each conflict $c \in C^b$, the following disjunctive constraints are imposed:

$$\textbf{Rule (b)} \begin{cases} w_{t_1} + a_{t_1} \leq w_{t_2} + a'_{t_2} \\ w_{t_1} + b_{t_1} \leq w_{t_2} + b'_{t_2} \end{cases} \vee \quad \begin{cases} w_{t_1} + a'_{t_1} \geq w_{t_2} + a_{t_2} \\ w_{t_1} + b'_{t_1} \geq w_{t_2} + b_{t_2} \end{cases} \tag{2}$$

Note that for both rules **(a)** and **(b)**, all the required data, i.e., parameters, must be fully known for each train type, the route and the considered multi-block section. This requires a pre-processing.

3. A mixed-integer linear programming formulation

The proposed model requires an off-line phase in which the set of admissible routes R_t are defined for each train $t \in T$ as well as all the potential conflicts $c \in C$. Additionally, and for the planned schedule, the starting and ending occupancy times are calculated for every conflict. Although the computational effort required by the pre-processing phase is high, this effort makes possible to resolve the subsequent on-line computing phase (i.e., the time at which conflicts take place) nimbly.

Assume that there are disturbances in the railway system affecting the arrivals of the trains to the station area. Let us denote by rt_t the actual entry time at the station area and that this is a source of conflicts that need to be solved. The first step is to calculate the difference between the actual time and the planned entry time $D_t^{in} = rt_t - pe_t^{in}$ for all $t \in T$. Then, every occupancy time in the system will be recalculated using the expression: $a'_{t,c} = a_{t,c} + D_t^{in}$, for all t, c and $b'_{t,c} = b_{t,c} + D_t^{in}$, for all t.

There are two vectors of rescheduling actions available. Delaying trains, $\mathbf{w} = (\cdots, w_t, \cdots)$, and re-routing trains, $\mathbf{y} = (\cdots, y_{t,r}, \cdots)$. $\mathbf{w}$ is a vector of continuous variables, where each component is the delay in minutes for a train. $\mathbf{y}$ is a vector of binary variables, where its components, $y_{t,r}$, are 1 if train t is assigned to route r, and 0 otherwise. Vectors of rescheduling and rerouting will be feasible if they avoid all conflicts in the network. To this end it will be shown now how conflicts can be expressed using disjunctive constraints.

Since each train uses a single route, only the subset of affected conflicts must be taken into account, being the remaining conflicts trivially accomplished. For each conflict $c \in C$, the auxiliary binary variable z_c will be used to indicate which of the blocks of the disjunctive constraints will be verified. This leads to the following formulation:

$$w_{t_1} + a'_{t_1,c} \geq w_{t_2} + b_{t_2,c} - Mz_c - M(2 - y_{t_1,r_1} - y_{t_2,r_2}), \forall c = (t_1, t_2, r_1, r_2, s) \in C^a : t_1 < t_2 \quad (3)$$

$$w_{t_1} + b_{t_1,c} \leq w_{t_2} + a'_{t_2,c} + M(1 - z_c) + M(2 - y_{t_1,r_1} - y_{t_2,r_2}), \forall c = (t_1, t_2, r_1, r_2, s) \in C^a : t_1 < t_2 \quad (4)$$

$$w_{t_1} + a_{t_1,c} \leq w_{t_2} + a'_{t_2,c} + Mz_c + M(2 - y_{t_1,r_1} - y_{t_2,r_2}), \forall c = (t_1, t_2, r_1, r_2, s) \in C^b : t_1 < t_2 \quad (5)$$

$$w_{t_1} + b_{t_1,c} \leq w_{t_2} + b'_{t_2,c} + Mz_c + M(2 - y_{t_1,r_1} - y_{t_2,r_2}), \forall c = (t_1, t_2, r_1, r_2, s) \in C^b : t_1 < t_2 \quad (6)$$

$$w_{t_1} + a'_{t_1,c} \geq w_{t_2} + a_{t_2,c} - M(1 - z_c) - M(2 - y_{t_1,r_1} - y_{t_2,r_2}), \forall c = (t_1, t_2, r_1, r_2, s) \in C^b : t_1 < t_2 \quad (7)$$

$$w_{t_1} + b'_{t_1,c} \geq w_{t_2} + b_{t_2,c} - M(1 - z_c) - M(2 - y_{t_1,r_1} - y_{t_2,r_2}), \forall c = (t_1, t_2, r_1, r_2, s) \in C^b : t_1 < t_2 \quad (8)$$

$$\sum_{r \in R_t} y_{t,r} = 1, \forall t \in T \quad (9)$$

$$y_{t,r} \in \{0, 1\}, \forall (t, r) \in T \times R : r \in R_t; \quad z_c \in \{0, 1\}, \forall c \in C = C^a \cup C^b \quad (10)$$

where M is an arbitrarily large enough constant, so that either of the two disjunctive constraints can be trivially satisfied.

The delay time of train t must be expressed in terms of the decision variables $(\mathbf{w}, \mathbf{y})$. Analogously to the calculation of D_t^{in}, the deviation in the exit time of the train station area, D_t^{out}, will be calculated. D_t^{in} and D_t^{out} may take negative values (arrival to the station area before the scheduled time) or positive (in case of delay). Our model will pursue the minimization of delays and then the non-negative values $h_t^{in} = \max\{D_t^{in} + w_t, 0\}$ and $h_t^{out} = \max\{D_t^{out} + w_t, 0\}$ will be taken into account. This gives rise to the following set of constraints:

$$D_t^{in} = rt_t - pe_t^{in}, \forall t \in T \quad (11)$$

$$D_t^{out} = -pe_t^{out} + rt_t + \sum_{r \in R_t} c_{t,r} y_{t,r}, \forall t \in T \quad (12)$$

$$D_t^{in} + w_t \leq h_t^{in}, \forall t \in T \quad (13)$$

$$D_t^{out} + w_t \leq h_t^{out}, \forall t \in T \quad (14)$$

$$0 \leq h_t^{in}, 0 \leq h_t^{out}, 0 \leq w_t, \forall t \in T \quad (15)$$

Constraint (11) computes the deviation, D_t^{in}, from the planned entry time to the station as the difference between the actual arrival time and the planned arrival time. Equation (12) computes the deviation D_t^{out} of train t from its scheduled departure time, the actual entry time rt_t, the travel time, $c_{r,t}$ on its route r and from its planned exit time pe_t^{out}. Finally, constraints (13)-(15) play the role of defining variables h_t^{in} and h_t^{out}.

The following constraint (16) states that a train cannot exit from its platform before its planned departure time dt_t. Let $c_{t,r}^p$ be the travel time for train t from the exterior lines to the platform within route r plus the *sojourn* time.

Then, there must hold that:

$$rt_t + w_t + \sum_{r \in R_r} c_{t,r}^p y_{t,r} \geq dt_t, \quad \forall t \in T \tag{16}$$

The TTP problem features a multi-objective nature. A first objective is to minimize the deviance from the planned timetable. This is, to minimize the sum of delays of incoming (arriving) and outgoing (departing) trains. The first term of the objective function corresponds to the delays of the trains with respect to the planned schedule. To this end all the delays in entering to the station area (h_t^{in}) as well as the delays of leaving the station area (h_t^{out}) are accounted for. A second term takes into account the allocation of non-preferred platforms. Let us assume that there exists a parameter $k_{r,t}$ that evaluates the assignment of platform in route r to train t. This parameter allows to penalize changes in the preassigned platform in the planned timetable. Then, the following multi-objective optimization problem can be stated:

$$\begin{array}{c}\text{Minimize } z(\mathbf{w}, \mathbf{y}) = \sum_{t \in T} \left[\left(u_t h_t^{in} + v_t h_t^{out} \right) + \lambda \sum_{r \in R_t} k_{r,t} y_{r,t} \right] \\ \text{subject to : (3) to (16).}\end{array} \tag{17}$$

We assume that the weights in the first of the objective function are positive, $u_t > 0$, $v_t > 0$ for all $t \in T$. They measure the priority of trains. λ, which must be greater or equal than zero, is a parameter so that it becomes $k_{r,t}$ in a delay equivalent penalty.

4. Numerical experiments

This section shows realistic computational experiments drawn from Atocha station in Madrid (Figure 3). This station area is the main infrastructure of the inter-regional rail network in Madrid's area. The railway company RENFE operates five lines, namely C2, C3, C4, C5, and C7, using Atocha station. As previously stated, the proposed approach has an off-line phase where all the routes and route sections that can be used by a line must be defined. In the numerical tests, the behaviour of the operator has been emulated and the usual 17 routes have been taken into account, which appear identified in Figure 3 with a color for each of them. Additionally, each route has been partitioned in 4 route sections (A,B,C, and D in Figure 3). A MATLAB program has been implemented, which automatically generates the conflict sets C^a and C^b for the trains in the tests. The running time of this code has an order of magnitude of seconds. Then, the MIP model has been coded in GAMS and solved using the commercial solver CPLEX.

Two time periods have been considered, the period from 5 a.m to 8 a.m and the all-day period. For each of these time periods, the planned schedule has been perturbed in two different ways. The first approach adds a log-normally distributed variable ($\mu = 1/6$ h and $\sigma = 1$h) to scheduled times. The second one adds a multinomial variable (which may take the values of $5, 4, 3, 2, 1$ minutes, each with probability 0.05, and 0 minutes with probability 0.75) to scheduled times . Table 1 displays the description of the considered case studies. Note that, S stands for the short time period. For instance, AtochaS1 refers to the first case study, which includes two lines and 33 trains. When referring to the same case study but for the whole day, Atocha1 is used instead. Depending on the perturbation form, either log-normal ($-logn$) or multinomial ($-mn$), two separate instances of the same case study are obtained, for a total of 12.

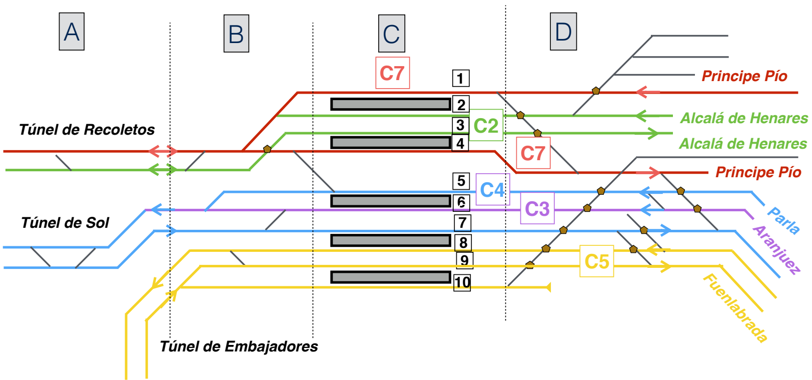

Fig. 3. Layout of Atocha station. Input/output block sections for trains in lines C2, C3, C4, C5, and C7.

Table 1. Case studies from the Atocha station.

Problem	Lines	# trains	Period	Problem	Lines	# trains	Period
AtochaS1	C2, C7	33	[5,8]	Atocha1	C2, C7	223	All day
AtochaS2	C2, C7, C4, C3	89	[5,8]	Atocha2	C2, C7, C4, C3	550	All day
AtochaS3	C2, C7, C4, C3, C5	132	[5,8]	Atocha3	C2, C7, C4, C3, C5	849	All day

We have given equal relevance to delays at the entrances and exits of the station area ($u_t = v_t = 1$). Since we are using the routes of the operator RENFE, re-routings do not produce unusual changes in the platforms for the passengers and because of that we have adopted $\lambda = 0$.

Table 2 shows the computational results. The first four columns of the table show the results for all the case studies featuring a short time period. Z is the objective function value (in hours), CPU is the computational time in seconds, and Rel. Gap the obtained relative gap. The last five columns of the table show the results for all the case studies featuring an all-day period. Z_i is the objective function value (in hours) for the first found feasible solution, CPU_i is the computational time in seconds for the first found feasible solution, Z_{600} is the objective function value (in hours) for the best found feasible solution after 600 seconds of computational time, and Rel. Gap$_{600}$ is the obtained relative gap for the best found feasible solution after 600 seconds of computational time. It can be observed that problems corresponding to the short time period are solved to optimality with a very low computational effort. For problems featuring an all-day period, a feasible solution with no conflicts can be found within 10 minutes of computational time. Because the time periods usually used in real time systems are smaller than three hours, the proposed approach is considered to be appropriate for real-time rescheduling purposes.

The mesoscopic approach seeks to reduce the complexity of the model by treating a set of block sections in a grouped way. The approach has the limitation that the number of conflicts grows non-linearly with the length of the planning interval.

This has been evidenced in the numerical tests where for the time interval of three hours an optimal solution can be found while for the whole day interval only a feasible solution is reached. For problems with a wide time interval or with a more intense railway traffic than in the analyzed station, pre-processing strategies should be used that remove conflicts between trains that are very distant in time or resort to metaheuristic resolution methods.

5. Conclusions

In this paper, we used a mesoscopic approach focused on multi-block sections to tackle the problem of real-time train platforming management. The collection of feasible routes for each line, as well as its decomposition into sections of lines, is calculated in an off-line process. These choices define the problem's granularity and the resulting computational cost. A mixed integer linear programming problem for addressing the potential conflicts, which are

Table 2. Computational results.

Case study	Z	CPU	Rel. Gap	Case study	Z_i	CPU_i	Z_{600}	Rel. Gap_{600}
AtochaS1-logn	42.93	0.66	0.00	Atocha1-logn	76.00	64.23	75.99	1.5
AtochaS1-mn	1.16	0.70	0.00	Atocha1-mn	8.34	61.1	8.26	25.1
AtochaS2-logn	10.32	2.53	0.00	Atocha2-logn	99.84	436.52	99.77	1.27
AtochaS2-mn	2.74	1.45	0.00	Atocha2-mn	18.53	247.40	18.45	11.27
AtochaS3-logn	14.06	2.41	0.00	Atocha3-logn	183.96	453.39	183.96	0.6
AtochaS3-mn	3.61	12.11	0.00	Atocha3-mn	27.27	447.2	27.27	7.8

calculated in the off-line process, is solved in an on-line phase. The performed numerical tests, which are based on the main infrastructure in an inter-urban railway network demonstrate the applicability of the proposed approach in real-time settings.

The approach presented is highly specialized for station management, being this a limitation for its application to general control areas. Computationally, it has been shown that, for problems of 100 trains and 10 platforms, the mesoscopic approach has a computational cost within the state-of-the-art, finding for the Atocha station exact solutions for a planning time interval of 3 hours. It has also been found that if the time interval were very wide (with several hundreds of trains), pre-processing techniques might be used to eliminate irrelevant conflicts (far away in time) and/or resort to heuristic methods for solving the proposed model.

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